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Fracture assessment of U-notched PMMA under mixed mode I/II loading conditions by means of local approaches.

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Abstract

The paper investigates the fracture behaviour of U-notched specimens made of polymethylmethacrylate at room temperature under mixed mode loading conditions. The specimens are flat and double notched. The notches bisectors lie on the same line that forms an angle β with the loading direction; this specimen shape allows to consider mixed mode loading conditions. With changing β, different contributions of mode I and mode II loading are achieved. The specimens net section, calculated perpendicularly to the load application direction, is maintained constant while the notch fitting radius changes ranging from 1 to 4 mm. The results have been analysed through both the averaged strain energy density method and the theory of critical distance assessing the use of coarse free mesh models for the application of these methods.

Keywords: Local Approaches; SED method; TCD method; Polymethylmethacrylate; U-notch.

1. Introduction

An accurate design of notched components against both static and fatigue failure represents a topic of high interest both in academia and industry. The increasing complexity of the geometry in components enhanced by the use of new technologies such as the additive manufacturing (AM) ones made also evident the necessity of accurate but also cheaper tools for the design of these component (Corigliano et al., 2019; Foti et al., 2020b) and even for simulating

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the entire process to realize a component in order to have a wider understanding of its mechanical behavior as a function of the process parameters (P Foti et al., 2021; Leoni et al., 2021, 2020b, 2020a). The use of local approaches, characterized by failure criteria that are not related to any particular geometry or loading condition, could represent a valid solution for the problems highlighted in the above. In literature different works are already available investigating the applicability of different local approached to the fracture assessment both in static and dynamic conditions of different materials having both ductile and brittle behavior (Ayatollahi et al., 2015; Susmel and Taylor, 2008a) such as steel welding (Foti and Berto, 2020a, 2020b, 2019a; Radaj et al., 2009; Song et al., 2018), steel (Berto and Barati, 2011; González et al., 2019), titanium alloys (Peron et al., 2018), polymers (Cicero et al., 2012; Peron et al., 2017; Razavi et al., 2018a), ceramics(Gómez and Elices, 2006; Taylor, 2004), rocks (Gómez and Elices, 2006; Justo et al., 2017; Zhou et al., 2018) manufactured both through conventional technique or innovative techniques such as the AM ones (Razavi and Berto, 2019). However, a limitation of the local approaches, in particular when dealing with complex geometries, is represented by the need of a finite element (FE) simulation whose accuracy can depend on the degree of refinement of the discretization in the model near the critical zone of the compsonent. The present work compares the accuracy of different local approaches, and in particular the strain energy density (SED) method and the theory of critical distances (TCD), in its point and line version, in assessing the fracture properties of polymethylmethacrylate (PMMA) U-notched specimens under mixed mode loadings. As a second objective of the work, two different numerical models have been considered in order to evaluate the applicability of these methods through free coarse mesh models in order to reduce the computational time and effort in applying these methods.

Nomenclature

E	Young modulus
K_{IC}	Fracture toughness
L	Critical length
R_0	Control volume characteristic length.
$\overline{W}$	Averaged strain energy density
$\overline{W}_c$	Critical averaged strain energy density.

Greek

2α	notch opening angle
λ_1	Mode I William's eigenvalue
ν	Poisson's ratio.
ρ	notch fitting radius
σ_0	inherent stress
σ_{UTS}	ultimate tensile strength

2. Theoretical background

2.1. Strain Energy Density Method

The strain energy density (SED) is an energetic local approach applied to investigate both fracture in static condition and fatigue failure (Aliha et al., 2017; Berto and Barati, 2011; Lazzarin et al., 2008; Lazzarin and Zambardi, 2002, 2001; Razavi et al., 2018b; Torabi et al., 2015). The method is based on the assumption that brittle fracture is determined by achievement of a critical value by the local SED, $\overline{W}$, averaged in a given control volume. Such a critical value of the averaged SED, $\overline{W} = \overline{W}_C$, has been proved to be independent of the notch opening angle and of the loading (Lazzarin et al., 2008; Lazzarin and Zambardi, 2002, 2001). Dealing with a material showing an ideally brittle behavior, the SED critical results to be equal to:

$$W_C = \frac{\sigma_{UTS}^2}{2E} \tag{1}$$

Being σ_{UTS} conventional ultimate tensile strength.

