Bull. London Math. Soc. 53 (2021) 740–746 doi:10.1112/blms.12455

A contractive Hardy–Littlewood inequality

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Abstract

We prove a contractive Hardy–Littlewood type inequality for functions from $H^p(\mathbb{T})$, $0 < p \leqslant 2$ which is sharp in the first two Taylor coefficients and asymptotically at infinity.

1. *Introduction*

The classical Hardy–Littlewood inequality [6] says that for $f(z) = a_0 + a_1 z + \cdots \in H^p(\mathbb{T})$, $0 < p \leqslant 2$, we have

$$\sum_{n=0}^{\infty} \frac{|a_n|^2}{(n+1)^{2/p-1}} \leqslant C_p ||f||_p^2. \tag{1.1}$$

In [4], the following more precise version of this inequality was conjectured.

Conjecture 1.1. For the function $f(z) = a_0 + a_1 z + a_2 z^2 + \cdots \in H^p(\mathbb{T}), 0 < p \leqslant 2,$ we have

$$\sum_{n=0}^{\infty} \frac{|a_n|^2}{c_{2/p}(n)} \leqslant ||f||_p^2, \tag{1.2}$$

where $c_\alpha(n) = \binom{n+\alpha-1}{n}$.

Despite vast numerical evidence, this conjecture is currently proved only for $p = \frac{2}{k}, k \in \mathbb{N}$ by Burbea [5], the case $p = 1$ being the famous Carleman inequality (see, for example, [8] for a simple self-contained proof).

In [3], inequality (1.2) was proved for the first two coefficients. Namely for the function $f \in H^p(\mathbb{T}), 0 < p \leqslant 2$, we have $|f(0)|^2 + \frac{p}{2}|f'(0)|^2 \leqslant ||f||_p^2$. In [2], by means of Wiessler's inequality [9], the authors proved the following strengthening of this result.

Theorem 1.2. *For the function $f(z) = a_0 + a_1 z + a_2 z^2 + \cdots \in H^p(\mathbb{T}), 0 < p \leqslant 2$ we have*

$$\sum_{n=0}^{\infty} \frac{|a_n|^2}{\Phi_{2/p}(n)} \leqslant ||f||_p^2, \tag{1.3}$$

where $\Phi_\alpha(n) = c_{[\alpha]}(n)(\frac{\alpha}{[\alpha]})^n$.

Note that $\Phi_\alpha(0) = c_\alpha(0) = 1, \Phi_\alpha(1) = c_\alpha(1) = \alpha$ but for $\alpha \notin \mathbb{N}$ these coefficients grow exponentially when n goes to infinity.

Received 1 May 2020; revised 21 November 2020; published online 14 January 2021.

2020 *Mathematics Subject Classification* 30H10 (primary), 30B10 (secondary).

This work was supported by Grant 275113 of the Research Council of Norway and by «Native towns», a social investment program of PJSC «Gazprom Neft».

In this paper, we prove the following theorem which gives us an inequality that is also sharp in the first two terms but for $n \geqslant 2$ the weight decays as in the Hardy–Littlewood inequality (1.1).

THEOREM 1.3. *For each* $0 < p \leqslant 2$, *there exists* $\varepsilon_p > 0$ *such that for all* $f \in H^p(\mathbb{T})$, $f(z) = a_0 + a_1 z + a_2 z^2 + \cdots$, *we have*

$$|a_0|^2 + \frac{p}{2}|a_1|^2 + \varepsilon_p \sum_{n=2}^{\infty} \frac{|a_n|^2}{(n+1)^{2/p-1}} \leqslant ||f||_p^2. \tag{1.4}$$

Note that the constant $\frac{p}{2}$ is optimal as can be seen from the function $f(z) = 1 + \varepsilon z, \varepsilon \to 0$.

The proof of this inequality is based on the following theorem which may be of independent interest.

