Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Engineering
Department of Geoscience and Petroleum

Ressi Bonti

Decision-Support Methods for Early Offshore Field Planning Based on Optimization and Proxy Modelling

Master's thesis in Petroleum Engineering
Supervisor: Professor Milan Stanko
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Summary

Field development planning is an essential phase in the life of an offshore field as many vital decisions, such as type of platform, production and drilling schedule, and capacity of processing facilities, are made during this stage. To address these challenges, SUBPRO, a research center for subsea production and processing, and BRU21, a research program on digitalization and automation solutions for the oil and gas industry, performed research to develop automated methods to provide decision-support in early field development.

This thesis picked up and continued the specialization project work of Bonti (2020). In the work of Bonti (2020), non-linear programming (NLP) optimization model was developed to determine drilling and production schedules and capacity of processing facilities that maximize project net present value (NPV). All wells were considered identical and part of a subsea network. This thesis studied the effect of uncertainties on the optimization results and developing and testing methods to determine the best field design considering such uncertainties. The effect of uncertain parameters such as initial oil in place, cost factor, well performance, and oil price was studied using simulation-based optimization and the Latin Hypercube Sampling (LHS) method.

Several field design methods using the results of the LHS simulations were tested, i.e., drilling schedule and processing capacities were selected using several criteria. The performance of the resulting design was then tested by repeating the LHS study but optimizing the production profile only and compared against the original results of the LHS. The analysis using the LHS method also evaluated the effect of 1) relaxing the upper bound in the number of producer wells allowed and 2) considering flexibility in the drilling schedule by having wells that can be "optional" to be drilled or not. The study showed that the base uncertainty model utilized an adequate number of producer wells (9). On the other hand, the inclusion of wells that are optional to be drilled or not significantly decreased the percentage difference of the NPV figures with respect to the base simulation results.

Stochastic programming (SP) was also employed to determine the optimal field design considering uncertainties. However, due to its computational complexity, it was not possible to include all uncertain parameters evaluated in the LHS. The optimal field design using the SP and the best methods based on the LHS results are very similar and have similar economic performances. Due to the complexity and the high running time required by the SP model, it is therefore recommended to use a design method based on LHS.

Lastly, the non-linear model was improved by including the distinct well performance in the production performance model and pipeline length in the cost model. The results and the computational performance of the revised model were compared against the model of Alkindira (2020), who employed piecewise linearization (PWL) using SOS2 variables instead of a non-linear formulation. The comparison showed that both models produced almost identical drilling schedule, production profile, and NPV figure. However, the non-linear model was superior to the linear model in running time, with 1.8 seconds compared to 21544.5 seconds. Therefore, the non-linear model seems to be a superior modeling and optimization approach to the PWL formulation.

Preface

The research presented in this thesis was conducted in the Department of Geoscience and Petroleum of the Norwegian University of Science and Technology (NTNU) as partial fulfillment of the requirements for the MSc degree in Petroleum Engineering. This thesis is a continuation of previous works done for the specialization project (Bonti (2020)).

I owe my greatest gratitude to my supervisor, Professor Milan Edward Wolf Stanko, whose knowledge, dedication, guidance, and encouragement have been invaluable throughout this study. Thank you for making it possible for me.

My appreciation also goes to Ph.D. candidate Leonardo Sales, Guowen Lei, and the rest of the research groups for the help and discussions over the last year.

Sincere acknowledgment to my parents and family for their countless prayers and supports. I would also like to thank Robin and Aditya, whose presence made my life in Trondheim easier. I finally want to thank you, Kharisma, who has been so tolerant and wonderful through all of this.

Trondheim, June 2021

Ressi Bonti Muhammad

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Chapter 1

Introduction

1.1 Background

Field development planning is an essential phase in the life of an offshore field as many vital decisions, such as type of platform, production and drilling schedule, and capacity of processing facilities, are made during this stage. Those decisions could have huge impacts on the economic aspects of the field. For instance, increasing the production rate could cause two contradicting outcomes, higher revenue or higher capital expense due to different production facilities. Thus, an organized effort of deciding them has to be made for the purpose of maximizing the value of the field.

It is often not possible to explore all possible scenarios to determine the most optimal concept due to time limitations and extended analysis time. Moreover, difficulties in integrating multiple disciplines and transferring important data between the discipline groups are also obstacles that slow down the process. To address these challenges, SUBPRO, a research center for subsea production and processing, and BRU21, a research program on digitalization and automation solutions for the oil and gas industry, performed research to develop automated methods to provide decision-support in early field development. The main idea of the method is to screen and search for the best combination of decisions but in an automated manner using numerical optimization.