As stated above the SED value considered by the method must be averaged in a so-called control volume. This control volume has a characteristic length R_0 that is assumed to be a material property evaluable under plane strain and plane stress conditions as reported by (Lazzarin and Berto, 2005a, 2005b) and by (Yosibash et al., 2004) dealing with cracks:

$$R_0 = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_{UTS}}\right)^2 \qquad \textit{plane strain} \tag{2}$$

$$R_0 = \frac{(5-3\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_{UTS}}\right)^2 \qquad \textit{plane stress} \tag{3}$$

While, in the case of a sharp V-notch the characteristic length R_0 can be determined as shown by (Lazzarin and Zambardi, 2001).

$$R_0 = \left[\frac{I_1 \cdot K_{1C}^2}{4\lambda_1(\pi-\alpha)EW_C}\right]^{\frac{1}{2(1-\lambda_1)}} = \left[\frac{I_1}{2\lambda_1(\pi-\alpha)}\left(\frac{K_{1C}}{\sigma_{UTS}}\right)^2\right]^{\frac{1}{2(1-\lambda_1)}} \tag{4}$$

On the other hand, the shape of the control volume depends on the local geometry having a sector-shaped cylinder for sharp notches and a crescent moon-like shape in the case of blunt notches. The position of the control volume, instead, has to be determined according to the loading conditions: dealing with sharp notches, the centre of the sector-shaped cylinder control volume is on the notch tip; dealing with blunt notches the control volume axis of symmetry has to be oriented in a way that the center of curvature of the notch and the first principal stress maximum lie belongs to it. An overview of the control volume of all the conditions enlisted above are provided in Figure 1 while for more considerations about the analytic frame of this method and its application we remand to (Berto and Lazzarin, 2014; Radaj, 2015; Radaj and Vormwald, 2013).

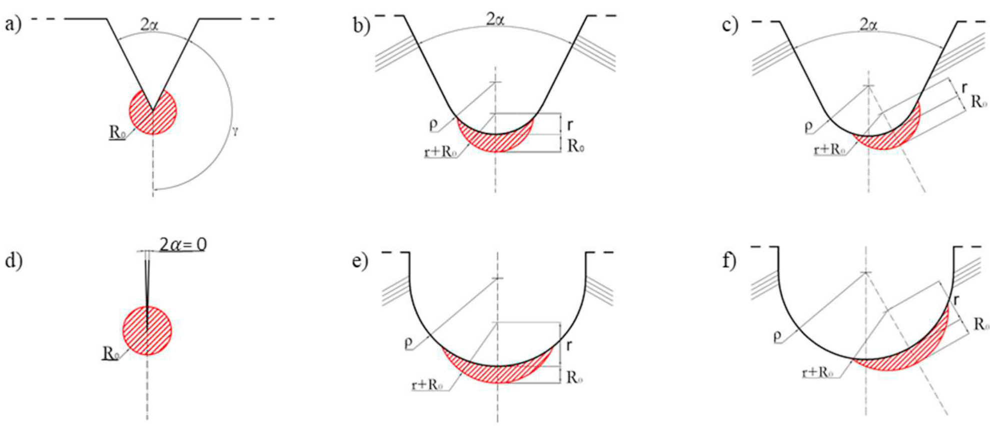

Figure 1: Control volume for a) Sharp V-notch; b) blunt V-notch under mode I loading; c) blunt V-notch under mixed mode loading; d) Crack; e) U-notch under mode I loading; f) blunt U-notch under mixed mode loading (Foti et al., 2021)

Regarding the application of the SED method in recent years significant effort have been devoted in the research to overcome one of its major drawbacks that limited its application to complex components (Campagnolo et al., 2020; Fischer et al., 2016; P. Foti et al., 2020; Foti et al., 2020a; Foti and Berto, 2019b). In the present work the SED method will be applied according to the so-called volume free procedure as proposed by (Foti et al., 2021)

2.2. Theories of Critical Distances

The Theory of Critical Distances (TCD) are a group of different methods that employ the mechanics of continuous media together with a characteristic parameter of the material named critical length, L, to predict the behavior of components with various geometry under different loading condition (Cicero et al., 2011; Susmel and Taylor, 2008b) for fracture and fatigue assessment (Susmel and Taylor, 2007; Taylor, 2008). The length parameter, L, is the so-called critical distance and it can be determined through the following equation.

