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Doctoral thesis

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Abstract

The ongoing transition to a more sustainable power system introduces challenges, but also opportunities. From the perspective of owners of hydropower assets, the rapidly transforming power sector gives rise to complex large-scale stochastic optimization problems that need to be formalized and solved. The aim of this thesis is to develop optimization models and methods for sustainable investment and operations in hydropower plants.

Investment and operational planning in hydropower plants is affected by several uncertain factors and numerous physical constraints. Various aspects are considered in the papers of this thesis, and each paper contributes with novel insights. In Paper I, we study maintenance and renewal of hydropower facilities under price uncertainty. In Paper II, we study operational planning under a joint model for the evolution of local inflow, availability of resources, i.e. system reservoir levels, and market prices. Paper III investigates the impact of short-term operational flexibility on long-term upgrade decisions. Finally, Paper IV addresses long-term risks associated with renewal and upgrading of existing hydropower facilities under limited long-term market information.

To represent, solve, and analyse the different aspects considered in the papers, we apply well-known tools from operations research and finance. In particular, we develop stochastic models for the evolution of exogenous factors which governs decision making. Moreover, we formulate the planning problems as Markov decision processes (MDPs), which is a framework for sequential decision making under uncertainty. Finally, based on structural properties of the MDPs, we employ real options analysis to obtain closed-form solutions, or approximate dynamic programming (ADP) for solving the MDPs near-optimally. To assess the quality of ADP solutions, we compute bounds for the optimal MDP value using established theory. Overall, the papers in this thesis contribute to the areas of finance and operations research by developing models, algorithms, and theory for solving large-scale optimization problems, with a specific focus on hydropower applications.

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Contents

Chapter 1

Introduction

In this chapter, we discuss business challenges and provide motivation for the research. Section 1.1 briefly explains the role of hydropower in electricity markets and introduces the business challenges. Section 1.2 presents the aims and objectives of the thesis. Section 1.3 provides additional information about the PhD.

1.1 Motivation and business challenges

Hydropower plants have generated clean and affordable electricity for many decades and are essential to a carbon-free and reliable electricity system. As the European Union and countries around the globe commit to net zero carbon emissions by 2050 (REN21, 2021), intermittent renewable energy sources, such as wind and solar, are expected to take an increasing part of power supply systems. Hydropower plants provide flexibility and storage to support the integration of a higher share of non-dispatchable renewable energy sources needed to meet climate goals. However, many hydropower plants rely on old technology from the large-scale hydropower projects in the mid-20th century (IRENA, 2015; EIA, 2017), as seen in Figure 1.1. The figure shows the distribution of hydropower capacity in Norway and the United States by the initial operating year. In addition to an aging hydropower fleet, most of the economically viable hydropower potential in developed regions, such as Europe, Canada, and the United States, is already exploited. Upgrading existing hydropower plants may therefore be more profitable than building new ones. Thus, the need for modernization of hydropower plants is growing.

Investment in renewal and upgrading projects of hydropower facilities is costly. To support these decisions, accurate and reliable mathematical models and methods for the calculation of the operational revenue associated with investment opportunities are crucial. Moreover, economic losses associated with unplanned maintenance can be substantial. To avoid the consequences of unplanned maintenance, owners of hydropower facilities often have several performance-enhancing activities under consideration. This may include tasks for improving existing assets

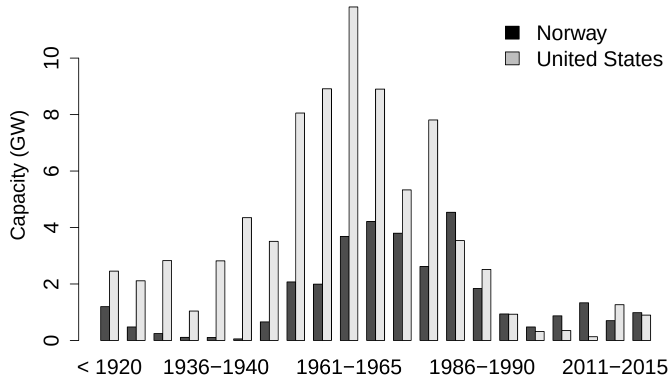

Figure 1.1: Hydropower capacity by initial operation year in United States and Norway. Data is provided by the U.S. Energy Information Administration (EIA, 2021) and the Norwegian Water Resources and Energy Directorate (NVE, 2021).

to extend the useful economic life, e.g. through surface treatment and hard coating of existing turbines, or renewal and upgrades where old components get replaced with new ones, possibly having higher efficiency or capacity (Goldberg and Lier, 2011).

In general, investment projects and plant operations interact and should therefore be coordinated. E.g. if renewal entails increased production capacity, a higher cash flow may be attainable by changing the operational pattern. Similarly, if production assets are partially damaged, the producer can deviate from the prevailing operating pattern to mitigate the damaging effects. From a quantitative business analysis point of view, in a competitive market-based setting, owners of hydropower assets typically aim at finding a coordinated schedule that maximizes the market value of existing and potentially new assets (Wallace and Fleten, 2003). This includes the estimation of the inherent flexibility to dynamically take profit-maximizing decisions in response to arriving exogenous information. Such information may be market-related, i.e. electricity prices, or firm-specific, e.g. inflows to reservoirs and signals about the technical state of plant components. Moreover, there are economic risks associated with undertaking large investment projects in hydropower, which are difficult to quantify. This is partly because the projects are irreversible and typically expected to last far beyond the horizon for which there exist electricity derivatives for controlling undesired risks.

1.2 Aims and objectives

This thesis aims at imparting knowledge about sustainable investment and operational planning in hydropower. Questions that we address include:

- How to coordinate the planning of a portfolio of possible performance-enhancing activities for deteriorating production assets under price uncertainty?

- Can the seasonal production schedule of a hydropower producer be improved by explicitly taking into account the co-dynamics between the level of available resources and prices in electricity markets dominated by hydropower?

- What is the value of short-term hydropower flexibility and how does it affect long-term capacity installments?

- How to make decisions regarding plant renewal and capacity investment under limited long-term information about market prices?

The thesis consists of four papers. The papers consider different aspects and aim at providing insights to the questions above. Paper I considers an hydropower operator of machinery with deteriorating performance, having the choice between performance-enhancing activities of different scales under price uncertainty. Paper II examines whether the production schedule of a local producer can be improved by taking into account that system prices are negatively correlated with the level of available resources in the system as a whole, which again is partly determined by the local producer's resources. Paper III studies capacity investment in hydropower and investigates how long-term investment decisions depend on medium-term capacity allocation and short-term capacity bids that respond to short-term price fluctuations. Paper IV proposes a framework for decision making for hydropower operators under limited long-term market information.

1.3 Project context

This PhD is a part of the research center HydroCen funded by the Research Council of Norway. The main objective of HydroCen is to enable the Norwegian hydropower sector to meet challenges and discover opportunities in the constantly evolving energy system. HydroCen consists of four work packages and the research areas include hydropower structures, turbine and generators, market and services, and environmental design. Within the HydroCen context, the main objective of the PhD is to develop models and methods for the calculation of future revenues for hydropower to support decisions for optimal investment in upgrading and expansion projects.

The remainder of this thesis is organized as follows: Chapter 2 provides the necessary background material. We provide a reference formulation that includes important aspects of the business problem and a description of the methods applied in the papers. Chapter 3 summarizes the papers and outlines their contributions. Chapter 4 contains concluding remarks. The four papers that constitute the thesis, Papers I-IV, then follows.

Chapter 2

Background

This chapter presents relevant academic literature and discusses modeling considerations and methodologies. We first present an overview over important features that affect hydropower planning and how these features can be modeled and calibrated. We then present a mathematical representation of the decision problem of coordinating investment and operations. This representation is meant to serve as a reference that accommodates all elements from the papers in this thesis. Finally, we present and discuss possible solution approaches.

2.1 Modeling uncertainty

There are several uncertain factors that affect hydropower planning. Table 2.1 gives an overview of important uncertain factors categorized into marked-related factors and firm-specific factors. Moreover, the sources of uncertainty are classified into short-term and long-term uncertainty. What separates long-term and short-term uncertainty is not well defined. However, changes in long-term factors are typically changes that are expected to last, e.g. market expectations of technology development, consumption growth, climate, etc. Conversely, changes in short-term factors are typically temporary, e.g. this year's total inflow does not tell much about the next year's inflow. The uncertain factors need to be represented appropriately to be applicable for sequential decision making. Typically, this involves the specification of a stochastic process and the estimation of its parameters from an empirical sample.

In the long run, there is uncertainty about the market structure, fuel prices of alternative power generation sources, consumption growth, technology innovation for power production assets in general, climate change, and political and regulatory effects, among other things. Expectations of these aspects can be represented by a geometric Brownian motion (GBM) with drift, which is a common model for the evolution of long-run energy prices (Pindyck, 1999). Price deviations from the long-run price expectation typically stem from short-term changes in demand which may be caused by intermittent supply disruptions and variations in

Table 2.1: Uncertain factors that affect hydropower planning.

	Market-related	**Firm-specific**
Short-term	Short-term changes in demand Intermittent supply disruptions Weather variations	Inflow to reservoirs Unplanned maintenance
Long-term	Market structure Fuel prices Consumption growth Interest rates Political and regulatory effects Climate change Technology developments	Investment costs Climate change Technology improvements Efficiency

weather. These effects can be alleviated by market participants reacting to changing conditions by modifying their inventory or reservoir levels. Therefore, such price deviations are expected to revert to the long-term price expectation, which makes a geometric Ornstein-Uhlenbeck process suitable for modeling short-term deviations (Schwartz and Smith, 2000). More sophisticated models for energy prices that better comply with empirical evidence exist, see e.g. Gambaro and Secomandi (2021).

In addition to market-related uncertainties, the hydropower producer faces firm-specific uncertainties, such as the inflow at the producer's location. Inflows often possess strong seasonal variations, serial correlations, and positive skewness (Prékopa and Szántai, 1978; Stedinger, 1980). However, inflow characteristics are also strongly dependent on the location where the producer is located. Other important firm-specific uncertain factors are the efficiency and risk of failure and unplanned maintenance. Both these aspects are typically not directly observed, but can be measured occasionally. Measurement data for the state of production assets is often scarce, which makes the specification and calibration of a model difficult. Nevertheless, some general characteristics apply, such as deterioration with time if no performance-enhancing activities occur. Moreover, the deterioration is often slow, but the impact in the event of failure is substantial.

In the papers of this thesis, we specify the uncertain factors as follows: All papers take a business perspective in restructured markets, assuming that firms are price-takers, which means that the price is exogenously given and that individual firms can not influence the market price by their decisions. In Paper I we apply the GBM for long-term price dynamics. In Papers III and IV we use the two-factor model by Schwartz and Smith

(2000) for the evolution of prices. In III and IV we specify a Gaussian process for inflow, similar to Gjelsvik et al. (2010), and also commonly used by practitioners. The choice of model is based on assessment of model fit to the local inflow data sets. The inflow model accounts for serial correlation by specifying a first order autoregressive process (AR-1) for normalized inflow. In Paper II we propose a modification of the two-factor price model in Schwartz and Smith (2000) that allows for additive price movements, and time-dependent cross-dependency with the resource availability of market participants. Our proposed model consists of a combination of linear time series models, including AR-1 processes, linear regression with AR-1 error structure, and exponential smoothing. We refer to Makridakis et al. (1998) for a survey on time series models. Regarding efficiency deterioration, we specify a deterministic function of time in Paper I.

2.1.1 Model calibration and risk-neutral valuation

After a stochastic process for the uncertain factors has been specified, its parameters need to be estimated from real data. We calibrate the price model to futures on electricity. By doing so, we can apply the risk neutral valuation approach (RN). This approach is referred to as *contingent claims analysis* in Dixit and Pindyck (1994). In a multistage setting, assuming that the markets are complete and arbitrage free, assets can be valued by replicating return and risk characteristics through dynamic trading in the underlying risky asset and a risk-free asset (Duffie, 2010). Considering financial derivatives, essentially this means that there exists a unique arbitrage-free price of a derivative that is the expected discounted payoff under the risk-neutral probability measure (Staum, 2007). This is sometimes also referred to as the equivalent martingale measure. Under this measure, the expected return on derivatives equal the risk-free interest rate. For real options applications in energy, assets can be valued similarly, where futures on the energy source are considered as the underlying. The asset market value can then be estimated by taking the expected sum (over the asset horizon) of a risky cash flow under a distribution where futures price dynamics are martingales, discounted at the risk-free interest rate (Nadarajah and Secomandi, 2021).

In reality, markets are incomplete, which means that some payoffs cannot be replicated by dynamic trading. In particular, managing an operational asset may depend on risk factors that are not traded, e.g. firm-specific inflow and failure risk. Mathematically, if markets are incomplete, then there exists a set of equivalent martingale measures for a given statistical measure of exogenous factors, as opposed to a unique risk-neutral measure under market completeness. Practically, this means the following: By using any of these measures in RN valuation of a real asset that depends on nontraded risks, the resulting value of the real asset is one that lies within the interval of values that do not create arbitrage opportunities by trading the asset and underlying futures on the energy source (Nadarajah and Secomandi, 2021). This implies that RN valuation still is useful for valuing real assets.

An alternative to RN valuation is dynamic discounted cash flow valuation (DDCF), which relies on the statistical measure, as opposed to an equivalent martingale measure. This approach is referred to as *dynamic programming* in Dixit and Pindyck (1994). DDCF valuation addresses risk differently than the RN paradigm. As explained above, in the RN paradigm, risk is accounted for by adjusting the probability distribution, resulting in an equivalent martingale measure where risky cash flows can be discounted at the risk-free rate. By contrast, The DDCF approach incorporates the risk premiums required by shareholders by adjusting the discount rate (Smith and McCardle, 1999). In theory, this makes discounting difficult, since the discount rate depends on the time when the cash flow occurs and the corresponding realization of uncertainties. Still, in practice, the discount rate should reflect the weighted average cost of capital (WACC).

For calibration of risk-neutral dynamics of prices, historical forward curves are needed. For electricity contracts, whose payoff is based on the average value over some period of time, smooth synthetic forward curves must first be constructed based on historically traded contracts (Fleten and Lemming, 2003; Benth et al., 2007). Using the synthetic forward curves, the estimation of two-factor price models typically involves Kalman filtering, to estimate latent long-term and short-term states, and maximum likelihood estimation of predictive distributions for futures prices seen from the day they are traded (Schwartz and Smith, 2000). We refer to Welch et al. (1995) for an introduction to the Kalman filter. Such forward-looking estimation is only possible when financially traded contracts with delivery at a future point in time exist. For nontraded risks, parameters of the stochastic process can be estimated based on the history of observations, while individual risk preferences must be modeled, see, e.g. Löhndorf and Wozabal (2021).

In Paper I, we estimate model parameters based on expert knowledge and implied methods. Furthermore, we apply DDCF using WACC as the discount rate. In the other papers we use the RN paradigm and consider planners that aim at maximizing the market value of their assets. In Paper III and IV we estimate the parameters by maximum likelihood and Kalman filtering using forward prices from synthetic forward curves (Goodwin, 2020). In Paper II we develop a new model for incorporating co-movements between firm-specific nontraded risks and forward prices. We apply a hybrid estimation approach using both historical observations and forward-looking data.

