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THE USE OF MATHEMATICAL PROGRAMMING TO DETERMINE OPTIMAL PRODUCTION AND DRILLING SCHEDULE IN AN OFFSHORE OIL FIELD, A CASE STUDY FROM THE BARENTS SEA

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ABSTRACT

A field with two neighboring reservoirs was discovered in the Barents Sea in 2013 and 2014. After a successful extended well test of an appraisal well in 2018 and initial field planning tasks, a preliminary drilling and production schedule was proposed based on cross-domain collaboration and group work involving several disciplines. In this paper, mathematical programming is employed to model and optimize the economic value of the project in order to determine the best drilling and production schedule for the field. The optimization includes some of the technical constraints considered by the field development team while also considering uncertainties such as reservoir size, productivity of well, and cost. These have been systematically evaluated by using simulation-based optimization (sampling). The results were that the use of mathematical programming allows the field planner to evaluate several scenarios within a reasonable time frame, thereby enabling rapid changes in the decisions to respond to new information and risk considerations in a dynamic environment. This paper illustrates the benefits of utilizing mathematical programming in early field planning to optimize the drilling and production schedule.

Keywords: Oil field development; Mathematical programming; Production and drilling scheduling optimization; Barents Sea; Case study

NOMENCLATURE

NPV	Net Present Value
NPD	Norwegian Petroleum Directorate
CAPEX	Capital expenditure
OPEX	Operational costs
LHS	Latin Hypercube Sampling
FPSO	Floating Production, Storage and Offloading
CPU	Central Processing Unit
GAP	Difference between the best lower and upper bounds
q_i	Flow rate, $i \in \{o = oil, g = gas, w = water\}$
N_p, G_p, W_p	Cumulative Production of Oil, Gas, Water
GOR	Gas-Oil Ratio
WC	Water Cut

INTRODUCTION

It is normal that a production and drilling schedule must be decided during the early stages of field planning even though there is limited and uncertain information. In subsequent stages of field development, these values are often frozen despite the availability of new information that might prompt adjustments to the original field development plan. As the process of creating a field development plan is complex and time-consuming it is often impractical to fully explore all alternatives and scenarios.

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Nowadays, oil and gas companies create field development plans based on cross-domain collaboration and group work from numerous related disciplines including subsurface, drilling, well completion, facilities, subsea, marine, pipeline, process, cost, economics, and the environment. In this process, each discipline provides their recommendations for development alternatives. For instance, a submarine pipeline engineer will determine suitable pipeline routes considering the seabed survey which typically does not match the shortest path, thus incurring in additional costs. Ideally, there should be enough time in field planning to thoroughly explore all scenarios and alternatives flagged by the experts (and their combinations) and narrow down the best alternatives before the detailed design phase starts. However, there are some challenges that hinder this: i) organizational and digital barriers when transferring information and digital twins between disciplines; ii) delays in information update (e.g. update of uncertainty due to new tests becoming available); and iii) the time required to evaluate multiple scenarios is often excessive.

The use of mathematical programming in field development problems started in the 1960s [1]. A review of relevant literature can be found in (Durrer and Slater, 1977 [2]; Sullivan, 1988 [3]; Tavallali, 2016 [4]; Khor et al., 2017 [5]; Nasir et al., 2021 [6]). Field development optimization has also been extensively investigated from the reservoir management and closed-loop optimization perspective [7–9]. When formulating the optimization problem, two main directions are usually followed: minimization of the investment cost [10–14], and maximization of the NPV [6, 9, 15–20]. In general, the methods targeting investment cost minimization deal with scheduling (e.g. drilling scheduling) and how to place the platforms, wells, manifolds, pipelines and other relevant production facilities. An example of this application is in Klose and Drexel, 2005 [21] that reviews and summarizes the location and distribution problem. The methods and works that target the maximization of the NPV are typically focused on increasing the revenue and the cash flow analysis, mainly by improving the production planning and well allocation. The advantage of mathematical programming methods is that they can determine and select the best scenarios considering a large number of available alternatives. However, there are few papers that presents real case applications instead of synthetic models for study [22]. This probably due to the difficulties in deploying these methods on real fields. As a result companies often opt for solutions such as commercial software that are incompatible with the proposed methods, they face difficulties with the inclusion of technical constraints to the optimization problem, or excessive computational time is required for the optimization.

Some of early-stage field development planning problem is formulated in a mixed-integer linear programming model which employed piecewise-linear function to approximate the model non-linearities. Examples can be found in the works from Angga [23] and Gonzalez [24]. With an efficient optimization model, it is possible to perform sensitivity studies for various scenarios and development alternatives.

On the other hand, planning the development of a field is usually made under risks and uncertainties that stem from both the surface and subsurface. Uncertainties that could change the project's profitability are the main concern in decision-making processes. Previous works have addressed uncertainty analysis applied to field development problems [9, 25–30]. For example, Awasthi et al. [9] formulated a two-stage stochastic programming model to compute an improved expected value of NPV for uncertainties in oil price and productivity indices. Gupta and Grossmann [29] present three examples of using different approaches (deterministic, complex fiscal rules and stochastic programming) to solve the uncertainty in oilfield development planning problem. Capolei et al. [30] used a sequential Monte Carlo algorithm to generate 100 permeability field realizations of a 2D reservoir when assessing the uncertain geological scenarios. Among the common approaches are sensitivity analysis, spider plots, decision trees, and probabilistic methods, all of which to some extent require probabilistic sampling to perform uncertainty management and deploy optimized field and reservoir management plans. However, the drawback of using probabilistic methods is their prohibitive computational time due to the requirement of multiple optimization runs for the various probable scenarios. Thus, as the computational complexity of probabilistic methods inherits the complexity of the optimization model. A crucial step before considering the uncertainty handling mechanism is to improve the efficiency of the optimization method. Finally, an ideal field development plan should not only include recommendations for an optimum development strategy with an implementation plan for a given scenario, but also provide good estimates regarding the risks involved in executing the proposed plan [31].

