Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Department of Economics

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FX Forward Hedging in Norwegian Equity Funds

Master's thesis in Financial Economics
Supervisor: Svein-Arne Persson
May 2022

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NTNU
Kunnskap for en bedre verden

Preface

This master thesis marks the end of a two-year master's in financial economics at NTNU Trondheim. The thesis has been very interesting and educational, where having the opportunity to utilize the knowledge gained throughout the studies has been very rewarding.

I would like to express my gratitude to my advisor, Svein Arne Persson for good advising throughout this process. I would also like to thank my fellow classmates for always being available for discussions regarding the thesis and for being good distractors during stressful days at school.

Mari Jore

Trondheim, May 2022

Abstract

With an increased interest in global investing, investors are facing uncertainty and risks regarding the foreign currency exposure. With uncertainties and volatility in the global market, exchange rates are volatile and difficult to predict. Many actively managed global funds are not hedged against currencies, and I will in this thesis investigate how FX Forward hedging influence the risk and return of the Norwegian equity fund, DNB Global Emerging Markets A.

Using historical monthly prices from 2008 to 2020 of the DNB Global Emerging Markets A fund, the monthly exchange rates between the Norwegian Krone and the four largest currencies traded in the fund, and the interest rates for the respective countries, the analysis creates FX Forward prices to create and forecast an optimal portfolio. Methods such as mean-variance optimizing and GARCH-modeling will be presented as a part of the analysis.

Over the time span investigated, the analysis has found hedging, in terms of mean-variance optimizing with FX Forwards, to not be crucially profitable. Forecasting the optimized mean-variance portfolio, results conclude that the investor should be indifferent in choosing between an uhedged and a hedged portfolio, however additional costs, time and resources is a factor that should be consider by the investor prior to making a decision.

Sammendrag

Med økt interesse i internasjonale investeringer, er investorer utsatt for usikkerhet og risiko ved deres eksponering for valuta. Med usikkerhet og volatilitet i det globale markedet er valutakurser volatile og uforutsigbare. Mange aktivt forvaltede aksjefond valuta-sikrer ikke deres globale aksjefond, og jeg vil derfor i denne analysen, undersøke om sikring ved bruk av valuta forwards vil ha en positiv påvirkning på det norske aksjefondet, DNB Global Emerging Markets A.

Ved å bruke månedlige, historiske priser fra 2008 til 2020 for DNB Global Emerging Markets A, månedlige valutakurser mellom den norske kronen og de relevante valutaene, samt den historiske renta for disse landene, skaper analysen forward kurser og prognoser for den optimale, valutasikrede porteføljen. Metoder som "mean-variance" optimering og GARCH modellering presenteres som en del av analysen.

Over tidsrommet som ble analysert, har analysen kommet frem til at sikring, ved bruk av "mean-variance" optimering av porteføljen, ikke skaper avgjørende resultater. Resultatene av prognosene for den optimerte porteføljen konkluderer med at investoren burde være likegyldig mellom å sikre eller ikke sikre porteføljen. I tillegg vil det kreves mer kostnader, tid og ressurser til å kontrollere sirkingen, som vil være noe ekstra å ta i betrakning under beslutningen.

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1 Introduction

Imagine planning the purchase of a large quantity of coffee beans from Costa Rica where the trade is done in US Dollars. Ahead of time, you have budgeted to purchase 1000 sacks of coffee beans at a price of $100 per sack. On March 8th, 2020, this corresponds to 9,253 NOK per sack. The total of the order would be $100,000, or 925,331 NOK on this day particular day. You wait a few days before placing the order, and the exchange rate has grown from 9.25331 on March 8th to 11.8104 on March 22nd. This was the last day you could place the order, you need this certain number of sacks, and see no other way out than purchasing. The total price of the purchase has now increased to 1,181,040 NOK. This means the order turned out 255,709 NOK more expensive than budgeted. With the use of derivatives or other hedging strategies, you could have secured the exchange rate at a lower rate, avoiding the additional costs. Global equity funds and international investors are exposed to similar situations daily, where the capital exposed to this exchange rate risk are often much higher.

With an increased interest in investing, simultaneously follows an increase in global investing and the amount of investors facing daily exchange rate volatility. According to T. E. Copeland et al. (2014), the decision of investing is essentially based on how much you choose to not consume in the present so that more can be consumed in the future, with the expectation that the value of the investment will rise. An investors optimal investment maximizes their expected satisfaction in which is gained from the additional consumption achieved. However, being an investor does not guarantee positive returns nor an increased room for consumption in the future. Investing includes being exposed to risk factors no matter which sector of the economy you are investing in, where the stock market is one of the financial markets with the highest associated risk. A common measure for this risk is volatility, where periods of high volatility can often be explained by shocks in the local or global economy such as natural disasters, elections, wars etc.

Studies by Levich (2001) and Solnik (1974) relating to diversification have confirmed gains from international diversification. Asset pricing models such as the Sharpe-Lintner CAPM and multi-factor models have been derived for international financial assets as a tool for global investors to investigate the relationship between systematic risk and expected return in their portfolios. However, despite efforts to improve portfolios, global investing expose investors to foreign currency in which again affects their risk and return profile. The exposure to the uncertainty regarding exchange rate volatility demands a decision by the investor of whether to hedge the faced currency exposure or to retain unhedged. Currency hedging is an investment strategy of using derivatives in an attempt to reduce the risk and volatility of fluctuations between currency pairs.

Global equity funds, such as the Norwegian equity fund, DNB Global Emerging Markets A, is exposed to daily exchange rate fluctuations. This is an actively managed fund in which has positions in emerging markets across the globe. With the largest shares of investments in China, India and South Korea, the four largest currencies they trade include US Dollar (USD), Hong Kong Dollar (HKD), South Korean Won (KRW) and the Indian Rupee (INR). This diverse portfolio has large shares in emerging markets where the currencies can face great volatility in times of shock. Most actively managed global Norwegian equity funds do not hedge currencies, whereas DNB Global Emerging Markets falls into this category.

An investors decision to expand their portfolio to include emerging markets is often a difficult decision to make. These markets are highly volatile and more unpredictable. Despite the volatile environments, the emerging markets are growing rapidly, creating larger and larger impacts on the global economy. When making an investment in a developing country, the transaction is mostly done in a different currency than the domestic currency and with the exchange rate risk present, the risk of missing out on potential capital in an investment is large. However, the volatility is just as large for the trade partner in the developing country. Controlling this risk would not only create a lower risk for the investor, but for the investee as well. With a lower risk for both partners, returns for both may be more stable, and the partners might become more willing to increase its economic involvement, positively affecting the global economy.

Several empirical research's have documented the gains from international diversification. This thesis will analyse the opportunities of currency hedging the most traded currencies in the Norwegian equity fund, DNB Global Emerging Markets A. The purpose of this empirical analysis is to investigate the hedging opportunities for and whether currency hedging will create a more efficient portfolio for the Norwegian equity fund, DNB Global Emerging Markets A. To explore this issue, the analysis will implement a 30 days and a 90 days FX forward to the portfolio to investigate historic performance of the two alternative portfolios compared with the unhedged portfolio. Further, assuming based on Campbell et al. (2010)'s paper, that fully hedging the FX Forwards will not be profitable, the analysis will use the FX forwards to create a mean-variance optimal portfolio with a combination of no hedging, the 30 days FX Forward and the 90 days FX Forward. To ensure reliability of the results, using these optimized portfolios, their volatilities will be further analyzed and forecasted with GARCH models. By investigating currency hedging opportunities for this fund, it will hopefully contribute to create the most mean-variance efficient portfolio.

The chapters of this thesis will include previous research, relevant theory used in analysis such as investment risk, modern portfolio theory and optional hedging strategies.

The research then continues to a description of the methods used, followed by a clear description of the data utilized, including the technical work performed prior to using the econometric models. Further, results of the analysis will be discussed in which will lead to a conclusion for the research question. Lastly, I will propose further research based on strengths and weaknesses identified in this research.

2 Literature Review

International finance and exchange rates provide for one of the most active and challenging areas of economic research. Despite the Capital Asset Pricing Model (CAPM) and studies of the CAPM suggesting international diversification to manage one's risk and return profile, fluctuations in the foreign exchange rate cause a risk in which arise when investing globally. With this risk present, the subject surrounding currency hedging becomes very central.

To get an overview of risk and finding the best methods to reduce the investment risk in international portfolios, Bender and Nielsen (2010) concluded that a successful investment process contains risk assessments based on different aspects of risk. Their argument states that risk measurement, risk monitoring and risk adjusted investment management are the proper aspects in risk management. Risk measurement involves using the correct tools accurately to quantify risk from various perspectives, risk monitoring includes tracking the output from the tools and flagging anomalies regularly, while risk-adjusted investment management use information from the two other risk aspects to align the portfolio with expectations and risk tolerance

Despite Bender and Nielsen concluding on the proper aspects of risk management, the question of how to manage risk, whether to hedge currencies in international portfolios and which methods to potentially utilize, is still an ongoing debate among researchers. Jorion and Glen (1993) found in their research that hedging in which includes forward contracts results in statistically significant improvements on the performance of portfolios that contain bonds. Their paper examined the benefits from currency hedging, both for speculative and risk minimizing motives, in both international bonds and equity portfolios. Investigating four approaches to currency hedging, conditional hedging strategies that allow the hedging coefficient to vary over time seem to yield substantially higher returns without additional risk. These strategies include setting the hedging coefficient to -1, depending on the sign of the forward discount, and each month, the decision variables are the amounts to buy or sell forward.

Currency risk and its volatility is impossible to fully remove, but it should be possible to reduce and minimize this risk. Campbell et al. (2010), found proof that for an international portfolio containing several currency transactions, the most effective risk minimizing will be where the investors short the currency in which is the highest positively correlated with the return, and long the currency in which is the most negatively correlated with the return. In their research, Campbell, Medeiros and Viciera have compared the volatility with full hedging, half hedging, zero hedging and optimal hedging. The results show how full hedging is sensitively dependent on the investors base currency and varies across all different currencies. Mean-variance optimal hedging on the other hand, they have found to reduce risk for all investors. This is an optimal portfolio where the currency position is optimized to find the most mean-variance efficient portfolio based on investors profile.

Bucher (2019) on the other hand, has in his analysis found an easier, more robust mean-variance approach to currency hedging in which can be utilized by international investors. He has named this method dynamic conditional currency hedging (DCCH) and is an alternative for full hedging. Compared with the well-known mean-variance approach, this method is said to be more robust against overfitting and better at risk minimizing currency positions. Similar to Campbell Medeiros and Viciera, Bucher use correlations between the currency pairs in which is predicted by using lagged implied exchange rate volatility. This allows the investors to dynamically adjust their hedging positions to ensure lower risk in comparison with other hedging alternatives. This approach also improves investors Sharpe ratio, especially in crisis situations such as in 2020. Bucher has found that this method provides the lowest volatility in comparison with all other considered hedging alternatives.