2.2 Hydropower planning MDP

Markov decision processes (MDPs) provides a framework for decision making under uncertainty (Puterman, 1994). In this section, we present a reference model which includes important features of the real-world problem of operations and investments in hydropower plants, and combines the aspects considered in the papers. Let $\mathcal{T} := \{0, 1, ..., T-1\}$ denote the set of time periods where T is the planning horizon. The MDP state at stage t is partitioned into endogenous and exogenous components, denoted $s_t \in \mathcal{S}_t$ and $\omega_t \in \Omega_t$, respectively. At stage t and state (s_t, ω_t), the decision maker chooses an action $x_t \in \mathcal{X}_t(s_t, \omega_t)$ and receives reward $r_t(x_t, s_t, \omega_t)$. The endogenous state gets updated according to a specified transition function $f(x_t, s_t, \omega_t)$, while the exogenous states gets updated according to a stochastic process, independently of the stage t action x_t. Stochastic processes that govern the evolution of these factors were discussed in Section 2.1. We denote by $\pi \in \Pi$ a policy, which is a collection of stage and state dependent decisions, which we denote by $\{X_0^\pi, X_1^\pi, ..., X_T^\pi\}$. The set Π is the set of feasible policies. Furthermore, we let s_t^π denote the stage t endogenous state reached by following policy π. The decision maker aims at maximizing the discounted accumulated expected reward by acting according to π. The MDP can be formulated as

$$V(s_0, \omega_0) = \max_{\pi \in \Pi} \mathbb{E}\left[\sum_{t \in \mathcal{T}} \gamma^t r_t(X_t^\pi(s_t^\pi, \omega_t), \omega_t) \middle| \omega_0 \right], \tag{2.1}$$

where γ is the discount factor. The endogenous state consists of the reservoir volume, capacity, and efficiency, $s_t = (s_t^{\text{res}}, s_t^{\text{cap}}, s_t^{\text{eff}})$. The exogenous state is given by the price factors long-term equilibrium (LE) and short-term deviation (SD), and inflow (I), $\omega_t = (\omega_t^{\text{SD}}, \omega_t^{\text{LE}}, \omega_t^{\text{I}})$. The stage t decision vector contains generation, spill, capacity, maintenance and renewal,

$x_t = (x_t^{\text{gen}}, x_t^{\text{spill}}, x_t^{\text{main}}, x_t^{\text{cap}}, x_t^{\text{ren}}) \in \mathcal{X}_t(s_t, \omega_t)$. The immediate reward can be written as

$$r_t(x_t, s_t, \omega_t) = \begin{cases} C_1 & \text{if } x_t^{\text{main}} > 0 \\ C_2(x_t^{\text{cap}}) & \text{if } x_t^{\text{ren}} > 0 \\ g_1(x_t^{\text{gen}}, s_t^{\text{res}}(x_t^{\text{gen}}); s_t^{\text{eff}}) g_2(\omega_t^{\text{SD}}, \omega_t^{\text{LE}}) & \text{otherwise} \end{cases} \quad (2.2)$$

If neither maintenance nor renewal occurs, the immediate reward is the product of generation quantity and price. The function $g_1(\cdot, \cdot; s_t^{\text{eff}})$ describes the generation function. This function is typically concave in reservoir volume, s_t^{res}, and discharge, x_t^{gen}, while s_t^{eff} determines the optimum power output point of the turbine. This function will be further described in the next section. The function $g_2(\cdot, \cdot)$ describes the transformation of the stochastic variables which determine the price. These are often are modeled on log-scale, e.g. Schwartz and Smith (2000). A cost is incurred when either maintenance or renewal is undertaken. Typically, C_2 is substantially higher than C_1. If renewal occurs, the cost also depends on the additional capacity installed.

We describe the endogenous states by separate functions f_1, f_2, f_3. In addition to representing physical constraints, such as energy coupling in time, these functions determine the future benefits of choosing actions x_t^{main} and x_t^{ren}, which entail immediate costs. The endogenous state updates can be written as

$$s_t^{\text{res}} = f_1(s_{t-1}^{\text{res}}, x_t^{\text{gen}}, x_t^{\text{spill}}, \omega_t^{\text{I}}) \quad (2.3)$$
$$s_t^{\text{eff}} = f_2(s_{t-1}^{\text{eff}}, x_t^{\text{ren}}, x_t^{\text{main}}) \quad (2.4)$$
$$s_t^{\text{cap}} = f_3(s_{t-1}^{\text{cap}}, x_t^{\text{cap}}) \quad (2.5)$$

We do not write these functions explicitly, but instead explain how they can be specified. Transition (2.3) ensures the energy balance. The reservoir volume at time t must be equal to the reservoir at time $t-1$ minus discharge x_t^{gen} and potential spillage x_t^{spill} plus incoming inflow which is some function of ω_t^{I}. Transition (2.4) describes the evolution of efficiency. The efficiency at time t is some function of the efficiency at time $t-1$ adjusted by deterioration, either deterministic or stochastic, and possible improvements coming from maintenance or renewals, x_t^{main} and x_t^{ren}, respectively. Transition (2.5) describes capacity upgrades, which ensures that more can be produced in each stage after renewal. The stage-t feasible action set is defined by

$$\mathcal{X}_t(s_t, \omega_t) = \{$$

$$s_t^{\text{res}} \geq R^{\text{low}} \tag{2.6a}$$
$$s_t^{\text{res}} \leq R^{\text{high}} \tag{2.6b}$$
$$x_t^{\text{gen}} \leq \min\{s_t^{\text{res}}, s^{\text{cap}}\} \tag{2.6c}$$
$$x_t^{\text{gen}}, x_t^{\text{spill}}, x^{\text{cap}} \geq 0 \tag{2.6d}$$
$$x_t^{\text{main}}, x_t^{\text{ren}} \in \{0, 1\} \tag{2.6e}$$
$$\}.$$

Constraints (2.6a)-(2.6b) are the minimum and maximum reservoir content, R^{low} and R^{high}, respectively. Spillage is defined in the transition (2.3) and ensures that (2.6b) is satisfied. Constraint (2.6c) ensures that generation in any stage must be less than both the generation capacity and the reservoir volume. Constraints (2.6d)-(2.6e) provide additional constraints on the decision variables.

The MDP is intractable because of a high-dimensional state space, which makes it difficult to evaluate the conditional expectations. Possible solution approaches involve reducing the problem complexity, which in some cases allows for analytic valuation, or searching for near-optimal policies using approximate dynamic programming algorithms. This will be discussed in Section 2.3. We first discuss some simplifications in the next section.

2.2.1 Energy equivalent reservoir

In practice, hydropower producers need to allocate water in multiple interconnected reservoirs in a watercourse. A power station is associated with each reservoir, and each station may consist of several generation units. In the formulation in the previous subsection and in the papers of the thesis, we aggregate the reservoirs and generating units and perform computations on an energy equivalent reservoir and power station. The aggregation is explained in the nominal work by Arvanitidits and Rosing (1970). This aggregation scheme has been applied to the Brazilian system (Maceiral et al., 2018), and by Norwegian hydropower producers (Flatabø et al., 1998), although the state-of-the-art today is to consider the more detailed interconnected system of reservoirs and corresponding power stations for hydro scheduling (Pereira and Pinto, 1991; Gjelsvik et al., 2010). The use of energy equivalents simplifies the modeling, while insights from the research in this thesis still are applicable to an interconnected system of reservoirs, either by heuristic disaggregation schemes or by detailed modeling of individual reservoirs and power stations in the watercourse.

A property of the MDP that should be noted is that without maintenance, renewal, and failure, the optimal MDP value is concave in the endogenous states and the action set is convex under certain assumptions regarding the generation function in (2.2). The proof of this property is provided in Paper II. This allows for effective solving of the pure operational MDP, which will be further explained in Section 2.3. Generally, the relation between discharge and power output, $g_1(x_t^{\text{gen}}, s_t^{\text{res}}(x_t^{\text{gen}}); s_t^{\text{eff}})$, is nonlinear. The function is defined by the water head, i.e. reservoir volume s_t^{res}, which again depends on the discharge volume x_t^{gen}. The function is often concave in discharge volume x_t^{gen} and reservoir volume s_t^{res}. Furthermore, the power output depends directly on the discharge, whose function is determined by the turbine efficiency curves. In our formulation, we let s_t^{eff} denote the optimum power output point. In the papers of this thesis, we assume a constant generation function, which means that the stage-t revenue is simply the product of the spot price and the generation volume. The dependency of output and discharge volume can straightforwardly be integrated in the MDP by approximating the concave discharge function by a piecewise linear function which can be added as constraints in the stage-t action set, which preserves the convexity of the pure operational MDP. The dependency between the generation output and reservoir volume, i.e. head variations, is more complicated to integrate, as it leads to a non-convex optimization problem. Methods and heuristics for handling this exists, see, e.g. Gjelsvik et al. (2010); Cerisola et al. (2012), and Hjelmeland et al. (2018).

2.3 Solution methods

We employ different solution approaches in each paper. The choice of solution approach can be guided by the MDP structure. In some cases, analytical valuation is possible (Dixit and Pindyck, 1994), and if not, computational methods are needed to obtain near-optimal policies (Bertsekas, 2007; Powell, 2011). This section introduces the methods that are used in the papers of this thesis.

2.3.1 Real options analysis

Real options analysis is a framework for valuing irreversible investment opportunities under uncertainty (Dixit and Pindyck, 1994). In this section, we illustrate real options analysis in its simplest form, namely, a one-time investment opportunity in a project that generates instantaneous profits $\omega_t Q$, i.e. price times quantity, over an infinite horizon upon capital outlay I. Before investment, zero profit is generated. Thus, we bypass explicit operations modeling as defined in the reference MDP and we are instead given an estimate of the instantaneous profit flow. The price is uncertain and evolves according to a GBM,

$$d\omega_t = \mu\omega_t dt + \sigma\omega_t dz,$$

where μ is the drift rate, σ is the volatility, and dz is normally distributed with zero mean and unit variance. Using RN valuation and r as the risk free interest rate, the value of the project if investing at time t can be written as

$$V(\omega_t) = \mathbb{E}\left[\int_t^\infty e^{-r(s-t)}\omega_s Q ds \middle| \omega_t\right] = \frac{\omega_t Q}{r-\mu}. \tag{2.7}$$

Thus, $V(\omega_t)$ follows the same dynamics as ω_t. We want to maximize the expected present value of the project, taking into account the timing flexibility,

$$F(V) = \max_\tau \mathbb{E}\left[e^{-r\tau}(V(\omega_\tau) - I)\right], \tag{2.8}$$

where τ is the unknown future time when the investment is undertaken. It can now be shown that the solution of this problem can be expressed in terms of a trigger V^* (Dixit and Pindyck, 1994). Above this trigger it is optimal to invest immediately, and below it is optimal to wait. If it is optimal to wait, the Bellman equation must hold (Bellman, 1957). In continuous time, this can be written as

$$rFdt = \mathbb{E}(dF), \tag{2.9}$$

which means that the total return of holding the investment opportunity must equal its expected change in value. Expanding dF using Ito's lemma, we obtain the following second-order differential equation

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \mu V F'(V) - rF = 0, \tag{2.10}$$

with boundary conditions $F(0) = 0$, $F(V^*) = V^* - I$, $F'(V^*) = 1$. The boundary conditions state that the option value is zero if $V = 0$, that the firm receives $V^* - I$ when investing, and that F needs to be continuous and smooth at the trigger V^*. Thus, the solution of the differential equation takes the form

$$F(V) = AV^{\beta}, \tag{2.11}$$

where $\beta > 1$ and dependent on μ, r and σ. By using the boundary conditions, the critical value for investment can be derived,

$$V^* = \frac{\beta}{\beta - 1} I. \tag{2.12}$$

The constant A can be found by inserting this expression into another boundary condition. This expression shows the opportunity cost of realizing the project. Since $\beta > 1$ entails $V^* > I$, whereas the traditional NPV approach recommend undertaking a project if $V > I$. For further details on real options analysis, we refer to Dixit and Pindyck (1994).

This example shows the basics of real options analysis. More sophisticated analysis is possible, e.g. mutually exclusive options (Décamps et al., 2006; Siddiqui and Fleten, 2010), timing and capacity optimization (Dangl, 1999; Bøckman et al., 2008), possibly in a competitive environment (Huisman and Kort, 2015). Still, the model governing uncertainty and the MDP structure needs to be relatively simple in order for analytical solutions to be attainable. E.g. problem-specific constraints, such as reservoir limits, or multi-factor models for prices is difficult to incorporate. In Paper I, we apply real options analysis to study mutually exclusive performance-enhancing projects under price uncertainty. In Paper III we apply real options analysis, together with a practice-based heuristic, to obtain semi-analytic policies for capacity investments in hydropower plants. In the next section, we present the general concept behind computational methods for solving MDPs.

2.3.2 Stochastic dynamic programming

If closed-form solutions are not attainable, numerical solution approaches must be applied. Similar to the real options approach, the Bellman's principle of optimality is important in the development of algorithms for solving MDPs (Bellman, 1957). The principle allows the optimal objective value to be expressed in terms of the current state s_t recursively. Thus, for each $(t, s_t, \omega_t) \in \mathcal{T} \times \mathcal{S}_t \times \Omega_t$, the MDP in (2.1) can be written as

$$\begin{aligned} V_t(s_t, \omega_t) = \max_{x_t \in \mathcal{X}_t(s_t,\omega_t)} & \{r_t(s_t, \omega_t, x_t) + \gamma W_t(s_{t+1}, \omega_t) : \\ & s_{t+1} = f(s_t, x_t, \omega_t)\} \\ W_t(s_{t+1}, \omega_t) = & \mathbb{E}\left(V_{t+1}(s_{t+1}, \omega_{t+1}) | \omega_t\right), \end{aligned} \tag{2.13}$$

with boundary condition $V_T(s_T, \omega_T) = 0$ for each s_T. The action set $\mathcal{X}_t(s_t, \omega_t)$ is defined in (2.6), and the value in (2.1) is given by $V_0(s_0, \omega_0)$ above. The stochastic dynamic programming equations can be solved by evaluation of the continuation function $W_t(\cdot, \cdot)$ for all possible states $s_t \in \mathcal{S}_t$ and $\omega_t \in \Omega_t$, traversing stages backwards. Thus, the endogenous and exogenous state space must be finite, or represented by a discretized lattice. However, enumeration of all possible states is computationally intractable in high dimensions. This is known as the curse of dimensionality. Therefore, approximation methods are often needed (Powell, 2011).

2.3.3 Stochastic dual dynamic programming

The stochastic dual dynamic programming algorithm (SDDP) was developed by Pereira and Pinto (1991), and is particularly suited for problems having high-dimensional endogenous state space. SDDP overcomes the curse of dimensionality by approximating the expected continuation function $W_t(s_{t+1}, \omega_t)$ by a piecewise linear function. The algorithm solves sub-problems iteratively until some convergence criteria is met (Philpott and Guan, 2008). At iteration k, the approximated version of model (2.13) is

$$\begin{aligned} V_t^k(s_t, \omega_t) = \max_{x_t \in \mathcal{X}_t(s_t,\omega_t), \theta_t} & \{r_t(s_t, \omega_t, x_t) + \theta_t : \\ & s_{t+1} = f(s_t, x_t, \omega_t), \theta_t \leq \alpha_t^k + \beta_t^k s_t\Big\}. \end{aligned} \tag{2.14}$$

Each iteration consists of two phases: First, in a forward pass, outcomes of the stochastic variable ω_t are realized in each stage. For each stage, a solution of (2.14) is computed and used as parameters in the problem being solved in the subsequent stage. Then, in a backward pass, new cuts $(\alpha_t^k, \beta_t^k, t \in \mathcal{T} \setminus \{T\})$ are constructed, which is a linear approximation of the continuation function at the values of the states s_t visited in the forward pass.

Since the cuts approximate θ_t from above (in maximization problems), any solution to problem (2.14) from the backward pass has a value that is an upper bound on the optimal value of the original problem (2.13). A lower bound can be estimated by a sample average approximation by simulating the policy after the convergence criteria is met. In order for the SDDP algorithm to be applicable, a number of assumptions are needed. First, the continuation function needs to be concave. Secondly, uncertainty needs to be stage-wise independent in order for the scenario tree not to grow exponentially fast. Still, time dependent randomness can be incorporated either by Markov chain discretization (MC-SDDP) or augmenting the state vector (TS-SDDP). The first approach allows for a rich representation of the data, but statistical properties with respect to the continuous state process is lost. The second approach limits the stochastic processes for exogenous states to be linear to keep the convexity requirement (Löhndorf and Shapiro, 2019). For further details on assumptions, convergence, and statistical properties of the SDDP algorithm, we refer to Philpott and Guan (2008) and Shapiro (2011).

SDDP is the state-of-the-art algorithm for managing hydropower reservoirs. We apply this algorithm in Paper II where we consider an MDP that complies with the SDDP requirements. Below, we describe alternative approximate dynamic programming algorithms that can handle non-convex action sets and more general stochastic processes for exogenous factors.

2.3.4 Reoptimization heuristic

The reoptimization heuristic (RH) is based on repeatedly solving deterministic optimization problems based on expectations of exogenous variables. This heuristic has been widely used by practitioners in various applications (Lai et al., 2010; Wu et al., 2012; Nadarajah and Secomandi, 2018). At stage t and state $(s_t, \omega_t) \in \mathcal{S}_t \times \Omega_t$, the heuristic uses the forecast $\hat{\omega}_{t,\tau} = \mathbb{E}(\omega_\tau | \omega_t)$ for $\tau > t$ and solves the following intrinsic programs (IP),

$$V_t^{IP}(s_t, \omega_t) = \max_{x_t \in \mathcal{X}_t^{IP}(s_t, \omega_t)} \left\{ \sum_{\tau \in \mathcal{T}_t} \gamma^\tau r_\tau(\hat{\omega}_{t,\tau}, s_\tau, x_\tau) : \right. \\ \left. s_{\tau+1} = f(s_\tau, x_\tau, \hat{\omega}_{t,\tau}), \quad \tau \in \mathcal{T}_t \right\}, \tag{2.15}$$

where $\mathcal{T}_t = \{t, t+1, , ..., T\}$ is the set of periods from time t to the end of horizon T, $x_t = (x_\tau,\ \tau \in \mathcal{T}_t)$, and $\mathcal{X}_t^{IP}(s_t, \omega_t)$ defines the feasible action set of the intrinsic problem solved at time t. After this problem has been solved at stage t, the first-stage solution of the stage-t program is implemented, the endogenous state gets updated before new information becomes available at stage $t+1$. The conditional expectation is now updated before a new intrinsic problem is solved. This procedure is repeated until the end of the horizon. After generating K sample paths and solving the intrinsic problems along each path, the value of the RH policy can be obtained as the sample average of rewards from each path. The RH policy is feasible and thus provides a lower bound on the optimal value of problem (2.13). In contrast to SDDP, where the lower bound converges in probability towards an upper bound, the performance of the RH-policy is unclear. In the next section, we discuss how to assess the performance.

