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Real options approach for a staged field development with optional wells

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ABSTRACT

With the decreasing average size of new discoveries in mature production areas, the uncertainties in the base of oil field investment decisions are continually increasing. Fewer appraisal wells, which allow to decrease the amount of subsurface uncertainty, are typically drilled before the development of a small field compared to large fields. In this context, novel solutions must be established to commercialize small discoveries under technical and market uncertainties. In such conditions, managerial flexibilities, which enable to change the course of the project in the event of new information acquisition, must be critically considered in the investment valuation process.

Combining the real options approach and decision analysis, we establish a novel model to identify the additional value created by a sequential drilling strategy for field development under oil price and resource uncertainty. In particular, we capture the sequence of the key investment and operating decisions pertaining to a marginal field development in cooperation with an oil industry partner, which corresponds to a synthetic yet realistic project case. By considering the flexibility in dividing the production well drilling into two stages, we adopt the least-squares Monte Carlo algorithm to evaluate the option to wait to expand the production by drilling additional wells.

Furthermore, we identify the conditions in which the staged (phased) development is preferable against standard development. We propose a decision rule to determine the optimal expansion timing based on the acquisition of new information on the reservoir and oil price uncertainty. Our results suggest that staged development carries large upside potential for the marginal field development under extensive reservoir uncertainty. In addition, partial hedging against the downside risks in the staged development can enhance the project's economy in a sufficiently significant manner to justify investment.

1. Introduction

The decreasing average size of new oil discoveries (Norwegian Petroleum Directorate, 2019) is a critical issue being encountered by oil and gas exploration and production (E&P) companies in mature production areas such as the Norwegian Continental Shelf (NCS). Exploration and development of smaller reservoirs that are located in more challenging formations requires the use of expensive technology and advanced engineering solutions to access it (Lund, 1999). Furthermore, fewer appraisal wells are typically drilled before the development of a small field than in the case of larger discoveries. This makes the investment and development decision base relatively more uncertain. Together with the adverse price environment, it led industry majors to reduce active exploration or even withdraw from such areas in recent years.[1] Nevertheless, smaller reservoirs may still represent substantial value and may be attractive investment opportunities if the decision-making process adequately addresses the field development risks and upside potentials. The realization of more informed decisions that exploit the data generated during the course of a project is critical to enable efficient and cost-effective hydrocarbon production in the presence of prominent downside risks. In their 2019 resource report, the Norwegian Petroleum Directorate emphasized the importance of continuing to find good solutions in order to make small discoveries commercial. This puts particular focus on flexible instruments that allow to react on the outcome of uncertain parameters by changing the course of the project.

In this study, we analyze a potential solution to the abovementioned problem, pertaining to the opportunity of sequentially investing in field

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[1] ExxonMobil, for example, sold their Norwegian assets to Vår for $4.5 bn in 2019 (see https://www.ft.com/content/f03fec96-e085-11e9-b112-9624ec9edc59).

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development within a staged drilling strategy. This framework allows the collection of additional information between the stages and tuning the development plan of the subsequent stages of the project. To incorporate this flexibility within the economic valuation, we propose a novel methodology that can serve as a decision support tool for oil companies. We put a particular focus on the relevance of this methodology for small field development cases. The key contributions of this study can be summarized as follows: (1) we evaluate staged drilling as a strategy to realize field development under prominent reservoir uncertainty; (2) we establish a methodology that allows optimize the production expansion decision during the production phase based on new information regarding two types of uncertainties pertaining to the estimated production rate and oil price; (3) we consider the complete range of probable outcomes of the technical uncertainty within the optimization and valuation procedure; (4) we derive investment threshold boundaries that allow the field operator to identify the optimal expansion decision.

In order to evaluate the investment under uncertainty with implied flexibility, we apply the real options approach (ROA) in combination with the decision analysis (DA). The use of this combination enables the decision maker to accommodate both technical and market uncertainties in the economic model representing a complex E&P project (Jafarizadeh and Bratvold, 2009). A static discounted cash flow (DCF) approach, traditionally used in the industry, cannot accurately capture multiple uncertainties and inherent flexibilities within the investment valuation. We demonstrate how the decision maker can optimize the drilling strategy based on the ROA by choosing staged development amid substantial reservoir uncertainty. To the best of our knowledge, such models have not been established for small or large field development.

Subsequently, we analyze the value of the flexibility created by the staged oil field development strategy and compare it to what we refer to as the standard development. In standard development, a predefined set of production and injection wells are planned to be drilled and completed prior to the production initiation in a field. At the same time, all the necessary facilities required to operate the field over the expected lifetime are established. This implies that measures to increase the hydrocarbon recovery during the operation of the field, such as drilling additional wells or improved oil recovery (IOR), may be identified, but are not included in the economic analysis at the time of the investment decision. In contrast, the basis for staged development is an initial development phase (Stage 1), which contains a number of predefined wells that is lower than that in case of the standard development, and facilities that must be in place at production start-up. In addition, the operator has an opportunity to expand the development by drilling additional production wells after a certain time of operation during Stage 1. This time is needed to gather and process additional information regarding the reservoir, which may help enhance the field development decision-making in the second stage of the project. Such a strategy is highly relevant in the event of prominent subsurface uncertainties. If the information generated during the initial stage indicates that the reservoir properties are inferior, the decision maker can avoid drilling superfluous wells. This flexibility provides a partial hedge against the reservoir risk and can be addressed in the valuation procedure at the stage of the investment decision. The proposed methodology allows the decision maker to exploit the benefits of the staged development approach and identify the potential to create additional value for both large and small fields. Special attention in this study is drawn to small discoveries, motivated by the fact that those are specifically sensitive to the downside risks.

Our methodology is built on several blocks. We first prepare the field production forecast, accounting for the uncertainty in the initial production rates. This framework allows us to realistically reflect the decision maker's knowledge regarding the reservoir characteristics and consider the possibility of encountering a "low reservoir performance" case that might lead to a negative overall project value. After identifying the field design basis, drainage strategy, and production forecast, we estimate the capital and operational expenses throughout the lifetime of the field. Next, we construct the future oil price curves by using the two-factor stochastic price model (Schwartz and Smith, 2000), calibrated using the Kalman filter and historical market data. We assume that the production at Stage 1 generates perfect information regarding the technical uncertainty. Based on these data and the conditions of the oil market, we consider that the operator can decide to expand the production by drilling additional wells within a predetermined period of time. Through the combined simulations of the production rate, cost profiles, and oil prices, we construct several sets of the expected yearly project cash flows associated with the respective expansion decisions. The waiting option to expand is formulated using the least-squares Monte Carlo (LSM) framework (Longstaff and Schwartz, 2001). To optimize the expansion decision within the LSM algorithm for American options by comparing the immediate exercise value with the estimated value from the continuation at each decision point, a regression function accommodating both the oil price and production rate parameters is used. Subsequently, we identify the optimal expansion timing (drilling for Stage 2) for each simulated case and establish a threshold boundary representing combinations of the production rate and oil price, which are considered to trigger the expansion decision. Finally, we calculate the values of the project under standard and staged development strategies and identify under which conditions the staged development is the preferred choice.

The objective of the present study is to propose a methodology to evaluate the staged development strategy, rather than examine a specific case, in which the project parameters and detailed reservoir characterization may impeded the understanding of the advantages of the flexibility. The value of the staged development significantly depends on the problem of interest. To confirm whether this strategy can create additional value, any decision maker can employ the project-specific inputs such as the development plan, expected production profiles, and costs. In Fedorov et al. (2020), we analyzed the potential of the staged development considering a benchmark reservoir model, Olympus (Fonseca et al., 2018), involving well control optimization based on several realizations of the reservoir model. Overall, our results lead to recommendations that can help facilitate and enhance the field development decision-making process. The proposed modeling approach is thus expected to be of both academic and industry value.

1.1. Literature review

This study aims to contribute to three different strands of the literature. The first strand pertains to the specifics of the economic assessment of small oil projects and decision-making under considerable subsurface uncertainty. The few contributions in this field include those of Laine et al. (1997), who consider an example of two Norwegian fields to model the deferral, expansion, and abandonment options and demonstrate that the option valuation techniques can add substantial value to marginal discoveries. Lund (1999) analyzes the investment in a small oil field on the NCS and emphasizes the importance of the operator's flexibility to change the project plan during the operating phase to enhance the overall project value, especially under notable uncertainties pertaining to the reservoir. Galli et al. (2001) examined a small satellite gas field in the North Sea by using a real option framework to evaluate the impact of drilling decisions on the project value. Armstrong et al. (2004) use information from production logging and a copula-based Bayesian updating scheme for real options valuation of small oil projects. Dias (2004) briefly discusses the possibility of phasing the investment in several stages, thereby providing an option to expand the production by drilling additional wells. This strategy is considered to address the high amount of subsurface uncertainty, which is a typical problem for small field development. Dias (2004) discusses a hypothetical method to analyze this flexibility as a sequence of actions that a decision maker should implement to apply the ROA to this problem.

Jafarizadeh and Bratvold (2015) highlighted the differences in the economic analysis of small and large discoveries by evaluating a waiting-to-invest option in two hypothetical exploration opportunities (large and small prospects). Jafarizadeh and Bratvold (2015) demonstrate that projects with smaller recoverable volumes, shorter lead times for development, and a steeper production decline are more sensitive to the variability in the oil prices and discount rate.

However, none of the existing studies, including that of Dias (2004), were focused on examining the effect of the staged approach on the value of an E&P project or clarifying the process to optimize the decision to drill optional wells under technical and market uncertainties for large or small oil fields. We contribute to this strand literature by building a formal model to quantify the value of a staged strategy, based on real options analysis and a production expansion optimization algorithm, and reporting the numerical results for a case study. The discussion of Dias (2004), in fact, underpins the value of our work and its contribution both to the industry and academia.

Second, this work aims to contribute to the strand of literature that combines the ROA and DA within a single valuation procedure. In the classic real options studies, methods to identify a market-traded portfolio that could replicate a real-world investment were adopted to perform a real option valuation. Introducing technical uncertainty to these methodologies is challenging and may lead to inaccurate valuation results (Jafarizadeh and Bratvold, 2009). Nevertheless, in the DA approach, the decision maker's beliefs regarding the project can be reflected by assessing the subjective probabilities for the uncertainties. The valuation for a risky project is typically implemented using a decision tree or dynamic program, thereby neglecting the market opportunities to hedge the price risks. In a pioneering contribution, Smith and Nau (1995) develop an integrated approach combining option pricing methods and a DA to accommodate the market uncertainty that can be hedged and technical uncertainty that cannot be hedged. Copeland and Antikarov (2001) and Brandão et al. (2005) employed traditional DA tools—binomial decision trees and binomial lattices—to solve real options problems. Specifically, the authors use a mix of DCF and risk-neutral methods; however, this approach is criticized by Smith (2005), who recommend the use of a fully risk-neutral approach leading to a single coherent valuation model that can be used to value projects with and without options. Comparing the competing methodologies, Smith (2005) analyzes an investment opportunity in an oil production project and concludes that "there is much to be gained from integrating the real options and DA approaches to project evaluation".

By applying the simulation-based risk-neutral valuation approach, we adopt the method presented by Smith and Nau (1995) and Smith (2005) and contribute to the literature on the integrated ROA and DA by considering the complete range of possible outcomes of the technical uncertainty in accordance with the decision maker's arbitrary probabilities within the valuation procedure. Unlike Dias (2002) and Santos et al. (2017a), we therewith capture not only the discretized representation of the technical uncertainty, as typically used for a decision tree and lattice model approach. Using a simulation approach instead, allows us to construct a better representation of how the reservoir risk affects the decision-making process within a field development case, which can yield more accurate valuation results.

Another key objective of this analysis is to examine the effect of new information on the decision-making. The data generated during the initial stage of the project are used to update the decision maker's knowledge regarding the reservoir to optimize the subsequent development strategy. This process might help generate additional value, which can be identified at the investment decision phase. Therefore, the third strand of literature that we aim to contribute to pertains to the value of information for natural resource projects when an ROA is adopted. Contributions to optimize timely decisions using new information include those of Chorn and Carr (1997), who examine the application of option pricing techniques to value information regarding the offshore gas field reserve volume and selection of the production strategy. Gallant et al. (1999) use "learning models"[2] to capture the changing expectations as new information is gained over the life of an E&P project. Dias et al. (1997) and Dias (2002) investigate the effect of the timing and "drilling games" with strategic interaction in E&P projects by explicitly modeling the value of learning. Cunningham and Begg (2008) analyze various scenarios of a sequential drilling program involving the use of new information. Notably, the authors provide an example of how the value of information can be proactively used in the construction of drilling strategies. Such an approach can help avoid the overspending on costly tests, which cannot change the initial beliefs regarding the project, and facilitate better decision-making. Santos et al. (2017b) introduce an uncertainty management method complementing techniques to acquire new information and add flexibility in the production system to reduce the downside risk within a robust production strategy. Kullawan et al. (2018) develop a discretized stochastic dynamic programming approach for sequential decisions in geosteering operations based on real-time information. In general, accounting for this information can help optimize the well trajectory and increase the economic value. Hanea et al. (2019) assess the value of learning created by the data generated within a sequential drilling strategy. Using a synthetic reservoir case, the researchers demonstrate how history matching and frequently updating the development strategy can enhance the field development.

Although we assume that the decision maker can acquire perfect technical information, we contribute to the abovementioned selection of literature by demonstrating how the information generated at the initial production phase can be used to optimize the drilling strategy and therewith, to increase the economic value. To this end, the reservoir uncertainty parameter is incorporated in the LSM regression. By introducing a threshold boundary representing the combinations of the production rate and oil price, which are considered to trigger the expansion decision, we directly show how the updated knowledge regarding the reservoir uncertainty influences the decision-making. Furthermore, we illustrate that the additional information that can be potentially acquired through the future production experience should not be ignored when finalizing the investment decision. This information is particularly important in the case of small field development involving subsurface uncertainties.

The remainder of this paper is organized as follows. Section 2 introduces the key features of the staged development and explains the limitations of the classic approach to evaluate an investment with flexibilities. Section 3 describes the formulation and development of the modeling approach to realize the project valuation of a staged development with an option to expand. Section 4 presents the case study in which the modeling approach is applied to a realistic problem. Section 5 presents the results, including those of a sensitivity analysis and robustness check. Section 6 presents the concluding remarks.

2. Background

This section describes the motivation for adopting the staged drilling strategy (see Section 2.1) including both the risks and benefits associated with the deferral of the second stage of an oil field development project. Table 1 summarizes the main features of the staged development strategy compared to the standard strategy. Furthermore, we explain the operator's decision-making process for the optimization of the production expansion once the technical uncertainty has been revealed. In Section 2.2, we discuss the essence of using a more advanced economic analysis compared to a static DCF approach to

[2] Gallant et al. (1999) defined a learning model as a depiction of how new information allows a decision maker to revise his/her initial belief regarding an uncertain event. Gallant et al. (1999) argue that the "learning model helps us take advantage of new information in the evaluation of a project's potential, not just in its execution".

Table 1
Key features of the staged development strategy compared to the standard one.

Risk	Staged development
Low reservoir/oil price scenario	Ability to mitigate downside risks by not drilling uneconomic wells
Timing	Loss of value because of waiting (time value of money) and probable oil loss caused by migration during Stage 1
Well placement	Improved well placement in Stage 2 based on production experience and acquired data, thereby enhancing the expected recovery
Capital expenditure (CAPEX)	Possibility to defer a significant amount of CAPEX until Stage 2

evaluate the investment with embedded options. We demonstrate the need to use a combination of the ROA and DA for cases in which the managerial flexibility is of significance for the project value.

2.1. Staged development with the expansion option

Decisions related to petroleum exploration and production are highly complex because of the large number of considerations involved in the process (Suslick et al., 2009). During the design phase, the project team must define, among other factors, the optimal number of development wells (both producers and injectors) and their placement. This decision is particularly challenging under technical and market uncertainties. Moreover, the investment in wells is considered to be irreversible as an operator cannot recover the drilling expenditures once they have been implemented. Consequently, by establishing a balance between the expected value creation by an additional well being drilled and the associated costs within prevailing uncertainties, the team must adopt a fundamental decision that cannot be changed during the project.

