Inter-source seismic interferometry by multidimensional deconvolution (MDD) for borehole sources

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Summary

Seismic interferometry (SI) is usually implemented by crosscorrelation (CC) to retrieve the impulse response between pairs of receiver positions. An alternative approach by multidimensional deconvolution (MDD) has been developed and shown in various studies the potential to suppress artifacts due to irregular source distribution and intrinsic loss. Following previous theories on SI by MDD, we extend it to retrieve the impulse response between pairs of source positions by invoking source and receiver reciprocity. We verify the theory using a simple two-layered model and show that the retrieved response by MDD is more accurate than that by CC, and furthermore, it is free of free-surface multiples. We discuss the necessary pre-processing required for this method. This inter-source SI approach creates a virtual acquisition geometry with both borehole sources and receivers without the need to deploy receivers in the borehole, which might be of interest to applications such as seismic while drilling (SWD).

Introduction

Seismic interferometry, also called "Green's function retrieval", refers to the general idea of retrieving the impulse response between two positions by processing the seismic responses measured between those two positions and the positions on the surrounding surface. When the new response is retrieved between two receiver positions, it is called inter-receiver SI. When the new response is retrieved between two source positions, it is called inter-source SI. While it is more common to use inter-receiver SI to create a virtual source at one of the physical receiver positions (Snieder, 2004, Schuster et al. 2004, Bakulin and Calvert 2006), it is sometimes also useful to turn a physical source into a virtual receiver. Curtis et al. (2009) applied the inter-source SI by CC to turn earthquakes into virtual seismometers.

SI is typically implemented by CC. Wapenaar and Fokkema (2006) derived the exact forms of SI by CC for both lossless acoustic and elastodynamic media. In order to simplify the exact formulas for a more straightforward implementation, they also showed the necessary approximations and assumptions involved. As a result,

although robust and simple, SI by CC in practice usually produces some spurious events in the retrieved response. A detailed analysis of those events is provided in Snieder *et al.* (2006). Alternatively, SI by MDD has the potential of correcting for a non-ideal source distribution and intrinsic losses by deblurring the correlation function by the interferomeric point-spread function (van der Neut and Bakulin 2009, van der Neut *et al.* 2011). Wapenaar *et al.* (2011) derived a form of inter-receiver SI by MDD for receivers in the subsurface and sources on the surface. We show that by invoking source and receiver reciprocity, one can obtain a similar form of inter-source SI by MDD to create a virtual acquisition geometry corresponding to borehole sources and receivers. We demonstrate the derivation by a numerical example and discuss the possible application limitations.

Theory

We start from Rayleigh's reciprocity theorem in equation①.

$$
\begin{aligned}
&\int_D (P_A Q_B - V_{i,A} F_{i,B} - P_B Q_A + V_{i,B} F_{i,A}) d^3 \mathbf{x} \\
&= \oint_{\partial D} (P_A V_{i,B} - P_B V_{i,A}) n_i d^2 \mathbf{x}
\end{aligned}
\qquad ①
$$

Capital letters indicate frequency domain. The subscripts A and B denote the state. Because multiplication in the frequency domain corresponds to convolution in the time domain, this is a reciprocity theorem of the convolution type (Wapenaar and Fokkema, 2006). Here P is the acoustic pressure, V_i the particle velocity in the i-th direction, F_i the external volume force density in the i-th direction, Q the volume injection rate density. An illustration of the two states A and B is given in Figure 1. The spatial domain *D* is enclosed by the surface ∂D, which includes the receiver surface ∂D_0 and extends downward into the subsurface. State A resembles the physical experiment and state B coincides with the physical medium inside *D* but is homogeneous outside. In Figure 1, $\mathbf{x}_S$ is the source position, and $\mathbf{x}_R$ and $\mathbf{x}$ are the receiver positions. The mathematical expressions for the terms in both states are shown in equation②.
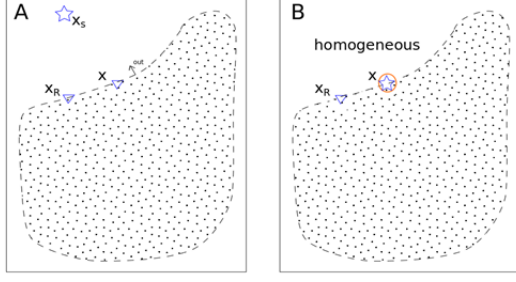
Figure 1. The configuration of state A and B for inter-receiver SI by MDD. The dotted line describes the surface boundary ∂D enclosing volume D and ∂D_0 is the virtual receiver surface. The orange circle denotes the virtual source. The star denotes source and the triangle denotes receiver.