In their paper from 2013, Chang et al. (2012) investigated currency hedging strategies using dynamic multivariate GARCH models. This paper examined the effectiveness of using futures contracts as hedging instruments of alternative models of volatility for estimating condiational variance and covariances, alternative currencies and alternative maturities of futures contracts. By estimating four multivariate volatility models and calculating optimal portfolio weights and optimal hedge ratios, they were able to identify the appropriate hedging strategies. The empirical results are of importance to currency hedgers who requires taking futures positions in order to reduce the risk. The final results shows that futures hedging strategies for the relevant currencies is not empirically crucial, and the effectiveness is revealed to not create large differences.

Despite many authors arguing for different hedging strategies using derivatives, T. Copeland and Joshi (1996) discussed the effectiveness of currency hedging to reduce risk in their paper "Why derivatives don't reduce FX risk". Throughout the paper, the pair discuss how hedging strategies appear so elegant in theory but don't work in practice. A study they conducted of nearly 200 large companies yielded evidence to doubt the effectiveness of FX Forwards hedging. Despite superbly designed and executed programs, cashflow volatility did not seem to reduce significantly for most firms. The authors have included one example of fourteen different barbed wires with the caption: "In the nineteenth century there were hundreds of different types of barbed wire available, each designed to deal with a particular kind of risk" (T. Copeland and Joshi (1996). This example describes how risk managing in all aspects of life needs to be individually designed for how to deal with each particular kind of risk, which might be the reason why the hedging techniques don't seem to create the positive outcomes that was expected. The conclusion of the paper suggest that hedging individual transactions may not work, but foreign exchange rate exposure on the company level should be measured and managed, just with other alternatives than derivatives.

The analysis wish to utilize a blend of these methods to create a more efficient method to currency hedge. By further investigating T. Copeland and Joshi (1996)'s statement of that derivatives don't reduce foreign exchange risk, the analysis begin with a full FX Forward hedge, and further use these FX Forwards in a mean-variance approach similar to Bucher (2019) as an alternative to a full hedge. Assuming that results will indicate a lower volatility using a mean-variance efficient portfolio, the analysis further use Bender and Nielsen (2010)'s theory of proper risk managing. In terms of using Generalized Autoregressive Heteroskedasticity (GARCH) models to quantify, fit and forecast volatility to follow the aspects of risk measurement, risk monitoring and risk adjusted investment management, respectively, the analysis investigate and forecast volatility for a proper view of the risk.

3 Theory

In the following chapter, the theories relevant for this analysis will be described. This includes risk and volatility, the foreign exchange market, portfolio theory and hedging alternatives.

3.1 Investment Risk

The financial market is a marketplace for the sale and purchase of all financial assets such as stocks, bonds, foreign exchange and derivatives. The price of assets and currencies, is a relationship between demand and supply. An increased demand for a currency will often increase the price of the asset and vice versa. The unpredictability of price changes cause uncertainty in the market, in which is the main root of all investment risk. All investors entering the financial market are faced with an unavoidable risk, whereas every investor has its own risk and return profile.

The financial market is unstable, and investors expect to be compensated for the additional risk they face. Risk can be identified as the uncertainty regarding whether an incident will occur or not. The term is often associated with negative outcomes, though it does surround positive outcomes as well. In financial theory, risk is measured by the volatility, or the standard deviation, of returns and cannot be directly observed in the market. This is to be estimated by using the observed market prices and investigating how much the prices move. With high price fluctuations, volatility is high, but we are unable to ascertain how high. Due to this unknown and uncertain factor, statistical models are tools in which are necessary to conduct a forecast. Modeling and forecasting volatility is helpful in developing more accurate models of asset and portfolio returns, as well as in applications for risk management of portfolios, including hedging strategies.

3.2 Measuring Volatility

When forecasting, we want to study the statistical properties of returns, given information available at time t-1 to further create a model of how returns evolve over time. A part of this study includes investigating the volatility of the time-series models in which often exhibit volatility clustering instead of a constant variance. When this effect is present, regular econometric tools and models are not strong enough to capture this feature, and models will be biased and mistakenly estimated. There exist many models for forecasting volatility such as moving average and implied volatility, but a more robust and more frequently applied model for measuring volatility is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The GARCH family of models belong to the category of conditional volatility models in which are based on using optimal exponential weighting of historical returns to create a volatility forecast. Volatility can be split into unconditional and conditional volatility where conditional volatility is defined as the historical volatility, based on what happened before, rather than volatility at one point in time.

3.3 Foreign Exchange Market

A diversified investor who has investments in different markets will be less vulnerable for the local, economic setbacks and unsystematic risk. In a world with floating exchange rates and global investing, the presence and strength of volatility and risk is crucial to all investors. The foreign exchange market consists of all currencies worldwide where every foreign exchange rate is measured in pairs. With a constantly changing market and society, the factors affecting currencies consistently change. Prediction of exchange rates is difficult, however univariate models such as the GARCH model are able to capture the conditional volatility, and with limitations, forecast the rate.

3.4 Modern Portfolio Theory

According to Cowell (2002), the most effective way to achieve a controlled portfolio diversification is by means of optimizing. Markowitz's mean-variance optimization is a part of modern portfolio theory in which rely on providing investors with a framework for analyzing risk-return trade-offs when making asset and portfolio allocation decisions. The fundamental goal of portfolio theory is to optimally allocate one's investments between different assets where the allocation is created based on expected return with respect to a selected risk profile. The theory describes how risk-averse investors will have a risk level in which is acceptable in relation to expected returns. To reach the desired combination, there are two different ways to approach the issue when building the portfolio:

1. One can maximize the expected return, given a chosen risk level
2. One can minimize risk, given a level of expected return.

The efficient frontier of risky assets was created by Markowitz in 1952 and reflects the two approaches mentioned (Bodie et al. (2018)). Markowitz published a formal model of portfolios selection in which embodies the principles of efficient diversification, and paved the way for his Nobel Prize in Economics in 1990. The frontier is the set of optimal portfolios that offers the highest expected return given a certain level of risk, or the lowest level of risk given a certain level of expected return. Figure (1) graphically represent the efficient frontier where the portfolios that lie within the frontier are viewed as efficient portfolios.

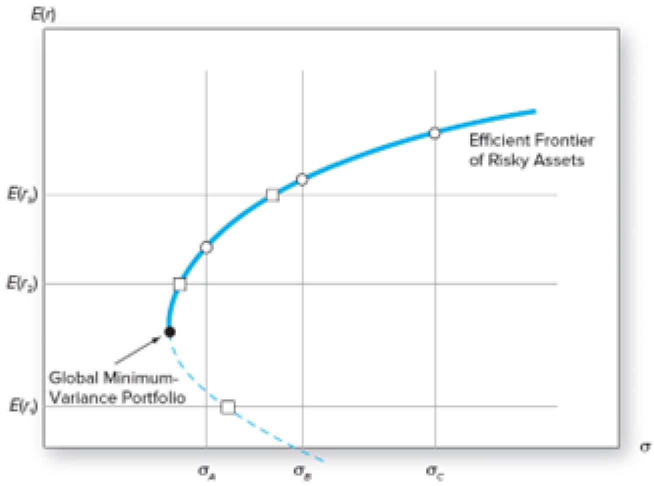

Figure 1: Efficent Frontier of Risky Assets, Markowitz (1952). Source: Bodie et al. (2018)

The rule of thumb expects that high expected return is the desired result while variance is unwanted. Given a choice between two portfolios with equal return, the investor will choose the portfolio with the lowest volatility. The expected return for a risky investment in which surpass the risk-free return is called the risk premium. This is a compensation for the additional risk the investor is entitled to take for the opportunity to gain a higher return on the investment. The risk premium can also be said to be the return the investor is willing to give up to secure a safer portfolio. According to Modern Portfolio Theory (MPT), if the goal is a portfolio with low volatility, the investor has to assume a lower return.

In an efficient market, one cannot gain a higher expected return without increasing risk. However, the investor may diversify their portfolio to spread the risk across the assets with different risk profiles. The mean-variance optimization includes as mentioned, finding the largest reward given a level of risk.

3.5 Hedging Alternatives

Ever since the beginning of global investing, investors have been exposed to exchange rate volatility where hedging instruments and forecasting have made it possible to manage this volatile environment. But the question is whether it is necessary to control the exposure and how to control it. The market constantly changes, and many researchers argue there is little to no reason and evidence that controlling the exposure will have a significant positive effect for an investor from the risk occurred when investing internationally. Many researchers against currency hedging argue that investors who are wanting to hedge against currency exposure may do so themselves by diversifying their own portfolios. Others argue that the hedging of the currencies is not to eliminate risk, but to control and reduce the risk you come upon when investing. Most researchers arguing for currency hedging justify their argument with the reduction of volatility of the return. Risk management reduce volatility of the cash flow, so managing risk may not only reduce volatility, but increase returns as well. Figure(2) visualize how hedging reduce the variability of the expected cash flows around the mean of the distribution. The illustration shows how the expected return of a hedged portfolio is higher than what it is for an unhedged portfolio.

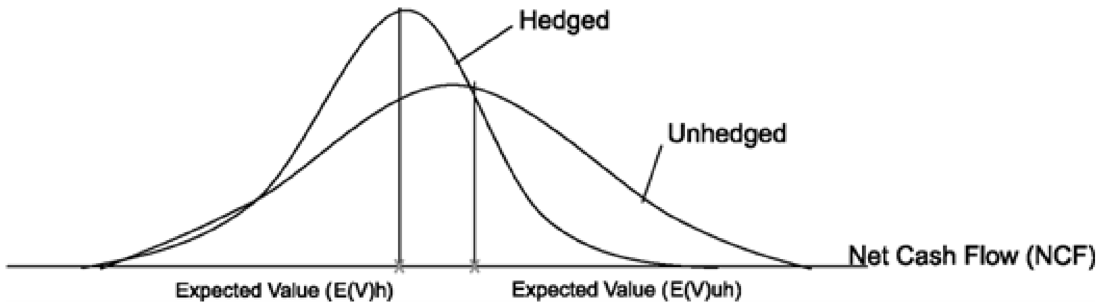

Figure 2: Comparison of expected net cash flow of a hedged and an unhedged position. Source:Ltd (n.d.)

3.5.1 Options

An option gives an investor the right, but not the duty to buy or sell a currency at a predetermined price. The advantage with this derivative is that it provides flexibility by that the owner may cancel the agreement if the exchange rate moves in a non-favorable direction. The seller of the option, the writer, is required to complete the transaction if the owner wishes to. By purchasing a call option if the currency is rising, and purchasing a put option if the value is falling, the investor will have the opportunity to hedge a currency if they are able to accurately evaluate the trend of the currency .

3.5.2 Forward and Futures Contracts

Futures contracts may be the simplest method for managing the risk occurred from uncertain fluctuations in the foreign exchange market. A futures contract is an agreement between a buyer and a seller to buy or sell a particular asset at a set date, at a set price. The underlying assets that may be involved in this contract may include stocks, bonds, currencies, exchange-traded funds (ETFs) etc. Futures are settled daily until the contract comes to an end. These contracts may be traded on public exchanges and is more accessible than other hedging instruments. Often, hedgers will short a certain amount of future contracts of a foreign currency if they have a long position in the underlying currency, and vice versa. To successfully hedge, the investor needs to know how many futures contracts that should be held for each unit of the underlying currency and the effectiveness of the ratio. The effectiveness of the hedge ratio evaluates the hedging performance of the strategy, while the ratio itself provides the investor with information of how many futures contracts they should hold.