$$\mathrm{L} = \frac{1}{\pi} \cdot \left(\frac{K_{IC}}{\sigma_0} \right)^2 \tag{5}$$

Where K_{IC} is the fracture toughness of the material and σ_0 is the inherent stress that is equal to the ultimate tensile strength dealing with brittle materials while it is greater than the ultimate tensile strength for ductile materials and should be properly determined. When the inherent stress is not available the critical length can be also be determined through experimental tests having different geometries finding the intersection of the stress curve as showed in Figure 2 c)

Different procedures are available for the application of these methods resulting in different failure criteria to assess fracture both in static and fatigue conditions. The procedures to apply the TCD considered in the present work are the Point Method (PM) and the Line Method (LM) whose definition is showed also in Figure 2 a) and b) respectively.

The PM establishes as a failure criterion that the stress evaluated along the notch tip at a length of $L/2$ reaches the value of σ_0. Such a condition can be expressed by the following equation under mode I loading:

$$\sigma_{\theta\theta}\left(\theta = 0; r = \frac{L}{2} \right) = \sigma_0 \tag{6}$$

The LM considers as failure criterion that the stress averaged along the notch tip in a length of $2L$ reaches the value of σ_0. Such a condition can be expressed by the following equation under mode I loading:

$$\frac{1}{2L} \int_0^{2L} \sigma_{\theta\theta}\left(\theta = 0; r \right) dr = \sigma_0 \tag{7}$$

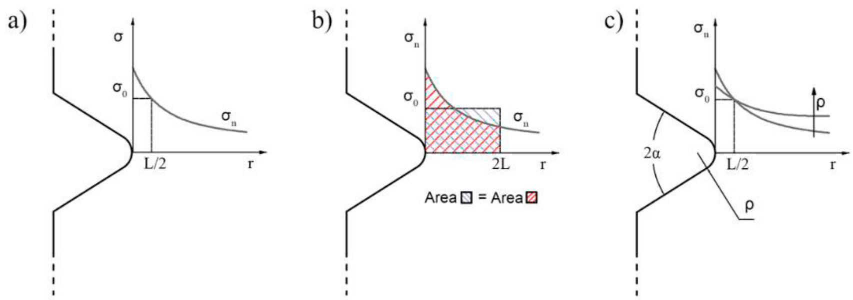

Figure 2: a) Definition of TCD Point method under mode I loading; b) definition of TCD Line method under mode I loading; c) Determination of the critical distance through experimental data having different geometries.

3. Materials and Methods

Static tensile tests in displacement control with a speed of $2\,mm/min$ were carried out on double U-notched specimens made of PMMA whose geometrical parameters are provided in Figure 3. The notch are oriented in the specimens according to a direction that forms an angle β with the loading direction. With changing β different contribution of mode I and mode II loading can be achieved; four different values of β have been considered. Three different values of the notch fitting radius ρ has been considered for each value assumed by the β angle for a total of 12 different cases. For each case, three different tests have been performed for a total of 36 static tests.

In order to apply the SED method and the TCD one the material properties needed to carry out the numerical simulation have been taken from literature (Berto and Lazzarin, 2014) and reported in table xx. An assumption of brittle behavior for this material let us approximate the inherent stress with the ultimate tensile stress resulting in the following in a value of the critical length for the application of the TCD method of $L = 0.246\ mm$ while the control volume radius for the application of the SED method has been taken from literature (Berto and Lazzarin, 2014) and is equal to $R_0 = 0.11\ mm$.