The methodology has been developed, studied, and implemented by several previous works to optimize the net present value (NPV) of the field development. Up to date, the developed methodologies are capable of determining the most optimum production profile, drilling schedule, and recovery mechanism that maximize the net present value (NPV) of the field. All previous works have something in common in which the non-linear equations (e.g., production deliverability equation) are approximated by performing a piecewise linearization (PWL). Unlike them, the specialization project work of Bonti (2020) has successfully implemented the automated field development to formulate the NPV optimization problem by using non-linear programming (NLP) techniques. However, the formulation made some simplifications, such as assuming that all the wells have similar performance and neglecting the gas and water production rate.

This thesis will pick up and continue the specialization project work by studying the uncertainties in the life of an offshore field, such as oil in place, cost factor, oil price, and well performance. The study aims to propose an optimal design of the field based on the uncertainties. Another work of this thesis is to improve the formulation to include the distinct well performance. The improved formulation will also consider a gas-oil ratio (GOR) and water cut (WC) curves, instead of fixed values, to calculate the gas and water production rates, respectively. The ultimate goal is to compare the results and performance of the non-linear formulation with the reference case from the work of Alkindira (2020) in which the PWL approximation is performed.

1.2 Objective

The primary objective of this project is to further develop the non-linear NPV optimization formulation developed by Bonti (2020) to make it more realistic, quantify the effect of uncertainty, and optimize its computational performance. The thesis is divided into the following four tasks:

1. Extend the non-linear formulation developed in the specialization project by including gas and water production rate in addition to the injection well factor and define it as the base formulation.
2. Compare the deterministic simulation performance between two different tools, Pyomo and GAMS. The most optimal tool will be utilized for further studies.
3. Study the effect of several uncertainties such as oil in place, well performance, cost factor, and oil price on the base model using the selected tool. The study will be performed using two methods, Latin Hypercube Sampling (LHS) and Stochastic Programming (SP). At the end of the study, an optimal design from each method will be proposed.
4. Further improve the base formulation by including the distinct well performance in the production performance model and pipeline length in the cost model. A comparison will be made with the work of Alkindira (2020) to quantify the difference between linear and non-linear programming approaches.

1.3 Case Study: The Alta-Gohta Field

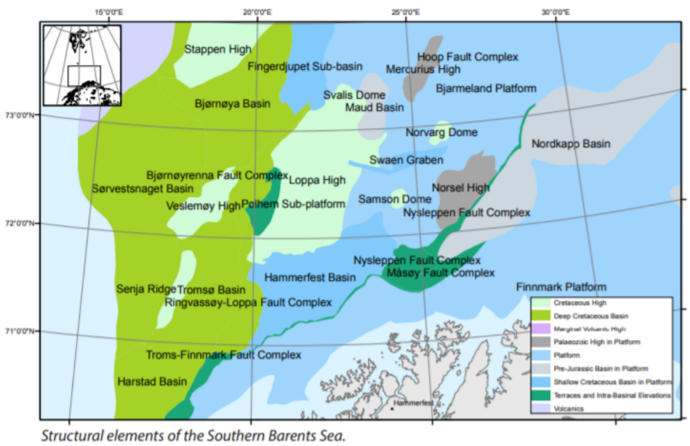


Figure 1.1: Location of Loppa High Area (Alkindira, 2020)

The field which is used as the case study for this thesis is located at the Loppa High Area, and it consists of two non-communicating reservoirs, reservoir 1 and reservoir 2. The illustration in Figure 1.1 shows the location of Loppa High Area. They are both saturated oil with the presence of gas cap. It has been estimated that Reservoir 1 has recoverable reserves of oil of 15.6 million Sm^3 and Reservoir 2 of 6.5 million Sm^3. While the production wells in reservoir 1 will be completed with a gas lift system, the wells in reservoir 2 will not use any artificial lift. The project horizon is 20 years long. In this work, taxes and royalties are neglected. The discount factor is 12 percent.

1.4 Programming Languages and Platforms Employed

There are several platforms available that can be used to formulate and solve the optimization problem. Two specific platforms that are utilized in this thesis are called GAMS and Pyomo. GAMS is a high-level modeling system for mathematical programming and optimization. It consists of a language compiler and a range of associated third-party solvers. GAMS is commercial software, and a license is needed to access its full feature. The features include a higher number of variables and constraints limit and access to the selection of solvers. All works in this thesis use GAMS version 34.2.0 and a community license.

On the other hand, Pyomo, which is also used in the specialization project of Bonti (2020), is a Python-based open-source optimization modeling language (Hart et al. (2017)). It supports the formulation of complex mathematical models for optimization applications which is usually associated with commercial algebraic modeling languages (AMLs) such as AMPL. As the definition implies, such capabilities are accessible within Python, an open-source high-level programming language with a large set of supporting libraries. Pyomo also supports a wide range of solvers to solve optimization problems. However, these solvers need to be installed manually or accessed through the NEOS web server, a free internet-based service for solving numerical optimization problems, as they are not available in the basic Pyomo package. Pyomo version 5.7 is used to solve some of the models in this thesis.

