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Cite as: AIP Conference Proceedings **2402**, 040009 (2021); https://doi.org/10.1063/5.0071391
Published Online: 15 November 2021

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Mathematical Models of Nonlinear Problems of Dynamics of Thin-Walled Structures under Aerodynamic Loading based on the Refined Timoshenko Theory

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Abstract. A class of dynamic problems is investigated using a programming environment, the mathematical models of these problems are described by complex nonlinear integro-differential equations. The following problems were investigated in the study: dynamics of thin-walled structures under aerodynamic loading, taking into account the hereditary properties of the material and geometric nonlinearity; dynamics of thin-walled structures under aerodynamic loading, taking into account the hereditary and inhomogeneous properties of the material.

INTRODUCTION

Studies conducted in [1-15] have shown that many non-stationary problems are reduced to solving systems of integro-differential equations with singular relaxation kernels. The main obstacle in solving these types of problems is the lack of numerical methods and algorithms for solving the problem, allowing an account for the rates of relaxation processes at the initial time points.

The results of studies of the hereditary properties of materials show that the rates of relaxation processes at the initial stages of testing are extremely high [14], and their direct measurement at the initial time points turns out to be impossible. Therefore, such processes are considered as dynamic, and their velocities are considered equal to infinity [14].

This phenomenon can be described using weakly singular functions that provide finite strains and stresses. Weakly singular functions describe well the rates of relaxation processes if they contain a sufficient number of rheological parameters. Such kernels are proposed by Yu. N. Rabotnov, A. R. Rzhanitsyn, M. A. Koltunov [15] and others.

In this article, when calculating the problems, a weakly singular Koltunov-Rzhanitsyn kernel with three rheological parameters of the form given in [16-30], is used.

PROBLEM FORMULATION

Let us consider a homogeneous thin-walled structure with hereditary material properties and flowed over on one side by a supersonic gas flow with an undisturbed velocity V, which is directed parallel to the Ox axis.

Let us construct a mathematical model of this problem based on the refined Timoshenko theory in a geometrically nonlinear formulation, taking into account shear strain and inertia of rotation.

Proceedings of the III International Conference on Advanced Technologies in Materials Science, Mechanical and Automation Engineering
AIP Conf. Proc. 2402, 040009-1–040009-6; https://doi.org/10.1063/5.0071391
Published by AIP Publishing. 978-0-7354-4156-9/$30.00

The equation of motion of a thin-walled structure with hereditary and inhomogeneous material properties without taking into account the radius of curvature of the middle surface, according to the generalized Timoshenko theory at overpressure has the following form

$$B_{11}(1-R_{11}^*)\frac{\partial\varepsilon_x}{\partial x}+B_{12}(1-R_{12}^*)\frac{\partial\varepsilon_y}{\partial x}+2B(1-R^*)\frac{\partial\gamma_{xy}}{\partial y}-\rho\frac{\partial^2 u}{\partial t^2}=0,$$
$$B_{22}(1-R_{22}^*)\frac{\partial\varepsilon_y}{\partial y}+B_{21}(1-R_{21}^*)\frac{\partial\varepsilon_x}{\partial y}+2B(1-R^*)\frac{\partial\gamma_{xy}}{\partial x}-\rho\frac{\partial^2 v}{\partial t^2}=0,$$
$$-2K^2\left[B_{13}(1-R_{13}^*)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial\psi_x}{\partial x}\right)+B_{23}(1-R_{23}^*)\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial\psi_y}{\partial y}\right)\right]--\frac{1}{R}\left[B_{22}(1-R_{22}^*)\varepsilon_y+B_{21}(1-R_{21}^*)\varepsilon_x\right]-$$
$$-\frac{\partial}{\partial x}\left\{\frac{\partial w}{\partial x}\left[B_{11}(1-R_{11}^*)\varepsilon_x+B_{12}(1-R_{12}^*)\varepsilon_y\right]+2B\frac{\partial w}{\partial y}(1-R^*)\gamma_{xy}\right\}- \quad (1)$$