THEOREM 1.4. *For* $0 < p \leqslant 2$, *there exists* $C'_p < \infty$ *such that for all* $f \in H^p(\mathbb{T})$, *we have*

$$||f(z) - f(0) - f'(0)z||_p^2 \leqslant C'_p\left(||f||_p^2 - |a_0|^2 - \frac{p}{2}|a_1|^2\right). \tag{1.5}$$

Since this theorem is obviously true for $p = 2$ we will prove it only for $0 < p < 2$. Moreover, the constants C'_p will be uniformly bounded except possibly for $0 < p < \varepsilon$ and $2 - \varepsilon < p < 2$. It is easy to see that in the former case nonuniformity is unavoidable but we do not know what happens when p is close to 2.

2. *Weak form of Theorem 1.4*

In this section, we will prove the following lemma.

LEMMA 2.1. *For every* $0 < p \leqslant 2$, *there exists a constant* γ_p *such that for all* $f \in H^p(\mathbb{T})$, *we have*

$$||f - f(0)||_p \leqslant \gamma_p \sqrt{||f||_p^2 - |f(0)|^2}. \tag{2.1}$$

In [1, Lemma 2.2], this is proved for $p \leqslant 1$ and in [7] this is proved for $1 < p \leqslant 2$ (in [7], this lemma is proved even for $f \in L^p$, but with $\gamma_p \to \infty$ as $p \to 1$). Nevertheless we present here a simple uniform proof of this lemma.

Proof. Without loss of generality, we may assume that $||f||_p = 1$. Let $n = [\frac{2}{p}]$, $\frac{1}{q} + \frac{n}{2} = \frac{1}{p}$. We can decompose the function f as a product $f = f_0 f_1 \ldots f_n$, $f_0 \in H^q(\mathbb{T})$, $f_1, \ldots, f_n \in H^2(\mathbb{T})$ such that $||f_0||_q = 1, ||f_k||_2 = 1, k = 1, \ldots, n$.

Let $f_k(z) = a_k + g_k(z)$, $g_k(0) = 0$. Note that $|a_k| \leqslant 1$, $\prod_{k=0}^{n} |a_k| = |f(0)|$. Therefore $|a_k| \geqslant |f(0)|$. By orthogonality, we have $||g_k||_2 \leqslant \sqrt{1 - |f(0)|^2}$ and this inequality is valid even for $k = 0$ since $||f_0||_2 \leqslant ||f_0||_q$.

We have the following formula for $f - f(0)$:

$$f - f(0) = g_n\left(\prod_{k=0}^{n-1} f_k\right) + g_{n-1}a_n\left(\prod_{k=0}^{n-2} f_k\right) + \cdots + g_1\left(\prod_{k=2}^{n} a_k\right)f_0 + g_0\left(\prod_{k=1}^{n} a_k\right). \tag{2.2}$$

For each of the first n summands, by the obvious estimate $|a_k| \leqslant 1$ and Hölder's inequality, we have H^p-norm is bounded by $\sqrt{1 - |f(0)|^2}$. For the last summand, we have $\prod_{k=1}^{n} |a_k| \leqslant 1$ and $||g_0||_p \leqslant ||g_0||_2 \leqslant \sqrt{1 - |f(0)|^2}$. Therefore by the triangle inequality (with the possible additional constant coming from the fact that $H^p(\mathbb{T})$ for $p<1$ is not a Banach space), we get $||f-f(0)||_p \leqslant \gamma_p\sqrt{1-|f(0)|^2}$. □

3. *Proof of Theorem 1.4 for functions without zeroes*

In this section, we will prove the following theorem.

THEOREM 3.1. *Let $0<p<2$ and $f\in H^p(\mathbb{T})$ has no zeroes in $\mathbb{D}$. Then the conclusion of Theorem 1.4 holds for this function f.*

For the proof of this theorem, we will need the following result which is Theorem 4.1 from [1].