Despite futures contracts being one of the simplest methods for managing volatility in the foreign exchange market, the contracts involve transaction costs in which needs to be taken into consideration to ensure profitability of the hedge. Forward contracts on the other hand, is also an agreement between a buyer and a seller regarding trading an underlying asset at a predetermined date and price. These contracts though are, opposed to futures, settled at maturity. The contracts are not as easily accessible as futures as they represent a private contract between a buyer and seller, but both parties would avoid transaction costs. Futures are therefore by nature often more used than forward contracts.

However, when using FX forwards, deciding the optimal choice time horizon for both forward contracts can make a crucial difference. When entering a 30 day forward contract, you agree on the price on the day of the agreement, and complete the transaction 30 days later. And when entering a 90 day forward contract, you again agree on the price on the day of the agreement, and complete the transaction 90 days later. Each investor has to evaluate how long they believe they will need to adjust the prices and settle a deal based on this.

As FX Forwards lock in exchange rates and are calculated based on spot prices, the client loses the ability to secure more advantageous deals which will correspond with a combination of both interest rate risk and exchange rate risk. Each investor has to evaluate whether they want to secure a currency with the chances of missing out on other opportunities, or to wait and hope for more advantageous opportunities.

4 Method

In the following chapter, the methodical decisions used to answer this thesis' problem statement will be discussed. The chapter will begin with elaborating the problem statement of the thesis, to further describe the assumptions that need to be held in order to perform the analysis. Thereafter, an elaboration on the model used in this research will be described.

4.1 Problem statement

Many Norwegian investors do not hedge their global equity funds facing daily foreign exchange risk. In this thesis, an analysis is performed on DNB's Global Emerging Markets A fund where currency hedging is currently not present. The analysis will consist of the research of whether hedging strategies such as FX Forwards with different time horizons will contribute to a efficient portfolio. Using different two different empirical models, this analysis views different approaches for the use of FX forwards to hedge currencies and control risk.

4.2 Full FX Forward Hedge

FX Forwards are contracts where an agreement between two parties is established to exchange a specified amount of currency at a pre-determined future date. The exchange rate for the transaction is agreed at the trade date, the time of entering the contract. The contracts are merely a function of the interest rates, the duration of the contract and the spot price.

$$FX_{Fwd}(T) = FX_{spot}(0)e^{(r_d - r_f)T} \tag{1}$$

Equation(1) exhibit the FX Forwards price calculation, where FX_{spot}represent the spot exchange rate, r_d represents the domestic interest rate, r_f represent the foreign exchange rate and T represent the time to maturity. To further find the current and historical prices for the new portfolios including the FX Forwards, the monthly gain or loss of the forward per share is added to the current price of the unhedged portfolio.

4.3 Mean Variance Portfolio Optimizing

Mean variance currency hedging is a method in the international world of currency hedging. As an alternative to holding an unhedged portfolio or a fully FX forward hedged portfolio, an investor may optimize its portfolio using mean variance currency hedging with FX forwards. Using an N x N covariance matrix of the FX Forward returns and an N x 1 vector of the mean return vector (R), the investor may choose a custom currency portfolio with weights according to their risk and return profile. This will spread risk across the portfolio. This custom portfolio will have the weights described in Equation (2).

$$w_{hedge} = \sum_{FX}^{-1} \sum_{R} \tag{2}$$

In the Markowitz framework, the investor wants to either minimize risk, given expected return, or maximize their expected portfolio return, given some risk measure. The formulation of minimizing the portfolio risk, is given by Equation(3).

$$\begin{aligned} &\min_{w} \frac{1}{2} w' \sum w \\ &s.t. \quad w'\mu = \bar{\mu} \\ &and \quad w'\mathbf{1} = 1 \end{aligned} \tag{3}$$

Where w is the custom portfolio weights described above, the parameter $\sum$ denotes the covariance matrix of the currency and the FX Forward returns, μ represent the mean return of the portfolio, while $\bar{\mu}$ represent the expected return of the portfolio.The first restriction requires that the sum of all weights and the mean return of the portfolio is equal to the expected return. **1** denotes a vector of all ones, indicating the second restriction which is that the sum of all weights has to equal **1**.

Equation (4) maximize expected return subject to that the variance is less than or equal to the target variance. Here, σ represent the maximum level of risk the investor is willing to take.

$$\begin{aligned} &\max_{w \epsilon \mathbb{R}^{FX}} \quad \bar{\mu}' w \\ &s.t. \;\; w' \sum w \leq \sigma_{target} \end{aligned} \tag{4}$$

These are simple, quadratic optimization problems in which can be solved by standard Lagrange multiplier methods.

4.4 Volatility Model

Modeling volatility is a demanding task as it is not directly observable, and as pointed out previously, one frequently applied model of volatility is the Autoregressive Conditional Heteroskedasticity (ARCH) model. This thesis will among other analyze the volatility of currencies on the Norwegian equity fund DNB Global Emerging Markets which calls for using an ARCH model to observe this feature. The model constitutes a complex framework of models that are able to cope with the features that apply for a time series model, as well as it captures essential features of returns such as volatility clustering. The ARCH models use the conditional volatility σ_t using the most recent returns, while unconditional volatility σ_t, is dependent on the entire sample.

4.4.1 Regression

As proposed by Engle in 1982, an ARCH model starts from the framework that we have a static, linear regression model where all Gauss-Markov assumptions are held, so the OLS estimators are BLUE[1]. The Gauss-Markov assumptions can be found in the appendix. A linear regression model consists of a straight line in which observes the relationship between two or more variables, obtained from a dataset. As it is rather unusual for a model to be estimated correctly using just one explanatory variable, most models must be extended to a multilinear regression model (MLR) that is presented in Equation (5).

$$y_t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + u_t \tag{5}$$

Here, β_0 is the constant term interpreted as the mean value for y_t when all x-values are equal to zero. An MLR model has k-1 explanatory variables x in which affects the dependent variable y. The strength of each x-variable on y is represented by the value of their respective β_k coefficients. u_t is an error term in which has the ability to catch up the effects that are not covered by the explanatory variables.

4.4.2 ARCH-Model

In traditional econometrics, we assume the variance of the error term, u_t to be homoscedastic, constant. But it is unlikely in the context of financial time series that the variance of the error terms is constant over time. The ARCH models give us the right to remove

[1] Best Linear Unbiased Estimator

this assumption of homoskedasticity and allow the variance to change over time. The model is able to describe how the unconditional variance of the errors evolve. Equation (6) represent the model we see in traditional econometrics.

$$y_t = \mu + \beta_1 x_t + \beta_2 x_t + +\beta_n x_t + u_t \tag{6}$$

Where the conditional variance of a zero mean is represented in Equation (7).

$$\sigma_t^2 = var(u_t | u_{t-1}, u_{t-2}, \dots) = E[u_t^2 | u_{t-1}, u_{t-2}, \dots] \tag{7}$$

But, when using an ARCH model, the conditional variance of the error term is allowed to depend on the immediate previous value of the squared residual.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{8}$$

Though, there are limitations to the ARCH(q) model such as the problem surrounding how to decide the number of lags of the squared residual. The q could become very large and non-negative constrains might be violated.

4.4.3 GARCH Model

To avoid overfitting and other constraints of the ARCH(q) model, a more general model called the Generalized Autoregressive Heteroskedastic (GARCH) model was developed by Bollerslev and Taylor in 1986, Brooks (2019). This model allows the conditional variance to be dependent on previous own lags. It includes a single lag of both the ARCH term and the conditional variance term. Therefore, the conditional variance equation can be changed to Equation (9).

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-0}^2 \tag{9}$$

Using the GARCH model, it is possible to interpret the current fitted variance as a weighted function of a long-term mean value, which is dependent on σ_0, information about volatility in the previous period, and the fitted variance from the model during the previous period .

The standard GARCH model assumes that positive and negative error terms have a symmetric effect on volatility. Here, good news and bad news will both have the same effect on the volatility. However, extensions of the model exist in which capture shocks in different ways.

The GJR, or the TGARCH model may argue that negative shocks have a stronger positive impact on the volatility than that of positive shocksBrooks (2019). This is often called leverage effects. Such a feature may be taken into account by formulating a GARCH model with asymmetric effects and is shown in Equation (10).

$$\begin{aligned} \sigma_t^2 &= \sigma_0 + \sigma_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + k_2 u_{t-1}^2 I_{t-1} \\ I_{t-1} &= \begin{cases} 1, & \text{if } u_{t-1} < 0 \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{10}$$

Here the hypothesis is that $k_2 > 0$, which reflects leverage effects. Good news, where the error term is larger than 0, has the impact α_1, while bad news where $u_{t-1} < 1$ impacts volatility by $\alpha_1 + k_2$. Symmetric news impact curve by $k_2 = 0$ which is easily tested against the alternative that $k_2 > 0$.

Another extension of this model, is the nonlinear EGARCH model, the general exponential GARCH model. This model is said to be fundamentally different from the other models as it does not put restrictions on its parameters to ensure nonnegative variances. Equation (11) shows the conditional variance equation for this model.

$$ln(\sigma_t) = \omega + \alpha * \frac{u_{t-1}}{\sqrt{\sigma t - 1}} + \beta * \frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}}} - \sqrt{\frac{2}{\pi}} \tag{11}$$

The natural logarithm on the left side of Equation (11) ensures the nonnegative variance as the exponential function is strictly positive. Positive shocks will have the same impact as negative shocks if $\alpha = 0$, but if $\alpha > 0$, positive shocks will in turn increase the conditional variance. If $\alpha < 0$, positive shocks will decrease the conditional volatility.

5 Data

The data used in this research has been retrieved from the Eikon database at NTNU. The dataset contains monthly observations from 31.01.2008-31.12.2020. By including both the market crash in 2008 and the crash in 2020, I will be able to observe whether hedging would have been beneficial during these times. Due to the risk of unwanted white noise in the dataset with daily data, monthly observations for all variables are used in contrast to daily observations.

5.1 Dependent Variable

The dependent variable is the return of the monthly closing price of DNB Global Emerging Markets A. DNB Global Emerging Markets A is a Norwegian global actively managed equity fund in which mainly invest in stocks in emerging markets across the world. As of December 31st of 2021, about 63% of investments are in China, India and South Korea, where the currency used in China is Hong Kong Dollar and US dollar. Historically, the US dollar has been frequently used in several of the countries and is one of the currencies historically traded the most in the portfolio.

5.2 Independent Variables

To investigate the relationship between exchange rate volatility on DNB's Global Emerging Markets A fund, FX Forwards and volatility, the independent variables included in the analysis are the four largest foreign exchange rates traded in the fund and the respective 30 and 90 days FX Forwards. The interest rates and historic shares of investment for each of the currencies of the relevant countries have been obtained to find the independent variables of the model.

The exchange rates included in the analysis consist of the currency pairs between the Norwegian Krone and the US Dollar, Hong Kong Dollar, South Korean Won and Indian Rupee. To investigate FX Forward hedging, forward prices for these currencies have been obtained by using the interest rates.