Other than the software or programming language itself, a careful selection of the solving engine is needed to solve the problems accordingly. For instance, the model built in this thesis is a mixed-integer non-linear programming problem, and hence, a strict non-linear programming (NLP) solver such as IPOPT could not be used. In this thesis, three MINLP solvers are selected based on their availability and difficulty in the implementation. The model that is formulated in Pyomo uses the KNITRO solver as it is available in NEOS and solves the problem very well. In comparison, the GAMS formulated model uses two solvers, which are GAMS/LINDO and DICOPT. The former is a bit more flexible in which it can be used to solve an uncertainty analysis model (i.e., solve the model multiple times in the same run). At the same time, the latter is only capable of simulating a deterministic model.

Chapter 2

Literature Review

This chapter has been reproduced from the specialization project work of Bonti (2020) and serves as a brief introduction to the following literature used in the thesis.

2.1 Production Potential

2.1.1 Definition of Production Potential

The production potential is defined as the maximum rate that a field can produce at a particular point in time (Stanko (2020a)). The maximum production rate is usually reached when the optimal condition of the production system is fulfilled (e.g., fully open choke, optimal injection rate from the injection well, and so on).

The production potential is determined by intersecting the inflow performance relationship (IPR) and tubing performance relationship (TPR) curve, as shown in Figure 2.1. Hence, the production potential changes over time due to the change in reservoir deliverability and production system. While reservoir deliverability is reduced in a similar trend to reservoir pressure (i.e., decreased with time), changes in the production system are usually made on purpose (e.g., drilling new wells, adjusting the choke opening, and well stimulation, etc.).

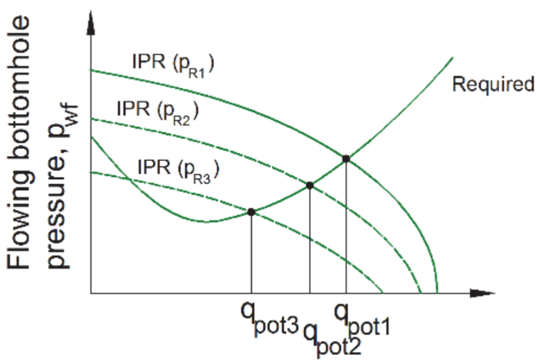


Figure 2.1: Illustration of production potential calculation using IPR-TPR (Stanko (2020a))

As discussed earlier, the reservoir deliverability is decreased with time in a similar trend to reservoir pressure. However, it is more dependent on the amount of fluid that has been produced over time than the time itself. Hence, it can be concluded that the production potential at a particular point in time is dependent on the cumulative fluid production up to that point in time (Stanko (2020a)).

The production potential curve is a curve that describes the relationship between the production potential and cumulative production. The curve can be very useful in planning the production schedule as it controls the production rate at any point in time, i.e., the field can produce at any rate which is no greater than the production potential. Figure 2.2 shows that the area below the curve is a feasible region for rates.

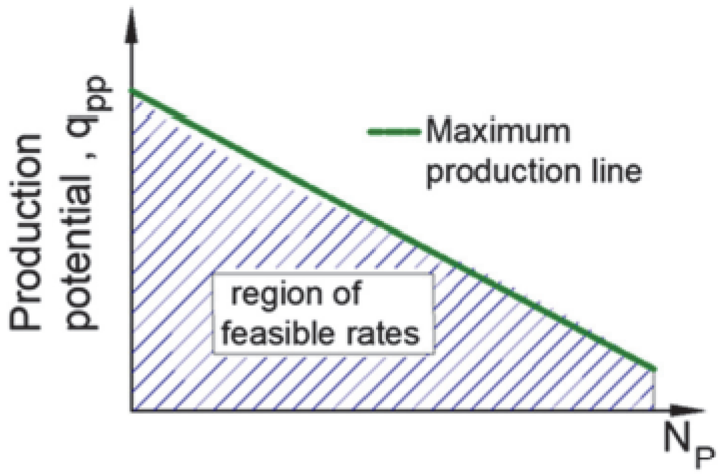


Figure 2.2: Illustration of production potential curve (Stanko (2020a))

There are two production modes in the oil/gas field: plateau mode and decline mode. In the plateau mode, the field is produced at a constant rate until the production potential can no longer support it. The production then continues in decline mode. On the other hand, the main objective of the decline mode is to produce as much as possible, which typically will follow the production potential since the beginning of the field. Producing in decline more is often not the most economical way to produce the field because production facilities must be sized for high rates and their cost often overweighs the gains of early production (Stanko (2020a)).

