UDC 004.41;004.424;519.64, 519.87:004.94:621.314](043)
DOI: 10.32626/2308-5916.2021-22.31-38

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DIGITAL CORRECTION FILTER IN PROBLEMS OF RECOVERY OF INPUT SIGNALS AND OBSERVING SYSTEMS' DATA IN ENERGY OBJECTS

The task of signal recovery is one of the most important for automated diagnostics and control systems of an energy object. When solving the inverse problems of recovering signals, images and other types of data, spectral distortions and losses occur (in some cases, very significant ones). They are primarily stipulated due to ill-posedness of these problems, which is the result of loss of information about the original signal due to strong (and even complete) suppression in the observed signal of a part of spectral components, which become indistinguishable against the background of errors and noise [1]. Besides, additional spectral distortions may occur in the process of solving recovery problems, which depend on specific methods used and their parameters. A method for building a digital correcting filter for processing the results of solving incorrect inverse problems is proposed, which effectively improves the quality of the solution. The method is based on the use of a singular decomposition of the matrix (SVD) of a system of algebraic equations that approximates the integral operator.

Key words: *inverse problems, signal recovery, digital filter, SVD decomposition, integral equations.*

Formulation of the problem. Methods and means of recovered signals' filtering correction and use of apriori information are intended to improve the quality of the inverse problems' solutions. It should be noted that there are many efficient algorithms for solving inverse problems, the idea of which is to use additional apriori information, for example, information about smoothness, monotonicity, or convexity of the solution [2-4]. However, the problem of ensuring the quality of obtained results remains relevant. Errors in the spectrum of the recovered signal naturally lead to its distortion and to a decrease in the accuracy of recovery. One of the most significant points is that, in the presence of fast change zones (jumps, peaks, etc.) in the original, truncation or sharp limitation of the spectrum leads to appearance of oscillations (Gibbs phenomenon) [5-7], which can have a very significant intensity and cover large areas. Often, these oscillations introduce significant distortions into the recovered signal, which mask important details of the recovered data and can lead to appearance of artifacts. The existing methods of handling the undesirable effects associated with the Gibbs phenomenon can themselves lead to appearance of additional distortions.

Inverse problems in most cases resolve themself into solving the systems of linear equations. The most time-consuming and complex problems arise precisely at this stage of solution.

Consider the classical inverse problem — the solution of the Fredholm integral equation of the 1st kind:

$$\int_a^b K(x,s)y(s)ds = f(x), \tag{1}$$

where $K(x,s)$, $f(x)$ are the given functions, $y(x)$ is the desired solution. We split the segment (a,b) by a uniform grid $\{x_i\}_{i=1}^n$, with a pitch h_x and pass on to the system:

$$\int_a^b K(x_i,s)y(s)ds = f(x_i), \quad i=\overline{1,n}. \tag{2}$$

Method description. To calculate integrals on the left side of the system (2), we use the formula for trapezoids on a uniform grid $\{s_j\}_{j=1}^m$, with a pitch h_s. The performed operations make it possible to obtain the matrix of the operator A, that approximates the integral operator in equation (1). Thus, the problem of solving the integral equation (1) resolves itself into to solving a system of algebraic equations:

$$AY = F. \tag{3}$$

where

$$A_{i,j} = \begin{cases} h_s K(x_i,s_j), & j=2,3,\ldots,m-1, \\ \dfrac{h_s}{2} K(x_i,s_j), & j=1,m, \end{cases} \qquad i=\overline{1,n},$$

$$F_i = f(x_i),\ i=\overline{1,n},\ Y_j = y(s_j),\ \ j=\overline{1,m}$$

are values of the unknown function at nodes of the splitting.

To solve system (3), we use the SVD *(singular)* decomposition of a matrix, which means that any matrix A of dimension $n \times m$ can be represented as:

$$A = U\Sigma V^T, \tag{4}$$

where U is the orthogonal $n \times n$ matrix, V is the orthogonal $m \times m$ matrix, Σ is the diagonal $n \times m$ matrix with diagonal elements $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_p \ge 0,\ p=\min\{n,m\}$. Schematically, such a decomposition can be represented as (Fig. 1).

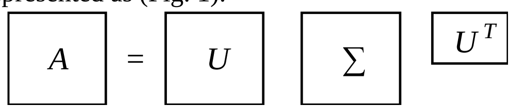

Fig. 1. *Matrix decomposition structure*

Columns U (or V respectively) are called left (right) singular vectors A, and σ_i — singular numbers. An important property of this decomposition is the fact that $rank(A) = r$ if and only if when $\sigma_r > 0, \sigma_{r+1} = ... = \sigma_p = 0$.

