

Mixed-Integer Minimization of the Cost function of the Unit Commitment problem for Isolated power systems*

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Abstract

Unit Commitment (UC) is a minimization problem that aims to schedule the required generating units in a power system over some time horizon to meet the demand based on minimizing the production cost. In this paper, we present a novel technique to minimize such functions based on Mixed-integer formulation, neglecting the time horizon and most of the constraints. This technique can be considered as a first step in a better and tighter mixed-integer formulation of the unit commitment problem, especially for isolated power systems that contain a small number of generating units. Data from isolated power systems on marine vessels are used to test this technique. The proposed technique requires more constraints and binary variables. However, the numerical results presented in this work, show that the proposed method gives more efficient results for low demand, and close results to those obtained from local minimizers when the demand is high. The computational time of the suggested method does not seem to be explicitly longer than the time taken by the local minimizers, especially for small isolated power systems.

1. INTRODUCTION

The *Unit Commitment* problem (UC) is an important topic in power systems. UC can be defined as the problem of finding (scheduling) the optimal number of generating units that must be activated to meet the total demand in a power system [14], over some time horizon [7](usually 24 hours), to minimize the total

production cost [5]. Deciding the output power level of each committed unit in each time slot is referred to in literature as "Economic Dispatch" [14]. The cost function in such formulations is the sum of fuel consumption functions, the start-up, and shut-down costs of all generating units [5]. The fuel consumption functions are, usually, given by second [7] or third order polynomials [6], depending on the type of the thermal generation unit. To simplify the formulation, such cost functions are approximated by *Piece-Wise Linear* (PWL) functions [5], [10], and [2]. Besides, linear *Mixed-Integer* (MI) constraints, which involve both real and integer (or binary) variables, are used to describe the restrictions on the operation of the power systems. The constraints, usually, include [2]: *Spinning reserve, Power balance, Ramping, Minimum Up and Down Time*. The demand on the power system is assumed known for each time slot (usually one hour) in the planning horizon. The authors in [2], [5], [11], and [13] introduce different MI formulations and solutions of UC for on-land power systems. MI solvers such as **CPLEX**, **OSL**, and **XPRESS** are usually used (see [8] and the references there in), because ordinary local minimizers can not solve such problems.

The focus of this work is on isolated power systems as, for example, the power systems in marine vessels. Power systems vary according to their size and purposes. In general, power systems can be classified into large (*on-land*) power systems, and small (*isolated*) ones. The on-land systems comprise, usually, large number of generating units of many different types. While, the isolated systems contain much lower number of units of less different types [4]. On the other hand, the demand variation in isolated systems is, probably, much higher than that on the on-land systems [4]. To our best knowledge, discussing MI formulation of the UC for isolated power systems is rare in literature, e.g., the authors in [12], exploit the MI formulation to solve the UC problem on off-shore oil platforms.

In order to formulate a UC problem suitable for isolated power systems, the start-up and shut-down costs were

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neglected (Start-up cost was discussed in a previous work by the authors in [1]). The constraints that span the planning horizon were neglected. So, we focused on the minimization of the fuel consumption functions of the thermal units. Spurred by the fact that the number of units used in isolated systems is not large, and the demand variation can be wide, we present, in this work, a novel MI formulation of the minimization problem in UC, suitable for isolated power systems. This novel technique is based on approximating the fuel consumptions by PWL functions. Then, binary variables are assigned for each unit in each linear block of the cost function. Finally, by using logic-based inequalities, the optimal values can be obtained as will be shown in the sequel. The computations were executed by CPLEX. The results obtained are compared with the results obtained by an ordinary local minimizer. The comparison shows that the proposed technique gives better results when the demand on the system is not high. In addition, the time required for such computations can be considered insignificant.

In the following section, the main minimization problem is introduced. In the third section, solutions are introduced for the case of identical units. In the fourth section, the case of different sizes units is discussed. In section 5, numerical computations and comparisons are shown. In the last section, some conclusions are drawn.