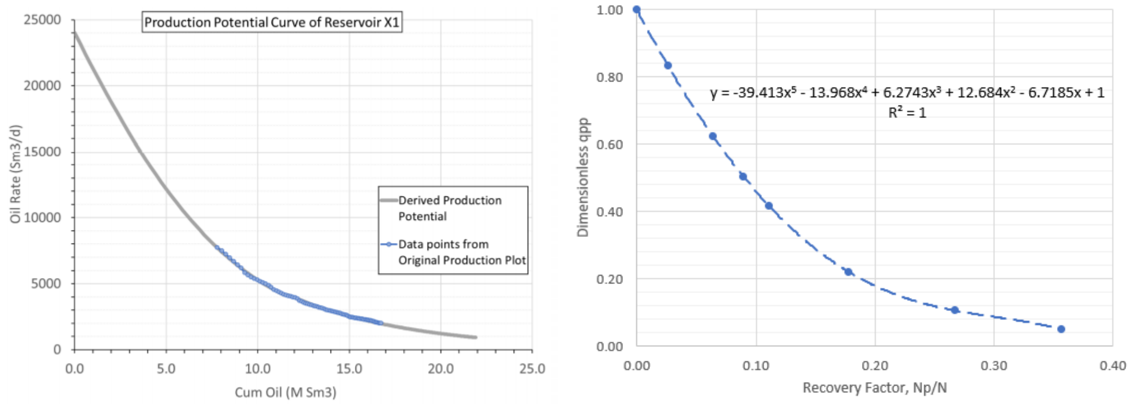
2.1.2 Construction of Dimensionless Production Potential Equation

Alkindira (2020) and Stanko (2020b) determined that, for many cases, production potential curves can be computed from curves of current dimensionless potential. The curves of current dimensionless potential are production potential curves where the cumulative production is normalized by initial oil or gas in place and production potential is normalized by the upper bound of the production potential.

The procedure in constructing the dimensionless production potential equation is straightforward if the data are available. The data required is a production potential curve (i.e., q_{pp} vs N_p) and the initial oil in place N.

Consider a production potential curve in Figure 2.3a. The first step is to extract some data

(q_{pp}, N_p) from the original curve. Secondly, divide the values of q_{pp} by the maximum production rate (i.e., when $N_p = 0$) and N_p by N. Finally, plot the new data and the resulting curve can be represented with a polynomial with a sufficient degree. In this case, a 5th-degree polynomial is used and Figure 2.3b shows the dimensionless equation with all the coefficients. It is important to note that the polynomial must intercept the y-axis at $q_{pp,dim} = 1$.



(a) Production Potential Curve (Alkindira (2020))

(b) Dimensionless Production Potential Curve and its Equation

Figure 2.3: An illustration of constructing a dimensionless production potential curve and equation

Therefore, the field production potential can be written as:

$$q_{pp,f} = q_{pp,max,f} \cdot \text{polynomial eq.} \tag{2.1}$$

where $q_{pp,f}$ is the field production potential and $q_{pp,max,f}$ is the maximum production potential of the field. In the following subsections, $q_{pp,max,f}$ will be discussed in more detail.

2.1.3 Effect of Multi-wells Production System on Production Potential

In an early phase of field planning, the wells are usually assumed to be standalone and identical. If this is the case, $q_{pp,max,f}$ is merely the maximum potential of a single well $q_{pp,max,well}$ multiplied by the number of wells available at a given time N_w. However, if a gathering system is applied, the previous calculation is no longer valid as the contribution of additional wells is no longer proportional. Thus, a modeling approach is suggested to capture this effect using a "pancake" factor (f_p) which represents the effect of adding wells to an existing gathering network. The new formulation can be stated as:

$$q_{pp,max,f} = q_{pp,max,well} \cdot N_w \cdot {f_p}^{(N_w-1)}, \quad \{f_p \in \mathbb{R}, 0 \leq f_p < 1\} \tag{2.2}$$

The value of f_p is always lower than 1, indicates that the effect of additional wells will be lower than the product of the maximum potential of a single well multiplied by the number of wells. For instance, let us consider a case where a field has one well with $q_{pp,max,f} = q_{pp,max,well}$ and $f_p = 0.95$. Instead of having a double production potential when an additional well is drilled, the field now has $q_{pp,max,f} = 1.9 \cdot q_{pp,max,well}$. A better illustration can be seen in Figure 2.4 where an approximated f_p is better fitted to the actual field rate than the one neglecting the effect of the gathering system.