Table 1. Material properties of PMMA (Berto and Lazzarin, 2014)

E [MPa]	ν	$K_{IC}\ [MPa \cdot m^{0.5}]$	$\sigma_{UTS}\ [MPa]$
2960	0.38	1.96	70.5

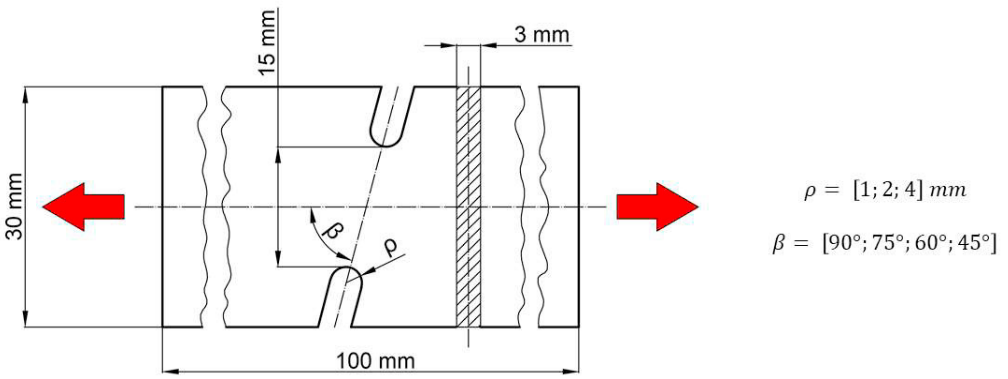

Figure 3: Schematic illustration of the notched specimens

4. Finite Elements Analysis

For the purpose of this work two different FE models were considered and reported in Figure 4. The model shown in Figure 4 a) represents the model that should lead to as less as possible errors in evaluating the stress field for the application of the TCD method and in as low as possible approximation in the control volume shape leading to an accurate estimation of the SED value according to (Foti et al., 2021). The model shown in Figure 4 b) has instead a free mesh with only a refinement along the notch edges and the notch fitting curve. The two models have been defined so that they have a remarkable difference in the number of elements to assess the possibility of both the methods considered in the present work to be applied through models having a rough discretization that requires a significantly lower effort by a designer but also lower computational resources.

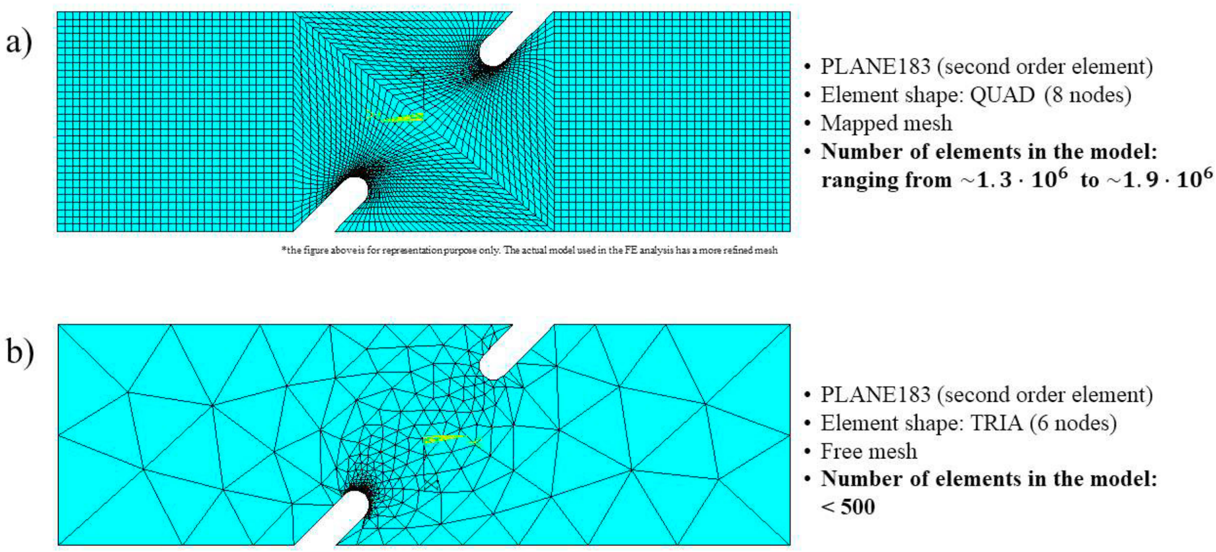

Figure 4: FE models with a) mapped mesh pattern for an accurate application of the SED and TCD methods b) free mesh pattern to assess the accuracy of the SED and TCD method.