A standard approach to decrease the reservoir uncertainty to enhance the quality of well placement and field design is to drill appraisal wells. However, this approach may not be suitable in certain projects in which the investment in the extensive appraisal program is considered to be inadequately high compared to the potential of information revelation (Dias, 2004). This facet is especially true in the case of small field development with marginal economy.

Under such conditions, a more optimal strategy to cope with the reservoir uncertainty may be to start drilling production wells based on the available information without an additional appraisal program. However, instead of drilling all the potential production wells, several of which likely cannot ensure a reasonable production rate, the operator can prioritize wells and drill them sequentially, i.e., develop an oil field in several stages. Under a staged development strategy, the decision maker first drills several wells in the locations that are less exposed to the reservoir risk. This is done to start the production at the initial stage of the field life, which may last from several months to years. During Stage 1, the operator gathers data and performs specific tests to optimize the Stage 2 drilling decision. Fig. 1 illustrates timeline and production profile for such a staged development strategy. After the final investment decision (FID) and authority approval of the plan for development and operation (PDO), the project enters the engineering and construction phase. The drilling of the production wells typically commences 1–2 years prior to the production initiation. After a certain period, the

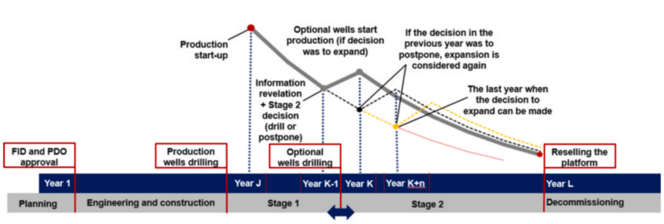


Fig. 1. Decision gates of a general project under the staged development strategy.

experiences accumulated through the drilling and production of these wells can be evaluated to update the underlying reservoir models. Based on the updated knowledge regarding the reservoir, managers can formulate the decision to drill additional production wells to increase the field production potential. Moreover, such new information allows the optimization of the number and placement of the optional wells as well the drilling timing for such wells. Alternatively, the decision maker might refrain from drilling optional wells and continue production via the same number of wells to avoid investing in wells that could potentially prove to be uneconomic. In this case, the process of considering the optional wells can be repeated after additional data have been collected. In this manner, the staged approach allows the mitigation of the downside risk, which could reduce the economic value of the project if the company commits to drill all the wells before actually starting to produce the field (Willigers et al., 2017). This phenomenon is particularly relevant for marginal fields, in which drilling costs may represent a large part of the capital expenditure and highly influence the overall value of the project.

After gathering and analyzing the data, managers can exploit the flexibility to expand the production by drilling optional wells only in favorable scenarios. In addition, the management may decide to wait to implement the Stage 2 drilling under unfavorable market conditions. Notably, in the case of large field development, the lifetime of which may be more than 30 years, holding this option for a long time could be reasonable. However, in the case of small fields, holding the expansion option for more than a couple of years may not make much sense due to the relatively short lifetime of the project. Moreover, the longer the optional wells remain undrilled, the higher is the potential loss of value through hydrocarbon migration away from the location of these optional wells towards the location of existing producers. The effect of this phenomenon has been described by Dias (2004) “as a dividend lost by the option holder”, thereby providing the holders with “a higher incentive to drill the optional well earlier”.

In addition, by not immediately drilling all possible wells, the operator may lose a certain part of the value owing to the depreciation effect of delayed production. This aspect is especially relevant if the reservoir performance is higher than expected. In practice, an E&P company must consider the additional investment costs that the expansion might require. This amount is mainly dictated by the state of the production system at the moment at which the decision to expand is made. The least costly approach is to fill the spare capacity of the facilities as soon as the field enters the production decline stage. According to certain other strategies, the field operator can design the field development process by ensuring that some additional capacity remains idle at Stage 1 to be able to increase the production rate while the end of the plateau during Stage 1 has not been reached.

Within a staged development strategy, field engineers and economists perform joint assessment to identify the production wells (and their location) to be drilled in Stage 1. Typically, these wells are expected to generate a sufficient amount of production and/or are less exposed to the reservoir uncertainty and thus can ensure more economical production not only at the initial stage, but throughout the whole lifetime of the project. In this context, the potential candidates for Stage 2 are the locations with the highest risk of losing value because of technical uncertainties.

Nevertheless, the field development is exposed to not only technical uncertainty but market conditions as well. The decision to expand the production during a drastic oil price downturn could disrupt the revenue, particularly of a small field, whose production lifetime might be limited to 5–8 years (Rasmussen, 2015), compared to a 30+ years lifespan of large discoveries. It must also be noted that the decision to expand could be beneficial amid a price surge, even if the cumulative production during Stage 1 highlights an inferior reservoir performance. In this context, the optimal expansion policy must be based on not only the reservoir information revelation but the market development as well. In general, the moment at which the managers can exercise the option is not limited to the point in time immediately after the information revelation. The price uncertainty might motivate the managers to postpone this decision. Assuming that the expansion decision can be made once a year, Fig. 1 shows that the operator can shift the last year of Stage 1, i.e., Year K-1, when the optional wells are drilled, and postpone Stage 2.

2.2. Limitations of the DCF approach for investment evaluation

As described in the introduction, E&P companies typically apply a DCF approach to evaluate investment opportunities. In the classical DCF approach, investment decisions are based on the net present value (NPV) calculated by discounting the cash flows using a risk-adjusted discount rate defined as

$$NPV = \sum_{t=t_0}^{T} \frac{\mathbb{E}[CF_t]}{(1+R_a)^t}, \tag{1}$$

where $\mathbb{E}(t)[CF_t]$ is the expected cash flow of period t, T is the number of periods, and R_a is the risk-adjusted discount rate. The risk-adjusted discount rate reflects both the time value of money and risk. It represents a compensation demanded by investors for the risks that are associated with holding an asset (Brealey et al., 2012). Most companies adopt a rate equal to their weighted average cost of capital, arguing that the investment should cover both the costs of debt and capital (Jafarizadeh and Bratvold, 2019). In practice, companies often apply a single discount rate to all their projects when evaluating the investment decisions or communicating with the authorities,[3] while ignoring the specific features of individual projects. Such an approach may result in incorrect valuation leading to poor decision-making, especially in the case in which the investment opportunities are exposed to various uncertainties.

The two main types of uncertainties that we focus on in the current study, reservoir and oil price uncertainty, have different risk natures. Our method adopts a risk-neutral pricing approach, which has been demonstrated to be the appropriate technique to perform the valuation from a methodological viewpoint (Smith and Nau (1995); Smith and McCardle (1999)). Notably, this approach distinguishes between market risks, hedgeable by trading securities, and private risks, which are project-specific and cannot be hedged with any market instrument. Taking a risk-neutral approach, instead of risk-adjusting the complete cash flow, the decision maker treats the uncertainties separately. In the present case, the risk-adjusted stochastic process is adopted for market risks (oil price) and true probabilities are considered for private risks (reservoir uncertainty) to calculate the expected value of the investment. The resulting cash flows are then discounted at the risk-free rate (Jafarizadeh and Bratvold, 2012) defined as

$$NPV = \sum_{t=t_0}^{T} \frac{\mathbb{E}[CF_t]}{(1+R_f)^t}, \tag{2}$$

where R_f is the risk-free discount rate.

This technique is illustrated by building the production profiles, as indicated in Section 3.1, based on the decision maker's beliefs regarding the probabilities associated with the technical uncertainty, and implementing the risk-neutral process for the future oil prices, as described in Section 3.3. Subsequently, the risk-neutral valuation procedure is implemented, as described in Section 3.4.

[3] Oil companies acting on the NCS must submit a PDO of a petroleum deposit to the Ministry of Petroleum and Energy of Norway before commencing the oil field development. The official guidelines, specifically, the Norwegian Petroleum Directorate PDO Guidelines https://www.npd.no/globalassets/1-npd/regelverk/forskrifter/en/pdo-and-pio.pdf recommend using a standard rate of 7% to justify the profitability of the project.

However, the difference between the traditional DCF and the proposed method is not only in how the risk is treated in the project cash flow. The DCF approach does not allow to accurately capture the managerial ability to change the course of the project. Moreover, this approach ignores the values of the embedded options, as it is based on a "static" view, in which the future decisions are assumed to depend only on the information available at present, while additional information that may be available at later stages is ignored. In contrast, the ROA can consider this flexibility and evaluate the additional value associated with it (Jafarizadeh and Bratvold, 2012). Different approaches, such as decision tree models, binomial lattices, or simulation-based approaches, can be used to perform the real options analysis for the project valuation. In this study, we apply a simulation-based approach, specifically, the LSM algorithm, which allows the representation the influence of the project parameters, uncertainties, and flexibility on the project performance. In Section 3.4 we motivate the choice for the simulation-based algorithm in detail and implement the risk-neutral procedure with the LSM algorithm to evaluate the project with an option to expand.

3. Methodology

This section describes the key components of the proposed method to evaluate the staged investment in a small offshore field with an option to expand under the presence of technical and market uncertainties. Fig. 2 shows the main building blocks and information flow of a modern development project. First, based on available data, a subsurface assessment is conducted to generate reservoir model(s) that allow to simulate the outcome of alternative drainage strategies in terms of production. At the same time, a design basis for the field development and the proposed drainage strategy are matured. Based on these, both a production forecast (see Section 3.1) and cost estimates (see Section 3.2) for the envisioned development are established, including the uncertainty estimates. These data can be combined with economic assumptions regarding oil prices (see Section 3.3), exchange rates, and other aspects to model the cash flows of the project. These data serve as an input to a real option valuation process of alternative drainage strategies. This is done by using the LSM algorithm (see Section 3.4), which allows to optimize the decision to expand the production by accounting for both the technical and price uncertainty. As the key objective is to compare the staged development strategy with the standard one, we perform a symmetric analysis of the two strategies based on the same price and production assumptions, to enable a fair comparison.

Furthermore, in this study, we focus on the introduction and implementation of the methodology to evaluate the opportunity to

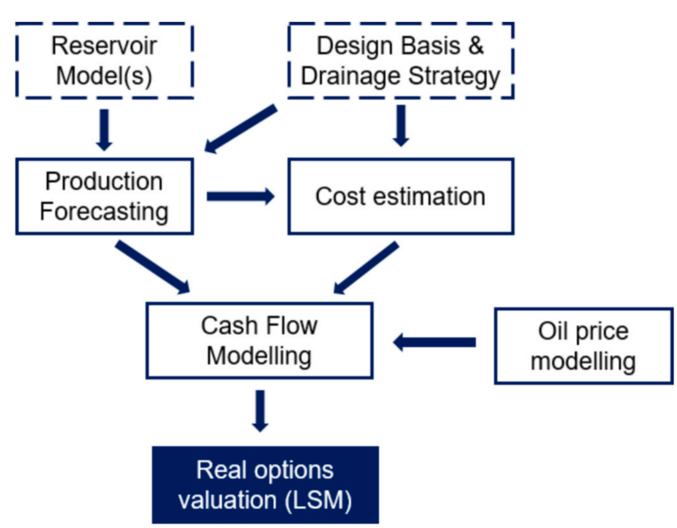


Fig. 2. Valuation procedure.

phase a field development into two stages with an option to expand. The considered case study pertains to a synthetic data set, described in detail in Section 4. We do not include any underlying reservoir model nor a specific design basis of the field development, and thus, the methodology can be easily adopted for other industry cases.

3.1. Production profiles

The modeling process begins with the estimation of the yearly production rate for each well for both standard (non-staged) and staged development strategies. It is vital to realistically assess the amount of underlying reservoir uncertainty that the operator is expected to deal with, before committing to develop the field. The field development decision-making process must aim to capture all probable outcomes of the uncertain parameters and exploit the available flexibility to respond to these outcomes.

We use the probability density functions of the initial production rate (in the first year of production) of each well as the input data to estimate the field production potential. Such probability density functions are typically generated by the reservoir and production engineers and represent the technical uncertainty that affects the production rates. Fig. 3 illustrates the potential probability distribution functions of the initial production rates for wells within the standard development strategy. Let S denote the number of wells drilled for Stage 1, with N being the total number of wells. Then, $N-S$ denotes the number of wells considered to be implemented in Stage 2. In the standard development strategy, all wells $i=1...N$ are drilled before the production start-up, whereas in the staged development, wells $i=1...S$ are drilled at Stage 1, and wells $i=S+1..N$ are candidates (with possibly adjusted positions) for Stage 2.

When examining an opportunity to phase the drilling strategy in two stages, the decision maker must identify the wells that should be drilled at the first stage and those that should be considered later, i.e., in Stage 2. Following the approach recommended by Dias (2004), we assume that the locations with the lowest risk and highest expected recovery are selected for Stage 1, whereas those that tend to be more exposed to risk and uncertainty are considered as candidates for Stage 2. In our case, the well reservoir risk and prioritization are defined by the distributions of the initial production rate.

We initiate the procedure with well $i=1$. We use the Monte Carlo simulation to generate the initial production rate samples in the complete range of the distribution provided by the field engineers as the

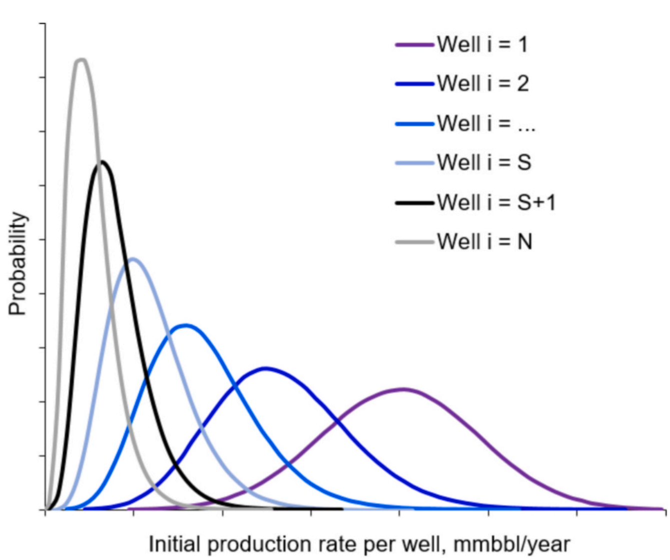


Fig. 3. Illustration of potential probability density functions of the initial production rates per well (standard development case).

input data. Considering the geological dependencies between the wells, we then proceed with sampling simulation cases for each subsequent well. In the standard development, in which all the wells $i=1..N$ are drilled immediately, the sampling of wells $i=2..N$ is explicitly dependent on the generated initial rate of the well $i-1$:

$$q_{0_i} = q_{0_{i-1}} k_i, \tag{3}$$

where q_{0_i} is the initial production rate of well i = 2..N, $q_{0_{i-1}}$ is the initial production rate of well i-1 and k_i defines the extent of geological dependency between wells i and $i-1$, which is randomly generated from a given distribution, $0<k_i<1$. This value describes the expected decrease/growth of the initial rate $q_{0_{i-1}}$ compared to q_{0_i}.[4]

To enable a comparison, we assume that the wells $i=1..S$, drilled in Stage 1 within the staged development, have the same properties as the respective wells under the standard development, and that the wells $i=2..S$ are dependent on the generated initial rate of well $i-1$, in the same manner as given by Eq. (3). However, with the data generated during Stage 1, the decision maker can update the reservoir model and thus the probability density functions for the initial rates and reassess the initial positions of the optional wells. Therefore, the uncertainty regarding the initial production rates is expected to decrease but is not fully eliminated.