2. Main Mathematical Formulation

To minimize the fuel consumption in thermal units for power systems on ships, the following cost function is usually used [6]:

$$\min_{p_i} \quad J = \sum_{i=1}^{I_G} F_{C_i}(p_i) \quad (1)$$

subject to:

$$\sum_{i=1}^{I_G} p_i = D \quad (2a)$$

$$\underline{P}_i \leq p_i \leq \bar{P}_i, \quad (2b)$$

where p_i denotes the output power level of unit i , D is the total demand, I_G is the number of the power generating units, and $\underline{P}_i, \bar{P}_i$ denote the minimum and maximum output power levels, respectively. Besides, $F_{C_i}(p_i)$ represent the instantaneous fuel consumption function given by[6]:

$$F_{C_i}(p_i) = b_{e_i}(p_i)p_i, \quad (3)$$

where $b_{e_i}(p_i)$ is the *brake specific fuel consumption* (BSFC) approximated usually by a polynomial, measured in g/KWh . The formulation in (1) is straightforward and intuitive, however, it does not drive the

scheduled power level p_i to be zero when there is no need for it.

To tackle this defect, the MI formulation is used for the same minimization problem, given by:

$$\min_{p_i, u_i} \quad J = \sum_{i=1}^{I_G} u_i F_{C_i}(p_i), \quad (4)$$

where u_i are binary variables to indicate the status of the units used, such that $u_i = 1$ if the unit is ON, and zero if it is OFF. We will first study the case of identical units and then go on to the case of different, or non-identical units.

3. IDENTICAL UNITS

In this section, we present a method to distribute the load optimally over N identical generating units. By this method, as will be shown, the units can be switched on or off, and their output levels can be determined, as well.

Because we are considering, in this section, identical units, the BSFC functions $b_{e_i}(p_i)$ are considered identical as well. So, let $b_e(p)$ denote the BSFC of any one of them. It is not difficult to prove that the optimal scheduled output power level is to distribute the load equally over the units when they are identical. However, if the demand can be provided by one unit, it would be more efficient to put all the demand on that unit, because the units are usually designed to be more efficient at full load than at partial load. This was motivated (see [4] and the references therein) based on the specifications of the medium-speed thermal unit that have been studied. Having said this, the minimization problem in (4) can be solved easily, because the optimal solutions are known.

3.1. Two Identical Units

First, we assume two thermal units only. If the demand (D) can be provided by one unit only, i.e., $D \leq \bar{P}$, the optimal solution can be described by the following if-statements:

$$[D \leq \bar{P}] \longrightarrow [u_2 = 0] \wedge [p_1^* = D] \wedge [p_2^* = 0] \quad (5a)$$

$$[D > \bar{P}] \longrightarrow [u_2 = 1] \wedge [p_1^* = p_2^* = D/2] \quad (5b)$$

$$[u_1 = 1] \text{ in both cases,}$$

where " \longrightarrow " and " \wedge " denote the logical **IF**, and **AND** operations. In [11], and [9] techniques to transform the logical statements (as in (5)) into linear inequalities are presented. To do that, we need to assume bounds on the demand D . Logically, in the design phase of power systems, the units used are chosen such that, $\underline{P} \leq D \leq 2\bar{P}$,

assuming a system comprising two identical thermal units only. Then, the if-statements (5) can be described by:

$$\begin{aligned} p_1 &= D - u_2 \frac{D}{2} \\ p_2 &= u_2 \frac{D}{2} \\ D - \bar{P} &\leq u_2 \bar{P} \\ D - \underline{P} &\geq u_2 (\bar{P} - \underline{P} + \varepsilon), \end{aligned} \quad (6)$$

where ε is an arbitrary positive small constant (For details on the procedure, consult [11]).