$$-\frac{\partial}{\partial y}\left\{\frac{\partial w}{\partial y}\left[B_{22}(1-R_{22}^*)\varepsilon_y+B_{21}(1-R_{21}^*)\varepsilon_x\right]+2B\frac{\partial w}{\partial x}(1-R^*)\gamma_{xy}\right\}+$$
$$+\frac{æp_\infty}{hV_\infty}\left(\frac{\partial w}{\partial t}+V\frac{\partial w}{\partial x}\right)+\rho\frac{\partial^2 w}{\partial t^2}=0,$$
$$\frac{B_{11}h^2}{12}(1-R_{11}^*)\frac{\partial^2\psi_x}{\partial x^2}+\frac{B_{12}h^2}{12}(1-R_{12}^*)\frac{\partial^2\psi_y}{\partial x\partial y}+\frac{Bh^2}{6}(1-R^*)\left(\frac{\partial^2\psi_x}{\partial y^2}+\frac{\partial^2\psi_y}{\partial x\partial y}\right)-$$
$$-2K^2B_{13}(1-R_{13}^*)\left(\frac{\partial w}{\partial x}+\psi_x\right)-\frac{\rho h^2}{12}\frac{\partial^2\psi_x}{\partial t^2}=0,(x\leftrightarrow y),(1\leftrightarrow 2).$$

Mathematical models obtained using the system of integro-differential equations (1) with corresponding boundary and initial conditions, simultaneously take into account the hereditary and inhomogeneous properties of the material of a thin-walled structure, shear strain and inertia of rotation, and elastic wave propagation.

Let the thin-walled structure be hinged at all edges. Satisfying the boundary conditions of the problem, we choose expressions for the functions $w=w(x,y,t)$, $\psi_x=\psi_x(x,y,t)$, $\psi_y=\psi_y(x,y,t)$ based on the polynomial approximation in the following form

$$w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$\psi_x(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}\psi_{xnm}(t)\cos\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$\psi_y(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}\psi_{ynm}(t)\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}.$$
$$u(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}u_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$
$$v(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}u_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}, \quad (2)$$

where $u_{nm}=u_{nm}(t)$, $v_{nm}=v_{nm}(t)$ are the unknown functions of time.

Introducing (2) to (1) and applying the procedure of the Bubnov-Galerkin method with respect to the unknowns $u_{kl}=u_{kl}(t)$, $v_{kl}=v_{kl}(t)$, $w_{kl}=w_{kl}(t)$,$\psi_{xkl}=\psi_{xkl}(t)$and $\psi_{ykl}=\psi_{ykl}(t)$ we obtain a system of integro-differential equations.

DISCRETIZATION AND METHOD OF SOLUTION

Introducing the following dimensionless quantities into the resulting system:

$$\frac{u_{kl}}{h},\frac{v_{kl}}{h},\frac{w_{kl}}{h},\ \frac{V_\infty t}{a},\frac{aR(t)}{V_\infty}$$