2.3.5 Information relaxations in stochastic dynamic programs

To assess the performance, an upper bound can be computed using information relaxation (IR) and duality theory (Brown et al., 2010). The approach is based on relaxing non-anticipativity constraints embedded in (2.13) and penalizing knowledge of the future. Let $\bar{\omega}^k := \{\bar{\omega}_0^k, \bar{\omega}_1^k, ..., \bar{\omega}_T^k\}$ denote a vector of realized stochastic variables in each stage $t \in \mathcal{T}$. Then we can define the following deterministic dynamic problem:

$$U_t^{IR}(s_t; \bar{\omega}^k) = \max_{x_t \in \mathcal{X}(s_t; \bar{\omega}_t^k)} \Big\{ r_t(s_t, x_t, \bar{\omega}_t^k) - q_t(x_t, \bar{\omega}_t^k, \bar{\omega}_{t+1}^k) + \gamma U_{t+1}^{IR}(s_{t+1}; \bar{\omega}_{t+1}^k) : s_{t+1} = f(s_t, x_t, \bar{\omega}_t^k) \Big\}, \tag{2.16}$$

with $U_T^{IR}(s_t; \bar{\omega}_0^k) = r_t(s_T, x_T, \bar{\omega}_T^k)$, and where $q_t(x_t, \bar{\omega}_t^k, \bar{\omega}_{t+1}^k)$ are dual penalties that penalizes knowledge of the future. Feasible dual penalties are those who do not penalize non-anticipative policies in expectation (Brown and Smith, 2014). An upper bound can now be attained by solving this problem for K Monte Carlo samples, and then take the sample average of revenues obtained from all paths. The simplest case for obtaining an upper bound is to set the dual penalty to zero, which leads to the perfect information upper bound. This is, however, often a very loose bound. Brown et al. (2010) show that when using an ideal dual penalty, the upper bound equals the optimal value of (2.13). However, since this is difficult to find, simpler penalties may be considered. Penalties that are linear in the decision variables are particularly useful in convex stochastic dynamic programs. Moreover, assuming stage-wise independent penalties simplifies computations further, and may still provide strong upper bounds. We apply the reoptimization heuristic and information relaxations in Paper III.

2.3.6 Information relaxation-based reoptimization heuristic

An information relaxation-based reoptimization heuristic (IRH) was proposed by Trivella et al. (2018). The heuristic combines decision making with the dual bound estimation process for obtaining a feasible policy. The heuristic solves IR problems, defined in (2.16), from which a distribution of first-stage decisions is obtained. By choosing a statistic of this distribution, e.g. mean or median, the decision is non-anticipative and feasible, and can thus be implemented in the first stage. In the next stage, the procedure is repeated and continues until the end of the horizon. The IRH-heuristic has shown promising performance compared to the RH-policy (Trivella et al., 2018). We apply this heuristic in Paper IV.

2.3.7 Optimality gap and sources of variance

Obtaining MDP policies typically involves a two-step procedure. First, a model for exogenous factors is specified and calibrated using data, then decisions are optimized under this model. Often, the calibrated models of data have continuous support. Some solution methods, such as SDP based methods and MC-SDDP, require discretization of the exogenous state space and estimation of transition probabilities. Approximating the continuous-state process with a discrete scenario tree, or lattice in the case of Markovian process, introduces a discretization error. Different approaches for the construction of discrete approximations exist. Moment matching aims at preserving the first four moments of multivariate continuous distributions (Høyland et al., 2003). Distance-based methods makes use of probability metrics and aims at minimizing some distance function between the discrete approximation and continuous distribution (Pflug, 2001; Heitsch et al., 2006). In path-based methods, clusters and transition probabilities are estimated based on simulations from the continuous state stochastic process (Heitsch et al., 2006). The relative difference between the performance of the solution of the approximated discretized version and the optimal continuous-state MDP value, is sometimes referred to as optimality gap (Kaut and Wallace, 2007).

Instead of pre-specifying a discrete scenario tree or lattice, samples from the stochastic process can be drawn while solving the MDP. Algorithms that rely on Monte Carlo simulations, such as TS-SDDP, the

RH heuristic, and the IRH heuristic, need sufficiently many samples to give a reasonable estimate of a lower bound of maximization problems. In TS-SDDP, the number of samples or iterations needed is determined based on a convergence criterion. For heuristics without convergence guarantees, the number of Monte Carlo simulations can be guided by the standard error of the performance of the heuristic policy. When a satisfactory low standard error is obtained, the estimated policy performance can be evaluated against an upper bound, as described in Section 2.3.5 and in Brown et al. (2010). Thus, an estimated optimality gap can be computed as the relative difference between the upper and lower bound on the optimal MDP value, see e.g. Lai et al. (2010).

We have now presented two error sources that stem from the discretization of the stochastic process and Monte Carlo simulations, respectively. Finally, we discuss different variance sources which stem from the two-step procedure for obtaining MDP policies. One can distinguish between two types of variance sources for MDPs, namely, internal variance and parameter variance (Mannor et al., 2007). Internal variance is the variance of the sample trajectory rewards, given a policy and with the parameters of the stochastic model being estimated and kept fixed. The parametric variance addresses that the stochastic model parameters are estimated from an empirical sample and are therefore subject to variance. Thus, parametric variance can be seen as the variance of the MDP values under different parameter estimates, i.e. the average accumulated discounted rewards by following a fixed policy subject to stochastic models with different parameter estimates. The parametric variance becomes increasingly important when data is scarce. A framework for decision making under limited knowledge about the evolution of exogenous factors, is robust optimization (see, e.g. Ben-Tal et al. (2009) or Bertsimas et al. (2011)). As opposed to stochastic programming, robust optimization requires only the support of data, instead of a distributional specification. The goal is to find solutions that perform well on worst-case realizations that belong to a predefined uncertainty set. However, a drawback with this approach is that it often leads to very conservative solutions.

In the papers of this thesis, we address optimality gaps and variance sources as follows: In Paper II we apply MC-SDDP and perform numerical experiments using a discrete lattice that approximate the continuous-state process. In Paper III and IV we apply the reoptimization heuristics and consider the standard error of the lower bound when determining the sample size needed to get a reasonable lower bound estimate. In Paper IV we address the parametric variance and limited long-term information by considering uncertain MDPs (Xu and Mannor, 2009).

2.4 Overview of model features and solution approaches

To summarize, Table 2.2 gives an overview of which features of the reference model and which solution approach that we apply in the four papers. The first two rows under exogenous states report whether cross-dependency between market-related and firm-specific factors is included or not, and if the parameters of the model are considered as known or unknown. The last three rows report how many factors that make up the dynamics of prices, inflows, and available resources in the system.

Table 2.2: Overview of solution approach and model features. Characters indicate whether cross-dependency is included (yes/no), and whether the parameters of the stochastic model are known or unknown for the decision maker (known/unknown). Numbers indicate the number of factors (1 or 2).

	Type	Paper I	Paper II	Paper III	Paper IV
Solution approach					
Analytic		✓		✓	✓
SDDP			✓		
RH				✓	
IRH					✓
Actions					
Generation	Cont.		✓	✓	✓
Capacity increase	Cont.			✓	✓
Renewal	Bin.	✓		✓	✓
Maintenance	Bin.	✓			
Endogenous states					
Reservoir volume	Cont.		✓	✓	✓
Max generation capacity	Cont.			✓	✓
Efficiency	Cont.	✓			
Exogenous states					
Cross-dependency			Y	N	N
Model parameters		K	K	K	U
Price	Cont.	1	2	2	2
Local inflow	Cont.		1	1	1
System hydrology	Cont.		1		

Chapter 3

Contributions

This chapter presents the four papers. For each paper, we explain the context, the approach we take, and we outline the contribution to the academic literature and industry practitioners.

3.1 Papers

Paper I: A Real Options Analysis of Existing Green Energy Facilities: Maintain or Replace?

Authors: Eirik Magnus Dønnestad, Stein-Erik Fleten, Andreas Kleiven, Maria Lavrutich, Amalie Marie Teige
Submitted to an international peer-reviewed journal.

This paper studies an operator of machinery with deteriorating efficiency. The operator considers performance-enhancing activities of different scales with uncertain associated profits, which stems from the price being uncertain. We analyse the setting of repowering of green energy facilities and specifically illustrate the implications of our model on a hydropower case study. The options available for the operator are i) a replacement option, which involves large costs and new machinery having better performance, and ii) a sequential compound option, which involves undertaking a smaller project, which makes the efficiency deteriorate more slowly, while still having the option to replace later. We apply real options analysis to characterize the optimal policy.

The paper is an application and extension of the model in Décamps et al. (2006) and contributes to the research on mutually exclusive options. Our framework recognizes maintenance as a temporary alternative to renewal, and we quantify the effect of having the replacement option embedded in the maintenance option. For practitioners, our analysis contributes with novel insights for operators of hydropower facilities. In particular, we demonstrate that a real options perspective on maintenance activities is valuable, and that long-term electricity price expectations can be used as a basis for deciding when and which activity to choose. Overall, our analysis highlights the importance of having several performance-enhancing activities under consideration when prices are uncertain, which is a setting that owners of hydropower assets can relate to.

My contribution to this paper is to complement and extend the analysis that was done in the Master's thesis of the co-authors. This included undertaking additional numerical experiments, extending the theory with comparative statics, rewriting the original Master's thesis into the original draft, and working on subsequent reviews.

Paper II: Co-movements between forward prices and resource availability in hydro-dominated electricity markets

Authors: Andreas Kleiven, Simon Risanger, Stein-Erik Fleten
Submitted to an international peer-reviewed journal.

This paper studies the dependency between system prices and resource availability in hydrodominated electricity systems. Hydropower producers are exposed to various risk factors. Market-related uncertainty, i.e. price risk, can be hedged through derivatives, while firm-specific uncertainty, i.e. volume risk, cannot be hedged. Still, a natural hedge exists, in the sense that periods with low inflow correlate with high prices and vice versa. Despite being aware of this relationship, common industry practice is to neglect it when establishing the seasonal production policy. The aim of this paper is to broaden the knowledge of the impact of neglecting this relationship and to learn about the potential loss. To do so, we formulate the production scheduling problem as an MDP and develop a novel price model which incorporates the effect of the system hydrological state and local inflow on prices. We then solve the MDP using SDDP, which is the state-of-the-art algorithm for hydropower production planning.

The paper contributes to the academic literature on pricing models by introducing a novel model with dependency between market-related and firm-specific uncertainties. It also contributes to research on hydropower reservoir management by analysing the effect of incorporating dependency between resource availability and prices when establishing the operational policy of a local producer. Furthermore, our analysis contributes with useful insights for industry practitioners. Our results show that producers underestimate their marginal water value if the negative relationship between prices and resource availability is ignored, which means that current water has a higher expected value in the future if co-movements are properly accounted for. Nevertheless, our numerical case study indicates that the potential expected gain is modest. Given these results and the complications that arise when incorporating this in commercial software, we are hesitant in advising practitioners to explicitly model the relationship between local inflow, system hydrology and prices.

My contribution to this paper is the conceptualization, model formulations, the illustrative example, theory, data collection, and calibration of the stochastic model. Co-author Simon Risanger was responsible for discretization of the stochastic process and implementation of the algorithm used to compute policies. Both prepared the original draft, analyzed the results, and worked on revisions. This paper is also a part of Simon Risanger's PhD dissertation.

Paper III: Revisiting Hierarchical Planning for Hydropower Plant Upgrades using Semi-analytical Policies and Reinforcement Learning

Authors: Andreas Kleiven, Selvaprabu Nadarajah, Stein-Erik Fleten
Working paper.

This paper considers long-term capacity investment in hydropower. Valuation of long-term investments typically relies on accurate mathematical models. Such models must incorporate various aspects, including long-term market dynamics, medium-term resource allocation, and short-term operational flexibility, which stems from the ability to quickly respond to price fluctuations. Given the complexity of the problem, hierarchical planning is commonly used, where short-term flexibility is simplified when making tactical resource utilization decisions and investments simplify the tactical aspect (Anthony, 1965). We formulate a novel investment model that integrates these aspects. Using properties of the price model combined with reinforcement learning, we obtain semi-analytical policies that unveil practical insights.

The paper contributes to investment analysis research by applying reinforcement learning and real options analysis to obtain semi-analytical policies. We find that investment models embedded with reinforcement learning that can estimate the value of tactical resource utilization and operational flexibility, may promote additional capacity installments in hydropower. Our framework also provides several insights for industry practitioners. First, we assess the performance of operational policies obtained from a heuristic that allows a straightforward integration of seasonal planning and intraweek scheduling. Second, we show how short-term operational models can be combined with long-term market price movements to evaluate investment alternatives.

My contribution to this work is conceptualization, model calibration, theory, implementation of the algorithm, analysing the results, and preparing the original draft for submission.

Paper IV: Robust Capacity Investment in Hydropower

Authors: Andreas Kleiven, Selvaprabu Nadarajah, Stein-Erik Fleten
Working paper.

This paper studies robust capacity investment in hydropower plants and addresses long-term risks associated with limited long-term information. We formulate a combined investment and operations problem as a Markov Decision process (MDP), where the goal is to find a policy that maximizes the market value of the asset. This requires calibration of a price model using risk-adjusted prices, i.e. futures contracts on electricity. However, since the expected lifetime of investment projects typically is longer than the maturity of electricity futures contracts, it is unclear how to estimate the model parameters for long maturities. Therefore, committing to one parametric stochastic process when data availability varies over time may lead to poor performance of the MDP-policy when applied to real data. We provide an empirical study for identifying the model parameters that have a major impact on long-run cashflows. Then, we define different criteria for decision making under unknown model parameters, including model-based, worst-case, and regret criteria.

The paper contributes to academic research on investment analysis by addressing the lack of long-term futures contracts, which is the empirical basis of the risk-neutral valuation framework. Moreover, we contribute to research on decision making in an uncertain environment by analysing decision criteria for making investments when a subset of parameters of the stochastic model is considered as unknown. We obtain near-optimal operational policies using an information relaxation-based heuristic, and we demonstrate that our approach may reduce the variability of cashflows associated with long-term investments. For industry practitioners, we identify economic risks that cannot be fully hedged and provide a modeling framework and solution approach to support long-term investment decisions in hydropower plants.

My contribution to this work is to collect data, the calibration of stochastic processes, empirical assessment, illustrative examples, theory, implementation of the algorithm, computational experiments, and preparation of the original draft.

3.2 Additional contributions

During my PhD, I also have contributed to two other projects. The research has been published in the following papers:

- Kleiven, A., I. Steinsland (2018). Inflow forecasting for hydropower operations: Bayesian model averaging for postprocessing hydrological ensembles. *The International Workshop on Hydro Scheduling in Competitive Markets. Springer, Cham.*
- Bakker, S. J., Kleiven, A., Fleten, S. E., Tomasgard, A. (2021). Mature offshore oil field development: Solving a real options problem using stochastic dual dynamic integer programming. *Computers & Operations Research.*

The first article is a result of my Master's thesis at the Department of Mathematical Sciences at Norwegian University of Science and Technology. I spent my first months of the PhD at the Department of Industrial Economics and Technology Management rewriting the thesis into an article. Although the application and methods are relevant for the thesis, only minor additional material was added to the existing material in the Master's thesis, which is why we have decided not to include this article in the dissertation. In the second article, I contributed with discussions and editing of the original draft and revisions. The work is methodologically relevant, but the application is not directly relevant. We therefore decided not to include the article in the PhD dissertation.

Chapter 4

Concluding Remarks and Future Perspectives

The papers in this thesis consider sustainable operations and investment in hydropower plants. We take a quantitative business analysis point of view when analysing emerging problems that stem from the transition to a more sustainable power system. To analyse these problems, we apply theory and concepts from the areas of statistics, finance, and operations research in designing novel stochastic processes, optimization models, and algorithms.