In this paper, we formulate the early-phase planning of the offshore Alta-Gohta oil field using mathematical programming with focus on determining well production allocation over time and the well drilling scheduling in order to maximize the field economic value. An initial field development plan (termed the *operator's plan* in this paper) was obtained with the traditional inter-disciplinary work used as the starting point for the study. Firstly the model is validated[1] by verifying its ability to reproduce the results obtained with traditional methods considering the same constraints. After the validation stage, the drilling sequence was removed from the mathematical model, and the model is optimized to find the optimal of the well drilling plan and production schedule. In addition, an uncertainty analysis was performed using this verified mathematical programming model. A total of 100 cases were evaluated using the Latin Hypercube Sampling (LHS) method considering the uncertainties in the reservoir size (initial oil in place), field productivity and cost. The main goal of this paper is to demonstrate the benefits of em-

[1] validating means a verifying of integrated reservoir-production model by using mathematical optimization.

ploying mathematical programming in the early-phase planning of a real offshore oil field, especially when uncertainty needs to be quantified.

The paper is organized as follows. The next section presents a description of the field studied in this work. Then, the next sections present the methodology and a mathematical programming model for field development optimization respectively. We demonstrate the benefits of applying mathematical optimization and uncertainty analysis to field development planning of the Alta-Gohta field in the following two sections. The final section presents the conclusions of the paper.

FIELD DESCRIPTION

It is estimated that the Barents Sea holds about 37% of the remaining Norwegian hydrocarbon resources. A significant percentage of this has yet to be proven. The undiscovered resources from the Barents Sea are estimated at 2505 million Sm^3 oil equivalent in 2019 [32].

The development of offshore hydrocarbon fields in the Barents Sea is challenging due to factors such as its metocean conditions, icing, the vulnerability of the Arctic environment, the remoteness and the difficulty to access existing gas transportation infrastructure. Offshore structures have stricter requirements and must be designed differently from those used in the North Sea and Norwegian Sea. New developments are therefore required to make large investments because of the lack of existing infrastructure to deliver oil and gas to the market.

There are two fields currently in operation in the Barents Sea, Snøhvit (gas) and Goliat (oil). Johan Castberg (oil) is a recently sanctioned field and is currently under development. The Wisting and Alta-Gohta fields are oil fields currently in the early-stage planning phase. Existing oil fields in the Barents Sea chose the concept of an FPSO linked to seabed templates with wells (producer and injector) [33]. Due to the long distance between discoveries and the nearest shoreline, the produced gas from oil fields is often re-injected into the reservoir for pressure support.

The Gohta and Alta reservoirs were discovered in 2013 and 2014 respectively. Their location is shown in Fig. 1, about 190 kilometers northwest of Hammerfest, the closest town on the Norwegian mainland. According to a report from the Norwegian Petroleum Directorate (NPD) [34], the estimated total surface volume of recoverable oil of two discoveries is 29.79 million Sm^3 (based on an update released on December 31st, 2019). The Alta reservoir is considered to be the larger of the two. The water depth in the license area is around 380 m, and the distance between the two discoveries is 15 km. The distance from the reservoirs to the shore is 160 km. Lundin Energy Norway is the operator of both fields. The proven reserves of these discoveries are not large enough to be economically viable to develop separately as stand-alone fields. An integrated field development plan with a common FPSO was considered and studied [35].

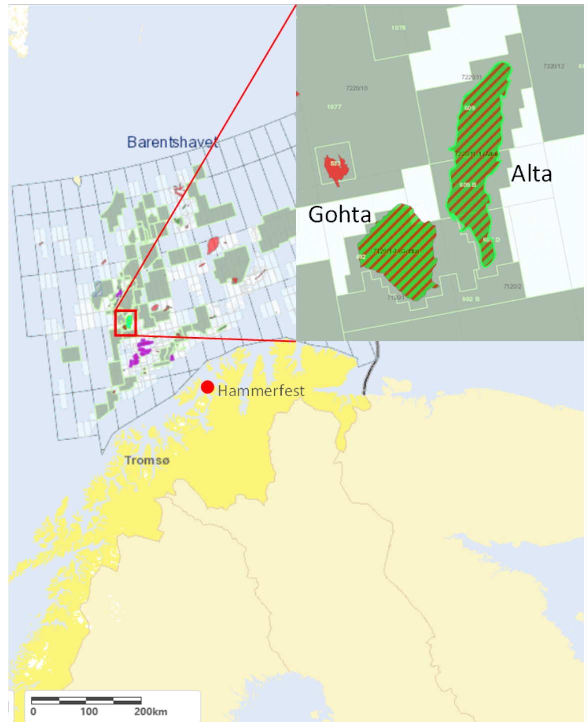

FIGURE 1. ALTA-GOHTA FIELD LOCATION. Ref: NPD

Last year, it was decided that the Alta-Gohta will not be developed as a stand-alone project because it is unprofitable (as stated in Lundin's year-end report 2019) [36, 37]. However, the authors believe it is still worthwhile to use it as a case study to assess the effectiveness of mathematical optimization in determining optimal development plans.

The original plan was to develop the field using 9 production wells with 2 or 3 subsea templates linked to the FPSO [35]. Here, 6 out of the 9 producers are in Alta and 3 wells in Gohta. The produced water and gas are designed to be re-injected into the reservoir with the purpose of pressure maintenance, but also because of the high cost of transporting the gas the long distance to shore.

METHODOLOGY

Hatvik et al. [38] reported that, since the discovery of the Alta-Gohta field, an integrated model of reservoir, well and network has been used to determine the best development alternative. In this study, the operator provides a field development plan (operator's plan) based on integrated teamwork involving multiple disciplines. The operator's plan has a specified drilling schedule and allocated production rate for each well during the whole lifetime of the field. In this plan, 9 producing wells are recommended to produce from these two reservoirs and their trajectories and locations are given. We will refer hereafter to this plan as the *preset* development plan and its drilling schedule as the 'original' schedule. We used the operator's plan as the basis for our analyses and as a baseline for comparison.