and retaining the previous notation, we obtain

$$\ddot{u}_{\rm kl}+\frac{M_E^1\pi^2k^2}{1-\mu_1\mu_2}(1-R_{11}^*)u_{\rm kl}+M_G\pi^2\lambda^2l^2(1-R^*)u_{\rm kl}=-\frac{M_E^1\mu_2\pi^2\lambda{\rm kl}}{1-\mu_1\mu_2}(1-R_{12}^*)v_{\rm kl}-$$
$$-M_G\pi^2\lambda{\rm kl}(1-R^*)v_{\rm kl}+\frac{M_E^1\pi}{\lambda\delta(1-\mu_1\mu_2)}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}{\rm ni}^2\mu_{\rm nik}\beta_{\rm mjl}(1-R_{11}^*)w_{\rm nm}w_{\rm ij}+$$
$$+\frac{M_E^1\mu_2\pi\lambda}{\delta(1-\mu_1\mu_2)}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}{\rm mij}\gamma_{\rm nik}\alpha_{\rm mjl}(1-R_{12}^*)w_{\rm nm}w_{\rm ij}+$$
$$+\frac{M_G\pi\lambda}{\delta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}\left({\rm mij}\gamma_{\rm nik}\alpha_{\rm mjl}+{\rm nj}^2\mu_{\rm nik}\beta_{\rm mjl}\right)(1-R^*)w_{\rm nm}w_{\rm ij},$$
$$\ddot{v}_{\rm kl}+\frac{M_E^2\pi^2\lambda^2l^2}{1-\mu_1\mu_2}(1-R_{22}^*)v_{\rm kl}+M_G\pi^2k^2(1-R^*)v_{\rm kl}=-\frac{M_E^2\mu_1\pi^2\lambda{\rm kl}}{1-\mu_1\mu_2}(1-R_{21}^*)u_{\rm kl}-$$
$$-M_G\pi^2\lambda{\rm kl}(1-R^*)u_{\rm kl}+\frac{M_E^2\pi\lambda^2}{\delta(1-\mu_1\mu_2)}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}{\rm mj}^2\beta_{\rm nik}\mu_{\rm mjl}(1-R_{22}^*)w_{\rm nm}w_{\rm ij}+$$
$$+\frac{M_E^2\pi\mu_1}{\delta(1-\mu_1\mu_2)}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}{\rm nij}\alpha_{\rm nik}\gamma_{\rm mjl}(1-R_{21}^*)w_{\rm nm}w_{\rm ij}+ \qquad (3)$$
$$+\frac{M_G\pi}{\delta}\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}\left({\rm nij}\alpha_{\rm nik}\gamma_{\rm mjl}+n^2j\beta_{\rm nik}\gamma_{\rm mjl}\right)(1-R^*)w_{\rm nm}w_{\rm ij},$$
$$\ddot{w}_{\rm kl}+M_p æ\delta\lambda\,\{\dot{w}_{\rm kl}=-K^2[(1-R_{13}^*)\left(M_G^{13}\pi^2k^2w_{\rm kl}+M_G^{13}\pi\lambda\delta k\psi_{\rm xkl}\right)+$$
$$+(1-R_{23}^*)\left(M_G^{23}\pi^2\lambda^2l^2w_{\rm kl}+M_G^{23}\pi\lambda^2\delta l\psi_{\rm ykl}\right)]+\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}w_{\rm nm}\,[\frac{M_E^1\pi}{\lambda\delta(1-\mu_1\mu_2)}a_{\rm nmijkl}(1-R_{11}^*)$$
$$++\frac{M_G\pi\lambda}{\delta}c_{\rm nmijkl}(1-R^*)+\frac{M_E^2\mu_1\pi\lambda}{\delta(1-\mu_1\mu_2)}f_{\rm nmijkl}(1-R_{21}^*)]u_{\rm ij}+$$
$$+\sum_{n,i=1}^{N}\sum_{m,j=1}^{M}w_{\rm nm}\,[\frac{M_E^1\mu_2\pi}{\delta(1-\mu_1\mu_2)}b_{\rm nmijkl}(1-R_{12}^*)+\frac{M_G\pi}{\delta}d_{\rm nmijkl}(1-R^*)++\frac{M_E^2\lambda^2\pi}{\delta(1-\mu_1\mu_2)}e_{\rm nmijkl}(1-R_{22}^*)]v_{\rm ij}$$
$$-\sum_{n,i,r=1}^{N}\sum_{m,j,s=1}^{M}w_{\rm nm}[\frac{M_E^1\pi^4}{32\lambda^2\delta^2(1-\mu_1\mu_2)}h_{\rm nmijrskl}(1-R_{11}^*)-$$
$$-\frac{M_E^1\mu_2\pi^4}{32\delta^2(1-\mu_1\mu_2)}p_{\rm nmijrskl}(1-R_{12}^*)+\frac{M_G\pi^4}{16\delta^2}g_{\rm nmijrskl}(1-R^*)+\frac{M_E^2\pi^4\lambda^2}{32\delta^2(1-\mu_1\mu_2)}q_{\rm nmijrskl}(1-R_{22}^*)-$$
$$-\frac{M_E^2\mu_1\pi^4}{32\delta^2(1-\mu_1\mu_2)}r_{\rm nmijrskl}(1-R_{21}^*)]w_{\rm ij}w_{\rm rs}-2æM_pM*\lambda\delta\sum_{n=1}^{N}n(\gamma_{n+k}-\gamma_{n-k})w_{\rm nl},$$
$$\ddot{\psi}_{\rm xkl}+\frac{M_E^1\pi^2k^2}{1-\mu_1\mu_2}(1-R_{11}^*)\psi_{\rm xkl}+\frac{M_E^1\mu_2\pi^2\lambda{\rm kl}}{1-\mu_1\mu_2}(1-R_{12}^*)\psi_{\rm ykl}+M_G\pi^2\lambda^2l^2(1-R^*)\psi_{\rm xkl}+$$
$$+M_G\pi^2\lambda{\rm kl}(1-R^*)\psi_{\rm ykl}+12M_G^{13}K^2\pi\lambda\delta k(1-R_{13}^*)w_{\rm kl}+12M_G^{13}K^2\lambda^2\delta^2(1-R_{13}^*)\psi_{\rm xkl}=0,$$
$$\ddot{\psi}_{\rm ykl}+\frac{M_E^2\pi^2\lambda^2l^2}{1-\mu_1\mu_2}(1-R_{22}^*)\psi_{\rm ykl}+\frac{M_E^2\mu_1\pi^2\lambda{\rm kl}}{1-\mu_1\mu_2}(1-R_{21}^*)\psi_{\rm xkl}+M_G\pi^2k^2(1-R^*)\psi_{\rm ykl}+$$
$$+M_G\pi^2\lambda{\rm kl}(1-R^*)\psi_{\rm xkl}+12M_G^{23}K^2\pi\lambda^2\delta l(1-R_{23}^*)w_{\rm kl}+12M_G^{23}K^2\lambda^2\delta^2(1-R_{23}^*)\psi_{\rm ykl}=0,$$
where $M_E^1=\frac{E_1}{\rho V_\infty^2}$, $M_E^2=\frac{E_2}{\rho V_\infty^2}$, $M_G=\frac{2B}{\rho V_\infty^2}$, $M_G^{13}=\frac{2B_{13}}{\rho V_\infty^2}$, $M_G^{23}=\frac{2B_{23}}{\rho V_\infty^2}$.