In case $\sigma_{r+1}, ..., \sigma_p$ are very small in comparison with $\sigma_1, ..., \sigma_r$, then for computational purposes it is assumed that the rank of the matrix A is equal to r, and it is said that A has an *effective rank* r. Applying decomposition (4) for system (3), we can obtain the solution sought in the form:

$$Y = V(m, 1:r)\Sigma^+(1:r, 1:r)U^T(1:r, n)F, \tag{5}$$

where Σ^+ is a diagonal matrix with diagonal elements $1/\sigma_1, 1/\sigma_2, ..., 1/\sigma_r$, or by replacing $C = \Sigma^+(1:r, 1:r)U^T(1:r, n)F$ we obtain:

$$Y = V_{m\times r}C_{r\times 1}. \tag{6}$$

It can be seen from relation (6) that we are looking for a solution to equation (1) in the so-called SVD basis, the basis functions of which are columns of a matrix $V_{m\times r}$ with rates from column *C*.

Building a corrective filter. The next task is to build an optimal filter for suppressing Gibbs phenomena, which can be observed in places where the sought-for function changes rapidly (jumps).

The idea behind filtering is as follows: for the signal decomposed on the basis $\{\varphi_k(x)\}_{k=1}^r$

$$y(x) \approx \sum_{k=1}^{r} c_k \varphi_k(x), \tag{7}$$

we look for the so-called *sigma factors* $\sigma(r,k)$ and multiply them by corresponding rates c_k, i. e.

$$\tilde{y}(x) \approx \sum_{k=1}^{r} \sigma(r,k) c_k \varphi_k(x), \tag{8}$$

suppressing high-frequency components of the signal in a way that improves the signal shape, reducing oscillations caused by the Gibbs phenomenon.

The Lanczos filter and $\left(\sigma(r,k) = \dfrac{\sin(\pi k / r)}{\pi k / r}\right)$ the Fejer filter $\left(\sigma(r,k) = \dfrac{r-k}{r}\right)$, which are widely known, are used for the Fourier's series [8]. To build a filter that is optimal for the SVD basis, let's consider a test case.

Let in equation (1) be as $K(x,s) = e^{-5(x-s)^2}$, $a = -1, b = 2, n = 100$, and the desired solution have the form:

$$y(x) = \begin{cases} 0, & x \notin (0,1), \\ 1, & x \in (0,1). \end{cases}$$

Having found the right side of equation (1), we will numerically recover the original signal (using the Matlab environment software modules). The results of programs can be seen in Fig. 2-4.

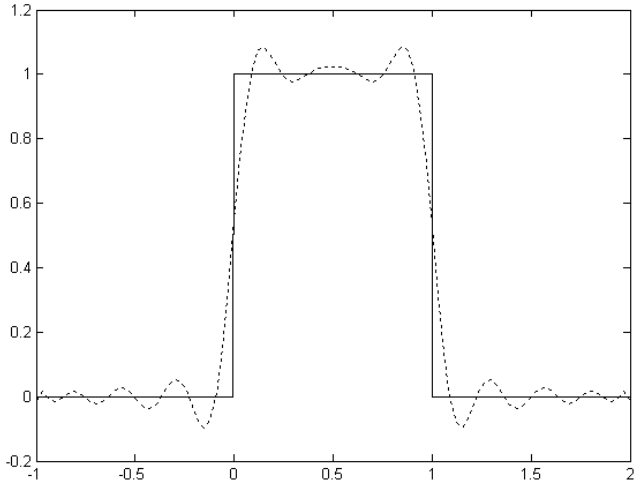

Fig. 2. *Original signal (—), the signal recovered using the Tikhonov regularization method (---)*

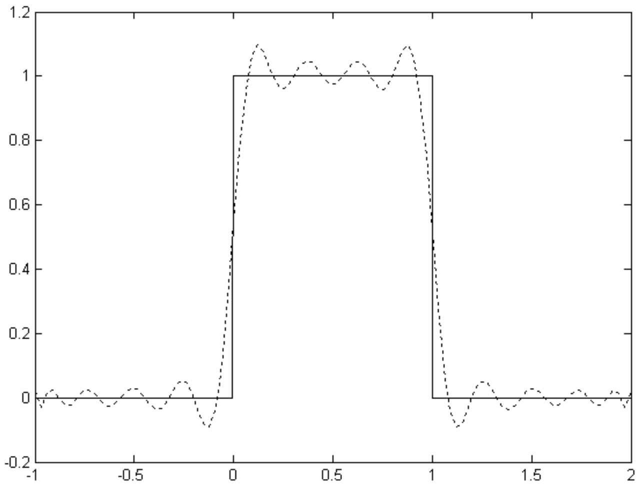

Fig. 3. *Original signal (—), the signal recovered using the SVD decomposition without filtering (---)*