Two important factors that determine the revenue and economic value of a petroleum asset are the oil price and production. In this paper, we focus only on the effect of production. Using the provided *preset* plan as a starting point, we address two questions: i) how many wells are necessary and in what sequence is it best to drill them to maximize the economic value of the project? ii) how to allocate the production in time and by well to maximize the economic value of the project? At the same time, producing all recoverable reserves within the production horizon.

The methodology proposed in this work is described as a workflow depicted in Fig. 2. It consists of four main parts:

1. Flag of relevant field development scenarios;
2. Development of a proxy model to estimate production profiles considering active well combination and initial oil in place;
3. Formulation and execution of the mathematical optimization model to maximize the Net Present Value (NPV);
4. Uncertainty analysis of the optimization results using simulation-based optimization (sampling).

The first step is to list all relevant well combinations. For a given field with a number of w wells, if all combinations are relevant there are 2^w scenarios that must be studied.

The second step concerns the generation of a proxy model for the integrated asset to predict production profiles. The proxy model consists of production potential curves representing the maximum oil production that a group of wells can deliver as a function of cumulative production and active well combination. The shape of the production potential versus cumulative production depends on the type of fluid and recovery mechanism acting in the reservoir. The value of the production potential is set as the upper bound of the field oil rate (decision variable) in the optimization. Production potential curves have been used extensively in the past, for example in previous works from Gupta and Grossmann, 2017 [29]; Lin and Floudas, 2003 [39]; Goel et al., 2006 [40]; Tarhan et al., 2009 [41]. Gonzalez showed that a production potential proxy model compared favorably against the output of a 3D reservoir model generated in Eclipse, with an average relative error of 8% [42]. Angga [23] found that a production potential proxy model could reproduce the output from a coupled model of material balance and network with excellent accuracy. Stanko [43] showed that a production potential proxy model compared favorably against the output of a 3D reservoir model coupled with a network model.

In this study, we generated a coupled model of network, well and reservoir using the commercial software IPM (Petex) [44] and modeled the reservoirs with two non-communicating tanks (using material balance equations). The model was validated against output from an integrated model of the well, network

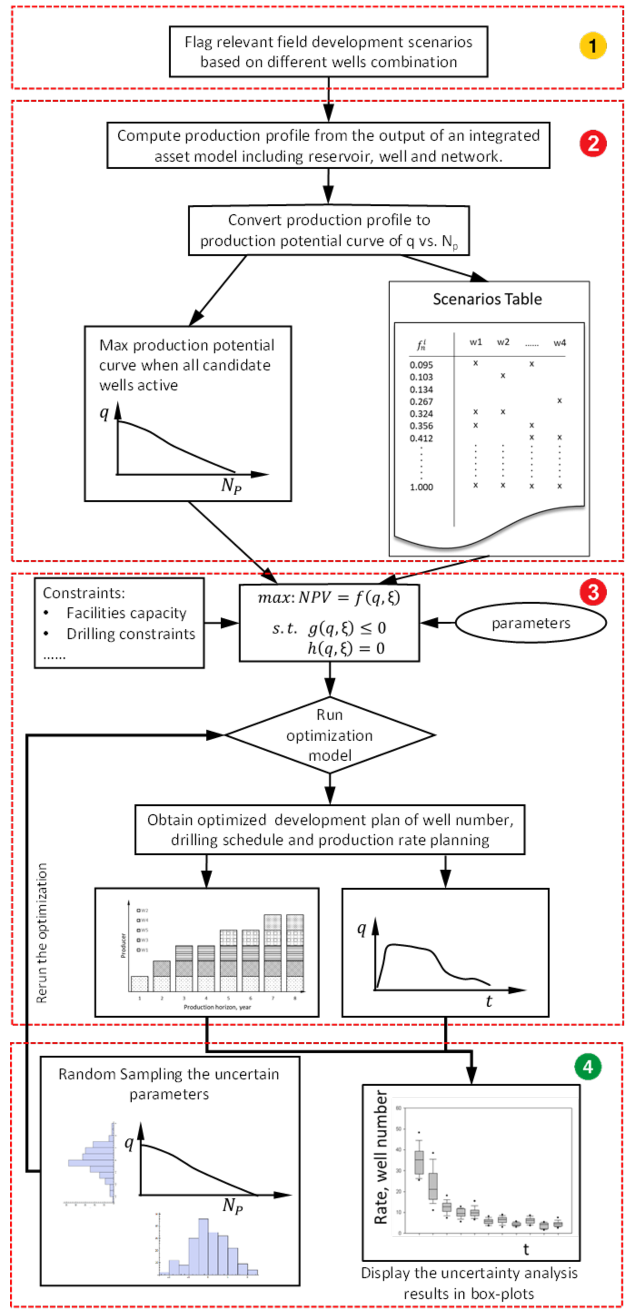

FIGURE 2. THE WORKFLOW OF FIELD DEVELOPMENT OPTIMIZATION

and reservoir that modeled the reservoirs using a 3D simulator. Due to confidentiality reasons, it was not possible to use this second integrated model directly in this study nor show its details. Production potential curves were extracted from the first model using the procedure outlined by Stanko [43].

Due to the fact that the two reservoirs, Alta and Gohta, are non-communicating and that they produce separately to the FPSO it is possible to generate production potential curves for each separately. The steps are:

1. Run a simulation with all wells active and producing as much as possible during the complete production horizon. Collect data of total oil rates and cumulative production;
2. Compute the maximum oil rate at initial time for all relevant well combinations. For each well combination considered, compute the ratio between the maximum oil rate at initial time of the combination and the maximum oil rate at initial time when all wells are active ($f_n^i = \frac{q_{pp,i}}{q_{pp,max}}$). Create a table listing all relevant well combinations and their corresponding fractional factor.

At a given time step, the maximum oil production that the reservoir can deliver is given by the production potential at the current cumulative production (found by interpolating in the table found in step 1) multiplied by the fraction of the particular well combination (found by performing a look-up in the table generated in step 2).