5.3 Data transformations

To create a valid model, the data is transformed into logarithmic returns of the daily rates of DNB Global Emerging Markets A and of each currency, i.e. $r_t = ln\frac{(P_t}{P_(t-1)}$. Where P and P_{t-1} are the closing prices for the relevant variables for days t and t-1, respectively.

FX Forward price are not observable in the market and has to be created using Equation(1) from Chapter 4 where the spot rate of the exchange rates, the interest rates of both the domestic and the foreign currency and the time horizon for the FX Forward is used.

$$FX_{Fwd}(T) = FX_{spot}(0)e^{(r_d - r_f)T} \tag{1}$$

Equation(1) exhibit the FX Forwards price calculation, where FX_{spot} represent the spot exchange rate, r_d represents the domestic interest rate, r_f represent the foreign exchange rate and T represent the time to maturity. The time horizons investigated is 30 days and 90 days.

The variable names in the analysis is described in Table (1)

Variables	Description
DNB	Monthly Prices of DNB Global Emerging Markets A
NOKUSD	Monthly Prices of USD/NOK exchange rate
NOKHKD	Monthly Prices of HKD/NOK exchange rate
NOKKRW	Monthly Prices of KRW/NOK exchange rate
NOKINR	Monthly Prices of INR/NOK exchange rate
rDNB	Monthly Return of DNB Global Emerging Markets A
rUSD	Monthly Return of USD/NOK exchange rate
rHKD	Monthly Return of HKD/NOK exchange rate
rKRW	Monthly Return of KRW/NOK exchange rate
rINR	Monthly Return of INR/NOK exchange rate
FX Forward 30	30 days FX Forward
FX Forward 90	90 days FX Forward
USD1	1 month FX price of USD
HKD1	1 month FX price of HKD
KRW1	1 month FX price of KRW
INR1	1 month FX price of INR
USD3	3 month FX price of USD
HKD3	3 month FX price of HKD
KRW3	3 month FX price of KRW
INR3	3 month FX price of INR

Table 1: Variable Descriptions

6 Descriptive Statistics

This chapter aims to study the characteristics of the monthly closing prices for DNB Global Emerging Markets A, NOK/USD exchange rate, NOK/HKD exchange rate, NOK/KRW exchange rate and the NOK/INR exchange rate as these are the variables used to create other variables and to conduct the analysis. The relevant information about the dataset will be described and discussed to create a clear and precise overview of the variables, their movements and how they behave.

	DNB	NOKHKD	NOKINR	NOKKRW	NOKUSD
Mean	280.384	0.918	0.121	0.627	7.138
Std.Dev	78.397	0.177	0.011	0.114	1.384
Min	142.920	0.648	0.093	0.467	5.055
Median	257.620	0.878	0.123	0.605	6.808
Max	519.480	1.347	0.143	0.853	10.444
N.Valid	155	155	155	155	155
Pct.Valid	100	100	100	100	100

Table 2: Descriptive Statistics

Table(2) present the descriptive statistics for the variables used in this thesis. The table shows an overview over number of observations, mean values of the variables, the minimum and maximum price level of each variable, as well as the standard deviations. The equity fund has over the period investigated, had a minimum value of 142.92 and a maximum value of 519.48, which is a natural difference regarding this time span. The analysis is viewing a 12 year period, so naturally the fund has grown.

6.1 Price Movements

Figure (13) in the appendix exhibits the historical monthly prices for DNB Global Emerging Markets A and the four currency pairs, as well as their respective monthly returns. As illustrated, the prices have steadily increased. In the early years of the time span observed of the fund, the price experienced a decrease, prior to a steady increase including dips during large economic events. Looking at year 2008 and 2020, we witness a particularly high variance amongst all assets, in which this is caused by the financial crisis of 2008 and the Covid-19 pandemic in 2020. The returns move similarly and a shock in the economy have created disturbances for all variables, piling all additional variance into the return of the fund. Due to this risk, the analysis is investigating whether hedging the fund with FX forward contracts will create a mean-variance efficient portfolio.

Commonly known, the stock market includes a slow and steady increase with sudden, rapid drops. As observed in the market prices of all included variables, there usually are not as rapid increases as there are drops, which is a sign the market tends to be negatively skewed. Though, for the exchange rate returns and some years for the portfolio, the opposite has been observed, indicating more volatile periods amongst all variables. The plotted monthly returns on the right side of Figure(13) shows means reverting to zero, and a relatively constant variance for all variables.

Table (3) in which summaries the descriptive statistics for the returns of DNB Global Emerging Markets A and all currency pairs ,follows the theory of mean reversion with a monthly mean of around 0% and a varying standard deviation among all variables, where we also observe a higher standard deviation for the return of DNB.

Statistic	N	Mean	St. Dev.	Min	Max
rDNB	155	0.005	0.046	−0.182	0.114
rUSD	155	0.003	0.028	−0.057	0.130
rHKD	155	0.003	0.028	−0.057	0.133
rKRW	155	0.002	0.024	−0.086	0.075
rINR	155	−0.001	0.031	−0.088	0.085

Table 3: Descriptive Statistics of Returns

Despite the plotted returns in Figure(13) appearing to be stationary and fluctuating around zero, they clearly show periods of high and low volatility in which confirms the presence of volatility clustering. The ACF plots of the returns and squared returns of DNB Global Emerging Markets A in Figure(3) show that for the returns, most of the correlation is located inside the 95% confidence interval, while the correlation for the squared returns lies outside the 95% confidence interval.[2] This confirms volatility clustering of the returns of the equity fund and is common for time series. This is a feature that univariate volatility models such as the GARCH model is designed to capture.

[2]Autocorrelation Function, Brooks (2019) p. 333. Describes how well the present value is related to its past values.

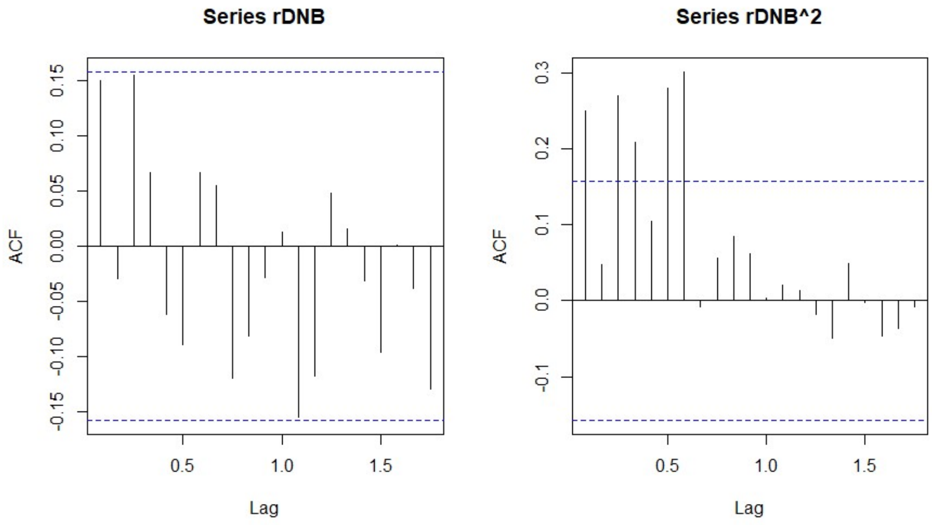

Figure 3: ACF Plot for Return of DNB and Squared Return of DNB

	rDNB	rUSD	rHKD	rKRW	rINR
rDNB	1	0.805	0.800	0.887	0.191
rUSD	0.805	1	1.000	0.951	0.406
rHKD	0.800	1.000	1	0.949	0.406
rKRW	0.887	0.951	0.949	1	0.246
rINR	0.191	0.406	0.406	0.246	1

Table 4: Correlation Matrix Between Returns of DNB Global Emerging Markets A Currency Pairs

The correlation matrix in Figure (4) shows the correlations between the unhedged fund and the monthly exchange rates of the exhibited currencies. We witness very high and both positive and negative correlations between the return of the fund and the different exchange rates, which is to be expected. A correlation coefficient of 1 suggests perfect correlation, while a correlation coefficient of -1, indicates perfectly negative correlation. Respectively, a correlation coefficient of 0 suggests no correlation. With the Norwegian Krone being a procyclical currency, the currency will often strengthen when stock markets increase and weaken in accordance with a declining stock market. Because Norway is an economy dependent on raw materials, exchange rates tend to move in the opposite direction of the return of Norwegian global investors. The correlation matrix shows that an increase in the return in the US Dollar, gives a negative response in the Norwegian stock market, and vice versa with the South Korean Won and Indian Rupee in which are two economies dependent on their exports, making their cycles move in accordance with the Norwegian krone. As witnessed in the graphs in Figure (13, in compliance with the correlation matrix, the return of the US dollar increase when the Norwegian stock fund is performing worse, while the South Korean Won and the Indian rupee both weaken when the return of the fund weakens.

In 1972, the Hong Kong dollar was pegged to the U.S. Dollar to protect the value of the currency. Exhibited in the table, these have a very strong correlation of 1. The exchange rate variables are only directly used in the portfolio optimization to create the FX forward prices and act as the unhedged currencies in the optimization. They are indirectly used in the GARCH models as this model use the unhedged portfolio and the two FX Forward portfolios for the analysis.

7 Model Analysis

The following chapter includes the results of the tests the data is required to pass to create a GARCH model on the data set.

7.1 ARCH Effects

The dynamics of the conditional volatility are important in different contexts, particularly in financial models. In these models, volatility movements itself is important to model to ensure efficiency of the model and estimations. The "ARCH effects", or the presence of heteroskedasticity, are often found in higher frequency financial data such as time series. Testing DNB Global Emerging Markets A for the existence of ARCH effects in the residuals is important prior to creating the GARCH model. The data has been tested using an "ARCH" test where the null hypothesis states no ARCH effects. Rejecting this test implies presence of heteroskedasticity and ARCH effects in the residuals.

ARCH LM-test

	DNB Returns
Chi-squared	29.16
df	12
p-value	0.003731

Table 5: ARCH LM-Test for DNB Returns

Table(5) indicates a rejection of the null hypothesis and the data is proven to contain ARCH effects. The analysis can then continue implementing a GARCH model.

7.2 Stationarity

The time series is required to exhibit stationarity where probability distribution of the series is stabile while the characteristic of the error term is constant. Often, financial data exhibit a connection between its observations, causing unit roots to be present. The price of the fund DNB Global Emerging Markets A today will be equal to yesterday's price, including a change in either a positive or negative direction. This creates a trend throughout the series in which makes the variables constantly change. If we are to perform a regression with non-stationary data, we risk biased results.

We have stationarity in the time-series model when the properties of the series are independent from the time of observation. This means that mean, variance and autocovariance are constant over time (Brooks 2014). Equation (12) mathematically show this statement derived.

$$\begin{aligned} &E(y_t) = \mu \\ &Var(y_t) = \sigma^2 \\ &Cov(y_t, y_{t+s}) = cov(y_t, y_{t-s}) \end{aligned} \tag{12}$$

To ensure no spurious results when performing the analysis, the data is tested for unit roots through an Augmented Dickey-Fuller test (ADF-test). At a 1% significance level, the hypothesis of a unit root in the returns of DNB Global Emerging Markets A can be rejected. The model is considered sufficient and will continue using the chosen variables.