THEOREM 3.2. *For $f\in H^p(\mathbb{T})$ with $||f||_p=1$, we have*

$$|f'(0)| \leqslant \kappa(p) = \begin{cases} 1, & p\geqslant 1, \\ \sqrt{\dfrac{2}{p}}(1-\frac{p}{2})^{1/p-1/2}, & 0<p<1. \end{cases} \tag{3.1}$$

Note that for all $0<p<2$ we have $\frac{p}{2}\kappa(p)^2<1$.

Proof of Theorem 3.1. Without loss of generality, we may assume that $||f||_p=1$, $f(z)=a_0+a_1z+\tilde f$. Note that $||\tilde f||_p\leqslant A_p$ for some absolute constant $A_p<\infty$ (for $p\geqslant 1$ we can take $A_p=4$).

We fix $0<\delta_p<\frac{1}{2}$ to be determined later and consider the following cases depending on the values of $|a_0|$ and $|a_1|$.

(i) $|a_0|<\delta_p$.
(ii) $\delta_p\leqslant|a_0|\leqslant 1-\delta_p, |a_1|<\delta_p$.
(iii) $\delta_p\leqslant|a_0|\leqslant 1-\delta_p, |a_1|\geqslant\delta_p$.
(iv) $1-\delta_p<|a_0|$.

In the first three cases, we will prove that $||f||_p^2-|a_0|^2-\frac{p}{2}|a_1|^2$ is greater than some absolute constant $\lambda_p>0$ from which, by the inequality $||\tilde f||_p\leqslant A_p$, the desired result follows.

In the first case, we have $||f||_p^2-|a_0|^2-\frac{p}{2}|a_1|^2\geqslant 1-\delta_p^2-\frac{p}{2}\kappa(p)^2$ which is positive if δ_p is small enough.

In the second case, we have $||f||_p^2-|a_0|^2-\frac{p}{2}|a_1|^2\geqslant 1-(1-\delta_p)^2-\delta_p^2=2(\delta_p-\delta_p^2)>0$.

For the third case, we will essentially repeat the proof of Lemma 1 from [3]. We have $U(z)=f^{p/2}(z)=a_0^{p/2}+\frac{p}{2}a_0^{p/2-1}a_1z+\cdots$ with $||U||_2=1$ (here we used that f has no zeros). Therefore

$$|a_0|^p+\frac{p^2}{4}|a_0|^{p-2}|a_1|^2\leqslant 1. \tag{3.2}$$

On the other hand, we have

$$\left(|a_0|^p+\frac{p^2}{4}|a_0|^{p-2}|a_1|^2\right)^{2/p}=|a_0|^2\left(1+\left(\frac{p|a_1|}{2|a_0|}\right)^2\right)^{2/p}>|a_0|^2\left(1+\frac{p|a_1|^2}{2|a_0|^2}\right), \tag{3.3}$$

where the last inequality is a Bernoulli's inequality $(1+t)^r>1+tr$ for $r>1, t>0$. Since we are on a compact set $\delta_p\leqslant|a_0|\leqslant 1-\delta_p$, $\delta_p\leqslant|a_1|\leqslant\kappa(p)$ and the functions are continuous, we actually have a nonzero loss in the Bernoulli's inequality

$$|a_0|^2\left(1+\left(\frac{p|a_1|}{2|a_0|}\right)^2\right)^{2/p} \geqslant |a_0|^2\left(1+\frac{p|a_1|^2}{2|a_0|^2}\right)+\lambda_p \tag{3.4}$$

for some $\lambda_p > 0$. Therefore $1 \geqslant |a_0|^2(1+\frac{p|a_1|^2}{2|a_0|^2})+\lambda_p = |a_0|^2+\frac{p}{2}|a_1|^2+\lambda_p$ as desired.

Now we turn to the fourth case which requires some additional ideas. Put $U(z) = f^{p/2}(z) = a_0^{p/2} + \frac{p}{2}a_0^{p/2-1}a_1 z + \tilde{U}(z) \in H^2(\mathbb{T})$, $||U||_2 = 1$.

Denote $|a_0|^2 = 1-\beta^2$, $||\tilde{U}||_2 = \varepsilon$. Our goal now is to prove that $||\tilde{f}||_p \lesssim (\beta^2+\varepsilon)$.