3.2. N Identical Units

In general, for N identical units, the if-statements (5) can be reformulated as follows:

$$[D \leq \bar{P}] \longrightarrow [u_2 = \dots = u_N = 0] \quad (7a)$$

$$\begin{aligned} [\bar{P} < D \leq 2\bar{P}] &\longrightarrow [u_2 = 1] \\ &\wedge [u_3 = \dots = u_N = 0] \end{aligned} \quad (7b)$$

⋮

$$[(n-1)\bar{P} < D \leq n\bar{P}] \longrightarrow [u_2 = \dots = u_N = 1]. \quad (7c)$$

Let $\delta_1, \dots, \delta_{N-1}$ be binary indicators to represent the following cases of the feasibility region:

$$[D \leq \bar{P}] \longleftrightarrow [\delta_1 = 1] \quad (8a)$$

$$[D \leq 2\bar{P}] \longleftrightarrow [\delta_2 = 1] \quad (8b)$$

⋮

$$[D \leq (N-1)\bar{P}] \longleftrightarrow [\delta_{N-1} = 1], \quad (8c)$$

subject to the conditions:

$$[\delta_1 = 1] \longrightarrow [\delta_{N-1} = \dots = \delta_2 = 1] \quad (9a)$$

$$[\delta_2 = 1] \longrightarrow [\delta_{N-1} = \dots = \delta_3 = 1] \quad (9b)$$

⋮

$$[\delta_{N-2} = 1] \longrightarrow [\delta_{n-1} = 1], \quad (9c)$$

where " \longleftrightarrow " denotes the logical operation if-and-only-if (**IFF**). Each if-statement in (8) can be described by a pair of linear inequalities in the form of [11]:

$$\begin{aligned} D - n\bar{P} &\leq (N-n)\bar{P}(1 - \delta_n) \\ D - n\bar{P} &\geq \varepsilon + (\underline{P} - n\bar{P} - \varepsilon)\delta_n, \\ &\forall n \in \{1, \dots, N-1\}, \end{aligned} \quad (10)$$

which is a generalization of the inequalities in (6). Whereas, each condition in (9) can be replaced by linear

inequalities, in the form [11]:

$$\begin{aligned} \delta_m + 1 - \delta_n &\geq 1 \\ \forall m \in \{n+1, \dots, N-1\}, \quad \forall n \in \{1, \dots, N-2\} \end{aligned} \quad (11)$$

Obviously, the generating units status indicators u_1, \dots, u_n can be determined from the indicators $\delta_1, \dots, \delta_{n-1}$ as:

$$\begin{aligned} u_1 &= 1 \\ u_n &= 1 - \delta_{n-1} \quad \forall n \in \{2, \dots, N\}. \end{aligned} \quad (12)$$

Finally, the optimal output power levels of each generating unit can be determined by using the unit status indicators as:

$$\begin{aligned} p_n &= u_n \frac{D}{n} - \sum_{m=n+1}^N u_m \frac{D}{m(m-1)} \\ \forall m \in \{n+1, \dots, N\}, \quad \forall n \in \{1, \dots, N-1\} \\ p_N &= u_N \frac{D}{N}. \end{aligned} \quad (13)$$

Actually, the constraints in (10) and (11), together with the relations in (12) and (13), describe the solution of the minimization problem in (4) subject to the constraints in (2). To elaborate, the optimal power levels are determined in advance, because the units are identical and there is no need to solve the problem itself. It is worth mentioning here, that the number of constraints can be shown to be equal to $\frac{N}{2}(N+5) + 1$. Thus, as mentioned before, the small number of generating units in isolated power systems motivates us to consider such a method.

4. DIFFERENT UNITS

In this section, we study different generating units in two cases. In the first case, K different units are considered. A method is presented to distribute the load optimally over the units, allowing switching the units on or off. In the second, sets or groups of different types of units are considered. For this case, two approaches are suggested.

4.1. Different Single Units

Assuming we have K different generating units, the minimization problem in (4) can be modified to:

$$\min_{p_k} J = \sum_{k=1}^K u_k F_{C_k}(p_k), \quad (14)$$

subject to the same constraints as in (2). The function $F_{C_k}(p_k)$, defined in (3), can be approximated by a polynomial of the third degree as mentioned earlier.