$$F_{i,A} = 0;$$
$$Q_A = S(\omega)\delta(\mathbf{x} - \mathbf{x}_S);$$
$$P_A = P(\mathbf{x} \mid \mathbf{x}_S);$$
$$V_{i,A} = -\frac{1}{j\omega\rho(\mathbf{x})}\partial_i P(\mathbf{x} \mid \mathbf{x}_S);$$

$$F_{i,B} = 0;$$
$$Q_B = \delta(\mathbf{x} - \mathbf{x}_R);$$
$$P_B = \overline{G}(\mathbf{x} \mid \mathbf{x}_R);$$
$$V_{i,B} = -\frac{1}{j\omega\rho(\mathbf{x})}\partial_i \overline{G}(\mathbf{x} \mid \mathbf{x}_R);$$

②

We follow the derivation in Wapenaar et al. (2011) by separating the wavefield at ∂D into outgoing and incoming components, and approximating the normal derivative of the wavefield in the high frequency regime by multiplying each component with a pseudo-differential operator $\mp jH$, where the minus-sign applies to outgoing waves, plus-sign to incoming waves. Because it is homogeneous outside D, $\overline{G}(\mathbf{x} \mid \mathbf{x}_R)$ is outgoing at x and further we can use $\overline{G}(\mathbf{x} \mid \mathbf{x}_R) = \overline{G}(\mathbf{x}_R \mid \mathbf{x})$. Substituting the expressions in ② into equation①, using the pseudo-differential operator to approximate the derivatives, and applying Sommerfield radiation condition gives

$$P(\mathbf{x}_R \mid \mathbf{x}_S)$$
$$=\oint_{\partial D_0} P^{in}(\mathbf{x} \mid \mathbf{x}_S)\left(\frac{-2}{j\omega\rho(\mathbf{x})}n_i\partial_i\overline{G}(\mathbf{x}_R \mid \mathbf{x})\right)d^2\mathbf{x}$$

③

Considering only the outgoing part of the field at x_R and applying decomposition at both sides of equation ③ gives

$$P^{out}(\mathbf{x}_R \mid \mathbf{x}_S) =\oint_{\partial D_0} P^{in}(\mathbf{x} \mid \mathbf{x}_S)\overline{G}_d^{out}(\mathbf{x}_R \mid \mathbf{x})d^2\mathbf{x}$$

④

where $\overline{G}_d^{out}(\mathbf{x}_R \mid \mathbf{x})$ is called the dipole Green's function. It corresponds to a dipole source at x and the field of this source observed at x_R is outgoing.

$$\overline{G}_d^{out}(\mathbf{x}_R \mid \mathbf{x}) = \frac{-2}{j\omega\rho(\mathbf{x})}n_i\partial_i\overline{G}^{out}(\mathbf{x}_R \mid \mathbf{x}) \qquad ⑤$$

In equation④, $P^{out}(x_R \mid x_S)$ denotes the waves propagating outward at x_R from x_S and $P^{in}(x \mid x_S)$ is the waves propagating inward at x from x_S. By invoking source and receiver reciprocity, it follows

$$P^{out}(\mathbf{x}_R \mid \mathbf{x}_S) = P^{in}(\mathbf{x}_S \mid \mathbf{x}_R);$$
$$P^{in}(\mathbf{x} \mid \mathbf{x}_S) = P^{out}(\mathbf{x}_S \mid \mathbf{x});$$