Consider $V(z) = U(z)(1-\frac{p}{2a_0}a_1 z)$. We have

$$V(z) = a_0^{p/2} - \frac{p^2 a_0^{p/2-2}}{4}a_1^2 z^2 + \tilde{U} - \frac{p}{2a_0}a_1\tilde{U}z = a_0^{p/2} + \tilde{V}. \tag{3.5}$$

Note also that by orthogonality it is easy to see from $||U||_2 = 1$ that $|a_1|, \varepsilon \lesssim \beta$. Therefore we can bound $||\tilde{V}||_2 \lesssim \varepsilon + \beta^2$. Thus, by Pythagoras's Theorem, we have

$$||V||_2 = \sqrt{|a_0|^p + ||\tilde{V}||_2^2} \leqslant \sqrt{|a_0|^p + O(\varepsilon^2+\beta^4)} = |a_0|^{p/2} + O(\varepsilon^2+\beta^4). \tag{3.6}$$

We will now apply Lemma 2.1 to the function $V^{2/p}$ (V has no zeros for small enough $\frac{|a_1|}{|a_0|}$, that is for small enough δ_p):

$$||V^{2/p}-a_0||_p \lesssim \sqrt{||V||_2^{4/p} - |a_0|^2} \leqslant \sqrt{|a_0|^2 + O(\varepsilon^2+\beta^4) - |a_0|^2} = O(\beta^2+\varepsilon). \tag{3.7}$$

Now we are going to connect $V^{2/p}-a_0$ and $\tilde{f}$:

$$\begin{aligned} V^{2/p} - a_0 &= U^{2/p}(1-\frac{p}{2a_0}a_1 z)^{2/p} - a_0 = (a_0 + a_1 z + \tilde{f})(1 - \frac{a_1}{a_0}z + O(\beta^2)) - a_0 \\ &= O(\beta^2) + \tilde{f} + \tilde{f}(a_1 z + O(\beta^2)) = \tilde{f} + O(\beta^2) + O(\beta)\tilde{f}. \end{aligned}$$

Therefore $||\tilde{f}|| = O(\beta^2+\varepsilon)(1+O(\beta))^{-1} = O(\beta^2+\varepsilon)$, as required.

Since $||U||_2 = 1$, we have

$$|a_0|^p + \frac{p^2}{4}|a_0|^{p-2}|a_1|^2 + \varepsilon^2 = 1. \tag{3.8}$$

Recall that in the end we want to prove that

$$|a_0|^2 + \frac{p}{2}|a_1|^2 + \varepsilon_p||\tilde{f}||_p^2 \leqslant 1. \tag{3.9}$$

By our bound for $||\tilde{f}||_p$, it is enough to prove that

$$|a_0|^2 + \frac{p}{2}|a_1|^2 + c_p(\beta^4+\varepsilon^2) \leqslant 1 \tag{3.10}$$

holds for some $c_p > 0$. Substituting the value of $|a_1|^2$ from (3.8), we get

$$|a_0|^2 + \frac{2}{p}|a_0|^{2-p}(1-\varepsilon^2-|a_0|^p) + c_p(\beta^4+\varepsilon^2) \leqslant 1. \tag{3.11}$$

Choosing $c_p \leqslant \frac{2}{p}(1-\delta_p)^{2-p}$, we can neglect terms with ε and we are left with the inequality

$$(1-\beta^2) + \frac{2}{p}(1-\beta^2)^{1-p/2}(1-(1-\beta^2)^{p/2}) + c_p\beta^4 \leqslant 1. \tag{3.12}$$

Expanding the left-hand side via Taylor's formula, we get

$$1 + \frac{p-2}{4}\beta^4 + c_p\beta^4 + O(\beta^6), \tag{3.13}$$

and it is smaller than 1 for $c_p < \frac{2-p}{4}$ and small enough β (that is small enough δ_p) since the constant in front of β^4 is negative. □

4. *Proof of Theorem* 1.4

In this section, we will finish the proof of Theorem 1.4 by taking into consideration the potential zeros of the function f.