7.3 Normality

Described in the Gauss Markov Assumptions, the error term needs to follow a normal distribution in the OLS regression, however, similar to the ARCH effects, the error terms need not to be normally distributed. An error term that does not exhibit normality is common for time-series and is another feature the GARCH model is designed to capture.

The Jarque-Bera test and the QQplot checks the returns for normality. A rejected Jarque-Bera test will indicate non-normality and an S-shaped QQplot will visually detect non-normality. The Jarque-Bera test is exhibited in Equation(13).

$$JB = \frac{n}{6}(S^2 + \frac{1}{4}(k-3)^2) \tag{13}$$

Where the null hypothesis is that the error term is normally distributed as followed

$$\begin{aligned} H_0 &: \mu = 0 \\ H_A &: Not\ H_0 \end{aligned} \tag{14}$$

If we are to reject the null hypothesis, the data does not contain normally distributed error terms. The results from our model are exhibited in Table (6) and show low p-values, allowing us to reject the null hypothesis. This indicates the data does contain normally distributed error terms.

Jarque-Bera Test	
	DNB Returns
X-squared	30.341
df	2
p-value	2.58e-07

Table 6: Jarque Bera Test for DNB Returns

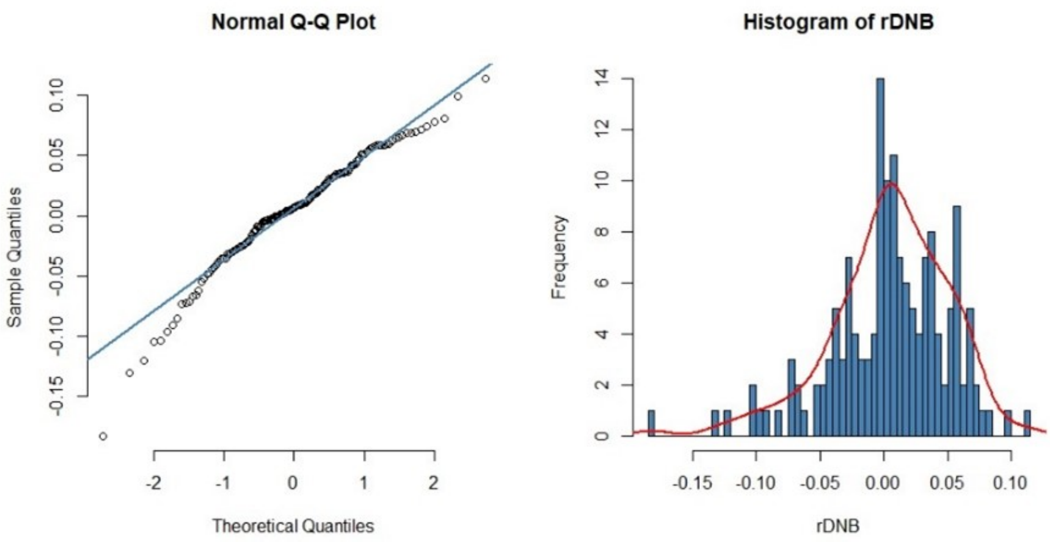

Figure 4: QQPlot and Histogram for DNB Returns

Table (6) and Figure(4) shows the results for the two tests for normality. Both tests exhibit that normality is not present in the time-series for the monthly return for DNB Global Emerging Markets A. The model can therefore continue with being estimated with a GARCH in which will capture the feature and create a normally distributed model.

7.4 Model Selection

The model has been approved for use with a GARCH model, but as described in Chapter (4), different extensions for the GARCH model exist. To ensure the retrieval of the best result, the model is to go through tests in order to determine the best extension of the model.

Prior to estimating the correct extension of the model, the data is investigated to find the best fitted ARIMA and GARCH model. Selecting the order of ARIMA to use in the model is done by creating an ACF and a PACF plot, combined with an EACF plot. Figure (5) shows the ACF and PACF results. The results of these plots are determined based on where the vertical lines for the lags falls underneath the blue plotted line. This output finds the best possible candidate models to be ARIMA(1,1,1) or ARIMA (0,1,1). Table (7) confirms the ACF and PACF plots, where the order of the circles again indicates the same models as the best fit.

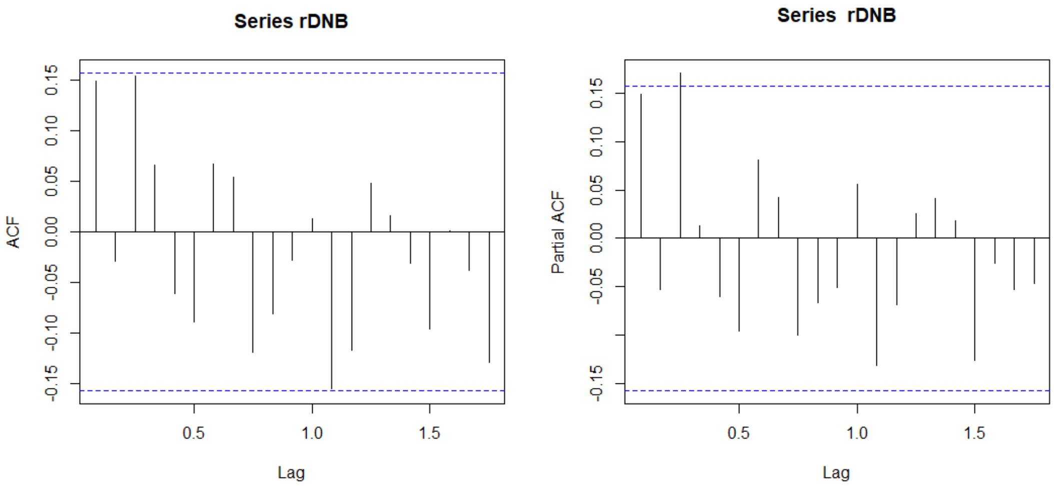

Figure 5: ACF and PACF for DNB Returns

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	o	o	o	o	o	o	o	o	o	o	o	o	o	o
1	x	o	o	o	o	o	o	o	o	o	o	o	o	o
2	x	x	x	o	o	o	o	o	o	o	o	o	o	o
3	o	x	x	o	o	o	o	o	o	o	o	o	o	o
4	o	x	x	o	o	o	o	o	o	o	o	o	o	o
5	x	x	x	o	x	o	o	o	o	o	o	o	o	o
6	x	x	x	x	x	o	o	o	o	o	o	o	o	o
7	x	x	x	o	x	o	o	o	o	o	o	o	o	o

Table 7: EACF for DNB Returns

Further, the candidate models are tested using tests of the coefficients, such as a conditional sum of squares and maximum likelihood. Using the lowest Aikakes Information Criteria (AIC) score, one form of maximum likelihood, to decide on the final model, ARIMA(1,1,1) has been selected as the best model. The results of these tests are included in Table (15) in the appendix.

In turn, to estimate the correct GARCH model,the same models as the best fit. the absolute value of the returns are tested with ACF, PACF and EACF again. The results of these are exhibited in Figure (6). The combined result of these tests suggest a GARCH(1,1) model.

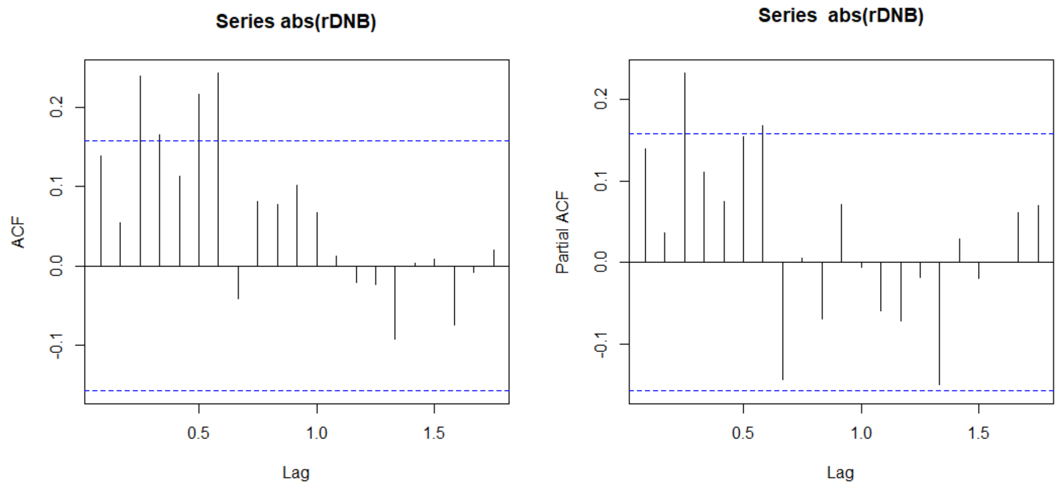

Figure 6: ACF and PACF of Absolute Values of DNB Returns

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	o	o	x	x	o	x	x	o	o	o	o	o	o	o
1	x	o	o	o	o	o	x	o	o	o	o	o	o	o
2	o	x	o	o	o	o	x	o	o	o	o	o	o	o
3	x	o	x	o	o	o	x	o	o	o	o	o	o	o
4	x	o	x	o	o	o	o	o	o	o	o	o	o	o
5	x	x	o	o	x	o	o	o	o	o	o	o	o	o
6	x	o	x	o	x	x	o	o	o	o	o	o	o	o
7	x	o	o	o	o	o	x	o	o	o	o	o	o	o

Table 8: EACF of Absolute Values of DNB Returns

Having decided to use ARIMA(1,1,1) and GARCH(1,1) to estimate the model, the model needs to be further tested for the possible extensions of the model. This is done by estimating the three different, SGARCH, GJRGARCH and EGARCH models and comparing the LogLikelihood and the Information Criteria values for all three models. These are measures of goodness of fit of a statistical model and a lower value is preferred. A lower value of the information criteria values indicates that there are either fewer explanatory variables or the model is a better fit, or both.

	SGARCH(1,1)	GJRGARCH(1,1)	EGARCH(1,1)
Log Likelihood	268.7011	273.8373	276.4239
Akaike	-3.3897	-3.4431	-3.4764
Bayes	-3.2719	-3.3056	-3.3390
Shibata	-3.3925	-3.4469	-3.4803
Hannan-quinn	-3.3418	-3.3872	-3.4206

Table 9: Model Selection for GARCH(1,1) for DNB Returns

Table (9) shows the Log Likelihood and Information Criteria values for the three extensions. All models show lower values for the EGARCH model, and the analysis will therefore continue using an EGARCH(1,1) model for estimation and forecasting.

8 Results

8.1 Empirical Method

This analysis is looking to empirically investigate whether including FX forward contracts with different time horizons create a more mean-variance efficient portfolio for the equity fund, DNB Global Emerging Markets A. To create an image of the situation, the analysis will begin by investigating the impact of the two different currency forwards on the historical risk and return of the fund where time horizons returns are utilized. To further implement the results, a mean variance optimization is performed to create a mean variance efficient portfolio for the investor where new weights in the different FX forwards are proposed. To investigate the issue further, the analysis will apply GARCH models for the unhedged returns and the returns of the different hedging options to investigate which model is forecasted to satisfy an investors risk return profile in the future.