We assume two potential scenarios (low and high cases) for the update of the expected initial rates of the Stage 2 wells. In the low-case scenario, which has a probability of x, the data generated during Stage 1 reveal that the optional wells have a lower production potential than expected. In the high-case scenario, which has an assigned probability of $1-x$, the new information and re-positioning of the optional wells allows to increase the expected initial rate of the optional wells against the standard development case. These probabilities depend on the operator's beliefs regarding their ability to alter the drilling program by improving the Stage 2 wells locations based on the data gathered during Stage 1. A higher probability of the high case corresponds to a higher benefit for the operator from the decreased subsurface uncertainty and increased expected initial production rate associated with the locations of Stage 2 wells.

The initial rate of well $i=S+1$ is calculated as follows:

$$q_{0_{i=S+1}} = \frac{\sum_{i=1}^{S} q_{0_i}}{S} k_i^*, \tag{4}$$

where $q_{0_{i=S+1}}$ is the initial production rate of well i = S+1, S is the number of wells drilled in Stage 1, k_i^* describes the expected decrease in the initial rate $q_{0_{i=S+1}}$ compared to the average initial rate of the Stage 1 wells (i = 1..S) of a specific sample case within the Monte Carlo simulation. k_i^* is also sampled from a distribution and has a respective probability of x and $1-x$ of having a low or high mean, defining the low/high scenario for the update.

Fig. 4 illustrates the possible results of updating the distribution of the initial rates for well $i=S+1$.

Subsequently, the initial rates of the remaining Stage 2 wells $i=S+2..N$ are then calculated using the relationship presented in Eq. (3), with the initial rates dependent on $q_{0_{i=S+1}}$.

When estimating the production rates for the Stage 2 wells, we consider the fact that the production potential of these wells decreases while we are waiting to drill them by assuming that $q_{0_{i=S+1}}$ is reduced by n% with each consecutive year of waiting to implement Stage 2.

After simulating all the samples for the initial production rates per well under staged and standard development, we estimate the production for the whole lifetime of the field. The production rate is assumed to

[4] As illustrated in Fig. 3, in the considered case, the expected value of k_i for each specific well is below 1, and thus, the expected value of q_{0_i} is less than $q_{0_{i-1}}$.

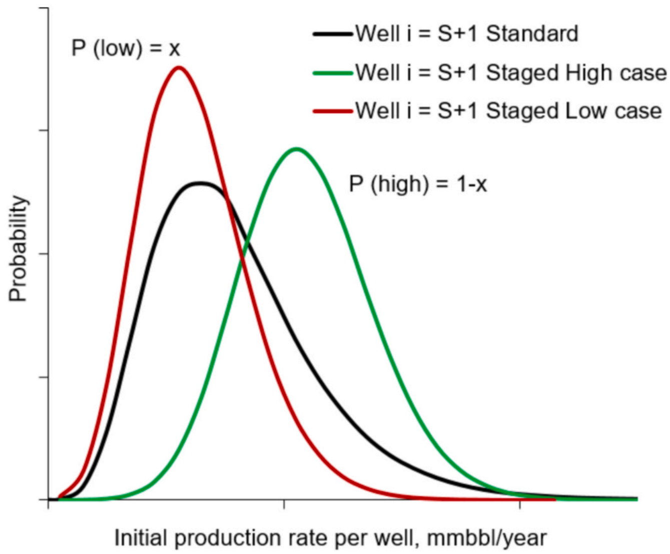


Fig. 4. Example of probability density functions of the initial production rates for well S+1 (standard development vs. staged development).

follow the general exponential equation (Fetkovich, 1980),

$$q_{i_t} = q_{i_{t-1}} e^{-a}, \tag{5}$$

where q_{i_t} is the production rate in year t, and a denotes the nominal decline rate.

The exponential decline is a special case of a hyperbolic decline curve introduced by Arps et al. (1945) and is widely used both by academic and industry professionals owing to its simplicity and ease of use. This case is based on the assumptions of constant bottom-hole pressure production and boundary-dominated flow (Fetkovich, 1980) and is often used as a first-order approximation to a production forecast, especially in a situation in which little or no observed production data are available. The decline rate in Eq. (5) is individual for each field design and may vary significantly among different cases, depending mainly on the reservoir depletion. The exponential form of the decline is only one of several options of mathematical approximation of the future production rate behavior. The proposed method is sufficiently flexible to incorporate any other production rate estimation approach. As described in Section 5.3.2, we assess alternative techniques to model the production decline, while adding the uncertainty about the decline rate.

The yearly total production of the field Q_t is equal to the sum of the yearly production rates per well:

$$Q_t = \sum_{i=1}^{N} q_{i_t}. \tag{6}$$

For simplicity, we assume that the production facilities are sufficiently large to handle even the high reservoir case without any limitations regarding the production rate, which is typical for small fields. This configuration allows us to avoid the technical complications resulting from adjusting the production profiles to a plateau rate.

The total recoverable reserves per well, Res_i, are equal to the sum of yearly production of the well:

$$Res_i = \sum_{t=1}^{t} q_{i_t}. \tag{7}$$

The total reserves of the field, Res, are equal to the sum of the reserves of each well, i.e.,

$$Res = \sum_{i=1}^{N} Res_i. \tag{8}$$

The fairly simple but sufficiently realistic representation of the technical uncertainty considered herein can be easily applied for other case studies and is considered to deliver a sufficient input for the main part of the methodology, that is, the economic modeling. To parameterize the a priori distributions of the initial recovery (q_0 and k_i in Eq. (3)) and sample production rates, only the input from the engineers, which is typically provided before making the FID, is required. This analytical approach to model the production uncertainty is in line with that adopted by Dias (2004), Armstrong et al. (2004) and Guedes and Santos (2016), who use a Monte Carlo simulation and binomial trees based on an input distribution to represent the reservoir uncertainty. Despite using a simplified approach to model the production uncertainty, we account for the geological dependencies between the wells by reproducing the declining marginal productivity of the new wells being drilled in the same reservoir. k_i in Eq. (3) is assumed to be less than 1, owing to which, the expected initial rate q_0 of well $i+1$ is less than q_0 of well i. In other words, more wells drilled in the same reservoir correspond to a smaller initial rate of a new well.

Nevertheless, we disregard the possible interference between wells that might reduce the production rate of the already existing wells. The expected production rate of well i in our model is not affected by the additional wells $i + \ldots$ being drilled. In general, the actual interference is highly dependent on the considered scenario and may considerably affect the value of the staged development. In this study, we consider this issue in a relatively limited fashion, while in Fedorov et al. (2020) we present an application of the introduced methodology involving a benchmark reservoir model simulation, where the interference is accounted for. Notably, the use of the staged development strategy can help avoid the drilling of wells that would interfere with one another. Specifically, the data generated in Stage 1 production allow the operator to optimize the positions of the new wells, thereby enhancing the value of the project. This strategy involves an iterative approach, in which the decision maker learns of the remaining uncertainty to update the drilling strategy. This learning effect was not considered in Fedorov et al. (2020); in contrast, in this study, we assume the acquisition of perfect information, which might either be a realization of a high or low case. Even though the actual reservoir properties and potentials for learning during the course of the project may be more complex, our economic valuation procedure can consider these aspects as long as the production profiles can be generated.

The advantage of the proposed approach is that we can capture the complete range of probable production rates per well, and not only the discretized values representing high, medium, and low cases, as typically done in DA problems using decision trees. With that we facilitate the process of replacing the scenario based thinking with a probabilistic approach. This allows more effective exploitation of the seismic data and assessment of the influence of the reservoir uncertainty on the optimization of the decision to expand the production by drilling optional wells. Performing the Monte Carlo simulation of the expected production rates based on the input distributions and running the whole valuation procedure, described in Sections 3.1–3.4, requires only a few seconds. The Monte Carlo simulation approach allows the consideration of several thousand production scenarios; comparatively, the reservoir simulation used in Fedorov et al. (2020) was limited to five realizations of the reservoir uncertainty because of the high computational demand of the production optimization.

3.2. Costs

The next step in the procedure is to estimate the cost considering the technical features of the production system. An appropriate estimate of the cost parameters and underlying uncertainties is critical for investment valuation. Underestimating the costs may lead to difficulties associated with cost overruns, and ultimately, a lower profitability than that expected by the stakeholders. However, exaggerating the cost estimates may cause the management to unnecessarily renounce a project.

The yearly capital investment ($CAPEX_t$) is considered to consist of drilling costs ($DRILLEX_t$), cost of the platform (PL_t) to be constructed and installed, and other associated costs (FC_t):

$$CAPEX_t = DRILLEX_t + PL_t + FC_t. \tag{9}$$

While overall drilling costs depend on the number of wells drilled, the platform cost component is determined by the chosen capacity size.

The yearly operating costs ($OPEX_t$) are related mainly with the maintenance of the platforms and wells, and the costs of day-to-day operation of the facilities (labor costs and maintenance). The OPEX are assumed to consist of a fixed (FO) and variable parameters, the latter of which depends on the yearly production rate of the field (Q_t) and a coefficient b, representing the relationship between the production rate and OPEX:

$$OPEX_t = FO_t + bQ_t. \tag{10}$$

We use the basic cost estimates for our synthetic case with the associated uncertainty estimation of the individual cost elements based on both the contractors' data and operator's own assessment, as discussed in Section 4.3.

Another key cost component that offshore operators incur corresponds to the decommissioning costs, which generate an "unavoidable negative cash flow" (Parente et al., 2006). Companies must cease their offshore operations, and depending on the regulators' requirements, must ensure that the offshore production does not cause environmental damage. The abandonment expenditure is expected to be fixed and consist of three cost elements: decommissioning planning, removal of the facility, and plugging and abandonment of the wells.

However, as a new platform is built in the considered field case, the operator may benefit from reselling the platform (or using it on other fields) once the field has been decommissioned, thereby generating an overall positive cash flow at the abandonment. Notably, the proposed methodology is sufficiently flexible to account for the additional cost components that may be significant in other case studies.

3.3. Oil price modeling

We now proceed with parametrizing the underlying market uncertainty, which, in the considered case, is represented by the oil price risk. Oil price is one of the main factors that drive the uncertainty in the economic value assessment of oil field development. As we apply the ROA, we use a stochastic price model that replicates the characteristics of the real market uncertainty. Considerable research has been performed on oil and gas price modeling, motivated by the desire to enhance the quality of investment valuation under price uncertainty. Early studies primarily used the geometric Brownian motion (GBM) approach for stochastic oil-price modeling, which is based on an analogy pertaining to the behavior of the oil prices and stocks in the capital markets (see, e.g., Cox et al. (1985); Smith and McCardle (1999)). Later, researchers noted a mean-reverting characteristic inherent to oil prices owing to the ability of producers to respond to the market conditions (see, e.g. Lund (1999)). Modeling the future oil price as a mean-reverting stochastic process allows short-term deviations from a constant long-term equilibrium. To better mimic the nature of oil markets, further research extended this approach by adding more levels of uncertainty, leading to the consideration of two or more factors in the model.

In this study, we assume that the future oil prices follow the two-factor stochastic price process proposed by Schwartz and Smith (2000). The two-factor price process allows to account for the mean reversion in short-term prices and uncertainty in the long-term equilibrium level to which prices revert. The equilibrium prices are modeled as Brownian motion, reflecting the expectations of the exhaustion of the existing supply, improved exploration and production technology, inflation, and political and regulatory effects. The advantage of this two-factor process is that it is relatively easy to calibrate while being based on realistic assumptions. Such a process has clear advantages over one-factor models owing to the uncertainty in both the short- and long-term factors, and other multi-factor models, which are highly difficult to calibrate and less intuitive to be communicated to industry representatives. The process proposed by Schwartz and Smith (2000) mimics not only the features of the physical commodity market with a mean-reverting nature, but the derivative market, in which the volatility of the near-maturity futures contracts is significantly higher than that of far-maturity futures contracts.

We denote P_t as the commodity price at time t, where

$$ln(P_t) = \xi_t + \chi_t. \tag{11}$$

ξ_t denotes the long-term equilibrium price level, and χ_t represents the short-term deviation from the equilibrium prices. ξ_t is modeled as Brownian motion with a drift rate μ_ξ and volatility σ_ξ and the following dynamics:

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi. \tag{12}$$

The short-term deviations from the equilibrium prices reflect the events in the market that affect the price in the short-term, but are smoothed in the long-term through the ability of market participants to adjust the production and inventory levels in response to market conditions. These short-term disequilibria are expected to dissipate with time. χ_t is therefore modeled as an Ornstein–Uhlenbeck process:

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi, \tag{13}$$

where κ is the mean-reversion coefficient, σ_χ is the volatility of the short-term factor, and $\boldsymbol{dz}_\xi$ and $\boldsymbol{dz}_\chi$ are the correlated increments of standard Brownian motion processes with $\boldsymbol{dz}_\xi \boldsymbol{dz}_\chi = \rho_{\xi\chi}\mathrm{d}t$.

As mentioned in Section 2.2, we use a risk-neutral valuation technique (Cox et al., 1985) to value an investment under multiple sources of uncertainty, instead of using a risk-adjusted discount rate. In other words, we risk adjust the individual uncertainties in the model. The short-term and long-term factors in the risk-neutral version of the two-factor price process can be described as

$$d\xi_t^* = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dz_\xi^*, \tag{14}$$

$$d\chi_t^* = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi^*, \tag{15}$$

where $\boldsymbol{dz}_\xi^*$ and $\boldsymbol{dz}_\chi^*$ are the correlated increments of standard Brownian motions such that $\boldsymbol{dz}_\xi^* \boldsymbol{dz}_\chi^* = \rho_{\xi\chi}\mathrm{d}t$, and λ_χ and λ_ξ represent the risk premiums subtracted from the drifts of each process.

In this case, the risk-neutral short-term factor reverts to $-\lambda_\chi/\kappa$. The drift of the long-term factor in the risk-neutral model is equal to $\mu_\xi^* = (\mu_\xi - \lambda_\xi)$.

As we use the Monte Carlo simulation to generate the prices and cash flows, we must discretize the price process. The discretization for the long-term component defined in Eq. (14) can be represented as

$$\xi_t^* = \xi_{t-1}^* + \mu_\xi^* \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t}. \tag{16}$$

The discretized version for the risk-neutral short-term component process is

$$\chi_t^* = \chi_{t-1}^* e^{-\kappa\Delta t} - (1 - e^{-\kappa\Delta t})\frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{(1 - e^{-2\kappa\Delta t})}{2\kappa}}, \tag{17}$$

where ε_ξ and ε_χ in Eqs. (16) and (17) are the standard normal random variables, correlated in each time period with the correlation coefficient $\rho_{\xi\chi}$.

The oil price model involves seven parameters (κ, σ_ξ, σ_χ, μ_ξ, λ_χ, $\rho_{\xi\chi}$, and λ_ξ), along with two initial values ξ_0 and χ_0 that must be estimated. As model parameters are not directly observable in the commodity markets, a tool to calibrate to these parameters must be used. Herein, we adopt the approach developed by Goodwin (2013),[5] using a Kalman filter and maximum likelihood estimation. The Kalman filter recursively computes the estimates for unknown parameters in the form of an a posteriori conditional distribution on a given data set of spot and/or futures prices and a measurement covariance matrix. The calibration performed for a market data set is described in Section 4.2. Section 5.3.3 describes the robustness check performed to test alternative price models and their influence on the obtained results.