This problem is not easy to solve, not only because of the non-linearity of the fuel consumption function, but due to the integer variables as well. Such problems belong to the class of *Mixed-Integer Nonlinear Programming* (MINLP). The authors in [8], describe different algorithms and methods to attack such problems, such as [8]: *Branch and Bound* (BB), *Outer Approximation* (OA), and *Extended Cutting Planes* (ECP). In spite of the availability of different codes to solve MINLP, one would rather try to approximate the formulation by a linear function to reduce the complexity and computational time, especially, when there are many constraints and binary variables. In UC problems, the fuel consumption function can be approximated by a piece-wise linear function [2], [5], and [12]. Let the piece-wise linear approximation of F_{C_k} be denoted by \tilde{F}_{C_k} , from here on. Of course, the higher the number of intervals chosen to split the domain of $F_{C_k}(p_k)$ are, the closer the approximation \tilde{F}_{C_k} gets to F_{C_k} . Let the domain $[\underline{p}_k, \bar{p}_k]$ of $F_{C_k}(p_k)$ be partitioned into L intervals as $\{[\underline{p}_k, p_k^{c,1}] \vee \dots \vee [p_k^{c,L-1}, \bar{p}_k]\}$, where $p_k^{c,1}, \dots, p_k^{c,L-1}$ are the break points, and " \vee " is the logical operation **OR**. The number of the line segments L can be chosen arbitrarily. However, the authors in [3], prove that the optimal number of line segments L used to realize an arithmetic function $f(x)$ defined on the interval $[a, b]$ by a PWL function $\tilde{f}(x)$ by using a numeric function generator, is proportional to [3]:

$$\frac{1}{4\sqrt{\sigma}} \int_a^b \sqrt{|f''(x)|}, \quad (15)$$

where $f''(x)$ is the second derivative of $f(x)$, and σ is the desired approximation error, such that [3]:

$$|f(x) - \tilde{f}(x)| \leq \sigma, \quad \forall x \in [a, b].$$

The new piece-wise linear function can be described by:

$$\tilde{F}_{C_k}(p_k) = \begin{cases} m_k^1 p_k + d_k^1, & \underline{p}_k \leq p_k \leq p_k^{c,1} \\ \vdots \\ m_k^L p_k + d_k^L, & p_k^{c,L-1} \leq p_k \leq \bar{p}_k, \end{cases} \quad (16)$$

where m_k^1, \dots, m_k^L , and d_k^1, \dots, d_k^L are the parameters of the line segments. Different from the UC formulation in [2], [5], and [12], we aim to tighten the formulation by using logic-based methods [9] to transform the non-linearities into binary variables with linear inequalities. Now let $\gamma_k^1, \dots, \gamma_k^{L-1}$ be binary variables assigned for unit k , that indicate whether p_k lies in the l th interval or not.

These binary indicators can be described by:

$$[p_k \leq p_k^{c,1}] \longleftrightarrow [\gamma_k^1 = 1] \quad (17a)$$

$$[p_k \leq p_k^{c,2}] \longleftrightarrow [\gamma_k^2 = 1] \quad (17b)$$

\vdots

$$[p_k \leq p_k^{c,L-1}] \longleftrightarrow [\gamma_k^{L-1} = 1], \quad (17c)$$

and the following conditions, as well [11]:

$$[\gamma_k^1 = 1] \longrightarrow [\gamma_k^2 = \dots = \gamma_k^{L-1} = 1] \quad (18a)$$

$$[\gamma_k^2 = 1] \longrightarrow [\gamma_k^3 = \dots = \gamma_k^{L-1} = 1] \quad (18b)$$