The papers provide several insights on complex decision problems that owners of hydropower assets can relate to. Paper I gives an introduction to considerations regarding investment activities in hydropower plants, with a focus on efficiency degradation and possible performance-enhancing activities which needs to be planned in an uncertain environment. We demonstrate how such activities can be valued properly and provide useful managerial insights. Paper II introduces the operational aspect. The paper examines whether a hydropower producer can improve water value estimates by accounting for a negative relationship between local inflow, system inventory levels, and prices. We present theory that identifies an opportunity cost by ignoring the negative relationship, and a numerical case study that suggest very small cost savings in practice. Paper III and IV introduce coordinated investment and operations planning. Paper III investigates the impact of short-term flexibility on long-term capacity investments. We propose an intuitive semi-analytical valuation framework, and show that the operational flexibility of production assets can significantly help promote additional capacity in hydropower. Paper IV focuses on downside risk and analyzes different decision criteria for establishing investment policies when the parameters of the stochastic model are unknown. Our analysis shows that our approach may reduce the variability of the cashflow associated with an irreversible long-term investment.

The papers present and address various aspects of the problem of planning preventive maintenance tasks, operations, upgrades and renewals in hydropower plants, which remains a challenging problem. The papers specifically address real options, computational challenges, and challenges related to heterogeneous data and stochastic modelling. The models and methods can be extended in several ways. An example of an aspect that has not been considered is the endogenous effect from operations on efficiency deterioration and failure probabilities, and thus investment policies. With an increased industry focus on condition monitoring of the mechanical state of generation assets and data collection, this could be an interesting future extension to the models in this thesis. A second possibility is to extend the models and analysis to imperfectly competitive electricity markets and incorporate an endogenous electricity price. A third direction is to consider bidding in sequential markets. The papers in this thesis have only considered participation in the day-ahead market, which is the main physical arena for trading power. However, there exist supplementing markets, and market structures are constantly evolving. Participation in multiple markets and the effect on medium-term hydropower schedules and long-term investments could be interesting to study. Furthermore, the concepts and tools we develop in the papers may have the potential for broader applications beyond managing hydropower assets, such as investment analysis in other renewable energy production assets, resource allocation problems, or portfolio management.

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Papers

I

Paper I

A Real Options Analysis of Existing Green Energy Facilities: Maintain or Replace?

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A Real Options Analysis of Existing Green Energy Facilities: Maintain or Replace?

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Abstract We consider an operator of machinery with deteriorating efficiency, facing the problem of optimally timing of either a minor (maintenance) investment or a major (replacement) investment under price uncertainty. If a maintenance investment is chosen, the efficiency of the machinery will deteriorate more slowly, and replacing later is still possible. The optimal decision rule is expressed in the form of thresholds for long-run prices, indicating that it may be rational to wait to see which of the large and small investment is the better choice. We relate the setting to repowering of green energy facilities, such as hydropower plants and wind farms. Our analysis provides several managerial insights. We characterize the conditions that govern whether the smaller investment should be considered at all, and we quantify the effect of having a replacement option embedded in a maintenance option. Our analysis demonstrates that the large investment may get postponed significantly in expectation, which recognizes maintenance as a temporary alternative to replacement.

Keywords Green energy · Maintenance · Real options · Replacement

1. Introduction

Owners of assets with deteriorating performance often have a range of possible actions to choose from in order to increase future expected profits. Replacements are often needed, either when the asset suffers from severe deterioration, the operating requirements change, or when new technology is available. Determining the time of replacing existing assets involves economic analysis of various drivers, such as the current value of the existing asset, operation and maintenance costs, and the cost and value associated with replacing the asset with an improved one [4,15,22]. As an alternative to replacing, maintaining existing assets can extend the useful economic life and often incurs lower immediate costs. Moreover, similar to replacement, many maintenance tasks have discretion over timing, uncertain benefits, and irreversible costs. Both maintenance and replacement actions can therefore be viewed as exercising real options [11], as opposed to the traditional view, which casts maintenance to be performed until marginal benefits equal marginal costs [5].

The problem of determining maintenance and replacement schedules under price uncertainty is of particular importance for operators of existing green energy facilities, such as wind farms and hydropower plants. Many hydropower plants in the European Union, United States, and Canada were built in the early to mid 20th century, and many of these plants suffer from inefficiencies in power production [13,23]. Moreover, the number of wind turbines in service are expected to increase over the coming decades [38], making it increasingly important to assess the value of performance-enhancing activities with uncertain benefits. Particular focus has been given to the replacement option, which is the process of replacing existing machinery or equipment with new ones that have higher capacity and/or efficiency. We refer to this process as *repowering*. Repowering leads to increased energy or power output, which has a positive effect on future profits for the operator. Eventually, repowering will appear as an attractive project to undertake, but such an investment is costly. In practice, different maintenance investment possibilities can be undertaken to postpone repowering. As an example, maintenance investments in hydropower plants include rehabilitation of existing turbines through surface treatment and hard coating [42,17], and changes in the prevailing operational pattern to mitigate the damaging effects already inflicted on

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the machinery.[1] Similar preventive tasks can be executed for wind turbines, where the upper cutoff point for high wind speeds is reduced to extend the economically useful life.

In this paper, we consider performance-enhancing activities, such as maintenance and replacement, as real options and analyze the range of flexibility that is offered by joint valuation of projects of different scales. We specify and analyze two mutually exclusive options: i) A replace-only option, and ii) a compound option where first maintenance is undertaken before replacement at a later point in time. The first investment alternative we consider is renewal/replacement. Upon renewal, the efficiency, or the profitability associated with the asset, is reset, while the market price is exogenous and unchanged. The second investment alternative we examine is a compound option where first maintenance can be undertaken, while keeping the replacement option alive. In our analysis, maintenance is an investment that reduces the efficiency deterioration rate and thus alters the drift of the underlying stochastic profit process. Our study provides a novel framework that incorporates the managerial flexibility which is present when an operator faces a deteriorating technical sub-system of existing facilities in the presence of uncertain market prices. In our framework, uncertain market prices are modelled as a Geometric Brownian motion (GBM), while having a deterministically declining efficiency, which has a negative effect on the profit stream.

The paper is organized as follows. We review the relevant literature in Section 1.1. In Section 2 we present real option models. In Section 3 we characterize the optimal values, optimal policies and we analytically compute some comparative statics. In section 4 we conduct numerical experiments, expanding the analytical work in Section 3. Concluding remarks are provided in Section 5.

1.1 Contributions in Light of Existing Literature

Early contributions to the real options literature that focus on replacement decisions include [33], [11, Chapter 4], and [32], among others. Further developments in this literature can be divided into two main categories, namely, capital replacement of physical assets ([44,36]), and asset renewals in general [1,35].[2] The former focuses on minimizing losses incurred by having an imperfect component, whereas the latter studies the problem of maximizing the net profit by balancing the revenue from the component with its operational and maintenance costs. Our work fits into the latter category. Differently from [1], only the profitability of the firm's infrastructure resets upon renewal, while we assume that the market price is unchanged, which is similar to [35]. In contrast to [35], we assume the efficiency to be deterministic, which allows us to value different investment alternatives analytically. We contribute to this literature by having the possibility to postpone renewal by investing in a smaller maintenance project.

Maintenance policies have traditionally been studied from an industrial engineering perspective, where maintenance often is optimized with criteria such as reliability, availability, work safety, and maintenance cost [16]. A limitation of traditional maintenance optimization models is that they often do not take into account market uncertainty. [24] addresses this limitation and proposes a methodology based on option pricing theory for joint scheduling of production and preventive maintenance under uncertain demand. Although the maintenance option in our framework has different characteristics, we view maintenance in a similar light as [24], and consider maintenance as a real option. However, unlike [24], we study the interaction between a maintenance option and a replacement option when the associated profits are uncertain, which is new to the real options literature on asset management.

By focusing on several options, we contribute to the stream of literature that studies mutually exclusive options. More specifically, we complement the literature that studies the types of problems introduced by [9] where the investment policy is not merely a simple trigger strategy, but may instead be governed by an investment region that is no longer a connected set. Examples of works that have studied these types of problems include [6] who analyze mutually exclusive projects under input price and output price uncertainty, and [2] who analyze a model with stochastic price and deterministic declining output flow and implications on choices of exit and new technologies involving different flow rates. In line with the literature on mutually exclusive options, we analyze how features of our model affect investment triggers when correctly accounting for the full set of choices available.

An important feature of our model is that exercising the maintenance option entails a change in the drift of the underlying profit flow process, which is similar to [26,20,21]. In this sense, we contribute to the real options literature where the firm has a one-time opportunity to boost the profit rate. [26] consider a firm which has the opportunity to innovate an ageing product while facing a declining profit stream. At any point in time, the firm can choose to continue operations or exit. A predefined change in drift

[1] The latter is often done in practice, where the operator lets go of potentially higher revenues, e.g. by reducing loads or starting the machinery less frequently, in order to maintain its facility and hence postpone a larger investment.

[2] The heuristic [1] employ, also used by [37], has been shown not to always lead to correct solutions, see e.g. [8,27].

that boosts the stream of profits if the firm chooses to innovate. The main findings are that the threshold for exiting decreases in volatility and that the threshold for investing might decrease in volatility if the profit boost from investing is sufficiently large. [21] extend the analysis and show monotonicity of exercise threshold in volatility numerically if the firm can choose the capacity when investing. Similar to this literature, we provide a comparative statics analysis. We examine how the maintenance option, which upon exercise reduces the deterioration rate, or equivalently, boosts the profit stream, affect investment triggers, waiting regions and expected hitting times.

Finally, our case study provides novel insights for real options applications in green energy. Existing literature on the topic include [29] who find that the investment behavior of professional developers of hydropower projects is consistent with real options theory. Moreover, [37] use a real option approach to value different technologies in the energy sector, including photovoltaics, wind, hydro, coal- and gas-fired power plants, among others, and [14] focus on capacity choice and investment timing in a case where a local load is to be served, and only surplus power is sold in the market. From this perspective, our study fits in the growing literature on real option valuation in green energy, see, e.g. [7] or the survey by [25]. We specifically study mutually exclusive projects with applications in green energy. Other papers that have studied this include [39] who consider a firm that may choose to deploy an existing green energy technology, or switch to an unconventional energy technology. Moreover, similar to us, [10] study mutually exclusive projects with different cost structures. In [10], both the cost and revenue of a project are stochastic, described by two distinct correlated geometric Brownian motions. Methodologically, our work is similar, but we focus on investment alternatives allowing to cope with the deteriorating efficiency of existing green energy facilities rather than cost uncertainty related to undertaking new projects. In contrast to our work, neither efficiency deterioration nor maintenance are considered.

2. Models

The operator can choose to undertake the following *projects*: Maintenance investment and replacement. Moreover, the firm can choose between the following *mutually exclusive options*:

1. Replace-only option: Replace the existing machinery by new machinery at a fixed cost I_R.
2. Compound option: First, invest in maintenance of existing machinery at a fixed cost $I_M < I_R$, followed by a replacement of the existing (maintained) machinery at cost I_R.[3]

The operator needs to carefully select which of these options to choose. The correct solution of this problem, joint valuation, takes the form of decision rules depending on thresholds for the price. We assume that the operator is price-taking and that investments are made instantaneously, meaning that there is no investment lag and no shutdown time associated with the execution of any of the projects.[4]

Cashflows from green energy facilities come from electricity generation. As variable operating costs for a green energy operator are typically very small, we consider variable costs to be negligible. We let the profit flow, $\pi(t)$, consist of three components: Electricity price $P(t)$, machinery efficiency $Q(t)$, and production quantity $R(t)$,

$$\pi(t) = P(t)Q(t)R(t). \tag{1}$$

We assume that the electricity price, $P(t)$, follows a GBM,

$$dP(t) = \alpha P(t)dt + \sigma P(t)dZ(t), \tag{2}$$

where α is the drift and $dZ(t)$ is the increment of a Wiener process. The volatility is denoted by $\sigma > 0$. The choice of a GBM is supported by [34] who indicates that applying a GBM for the price of a commodity is an appropriate choice when considering long-term investments. Similarly, [14] argue that although using a GBM to model price dynamics ignores short-term mean reversion in prices, the short-term mean reversion has a minor influence on long-term investment decisions. Alternatives to GBM for modeling electricity prices are discussed in [31].

The second component, the production quantity, is denoted by $R(t)$. The supply of green energy facilities, e.g. wind for wind farms or inflow to water reservoirs for hydro producers, are by nature stochastic. However, there is typically a very small memory effect in supply, meaning that this year's

[3] Maintenance required to keep the machinery available on a day-to-day basis is not considered as a maintenance investment in our model. We consider any other activity that enhances the performance of existing machinery, such as e.g. surface treatment, coating of turbine blades, or actively protecting components by reducing maximum load in certain periods, as a maintenance investment. We account for the latter activity by allowing a certain fraction of profit to be lost.

[4] In practice these shutdown times vary with the project size, but we regard them as negligible in our analysis as they typically are short compared to expected project lifetimes.

supply is a poor predictor of the next year's supply. Therefore, we consider the instantaneous production quantity to be deterministic and normalized to 1. Thus, the production quantity is given by

$$R(t) = \begin{cases} 1 & t \leqslant \tau_1, \\ 1-k & \tau_1 < t \leqslant \tau_2, \\ 1 & t > \tau_2, \end{cases} \tag{3}$$

where τ_1 is the time when a maintenance investment is undertaken, and τ_2 is the time when replacement is undertaken. These stopping times are unknown in advance. The parameter k captures the loss of revenues from changed operational pattern by undertaking the maintenance investment, where k determines the lost fraction of revenues between τ_1 and τ_2.[5]

The third component that determines the profits in our model is the efficiency of the machinery, $Q(t)$, which we define as

$$Q(t) = \begin{cases} Q_E e^{-\gamma_E t} & t \leqslant \tau_1, \\ Q_E e^{-\gamma_M t} & \tau_1 < t \leqslant \tau_2, \\ Q_R e^{-\gamma_E t} & t > \tau_2, \end{cases} \tag{4}$$

where γ_E is the efficiency deterioration rate of the existing and replaced machinery, and $\gamma_M < \gamma_E$ is the efficiency deterioration rate after maintenance. Parameters Q_E and Q_R are the initial efficiency of the existing and replaced machinery, respectively. By solving the differential equation in (2), using expressions for $R(t)$ and $Q(t)$ in (3) and (4), and inserting into (1), we obtain instantaneous profits using the original machinery, the original machinery after a maintenance investment, and using a replaced, i.e. new machinery, respectively,

$$\pi(t) = \begin{cases} Q_E p e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} & t \leqslant \tau_1, \\ Q_E p(1-k) e^{-\tau_1(\gamma_E-\gamma_M)} e^{(\alpha-\gamma_M-\frac{\sigma^2}{2})t+\sigma Z(t)} & \tau_1 < t \leqslant \tau_2, \\ Q_R p e^{\tau_2\gamma_E} e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} & t > \tau_2, \end{cases}$$

where $P(0) = p$. The factor $e^{-\tau_1(\gamma_E-\gamma_M)}$ adjusts for the deterioration process before maintaining at time τ_1, and the factor $e^{\tau_2\gamma_E}$ resets the deterioration process when the machinery is replaced at time τ_2. Figure 1 illustrates price simulations and corresponding profit flow simulations. In the upper panel, three price scenarios are generated from the GBM in (2), and the dashed line is the expected price. The dashed line in the second upper panel is the efficiency, $Q(t)$, and the second lower panel shows the production quantity $R(t)$. The profit flow, $\pi(t)$, in the bottom panel is the product of price, efficiency and production quantity. Lost profit from changed operations after maintenance, k, is apparent by the vertical downward shift at τ_1, seen in the production panel. Moreover, we observe that profits are expected to increase at a higher rate (dashed line) between the stopping times τ_1 and τ_2, because $\gamma_M < \gamma_E$. Using instantaneous profits, we can formulate the optimal stopping problem for an operator of machinery with deteriorating efficiency, facing the problem of optimally timing of the maintenance investment project or the replacement project. We first specify the optimal stopping problem for each of the alternatives, and then incorporate both in the same framework.

The optimal stopping problem for the replace-only option can be formulated as

$$\begin{aligned} F_R(p) = \sup_{\tau_2} \mathbb{E}\Bigg[\int_0^{\tau_2} e^{-\rho t} Q_E p e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_R e^{-\rho\tau_2} \\ + \int_{\tau_2}^{\infty} e^{-\rho t} Q_R p e^{\tau_2\gamma_E} e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t \,\Big|\, P(0) = p \Bigg]. \end{aligned} \tag{5}$$

We assume that the decision-maker discounts the future profit at a constant exogenous rate, $\rho > \alpha - \gamma_E$. This assumption ensures that it would never be optimal to delay exercise either of the options forever, as the expected growth would exceed the discount factor.

The optimal stopping problem for the compound option is given by

[5] Changed operational pattern means that the operator chooses to deviate from the optimal production policy to reduce the deterioration rate of the machinery.