3.4. Option to expand at stage 2 and LSM algorithm implementation

In this study, we focus on the valuation of the flexibility to drill additional wells during the production phase. We assume that the field operator holds an option to expand the production from the moment of the reservoir information revelation, which occurs as soon as the necessary production data are collected and processed. This point in time serves as the option holder's lower time constraint. The production experience in Stage 1 is assumed to generate perfect information regarding the remaining reserves. Knowing the initial rates per each well, the decision maker can estimate the future field production profile by using the exponential decline curve. After the reservoir information revelation, the operator has a certain number of years to exercise the option. Drilling activities are assumed to be performed by the end of the year in which the decision to expand was made. If the optional wells are drilled, the field production potential increases from the following year (see Fig. 1 for an example of how the decision to drill optional wells can affect the production profile). Managers are assumed to be able to reevaluate this decision once a year after the information revelation. This assumption is considered to be realistic for oil field development in terms of the time needed to process the data for decision support. The upper time constraint of the option, i.e., the moment at which drilling optional wells is considered to be no longer reasonable, is defined by the operator, based on the development strategy. In the case of small field development, owing to the reservoir depletion, this point may be limited to a few years after the decision to expand can be made for the first time. This means that we evaluate an American call option that can be exercised at predetermined discrete points in time.[6]

On the basis of the information regarding the initial production rates per well and then-current state of the oil market, the operator optimizes the expansion decision by choosing whether to exercise the expansion option at a given point in time. The valuation for problems such as the considered one, involving midway decisions that change the course of a project, is typically performed in a backward fashion (Jafarizadeh and Bratvold, 2009). This means that one first determines the optimal exercise strategy at the last decision point in time. Proceeding backwards in time, an optimization algorithm determines the optimal strategy for precedent choices. To this end, we apply a LSM simulation approach, which is "a state-of-the-art approximate dynamic programming approach used in financial engineering and real options analysis to value and manage options with early or multiple exercise opportunities" (Nadarajah et al., 2017). This approach is considered to be well suited for investment valuation problems in which the investment decision depends on multiple sources of uncertainties and involves multiple decision points. Notably, the LSM approach does not suffer from the curse of dimensionality (Longstaff and Schwartz, 2001), (Willigers and Bratvold, 2009). It is considered to be computationally efficient, flexible, and transparent as it is based on a simple least-squares regression. Real option valuation methods based on the LSM approach have been compared and verified by Nadarajah et al. (2017) and used in several oil and gas applications (Jafarizadeh and Bratvold, 2009), (Willigers and Bratvold, 2009), (Hong et al., 2018).

In our model, we first determine the expected yearly cash flows by combining the simulated production and cost profiles as well as the trajectories for the oil prices based on the risk-neutral process. To this end, we generate several sets of cash flows, where each set represents the simulated cash flows associated with the respective time when the decision to expand is made (i.e., project cash flow given that the optional wells are drilled in Year K, Year K-1, etc.). These cash flows serve as the main input for the LSM algorithm. At each decision point t_n, the algorithm compares the estimated value associated with the decision to drill optional wells now, denoted by $\Pi\left(t_n, P_{t_n}, Q_{0(expand)}\right)$, with the estimated value from the continuation of Stage 1, expressed as $\Phi\left(t_n, P_{t_n}, Q_{0(Stage1)}\right)$. At time t_n both parameters Π and Φ are unknown and are equal to the expected conditional values $\mathbb{E}^*_{t_n}$ of Π and Φ. Based on our assumptions regarding the oil price process and production rates, we can estimate the expected value of Π and Φ conditional on the current information regarding the oil price P and sum of initial production rates of wells Q_0.

$$F = max\left\{\mathbb{E}^*_{t_n}\left[\Pi\left(t_n, P_{t_n}, Q_{0(expand)}\right)\right], \mathbb{E}^*_{t_n}\left[\Phi\left(t_n, P_{t_n}, Q_{0(Stage1)}\right)\right]\right\} \tag{18}$$

Notably, the option can only be exercised at discrete time steps, in the interval between Year $K-1$—when the reservoir information is gathered and processed—and Year $K+n$—when the decision to expand can be last made (Fig. 1). The optimal value function F at time step t_n can be obtained using the following Bellman equation (Rodrigues and Rocha Armada, 2006): The algorithm starts with the last decision point and maximizes the expected project value along each simulated path. The algorithm determines the optimal decision by comparing the expected project values associated with the decision to drill optional wells now and the decision to keep the option unexercised by performing production using only the wells drilled in Stage 1 until the field shut-down. We then consider these decision policies in the precedent years and find the optimal expansion time (if undertaking the expansion is optimal) for each path of the simulation.

The main challenge in this process is to determine the expected conditional values of Π and Φ. To identify these values, we adopt the technique recommended by Longstaff and Schwartz (2001), who use linear regression to estimate the expected value of the future cash flows conditional on the current information on the state variables (in our case, the simulated oil prices and sum of the initial production rates per well), defined as

$$\mathbb{E}^*_{t_n}\left[\Pi\left(t_n, P_{t_n}, Q_{0(expand)}\right)\right] = \alpha_1 P_{tn} + \alpha_2 Q_{0(expand)} + \alpha_3 {P_{t_n}}^2 + \alpha_4 Q^2_{0(expand)} + \alpha_5 P_{tn} Q_{0(expand)}, \tag{20}$$

where $\alpha_{1\ldots5}$ denote the regression coefficients. The same applies to the estimation of expected value of Φ, except the Q_0 parameter used in the regression. In case of Φ, we should use the sum of initial rates of Stage 1 wells only. Any additional risk factors that can affect the expected continuation values can be easily added to the regression model (Willigers and Bratvold, 2009).

As suggested by Longstaff and Schwartz (2001), we use only in-the-money paths to estimate the regression parameters "since it allows us to better estimate the conditional expectation function in the region where exercise is relevant". According to this principle, in Year $K+n-1$, we consider the project values of only the simulated samples for which the optimal decision was to expand in Year $K+n$ and regress them on the current oil price and sum of initial production rates per well.

To calculate the regression coefficients α_i in Eq. (20) based on the in-the-money paths at each time step, we employ the MATLAB "backslash"

[5] Goodwin (2013) uses a MATLAB function to estimate the parameters of the two-factor process. This approach is computationally efficient and allows the process calibration by using a large data set of historical prices, which is required as an input.

[6] This type of an option is also known as the Bermuda option.

solver,[7] following the approach recommended by Jafarizadeh and Bratvold (2015). The fitted values from this regression are used as an estimate for the expected conditional values of Π and Φ, respectively. If the immediate exercise value is higher than the estimated Stage 1 continuation value, the optimal decision is to exercise the option; otherwise, holding the option is optimal.

Moving backwards in time, we calculate the maximum value at each time step. In our case, the algorithm stops after evaluating the decision in Year $K-1$, in which the data regarding the reservoir are gathered and the decision whether to exercise the option can be made for the first time. The cash flows for the years in which the decision maker does not have any flexibility, in our case, before Year $K-1$, are evaluated based on a simple DCF procedure. To ensure the consistency of the results, the DCF evaluation is based on the same simulated oil price paths as the real options procedure. Next, we calculate the overall project value by summing the cash flows that the project generates before Year $K-1$ and the values resulting from the LSM algorithm from Year $K-1$, all discounted with the risk-free rate.

If the traditional DCF approach is used to evaluate a project with flexibility, the value of waiting to expand cannot be reflected. This additional value stems from the fact that the operator can optimize the decision to drill optional wells based on the new knowledge. To the best of our knowledge, models that allow to evaluate the staged development with an option to drill additional wells have not been reported. Previous contributions primarily considered the value of the option to expand production by connecting a tie-in field (Fleten et al., 2011) or initiating an IOR solution (Hong et al., 2018). Notably, the focus of these contributions was on the upside potential that can be exploited by a production increase, while the main focus of this work is to illustrate how the staged development strategy allows the mitigation of the prominent downside risk, especially in the case of a marginal project.

4. Case study

We now illustrate the implementation of the proposed method to evaluate an investment in a small oil field based on a synthetic yet realistic industry case. The relevant technical inputs for the model are selected to ensure that a realistic case can be considered, taking into account the main features of an offshore field development project, while disregarding components that are considered to be of secondary importance and likely to render the process demonstration less intuitive and transparent. The goal of the case study is not to analyze a specific project investment, but rather to illustrate the valuation algorithm based on the synthetic case, and to highlight that the workflow is sufficiently flexible to be easily implemented for other project cases.

Following the proposed modeling procedure, we first build 30,000 simulation samples[8] of production profiles per well for each of the expansion scenarios (five sets of 30,000 paths), which serve as the input for the cost profile estimation. Next, we establish the project's cash flows based on the simulated production profiles and oil price paths to run the LSM algorithm to identify an optimal strategy for the staged development.

Fig. 5 illustrates the flexibility that an operator has in the considered project case. Year 7 is the year in which the data regarding the reservoir are gathered, and the decision to expand can be made for the first time. The operator is assumed to hold the expansion option for a timespan of three years, reevaluating the decision once a year within this period of time. Such a restriction for the number of decision points is consistent with the project's expected lifespan and other features such as the loss of migrating oil and time value of money. In this configuration, the LSM algorithm is applied from Year 9 (when the optional wells can be drilled last) working backwards to Year 7. Reducing the time needed for the initial stage of the project to make a decision regarding Stage 2 drilling would clearly increase the value of the staged development. We, however, remain conservative, thereby providing an operator more than sufficient time to gather and process the Stage 1 production data.

4.1. Production profiles

The case study mimics the production and cost profiles for the standalone development of an offshore oil discovery with relatively low reservoir properties, allowing an ultimate recovery factor of the order of only 20%. The main components of the field design consist of a single platform and subsea templates. The oil is offloaded to a tanker and transported to the mainland. A part of the gas production is used to generate power for the platform, and the remaining part is re-injected into the reservoir.

Overall, 6 oil production, 3 water injection, and 1 water production wells are planned for the standard (non-staged) development. In contrast, in the staged development, 4 oil production, 2 water injection, and 1 water production wells are drilled in Stage 1, with an option to drill the remaining 2 oil production and 1 water injection wells in Stage 2.

We estimate the initial production rates per well, accounting for the geological dependencies by using the Monte Carlo simulation, as described in Section 3.1. The procedure starts with assigning the initial rate for Well 1, followed by the remaining wells, applying Eq. (3) for Wells $i = 2..6$ and Wells $i = 2..4$ in the standard development and staged development cases, respectively. The input parameters used in the Monte Carlo simulation based on the input from the field engineers are presented in Table 2.

In general, the expected initial rate of Wells $i = 2..6$ decreases because of the factor k_i in Eq. (3), which is less than 1. In our case, the expected value of the initial rate of Well 2 is 33.3% lower than the initial rate q_0 of Well 1. Nevertheless, given the simulation inputs, Well 2 may have a greater initial rate than Well 1. Fig. 6 shows the probability distributions for the initial rates of Wells $i = 2..6$ with a given initial rate of Well 1 for two example cases within the standard development case. For cases a and b we assume that q_0 of Well 1 equals 5.2 and 2.8 mmbbl/y, respectively. If the initial rate of Well 1 is relatively high (as for case a in Fig. 6), Wells $i = 2..6$ are more likely to produce at a higher than average rate owing to the geological dependencies between the wells. The opposite holds for case b.

To account for the learning effect during Stage 1 within staged development, we generate samples of the initial rates of the first optional well, i.e., Well 5, by considering the dependency presented in Eq. (4). As discussed in Section 3.1, the information revealed during Stage 1 might lead to either an increase or decrease in the expected initial production rate for optional wells compared to the respective a priori initial rates. We assign the probabilities for the high and low cases to be 60% and 40%, respectively. Given these probabilities, the factor k_i^* in Eq. (4) is sampled from a normal distribution with mean 0.45 and standard deviation 0.015 in 60% of the simulated cases; in the remaining 40% of the cases, a distribution with mean 0.26 and standard deviation 0.07 is used (see Table 2). The initial rate for Well 6 within the staged development is sampled using Eq. (3) based on the generated values for Well 5.

Although the low case has a relatively high assigned probability for our case, the expected overall recovery from Wells 5 and 6 under the staged development is expected to slightly increase. However, this does not necessarily mean that the staged development (given that Wells 5 and 6 are always drilled) corresponds to an increased recoverable volume in all the cases, compared to that in the standard development. Our simulation results show that in 34.2% of the cases, the total reserves under the standard development strategy are larger than those under the

[7] The backslash solver yields m unknowns for n system of equations when $n = m$. If $n > m$, this function uses the linear least-squares regression to estimate m (Jafarizadeh and Bratvold, 2015).

[8] 30,000 iterations proved to be computationally reasonable and produce a consistent and stable result that deviated insignificantly throughout several simulations.

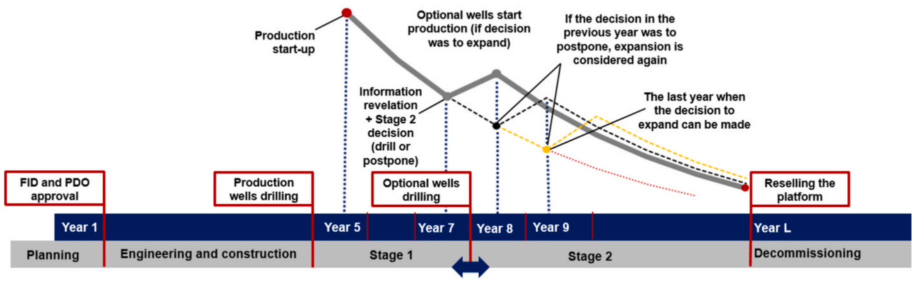


Fig. 5. Decision gates of the project under the staged development strategy.

Table 2
Input parameters for the Monte Carlo simulation.

	Distribution	Mean	Unit	Std. dev.
Initial production rate, Well 1	Normal	4.0	million bbl/year	0.9
k_i	Normal	0.667	–	0.12
k_i^* high case	Normal	0.45	–	0.015
k_i^* low case	Normal	0.26	–	0.07

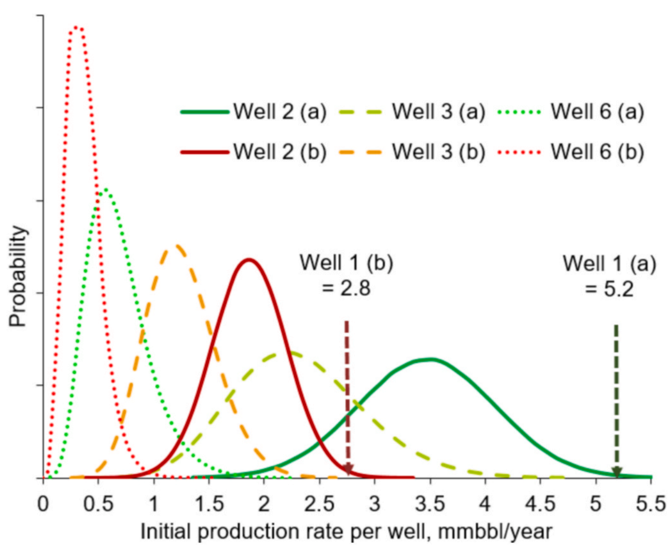


Fig. 6. Probability density functions of the initial production rates per well with a fixed initial rate for Well 1 (cases *a* and *b*) for the standard development case.

staged development.

Additionally, we consider that the production potential of the Stage 2 wells decreases by 1% yearly from Year 7 (when the expansion decision is made for the first time) owing to the loss of the migrated oil. Consequently, in addition to the time value of money and depreciation effects, the decision maker has another incentive to drill Stage 2 wells earlier.

Assuming that the production rate follows an exponential decline function and is not limited by the capacity constraint, we simulate the production profiles. Given the initial production rate distributions by each well, we estimate 30,000 possible production profile realizations for each expansion scenario by using Eq. (5) with a nominal decline rate of 22.5%. In Section 5.2, we perform a sensitivity analysis to examine the influence of the decline rate assumptions on the obtained results. The minimal production rate per well is set as 0.05 million bbl per year. Once this threshold is reached, a well is assumed to stop production. Fig. 7 illustrates the confidence bands of the field production profiles for the case of the staged development, given that the expansion takes place in Year 7 in each case.

Summing the production profiles of all wells, we obtain the total recoverable reserves for all simulation samples. Those are shown in Fig. 8a and Fig. 8b for all production wells, with the confidence bands resulting from the Monte Carlo simulation. The results show that the additional expected recovery decreases as more wells are drilled, indicating that the P50 value is continuously decreasing from Well 1 to Well 6 in the standard development case. In the case of the staged development strategy (Fig. 8b), the expected recoverable volume for the optional wells drilled in Stage 2 increases compared to the respective values under the standard development.