5. Results and Discussions

The load vs displacement curves of the performed experimental tests are reported in Figure 5 a) while the picture of the broken U-notched specimens considered in the present work are reported in Figure 5 b)

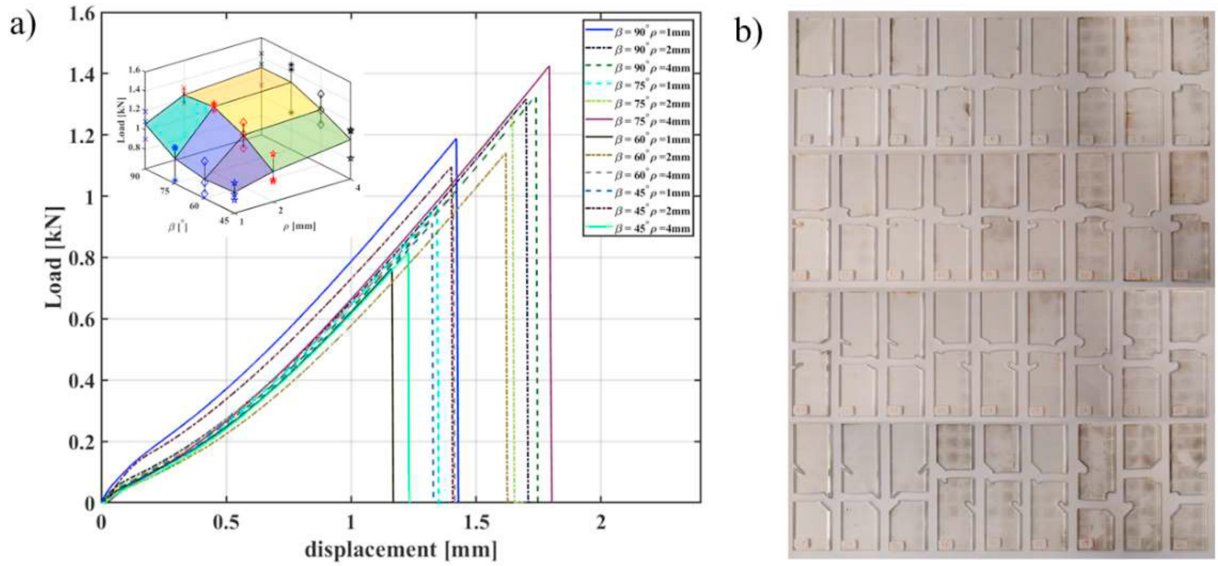

Figure 5: a) Load vs Displacement curve b) Fractured specimens after the experimental test