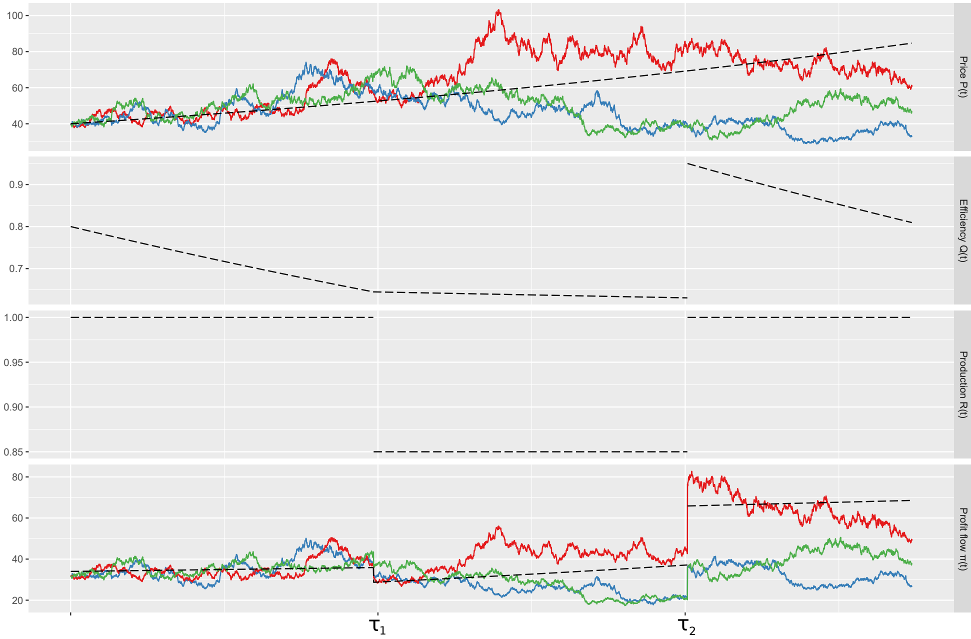


Fig. 1: Illustration of dynamics of prices and profits, with numerical values $P_0 = 40$, $Q_E = 0.80$, $Q_R = 0.95$, $\alpha = 0.025$, $\gamma_E = 0.04$, $\gamma_M = 0.004$. Colored lines in the upper panel represent price simulations, and the profit flow for each simulated price path is illustrated in the lower panel. Times τ_1 and τ_2 are times where maintenance and replacement takes place, respectively, and are unknown in advance.

$$\begin{aligned} G(p) = \sup_{\tau_1,\tau_2>\tau_1} \mathbb{E}\Bigg[&\int_0^{\tau_1} e^{-\rho t} Q_E p e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_M e^{-\rho\tau_1} \\ &+ \int_{\tau_1}^{\tau_2} e^{-\rho t} Q_E p(1-k) e^{-\tau_1(\gamma_E-\gamma_M)} e^{(\alpha-\gamma_M-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_R e^{-\rho\tau_2} \\ &+ \int_{\tau_2}^{\infty} e^{-\rho t} Q_R p e^{\tau_2\gamma_E} e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \,\Big|P(0)=p\Bigg]. \end{aligned} \tag{6}$$

Note that in this problem, the maintenance action must be undertaken before renewal/replacement. The next problem takes into account that the maintenance action can be skipped:

$$\begin{aligned} H(p) = \sup_{\tau_1,\tau_2\geqslant\tau_1} \mathbb{E}\Bigg[&\int_0^{\tau_1} e^{-\rho t} Q_E p e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_M e^{-\rho\tau_1} \mathbb{1}_{\{\tau_1<\tau_2\}} \\ &+ \int_{\tau_1}^{\tau_2} e^{-\rho t} Q_E p(1-k) e^{-\tau_1(\gamma_E-\gamma_M)} e^{(\alpha-\gamma_M-\frac{\sigma^2}{2})t+\sigma Z(t)} \mathrm{d}t - I_R e^{-\rho\tau_2} \\ &+ \int_{\tau_2}^{\infty} e^{-\rho t} Q_R p e^{\tau_2\gamma_E} e^{(\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)} \,\Big|P(0)=p\Bigg]. \end{aligned} \tag{7}$$

The firm can either (1) choose $\tau_1 < \tau_2$ which corresponds to the compound option or (2) choose $\tau_1 = \tau_2$ which corresponds to the replace-only option. If $\tau_1 = \tau_2$ the second term in (7), which is the time spent using the machinery between the maintenance investment and replacement investment, becomes zero, and the firm does not have to pay any maintenance investment costs, which is ensured by the indicator function $\mathbb{1}_{\{\tau_1<\tau_2\}}$. The only difference between (6) and (7) is that the firm can choose $\tau_1 = \tau_2$ and replace directly without maintaining first.

3. Characterization of Optimal Policies and Values

This section presents the solutions to (5), (6), and (7). We first analyze the replace-only option and the compound option separately, and then provide an analysis when both are considered in the same framework, as defined in (7). For ease of notation, we define $\mu_E = \rho - \alpha + \gamma_E$ and $\mu_M = \rho - \alpha + \gamma_M$. Similar to [1], we aim at identifying the economic conditions that trigger a renewal to restore the economic potential of the machinery. In their framework, the option to renew the asset appears attractive when the revenue stream from the existing one is low, similar to a put option. This is because exercising the renewal option entails the output revenue being restored to the original value. However, in our case, a renewal appears attractive when the profit associated with the existing machinery is high, i.e. we view the replacement option as a call option, as opposed to [1]. The difference is that, in our case, it is the price that drives the profitability of any investment alternative. Replacing when the price is low will not appear attractive as the firm has to pay a sunk cost, which will not be covered by the profit stream of the renewed machinery.

3.1 Replace-Only Option

Proposition 1 gives the value of the option to replace the machinery defined in (5).

Proposition 1 *It is optimal for the firm to replace its machinery as soon as $P(t)$ reaches the optimal threshold, given by*

$$p_R^* = \frac{\beta_E}{\beta_E - 1} \cdot \frac{\mu_E}{Q_R - Q_E} I_R, \tag{8}$$

where

$$\beta_E = \frac{1}{2} - \frac{\alpha - \gamma_E}{\sigma^2} + \sqrt{\left(\frac{\alpha - \gamma_E}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{9}$$

Thus, the value of the option to replace the existing machinery is given by

$$F_R(p) = \begin{cases} A_1 p^{\beta_E} + \dfrac{Q_E p}{\mu_E} & \text{if } p < p_R^*, \\ \dfrac{Q_R p}{\mu_E} - I_R & \text{if } p \geqslant p_R^*, \end{cases} \tag{10}$$

where

$$A_1 = \frac{I_R}{\beta_E - 1}\left[\frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_R - Q_E}{\mu_E} \cdot \frac{1}{I_R}\right]^{\beta_E}. \tag{11}$$

This option represents a single investment opportunity, and therefore closely resembles the solution of a standard real option problem, e.g. as in [11]. The values in the continuation region, $p < p_R^*$, and in the stopping region, $p \geqslant p_R^*$ are given in (10). In the waiting region, the last term is the perpetual profit without any investment, whereas the first term represents the value of the option to improve the efficiency once the profit is large enough. When the replacement option is exercised, the producer operates with increased efficiency of the machinery, Q_R, and with a deterioration rate γ_E.

3.2 Compound Option: Maintenance before Replacement

The second option, the compound option, is a constrained sequential option where the maintenance project needs to be undertaken before the replacement project. The optimal stopping problem is formulated in (6). We solve this problem backwards, where Proposition 2 gives the value of the replacement option, provided that the maintenance project already is undertaken.

Proposition 2 *With the maintenance investment option already exercised, it is optimal for the firm to replace its existing machinery as soon as $P(t)$, $t > \tau_1$, reaches the optimal threshold given by*

$$p_{M,R}^* = I_R \frac{\beta_M}{\beta_M - 1} \cdot \frac{\mu_E \mu_M}{Q_R \mu_M - (1-k) Q_E \mu_E}, \tag{12}$$

where

$$\beta_M = \frac{1}{2} - \frac{\alpha - \gamma_M}{\sigma^2} + \sqrt{\left(\frac{\alpha - \gamma_M}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{13}$$

Thus, the value of the option to replace the existing machinery, after having maintained it, is given by

$$G_R(p) = \begin{cases} B_2 p^{\beta_M} + \dfrac{(1-k)Q_E p}{\mu_M} & \text{if } p < p^*_{M,R}, \\ \dfrac{Q_R p}{\mu_E} - I_R & \text{if } p \geqslant p^*_{M,R}, \end{cases} \tag{14}$$

where

$$B_2 = \frac{I_R}{\beta_M - 1}\left[\frac{\beta_M - 1}{\beta_M} \cdot \frac{Q_R\mu_M - (1-k)Q_E\mu_E}{\mu_E\mu_M} \cdot \frac{1}{I_R}\right]^{\beta_M}. \tag{15}$$

The solution in Proposition 2 is very similar to the replace-only option, but in this case the profit flow in the continuation region $p < p^*_{M,R}$ is affected by the maintenance investment option being exercised beforehand. The value in the stopping region $p \geqslant p^*_{M,R}$ coincide with the perpetual revenues of the replace-only alternative in the stopping region in (10), i.e. the value when $p > p^*_R$. As the sequential investment alternative eventually will lead to a replacement of the machinery, the trade from going from a maintained state to a replaced state of the machinery must entail a net positive increase in the operating profits. If not, the option to replace after maintenance investment will have no value. This can be expressed as follows:

$$(1-k)Q_E\mu_E < Q_R\mu_M. \tag{16}$$

The value of the option to invest in the first stage, i.e. to undertake a maintenance investment on the existing machinery, is presented in Proposition 3.

Proposition 3 *It is optimal for the firm to invest in maintenance of its existing machinery as soon as $P(t), t < \tau_1$, reaches the optimal threshold p^*_M which implicitly solves the equation given by*

$$B_2\frac{\beta_E - \beta_M}{\beta_E}{p^*_M}^{\beta_M} + \frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_E\left((1-k)\mu_E - \mu_M\right)}{\mu_E\mu_M}p^*_M - I_M = 0, \tag{17}$$

where β_E and β_M are given in (9) and (13), respectively. Thus, the value of the option is given by

$$G_M(p) = \begin{cases} B_1 p^{\beta_E} + \dfrac{Q_E p}{\mu_E} & \text{if } p < p^*_M, \\ G_R(p) - I_M & \text{if } p \geqslant p^*_M, \end{cases} \tag{18}$$

where

$$B_1 = B_2\frac{\beta_M}{\beta_E}{p^*_M}^{\beta_M - \beta_E} + \frac{Q_E}{\beta_E} \cdot \frac{(1-k)\mu_E - \mu_M}{\mu_E\mu_M}{p^*_M}^{1-\beta_E}, \tag{19}$$

and where $G_R(p)$ and B_2 is given by (14) and (15), respectively.

It is worth pointing out that the value in the stopping region (18) is not a linear function of profit, reflecting the fact that it is an option itself. This option value is given by the option value to replace after having maintained, which is the option value in (14). We also note that the solution to the characteristic equation, β_E, differs from β_M in Proposition 2 due to the change in degradation rate from the degradation rate in a maintained state, γ_M, to the degradation rate of the non-maintained state, or equivalently, replaced state, γ_E. Since β_M is governed by the smallest degradation rate, it follows that $\beta_E > \beta_M$.

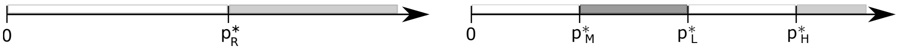
3.3 Joint Framework for the Compound Option and the Replace-Only Option

We now compare the two investment alternatives to determine when it is optimal for the firm to choose to invest sequentially or simply replace the machinery. The problem is formulated in (7). The following proposition gives the condition for when the replace-only alternative always dominates the sequential investment alternative.

Proposition 4 *It will be optimal to replace directly if the following holds for all p:*

$$\frac{I_R}{\beta_E - 1}\left[\frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_R - Q_E}{I_R\mu_E}p\right]^{\beta_E} - B_2\beta_M p^{\beta_M} - Q_E\frac{(1-k)\mu_E - \mu_M}{\mu_E\mu_M}p + I_M \geqslant 0, \tag{20}$$

where B_2 is given by (15).



(a) Relevant triggers if Proposition 4 holds. Between 0 and p_R^*, it is optimal to wait (white). For higher values it is optimal to replace (light grey).

(b) Relevant triggers if Proposition 4 does not hold. Between 0 and p_M^*, and between p_L^* and p_H^* it is optimal to wait (white), between p_M^* and p_L^* it is optimal to perform the maintenance investment (dark grey), and for values greater than p_H^* it is optimal to replace (light grey).

Fig. 2: Continuation and stopping regions when considering the compound option and the replace-only option jointly. Relevant thresholds depend on whether Proposition 4 holds or not.

The proof of Proposition 4 is provided in A.3. To determine whether the replace-only alternative is the dominant choice, the option values for the different regions need to be taken into account. If the replace-only option has a higher value than the compound option in the region before any thresholds are reached, the replace-only option will always have the higher value. If Proposition 4 holds it will never be optimal for the producer to choose the sequential investment alternative, meaning that the threshold for replacement is given by Proposition 1, and the value of $H(p)$ defined in (7) coincides with $F_R(p)$ in (10). Relevant thresholds if Proposition 4 holds are illustrated in Figure 2a.

Relevant thresholds when Proposition 4 does not hold are illustrated in Figure 2b. The solution space is divided into four different regions, $(0, p_M^*)$, $[p_M^*, p_L^*)$, $[p_L^*, p_H^*)$, and $[p_H^*, \infty)$. The threshold p_M^* is given by (12), whereas the remaining thresholds will be defined below. In the first and the third regions, it is optimal to wait, whereas the second and fourth regions are stopping regions, where investments are undertaken immediately. We refer to these four regions as waiting, maintenance, inaction, and replacement regions, respectively. The value of the option $H(p)$ can be written as

$$H(p) = \begin{cases} B_1 p^{\beta_E} + \frac{pQ_E}{\mu_E} & p < p_M^* \\ B_2 p_M^{\beta_M} + \frac{(1-k)Q_E p_M}{\mu_M} - I_M & p_M^* \leqslant p < p_L^* \\ C p^{\beta_E} + D p^{\beta_E^-} + \frac{Q_E p}{\mu_E} & p_L^* \leqslant p < p_H^* \\ \frac{Q_E p}{\mu_E} - I_R & p \geqslant p_H^* \end{cases} \tag{21}$$

In the first region, the waiting region $(0, p_M^*)$, the option value $H(p)$ coincides with the value of the option in the continuation region in (18). It is optimal to wait until the investment threshold p_M^* is reached, and then to perform the maintenance action. This is the same continuation region as in the standard model proposed by [11].

In the second region, the maintenance region $[p_M^*, p_L^*)$, it is optimal to perform the maintenance action immediately in order to reduce the degradation rate and retain the option to replace it. In this case, the value of H is given by the value of the option in the stopping region in (18), or equivalently, the difference between the value in the continuation region in (14) and the maintenance investment cost.

The third region, the inaction region $[p_L^*, p_H^*)$, is defined by two thresholds p_L^* and p_H^* that form an intermediate region of inaction around an indifference point. We show that the indifference point always is a part of the inaction region, where it is optimal for the operator to wait, in A.4. It follows that $H(p)$ on the interval $[p_L^*, p_H^*)$, is of the form $Cp^{\beta_E} + Dp^{\beta_E^-} + \frac{Q_E p}{\mu_E}$. The first two terms represent the value of waiting without having made any irreversible decisions yet. More specifically, the first term represents the option to replace directly if the price increases to p_H^*, whereas the second term represents the option to invest sequentially if the price decreases to p_L^*. The coefficients C and D, as well as the optimal stopping thresholds p_L^* and p_H^* can be found by solving the value matching and smooth pasting conditions. A feature that follows from the existence of the inaction region $[p_L^*, p_H^*)$ is that it can be optimal for the firm to undertake an investment even though the price falls. It is optimal to exercise the maintenance investment option when the price falls to p_L^*, because p_L^* is higher than p_M^*, above which it would be optimal to perform the maintenance action in the constrained sequential alternative. Moreover, it is too costly to wait until the price reaches the upper threshold p_H^* and then invest in replacement due to the time value of money. The prerogative to choose between two different projects, instead of being confined to either one of them, also increases the demand for information and creates an additional incentive to delay investment. Thus, in this particular region, it is optimal to delay the investment even though it would be optimal to invest if only the compound option was available.

In the fourth region, the replacement region $[p_H^*, \infty)$, it is optimal to replace immediately, and the option value coincides with the option value in the stopping region in (10).