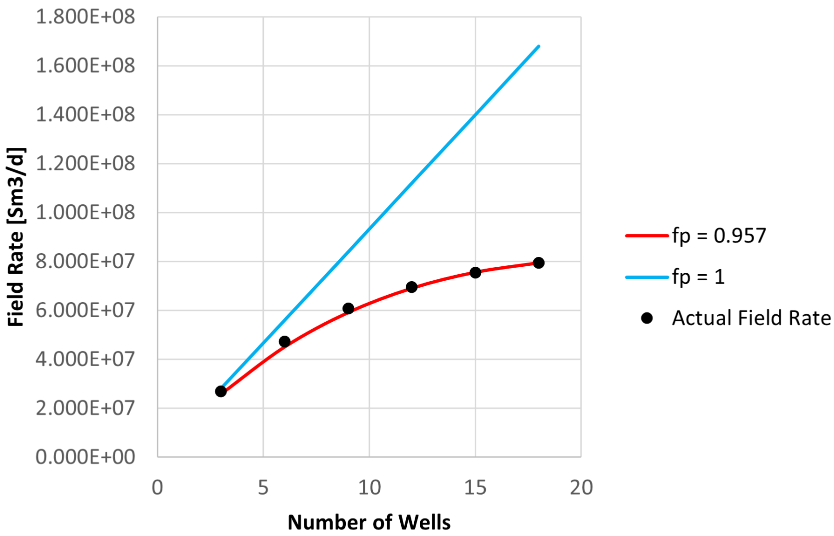


Figure 2.4: Illustration of pancake factor curve fitting

The assumption of identical wells is often appropriate for early phases of field planning where there is limited information about the field.

2.2 Previous Works

As mentioned in the previous chapter, the work in this thesis is based on previous work on automated field development methodology. The first one, Gonzalez (2020), maximized the NPV of the Wisting field case by finding the most optimum production and drilling schedule. Gonzalez performed some analysis to study the effect of some uncertainties such as cost values, reservoir size, well productivity, and layout of the production system, using probability trees. Based on the results of her work, Gonzalez concluded that the optimal production profile was mainly influenced by the recovery method used. At the same time, the drilling schedule was highly affected by the cost of drilling a well. Furthermore, it was controlled by the maximum number of wells allowed to be drilled each year until the maximum number of wells in the field is reached.

The next one was the work from Angga (2019), where he applied the same methodology to maximize two separate objective functions, the plateau duration and NPV, of a three reservoirs synthetic field case. Angga continued the works of Gonzalez by including the recovery mechanism of each reservoir as one of the decision variables and added constraints in the scheduling of injection wells. Angga also performed some uncertainty analysis using Latin Hypercube Sampling (LHS) and probability trees, which took the uncertainties of oil in-place, oil price, and development and operational costs. One of the conclusions that Angga made was that the inclusion of both drilling schedule and recovery method as decision variables alongside the production profile prolonged the plateau duration. Furthermore, it also significantly increased the NPV of the model compared to the one that only used production profile as the decision variables. He also showed that quantifying uncertainty using probability trees and LHS gives similar results.

Finally, Alkindira (2020) further developed the method by considering that wells had distinct performance and a given well combination had unique production deliverability. By doing so, Alkindira increased the complexity of the problems while also increasing the accuracy of the solution.

Alkindira formulated a model that maximized the NPV of a two non-communicating reservoirs case by optimizing the production and drilling schedule. To understand the effect of distinct performance of the well, Alkindira compared the results of a reference case in which the drilling schedule was fixed with the results of the base case. At the end of the study, Alkindira stated that the drilling schedule was an important decision variable as it slightly increased the NPV and allowed the field to drill fewer producer wells than the reference case.

Other than the base formulation, Alkindira also performed NPV optimization for some extreme cases. It was done by considering the possible changes in water cut (WC) and gas-oil ratio (GOR) value of the reservoirs to some extreme values in which she maximized or minimized the production of gas and water. Based on the results, Alkindira concluded that a case where the water and gas productions were simultaneously minimized has a slightly higher NPV than the base case, while an extreme case in which a field was producing high water and gas yielded a lower NPV than the base case. Similar to the previous two works, an uncertainty study was also performed in her work, where the probability tree method was chosen to understand the effect of some uncertainty parameters. The uncertainties include oil in place, water and gas production profiles, cost figures, production potential, and oil price. However, the level of complexity in the formulation forced her to limit the running time of each case in the uncertainty analysis to accommodate the project time restriction.

As it is based on Gonzalez (2020) and Angga (2019), the model that Alkindira formulated in her thesis utilized an identical approach where she represented the non-linear equations (e.g., production deliverability equation, gas and water cumulative production) by using Piecewise linearization (PWL). Such a method made the problem categorized as a mixed-integer linear programming (MILP) problem instead of mixed non-linear programming (MINLP) problem. In general, the piecewise linear models are more complex than employing the original nonlinear function. On the other hand, the nonlinear approach sometimes can converge to local optimal, and additional heuristics are usually required to find the global optimal. Additionally, MINLP problems are often challenging to solve. A comparison is needed to understand the benefits and drawbacks of using such a method.

Provided in Table 2.1 is the comparison between the works of Gonzalez, Angga, and Alkindira.