⑥

where $P^{in}(x_S \mid x_R)$ denotes the waves propagating inward from x_R to x_S and $P^{out}(x_S \mid x)$ denotes the waves propagating outward from x to x_S. We see that both $P^{out}(x_R \mid x_S)$ and $P^{in}(x_S \mid x_R)$ are the waves that are last reflected from inside ∂D and $P^{in}(x \mid x_S)$ and $P^{out}(x_S \mid x)$ are the direct waves and the waves last reflected from outside ∂D by, for example, the free surface. Substituting these two expressions into equation④ and rewriting it in a matrix form following the same convention in Wapenaar et al. (2008), we have

$$P^{in}(\mathbf{x}_S \mid \mathbf{x}_R) =P^{out}(\mathbf{x}_S \mid \mathbf{x})\overline{G}_d^{out}(\mathbf{x}_R \mid \mathbf{x}) \qquad ⑦$$

To make the formula more intuitive, x_R is renamed as $x_{S'}$ and x_S is renamed as $x_{R'}$, then equation⑦ becomes

$$P^{in}(\mathbf{x}_{R'} \mid \mathbf{x}_{S'}) =P^{out}(\mathbf{x}_{R'} \mid \mathbf{x})\overline{G}_d^{out}(\mathbf{x}_{S'} \mid \mathbf{x}) \qquad ⑧$$

where $P^{in}(x_{R'} \mid x_{S'})$ denotes the waves propagating inward from $x_{S'}$ to $x_{R'}$ and $P^{out}(x_{R'} \mid x)$ denotes the waves propagating outward from x to $x_{R'}$. The corresponding notation in both states is shown in Figure 2, where one of the sources is turned into a virtual receiver.

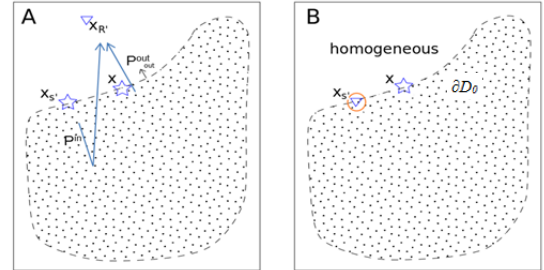Figure 2. Configurations of state A and B for inter-source SI by MDD. The dotted line describes the boundary ∂D enclosing the volume, D and ∂D_0 is the virtual receiver surface. The orange circle denotes the virtual receiver. The star denotes source and the triangle denotes receiver.

The unknown Green's function between two source positions can be estimated with a standard Damped Least Square approach (Menke, 1989)

$$\overline{G}_d^{out} = (P^{out\dagger}P^{out} + \varepsilon^2 I)^{-1} P^{out\dagger}P^{in} \qquad ⑨$$

where † denotes conjugate transpose and ε^2 is the damping factor to control the model smoothness. The inversion is carried out for each frequency, and then the result is inverse Fourier transformed back to time.

According to the description of $P^{in}(x_{R'}|x_{S'})$ and $P^{out}(x_{R'}|x)$, the first term includes the multiples that propagate inward from $x_{S'}$ and the second term includes the multiples that propagate outward from x. In the absence of multiples, equation ⑧ can be written as

$$P_s(\mathbf{x}_{R'} \mid \mathbf{x}_{S'}) = P_d(\mathbf{x}_{R'} \mid \mathbf{x}) \overline{G}_d^{out}(\mathbf{x}_{S'} \mid \mathbf{x}) \qquad ⑩$$

where P_s denotes the primary reflected waves and P_d the direct waves.

Numerical example

To illustrate the theory, a simple 2D two-layered model, shown in Figure 3a, is used. We used 51 shots (only 17 are shown) distributed with an average spacing $\Delta x=30$ m along a modeled drilling trajectory. 101 receivers (only 25 are shown) are evenly placed with a spacing $\Delta x=25$ m at the depth $z=5$ m. The source is modeled with a Ricker wavelet of a central frequency of 15 Hz. The modeled acoustic pressure field and the vertical velocity field are computed using an acoustic Finite Difference scheme (courtesy to Espen Birger Raknes). Figure 3b shows an example of a shot gather with a sampling interval of 2 ms. The total recording time is 4 s. Because of the simplicity of the model used, the direct wave (noted by the orange dashed line) and the primary reflected wave (noted by the green dashed line) can be easily separated from the multiples (noted by the purple dashed line) by time-windowing in this example.