3.4 Comparative Statics

In this section, we focus on the degradation rate parameters, γ_E and γ_M. Comparative statics and relevant conditions are formalized in Proposition 5 and 6.

Proposition 5 *An increase in γ_E leads to an increase in the threshold for replacement before and after maintenance:* $\frac{\partial p_R^*}{\partial \gamma_E} > 0$ *and* $\frac{\partial p_{M,R}^*}{\partial \gamma_E} > 0$.

Proposition 6 *Under the condition in (16), an increase in γ_M and γ_E leads to a decrease in the threshold for replacement after maintenance and the threshold for maintenance, respectively:*

a. $\frac{\partial p_{M,R}^*}{\partial \gamma_M} < 0$

b. $\frac{\partial p_M^*}{\partial \gamma_E} < 0$ *if and only if* $K_2 p_M^* > K_1 I_M$*, where* K_1 *and* K_2 *are provided in (78)-(79).*

Extensive numerical testing shows that the condition in Proposition 6 is met for reasonable parameter values. These results complement existing analytical results on the impact of changes in the drift parameter of the underlying stochastic process. These results include [26], where the investment threshold is monotonic in the drift, and [21], where the return function is strictly increasing in the post-investment drift rate. A difference between our model and the above-mentioned studies is that we consider two performance-enhancing projects that can boost profit, while [26] and [21] consider one project that boosts profit and the possibility to exit operations. Therefore, our results show that monotonicity in investment thresholds, with respect to the drift, are preserved also in the situation when the firm instead has the option to invest in a larger project, as opposed to exit. In the next section, we analyze the sensitivity of the thresholds with respect to other model parameters numerically.

4. Numerical Illustrations

In this section, we examine the implications of our model in a hydropower example. We analyze expected hitting times and study how our results are affected by changes in selected parameter values. Furthermore, we examine the conditions under which the optimal choice transitions from the dichotomous environment, i.e. if Proposition (4) does not hold, to when the replace-only choice is dominant over the entire state space, or vice versa.

4.1 Parameter Choices

Our baseline parameter values are given within a Norwegian hydropower context. When considering the efficiency of the existing machinery, we consider a mid-life machinery that has experienced some efficiency decay but is still some time from reaching its economic lifetime. We set 0.91 as a baseline value. The efficiency of a new machinery reflects the state of the art for this technology. This parameter varies depending on the type of machinery and on how the machinery is designed to operate with different loads. According to [30], a suitable value for Q_R is 0.95, which also gives a realistic difference between Q_E and Q_R.[6].

The degradation rate for machinery in the hydropower industry is quite low compared to other energy generating industries. In the appraisal of applications from Norwegian hydropower producers, the regulator, the Norwegian Water Resources and Energy Directorate (NVE), uses a guiding degradation rate of 0.00087[7], which is a suitable choice as a baseline value. A suitable value for the post-maintenance parameter γ_M is significantly harder to find because of the lack of empirical studies on the subject. Thus, we opt for a value which gives an obvious reduction in the degradation rate so that the firm might be willing to perform a maintenance investment. Still, the reduction cannot be too large as this would mean that the machinery virtually does not degrade, which contradicts industry observations. With this in mind, we set the value for γ_M equal to 0.0005.

The investment costs, I_R and I_M, are highly dependent on the specific hydropower plant due to the high level of idiosyncracy. However, some general characteristics of the relationship between the two do exist. First, the value of I_M should be significantly lower than I_R. This is because of the difference in the physical characteristics of the two investments. A replacement requires a brand new machinery to be made, whereas a maintenance investment is a significantly less extensive procedure. Moreover, a replacement typically means that the plant is unavailable for a longer period compared to maintenance, which means that there is a higher cost associated with production loss. To quantify the

[6] The numerical values for Q_E and Q_R are based on a Francis turbine subject to Norwegian weather and market conditions.

[7] https://www.nve.no/Media/5330/veileder-elsertifikater-ou_vannkraftverk_09-02-2017.pdf

suitable cost levels, we have consulted several experts on the area. Based on these discussions, and taking the limitations above into account, we have set I_R equal to 30 MNOK, I_M is set to 1.75 MNOK, and fraction of profits lost to changed operations in the maintenance project, k, is set to 0.005. In the setting of a hydropower producer with storage reservoirs, changed operational pattern, e.g. using the machinery for production through periods with low prices to avoid unnecessary starts and stops, implies a relatively small change in the production schedule.

Estimation of parameters σ and α often demands an in-depth analysis of different economical and site-specific factors. We use the work of [3] as a basis for σ, and set α to reflect the expected rate of inflation. For the volatility, we choose $\sigma = 0.2$ as our baseline, whereas the drift rate, α, is set to 0.025. The discount rate for a given repowering project, ρ, can vary significantly, depending on the plant's risk characteristics and financing. [3] argue for a discount rate of 7% on an investment in a setting similar to ours. However, in recent years, the discount rate has shown a downward trend [12]. A survey performed by the consulting and accounting firm [18] proposes a guiding discount rate of 5.75% for a levered hydropower firm. Their results were obtained by consulting incumbents in the Nordic hydropower industry. Since the latter study is more up to date, we choose a discount rate of 6%.

Table 1 summarizes the baseline parameter values which are used in the analysis.

Parameter Description	**Symbol**	**Baseline Value**
Discount rate	ρ	0.06
Starting efficiency of the existing machinery	Q_E	0.91
Starting efficiency of a new machinery	Q_R	0.95
Degradation rate of original machinery	γ_E	0.00087
Degradation rate of maintained machinery	γ_M	0.0005
Investment cost of replacement	I_R	30
Investment cost of maintenance	I_M	1.75
Fraction of profits lost to changed operations	k	0.005
Volatility of gross profit	σ	0.2
Growth rate of gross profit	α	0.025

Table 1: Baseline parameter values

After solving (5)-(7), the thresholds for maintenance, lower threshold for waiting, the threshold for replacing directly if options are valued separately, and the threshold for replacing directly if options are valued jointly for the baseline values are: $p_M^* = 32.1,\ p_L^* = 37.5,\ p_R^* = 69.5,\ p_H^* = 69.7$, respectively.

4.2 Value Functions

Figure 3 indicates how far the replace-only option value, $F_R(p)$, and the compound option value, $G_M(p)$, are from the joint value of the options. In Figure 3, the relative difference, $\frac{(H(p)-V(p))-(F_R(p)-V(p))}{H(p)-V(p)}$ and $\frac{(H(p)-V(p))-(G_M(p)-V(p))}{H(p)-V(p)}$ are represented by black and grey curves, respectively. We adjust the option values by $V(p) = \frac{Q_E p}{\mu_E}$, which is the profit generated by doing nothing. The value of doing nothing enters in all values, see (10), (18), and (21), and by adjusting for this value we can analyze the additional profit generated by considering the replace-only option and compound option, respectively. Several features can be observed. First, the compound option adds value in the region $p < p_H^*$. This can be seen by studying the black solid line in Figure 3, showing a positive relative difference between the joint option value, $H(p)$, and the replace-only option value $F_R(p)$ in the region $p < p_H^*$. This highlights the added value of having a smaller investment project in the portfolio. Second, having the option to replace directly adds value to the compound option in the region from $p > p_L^*$. In this region, the relative difference between the joint value $H(p)$ and the compound option value $G_M(p)$ is positive. Third, separate valuation of the replace-only option and the compound option leads to suboptimal investment thresholds. At the point $\bar{p}$ we observe that the compound option value and the replace-option value are equal. Below this, the firm would maintain immediately, and above this point the firm would wait and replace if the price reaches p_R^* under separate valuations. Under joint valuation we find triggers $p_H^* > p_R^*$ and $p_L^* < \bar{p}$ for our base case parameter values.

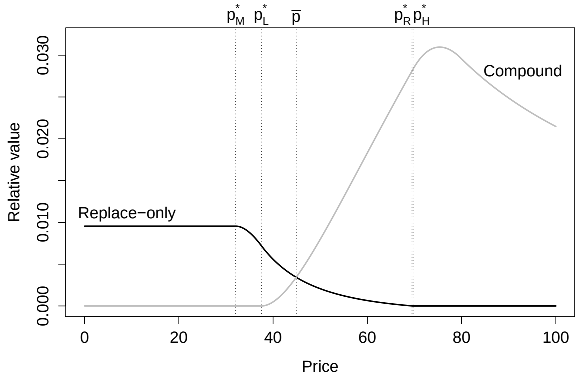
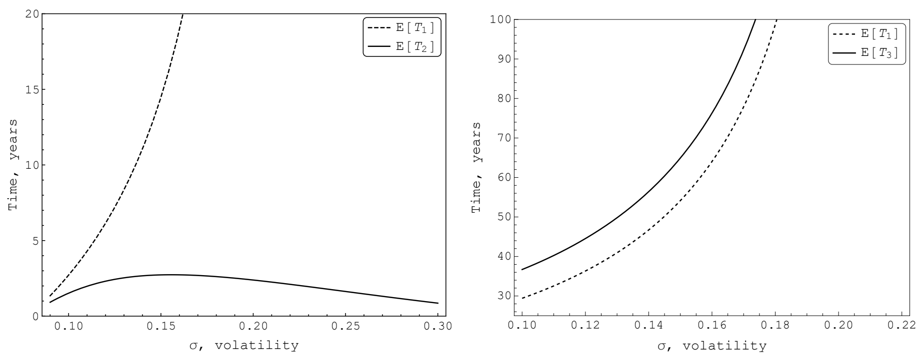


Fig. 3: Relative difference in value. The black line is $\frac{(H(p)-V(p))-(F_R(p)-V(p))}{H(p)-V(p)}$ and the grey line is $\frac{(H(p)-V(p))-(G_M(p)-V(p))}{H(p)-V(p)}$, where $V(p) = \frac{Q_E p}{\mu_E}$ is the profit stream if neither the replace-only nor the compound option is exercised. Price thresholds are plotted in vertical dotted lines.

4.3 Expected Hitting Times

Figure 4a shows the expected time to hit the threshold p^*_R, i.e. the threshold for replacement in the replace-only alternative, $\mathbb{E}[T_1]$, and the expected time to exit the inaction region, $\mathbb{E}[T_2]$, i.e. hitting either p^*_L from above or p^*_H from below, given a current price in between the thresholds.[8]



(a) The expected time to hit the replace-only threshold (dashed line) and to exit the inaction region (solid line) for $p = 50$.

(b) The expected time to hit the replace-only threshold (dashed line) and to replace after having maintained (solid line) for $p = 30$.

Fig. 4: Expected hitting times as a function of volatility for the following parameter set: $\rho = 0.06$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$.

We observe that the expected time to exit the inaction region, for the current price $p = 50$, first increases in volatility and then decreases. This can be explained by the fact that both the replacement option and compound option become more valuable as σ increases. This means that the probability of hitting the replacement threshold first decreases with σ for the initial price below this threshold, and the probability of hitting the maintenance threshold first increases with σ for the initial price above this

[8] We follow the approach presented in [43], and calculate the expected times to hit the investment thresholds. In Figures 4a-4b, $\mathbb{E}[T_1] = \frac{1}{-\sigma^2/2+\alpha-\gamma_E} \ln\left(\frac{p^*_R}{p}\right)$, $\mathbb{E}[T_3] = \frac{1}{-\sigma^2/2+\alpha-\gamma_M} \ln\left(\frac{p^*_{M,R}}{p}\right)$, and the expected time to exit the inaction region is $\mathbb{E}[T_2] = \frac{1}{0.5\sigma^2-\alpha+\gamma_E}\left(\ln\left(\frac{p}{p^*_L}\right) - \ln\left(\frac{p^*_H}{p^*_L}\right)\left(1-\left(p/p^*_L\right)^{1-2(\alpha-\gamma_E)/\sigma^2}\right) / \left(1-\left(p^*_H/p^*_L\right)^{1-2(\alpha-\gamma_E)/\sigma^2}\right)\right)$.

threshold. Moreover, we observe that having the option to replace directly delays the expected time for the firm to act by 2.4 years for our baseline of $\sigma = 0.2$, compared to only having the compound option. In addition, if the firm does not have the option to maintain, the expected time until the investment is 80 years. Figure 4b shows the expected time to replace with and without the compound option as a function of volatility, at the current price $p = 30$. At this price, it is optimal to exercise the compound option immediately for values of σ between 0 and 0.20. Figure 4b shows that the expected time spent in the maintained state is $\mathbb{E}[T_3] = 65$ for $\sigma = 0.15$, while the expected time to replace, without the option to maintain, is $\mathbb{E}[T_1] = 54$. Hence, a large investment is expected to be delayed significantly with the replacement option embedded in the maintenance option, compared to only having the option to replace.

4.4 Sensitivity Analysis of Investment Thresholds

As comparative statics for model parameters are difficult to obtain analytically, we perform the sensitivity analysis using numerical illustrations for reasonable parameter values. In the subsequent figures, we use dark grey and light gray shading to illustrate the stopping region for the compound option and the replace-only option, respectively. The investment threshold when the replace-only option is dominant over the entire state space is p_R^*. The maintenance region lies between p_M^* and p_L^*. The region above p_H^* is the replacement region.

We start by examining the effect of volatility, σ in Figure 5.

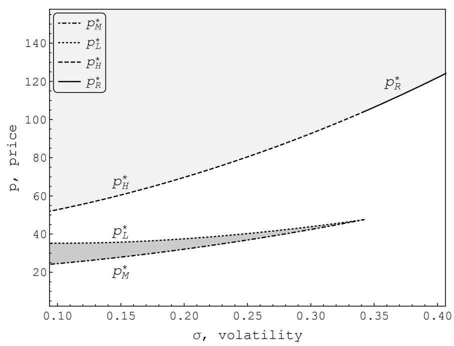


Fig. 5: The effect of varying volatility for the following parameter set: $\rho = 0.06$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region.

As can be seen, the second inaction region $[p_L^*, p_H^*)$ increases in volatility. Interestingly, both p_L^* and p_H^* increase in volatility, until the replace-only alternative becomes dominant (around $\sigma = 0.34$). Thus, it can be optimal for the hydropower firm to undertake an investment even though the price falls. [19] find a similar behavior when considering mothballing and exit options. However, unlike in their analysis, where the lower threshold decreases in volatility, in our model, p_L^* increases in σ. This is due to the added value of the replacement option that is available after having exercised the maintenance option.

The effect of changing the initial efficiency of the existing machinery, Q_E, is shown in Figure 6.

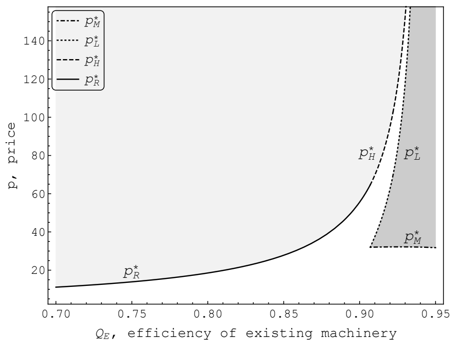
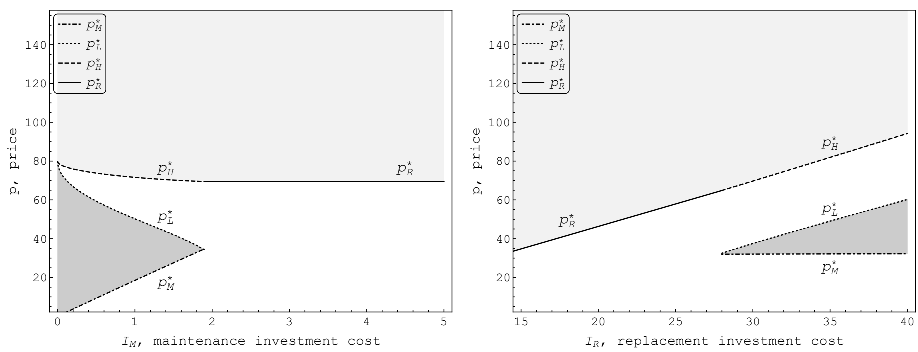


Fig. 6: The effect of varying pre-investment machinery efficiency for the following parameter set: $\rho = 0.06$, $\sigma = 0.20$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region.

In contrast to the volatility, a lower efficiency of the machinery makes the replace-only option dominant. To understand this, note that if the efficiency is already low, the payoff from replacing and restarting the degradation process dominates that of the maintenance investment, which only slows down degradation, even though the cost is higher. In addition, we observe that when the dichotomous environment is prevailing, all thresholds except p_M^* experience a significant increase when Q_E approaches Q_R. This is because when the net benefit of replacing the machinery is smaller, the firm requires a drastically higher price level before it is profitable to replace. However, the same effect has little influence on the threshold to perform the maintenance investment, p_M^*. This can be explained by two contradicting incentives. On the one hand, the firm has an incentive to invest in maintenance earlier because reducing the degradation rate on a machinery with higher efficiency extends its economic lifetime more substantially, and hence delays the subsequent replacement. On the other hand, the threshold is indirectly affected by the replacement option through the implicit equation (17). This gives the hydropower producer an incentive to delay the investment because replacement is no longer as imminent with such a high efficiency of the initial machinery. The dominating effect is the former, which leads to a reduction in the threshold.