Table 2.1: Comparison between the previous three works

Works of	Objective Function(s)	Variables	Case	Improvement
Gonzalez	NPV	Production profile, Drilling Schedule	Wisting field	
Angga	Plateau duration, NPV	Production profile, Drilling schedule, Recovery mechanism	Safari field - 3 reservoirs	Inclusion of recovery mechanism as decision variables
Alkindira	NPV	Production profile, Drilling schedule	Barent sea - 2 reservoirs	Considering distinct well performance

2.3 Mathematical Programming

2.3.1 Non-linear Programming

Definition. Non-linear Programming (NLP) problem is a mathematical problem in which the objective function is non-linear and/or the feasible region is defined by non-linear constraints (Bradley et al. (1977)). Non-linearities are sometimes important to represent a problem correctly so that convergence can be reached. A general form of a non-linear minimization program is stated as:

$$\begin{aligned} &\text{Minimize} && f(x) \\ &\text{Subject to:} && g_i(x) \leq a_i \quad (i = 1, 2, ..., n) \\ &&& h_i(x) = b_i \quad (i = 1, 2, ..., n) \\ &&& x \subseteq \mathbb{R} \end{aligned} \tag{2.3}$$

with x in the equations denote the vector of n decision variables that is $x_1, x_2, ..., x_n$ or can be written as $x = (x_1, x_2, ..., x_n)$.

Interior Point: Method to solve NLP problems. The interior-point (or barrier) method was first introduced by Kachiyan in the late 1970s. However, this method was impractical and could not compete with the simplex method that had been around since 1951. In 1984, a new method with the same complexity as the previous one and also had a good practical performance was developed by Karmarkar. Ten years later, a sub-class named the primal-dual method arose and turned out to be more effective than barrier sub-class, as reported by Schmidt (2015). The main difference between them is that a single iteration is performed in primal-dual as there is no distinction between inner and outer iterations. Furthermore, the primal and dual iterations do not have to be feasible. The differences made primal-dual a preferred method due to its effectiveness and accuracy.

Consider a non-linear problem below:

$$\begin{aligned} &\min && f(x) \\ &\text{s.t.} && c(x) = 0 \\ &&& x \geq 0 \end{aligned} \tag{2.4}$$

where the objective function f and constraints c are assumed to be smooth functions. In order to eliminate the inequality constraints, the barrier method is used in which Equation 2.4 is replaced with a series of barrier problems written as:

$$\min \quad \varphi_\mu(x) := f(x) - \mu \sum_{i=1}^{n} \ln x_i \quad \text{s.t.} \quad c(x) = 0 \tag{2.5}$$

with the barrier parameter ($\mu > 0$) needs to be driven to zero. Equation 2.5 is equivalent to the following primal-dual system that has been applied with a homotopy method:

$$\begin{aligned} \nabla f(x) - \nabla c(x)^T \lambda - z &= 0 \\ c(x) &= 0 \\ XZe - \mu e &= 0 \end{aligned} \tag{2.6}$$

with $\nabla c(x)$ is the Jacobian matrix of the constraint vector c and e is the vector of all ones with suitable dimesion. The capital letters (X and Z) denote diagonal matrices made up of the corresponding vectors, x and z. λ and z are the dual variables that correspond to Lagrange multipliers of the equality constraints and variable bounds, respectively.

The following Karush-Kuhn-Tucker (KKT) system of equations presented in Equation 2.6 is then solved to yield the search directions $(\Delta x^{(k)}, \Delta\lambda^{(k)}, \Delta z^{(k)})$:

$$\begin{bmatrix} H(x^{(k)}) & -\nabla c(x^{(k)})^T & -I \\ \nabla c(x^{(k)}) & 0 & 0 \\ Z^{(k)} & 0 & X^{(k)} \end{bmatrix} \begin{pmatrix} \Delta x^{(k)} \\ \Delta\lambda^{(k)} \\ \Delta z^{(k)} \end{pmatrix} = - \begin{pmatrix} \nabla_x \mathcal{L}(y^{(k)}) \\ c(x^{(k)}) \\ X^{(k)} Z^{(k)} e - \mu^{(k)} e \end{pmatrix} \tag{2.7}$$

with $\nabla_x \mathcal{L}$ and H denote the Lagrangian gradient and Lagrangian Hessian, respectively.

The search directions are then used to compute the primal and dual step lengths ($\alpha_{pri}^{(k)}$ and $\alpha_{dual}^{(k)}$) using the following fraction-to-boundary rule:

$$\begin{aligned} \alpha_{pri}^{(k)} &:= \max_{\alpha \in (0,1]} \{x^{(k)} + \alpha \Delta x^{(k)} \geq (\tau - 1) x^{(k)}\} \\ \alpha_{dual}^{(k)} &:= \max_{\alpha \in (0,1]} \{z^{(k)} + \alpha \Delta z^{(k)} \geq (\tau - 1) z^{(k)}\} \end{aligned} \tag{2.8}$$

where $\tau \in (0, 1)$ is the fraction-to-boundary parameter.