Next we apply the inter-source SI using MDD according to equation ⑩ by first Fourier transforming the separated direct waves and primary reflected waves into the frequency domain, then sorting the data into the matrix form, $P_s(x_{R'}|x_{S'})$ and $P_d(x_{R'}|x)$, where each column accommodates a fixed source position and variable receiver positions for a given frequency. The dipole Green's function is estimated by using equation⑨. Inverse Fourier transforming back to time gives the virtual responses that would be recorded at the source positions. Figure 4d shows an example of the virtual shot gathers.

In order to demonstrate the effect of not separating the outgoing and incoming multiples for use in the term $P^{in}(x_{R'}|x_{S'})$ and $P^{out}(x_{R'}|x)$, we then used the direct waves for the first term, the primaries and all multiples for the second term in equation ⑨. An example of the virtual shot gathers retrieved in this case is shown Figure 4c.

To compare, the reference response and the virtual response retrieved by the common CC approach are also computed. The reference response, shown in Figure 4a, is the directly modeled vertical velocity field (direct arrivals removed) that are recorded at the source positions (the virtual receivers) due to the source at $x=1370$ m. For the CC approach, the virtual response $V_z(x_B|x_A)$ is computed by crosscorrelating the vertical velocity field $V_z(x_A|x)$ and $V^*_z(x_B|x)$ for all sources x and then summing the result. Repeat this for all the source and virtual receiver pair. By taking the causal part of the result and applying an overall scaling factor to all traces, we obtain the response shown in Figure 4b.

Figure 5 shows a trace by trace comparison for every fifth of the retrieved responses. One should note that the retrieved response by CC has the effect from a free surface while the result by MDD is the wavefield that would be observed if the medium outside the volume *D* is homogeneous. Overall, we observe that the arrival time, waveform and amplitude match between the reference response and the retrieved response is the best for MDD without multiples, which is tailored exactly according to the theory.
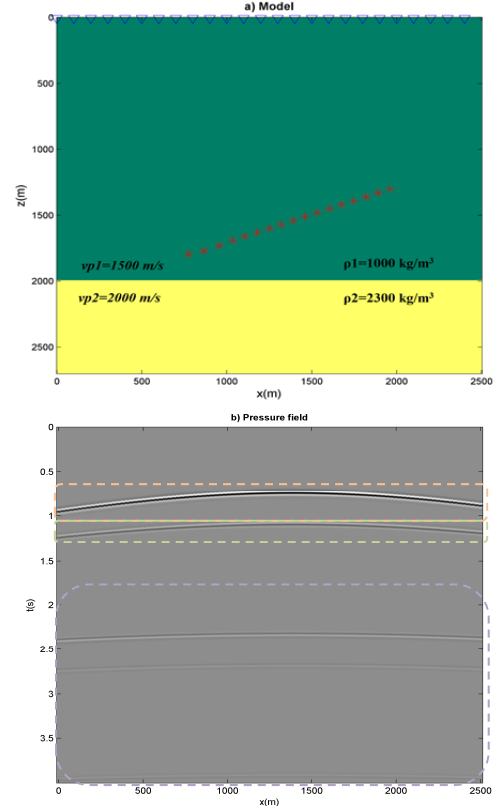

Figure 3. a) The P wave velocity model. The red stars denote the sources and the blue triangles denote the receivers. Every third sources and every fourth receivers are drawn. b) The shot gather of source at x=1370 m.