8.1.1 Portfolio Optimization

The monthly prices for the DNB Global Emerging Markets A fund from January of 2008 until December of 2020 are again visualized in Figure(7). The figure exhibits high variance and a mean value that is not equal to zero, indicating that this time series is non-stationary with a varying mean and variance. By using FX Forwards, the model may be able to moderate this varying mean and variance to create a less volatile environment.

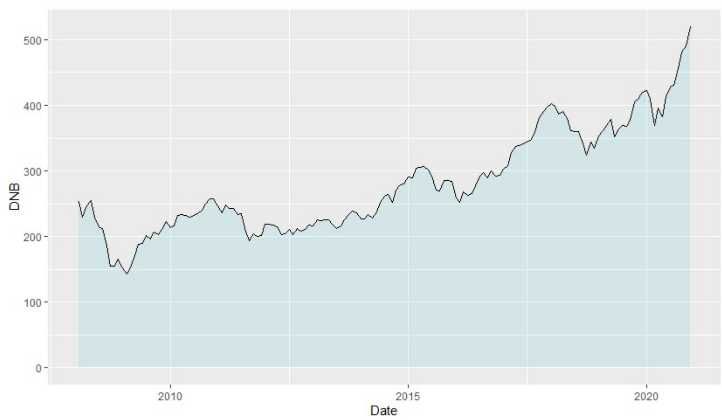

Figure 7: Monthly closing prices for DNB Global Emerging Markets from 2008 to 2020

By using monthly closing prices of the equity fund, DNB Global Emerging Markets A, NOK/USD, NOK/HKD, NOK/KRW, NOK/INR, and monthly interest rates for the respective countries, the analysis has been performed. The FX Forwards described earlier are used to create two new hedged portfolios.

To find the current and historic prices for the hedged portfolio including the FX Forwards, the monthly gain or loss of the forward in question, multiplied by the weight invested in each currency is added to the original DNB Global Emerging Markets A fund price. To visually see the difference in the value of the FX Forwards and the original price, Figure(8) is included.

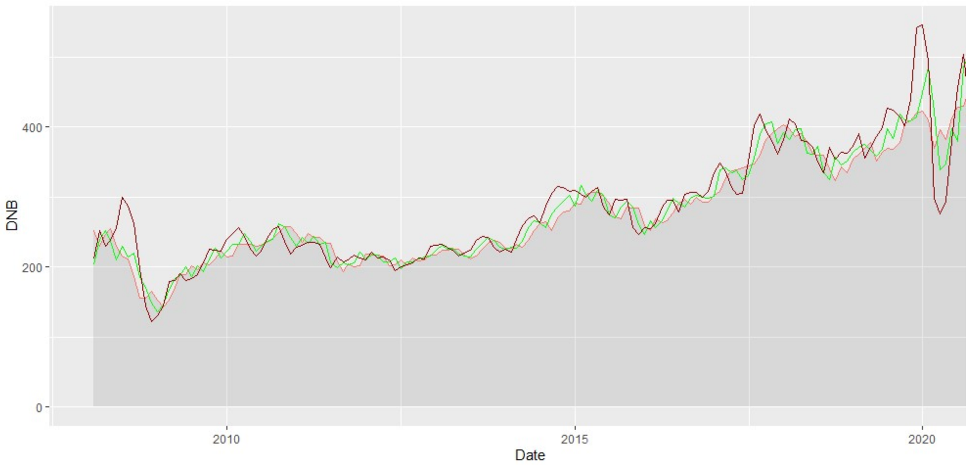

Figure 8: Monthly closing prices DNB vs FX Forwards

Figure (8) represents the historical monthly prices of DNB Global Emerging Markets and the estimated historical prices of the fund with FX Forwards with the two different time horizons on all currencies. The orange line represent the unhedged portfolio, the green line denotes the 30 day FX Forward and the red line represent the 90 day FX Forward. The graph shows that the three price levels follow each other closely, however, the graph shows tendencies to that the hedged portfolios exhibit more volatile environments than what the original, unhedged DNB Global Emerging Markets A does.

The statistics of the returns of the various portfolios are highlighted in the Table(10).

	DNB	FX Forward 30	FX Forward 90
Annualized Returns	0.0437	0.040	0.0044
Annualized Std	0.1610	0.222	0.3016
Annualized Sharpe Ratio	0.2406	0.158	-0.0013

Table 10: Statistics of Returns for DNB Global Emerging Markets & FX Forwards

The unhedged fund reveal the largest annualized returns, the lowest standard deviation and the largest Sharpe ratio, with a total return over the period investigated of 0.7196. The 90 days FX forward exhibit a negative Sharpe ratio, indicating expected returns are expected to be negative. The Sharpe ratio describe how much excess return you receive for the volatility and risk of holding a riskier asset, compared to a risk-free asset. The Sharpe Ratio will help you determine investment choice in which will give you the highest returns when considering risk. When comparing portfolios, if the portfolios have the same risk, the investor will choose the portfolio with the higher Sharpe ratio as it will give them a higher ratio of return against risk.

Compared with the 30 days FX forward and the 90 days FX forward, the original, unhedged fund is found to be the most favorable as it historically has created higher returns and a lower risk. Though, there might always exist a more optimal portfolio, and the analysis will therefore continue with optimizing a mean-variance portfolio to investigate whether a combination of both considered FX forwards creates a hedged mean-variance efficient portfolio in which is more efficient than the unhedged.

Portfolio optimization is as described previously, the process of deciding the best portfolio according to some objective, here minimum variance and mean variance, with necessary restrictions included. In the mean variance optimization of this thesis, the spot price for each the forward prices found previously are separated into the different currencies based on their weights and added the gain or loss to the original price. This way, the optimization process will include creating an optimal portfolio based on FX Forwards for all considered currencies, combined with the original portfolio.

Optimizing a portfolio to obtain minimum risk and a maximum return with the constraints of a full investment, only long positions, and a maximum investment of 0.7 in one asset, the optimization is very limited but suggest the weights in Table (11).

USD	HKD	KRW	INR	USD1	HKD1	KRW1	INR1	USD3
0.010	0.304	0.008	0.024	0.018	0.006	0.056	0.208	0.066

HKD3	KRW3	INR3
0.00	0.078	0.222

Table 11: Currency Weights From Portfolio Optimization

Which again provide us with the statistical results in Table (12).

	Minimum Variance	Mean Variance
Annualized Returns	0.0482	0.0482
Annualized Standard Deviation	0.0979	0.0979
Annualized Sharpe Ratio	0.4422	0.4422

Table 12: Annualized Statistics for Minimum Variance and Mean Variance Portfolios

Optimizing a mean variance portfolio as well, two very similar results are obtained. These suggested optimized portfolios consist of utilizing all but one FX forward, the 90 days FX forward on the Hong Kong Dollar, and provide the new portfolio with an increase in return, a decrease in risk and a higher Sharpe ratio. Optimizing the portfolio, risk has been diversified and spread across the portfolio explaining how the risk and returns have changed. However, to further view whether the mean-variance optimized portfolio in fact is the most efficient, the analysis will forecast the portfolios.

8.1.2 GARCH Models

Further investigating the volatility, details and expected risk and returns of the hedged and unhedged portfolios, GARCH models will help model volatility and further forecast the portfolios. As described earlier, univariate models such as the GARCH framework captures features in which traditional econometric models are not able to capture. The unhedged fund has already been proven to pass the requirements to be modeled through the GARCH framework, while quick tests will determine whether the requirements are passed for the hedged portfolio as well. The hedged portfolio is tested for heteroskedasticity, stationarity and normality. All tests are significant and the thesis can continue using this framework to further investigate the volatility of the unhedged and hedged portfolios as well as provide forecasts for the portfolios.

Estimation Results

Estimating the GARCH models, the model selection suggest the use of an EGARCH(1,1) model for both portfolios. Two different EGARCH(1,1) models are estimated and forecasted for the returns of the unhedged DNB Global Emerging Markets A fund and for the returns of the proposed hedged portfolio.

The approach to estimate an EGARCH(1,1) model for the nominal monthly returns on DNB Global Emerging Markets A, y_t and the hedged portfolio, is to first identify the mean equation, to further find the variance equation, the conditional volatility σ^2. The mean model is given by Equation (15), and the variance model is given by Equation(16).

$$y_t = \mu + u_t \quad \text{where} \quad u_t \sim \mathcal{N}(0, \sigma_t^2) \tag{15}$$

$$\sigma_t^2 = \alpha_0 + \omega_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{16}$$

The parameters in the mean equation are defined similar to the OLS model, where μ is the intercept. The variance equation however, identifies that the variance σ_t^2 of the time series today is equal to a constant, ω, in addition to some amount of α of the previous residual ϵ_{t-1}, plus some β of the previous variance σ_{t-1}^2.

The results of the EGARCH(1,1) model for the unhedged DNB Global Emerging Markets A and the hedged portfolio are given by Table(13)

	DNB Global Emerging Markets A	Optimal Portfolio
μ	0.005401	0.005401
	(0.0076037)	(0.076037)
ar1	-0.510693^{*}	-0.510693^{*}
	(0.017055	(0.017055)
ma1	0.671627^{***}	0.671627^{***}
	(0.000119)	(0.000119)
ω_1	-0.822245^{***}	-0.822245^{***}
	(0.00)	(0.000)
α_1	-0.331677^{***}	-0.331677^{***}
	(0.00)	(0.000)
β_1	0.874398^{***}	0.8743978^{***}
	(0.00)	(0.000)
γ_1	0.068189	0.068189
	(0.265256)	(0.265257)
Log likelihood	276.4239	276.4239
AIC	−3.4764	−3.4764
ARCH LM	(0.3967)	(0.3967)
Goodness-of-Fit	(0.5031)	(0.5031)

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 13: EGARCH(1,1) Estimations of DNB Global Emerging Markets A & Optimal Portfolio

In Table (13), μ, the intercept, is interpreted as the expected mean returns. The expected mean return of the fund is 0.0054. With a proof of mean reversion still being present, the model follows the financial theory of that asset prices and historical returns eventually revert to their long-term mean.

In the variance equation, ω represent the variance intercept of the EGARCH(1,1) model, and α_1 represent the past errors and how volatility reacts to new information, while β_1 is the estimated parameter of the lagged variance, a β_1 of 0.874398 is close to 1, which means the variance at time t is highly correlated with the variance at time t-1. The Ljung-Box test on the standardized residuals and the standardized squared residuals has a significant p-value, meaning the models have captured the auto-correlation that was present prior to implementing the EGARCH(1,1) models. ARCH LM tests on the residuals confirms there are no remaining ARCH effects in the models.

From the results in Table(9), we can identify the variance, and in turn the volatility of the portfolios. The volatility persistence, a measure by the sum of α and β is quite high and can be found with equation(17). The volatility persistence for both models is visualized in Figure(9).