Let $f \in H^p(\mathbb{T}), ||f||_p = 1$. Write it as $f = Bg, ||g||_p = 1$, g has no zeros, $B = \prod_{n=1}^N \frac{z-w_n}{1-z\bar{w}_n}$ (obviously, it is enough to consider finite Blaschke products). Let $g(z) = a_0 + a_1 z + \tilde{g}(z)$, $B(z) = b_0 + b_1 z + \tilde{B}(z)$. We know that $|a_0|^2 + \frac{p}{2}|a_1|^2 + \varepsilon_p||\tilde{g}||_p^2 \leqslant 1$ and we want to prove the same bound for f (with possibly smaller ε_p).

Note that if $|f(0)| < \delta_p$, then as in the first case of the proof of Theorem 3.1 we can prove the desired inequality. Therefore we can assume that $|f(0)| \geqslant \delta_p$. Since $|f(0)| \leqslant |w_n|$ for all n, we have that $|w_n| \geqslant \delta_p$.

Put $f_k(z) = g(z)\prod_{n=1}^k \frac{z-w_n}{1-z\bar{w}_n}$. Note that $|f_k(0)| \geqslant |f_N(0)| = |f(0)| \geqslant \delta_p$. We will now show that each factor $\frac{z-w_k}{1-z\bar{w}_k}$ decreases $|f(0)|^2 + \frac{p}{2}|f'(0)|^2$ by at least $c_p(1-|w_k|)$ for some $c_p > 0$, that is

$$|f_{k-1}(0)|^2 + \frac{p}{2}|f'_{k-1}(0)|^2 \geqslant |f_k(0)|^2 + \frac{p}{2}|f'_k(0)|^2 + c_p(1-|w_k|). \tag{4.1}$$

This inequality can be extracted from the proof of Lemma 1 in [3] but for the reader's convenience we outline the argument here. For simplicity, let us set $f_{k-1}(0) = a, f_{k-1}(0)' = b, w_k = w$. We have

$$\begin{aligned}
|f_k(0)|^2 + \frac{p}{2}|f'_k(0)|^2 &= |aw|^2 + \frac{p}{2}\left|a - a|w|^2 - wb\right|^2 \\
&\leqslant |aw|^2 + \frac{p}{2}|a|^2(1-|w|^2)^2 + p|a||b||w|(1-|w|^2) + \frac{p}{2}|b|^2|w|^2 \\
&= |a|^2 + \frac{p}{2}|b|^2 - (1-|w|)(1+|w|)\Big(|a|^2 + \frac{p}{2}|b|^2 - \frac{p}{2}|a|^2(1-|w|^2) - p|a||b||w|\Big).
\end{aligned} \tag{4.2}$$

Since $\frac{p}{2}|b|^2 - p|a||b|w| \geqslant -\frac{p}{2}|a|^2|w|^2$, we have

$$\begin{aligned}
&(1+|w|)\Big(|a|^2 + \frac{p}{2}|b|^2 - \frac{p}{2}|a|^2(1-|w|^2) - p|a||b||w|\Big) \\
&\qquad \geqslant (1+|w|)|a|^2\Big(1 - \frac{p}{2}\Big) \geqslant |a|^2\Big(1-\frac{p}{2}\Big).
\end{aligned} \tag{4.3}$$

Combining (4.2), (4.3) and the fact that $|a| = |f_{k-1}(0)| \geqslant \delta_p$, we get

$$|a|^2 + \frac{p}{2}|b|^2 \geqslant |f_k(0)|^2 + \frac{p}{2}|f_k(0)'|^2 + (1-|w|)\Big(1-\frac{p}{2}\Big)\delta_p^2 \tag{4.4}$$

and we obtain the desired estimate with $c_p = (1-\frac{p}{2})\delta_p^2$.