In Figure 7a and 7b, the value of I_M and I_R vary, respectively.



(a) For $I_R = 30$. (b) For $I_M = 1.75$.

Fig. 7: The effect of varying investment costs for the following parameter set: $\rho = 0.06$, $\sigma = 0.20$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region.

Increasing the replacement cost I_R increases all thresholds. In addition, the replace-only alternative is dominating only for low values of I_R. This is because the value gained from the maintenance project before an eventual replacement is not high enough compared to directly replacing the machinery for low replacement cost. As the maintenance investment cost I_M increases, the inaction region becomes larger and the investment threshold p_M^* increases. The thresholds p_L^*, and p_H^*, however, decline with I_M. This is because for larger I_M, the replace-only option becomes more attractive.

Figure 8 shows the effect of changing the discount rate ρ.

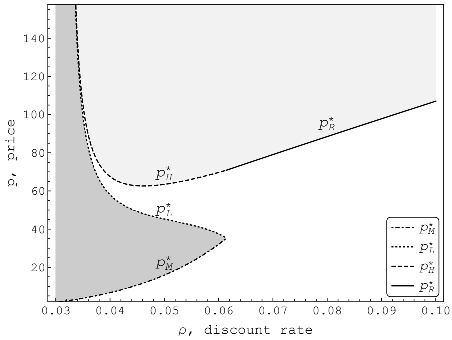
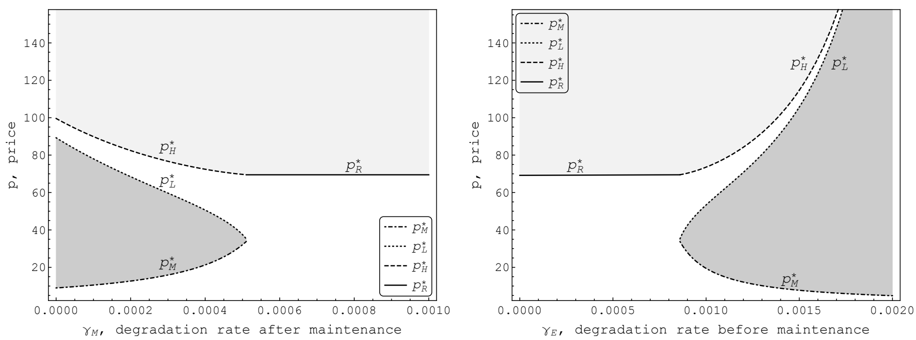


Fig. 8: The effect of varying discount rate for the following parameter set: $\sigma = 0.20$, $Q_E = 0.91$, $Q_R = 0.95$, $\gamma_E = 0.00087$, $\gamma_M = 0.0005$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region.

We observe that increasing ρ effectively devalues the sequential investment, making the single investment choice dominant for higher values of ρ. A higher discount rate dampens the relative importance of the change in drift after the maintenance investment, so the sequential alternative loses its attractiveness, and thus the gain of a lifetime extension is discounted too much to be a viable choice for the firm. Therefore, the maintenance investment region shrinks as a result of an increase in the threshold p_M^* and a decline of p_L^*. In the case of the replace-only threshold, however, it is not as clear-cut. In fact, the threshold p_H^* deceases for low values of ρ and increases for large values of ρ. This happens due to two opposing effects. On the one hand, the replacement option becomes more attractive than the maintenance option as ρ increases. On the other hand, however, increasing the cost of capital reduces the value of the expected future cash flows from replacement relative to the expected future cash flows from continuing current operations. Similar opposing effects for the discount rate have been found in the literature, e.g. [28] who found that the entry timing of a firm who considers entering a market with an active incumbent, is non-monotonic in discount rate. For large values of ρ the discounting effect dominates, and the firm is incentivized to replace the machinery earlier, making it the more valuable option. It is also worth mentioning that the waiting region $[0, p_M^*)$ and the inaction region $[p_L^*, p_H^*)$ are expanding with ρ. This is caused by an increased value in the option to invest in either of the two alternatives and hence increases the opportunity cost of investing immediately.



(a) For $\gamma_E = 0.00087$.

(b) For $\gamma_M = 0.0005$.

Fig. 9: The effect of varying degradation rate for the following parameter set: $\rho = 0.06$, $\sigma = 0.20$, $Q_E = 0.91$, $Q_R = 0.95$, $I_R = 30$, $I_M = 1.75$, $k = 0.005$, and $\alpha = 0.025$. The compound option is exercised in the dark grey region, the replace-only option is exercised in the light grey region, and white is the waiting region.

The effects when changing the degradation rates γ_E and γ_M are shown in Figure 9a and 9b, respectively. These Figures supplement Propositions 5 and 6 by illustrating the investment thresholds based on a realistic set of parameter values for a hydropower operator. We observe that when γ_M is low, the dichotomous environment dominates, whereas for large γ_M the firm will choose to replace directly. This is because the more efficient the maintenance investment is, implying a smaller γ_M, the more valuable the maintenance option becomes. Varying γ_E has the opposite effect, there the dichotomous environment dominates when γ_E is large. These effects can be explained by the increased benefits of maintenance for low γ_M and large γ_E, which leads to an increase of the investment thresholds which define the inaction region. At the same time, p^*_M decreases drastically under the dichotomous regime. Both of these changes can be explained by the attractiveness of operating after the maintenance project when the relative difference between γ_E and γ_M escalates. By executing the maintenance project earlier, the benefit is reaped sooner and the time until a replacement is required is prolonged due to the decelerated degradation rate. When γ_E increases, the inaction region, $[p^*_L, p^*_H)$, shrinks, while the maintenance region, $[p^*_M, p^*_L)$, expands rapidly. Moreover, note that p^*_R is independent of γ_M, since this parameter only relates to the maintenance project in the sequential investment alternative. We also see that p^*_R is quite insensitive to changes in γ_E. This is most likely due to the drift rate being dominated by the profitability growth α.

4.5 Limitations

In this paper, we provide a novel perspective on managing assets with deteriorating performance by quantifying the effect of correctly accounting for the mutually exclusive mitigation options. We emphasize the real options perspective within the field of maintenance and renewal. To keep the model tractable, we make several assumptions, e.g., deterministic efficiency deterioration and Gaussian relative changes in long-term prices. However, it is valuable to extend the current framework to account for potential other real-world features, such as stochastic efficiency deterioration, or breakdown risk. This can be done by, for example, assuming that the degradation rate follows a GBM with negative drift, and that the breakdown risk is represented by a Poisson jump process or a gamma process [41]. Intuitively, the additional source of uncertainty in efficiency will make the option to wait more valuable in line with the standard real options theory. However, such extensions will require numerical solutions. Another interesting extension is to add other options to the investment portfolio, for example, sequential maintenance options. We demonstrate our framework on a hydropower example, and the framework allows to provide insights for investment decisions in other power generating industries, e.g., wind power. In order for the model to fit other particular industries, the numerical values will have to be adjusted. Compared to hydro-specific estimates, this will likely imply slightly higher values of the efficiency deterioration [40] and in the case of stochastic deterioration, also a higher volatility of the efficiency deterioration process in the case of wind energy.

5. Conclusions

This paper examines the decisions of a firm concerning a potential maintenance or replacement of machinery within a real options framework. We present a tractable model, applicable to general asset management, where we examine the conditions for when it is optimal to undertake investments, and possibly switch from the minor maintenance project to the major replacement project. The paper contributes to the literature on replacement options in mutually exclusive investment projects within the real options framework.

We find that there is a possibility that the investment region is dichotomous. That is, the investment region is no longer a connected set, similar to the findings in [9] and [19]. We demonstrate implications of our model by studying investment alternatives for a hydropower producer facing a deteriorating efficiency of its generation units. Our analysis shows that hydropower producers are likely to operate in an environment where the dichotomous investment environment is present. We further find that the dichotomous environment is more likely to be present when the maintenance option is valuable, and that the maintenance investment becomes preferable when the volatility, the discount rate, the maintenance investment cost, and the deterioration rate after maintenance are low, and when the replacement cost and the deterioration rate before maintenance are high. By analyzing expected hitting times, we find that that the replacement decision may be delayed significantly in expectation with the replacement option embedded in the maintenance option. Furthermore, for intermediate values of the current price, the expected time for the producer to act is first increasing in volatility, then decreasing, when the sequential investment alternative and replace-only alternative are valued jointly. However, if the alternatives are valued separately, the inaction region does not exist, and the producer would undertake maintenance investment immediately. This shows the importance of properly identifying the potential alternatives available in the project portfolio.

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A. Proof of Propositions

A.1 Proposition 2

The stopping value is given by

$$G_R(p) = \mathbb{E}\left[\int_0^\infty e^{-\rho t} Q_R p e^{-\left((\alpha-\gamma_E-\frac{\sigma^2}{2})t+\sigma Z(t)\right)} \mathrm{d}t\right] - I_R = \frac{Q_R p}{\mu_E} - I_R. \tag{22}$$

(23)

In the continuation region, the problem is almost the same as in Proposition 1. The differences between the two are that the initial condition and drift of the profit flow. Using parameters from the profit flow after maintenance at time τ_1, we obtain the following expression for the value of the replacement option in the continuation region,

$$G_R(p) = B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M}, \tag{24}$$

where

$$\beta_M = \frac{1}{2} - \frac{\alpha-\gamma_M}{\sigma^2} + \sqrt{\left(\frac{\alpha-\gamma_M}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{25}$$

To find the optimal stopping value, p_M^*, the value matching and smooth pasting conditions must be met. These are given by the following expressions:

Value matching:

$$B_2 {p_M^*}^{\beta_M} + \frac{(1-k)Q_E p_M^*}{\mu_M} = \frac{Q_R p_M^*}{\mu_E} - I_R. \tag{26}$$

Smooth pasting:

$$B_2 \beta_M {p_M^*}^{\beta_M - 1} + \frac{(1-k)Q_E}{\mu_M} = \frac{Q_R}{\mu_E}. \tag{27}$$

Solving these equations to find p_M^* and B_2, yields

$$p_M^* = \frac{\beta_M}{\beta_M - 1} \cdot \frac{\mu_E \mu_M}{Q_R \mu_M - (1-k)Q_E \mu_E} \cdot I_R, \tag{28}$$

$$B_2 = \frac{I_R}{\beta_M - 1}\left[\frac{\beta_M - 1}{\beta_M} \cdot \frac{Q_R \mu_M - (1-k)Q_E \mu_E}{\mu_E \mu_M} \cdot \frac{1}{I_R}\right]^{\beta_M}. \tag{29}$$

Thus, the value of the option to replace in the sequential alternative is given by

$$G_R(p) = \begin{cases} B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M} & \text{if } p < p_M^*, \\ \frac{Q_R p}{\mu_E} - I_R & \text{if } p \geqslant p_M^*. \end{cases} \tag{30}$$

A.2 Proposition 3

In the stopping region, one pays the investment cost to obtain the second option. Thus, the value of the option is given by

$$G_M(p) = B_2 p^{\beta_M} + \frac{(1-k)Q_E p}{\mu_M} - I_M. \tag{31}$$

In the continuation region, the Bellman equation must hold. This equation is given by

$$\rho G_M dt = \mathbb{E}[dG_M] + Q_E p_0 dt. \tag{32}$$

Solving this equation for the homogeneous and the particular solution yields the following expression for the option value:

$$G_M = B_1 p^{\beta_E} + \frac{Q_E p}{\mu_E}, \tag{33}$$

where

$$\beta_E = \frac{1}{2} - \frac{\alpha-\gamma_E}{\sigma^2} + \sqrt{\left(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}. \tag{34}$$

At the investment threshold, p_M^*, the following value matching and smooth pasting conditions must hold:

Value matching:

$$B_1 {p_M^*}^{\beta_E} + \frac{Q_E p_M^*}{\mu_E} = B_2 {p_M^*}^{\beta_M} + \frac{(1-k)Q_E p_M^*}{\mu_M} - I_M. \tag{35}$$

Smooth pasting:

$$B_1 \beta_E {p_M^*}^{\beta_E - 1} + \frac{Q_E}{\mu_E} = B_2 \beta_M {p_M^*}^{\beta_M - 1} + \frac{(1-k)Q_E}{\mu_M}. \tag{36}$$

The expression for p_M^* cannot be solved analytically, but implicitly solves the following equation:

$$p_M^{*\,\beta_M} B_2 \frac{\beta_E - \beta_M}{\beta_E} + p_M^* \frac{\beta_E - 1}{\beta_E} \cdot \frac{Q_E((1-k)\mu_E - \mu_M)}{\mu_E \mu_M} - I_M = 0. \tag{37}$$

Given the value of p_M^*, one can calculate the value of B_1 as

$$B_1 = B_2 \frac{\beta_M}{\beta_E} \cdot p_M^{*\,\beta_M - \beta_E} + \frac{Q_E}{\beta_E} \cdot \frac{(1-k)\mu_E - \mu_M}{\mu_E \cdot \mu_M} p_M^{*\,1-\beta_E}. \tag{38}$$

Thus, the value of the option is given by

$$G_M(p_0) = \begin{cases} B_1 p^{\beta_E} + \dfrac{Q_E p}{\mu_E} & \text{if } p < p_M^*, \\ B_2 p^{\beta_M} + \dfrac{(1-k)Q_E p}{\mu_M} - I_M & \text{if } p \geqslant p_M^*. \end{cases} \tag{39}$$

A.2.1 Proof of Unique Solution for p_M^*

The implicit solution for p_M^* in the sequential alternative, given by (37), is of the following form:

$$\Psi(p_M^*) = A p_M^{*\,\beta_M} + B p_M^* - C = 0. \tag{40}$$

To prove the existence of a unique solution for p_M^* we start by defining the domain of the above function, which is restricted to positive values only, i.e. $p_M^* \in [0, \infty\rangle$. We also know that β_M is the positive root of the quadratic equation given by (13), and is thus greater than 1 (see [11]).

We can prove that the constants A, B and C are strictly positive. A consists of two terms, namely $(\frac{\beta_E - \beta_M}{\beta_E})$ and the constant B_2 defined by (15). First, we know that $\beta_E > \beta_M$ due to the fact that $\gamma_E > \gamma_M$. This means that $(\frac{\beta_E - \beta_M}{\beta_E})$ is always positive. In order for B_2 to be positive, we must assume that

$$Q_R \mu_M > (1-k) Q_E \mu_E. \tag{41}$$

This inequality signifies that the net benefit of replacing the machinery after first having upgraded it is positive. Combined, these two parts yield that A is always positive. Furthermore, to assure a positive B, we require that

$$(1-k)\mu_E \geqslant \mu_M. \tag{42}$$

This is the same as assuming that the net benefit from upgrading the pre-existing machinery is either zero or strictly positive, which must be true, otherwise the option would have no intrinsic value. The last constant, C, represents the investment cost of upgrading and is by definition always strictly greater than zero. As we know that the constants are always positive, we can take the derivative of (40) to show that the function is monotonically increasing

$$\Psi'(p_M^*) = A \beta_M p_M^{*\,\beta_M - 1} + B. \tag{43}$$

Since we have already confirmed that $\beta_M > 1$, this is a monotonically increasing function for $p_M^* \in [0, \infty\rangle$. By applying the intermediate value theorem, we therefore know that (40) has a unique solution for p_M^*.

A.3 Proposition 4

We know that when all thresholds are reached, the value of the sequential option is I_M to the right relative to the option to replace directly. It is also known that the derivative of the option value in the stopping region of G_M is less than the derivative of the option value in the stopping region for F_R. From value matching and smooth pasting, we know that the values in the continuation regions will always converge towards the values in their respective stopping regions in terms of both values and derivatives. Using this, and the fact that the first derivatives of all option values in the continuation regions are strictly positive, it can be shown that the sequential option will first converge towards a less steep function and thereafter converge towards the right-shifted parallel line. It will therefore never cross the option value which converges towards the stopping value of the replace-only option.