This proxy model captures the production interdependency of the wells and the effect of reservoir pressure decline on production. Moreover, it assumes that all produced gas is re-injected.

The workflow for creating production potential curves and the scenarios table is described in Appendix B.

The third step consists of setting up and running an optimization model to automatically determine the optimal drilling scenario and production schedule. The objective function of the optimization model is maximizing the project's NPV, which is the sum of the discounted cash flows during the lifetime of the field. Apart from the two variables of drilling schedule and production profile, which are constrained by the outputs from the second step, additional constraints of the field architecture and the parameters of cost and economics are also included in the optimization model.

The fourth step in the workflow is the evaluation of uncertain parameters such as cost, initial oil in place in both reservoirs and well productivity. We did not consider the uncertainty on the oil price and it was assumed constant during the production horizon. Each uncertain parameter has a probability distribution function associated to it. To quantify the effect of uncertainty on the optimization results, random samples were drawn using LHS to generate the uncertain scenarios. The optimization was run for each of the scenarios and a frequency analysis was performed on the results.

The use of production potential curves as a proxy model for production performance will probably not be appropriate for complex production systems such as compartmentalized reservoirs, highly heterogeneous reservoirs, or capturing realistic injection processes. Another limitation of the current implementation of this approach is that reservoir pressure is not modeled explicitly and we not track the production and cumulative production of individual wells, but rather the production and cumulative production of the total number of wells producing from the reservoir.

However, the authors believe that this approach is still appropriate for early phases of field development when limited data is available, and reservoir models are highly uncertain, under construction or unavailable. Moreover, this approach allows the formulation of an efficient optimization model that has high computational efficiency, low runtime and therefore is suitable to use in extensive uncertainty analyses.

MATHEMATICAL PROGRAMMING

In field development optimization, we intend to obtain the optimal yearly wells combination, drilling sequence, and production allocation, by maximizing the NPV of the Alta-Gohta field development. In this work, we apply a mixed-integer linear programming model to optimize the early-phase development plan for the Alta-Gohta field. We present a nonlinear conceptual model for the problem, which contains a high-level mathematical formulation.

The indexes, sets, parameters, and variables used in the nonlinear model are listed in Appendix A. The objective function and constraints are described below.

The objective is defined as:

$$\max\ NPV = \sum_t \frac{p_o \cdot q^f_{o,t} - CAPEX(t) - OPEX(t)}{(1+\gamma)^t} \tag{1}$$

whereas the constraints are the following:

$$CAPEX(t) = CAPEX_{Facility}(t) + CAPEX_{Well}(t), \forall t \in \mathscr{T} \tag{2}$$

$$OPEX(t) = OPEX_{Rate}(t) + CAPEX_{Non-rate}(t), \forall t \in \mathscr{T} \tag{3}$$

$$q^f_{i,t} = \sum_{r=1}^{R} q^r_{i,t}\ ,\ \forall i \in \{o,g,w\}, \forall\ t \in \mathscr{T}\ ,\ \forall\ r \in \mathscr{R} \tag{4}$$

$$D^{max} \geq \sum_{w \in \mathscr{W}} x^r_{w,t} - \sum_{w \in \mathscr{W}} x^r_{w,t-1},\ \forall\ t \in \mathscr{T} \setminus \{0\}, \forall r \in \mathscr{R} \tag{5}$$

$$x^r_{w,t-1} \leq x^r_{w,t}\ ,\ \forall\ t \in \mathscr{T} \setminus \{0\}, \forall r \in \mathscr{R} \tag{6}$$

$$q^f_{i,t} \leq q^{f,max}_i\ ,\ \forall i \in \{o,g,w\}, \forall\ t \in \mathscr{T} \tag{7}$$

$$q^{f,max}_l \geq q^f_{o,t} + q^f_{w,t}\ ,\ \forall\ t \in \mathscr{T} \tag{8}$$

$$N^r_{p,t} = N^r_{p,t-1} + q^r_{o,t-1}\ ,\ \forall\ t \in \mathscr{T} \setminus \{0\}\ ,\ \forall r \in \mathscr{R}, (N^r_{p,0} = 0) \tag{9}$$

$$G^r_{p,t} = G^r_{p,t-1} + q^r_{g,t-1}\ ,\ \forall\ t \in \mathscr{T} \setminus \{0\}\ ,\ \forall r \in \mathscr{R}, (G^r_{p,0} = 0) \tag{10}$$

$$W^r_{p,t} = W^r_{p,t-1} + q^r_{w,t-1}\ ,\ \forall\ t \in \mathscr{T} \setminus \{0\}\ ,\ \forall r \in \mathscr{R}, (W^r_{p,0} = 0) \tag{11}$$

$$G^r_{p,t} = f^r_G(N^r_{p,t})\ ,\ \forall\ t \in \mathscr{T}, \forall r \in \mathscr{R} \tag{12}$$

$$W^r_{p,t} \quad = f^r_W(N^r_{p,t}) \,, \forall\, t \in \mathscr{T}, \forall r \in \mathscr{R} \tag{13}$$
$$q^r_{o,t} \quad \leq f^i_n(\mathbf{x}^r_t) \cdot f^r_q(N^r_{p,t}) \,, \forall\, t \in \mathscr{T}, \forall r \in \mathscr{R} \tag{14}$$

The objective function to be maximized is defined in Eq. (1), which is the NPV of the Alta-Gohta oil field. Since the only sales product is oil, the revenue comes from the production rate of oil solely, and is also affected by the oil price. The two items referring to the cost are the capital expenditures (CAPEX Eq. (2)) such as the cost of facilities ($CAPEX_{Facility}$), well construction costs ($CAPEX_{Well}$) and operational costs ($OPEX$ Eq. (3)) such as well maintenance, topside maintenance, personnel, among others. $CAPEX_{Well}(t)$ is a linear function of the number of producing wells $\sum x^r_{w,t}$ drilled at a given time t. The OPEX is divided into non-rate dependent $OPEX_{None-rate}$ that is a linear function of the number of producers in time t and rate-dependent $OPEX_{Rate}$, a linear function of the oil rate q_o, gas rate q_g and water rate q_w in time t. $CAPEX_{Facility}$ is a linear function of the maximum rates of oil, gas and water and the maximum number of producers. Therefore, the revenue and cost functions depend on the number of producers and the production rates of oil, gas and water.