\vdots

$$[\gamma_k^{L-2} = 1] \longrightarrow [\gamma_k^{L-1} = 1]. \quad (18c)$$

Each if-statement in (17) can be represented by a pair of linear inequalities as:

$$\begin{aligned} p_k - p_k^{c,l} &\leq (\bar{p}_k - p_k^{c,l})(1 - \gamma_k^l) \\ p_k - p_k^{c,l} &\geq \varepsilon + (\underline{p}_k - p_k^{c,l} - \varepsilon)\gamma_k^l \\ \forall l &\in \{1, \dots, L-1\}, \quad \forall k \in \{1, \dots, K\}. \end{aligned} \quad (19)$$

where ε is an arbitrary positive number, as before. Whereas, each condition in (18) can be replaced by linear inequalities as:

$$\begin{aligned} \gamma_k^m + 1 - \gamma_k^l &\geq 1 \\ \forall m &\in \{l+1, \dots, L-1\}, \quad \forall l \in \{1, \dots, L-2\} \\ \forall k &\in \{1, \dots, K\}. \end{aligned} \quad (20)$$

Then, $\tilde{F}_{C_k}(p_k)$ can be described, by using the binary indicators $\gamma_k^1, \dots, \gamma_k^{L-1}$, as:

$$\begin{aligned} \tilde{F}_{C_k}(p_k) &= \sum_{l=1}^{L-1} (m_k^l - m_k^{l+1}) p_k \gamma_k^l + m_k^L p_k \\ &+ \sum_{l=1}^{L-1} (d_k^l - d_k^{l+1}) \gamma_k^l + d_k^L. \end{aligned} \quad (21)$$

Taking one term of the proposed cost function in (14) after replacing the function $F_{C_k}(p_k)$ by $\tilde{F}_{C_k}(p_k)$, we get $u_i \tilde{F}_{C_k}(p_k)$. This term can be expressed from (21) as:

$$\begin{aligned} u_i \tilde{F}_{C_k}(p_k) &= \sum_{l=1}^{L-1} (m_k^l - m_k^{l+1}) p_k \gamma_k^l u_k + m_k^L p_k u_k \\ &+ \sum_{l=1}^{L-1} (d_k^l - d_k^{l+1}) \gamma_k^l u_k + d_k^L u_k. \end{aligned} \quad (22)$$

The terms in (22) with more than one binary variable must be simplified. The product of two binary variables is equivalent to the logical **AND**. The authors in [11]

suggests replacing the product of two binary variables as $\gamma_k^l u_k$ with another binary variable, and let it be $\gamma_{u_k}^l$, with the following constraints:

$$\begin{aligned} -\gamma_k^l + \gamma_{u_k}^l &\leq 0 \\ -u_k + \gamma_{u_k}^l &\leq 0 \\ \gamma_k^l + u_k - \gamma_{u_k}^l &\leq 1. \end{aligned} \quad (23)$$

Moreover, the product of a function and a binary variable, as $p_k u_k$, can be replaced by another variable, and let it be p_{u_k} , with the following constraints:

$$\begin{aligned} \underline{P}_k u_k &\leq p_{u_k} \leq \bar{P}_k u_k \\ p_k - \bar{P}_k(1 - u_k) &\leq p_{u_k} \leq p_k - \underline{P}_k(1 - u_k). \end{aligned} \quad (24)$$

Finally, the product of three quantities, as $p_k \gamma_k^l u_k$, is equivalent to $p_k \gamma_{u_k}^l$, which can also be replaced by another variable, let it be $p_{(\gamma^l, u)_k}$, with the following constraints:

$$\begin{aligned} \underline{P}_k \gamma_{u_k}^l &\leq p_{(\gamma^l, u)_k} \leq \bar{P}_k \gamma_{u_k}^l \\ p - \bar{P}_k(1 - \gamma_{u_k}^l) &\leq p_{(\gamma^l, u)_k} \leq p - \underline{P}_k(1 - \gamma_{u_k}^l). \end{aligned} \quad (25)$$

Inserting these new variables in (22), we get:

$$\begin{aligned} u_k \tilde{F}_{C_k}(p_k) &= \sum_{l=1}^{L-1} (m_k^l - m_k^{l+1}) p_{(\gamma^l, u)_k} + m_k^L p_{u_k} \\ &\quad + \sum_{l=1}^{L-1} (d_k^l - d_k^{l+1}) \gamma_{u_k}^l + d_k^L u_k. \end{aligned} \quad (26)$$