Table 3 summarizes the total recoverable reserves of the field under the standard development and staged development strategies. For the staged development, the results for three cases are presented. The confidence bands presented in the first row result from the expansion optimization, discussed in Section 5. The LSM algorithm indicates that in 24.7% of the simulated cases, the decision maker should not drill additional wells and should produce only with the Stage 1 wells. It follows that the recoverable reserves under the two strategies are similar (the mean for the staged development is even slightly lower), thereby enabling a fair comparison of the economic value for the two strategies. The values in the last two rows are limited to extreme points when optional wells are drilled in all and none of the simulated cases. If optional wells are always drilled, the field production potential increases; however, the production of these reserves might be uneconomical.

4.2. Oil price simulation

As mentioned in Section 3.3, we calibrate the oil price process

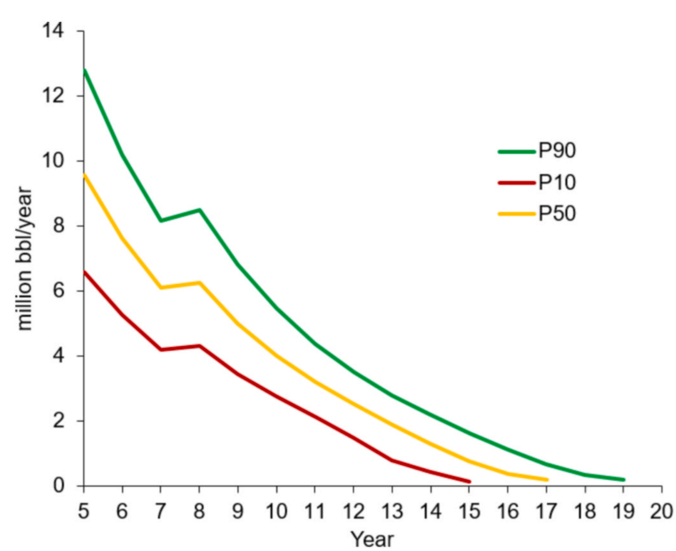
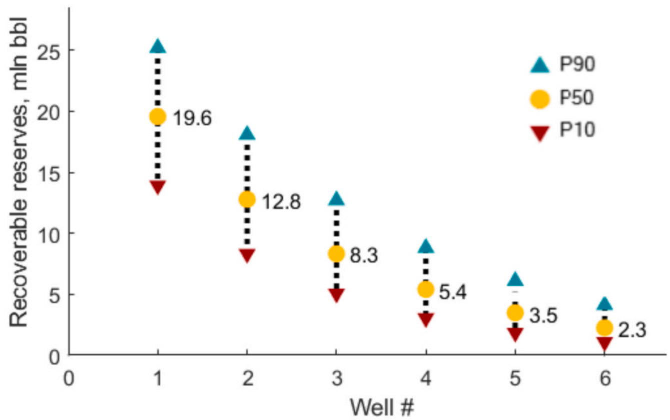
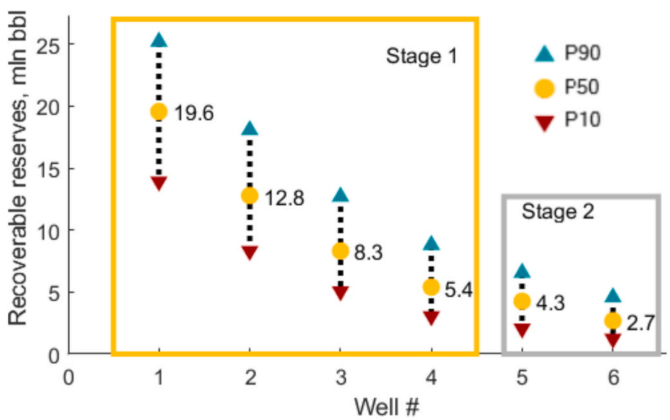


Fig. 7. Confidence bands of the expected production profile under the staged development (expansion in Year 7).



(a) Standard development



(b) Staged development without optimization

Fig. 8. Total recoverable reserves per well.

Table 3
Confidence bands of the recoverable reserves, mmbbl

	P10	P50	Mean	P90
Optimized staged (optional wells are drilled when it is optimal)	33.8	52.0	52.8	72.4
Standard	35.2	52.0	53.0	72.2
Staged (optional wells are drilled in all the 30,000 cases)	36.4	53.2	53.9	72.5
Staged (optional wells are drilled in none of the 30,000 cases)	31.8	46.4	46.9	62.6

parameters based on the historical market data by using the Kalman filter. We use Thomson Reuters weekly (averaged daily) data pertaining to the ICE Brent historical futures contracts from March 2006 (the first available forward curve with 81 month maturity) to November 2019, along with the dated Brent FOB North Sea spot price for the same period. Subsequently, we average the monthly maturity contracts with mid- and long-range maturities to construct forward curve vectors with maturities of 1, 2, 3 … 12, 14 … 18, 21, 25, 28, 34, 40, 46, 55, 66, and 78 months to decrease the computational effort for the Kalman filter algorithm. The overall data set for the model calibration forms a 715*24 matrix.

The resulting oil price process parameters are reported in Table 4. Fig. 9 illustrates the confidence bands and expected value for the risk-neutral price process based on the 30,000 simulated paths. The thin colored lines represent examples of the simulated price paths used for our valuation procedure. The resulting expected (mean) price increases from 56 USD/bbl in 2021 (Year 2 in our procedure) to 65 USD/bbl in 2040 (Year 21).

Table 4
Calibrated parameter values used for the Schwartz–Smith two-factor price process simulation.

ξ_0	4.0	χ_0	0.1
σ_ξ	13%	σ_χ	58%
μ_ξ^*	−0.4%	$\rho_{\xi\chi}$	0.07
κ	0.52	λ_χ	9.82%

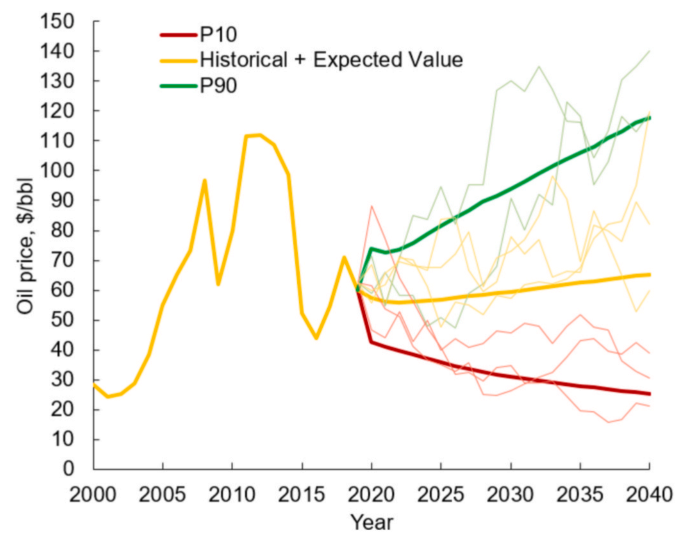


Fig. 9. Historical Brent crude oil prices, confidence bands, and simulation examples for the risk-neutral price process.

4.3. Costs and abandonment

As mentioned in Section 3.2, the main components of the CAPEX are the facility cost and the cost of production drilling. The former is assumed to consist of fixed and variable components that depend on the chosen capacity of the platform. For the considered case, the facility cost is payable over several years, as indicated in Table 5 and Fig. 10. The cost of drilling and including one oil production well in the production system is equal to 48 MM USD. The CAPEX is assumed to have a random component that can increase/decrease the cost estimate within a 15% range for a specific simulated cost profile. The resulting confidence intervals for the CAPEX estimates are presented in Table 5. In the staged development case, Wells 5 and 6 remain undrilled in Year 4. The decision maker can consider whether to drill those wells in Year 7, 8 or 9 or to not drill these wells at all.

The main cost elements of OPEX pertain to the storage vessel leasing, facility staffing and operations, and fuel costs. The fixed component in

Table 5
P10–P50–P90 CAPEX estimation under the staged and standard development strategies, MM USD.

	Year 2	Year 3	Year 4	Year 7, 8 or 9
Staged (optional wells are drilled)	85-96-108	32-36-40	602-683-765	127-145-162
Standard	85-96-108	32-36-40	729-828-927	0-0-0
Staged (optional wells are not drilled)	85-96-108	32-36-40	602-683-765	0-0-0

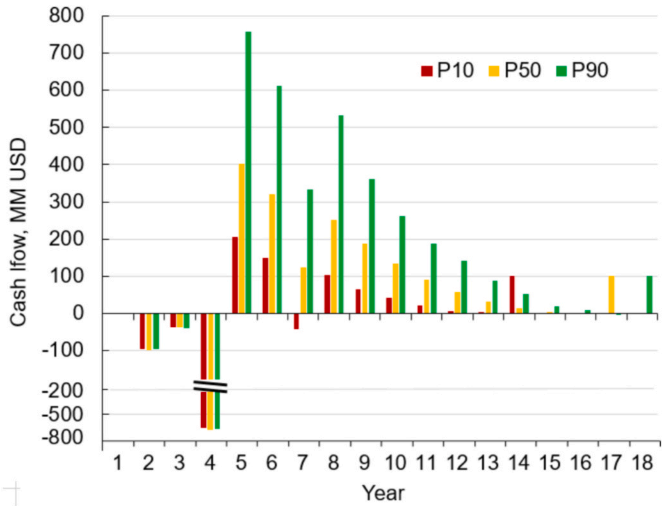


Fig. 10. Project cash flows under the staged development strategy (optional wells are drilled in Year 7).

Eq. (10) equals 84 MM USD, with the *b*-coefficient set as 0.6.

Moreover, we must consider the end of the field lifetime as it bears notable implications for the field management strategy and project cash flow. To this end, we first introduce a decision rule to cease the production and abandon the field. Once the project cash flow reaches negative values owing to the reservoir depletion, only an oil price upturn can enhance the revenue from the negative to positive values. By performing simulations, we empirically determine a threshold, which is considered to be a "point of no return" for the project development. The production is assumed to be ceased as soon as the cash flow falls below -0.93 MM USD. Considering the calibrated parameters for the oil price process, the probability to reach a positive cash flow after reaching this threshold is reasonably low (less than 0.5%). One year after the production is halted, the company incurs certain losses owing to the decommissioning. However, the company can resell the production platform, which is expected to generate an overall revenue of 100 MM USD in the last year of the project life. Fig. 10 illustrates the confidence bands for the project's expected yearly cash flows. Unfavorable price and production scenarios that represent the P10 case force the oil company to shut down the field and resell the platform early.

5. Results

The key issue that the decision maker encounters in the considered case is whether to invest in drilling all the wells before the oil production start-up or to opt for the staged development strategy. The goal of this study is to evaluate whether the staged development creates value and to clarify the conditions in which the staged development is the preferred strategy.

We first study the staged development case with the option to expand. Based on the input from the technical part of the workflow, we use the LSM algorithm to optimize the expansion decision. The simulation with 30,000 trials for each development strategy (expand in Years 7, 8, or 9/do not expand/perform standard development) generates five sets of 30,000 production and cost profiles as well as risk-adjusted paths for oil prices. The value of the investment at a respective decision node is calculated by determining the expected project value discounted at the risk-free rate of 2.5%.

As mentioned in Section 3.4, the LSM algorithm works in a backward manner, starting from Year 9 and then moves to the precedent time steps, i.e., Years 8 and 7, to optimize the expansion policy for each simulation case.

Fig. 11 illustrates the results related to the optimal timing of exercising the option to expand, as obtained using the LSM algorithm. In 68.5% of all simulated cases, it is optimal to drill optional wells as soon as the reservoir information is gathered, i.e., in Year 7. In these cases, the field production potential and oil prices favor the early expansion. However, the decision maker is recommended to defer the drilling of the optional production wells until Years 8 and 9 in 4.0% and 2.8% of the cases, respectively. Finally, in 24.7% of the cases, the optimal decision is to leave the option to expand unexercised and avoid drilling optional wells. These cases represent the realizations of unfavorable production and/or price scenarios.

By identifying the optimal expansion decision for each simulated case, we sum the value that the project generates before the optional wells are drilled (if the expansion is optimal) and that over Stage 2. As mentioned in Section 3.4, the former value is evaluated using a simple DCF approach, whereas the latter value is the result of the risk-neutral real option valuation procedure.

Subsequently, we derive the value of the project when using the standard development strategy, which serves as a reference point. As the standard development considers an irreversible commitment at Year 4 to drill all the production and injection wells, the value can be evaluated using a simple DCF approach. To this end, we adopt the same data set (Monte Carlo simulation for the production rates for Wells 1..4, cost profiles, and oil price paths).

Table 6 summarizes the results of the project valuation for the staged and standard development case. Owing to the flexibility to optimize the expansion decision, the staged development corresponds to increased values of all the confidence bands and, most importantly, the expected value of the project. In Section 5.3.1, we discuss the potential to further increase the value of the flexible project by accounting for additional flexibility to choose the number of optional wells.

The choice between staged and standard development is affected by several factors. A key factor is the amount of the additional oil production generated in Stage 2. As mentioned previously, the decision maker can improve the drainage strategy by adjusting the locations and design of the Stage 2 wells by using the information gathered during Stage 1. This process helps increase the expected recovery for Wells 5 and 6 for the high case in comparison with that for the standard development. However, for the low case, the optional wells exhibit a lower recovery than expected. The assigned probabilities to the high and low cases represent key elements in the choice between the staged and standard development. To isolate the value of flexibility, we select the parameters to update the recoverable volumes of the Stage 2 wells (60% high case) such that the P50 recoverable reserves of the flexible project

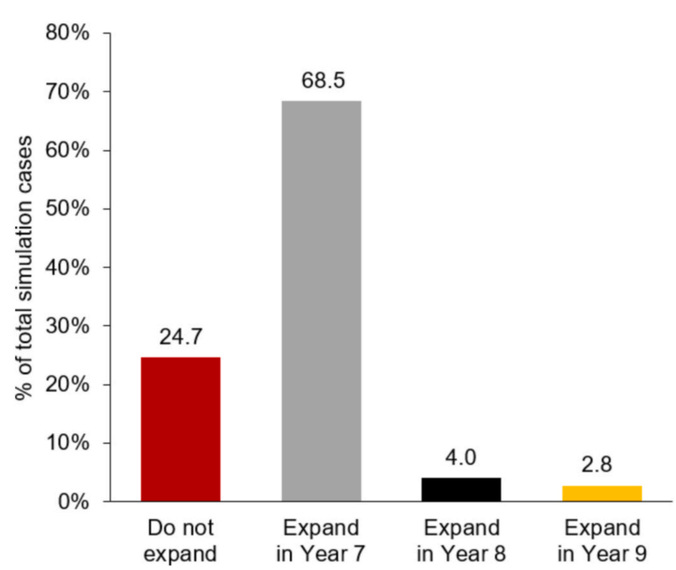


Fig. 11. Optimal expansion timing as a percentage of the total number of simulation cases.

Table 6
Confidence bands of the pre-tax values of the flexible and inflexible projects, MM USD.

	P10	P50	Expected value (mean)	P90
Optimized Staged	−164	675	885	2186
Standard	−188	655	851	2127

(under the optimized staged development strategy) are equal to those under the standard development. This configuration helps realize a fair comparison between the staged and standard development because the amount of value created solely by the flexibility can be identified without considering the increase in the recoverable volume with the staged development.

Fig. 12 shows the evolution of the values of the two strategies as a function of the probability of the high case for updating the initial rate of the Stage 2 wells under the staged development scenario. The probability of the high case must be at least 35.5% (as shown by the red arrow in Fig. 12) for the staged development to be the preferred option. Specifically, if the engineers expect the information gathered during Stage 1 to ensure an expected total recoverable volume of at least 51.9 mln bbl, the staged development strategy is the optimal choice. If not, the value lost during the wait for the information revelation cannot be compensated by the production increase in Stage 2 or ability to partly hedge against the downside risk by not drilling optional wells.