Integration of system (3) was conducted by the numerical method. In this case, a weakly singular Koltunov-Rzhanitsyn kernel of the form given in [10-30] was used as the relaxation kernel.

NUMERICAL RESULTS AND DISCUSSION

Further, the results of calculating a thin-walled structure (with hereditary and inhomogeneous material properties), flowed over by a supersonic gas flow are presented (Figs. 1, 2). The gas flow rate is 1000 m/s. Unless other data are specified, the following values are taken as initial data: A=0.02, A_{11}=0.03, A_{12}=0.04, A_{21}=0.05, A_{22}=0.06, A_{13}=0.07, A_{23}=0.08, M_E=4.71, M_P=0.003.

The curves in Fig. 1 correspond to different cases of the hereditary properties of the material: curve 1 corresponds to the case when the hereditary properties of the material are taken into account only in shear directions (1 - A=A_{13}=A_{23}=0.0099, A_{ij}=0, i=1,2, j=1,2); curve 2 corresponds to the case when the hereditary properties of the material are the same in all directions (2 - A=A=A_{13}=A_{23}=0.0099, A_{ij}=0, i=1,2, j=1,2); curve 3 corresponds to the case when the hereditary properties of the material are different in all directions (3 A=0.0099; A_{11}=0.02; A_{12}=0.03; A_{21}=0.04; A_{22}=0.05; A_{13}=0.06; A_{23}=0.07 – an inhomogeneous formulation). As seen from Figure 1, an account for the hereditary properties of the material in all directions simultaneously leads to more intense damping of the oscillation amplitude and a phase shift to the right.