Hence, the final formulation of the minimization problem can be stated as:

$$\min_{p_{u_k}, u_k} J = \sum_{k=1}^K u_k \tilde{F}_{C_k}(p_k) \quad (27)$$

subject to the constraints in (19), (20), (23), (24), and (25). Note that the decision variables of this problem-in which we are interested- comprise u_k and p_{u_k} . So, the constraint in (2a) must be modified to:

$$\sum_{k=1}^K p_{u_k} = D, \quad (28)$$

and added to the set of the constraints of the final problem in (27).

The increase of the number of the binary variables and constraints, probably, adds to the complexity of the MI problem. By this approach, the numbers of the additional binary variables γ_k^l , $\gamma_{u_k}^l$, $p_{(\gamma^l, u)_k}$, and p_{u_k} required are $L-1$, $L-1$, $L-1$, and 1, respectively, for each unit k , assuming L is the same for all units. So, one needs

$3L-2$ binary variables for each unit. While, the numbers of constraints in (19), (20), (23), (24), and (25) are $2(L-1)$, $\frac{(L-2)(L-1)}{2}$, $3(L-1)$, 4, and $4(L-1)$, respectively, which add up to $(L-1)(\frac{L}{2} + 4) + 4 = \frac{L}{2}(L+7)$, for each unit. Increasing L will increase the accuracy of the results. On the other hand, increasing L will increase the number of the integer variables and constraints required, this is a trade-off one has to make. Since, as mentioned earlier, the number of units used is not so large, the number of constraints would not be a problem.

4.2. Sets of Different Units

In most power systems on marine vessels, there are sets of 2-4 different types of units, only. Let us assume that we have K types of units. Let the number of units used of each type be N_1, \dots, N_K . Let also the cost function of the n th unit of type k be $F_{C_n}^k(p_n^k)$, where p_n^k is the power generated by n th unit of type k , $\forall n \in \{1, \dots, N_k\}$, $\forall k \in \{1, \dots, K\}$. Then, we know that $F_{C_n}^k$ is the same $\forall n \in \{1, \dots, N_k\}$. In this work, we suggest two approaches to minimize the fuel consumption in this case:

1. Direct Approach.

2. Grouping Approach.

In the first approach, the fuel consumption functions $F_{C_n}^k(p_n^k)$ are minimized, as was done before, by the formulation in (27). However, this approach may require too many constraints and binary variables. Hence, in the second approach, both the strategy presented for different single units, and that for N identical units can be exploited together. The idea, we propose here, is to treat all units of the same type as one unit with one BSFC, find the optimal load sharing from (27) for each type k , and then distribute load over the identical n_k units. To elaborate, let the the minimization problem be formulated as:

$$\min_{p_{u,k}, \mathbf{u}_k} \mathbf{J} = \sum_{k=1}^K \mathbf{u}_k \tilde{\mathbf{F}}_{C_k}(p_k), \quad (29)$$

subject to the constraints in (19), (20), (23), (24), and (25). $\tilde{\mathbf{F}}_{C_k}$ is the piece-wise linear approximation of \mathbf{F}_{C_k} , which is given by:

$$\mathbf{F}_{C_k} = N_k F_{C_n}^k, \quad \forall n \in \{1, \dots, N_k\}, \quad \forall k \in \{1, \dots, K\}. \quad (30)$$