For all cases studied, it was assumed that the number of injectors is present from year zero and remain constant in time. The presence of injectors adds constant amounts to $CAPEX_{Facility}$, $CAPEX_{Well}$ and $OPEX_{None-rate}$.

The constraint in Eq. (4) denotes the field production as the total production coming from the different reservoirs. The constraint in Eq. (5) limits the number of yearly drilled wells to the maximum yearly drilling capacity of the field. Eq. (6) ensures that the well remains opened after being activated. The constraint in Eq. (7) bounds the total commingled fluids from both fields by the maximum production rates at the processing facilities for oil, gas, water, and the total liquid. The gas rate and water rate are back-calculated using Eq. (10) and Eq. (11) respectively, in addition to the function of cumulative oil rate in Eq. (12) and Eq. (13).

In the production potential approach, constraints are placed on the total production of a group of wells, not on individual wells, and we do not track the production of each well. To calculate the flow rate of gas and water, we use curves of cumulative gas production and cumulative water production versus cumulative oil production, assuming that GOR and WC are functions of cumulative oil production only and are not affected by the well combination.

The constraint presented in Eq. (14) specifies the upper bound on the oil rate at time/year t. The term $f^r_q(N_{p,t})$ denotes the maximum production rate as a function of cumulative production $N_{p,t}$, while $f_n(\mathbf{x}^r_t)$ is the well combination factor of the field at time/year t for a given combination $\mathbf{x}^r_t$, which is illustrated in the Appendix B of scenarios table in Fig. 10. It ensures that the production does not exceed the maximum feasible oil rate the production system can deliver. Depending on the mechanisms of

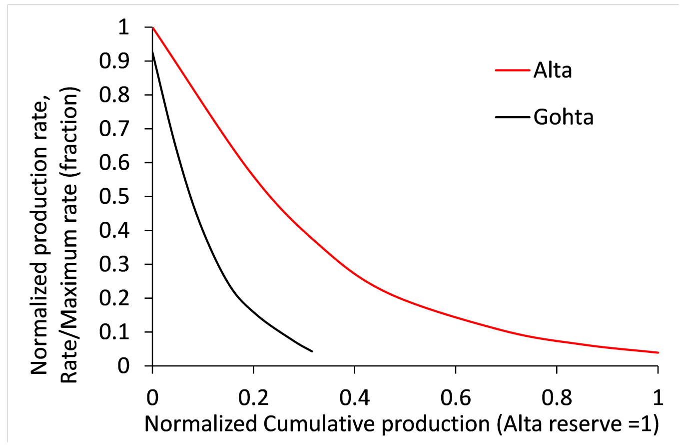

FIGURE 3. NORMALIZED MAXIMUM PRODUCTION POTENTIAL CURVES OF ALTA AND GOHTA FIELD

fluid communication between reservoir, the upper bound of the maximum production potential curve can be defined either individually or commingled. In the Alta-Gohta field, the maximum production rates were specified for Alta and Gohta separately, meaning that Eq. (14) is imposed for each reservoir separately.

The production potential of a specific well group is automatically calculated based on this fractional value and as a function of cumulative production. It should be kept in mind that the production potential curves are non-linear and the well combination involves using binary and integer variables, thus the optimization is a mixed-integer nonlinear problem, which is hard to solve. We employ the piecewise-linear functions to linearize these nonlinear functions and yield a computationally feasible mathematical formulation for the problem.

ALTA-GOHTA FIELD DEVELOPMENT OPTIMIZATION

The proposed workflow and mathematical model are applied to optimize the Alta-Gohta field development. Data to create the scenarios table are taken from the operator's plan. As previously mentioned, this plan employed an integrated model of the well, network and reservoir that modeled the reservoirs using a 3D simulator. More detailed information of the integrated model can be found from the works of Alkindira [45]. This integrated model was validated against output from the operator's original model. Production potential curves for Alta and Gohta were extracted from this model and are presented in Fig. 3 (curves have been normalized due to confidentiality reasons).

The linear cost functions were derived from cost data provided by the operator. We then validated the mathematical optimization model against the operator's plan. We have considered the following assumptions when building the models:

- There is no underground flow communication between Alta and Gohta;

- The production from Alta is hydraulically decoupled from the production of Gohta, as they are transported through separate risers to the FPSO;
- The producing GOR and WC of each reservoir are a function of the cumulative oil production from the reservoir;
- Each well and well combination has distinct production performance;
- Once a well is drilled, the production will start in the same year and the drilled well will not be shut-in;
- All produced gas and water will be re-injected into the reservoir for the purpose of pressure support. It is assumed that a fixed number of injectors are available for this task from production start. The number of injectors and location is not considered in the optimization;
- There are a total of 3 subsea templates to place producing wells. Moreover, there are specific wells that are allowed to be drilled in each template. The costs of these templates are included in the mathematical model in the term $CAPEX_{Facility}$.

The parameter values of oil, gas, water processing capacities used in the optimization are not listed here for a confidential reason, but constraints of oil price, maximum number of wells per year, and discount rate are provided in Tab. 1. The optimization model was formulated using the software AMPL [46] and solved using the Gurobi solver [47], the computational performance is tested on a computer of Intel(R) Core(TM) i7-8565U CPC @ 1.80 Hz 1.99 GHz 64 bytes.