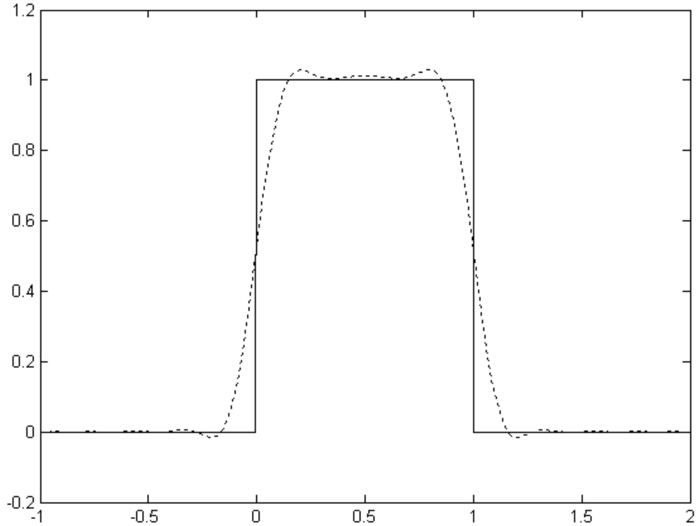

Fig. 4. *Original signal (—), the signal recovered using the SVD decomposition with Lanczos filter (---)*

As can be seen from Fig. 4, the Lanczos filter enables suppressing the Gibbs phenomena, but the shock front is recovered worse in this case, and it also does not provide reliable information about the jump signal height. We will look for a new filter in the form of the third degree linear function:

$$\sigma(r,k) = a_0 + a_1\left(\frac{k}{r}\right) + a_2\left(\frac{k}{r}\right)^2 + a_3\left(\frac{k}{r}\right)^3. \quad (9)$$

Let's build the functional:

$$W(a) = \sum_{j=1}^{23}\left(y(x_j) - \tilde{y}_a(x_j)\right)^2 + \\ + \sum_{j=43}^{58}\left(y(x_j) - \tilde{y}_a(x_j)\right)^2 + \sum_{j=78}^{100}\left(y(x_j) - \tilde{y}_a(x_j)\right)^2. \quad (10)$$

where $y(x)$ is the original signal, $\tilde{y}_a(x)$ is the signal recovered using the SVD decomposition and the applied filter, $a = \{a_0, a_1, a_2, a_3\}$.

To find ratios, $a_i, i = \overline{0, 3}$ we minimize the functionality (10), this can be done using the *fmincon* function built into the Matlab environment. The results were as follows:

$$a_0 \approx 0.990118; \; a_1 \approx 0.001268;$$

$$a_2 \approx -0.00329; \; a_3 \approx 0.000073.$$

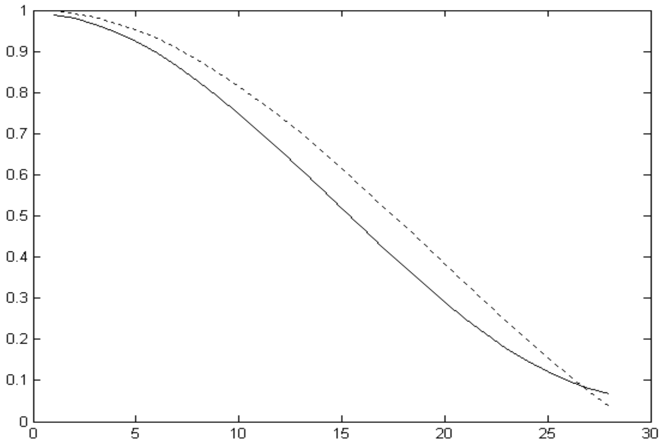

Fig. 5. *Lanczos filter (---) and the filter built for the test problem (—)*

As can be seen from Fig. 5, the filter formed, when compared with the Lanczos one, has better suppressed the Gibbs oscillations, but at the same time the jump front was restored less clearly. Applying the obtained filter for a large number of other problems' examples made it possible to find out that the solution, as in the test example, successfully recovers the original signal at a certain distance from the jump. This fact makes it possible, by finding the maximum derivative of the obtained solution (determining the place of the jump), to finish building the function in the intervals close to the jump, approximating the function on well-recovered intervals. The results of this operation can be seen in Fig. 6.