Furthermore, \mathbf{u}_k is the binary variable that indicates whether all of the N_k units of type k are ON or not. The reason why the cost function is formulated as in (30) is that this function, as seen from (3), results from multiplying the BSFC by the the output power level. The

constraints of the problem in (29) will be also (19), (20), (24), (23), and (25), after replacing each u with \mathbf{u} . The demand constraint in (28), however, has to be changed into:

$$\sum_{k=1}^K N_k p_{\mathbf{u}_k} = D, \quad (31)$$

where, $p_{\mathbf{u}_k}$ is the output level of each unit of type k . The outcome of the minimization problem in (29) will be the decision variables $p_{\mathbf{u}_k}$, and \mathbf{u}_k . To distribute the load on the identical N_k units, it suffices to add the constraints in (10), (11), (12) and (13) repeated K times. Each time the demand D , and number of identical units N are replaced with $N_k p_{\mathbf{u}_k}$, and N_k , respectively. Moreover, the first binary indicator u_1 of the first unit of the set of the N_k units of type k is set to \mathbf{u}_k , in (12), instead of 1.

As mentioned before, the number of the constraints required to distribute the load over N identical units is $\frac{N}{2}(N+5)+1$. While the number of the constraints needed to distribute the load over the different units is $(L-1)(\frac{L}{2}+4)+4$, for each unit, assuming same number of intervals is used for all fuel consumption functions. Thus, for the minimization problem in (29), with K different types of units, and N_k identical units of each type, the total number of constraints N_{con} required can be determined by:

$$N_{con} = \sum_{k=1}^K [(L_k - 1)(\frac{L_k}{2} + 4) + \frac{N_k}{2}(N_k + 5) + 5], \quad (32)$$

where L_k is the number of line segments used to discretize the cost function \mathbf{F}_{C_k} .

5. NUMERICAL RESULTS

In this section, numerical solutions for the optimization problem are presented in different cases. Three different types of generating units were assumed. The BSFC function of each one is assumed to be a quadratic function of the form $ap^2 + bp + c$. The approximation error σ used for the PWL approximation was assumed to be $0.4Kg/KWh$. The coefficients of the BSFC, the maximum and minimum allowable power levels of each unit are listed in Table 1 [4]. The numbers of segments, as obtained from (15), of the PWL approximation of the units are also listed in Table 1. Two examples were considered:

1. **Example I:** When there are one unit only of each type.
2. **Example II:** When there are 3 units of Type I, 4 units of Type II, and 2 Units of Type III.

From Table 1, we can see that the total capacity of the power systems assumed in example I and example II is

Table 1: The specifications of the generation units used in numerical solutions [4]

	Type I	Type II	Type III
$\bar{P}[KW]$	3300	2200	1100
$\underline{P}[KW]$	600	400	200
$a[g/KW^3h] \times 10^{-4}$	0.23406	0.52662	2.1065
$b[g/KW^2h]$	-0.1035	-0.1553	-0.3105
$c[g/KWh]$	298.015	298.015	298.015
L	10	8	5

6600KW and 20900KW, respectively.

The first example was solved twice. Once, the MI problem in (27) subject to the constraints in (19), (20), (23), (24), (25), and (28) was solved by using IBM ILOG CPLEX Optimization Studio V12.5. While, the second solution was obtained by applying the local minimizer *fmincon* in MATLAB for the problem in (1) subject to the constraints in (2) for comparison. The results are shown in Table 2. Obviously, the local minimizer does not drive the units to OFF state. Definitely, the optimum power levels with the MI formulation make the total cost less, when the demand is low ($< 3300KW$ approximately). While, when the demand increases, the local minimizer gives slightly more efficient results and less total cost function. The differences in the optimal output power levels and total cost between the two formulations, when the demand increases, are due to approximating the fuel consumption function $F_{C_i}(p_i)$ by PWL. Certainly, if the number of the line segments is increased (the approximation error σ decreased), closer results will be obtained, but this will be at the expense of the number of binary variables and constraints, and hence, the complexity of the problem. Besides, the computational time of the MI formulation can be considered negligible.