Conclusions

By applying source and receiver reciprocity, we extend the method by Wapenaar et al. (2011) to turn sources in the subsurface into virtual receivers by using a multidimensional deconvolution (MDD) approach. This creates a virtual acquisition geometry with both sources and receivers in the subsurface. We tested the formula by a numerical example and compare the result against the crosscorrelation (CC) approach. The retrieved response by MDD matches well in terms of arrival time, waveform and amplitude with the directly modeled reference response. We find that by only using the direct wave and the primary reflected wave when they can be separated from the multiples result in a more accurate result. If all events should be used in this method, a separation of incoming and outgoing multiples at the sources needs to be considered.
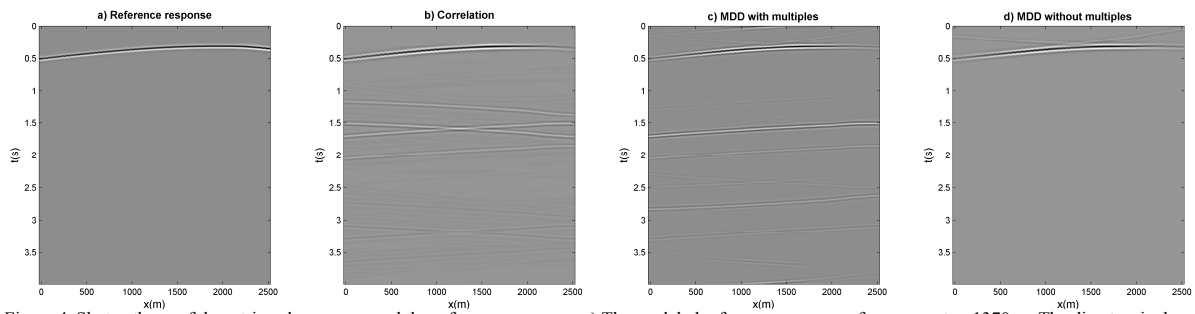
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Figure 4. Shot gathers of the retrieved responses and the reference response. a) The modeled reference response of a source at x=1370 m. The direct arrivals are removed. b) The response obtained by CC. c) The response obtained by MDD when the multiples are not separated in the input. d) The response obtained by MDD when only the direct wave and the primary reflected wave are used as input.
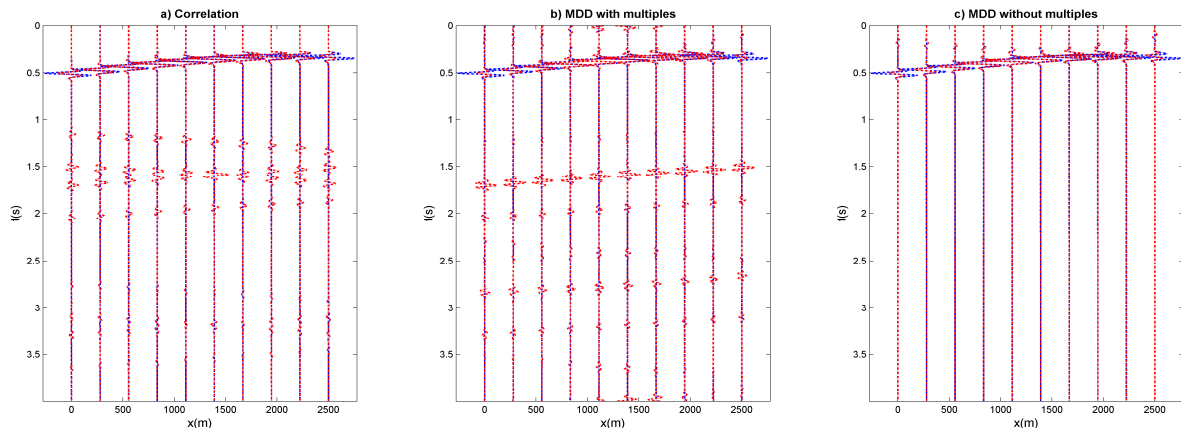

Figure 5. Comparison of the retrieved responses with the reference response. a) Result of interferometry by CC (red traces) compared with the directly modeled referense response (blue traces). Every third traces are plotted. b) Result of inter-source SI by MDD when multiplesare not separated. c) Result of inter-source SI by MDD without multiples.

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