TABLE 1. INPUT DATA

p_o :	60	$USD/barrel$
D^{max} :	3	$well/year$
γ :	0.08	

The optimization model is validated by assessing how accurately the proposed mathematical model reproduces the results of the operator's plan. To do this, we input the 'original' drilling schedule given in the *preset* plan to the optimization model, but keep the production rate as a variable. Fig. 4 is a comparison of the production profile between the *preset* plan and the optimization results. The figure shows that the results are comparable, except for the high rates predicted by the optimization model at the beginning and an early reduction in rate from Alta at the end of the field life. The reason for this difference could be that the time steps used in the optimization model and in the *preset* studies are different (1 year versus 1 month respectively). However,

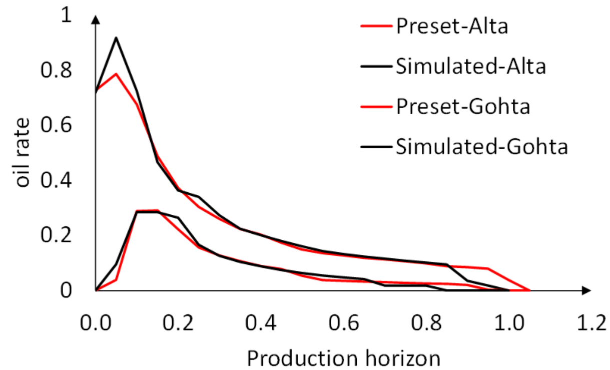

FIGURE 4. A COMPARISON OF THE OIL PRODUCTION PROFILE BETWEEN PRESET MODEL AND OPTIMIZATION MODEL WITH SAME DRILLING SEQUENCE, THE PRODUCTION RATE(Y-AXIS) AND HORIZON(X-AXIS) ARE NORMALIZED TO THE MAXIMUM PROCESSING CAPACITY AND LIFETIME

the time steps were kept sparse as reductions in the time step in the optimization model increases significantly the computational time.

After the model was validated, we rerun the optimization including the drilling schedule as a variable in addition to the production profile. The purpose of this simulation is to check whether the mathematical model can find a better field development plan than the present in terms of NPV. The computational performance of this simulation and the previous one is given in Tab. 2. As can be seen from the table, there is an increase in the number of constraints, variables and CPU time when the drilling schedule is included as a variable. A solution with slightly higher NPV than the original was found (100.72% vs 100%).

TABLE 2. COMPARISON OF COMPUTATION PERFORMANCE

	With drilling schedule	Without drilling schedule
Constraints:	4931	5275
Variables:	8782	8790
CPU time:	8.32 seconds	1834.51 seconds
GAP:	0%	0%
NPV:	100%	100.72%

The production profiles obtained are shown in Fig. 5. As seen in the figure, the Gohta oil production profile is same in both cases, but Alta has a slightly different profile (areas with the largest differences are marked with a red box in the plot).

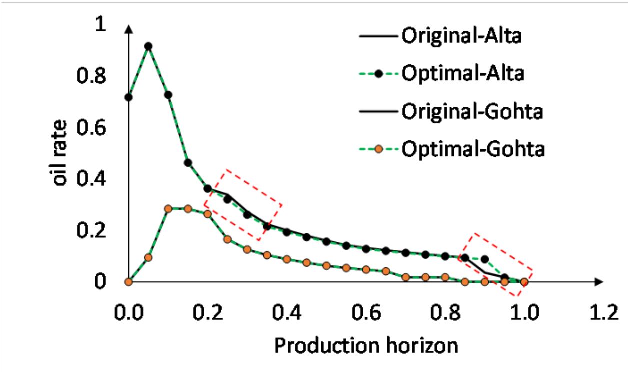

FIGURE 5. THE PRODUCTION PROFILE FROM THE OPTIMIZED MODEL AND ORIGINAL WELL SCHEDULE, SAME NORMALIZATION AS FIGURE. 4

In Tab. 3, we list the wells drilled in Alta and Gohta each year from the 'original' schedule and the optimization model. Only 8 wells were selected in the optimal plan, instead of 9 as suggested in the *preset* plan. The candidate well $w1$ from Alta field was inactivated by the optimization model, but the results reached a similar production profile (Fig. 5). This means that the operator could potentially produce a similar oil rate with one well less and thus generate a higher project NPV. Besides that, the predrilled wells from Alta field were changed from $w1$, $w2$, $w4$ to $w2$, $w3$, $w5$. The first well to drill in Gohta also changed from $w7$ to $w8$.

TABLE 3. DRILLING SEQUENCE

Year	Original		Global *Optimal*	
	Alta	Gohta	Alta	Gohta
predrill	$w1, w2, w4$	/	$w2, w3, w5$	/
1^{st} *Year*	$w1,w2,w4,w5,w6$	$w7$	$w2, w3, w4, w5$	$w8$
2^{nd} *Year*	$w1, w2, w4, w5, w6$	$w7, w8, w9$	$w2, w3, w4, w5$	$w7, w8, w9$
3^{rd} *Year*	$w1, w2, w3, w4, w5, w6$	$w7, w8, w9$	$w2, w3, w4, w5, w6$	$w7, w8, w9$
4^{th} *Year*	...	...	...	...
5^{th} *Year*	...	...	...	...

UNCERTAINTY ANALYSIS

The following uncertainties are considered in this study: i) the well productivity; ii) the initial oil in place in Alta and Gohta; and iii) the costs (CAPEX and OPEX). Among these uncertainties, the well productivity and the initial oil in place are of geological origin, because they depend on the reservoir permeability, thickness, porosity, and oil saturation, among other factors. The uncertainties in cost are economic uncertainties.

In this study, we performed an uncertainty analysis using simulation-based optimization (repeating the optimization with different inputs). The samples of the uncertain variables are obtained using LHS as it has been reported [48]. This has the advantage of achieving convergence with less samples than other methods (for example Monte Carlo).

A total of 100 samples were generated using LHS. These samples were used as the input data for the optimization problem, resulting in 100 optimal solutions. We assume these parameters exhibit the following probability distributions: i) a log-normal distribution in the initial oil in place; ii) a normal distribution in the cost; and iii) a uniform distribution in the productivity. However, instead of using the value of the variable directly, in the optimization model we employed uncertainty factors, a ratio between the uncertain value and the base case ($u_c = \frac{random\ value}{Base\ value}$). The cumulative probability distribution function of each uncertainty factor for each variable is presented in Fig. 6.