The new search directions are computed using the following equations:

$$\begin{aligned} x^{(k+1)} &= x^{(k)} + \alpha_{pri}^{(k)} \Delta x^{(k)} \\ \lambda^{(k+1)} &= \lambda^{(k)} + \alpha_{dual}^{(k)} \Delta\lambda^{(k)} \\ z^{(k+1)} &= z^{(k)} + \alpha_{dual}^{(k)} \Delta z^{(k)} \end{aligned} \tag{2.9}$$

The barrier parameter $\mu^{(k)}$ can also be updated to a different value while ensuring that the sequence is converging to zero. The iterations stop when the following condition is met:

$$e^{(k)} := \max\{\| \nabla_x \mathcal{L}(y^{(k)}) \|_\infty, \| c(x^{(k)}) \|_\infty, \| X^{(k)} Z^{(k)} e \|_\infty \leq \epsilon \tag{2.10}$$

2.3.2 Mixed-Integer Non-linear Programming

Definition. Mixed-Integer Non-linear Programming (MINLP) problem is a mathematical problem that addresses the non-linear problem with both continuous and integer decision variables (Sahinidis (2019)). It is crucial for problems that have at least one restricted integer variable. For instance, one can model complex chemical processes with non-linear equations while integer variables model discrete decisions. The general form is similar to Equation 2.3 with an addition of integer restriction for decision variables.

$$\begin{aligned} &\text{Minimize} && f(x, y) \\ &\text{Subject to:} && g_i(x, y) \leq a_i \quad (i = 1, 2, ..., n) \\ & && h_i(x, y) = b_i \quad (i = 1, 2, ..., n) \\ & && x \subseteq \mathbb{R},\ y \subseteq \mathbb{Z} \end{aligned} \tag{2.11}$$

Branch and Bound: Method to solve MINLP problem. The branch and bound method used for MINLP problems is based on the same idea used in Mixed-Integer Linear Programming (MILP) problems. First, the problem is relaxed to be an NLP problem and solved. If it yields an integer solution, the procedure stops instantly. Otherwise, the solution becomes the upper bound to the optimal solution. Then, a tree branch enumeration is performed in which each node of the tree represents different subset relaxation of the integer constraints. When an integer solution is found, it becomes the lower bound to the optimal solution.

In the 1990s and early 2000s, several branch and bound improved methods are introduced. One of them is the Quesada-Grossman algorithm which Quesada and Grossman developed in 1992. The main difference between them is that in the Quesada-Grossman algorithm, the nodes remain as LP subproblems that can be easily updated. If an integer solution is found, the NLP subproblem is then solved, as shown in Figure 2.5. Consequently, stronger upper bounds are generated to reduce the branch and bound enumeration (Quesada and Grossmann (1992)).

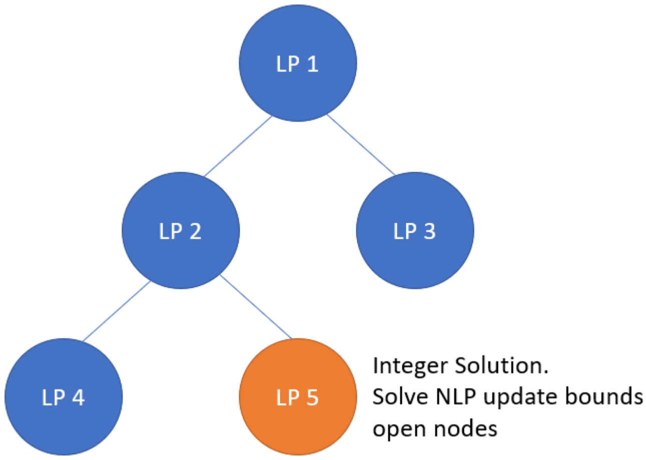


Figure 2.5: Illustration of Quesada-Grossman algorithm

2.4 Latin-Hypercube Sampling

Sampling is a process used in statistical analysis in which a predetermined value is taken from its probability distribution. There are many selections of sampling methods and Latin-Hypercube Sampling (LHS) being one of them. LHS method is an improvement from the conventional sampling method known as random sampling or Monte Carlo Simulation (MCS), in which it can provide a more accurate sampling in fewer iterations. The main principle of LHS is to divide the cumulative curve into equal intervals on the cumulative probability scale (0 to 1.0). A sample is then randomly taken from each interval of the probability distribution. By doing so, sampling is forced to represent values in each interval, and at the same time, is forced to recreate the probability distribution (UiO (2019)).

As an illustration, let us consider an example where five random samples are expected from a cumulative density function (CDF) curve. The distribution is a collection of numbers between 0 and n. First, the CDF curve must be divided into five equal intervals. The first sample must be taken from the interval between (0,$n/5$). The second sample would be taken from the interval between ($n/5$,$2n/5$) and so on. The approach being used during the LHS is called "sampling without replacement". Once a sample is taken from a particular interval, it is not sampled from again as its value is already represented in the sample set. The above illustration is shown in Figure 2.6.