Furthermore, our methodology allows to derive the investment boundaries at which it is optimal to exercise the expansion option. These investment boundaries are defined as functions of the sum of initial production rates per well, as indicated by the data collected during Stage 1 and the observed oil price in the market when the decision is made. The threshold boundaries result from hyperbolic fitting for the boundary combinations of the production rates/oil price (dots in Fig. 13) that result in the decision to exercise the option in the LSM algorithm. Fig. 13 illustrates the derived investment boundaries for each year when the expansion is possible. If the combination of the current oil price and sum of the initial production rates lay to the upper right of the respective investment boundary, it is optimal to exercise the expansion option in this year. If the combination lays to the lower left, it is optimal to wait and revisit the decision one year later. The continuation region becomes wider as the investment boundary moves to the upper right with each year while the decision maker defers the decision to expand. Higher oil prices are then needed to justify an expansion later in time.

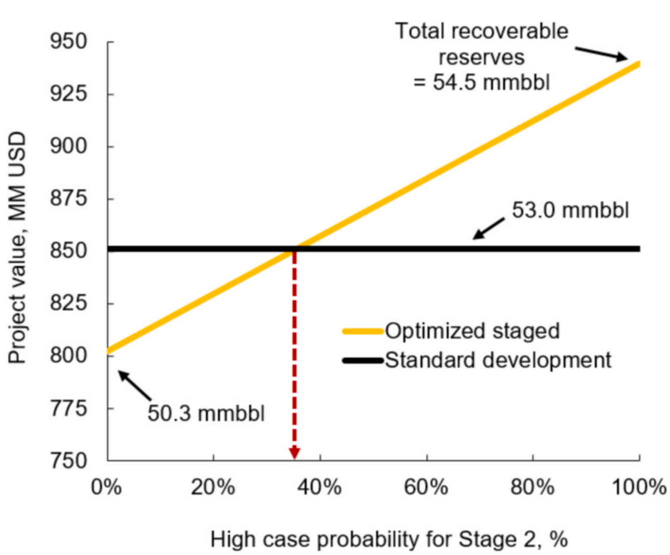


Fig. 12. Project value under the staged and standard development depending on the probability of the high case for the Stage 2 wells.

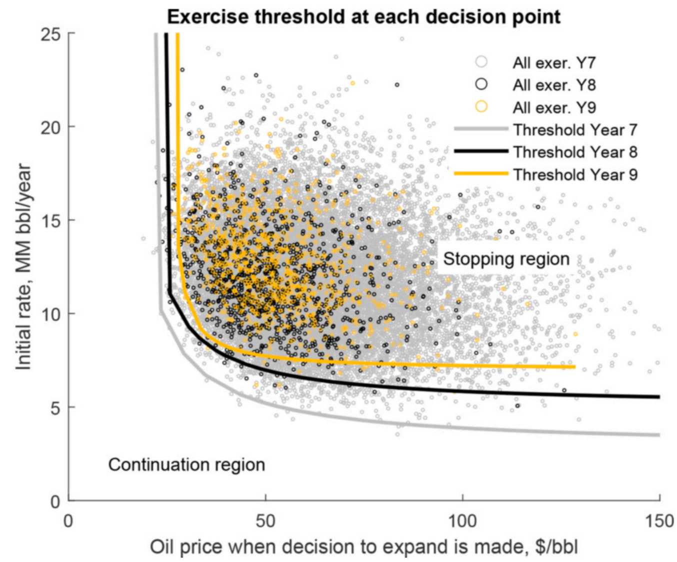


Fig. 13. Results of the expansion timing optimization performed using the LSM algorithm along with the threshold boundaries.

5.1. Downside risk mitigation

A key advantage of the staged development strategy is the possibility to leave the expansion option unexercised if the reservoir information indicates an excessively low potential production inflow in the case of drilling more wells and/or the oil prices are at a level, which does not trigger investment. In other words, this strategy allows one to partly hedge against the downside risk in the unfavorable reservoir/oil price scenario by not drilling the uneconomical wells. We, however, should also note that for the case of real options, unlike financial options, a perfect hedge is almost never possible. This is due to the fact that multiple sources of uncertainty must be considered, and that the decisions have long-lasting impacts. Moreover, the optimization algorithm proposes an optimal decision given the then-current knowledge, while the project remains exposed to certain risks in the future that are not hedged. In this regard, we acknowledge that the staged development can provide only a partial hedge.

In a low reservoir/oil price scenario, the project value is at risk, making the decision maker more likely to reject the investment when no opportunity exists to react to the outcome of the uncertainty. Fig. 14 shows the project value distribution in the low reservoir/price scenario under the staged and standard development. Only the simulated cases in which the sum of the initial production rates per well for the staged development is less than 7 mmbbl and the oil price is less than 40 USD/bbl in Year 7 are reflected (in these cases, the decision not to drill Stage 2 wells is optimal). The staged development strategy in these cases tends to decrease the project value losses compared to that in the standard development case.

Performing a simulation with 30,000 samples allows us to capture a wide range of probable combinations of production and cost profiles, oil price paths, and scenarios of information revelation after Stage 1. Fig. 15 illustrates the probability that the project value under the standard development (right axis) is below a certain level (horizontal axis). The yellow line (left axis) in Fig. 15 reflects the additional average project value if the staged development is implemented instead of the standard development for the same simulation cases (under the same initial production rates for the Stage 1 wells, oil prices, etc.).

The average increase in the project value owing to the staged development if the project value under the standard development is below −500 MM USD is 82 MM USD. Overall, in 81% of the simulated cases in which standard development results in a negative value, using the staged development increases the project value. In the simulated

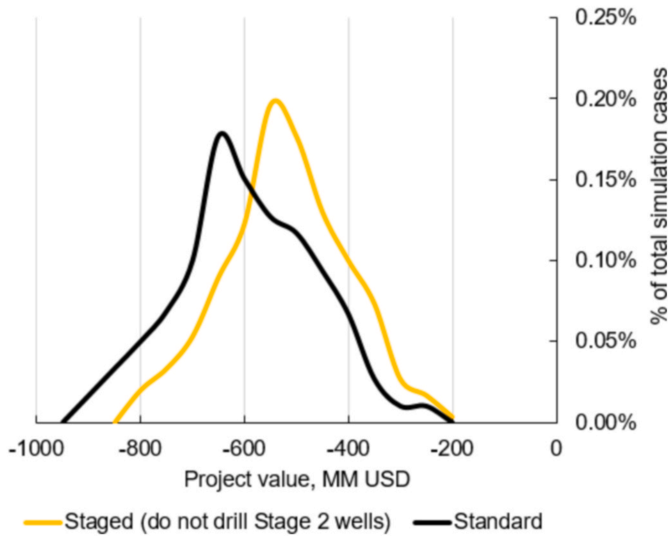


Fig. 14. Project value distribution under staged and standard development in the low reservoir/price scenarios.

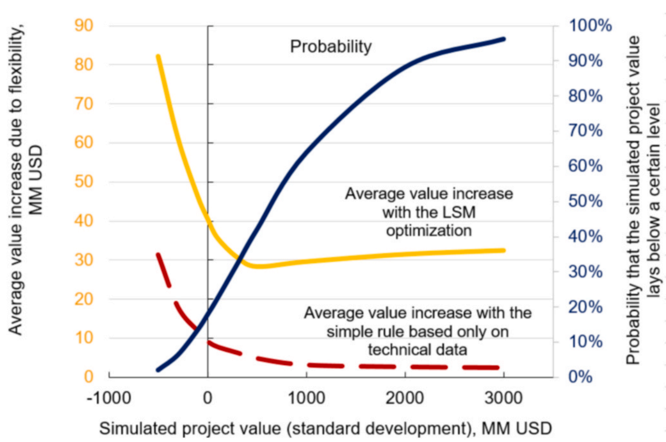


Fig. 15. Project value increase in the staged strategy against the standard development strategy.

cases in which the expected project value under the standard development is high (in the case of a large reservoir or under high oil prices), the effectiveness of the sequential drilling decreases. However, for the range of simulated project values below 3000 MM USD, the staged development adds value in 59% of the cases.

Furthermore, we compare the results from the optimization based on the LSM algorithm with a simple approach based on a now-or-never decision. Consider that the managers decide to drill Wells 5 and 6 in Year 7 only if the information indicates that the associated compound initial rate would exceed 0.7 mln bbl/year. By following this rule, the decision maker disregards the market uncertainty and value of waiting, focusing only on the technical part of the problem. The red dashed line in Fig. 15 (left axis) reflects the average NPV added by such a strategy within the staged development. Although such an optimization approach also allows the hedging of a certain part of the downside reservoir risk, only 55% of the simulated cases with the negative project value under the standard development are improved owing to this strategy implementation (compared to 81% when using the LSM algorithm). In the worst cases (when the project value is below −500 MM USD), this flexibility adds, on average, 31 MM USD. For the range of project simulated values below 3000 MM USD, the average increase in the value is approximately 3 MM USD.

These findings are in agreement with those reported in Fedorov et al. (2020), in which we performed a benchmark reservoir model simulation to identify the value of the staged development in the presence of prominent reservoir uncertainty. However, as mentioned in the introduction section, whether the staged development creates additional value depends on the considered problem. In this context, the objective of this study is to establish a methodology that can be easily used for other project cases to assess whether the staged approach would be beneficial.

5.2. Sensitivity analysis

This section describes the sensitivity of the results to several key factors. We focus on the effect of the changes in the production decline rate, oil price volatility, and drilling costs.

In our case study, we assume that the production profiles follow an exponential decline curve with a constant nominal decline rate of 22.5%, based on the input from the field engineers. Nevertheless, it is desirable to identify the change in the results under a different reservoir depletion rate. To maintain the recoverable reserves mean in line with the reference case (approximately 53 mmbbl), we must adjust the initial production rate of wells while reducing/increasing the nominal decline rate. Thereby, a steeper decline would mean a higher production rate at earlier stages to reach the same cumulative production by the end of the field lifetime.

Fig. 16 illustrates that a lower production decline corresponds to a higher benefit of the staged development, as mid- and late life decisions influence the project value more significantly. Notably, we adopt conservative values in our analysis: the added value by the staged development in our reference case is 4% under a nominal decline rate of 22.5%; nevertheless, under a decline rate of 8%, the staged development can add 14% of the value compared to that of the standard development.

Although the calibration based on the historical market data provides valuable knowledge regarding the probable evolution of future oil prices based on the obtained information, a perfect estimate cannot be achieved. In other words, the historical data are based on past expectations in the market and are thus backward looking in nature. Consequently, these data cannot fully reflect the possible changes in the future, and the parameters used to build a stochastic price process may change with time. In such scenarios, a sensitivity analysis must be conducted to illustrate how the probable changes in the underlying uncertain parameters may affect the investment decisions and project value.

According to the sensitivity analysis, the project value depends on variations in both the short- σ_χ and long-term σ_ξ factors used in the oil price modeling. However, the sensitivity to σ_χ is rather small, primarily because of the long build-up phase of the project (4 years), which decreases the effect of the short-term price variations on the project. In

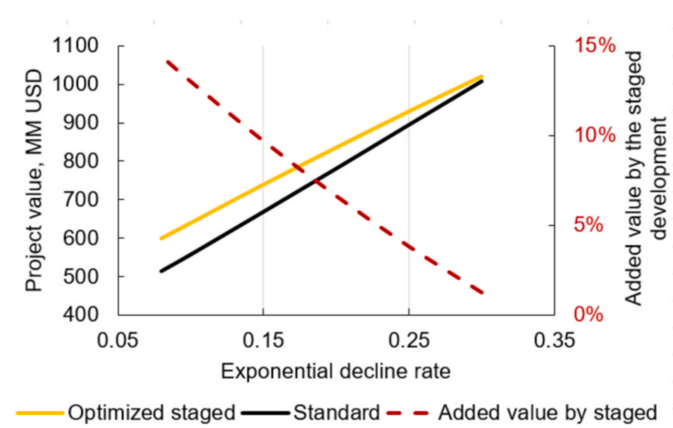


Fig. 16. Sensitivity of the project value to changes in the exponential decline rate (with adjusted initial production rates).

contrast, the long-term factor is more notable because the field production spans over a time horizon that starts in five years and ends 12–18 years from the investment decision (depending on the optimal shut-down time). These results regarding the sensitivity of the petroleum production investment (even in a small oil field) to the long-term volatility are in agreement with the findings reported by Jafarizadeh and Bratvold (2015). In particular, Jafarizadeh and Bratvold (2015) argue that the uncertainty in the long-term price equilibrium makes the Schwartz–Smith oil price model particularly suitable for valuing long-term investments affected by the commodity prices.

As illustrated in Fig. 17, the project value increases with σ_ξ, as well as the difference between the value of the flexible and non-flexible projects. This phenomenon is a result of the shift in the expansion timing (see Fig. 18). The yellow line in Fig. 18 illustrates that a higher long-term volatility corresponds to a higher willingness of a decision maker to postpone the production expansion decision while awaiting higher oil prices.

Another parameter that considerably influences both the project values and expansion policy is the drilling cost of the production well. Because we consider a fixed value of 48 MM USD, the decision maker has a certain incentive to wait for the information revelation before committing to drill all the production wells. Fig. 19 shows that the values of the flexible and non-flexible projects diverge as the drilling cost of the production well grows. This phenomenon occurs owing to the increasing amount of simulation cases in which the optimal decision is to retain only Stage 1 wells and not expand to prevent extensive spending on drilling. The flexibility to leave the expansion option unexercised allows one to hedge against this risk.

Notably, if these expenditures are excessively low, the added value of the staged development strategy decreases, as the decision maker tends to drill optional wells even in the low-case scenario. In contrast, increasing drilling expenditures may increase the hesitance of the decision maker in investing in Stage 2 wells. As shown in Fig. 20, a larger well cost corresponds to a higher likelihood that the expansion is not optimal.

5.3. Robustness check

We performed certain additional analyses on the proposed modeling approach by including a higher flexibility in the drilling strategy (see Section 5.3.1), accounting for additional uncertainty in the production rates (see Section 5.3.2), and testing different assumptions on how the future oil price is modeled (see Section 5.3.3).

5.3.1. Altering the number of optional wells in stage 2

The proposed modeling approach involves an assumption that the operator can choose whether to drill two optional wells in Stage 2 or to

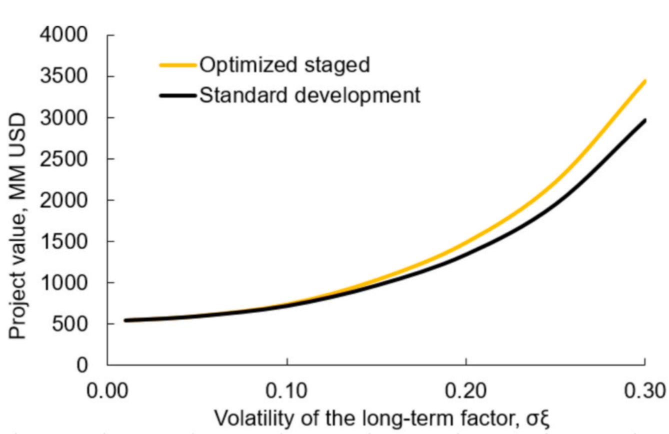


Fig. 17. Sensitivity of project value to changes in the long-term oil price volatility factor.σ_ξ

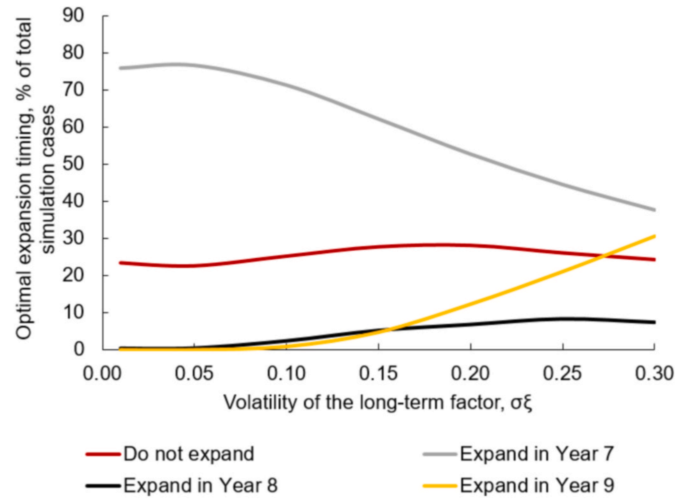


Fig. 18. Shift in optimal expansion timing with changes in the long-term oil price volatility factor.σ_ξ

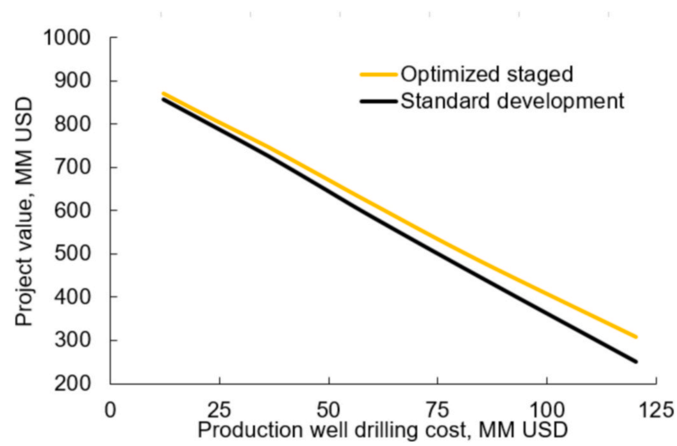


Fig. 19. Sensitivity of project value to changes in the production well drilling cost.