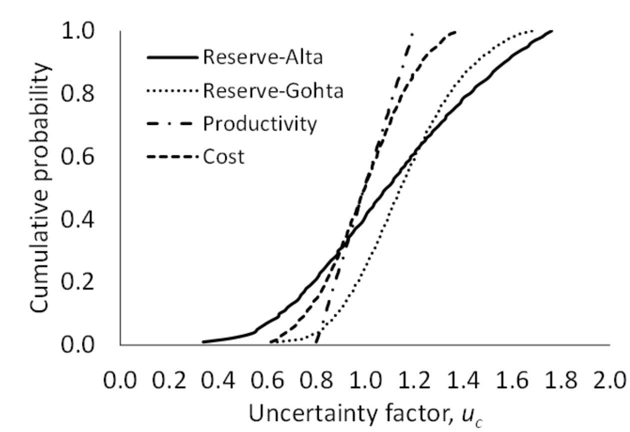

FIGURE 6. THE CUMULATIVE DISTRIBUTION OF THE UNCERTAINTY FACTOR OF OIL IN PLACE OF ALTA AND GOHTA, WELL PRODUCTIVITY AND COST

To present the result of the uncertainty analysis we use box-and-whisker plots, which graphically depict groups of numerical data through their quartiles[2]. Fig. 7 presents the drilling schedule. Despite the uncertainty in the input, 3 wells are assumed to be always pre-drilled in Alta before production. The distribution of pre-drilled wells found by the optimizer is presented in Fig. 8. Pre-drilling wells $w3$, $w4$, $w5$ from Alta is the most probable (28%) out of the 100 optimal solutions.

[2] the minimum-0^{th} percentile, the maximum-100^{th} percentile, the sample median(in red)-50^{th} percentile, and the first 25^{th} percentile and third quartiles-75^{th} percentile

The median value is drilling 6 wells in Alta after Year-6, instead of 5 stated in the global *optimal* in previous section, obtained by using only the values of the base case. This value is closer to the *preset* plan. In Gohta, all 3 wells have to be drilled in the first 3 years, but there is some variation in drilling sequence in Year-1 or Year-2.

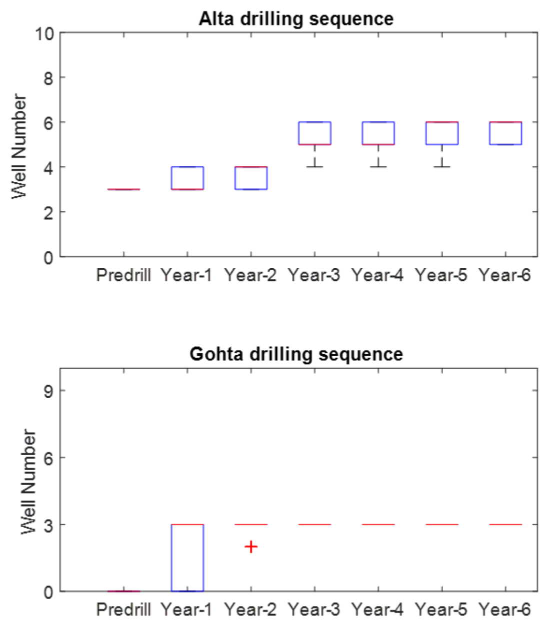

FIGURE 7. DRILLING SCHEDULE UNDER UNCERTAINTY ANALYSIS

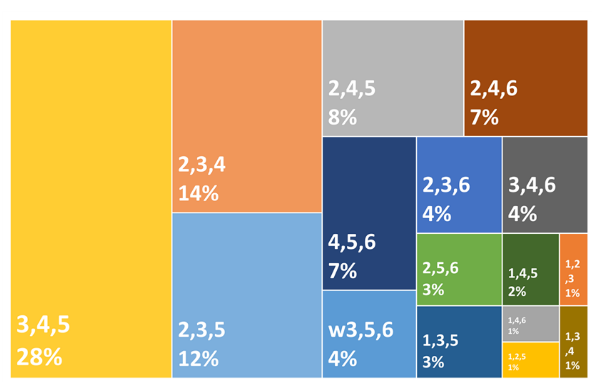

FIGURE 8. PREDRILL WELLS UNCERTAINTY

Figure 9 shows the field production profile. Most cases give a production plateau of 3-4 years, but there are some outliers in different periods from the production horizon.

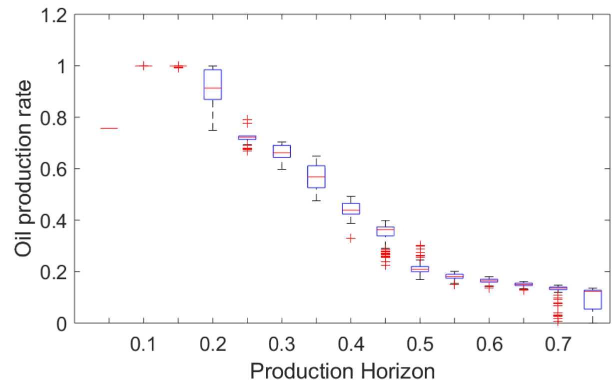

FIGURE 9. ALTA-GOHTA TOTAL FIELD OIL PRODUCTION PROFILE UNDER UNCERTAINTY ANALYSIS, SAME NORMALIZATION AS FIGURE. 3

CONCLUSION

In this paper, we presented a real-world application of mathematical programming to determine an optimal drilling and production schedule in early field development of an offshore oil field in the Barents Sea. Based on the study, we have the following observations:

1. The mathematical programming model developed satisfactorily reproduces the results previously obtained by the multi-disciplinary field development team (the operator's plan) when including their design constraints.
2. The mathematical programming model proposed has low running time and is computationally efficient. Therefore, it allows the evaluation of several scenarios in a short time and can perform exhaustive uncertainty evaluation.
3. If uncertainty is neglected, the optimization results show that the NPV of the Alta-Gohta could be increased slightly if one producer well is not drilled and the drilling sequence is adjusted slightly.
4. When considering geological uncertainties (in place volumes and well productivity) and economic uncertainties (CAPEX and OPEX), all cases show that 3 pre-drilled wells are required in the Alta reservoir and all 3 wells must be drilled in Gohta. Also, almost all cases indicate that a production plateau duration of 3-4 years is likely.