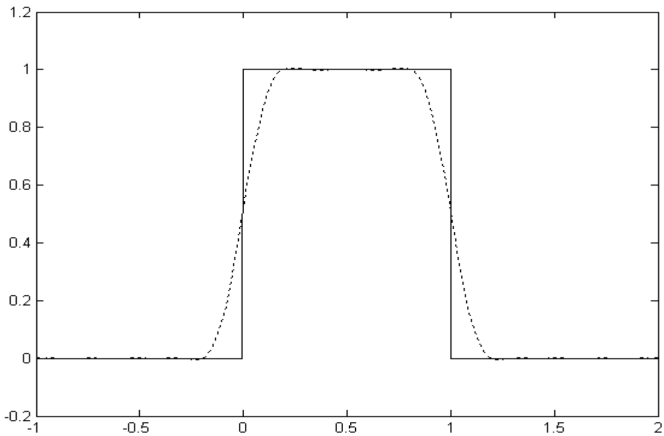

Fig. 6. *Original signal (—), the signal recovered by means of the SVD decomposition using the built filter (---)*

In this case, the mean-root square error turned out to be 0.01316, which is much less than the errors of the Tikhonov's method (0.972843) and the singular value decomposition method with Lanczos filtering (1.226638), which indicates the efficiency of the built algorithm for such classes of problems. Figure 7 shows the result of solving the problem of this type, which also confirms advantages of the considered method.

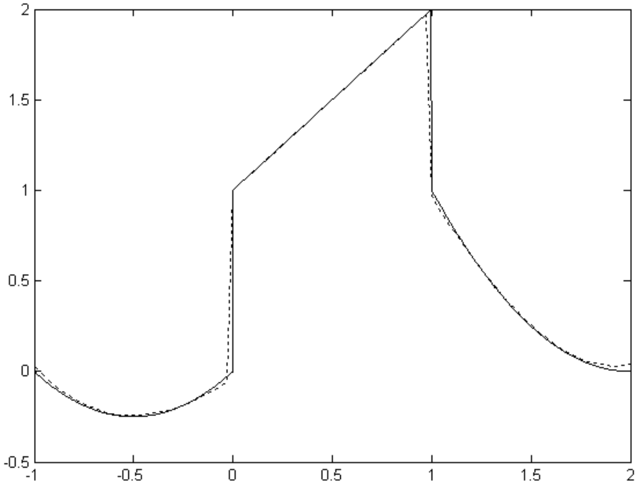

Fig. 7. *Original signal (—), recovered signal (---)*

Conclusions. A method has been developed to improve the accuracy and quality of recovering an energy object control system' input signals containing sharp jumps; which based on the use of optimized spectral filtering and apriori information. In comparison with widely known methods of filtering by Fejer and Lanczos, the proposed method makes it possible to more accurately recover signals outside the jump zones, as well as to extrapolate them within these zones.

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ЦИФРОВИЙ КОРЕКТУЮЧИЙ ФІЛЬТР У ЗАДАЧАХ ВІДНОВЛЕННЯ ВХІДНИХ СИГНАЛІВ І ДАНИХ СИСТЕМ СПОСТЕРЕЖЕННЯ ЕНЕРГЕТИЧНИХ ОБ’ЄКТІВ

Завдання відновлення сигналу є одним з найважливіших для автоматизованих систем діагностики та керування енергетичним об'єктом. При вирішенні зворотних завдань відновлення сигналів, зображень та інших видів даних мають місце спектральні спотворення та втрати (у деяких випадках дуже значні). Вони зумовлені насамперед некоректністю цих завдань, яка є результатом втрат інформації про вихідний сигнал внаслідок сильного (і навіть повного) придушення у спостережуваному сигналі частини спектральних компонентів, які стають нерозрізняними на тлі помилок та шумів. Крім того, у процесі вирішення завдань відновлення можуть виникати додаткові спектральні спотворення, які залежать від конкретних методів, що застосовуються, та їх параметрів. Запропоновано метод побудови цифрового коригувального фільтра для обробки результатів розв'язання некоректних обернених задач, що дозволяє ефективно підвищувати якість рішення. Метод заснований на застосуванні сингулярного (SVD) розкладання матриці системи рівнянь алгебри, що апроксимує інтегральний оператор.

Ключові слова: *обернені завдання, відновлення сигналів, цифровий фільтр, SVD-розкладання, інтегральні рівняння.*

Отримано: 8.10.2021