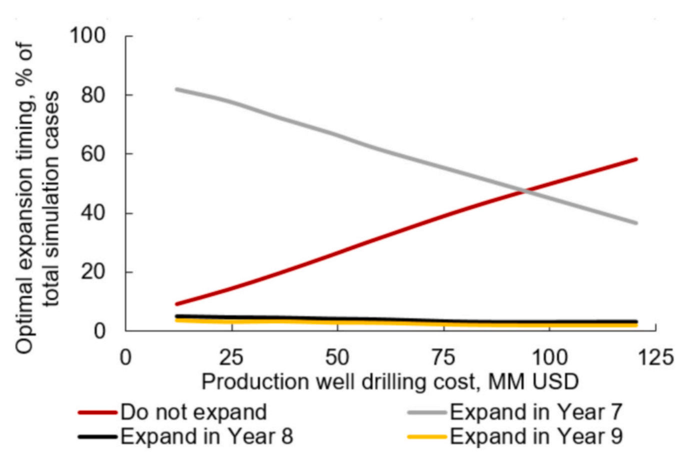


Fig. 20. Shift in optimal expansion timing with changes in the production well drilling cost.

drill no wells. In reality, the operator might not be limited to a fixed number of wells that must be drilled after the initial stage of the project. The staged development allows for a higher flexibility by enabling the decision maker to respond to the information generated at Stage 1.

Therefore, we conduct a robustness check of the model and investigate whether altering the number of wells in Stage 2 considerably influences the expected value.

Notably, the additional data generated during Stage 1 might indicate that several optional wells have little potential to create additional value if drilled. To conduct the robustness check, we first introduce an opportunity to choose only one optional well of the two to be drilled in Stage 2. To account for this flexibility, the number of simulation sets must be increased, as in this case, the operator can choose between three alternative policies each year instead of two: to drill only one optional well with the highest expected recovery now, to drill two wells now, or postpone the drilling decision until next year. The LSM algorithm enables the direct implementation of this aspect in the analysis. In only 1.6% of the simulated cases the optimal decision is to drill only one of the optional wells. Table 7 presents the results of the project valuation accounting for the additional flexibility. Incorporating the possibility to choose the best well or drill both optional wells in the valuation procedure leads to only a minor increase in the expected value of the project by 0.02%. For the considered case, in most of the simulated cases, both optional wells exhibit sufficient production potential to ensure a positive additional cash flow if drilled.

Furthermore, the updated knowledge regarding the subsurface after Stage 1 might indicate that the operator can drill more than two optional wells owing to a higher than expected reservoir performance. As mentioned previously, in this case study, we assume that the expected recovery under the staged and standard drilling strategies is equal to enable a fair comparison. However, for this robustness check, we assume that in 20% of the simulated cases that correspond to the high case for Stage 2, the operator can drill three additional wells instead of two. The initial production rate of the third optional well is modeled in the same manner as the second well by using Eq. (3). By including this flexibility in the proposed model, we can optimize the number of optional wells drilled (i.e., only the best one, the two best, or all three[9]). The results in Table 7 demonstrate that accounting for this flexibility increases the expected value of the project under the staged development by 0.8% (6.9 MM USD). However, this increase can be primarily attributed to the additional expected recovery (+0.6%) yielded by the third optional well, which is optimal to be drilled in 9.1% of all simulated cases.

Fig. 21 illustrates the optimal expansion policy accounting for an opportunity to choose the number of optional wells, as a percentage of all simulated cases. Comparing the results presented in Fig. 11 for the reference case of the staged development with those presented in Fig. 21, it can be noted that accounting for the flexibility in the number of optional wells leads to minor changes. Overall, the percentage of the simulated cases in which optional wells remain undrilled decreases because the decision maker can now choose to drill only one of the optional wells that creates the highest value, rather than having to drill both. Moreover, it is optimal to drill three wells in 78.1% of the cases, when the operator is able to add a third well, whereas in 6.3% of the cases, it is optimal to drill the two best wells out of the three available. In 0.7% of the cases, only one well is drilled, whereas in 15.0% of the cases, the optimal decision is not to drill optional wells at all (most likely because of the low oil prices in these simulated cases).

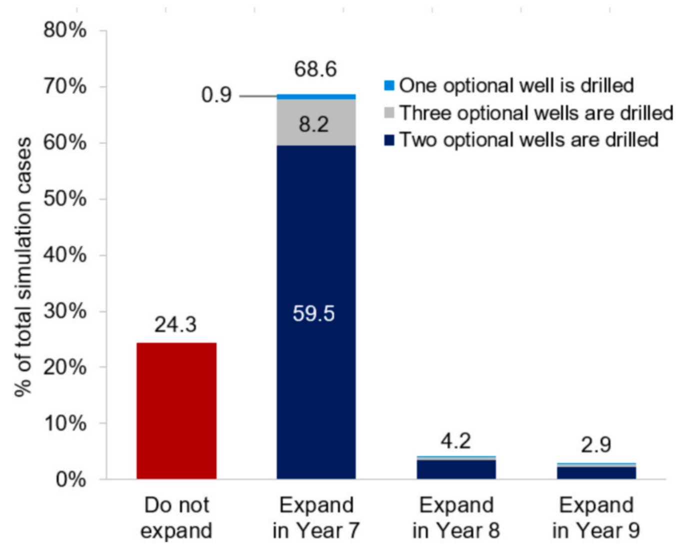


Fig. 21. Optimal expansion timing, accounting for the flexibility in optimizing the number of optional wells.

Table 7
Impact on the project value due to adding more flexibilities.

	Expected value (MM USD)	Change vs. reference case (MM USD)	Change vs. reference case (%)
Staged development, reference case (two optional wells without an opportunity to decrease/ increase the number)	885.14	–	–
Possibility to choose one best or drill both optional wells	885.29	0.15	+0.02
Possibility to add a third well and choose optimal number of wells (one/two/ three)	892.06	6.92	+0.78

[9] Because 60% of all simulated cases correspond to a high case, the decision maker can decide to drill three wells in 12% of the total simulated cases.

Thus, the proposed methodology can be directly used to account for the flexibility in choosing an optimal number of optional wells. In the considered case study, none of the additional flexibilities in terms of the choice of optional wells, added a significant value. However, in other case studies, such flexibilities may generate substantial additional value under a higher uncertainty regarding the expected production rate of the optional wells and/or a higher number of wells.

5.3.2. *Alternative approaches to model the production rate*

In the proposed methodology, we assume that the production profiles follow a decline curve with a constant nominal decline rate. Thus, the operator can build a production forecast until the field shutdown once the initial rate of each well has been sampled via the Monte Carlo simulation. As described in Section 4.1, an exponential decline equation is used to model the production rates of each well. This approximation suits the expected drainage from the considered reservoir case. The nominal decline rate is considered to be fixed at 22.5% based on the input from the field engineers. However, several alternative approaches can be used to mathematically represent the future production rate. A general discussion regarding the decline curve analysis methods was provided by Höök et al. (2009). Three variations of the decline curve presented in (Arps et al., 1945) exist: hyperbolic, harmonic, and exponential. The latter two curves represent a special case of the hyperbolic decline equation. Höök et al. (2009) indicated that “the disadvantage of the exponential decline curve is that it sometimes tends to underestimate production far out in the tail part of the production curve, as the decline often flattens out toward a more harmonic and hyperbolic behavior in that region”. Höök et al. (2009) show that a difference between the exponential and hyperbolic fitting emerges after approximately 15 years of production. Therefore, the difference between these approaches is expected to be marginal in the considered case of a marginal field, whose production lifetime is limited to 12 years on average. Thus, performing the sensitivity check of alternative deterministic approaches is not expected to lead to significantly different results.

Nevertheless, it is interesting to identify how accounting for the uncertainty in the production forecast affects the obtained results. A straightforward way to add uncertainty to the exponential decline model is to assume that the nominal decline rate in Eq. (5) might deviate from the expected mean. The decline rate might either be the same for all wells or modeled for each individual well. For this robustness check, we consider a normal distribution with the same mean of 0.225 as in the base case and standard deviation of 0.1 to sample the nominal decline rate for each year for each simulation case. This configuration can generalize the production rate decline for the complete field, leading to proportional depletion of all the wells.[10]

When accounting for the uncertainty in the production decline, the project's expected value under the standard development remains the same (851 MM USD) as the expected mean of the nominal decline remains unchanged. However, under the staged development, the operator can proactively exploit the flexibility to optimize the expansion policy when the additional uncertainty is accounted for. In such conditions, the decision maker can benefit more from learning while waiting for the expansion. In the considered case, the project value under the staged development increases from 885 MM USD to 901 MM USD when the nominal decline rate has a standard deviation of 0.1. The additional value created by the staged development will further increase with the amount of the uncertainty in the decline rate.

A classic approach to address the uncertainty in production rates in the real options research is to assume that the production rate follows a stochastic process such as GBM (Smith and McCardle, 1998), (Jafarizadeh and Bratvold, 2009). The GBM allows to account for the probable deviations from the expected decline rate while implementing the uncertainty in the production profile modeling. When using the GBM process, the current production depends on the previous year's rate and a probabilistic element representing the variability, defined as

$$q_{t_n} = q_{t-1_n} e^{(-\mu - 0.5\sigma)\Delta t + \sigma N(0,1)\sqrt{\Delta t}}, \tag{21}$$

where μ is the expected decline rate, and σ is the volatility.

Fig. 22 shows sample paths of the production profiles by considering a (1) fixed exponential decline, (2) exponential decline with uncertainty, and (3) GBM model. In the GBM model, we assume an expected decline rate (μ) of 22.5%, and the volatility (σ) is set as 0.25 to ensure that the confidence bands of the recoverable reserves are similar to those in the base case to perform a fair comparison with the reference case results.

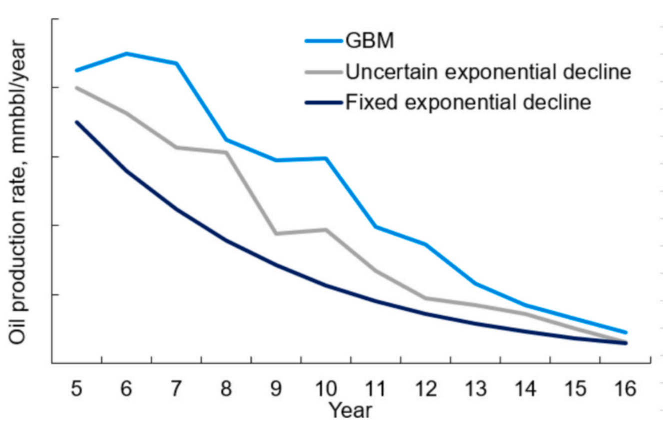


Fig. 22. Production profile simulation examples (expected nominal decline rate = 22.5%).

[10] This approach can be straightforwardly extended to account for different decline rates of the wells' production rate.

The project value under the standard development remains at 851 MM USD, while the value of the staged development increases to 910 MM USD owing to the slightly higher variability of the production rates. This further increases the real option value owing to the opportunity to respond to the outcome of more uncertain conditions.

We also performed a sensitivity analysis using the three models to illustrate how the added value created by the staged development evolves with changes in the nominal decline rate (the initial production rate of all the wells is changed in accordance to ensure a constant mean of the recoverable volume (approximately 53 mmbbl). Fig. 23 illustrates the added value by the staged development as a function of the expected decline rate when the three different approaches are used. As was already show in Fig. 16, the added value increases with the decreasing expected decline rate. Considering the uncertainty in the decline curve using the normal distribution with a standard deviation of 0.1 adds, on average, 1% of the project value, with an additional 1% achieved when using the GBM process to model the production rate. In relative terms, accounting for the uncertainty is more important under higher expected decline rate.

5.3.3. *Testing alternative oil price models*

In this section, we perform a robustness check of our oil price modeling approach. As mentioned in Section 3.3, two standard ways to model oil prices used in the literature are to assume that oil prices follow (1) a GBM or (2) a mean-reverting (MR) process with constant long-term equilibrium. The Schwartz-Smith two-factor process that we applied in our methodology has clear advantages in comparison to these two classic approaches in terms of the ability to mimic conditions of the real oil market. It is, however, interesting to see how the choice of the oil price model can affect our results. Some earlier contributions already compared effects of using different types of oil price processes on the value of real option. See, for example, Al-Harthy (2007) and Xu et al. (2012). Xu et al. (2012) show that many of the simulated price paths using a GBM model are at much higher than realistic oil price levels. The price range becomes wider with the increase of the time horizon, which leads to overvaluation of the long-term options.

In order to perform a robustness check, we use the same historical data as in Section 4.2 to estimate parameters for the simulation of a risk-neutral version of a GBM process, given by

$$P_{t_n} = P_{t-1_n} e^{(r-\delta-0.5\sigma)\Delta t + \sigma N(0,1)\sqrt{\Delta t}}, \tag{22}$$

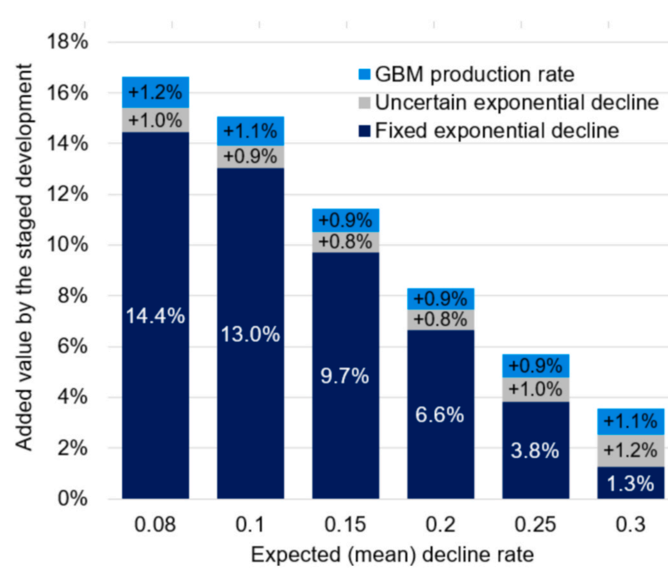


Fig. 23. Added value by the staged development depending on the field's expected production decline rate.

where r is the risk-free rate, δ is the convenience yield and σ is the volatility.