Let us, therefore, consider the option values where both alternatives are in the first inaction region. In the case where F_R is more valuable, the following inequality will hold:

$$F_R^{\mathcal{C}} - G_M^{\mathcal{C}} \geqslant 0 \tag{44}$$

Inserting the relevant expressions from (10) and (18), yields

$$\left[A_1 p^{\beta_E} + \frac{p Q_E}{\mu_E} \right] - \left[B_1 p^{\beta_E} + \frac{p Q_E}{\mu_E} \right] = A_1 p^{\beta_E} - B_1 p^{\beta_E} \geqslant 0. \tag{45}$$

The inequality simplifies to

$$A_1 - B_1 \geqslant 0. \tag{46}$$

We now substitute these parameters by their expressions given in Eqs. (11) and (19)

$$\frac{I_R}{\beta_E-1}\left[\frac{\beta_E-1}{\beta_E}\cdot\frac{Q_R-Q_E}{\mu_E}\cdot\frac{1}{I_R}\right]^{\beta_E}-\left[B_2\frac{\beta_M}{\beta_E}p_M^{*\ \beta_M-\beta_E}+\frac{Q_E}{\beta_E}\cdot\frac{(1-k)\mu_E-\mu_M}{\mu_E\mu_M}p_M^{*\ 1-\beta_E}\right]\geqslant 0. \tag{47}$$

By reformulation,

$$I_R\frac{\beta_E}{\beta_E-1}\left[\frac{\beta_E-1}{\beta_E}\cdot\frac{Q_R-Q_E}{I_R\mu_E}p_M^*\right]^{\beta_E}-B_2\beta_M p_M^{*\ \beta_M}-Q_E\frac{(1-k)\mu_E-\mu_M}{\mu_E\mu_M}p_M^*\geqslant 0. \tag{48}$$

Thus, if (48) holds, replace only will be the dominant choice in the entire state space.

A.4 The indifference point

The indifference point, $\tilde{p}^*$, never belongs to any of the stopping regions and will always be located between p_L^* and p_H^*. We show this in this subsection. The point is implicitly given by the following equation:

$$B_2\tilde{p}^{*\ \beta_M}+\frac{(1-k)Q_E\mu_E-Q_R\mu_M}{\mu_E\mu_M}\tilde{p}^*-(I_M-I_R)=0, \tag{49}$$

where β_M and B_2 are given by (13) and (15), respectively. When contemplating investment, the firm will select the alternative which generates the highest net expected profit, given the current price p. The value of investment is therefore the highest stopping value of the two alternatives, i.e. $\max\{G_M^S, F_R^S\}$. When the two alternatives are equally valuable, it is called the indifference point. This point is given as the solution to

$$B_2\tilde{p}_0^{*\ \beta_M}+\frac{(1-k)Q_E\tilde{p}^*}{\mu_M}-I_M=\frac{Q_R\tilde{p}^*}{\mu_E}-I_R. \tag{50}$$

Rearranging, we get

$$B_2\tilde{p}^{*\ \beta_M}+\left[\frac{(1-k)Q_E\mu_E-Q_R\mu_M}{\mu_E\cdot\mu_M}\right]\tilde{p}^*-(I_M-I_R)=0. \tag{51}$$

For values of p below the indifference point, the value of the sequential option exceeds that of the replace-only option, and vice versa for values above the indifference point.

The intuition for why the indifference point never belongs to the stopping region is quite instructive. We start with the heuristic argument put forward by [11] to justify the smooth pasting condition. Suppose that the current profit is equal to the indifference point. Then, by waiting for a small time dt, the firm can observe the evolution of the profit without having to make any decisions. The intuitive idea is that by waiting a little longer, the firm can observe the next step of p and choose to invest on either side of $\tilde{p}^*$. The resulting average pay-off is thus greater than the payoff obtained by investing at the indifference point itself since the payoff at this point is not differentiable. This is an implication that follows directly from Jensen's inequality, which states that, given a convex function, equally spaced changes in p_0 give rise to unequally spaced changes in $V(p_0)$. In particular, $V[\mathbb{E}(p)]\leqslant\mathbb{E}[V(p)]$. This remains true even though the average payoff must be discounted because it occurs at a later time dt. The reason is that, for a Brownian motion, the movements are proportional to $\sqrt{dt}$, which is valid for the expected payoff. However, the cost due to discounting is of magnitude dt, and thus when dt is small, the $\sqrt{dt}$-term dominates. The result is that the firm is better off by waiting for more information, which gives rise to an inaction region. Thus, whenever the inequality given by Proposition 4 does not hold, in contrast to [11], the stopping region is dichotomous, and the optimal investment decision is not governed by a simple trigger strategy.

A.5 Values for C, D, p_L^* and p_H^*

To find the values for C, D, p_L^* and p_H^*, value matching and smooth pasting conditions must be met at the two thresholds. The conditions at p_L^* are given by

Value matching:

$$Cp_L^{*\ \beta_E}+Dp_L^{*\ \beta_E^-}+\frac{Q_Ep_L^*}{\mu_E}=\frac{Q_E(1-k)p_L^*}{\mu_M}+B_2p_L^{*\ \beta_M}-I_M. \tag{52}$$

Smooth pasting:

$$\beta_ECp_L^{*\ \beta_E-1}+\beta_E^-Dp_L^{*\ \beta_E^--1}+\frac{Q_E}{\mu_E}=\frac{Q_E(1-k)}{\mu_M}+\beta_MB_2p_L^{*\ \beta_M-1}. \tag{53}$$

Rearranging (53), we get

$$C=\left[\frac{Q_E(1-k)}{\mu_M}+\beta_MB_2p_L^{*\ \beta_M-1}-\frac{Q_E}{\mu_E}-\beta_E^-Dp_L^{*\ \beta_E^--1}\right]\frac{p_L^{*\ 1-\beta_E}}{\beta_E}. \tag{54}$$

Inserting this into (52) and rearranging, yields

$$D=Q_E\frac{\beta_E-1}{\beta_E-\beta_E^-}\cdot\frac{\mu_E(1-k)-\mu_M}{\mu_E\mu_M}p_L^{*\ 1-\beta_E^-}+B_2\frac{\beta_E-\beta_M}{\beta_E-\beta_E^-}p_L^{*\ \beta_M-\beta_E^-}-I_M\frac{\beta_E}{\beta_E-\beta_E^-}p_L^{*\ -\beta_E^-}. \tag{55}$$

By using the expression for D given by (55) in (54), we get

$$C=Q_E\frac{\beta_E^--1}{\beta_E^--\beta_E}\cdot\frac{\mu_E(1-k)-\mu_M}{\mu_E\mu_M}p_L^{*\ 1-\beta_E}+B_2\frac{\beta_E^--\beta_M}{\beta_E^--\beta_E}p_L^{*\ 1-\beta_E}-I_M\frac{\beta_E^-}{\beta_E^--\beta_E}p_L^{*\ -\beta_E}. \tag{56}$$

On the other end of the interval, the conditions at p_H^* are given by
Value matching:

$$Cp_H^{*\ \beta_E} + Dp_H^{*\ \beta_E^-} + \frac{Q_E p_H^*}{\mu_E} = \frac{Q_R p_H^*}{\mu_E} - I_R. \tag{57}$$

Smooth pasting:

$$\beta_E C p_H^{*\ \beta_E - 1} + \beta_E^- D p_H^{*\ \beta_E^- - 1} + \frac{Q_E}{\mu_E} = \frac{Q_R}{\mu_E}. \tag{58}$$

Rearranging (58), we get

$$C = \left[\frac{Q_R - Q_E}{\mu_E} - \beta_E^- D p_H^{*\ \beta_E^- - 1}\right] \frac{p_H^{*\ 1-\beta_E}}{\beta_E}. \tag{59}$$

Inserting this in (57) and solving for D, yields

$$D = \frac{\beta_E - 1}{\beta_E - \beta_E^-} \cdot \frac{Q_R - Q_E}{\mu_E} p_H^{*\ 1-\beta_E^-} - I_R \frac{\beta_E}{\beta_E - \beta_E^-} p_H^{*\ -\beta_E^-}. \tag{60}$$

By using the expression for D given by (60) in (59), we get

$$C = \frac{\beta_E^- - 1}{\beta_E^- - \beta_E} \cdot \frac{Q_R - Q_E}{\mu_E} p_H^{*\ 1-\beta_E} - I_R \frac{\beta_E^-}{\beta_E^- - \beta_E} p_H^{*\ -\beta_E}. \tag{61}$$

The expressions for C and D in both ends of the inaction region can be generalized by using the following expressions:

$$M_{i,j}(p) = \frac{\beta_i - 1}{\beta_i - \beta_j} \cdot \frac{Q_R - Q_E}{\mu_E} p^{1-\beta_j} - I_R \frac{\beta_i}{\beta_i - \beta_j} p^{-\beta_j}, \tag{62}$$

$$N_{i,j}(p) = Q_E \frac{\beta_i - 1}{\beta_i - \beta_j} \cdot \frac{\mu_E(1-k) - \mu_M}{\mu_E \mu_M} p^{1-\beta_j} + B_2 \frac{\beta_i - \beta_M}{\beta_i - \beta_j} p^{\beta_M - \beta_j} - I_M \frac{\beta_i}{\beta_i - \beta_j} p^{-\beta_j}. \tag{63}$$

By setting equal the two expressions for both constants, it is possible to rearrange the initial system to
For C:

$$N_{21}(p_L^*) = M_{21}(p_H^*). \tag{64}$$

For D:

$$N_{12}(p_L^*) = M_{12}(p_H^*). \tag{65}$$

These expressions can now be used to obtain the thresholds p_L^* and p_H^* by using a numerical solution procedure.

A.6 Proposition 5

We start with the first part. To prove this, we follow [28]. Taking the derivative of expression (8) with respect to γ_E gives

$$\frac{\partial p_R^*}{\partial \gamma_E} = \frac{I_R}{Q_R - Q_E} \frac{-\mu_E \frac{\partial \beta_E}{\partial \gamma_E} + \beta_E(\beta_E - 1)}{(\beta_E - 1)^2}, \tag{66}$$

where

$$\begin{aligned} \frac{\partial \beta_E}{\partial \gamma_E} &= -\frac{\frac{\alpha - \gamma_E}{\sigma^2} - \frac{1}{2}}{\sigma^2 \sqrt{(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}} + \frac{1}{\sigma^2} \\ &= \frac{\beta_E}{\sigma^2 \sqrt{(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}}. \end{aligned} \tag{67}$$

Rearranging and using the expression for β_E, $\sqrt{(\frac{\alpha-\gamma_E}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} = \sigma^2 \left(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2}\right)$, gives

$$\begin{aligned} \frac{\partial p_R^*}{\partial \gamma_E} &= \frac{I_R}{(Q_R - Q_E)} \left(\frac{\beta_E}{\beta_E - 1} - \frac{\mu_E \frac{\partial \beta_E}{\partial \gamma_E}}{(1-\beta_E)^2}\right) \\ &= \frac{I_R}{(Q_R - Q_E)} \left(\frac{\beta_E}{\beta_E - 1} - \frac{\mu_E \frac{\beta_E}{\sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}}{(1-\beta_E)^2}\right) \\ &= \frac{I_R \beta_E}{(Q_R - Q_E)} \left(\frac{(\beta_E - 1)(\sigma^2 \beta_E - \frac{\sigma^2}{2} + \alpha - \gamma_E) - \mu_E}{(1-\beta_E)^2 \sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}\right) \\ &= \frac{I_R \beta_E}{(Q_R - Q_E)} \left(\frac{\frac{\sigma^2}{2}\beta_E^2 + (\alpha - \gamma_E - \frac{\sigma^2}{2})\beta_E - \rho + \frac{1}{2}(\beta_E - 1)^2 \sigma^2}{(1-\beta_E)^2 \sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}\right) \\ &= \frac{I_R \beta_E \sigma^2}{2(Q_R - Q_E)(1-\beta_E)^2 \sqrt{(\alpha - \gamma_E - \frac{\sigma}{2})^2 + 2\rho}}. \end{aligned} \tag{68}$$

Differentiating (12) with respect to γ_E gives

$$\frac{\partial p^*_{M,R}}{\partial \gamma_E} = \frac{\mu_M I_R Q_R \beta_M^2}{(\beta_M - 1)(Q_R\mu_M - (1-k)Q_E\mu_E)^2}, \tag{69}$$

which is greater than zero for all $\beta_M > 1$, which shows the second part of the proposition.

A.7 Proposition 6

Differentiating (12) with respect to γ_M gives

$$\frac{\partial p^*_{M,R}}{\partial \gamma_M} = I_R\mu_E \left(\frac{-\mu_M(Q_R\mu_M - (1-k)Q_E\mu_E)\frac{\partial \beta_M}{\gamma_M} - Q_E\mu_E(1-k)\beta_M(\beta_M - 1)}{(Q_R\mu_M - (1-k)Q_E\mu_E)^2(\beta_M-1)^2} \right) < 0, \tag{70}$$

since $\frac{\partial \beta_M}{\gamma_M} > 0$, $\beta_M > 0$ and $Q_R\mu_M - (1-k)Q_E\mu_E > 0$ if the condition in (16) is met. For the maintenance threshold, let f denote the implicit equation (17). We have

$$0 = \frac{df}{d\gamma_E} = \frac{\partial f}{\partial \gamma_E} + \frac{\partial f}{\partial p^*_M}\frac{\partial p^*_M}{\partial \gamma_E}, \tag{71}$$

which implies

$$\frac{\partial p^*_M}{\partial \gamma_E} = -\frac{\frac{\partial f}{\partial \gamma_E}}{\frac{\partial f}{\partial p^*_M}}. \tag{72}$$

Computing the denominator

$$\frac{\partial f}{\partial p^*_M} = B_2\beta_M\frac{\beta_E - \beta_M}{\beta_E}p_M^{*\beta_M - 1} + \frac{\beta_E - 1}{\beta_E}Q_E\frac{\mu_E(1-k) - \mu_M}{\mu_E\mu_M}, \tag{73}$$

which is positive given the precondition in (16). We are left to show that $\frac{\partial f}{\partial \gamma_E}$ is positive when the condition holds. Differentiating f with respect to γ_E gives

$$\frac{\partial f}{\partial \gamma_E} = \frac{1}{\mu_E^2\beta_E^2}\left(a_1 + a_2 + a_3\right), \tag{74}$$

where

$$a_1 = \frac{\partial \beta_E}{\partial \gamma_E}\mu_E\left(\beta_M p_M^{*\beta_M}\mu_E B_2 + Q_E p^*_M\left(\frac{\mu_E(1-k) - \mu_M}{\mu_M}\right)\right),$$
$$a_2 = -\frac{\partial B_2}{\partial \gamma_E}p_M^{*\beta_M}\mu_E^2\left(\beta_E\beta_M - \beta_E^2\right),$$
$$a_3 = Q_E p^*_M(\beta_E^2 - \beta_E).$$

Differentiating B_2 with respect to γ_E gives

$$\frac{\partial B_2}{\partial \gamma_E} = -\frac{Q_R}{\mu_E^2}\left(\frac{\beta_M - 1}{\beta_M}\left(\frac{\mu_M Q_R - (1-k)Q_E\mu_E}{I_R\mu_E\mu_M}\right)\right)^{\beta_M - 1} = -\frac{Q_R\beta_M\mu_M}{\mu_E(\mu_M Q_R - (1-k)Q_E\mu_E)}B_2 \tag{75}$$

Inserting for $\frac{\partial B_2}{\partial \gamma_E}$ and $\frac{\partial \beta_E}{\gamma_E}$ given by (67) into (74), and eliminating $B_2 p_M^{*\beta_M}$ using the implicit equation f gives

$$\frac{\partial f}{\partial \gamma_E} = K_0(K_2 p^*_M - K_1 I_M), \tag{76}$$

where

$$K_0 = \frac{1}{\mu_E\beta_E(\beta_E - \gamma_E)(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2})}, \tag{77}$$

$$K_1 = \beta_M\beta_E\left(\left(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2}\right)(\beta_E - \beta_M)C_2 - \frac{\mu_E^2}{\sigma^2}\right), \tag{78}$$

$$K_2 = \frac{\mu_E^2}{\sigma^2}C_1(\beta_E - \beta_E\beta_M) + \left(\beta_E - \frac{1}{2} + \frac{\alpha - \gamma_E}{\sigma^2}\right)(\beta_E - 1)(\beta_E - \beta_M)(Q_E + C_1C_2\beta_M), \tag{79}$$

where

$$C_1 = \frac{Q_E((1-k)\mu_E - \mu_M)}{\mu_E\mu_M},$$
$$C_2 = \frac{Q_R\mu_E\mu_M}{\mu_M Q_R - (1-k)Q_E\mu_E}.$$

Hence, $\frac{\partial f}{\partial \gamma_E} > 0$ if and only if

$$K_2 p^*_M > K_1 I_M, \tag{80}$$

and combined with (72), (73), and (16), this implies $\frac{\partial p^*_M}{\partial \gamma_E} < 0$, which shows the second part of the proposition.

Paper II

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Andreas Kleiven, Simon Risanger, Stein-Erik Fleten

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II

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Paper III

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Andreas Kleiven, Selvaprabu Nadarajah, Stein-Erik Fleten

Working paper.

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III

Paper IV

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