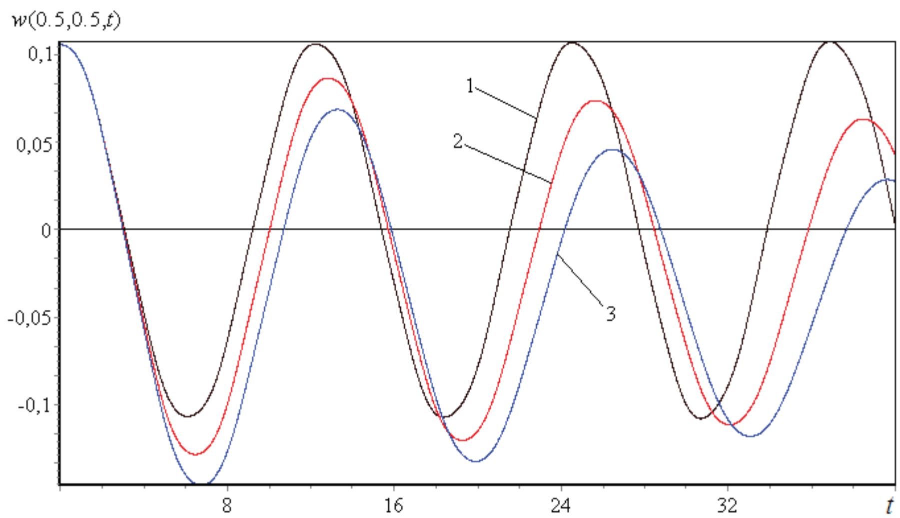

FIGURE 1. Time dependence of functions *w*.

The curves in Figure 2 correspond to the cases when the dynamic process is considered with (curve 1) and without taking into account the elastic wave propagation (curve 2).

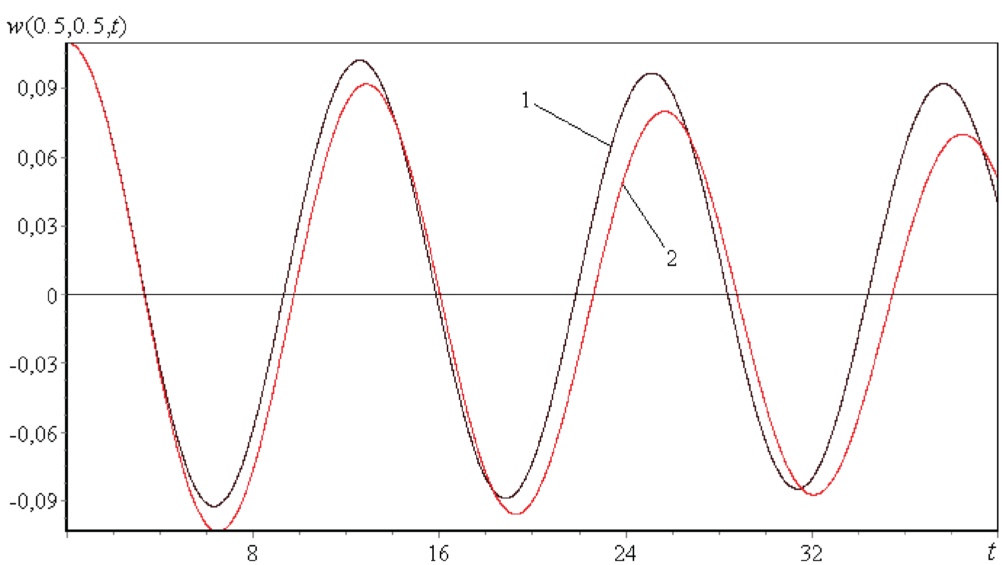

FIGURE 2. Time dependence of functions *w*.

As seen from Fig. 2, in the case when the process is considered without the elastic wave propagation, a more intense decrease in the value of function w is observed compared to the case when the process is considered taking into account the elastic wave propagation.

CONCLUSIONS

On the basis of the refined Timoshenko theory, mathematical models of the problems of the dynamics of thin-walled structures under aerodynamic loading were constructed, taking into account the hereditary and inhomogeneous properties of the material.

The main resolving nonlinear integro-differential equations of the problem of the dynamics of thin-walled structures under aerodynamic loading were obtained on the basis of the Timoshenko theory, taking into account the hereditary and inhomogeneous properties of the material.

Nonlinear problems of the dynamics of thin-walled structures under aerodynamic loading were investigated taking into account the hereditary and inhomogeneous properties of the material according to the refined Timoshenko theory.

In the calculations, it is necessary to choose the appropriate theory (linear and non-linear ones) depending on the various geometric and physical parameters of thin-walled structures.

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