Similarly, we calibrate a risk-neutral version of a MR process, given by

$$P_{t_n} = e^{\gamma}, \tag{23}$$

where

$$\gamma = ln(P_{t-1_n})e^{-\eta\Delta t} + \left(ln\left(\overline{P}\right) - \frac{\mu - r}{\eta}\right)(1 - e^{-\eta\Delta t}) - \left(1 - e^{-2\eta\Delta t}\right)\frac{\sigma^2}{4\eta} + \sigma\sqrt{\frac{(1 - e^{-2\eta\Delta t})}{2\eta}}N(0,1), \tag{24}$$

where η is the speed of mean reversion, μ is the risk-adjusted rate and $\overline{P}$ is the long-term mean of the price.

Parameters for both the GBM and MR processes resulting from the calibration are presented in Table 8. As before, the risk-free rate is set to 2.5%, the convenience yield is 2.0%, while the volatility equals 28.1%. The risk-neutral version of the MR process reverts to the long-term mean price of 74.7 $/bbl with the speed of 0.358. The risk-adjusted rate is assumed to be equal to 8.0%.

Assuming that future oil prices follow a GBM process significantly increases the project's expected values. This result is expected and supports the findings of Xu et al. (2012) as using a GBM process leads to the fact that some price paths reach unrealistically high levels, especially in the long run. This phenomenon is illustrated in Fig. 24, that shows that the P90 values of the prices simulates with a GBM are much higher than the respective values of the MR or the Schwartz-Smith processes. Therefore, this also results in a higher mean of the simulated price paths.

Jafarizadeh and Bratvold (2012) argues that using a GBM for modeling commodity prices that, in fact, have a mean-reverting nature might lead to an overestimation of option values. Our results stated in Table 9 support this and show that the staged development adds 10% of the project value when the GBM price process is used, which is much higher than for the cases where we used other oil price process models. Another reason for the higher value of flexibility is the fact that in case of the GBM, also the number of simulated cases with very low oil prices increases. The decision maker benefits from being able to avoid drilling optional wells in these scenarios, and therewith, the value of the staged development increases. This is illustrated in Table 10 that shows that in 46.7% of the simulated cases using a GBM, the optimal decision is to keep the number of wells unchanged after Stage 1.

Also in case of the MR process assumption, the project's expected value of both the staged and the standard development is higher than in the reference case. In this case the increase is mostly due to a higher price mean and a much higher (more than 40$/bbl) P10 confidence band compared to both the GBM and Schwartz-Smith models. This reflects that the simulation based on the simple MR process does not properly capture low price scenarios. Due to that, the decision maker has more incentives to drill optional wells, expecting a price range that favors the expansion. This is confirmed by the results shown in Table 10. A large part of the downside risk is, therefore, eliminated, which decreases the percentage of added value due to the staged development.

Our conclusion is that the Schwartz-Smith two-factor process allows to more realistically capture the oil price risk and probable developments of the future prices compared to the GBM and MR models. Modeling the investment problem based on underlying two-factor model also presented more conservative, but more realistic results for our case. This is in line with the discussion in Jafarizadeh et al. (2012), who argue that the Schwartz-Smith model is suitable for analyzing long-term investments, leading to accurate assessments both of the project and option values and remains simple enough to calibrate and simulate.

Table 8
Calibrated parameter values of the risk-neutral versions of GBM and MR price processes.

r	2.5%	η	0.358
Δ	2.0%	$\overline{P}$	74.7
Σ	28.1%	μ	8.0%

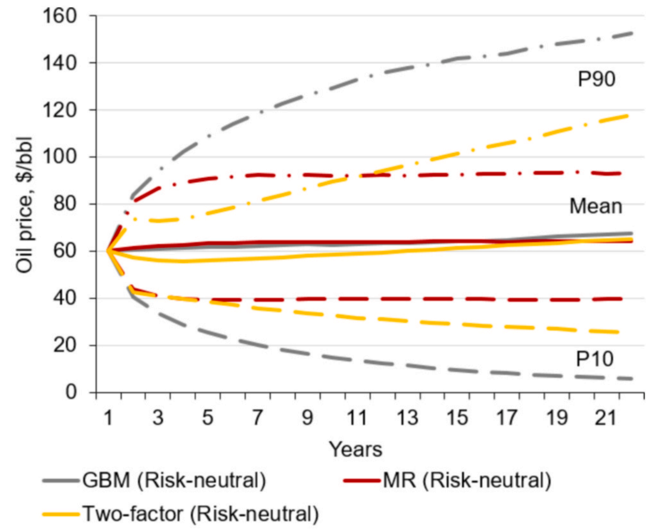


Fig. 24. Confidence bands and expected values based on the price simulation using a GBM, MR and Schwartz-Smith two-factor process, respectively.

Table 9
Impact on the project value due to the use of different oil price processes assumptions. Values are stated in terms of MM USD.

	Reference case (risk-neutral two-factor process)	GBM (risk-neutral process)	Mean-reverting (risk-neutral process)
Standard development	851	1155.2	1086.7
Staged development	885 (+4.0%)	1271.0 (+10.0%)	1120.1 (+3.2%)

Table 10
Results of expansion optimization based on different oil price processes, % of total cases simulated.

	Reference case (risk-neutral two-factor process)	GBM (risk-neutral process)	Mean-reverting (risk-neutral process)
Do not expand	24.7%	46.7%	15.8%
Expand in Year 7	68.5%	37.8%	72.9%
Expand in Year 8	4.0%	5.7%	2.7%
Expand in Year 9	2.8%	9.8%	8.6%

6. Conclusions

This paper presents a novel methodology to evaluate the investment in a small offshore oil field under technical and price uncertainty, while addressing the managerial flexibility to phase the drilling strategy in two stages. By investing sequentially, the operator can gather additional information regarding the reservoir uncertainty during the initial stage of the project. To optimize the production expansion decision, we implement the least-squares Monte Carlo (LSM) algorithm for the real options valuation, modeling the oil price as a two-factor oil price process and accounting for the production rate uncertainty. By applying the methodology on a synthetic project case, we demonstrate that accounting for such flexibility as staged development is crucial for the valuation of an investment in a marginal discovery.

The proposed methodology is sufficiently flexible to be applied to other case studies and can generate results that can be easily communicated to managers of oil E&P companies. The modeling process yields recommendations for the managers, which can facilitate the decision-making process. The operator's investment policy can be optimized based on the developed algorithm and threshold boundaries. We identify the key features that may affect the choice between the standard and staged field development strategy and calculate the associated project values. Furthermore, we perform a sensitivity analysis to clarify the influence of the drilling costs, production decline rate, and oil price process parameters on the optimal decision and values of the project. In addition, by performing a robustness check of the modeling approach, we demonstrate that the considered assumptions regarding the oil price process are realistic, and the methodology can potentially account for more flexibilities in terms of the drilling strategy and production decline rate assumptions.

Certain limitations encountered in this study, such as those pertaining to the simplistic reservoir modeling, may be avoided by introducing a realistic reservoir simulation that can track the geological dependencies between possible production well locations and assess the effect of Stage 1 production data on the reservoir model based on history matching and Bayesian updating. We have partly addressed this aspect in Fedorov et al. (2020) and consider the methodology to be further developed in cooperation with the industry partners to account for even more flexibilities, e.g., possibility of optimizing the number of wells drilled in Stage 2.

Moreover, future work may be aimed at analyzing various approaches to include managerial flexibility in the development strategy of a small offshore field. This flexibility may stem from using a floating production storage and offloading vessel (the cost of which is typically higher than that of a platform) that can be re-positioned given the production experience during the initial stage of a project or used for another discovery in case of a failure with the existing field. Among other potential measures to create additional value during the course of a project, one may include the possibility of IOR solution and additional capacity for a probable tie-in. Furthermore, the concept of imperfect information regarding the technical data generated in Stage 1 may be incorporated in the proposed methodology.

CRediT author statement

Semyon Fedorov: Conceptualization, Methodology, Software, Data curation, Writing – original draft preparation Visualization, Verena Hagspiel: Supervision, Reviewing and Editing, Data curation. Thomas Lerdahl: Supervision., Reviewing and Editing, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

Al-Harthy, M.H., 2007. Stochastic oil price models: comparison and impact. Eng. Econ. 52, 269–284.
Armstrong, M., Galli, A., Bailey, W., Couët, B., 2004. Incorporating technical uncertainty in real option valuation of oil projects. J. Petrol. Sci. Eng. 44, 67–82.
Arps, J.J., et al., 1945. Analysis of decline curves. Transactions of the AIME 160, 228–247.
Brandão, L.E., Dyer, J.S., Hahn, W.J., 2005. Using binomial decision trees to solve real-option valuation problems. Decis. Anal. 2, 69–88.
Brealey, R.A., Myers, S.C., Allen, F., Mohanty, P., 2012. Principles of Corporate Finance. Tata McGraw-Hill Education.
Chorn, L., Carr, P., et al., 1997. The value of purchasing information to reduce risk in capital investment projects. SPE Hydrocarbon Economics and Evaluation Symposium. Society of Petroleum Engineers.
Copeland, T., Antikarov, V., 2001. Real Options. Texere, New York.
Cox, J.C., Ingersoll Jr., J.E., Ross, S.A., 1985. An intertemporal general equilibrium model of asset prices. Econometrica: Journal of the Econometric Society 363–384.
Cunningham, P., Begg, S., 2008. Using the value of information to determine optimal well order in a sequential drilling program. AAPG Bull. 92, 1393–1402.
Dias, M., et al., 1997. The timing of investment in e&p: uncertainty, irreversibility, learning, and strategic consideration. SPE Hydrocarbon Economics and Evaluation Symposium. Society of Petroleum Engineers.
Dias, M.A.G., 2002. Investment in information in petroleum, real options and revelation. Proceedings of the 6th Annual International Conference on Real Options. Real Options Group at Cyprus (Cyprus).
Dias, M.A.G., 2004. Valuation of exploration and production assets: an overview of real options models. J. Petrol. Sci. Eng. 44, 93–114.
Fedorov, S., Hagspiel, V., Lerdahl, T., Idris, A., Ghahfarokhi, A.J., et al., 2020. Staged development of a marginal oil field under technical and market uncertainty: the olympus case. SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers.
Fetkovich, M.J., 1980. Decline curve analysis using type curves. J. Petrol. Technol. 32, 1065–1077.
Fleten, S.E., Gunnerud, V., Hem, Ø.D., Svendsen, A., 2011. Real option valuation of offshore petroleum field tie-ins. Journal of real options 1, 1–17.
Fonseca, R., Della Rossa, E., Emerick, A., Hanea, R., Jansen, J., 2018. Overview of the olympus field development optimization challenge. ECMOR XVI-16th European Conference on the Mathematics of Oil Recovery. European Association of Geoscientists & Engineers, pp. 1–10.
Gallant, L., Kieffel, H., Chatwin, R., et al., 1999. Using learning models to capture dynamic complexity in petroleum exploration. SPE Hydrocarbon Economics and Evaluation Symposium. Society of Petroleum Engineers.
Galli, A., Jung, T., Armstrong, M., Lhote, O., et al., 2001. Real option evaluation of a satellite field in the north sea. SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers.
Goodwin, D., 2013. Schwartz-smith Two-Factor Model in the Copper Market: before and after the New Market Dynamics.
Guedes, J., Santos, P., 2016. Valuing an offshore oil exploration and production project through real options analysis. Energy Econ. 60, 377–386.
Hanea, R., Casanova, P., Hustoft, L., Bratvold, R., Nair, R., Hewson, C.W., Leeuwenburgh, O., Fonseca, R.M., et al., 2019. Drill and learn: a decision-making work flow to quantify value of learning. SPE Reservoir Eval. Eng. 22, 1–131.
Hong, A., Bratvold, R.B., Lake, L.W., et al., 2018. Fast analysis of optimal ior switch time using a two-factor production model and least-squares Monte Carlo algorithm. SPE Norway One Day Seminar. Society of Petroleum Engineers.
Höök, M., Söderbergh, B., Aleklett, K., 2009. Future Danish oil and gas export. Energy 34, 1826–1834.
Jafarizadeh, B., Bratvold, R., et al., 2012. Two-factor oil-price model and real option valuation: an example of oilfield abandonment. SPE Econ. Manag. 4, 158–170.
Jafarizadeh, B., Bratvold, R.B., 2015. Oil and gas exploration valuation and the value of waiting. Eng. Econ. 60, 245–262.
Jafarizadeh, B., Bratvold, R.B., 2019. Exploration economics: taking opportunities and the risk of double-counting risk. Mineral Economics 1–13.
Jafarizadeh, B., Bratvold, R.B., et al., 2009. Taking real options into the real world: asset valuation through option simulation. SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers.
Kullawan, K., Bratvold, R., Bickel, J., 2018. Sequential geosteering decisions for optimization of real-time well placement. J. Petrol. Sci. Eng. 165, 90–104.
Laine, J., et al., 1997. Option valuation of field development projects. SPE Hydrocarbon Economics and Evaluation Symposium. Society of Petroleum Engineers.
Longstaff, F.A., Schwartz, E.S., 2001. Valuing american options by simulation: a simple least-squares approach. Rev. Financ. Stud. 14, 113–147.
Lund, M.W., 1999. Real options in offshore oil field development projects. 3rd Annual Real Options Conference (Leiden).
Nadarajah, S., Margot, F., Secomandi, N., 2017. Comparison of least squares Monte Carlo methods with applications to energy real options. Eur. J. Oper. Res. 256, 196–204.
Norwegian Petroleum Directorate, 2019. Resource report. URL: https://www.npd.no/en/facts/publications/reports2/resource-report/resource-report-2019/.
Parente, V., Ferreira, D., dos Santos, E.M., Luczynski, E., 2006. Offshore decommissioning issues: deductibility and transferability. Energy Pol. 34, 1992, 2001.
Rasmussen, V., 2015. Field Development of a Marginal Oil Field on the Norwegian Continental Shelf. Master's thesis. University of Stavanger, Norway.
Rodrigues, A., Rocha Armada, M.J., 2006. The valuation of real options with the least squares Monte Carlo simulation method. Available at: SSRN 887953.

Santos, S.M., Gaspar, A.T., Schiozer, D.J., 2017a. Value of information in reservoir development projects: technical indicators to prioritize uncertainties and information sources. J. Petrol. Sci. Eng. 157, 1179–1191.

Santos, S.M., Gaspar, A.T.F., Schiozer, D.J., 2017b. Risk management in petroleum development projects: technical and economic indicators to define a robust production strategy. J. Petrol. Sci. Eng. 151, 116–127.

Schwartz, E., Smith, J.E., 2000. Short-term variations and long-term dynamics in commodity prices. Manag. Sci. 46, 893–911.

Smith, J.E., 2005. Alternative approaches for solving real-options problems: (comment on brandão et al. 2005). Decis. Anal. 2, 89–102.

Smith, J.E., McCardle, K.F., 1998. Valuing oil properties: integrating option pricing and decision analysis approaches. Oper. Res. 46, 198–217.

Smith, J.E., McCardle, K.F., 1999. Options in the real world: lessons learned in evaluating oil and gas investments. Oper. Res. 47, 1–15.

Smith, J.E., Nau, R.F., 1995. Valuing risky projects: option pricing theory and decision analysis. Manag. Sci. 41, 795–816.

Suslick, S.B., Schiozer, D., Rodriguez, M.R., 2009. Uncertainty and risk analysis in petroleum exploration and production. Terra 6, 2009.

Willigers, B.J., Begg, S., Bratvold, R.B., 2017. Hot spot hunting: optimising the staged development of shale plays. J. Petrol. Sci. Eng. 149, 553–563.

Willigers, B.J., Bratvold, R.B., et al., 2009. Valuing oil and gas options by least-squares Monte Carlo simulation. SPE Proj. Facil. Constr. 4, 146–155.

Xu, L., Sepehrnoori, K., Dyer, J., Van Rensburg, W.C., et al., 2012. Application of real options to valuation and decision-making in the petroleum e&p industry. SPE Hydrocarbon Economics and Evaluation Symposium. Society of Petroleum Engineers.