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta} \tag{17}$$

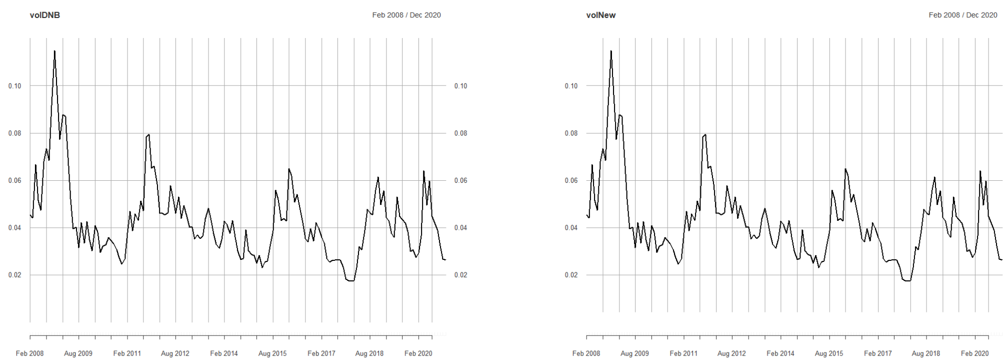

Figure 9: GARCH(1,1) Volatility of DNB Global Emerging Markets A

As discussed previously, the variance of the timeseries of DNB Global Emerging Markets increase during shocks in the economy. The volatility exhibit these shocks more clearly, and as mentioned in Chapter 6, the volatility drastically increase during the financial crisis of 2008 and during the covid-19 pandemic of 2020. When examining the volatility, we see that there clearly are other shocks that effect the volatility in this fund as well. Keeping in mind this fund is focusing on emerging markets, the fund will be affected by changes and shocks to the economy of the respective countries and its surrounding areas. In 2011, the tsunami in Japan occurred, and it is natural to assume that the volatility jump in 2011 is mainly caused by this. Further, in 2015, China devalued its currency and caused shock waves through the world economy, causing the funds volatility to pike.

Continuing the analysis, forecasting the risk and return for the equity fund, DNB Global Emerging Markets A fund and the hedged mean-variance optimized portfolio, is performed by continuing using the EGARCH(1,1) model estimated. By splitting the dataset into a training and a test set, the first portion of the dataset is trained to further test the forecast against the test part of the data set. When fitting the EGARCH(1,1) model to the training set, the output in Figure (10) is achieved. The figure represents both portfolios. The QQ plot for normality shows scatterplots in which follows the line of normal distribution better than previously. The plot for empirical density of standardized residuals confirms the normality statement and results in the QQplot, where it shows a very slight skewness. For a final confirmation of normality, the model was tested using a Jarque-Bera test for normality. The test rejects the hypothesis of non normality, opposed from prior to using a GARCH model. The results of the test is included in Table(16) are in the appendix.

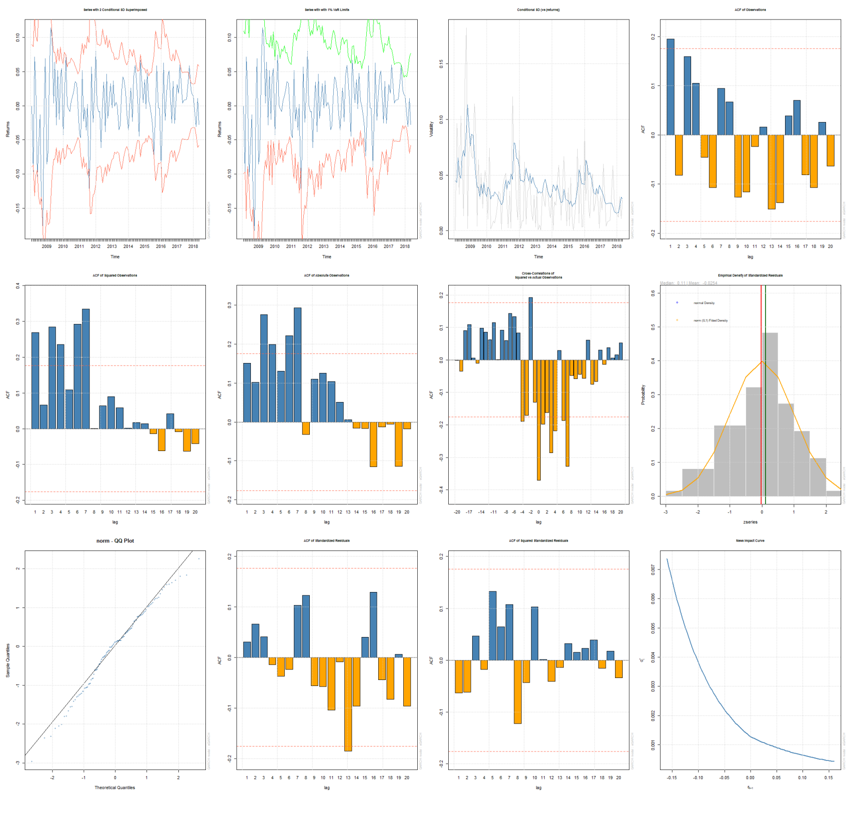

Figure 10: EGARCH(1,1) Fit for DNB Global Emerging Markets A & Optimized portfolio

As mentioned previously, the EGARCH model has a new impact curve

Figure (11) shows four different forecasting plots that represents both models.The predicted conditional volatility at time t+h is the sigma of the forecast, while the series represent the conditional mean at time t+h. Because the mean model on R_t is constant, the predicted mean is in turn observed to be constant, hence the straight lines identifying the forecasting lines. The model seems to be well fitted where a slight reduction in volatility is forecasted to occur. The forecasted risk, returns and price of the portfolio is represented in Table (14).

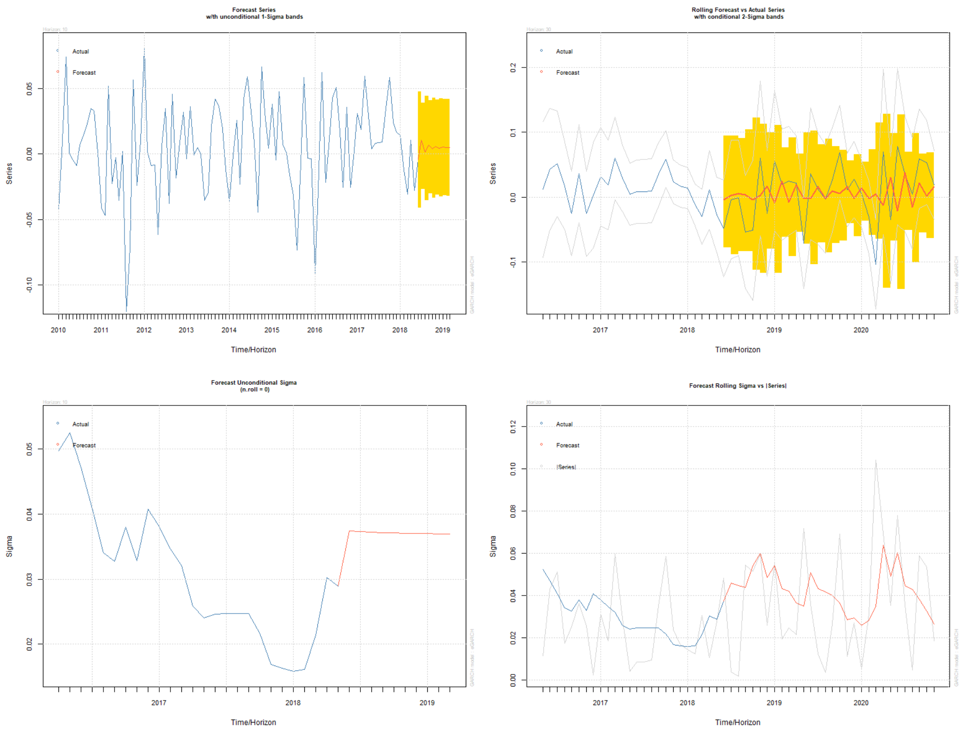

Figure 11: EGARCH(1,1) Forecast Plot

The forecast results of the two portfolios have almost identical results using an EGARCH(1,1) model. Corresponding with the plots above, the volatility slowly decline combined with a slight increase in the returns for both portfolios. Though, as the current price of DNB Global Emerging Markets is higher than the current price of the optimal portfolio, the forecasted price level for DNB Global Emerging Markets is slightly higher.

	DNB Global Emerging Markets			Optimal Portfolio		
	Return	Risk	Price	Return	Risk	Price
T+1	-0.003866	0.03737	490.301	-0.003866	0.03737	485.9445
T+2	0.010440	0.03729	490.3522	0.010440	0.03729	485.9952
T+3	0.001708	0.03723	490.3606	0.001708	0.03723	486.0035
T+4	0.007037	0.03716	490.3951	0.007037	0.03716	486.0378
T+5	0.003785	0.03711	490.4137	0.003785	0.03711	486.0561
T+6	0.005770	0.03706	490.442	0.005770	0.03706	486.0842
T+7	0.004558	0.03702	490.4643	0.004558	0.03702	486.1063
T+8	0.005298	0.03699	490.4903	0.005298	0.03699	486.1321
T+9	0.004847	0.03696	490.5141	0.004847	0.03696	486.1557
T+10	0.005122	0.03693	490.5392	0.005122	0.03693	486.1806

Table 14: EGARCH(1,1) Forecast for DNB Global Emerging Markets A Optimal Portfolio

9 Discussion of Results

The purpose of this analysis has been to research and investigate whether the use of FX forwards in the global equity fund DNB Global Emerging Markets A could stabilize the portfolio through financial hedging strategies. Using methods and models such as mean-variance optimizing and GARCH models, the fund was investigated for how a full FX Forward hedge on selected currencies historically would have impacted the portfolio. Additionally, the model was investigated for the options for an optimized mean-variance efficient portfolio by diversifying exposure to the different FX Forwards. Further, the analysis forecasts the original and the hedged portfolio's conditional volatility and compare the forecasts for the two portfolios.

Creating the FX forwards and comparing the currency hedged FX Forward portfolios with the original, unhedged portfolio, results immediately indicated that fully hedging the currency exposure with FX Forwards creates lower the returns, a higher annualized standard deviation and a lower Sharpe ratio. Since an optimal portfolio choice for most investors include a reduction in standard deviation and increase in Sharpe Ratio, these results support not using FX Forward to hedge the portfolio. Similar to Campbell et al. (2010) who stated that full FX Forward hedging is sensitively dependent on the base currency and varies across all risk for all investor, this initial analysis found similar results. With the measurements used in this analysis, full FX Forward hedging is not rewarding at this point.

Due to the immediate results indicating that a full hedge with either time horizons of FX forwards is not the best option for DNB Global Emerging Markets A, the analysis has used this information for further research. This study optimized a mean-variance efficient portfolio in which includes hedging in both 30-days FX forwards, 90-days FX forwards and no hedging at all. This optimization, in which was performed with the purpose of maximizing expected return given some risk measure, came to the conclusion that diversifying the portfolio by utilizing a small fraction in all but one of the strategies, will be the optimal choice for the investor, DNB Bank. With the perfect correlation coefficient between the USD and the HKD, this may be a reason of why the optimization suggests a fraction of zero in the 90 days FX forward for the HKD. The results of this optimization reduce the risk from 0.1610 to 0.0979 while increasing returns by 0.45%. A higher reported Sharpe ratio supports the new, optimized portfolio. Parallel with Campbell et al. (2010) who further found that a mean-variance optimization of the portfolio will create the best portfolio for the individual investor, the analysis found that mean-variance optimizing the portfolio using the FX Forwards minimize risk and maximize the return for the portfolio.