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Appendix A: Sets, Parameters and Variables

Indexes and sets:

$w \in \mathscr{W}$:	set of candidate wells
$t \in \mathscr{T}$:	set of years in the field lifetime
$i \in \mathscr{I}$:	set of well combination scenarios
$r \in \mathscr{R}$:	set of reservoirs within the field f
$f \in \mathscr{F}$:	set of fields

Decision variables:

$x_{w,t}^r$:	well status of well w from reservoir r at time t: 1 when the well is active, and 0 when it is shut-in. The vector containing the status variables of all wells is denoted by $\mathbf{x}_t^r$
$q_{o,t}^f$:	oil production rate from field f at time t
$q_{g,t}^f$:	water production rate from field f at time t
$q_{w,t}^f$:	gas production rate from field f at time t
$q_{o,t}^r$:	oil production rate from reservoir r at time t
$q_{g,t}^r$:	water production rate from reservoir r at time t
$q_{w,t}^r$:	gas production rate from reservoir r at time t
$N_{p,t}^r$	Cumulative oil prod. of reservoir r at time t
$G_{p,t}^r$	Cumulative gas prod. of reservoir r at time t
$W_{p,t}^r$	Cumulative water prod. of reservoir r at time t

Parameters:

γ	Discount rate
p_o :	Oil price
$q_o^{f,max}$:	Maximum Oil processing capacity
$q_g^{f,max}$:	Maximum Gas processing capacity
$q_w^{f,max}$:	Maximum Water processing capacity
$q_l^{f,max}$:	Maximum total liquid processing capacity
D^{max} :	Maximum drilling capacity per year
$f_n^i(\mathbf{x}_t^r)$:	Fractional ratio of total production potential of the field as a function of well combination $\mathbf{x}_t^r$
$f_q^r(N_{p,t}^r)$:	Field potential as a function of the cumulative oil production $N_{p,t}^r$
$f_G^r(N_{p,t}^r)$:	Cumulative gas production as a function of the cumulative oil production $N_{p,t}^r$
$f_W^r(N_{p,t}^r)$:	Cumulative water production as a function of the cumulative oil production $N_{p,t}^r$

Appendix B: Workflow of drilling scenario creation

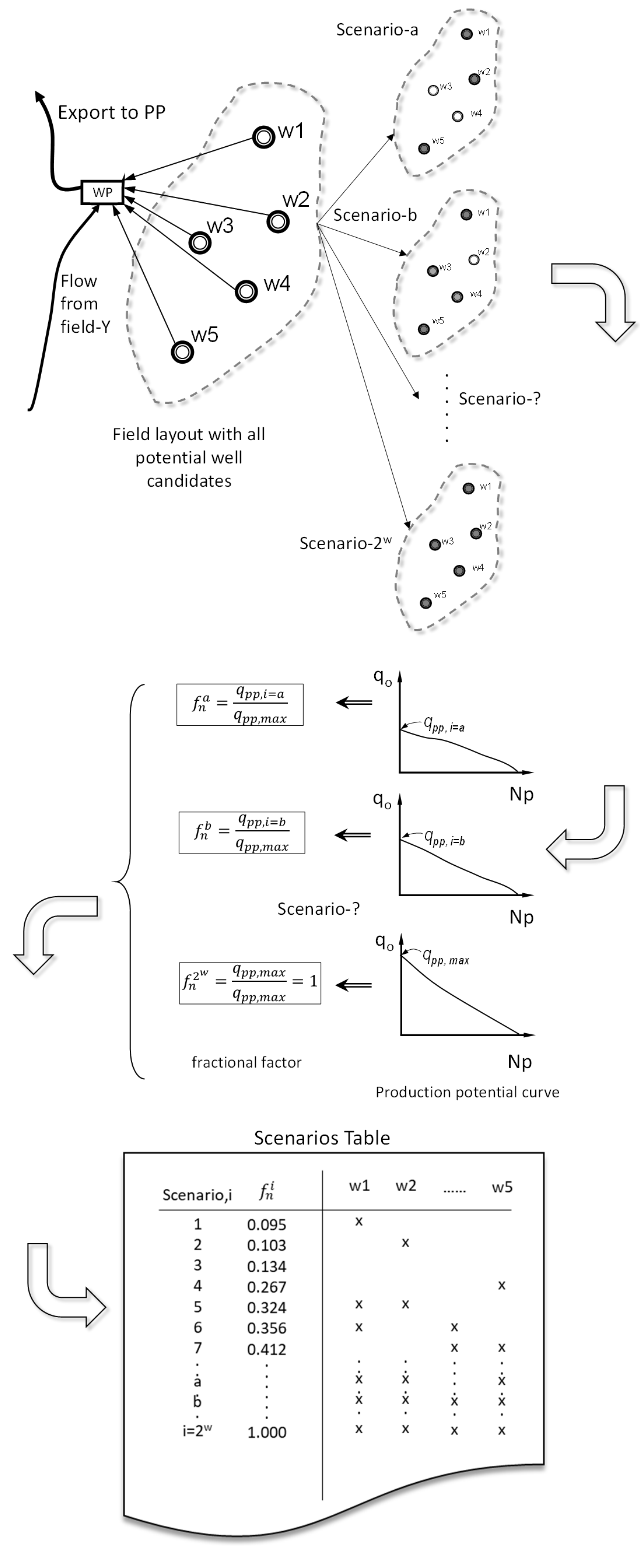

FIGURE 10. THE WORKFLOW OF CREATE SCENARIOS OF FIELD DEVELOPMENT