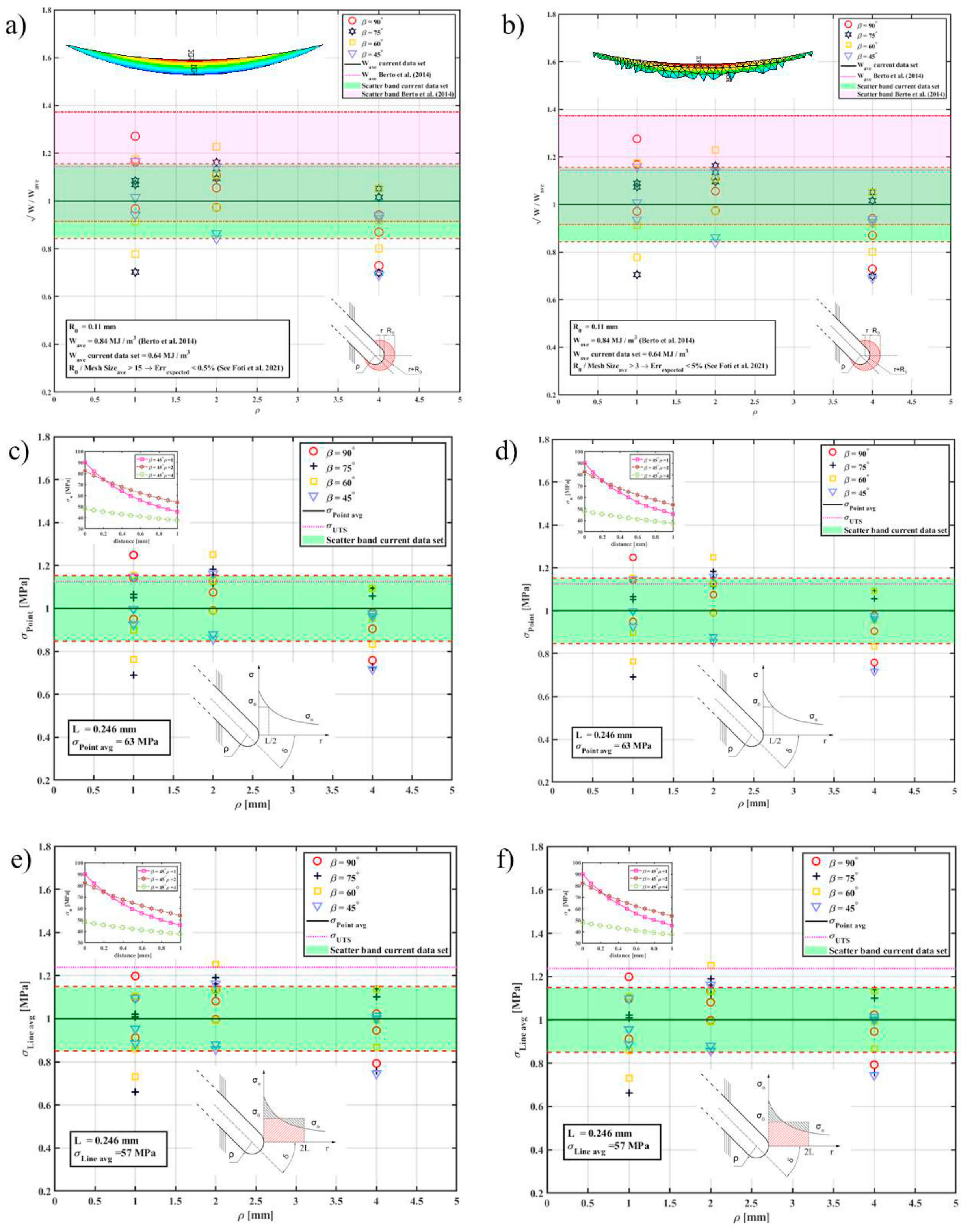

Figure 6: Summary of the experimental data through a) SED method applied with model A; b) SED method applied with model B; c) TCD PM applied with model A; d) TCD PM applied with model B; e) TCD LM applied with model A; f) TCD LM applied with model B.

The results summarized in terms of SED value and stress evaluated according to the TCD method are reported in Figure 6. As it is possible to notice all the three methods considered in the present work are able to summarize in a narrow scatter band defined in the present work through the standard deviation of the dataset considered. A comparison with available data in literature (Berto and Lazzarin, 2014) has been provided. For the SED method a scatter band was already available as it is possible to see from Figure 6 a) and b) while for the TCD method, assuming a brittle behavior for the material, the inherent stress has been approximated with the ultimate tensile stress of the material. As it is possible to see from Figure 5 a) the material is showing indeed a brittle behavior. The non-linear portion of the curve has to be addressed to the consideration of the global displacement of the specimens instead of the local displacement at the notch neighborhood, even if the concentration of stresses in the notch vicinity results in a plastic deformation localized around the notch (Torabi et al., 2016). In order to provide a meaningfull comparison between the methods, the data from literature have been taken from the same source. As it is possible to see from Figure 6 both the SED and the TCD method in its point version provide an assessment for the present dataset having a comparable discrepancy with the data from literature (see difference between mean value of the present dataset and the value from (Berto and Lazzarin, 2014). As regard the TCD method in its line version an higher difference have been noticed with the theoretical critical value derived from literature considering the assumption done in the present work.

6. Conclusions

In this work the fracture behavior of PMMA specimens weakened by blunt U-notches under mixed mode I/II conditions has been investigated through both the SED method and the TCD method in its point and line version. Furthermore, the methods considered in this work have been applied considering two different numerical models having a remarkable difference in terms of number of elements used for the discretization of the studied components.

The results of the study show that the average value of the SED value and the stress calculated according to the TCD method in its point version are in a good agreement with other data already available in literature and results in comparable results when applied to the experimental tests provided in this work. As regard the TCD in its line version, the results showed that the application of the method to the dataset considered results in a synthesis comparable with the other methods considered here, i.e., the data have a similar distribution around their averaged value, but an higher difference has been found with data already available in literature. As regard the application of the methods through the two different numerical models presented in this work no appreciable difference has been found.

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