We have

$$|b_0| = \prod_{n=1}^N |w_n| = \exp\left(\sum_{n=1}^N \log|w_n|\right) \geqslant \exp\left(-C_p\sum_{n=1}^N(1-|w_n|)\right), \tag{4.5}$$

where $C_p < \infty$ since all w_n are bounded away from 0. By orthogonality, we have

$$||\tilde{B}||_p \leqslant ||\tilde{B}||_2 \leqslant \sqrt{1-|b_0|^2} \leqslant \sqrt{1-\exp\left(-C_p\sum_{n=1}^N(1-|w_n|)\right)} \leqslant \sqrt{C_p\sum_{n=1}^N(1-|w_n|)}. \tag{4.6}$$

Let us now write $f(z) - f(0) - f'(0)z$ in terms of B and g:

$$f(z) - f(0) - f'(0)z = b_1 a_1 z^2 + B(z)\tilde{g}(z) + \tilde{B}(z)(a_0 + a_1 z). \tag{4.7}$$

Since Blaschke products are unimodular, we have $||B\tilde{g}||_p = ||\tilde{g}||_p$. Since $|a_0| \leqslant 1, |a_1| \leqslant \kappa(p)$, the last term has H^p-norm at most $\alpha_p||\tilde{B}||_p$ for some $\alpha_p < \infty$. Finally, for b_1 we have again by orthogonality

$$|b_1| \leqslant \sqrt{1 - |b_0|^2} \leqslant \sqrt{C_p \sum_{n=1}^{N}(1 - |w_n|)}. \tag{4.8}$$

Collecting everything we get

$$||f(z) - f(0) - f'(0)z||_p \leqslant A_p\left(||\tilde{g}||_p + \sqrt{\sum_{n=1}^{N}(1 - |w_n|)}\right). \tag{4.9}$$

On the other hand by (4.1)

$$|f(0)|^2 + \frac{p}{2}|f'(0)|^2 \leqslant |a_0|^2 + \frac{p}{2}|a_1|^2 - c_p\sum_{n=1}^{N}(1 - |w_n|) \tag{4.10}$$

and by Theorem 3.1

$$|a_0|^2 + \frac{p}{2}|a_1|^2 + \varepsilon_p||\tilde{g}||_p^2 \leqslant 1. \tag{4.11}$$

Now it is easy to see from (4.9), (4.10), (4.11) and the trivial inequality $(x+y)^2 \leqslant 2x^2 + 2y^2$ that for some $\varepsilon'_p > 0$, we have

$$|f(0)|^2 + \frac{p}{2}|f'(0)|^2 + \varepsilon'_p||f(z) - f(0) - f'(0)z||_p^2 \leqslant 1, \tag{4.12}$$

as required.

5. *Proof of Theorem* 1.3

In this section, we will deduce Theorem 1.3 from Theorem 1.4.

We can rewrite inequality (1.1) as

$$\frac{1}{C_p}\sum_{n=0}^{\infty}\frac{|a_n|^2}{(n+1)^{2/p-1}} \leqslant ||f||_p^2. \tag{5.1}$$

Applying this to the function $\tilde{f}(z) = f(z) - f(0) - f'(0)z$, we get

$$\frac{1}{C_p}\sum_{n=2}^{\infty}\frac{|a_n|^2}{(n+1)^{2/p-1}} \leqslant ||\tilde{f}||_p^2. \tag{5.2}$$

Combining it with the bound from Theorem 1.4, we get

$$|a_0|^2 + \frac{p}{2}|a_1|^2 + \frac{1}{C_pC'_p}\sum_{n=2}^{\infty}\frac{|a_n|^2}{(n+1)^{2/p-1}} \leqslant ||f||_p^2. \tag{5.3}$$

Acknowledgements. I would like to thank my advisor Kristian Seip for introducing me to this problem and for his constant support and encouragement, as well as for carefully reading this note and spotting some inaccuracies in the first version of it. I also would like to thank Dmitriy Stolyarov and Pavel Zatitskiy for fruitful discussions. I thank the anonymous referee for helpful comments that improved the exposition.

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