However, as researches such as T. Copeland and Joshi (1996) conclude that derivatives don't reduce FX risk, the analysis further forecasted and compared the hedged, optimized portfolio with the original, unhedged portfolio. Using the suggestions from Bender and Nielsen (2010) to utilize the correct tools to quantify risk, the continued investigation of the two funds found unexpected results. The further analysis included using GARCH models to quantify, fit and forecast to cover the proper aspects in risk management; risk measurement, risk monitoring and risk adjusted investment management, respectively.

The further analysis of the two portfolios included forecasting two EGARCH (1,1) models. Using EGARCH models for both portfolios, the final models reported close to identical results with an expected decline in the volatility, and anticipated increase in price and returns of both portfolios. When investigating the correlation coefficient between these two portfolios, it is equal to 1, indicating perfect correlation. Historically, the returns have been very close to each other, but not identical, however, with a perfect correlation, the portfolios will follow the same pattern, with minimal differences. Therefore, the two portfolios are not identical viewing their historic risk and return, but forecasting them using an EGARCH(1,1) model, the results turns out very close to identical.

The forecasts shows expected means in which are close to zero. In line with theory, as the GARCH model framework is based on the mean equation, it has a mean reverting variance process where means are to revert to their long term meansEngle (2001). If we view this in line with the output of the GARCH-model, we witness the sum of α_1 and β_1 is less than one, corresponding to a mean reverting process. However, with the sum being close to 1, this mean reversion process is slow.

One setback of this analysis was the mean absolute error of the forecasted values (MASE) of the forecasts. The MASE value is preferred to have a value below 1, though the forecasts of this model presents MASE values of 3.022373 and 3.110797. This is obtained as a 3.022373 and 3.110797 approximation of the true results, respectively. However, with the forecasts being almost identical and the goal of the analysis is to compare the two models, I have made the assumption that the comparison shows similar results as it would have with different MASE values.

10 Conclusion

Witnessing the high volatility presence in the global equity market, this thesis investigated opportunities for controlling this volatility in the Norwegian equity fund DNB Global Emerging Markets A.

After investigating fully hedging the Norwegian equity fund, DNB Global Emerging Markets A with FX forwards of different time horizons, the analysis mean-variance optimized the portfolio to include a diversification of both FX forwards and unhedged currencies. The analysis further continued with forecasting and comparing the hedged and the unhedged portfolio, where the two forecasts provided close to identical results.

With the results from outputs and tests retrieved in the analysis, the investor should be indifferent about whether to use the suggested mean-variance optimization hedging or the original unhedged portfolio. However, implementing the use of FX forwards in a portfolio demands extra costs, time, planning and resources. With the two portfolios creating identical risk and return, the extra time and resources would cause unnecessary spending of capital for identical results. Staying put with the hedged portfolio would therefore be more sustainable for the portfolio in the current situation with the hedging strategies investigated. In line with Chang et al. (2012), final results shows that the hedging strategies for the relevant currencies for this fund is not empirically crucial, where the effectiveness is revealed to not create large differences. Keep in mind that the weights calculated have taken into considerations only these currencies. The fund does trade other currencies, but for the ease of this study these other currencies and weights have not been considered.

If the fund is to implement the use of currency hedging, further research is necessary to create an even more optimal portfolio. Suggestions of this entails creating a mean-variance optimal portfolio only using unhedged and only one of the time horizons for the FX forwards. In addition, this analysis only included the largest currencies, so a further research should include all traded currencies. Costs should also be considered in the further research.

Both Campbell et al. (2010) and Bucher (2019) utilized correlations between the currency pairs to hedge the currencies by going short currency pairs that are positively correlated with the return, and long the currency pairs that are negatively correlated with the return. This would have been very interesting to include in the analysis as an option, and is therefore suggested for further research.

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Appendix

A OLS Assumptions

The data used in this analysis is time series data, meaning the data is observed over time. To ensure efficiency, preciseness and consistency of the OLS model and the data investigated, Brooks (2019) lists a set of Gauss-Markov and classical linear model assumptions that needs to be held. If these assumptions are held, the estimations are said to be BLUE (Best Linear Unbiased Estimator) where “best” indicates that the method provides us with the lowest possible estimate for the variance compared with other linear unbiased estimators.

A.1 Linear in parameters

The first assumption set for a time series regression states that the series follows a model that is linear in parameters. There needs to be a linear relationship between the dependent and independent variables. If this assumption is not held, there is a possibility the beta coefficients are biased. If parameters are found to not be linear, one can attempt to avoid the issue by implementing natural logarithms or exponential forms of the parameters.

A.2 No perfect collinearity

In the assumption of no perfect collinearity, the model requires no independent variable to be constant nor a perfect linear combination of the other variables. If two variables have a perfect linear relationship, one x-variable might predict the other with high precision. If there exist multicollinearity in the model, OLS will not be able to estimate the unique regression coefficients for the variables in focus, and the standard errors will be infinite. Perfect multicollinearity is rather rare, though, if this is the case, one can drop one of the parameters as they are assumed to absorb the same information.

A.3 Zero conditional mean

$$E(u_t x_t) = 0, \quad t = 1, 2, \dots, n \tag{18}$$

The error term is a variable in which is produced when the model is not a complex representation of the relationship between the dependent and the independent variables. It absorbs the residual values in which are left out, but we do assume that these residuals are random and have a zero conditional mean. Regardless of the value of the explanatory variable, the error term should for each period "t" have an expected value of zero. The error term doesn't necessarily need to have a value equal to zero, but the term needs to be constant. The assumption is important in order to present causal relationships in the regression. If this assumption is not held, a systematic important factor might be left out and the estimated beta coefficients will be biased.

A.4 Homoskedasticity

The assumption of homoskedasticity requires the error terms to have a constant variance. With a constant variance, the disturbance not absorbed in the model is of equal size in all observations and estimations:

$$Var(u_t x_t) = \sigma^2, \quad t = 1, 2, \ldots, n \tag{19}$$

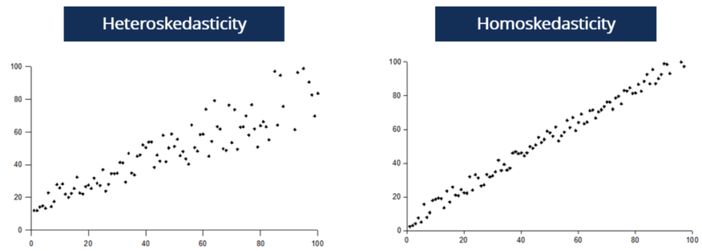


Figure 12: Heteroskedasticity vs. Homoskedasticity. Source: Heteroskedasticity *(2022)*

Equation (19) above shows the assumption, while Figure (12) shows the difference between a homoscedastic and a heteroskedastic model. In the heteroskedastic model, variance increase with an increase in the value of the variable.

A.5 No serial correlation

In a simple static regression model:

$$y_t = \beta_0 + \beta_1 x_t + u_t \tag{20}$$

Serially correlated errors imply that we relax the assumption that the error term in one period (t) is independent of the error term in another period. With that, we assume:

$$cov(u_t, u_s x) \neq 0, \quad s \neq t \tag{21}$$

And

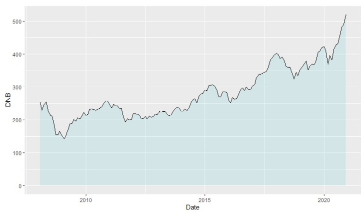
$$corr(u_t, u_s) = 0, \quad for \;\; all \;\; t \neq s \tag{22}$$

Though, some time series variables such as stock prices, are often serially correlated as the current stock price will be dependent on historic stock prices. The issue in this case of serially correlated error terms may be solved by viewing the percentage change in prices, rather than viewing the price itself.

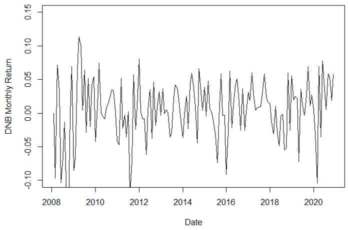
A.6 Normality

The last assumption states that through assumptions 1-5, the OLS estimators are normally distributed. The errors u_t are to be independent of x and are identically distributed as $\mathcal{N}(0, \sigma^2)$.

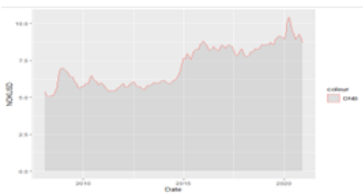
B Price and Return Movements



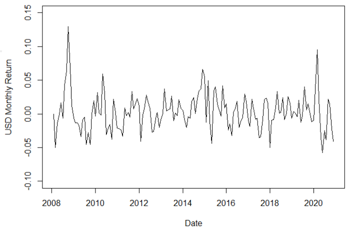
(a) DNB Prices



(b) DNB Return

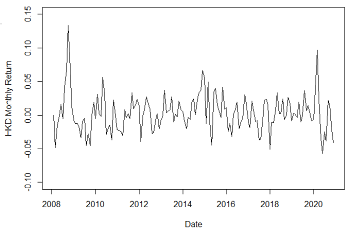


(c) NOK/USD



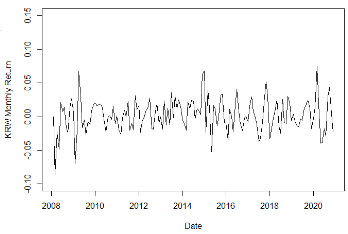
(d) NOK/USD Return

(e) NOK/HKD

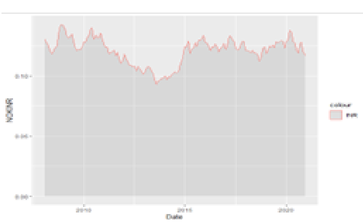


(f) NOK/HKD Return

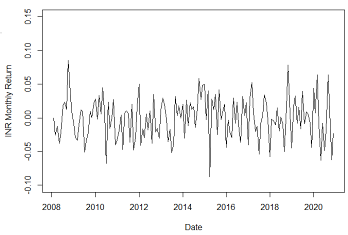
(g) NOK/KRW



(h) NOK/KRW Return



(i) NOK/INR



(j) NOK/INR Return

Figure 13: Left side: Monthly closing prices DNB Global Emerging Markets A and exchange rates. Right side: Monthly returns for DNB Global Emerging Markets A and exchange rates

C Model Selection

Aikaike's Information Criteria of the models ARIMA(1,1,1) and ARIMA(0,1,1)

Statistic	ARIMA(1,1,1)	ARIMA(0,1,1)
df	3	2
AIC	-502.0064	-500.566

Table 15: AIC results from ARIMA(1,1,1) ARIMA(0,1,1)

D Test of Normality of EGARCH(1,1) Outputs

	Jarque-Bera Test	
	DNB Forecast	Hedge Forecast
X-squared	1.738	1.738
df	2	2
p-value	0.4194	0.4194

Table 16: Jarque Bera Test for GARCH Model of DNB Returns and the hedged portfolio

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