In the second example, the minimization problem was solved three times. The first is by the local minimizer. The second is by the MI formulation by the direct approach in (27). And the third is by the MI formulation by the grouping approach in (29) subject to the constraints in (10), (11), (12),(13), (19), (20), (23), (24), and (25). The functions \mathbf{F}_{C_1} , \mathbf{F}_{C_2} , and \mathbf{F}_{C_3} were approximated by PWL with $\sigma = 0.4Kg/KWh$, also. The number of segments, obtained from (15), for them were 21, 20, and 9, respectively. The solutions were obtained for three values of the demand only, and the results are shown in Table 3. As can be seen from Table 3, MI formulation by direct approach gives the more efficient results when the demand is low ($\leq 10000KW$), as re-

Table 2: The numerical results of Example I by using MI, and a local minimizer.

D [KW]	Proposed MI minimization							Local minimizer					
	u_1	u_2	u_3	$p_{u,1}$ [KW]	$p_{u,2}$ [KW]	$p_{u,3}$ [KW]	J^* [Kg/h]	Time [sec]	p_1 [KW]	p_2 [KW]	p_3 [KW]	J^* [Kg/h]	Time [sec]
1000	0	1	0	0	1000	0	195.4	0.8	533.3	333.3	133.3	251.8	~
2000	1	0	0	2000	0	0	369.7	0.8	600	836.5	563.5	425.1	~
3000	1	0	1	2260	0	740	551.4	0.6	1678	1122	200	581.4	~
4000	1	1	0	2475	1525	0	738.7	0.8	2000	1334	666	738.5	~
5000	1	1	1	2490	1750	760	930.1	0.8	2500	1667	833	928.7	~
6000	1	1	1	3030	1975	995	1192	0.7	3000	2000	1000	1189	~

Table 3: The numerical results of Example II by using mixed-integer, and a local minimizer.

D	MI-Direct Approach			MI-Grouping Approach			Local minimizer		
	5000	10000	15000	5000	10000	15000	5000	10000	15000
p_1 [KW]	2040	2220	2490	600	2600	2395.5	600	1692.33	2367.8
p_2 [KW]	0	2220	2440	0	2600	2395.5	600	1692.33	2367.8
p_3 [KW]	2220	2220	2490	0	0	2395.5	600	1692.33	2367.8
p_4 [KW]	0	1300	1525	2100	1533.3	2093.3	411.75	1130.75	1579.5
p_5 [KW]	0	1300	1525	2100	1533.3	2093.3	411.75	1130.75	1579.5
p_6 [KW]	0	0	1525	0	1533.3	2093.3	411.75	1130.75	1579.5
p_7 [KW]	0	0	1525	200	0	0	411.75	1130.75	1579.5
p_8 [KW]	740	0	740	0	200	766.7	776.5	200	789.3
p_9 [KW]	0	740	740	0	0	766.7	776.5	200	789.3
J^* [Kg/h]	919.9	1839.8	2767.1	1031.5	1906.2	2762.9	1125.7	1920.0	2762.1
Time [sec]	2.4	2.0	1.9	1.6	1.5	1.5	~	~	~
Constraints	808			895			10		
Variables	217			184			9		

marked from Example I. When the demand is higher, the difference in the optimal solutions obtained by the local minimizer and MI by direct approach can be considered insignificant. One would see, as well, that the MI formulation by the grouping approach gives less efficient (but still satisfactory) results than those obtained from the MI formulation by the direct approach but with fewer variables (binary and continuous). The computational time is worst for the MI formulation with the direct approach due to the utmost number of (binary) variables. However, it still can be considered small.

6. CONCLUSION

In this paper, we presented a tight MI formulation of the minimization problem suitable for the UC of the isolated power systems. This MI formulation is based

on PWL approximation of the cost function. Binary variables are introduced to indicate the status of each unit, and in each linear block. Then, logic-based inequalities were used as constraints to solve the minimization problem. Three methods were presented based on this technique. One for N identical units, another for K different units, and the last for groups of different units. The numerical results showed that the proposed MI formulation is more efficient than local minimizers, especially, when the demand on the system is low. The grouping approach proposed for sets of different units, can be used to give satisfactory results with less number of variables. Furthermore, the computational time of the proposed technique is small. Thus, the proposed MI formulations are more adequate for isolated power systems with small number of generating units and wide variations in the demand.

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