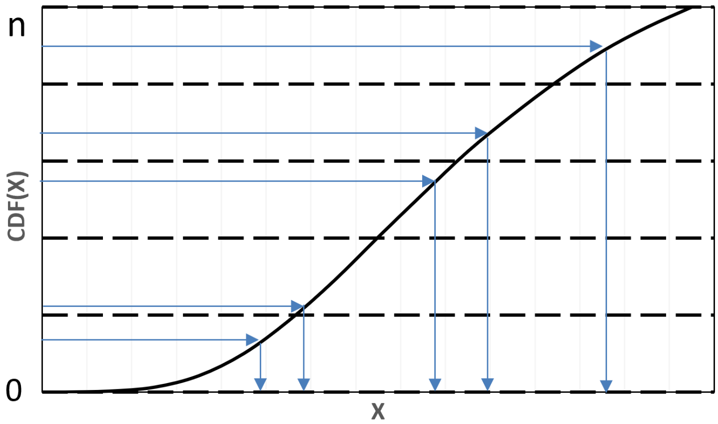


Figure 2.6: Illustration of LHS method

The LHS method can also be used to sample from multiple variables. However, it is important to ensure that the values sampled from one variable are independent of those samples from other variables. It is done by randomly selecting the interval to draw a sample from each variable. For instance, at the first iteration, variable x may be sampled from interval five while variable y is sampled from interval one. At the second iteration, variable x is sampled from interval three and variable y is sampled from interval two, and so on. As a result, the variables will not correlate with each other and maintain the desired randomness. The above illustration is shown in Figure 2.7.

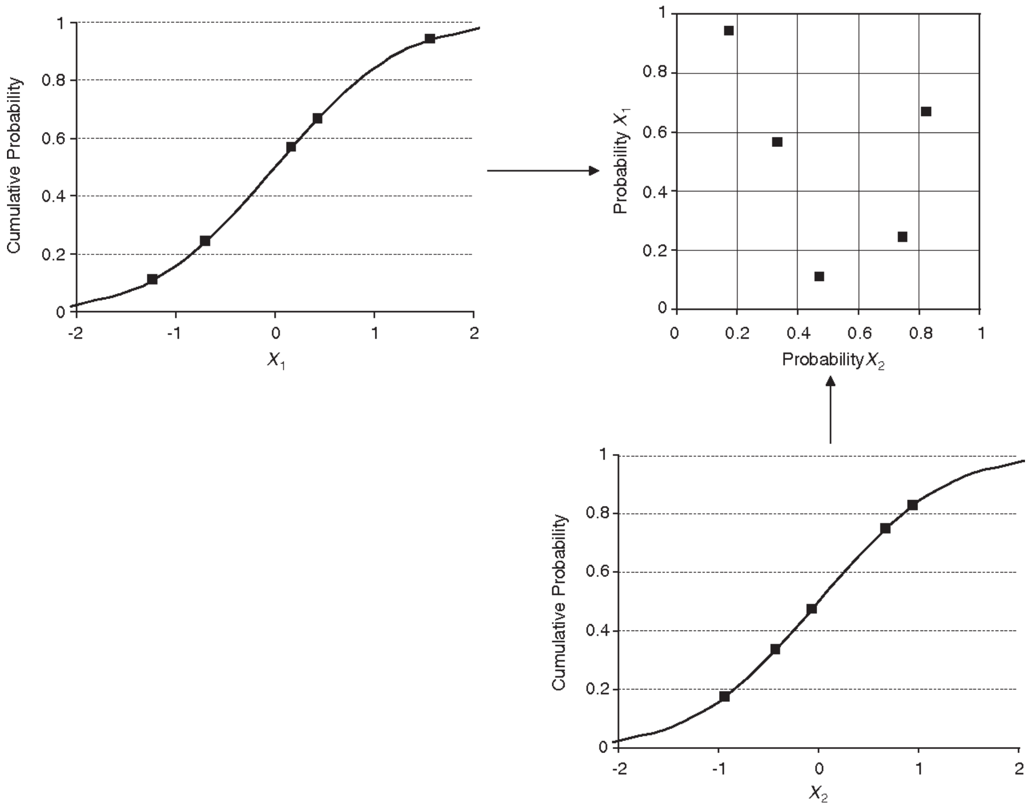


Figure 2.7: Illustration of LHS method implementation on multiple variables

2.5 Stochastic Programming

Stochastic programming refers to a collection of methods for maximizing or minimizing an objective function when randomness is present (Lauren (2015)). The main idea of stochastic programming is to require the decision-maker to decide one or more variables now and then to minimize the expected costs based on that decision. There are two forms of stochastic programming: two-stages and multi-stages. The former resolved all the uncertainties in the second stage and recourse actions can be taken given this new information. For the latter, uncertainties that are resolved per stage are based on the definition in the formulation. In this thesis, the only form that will be considered is the two-stages stochastic programming and to understand things clearly, an example about the newsvendor problem presented in Birge and Louveaux (2011) will be reproduced below.

The newsvendor has to decide early in the morning the number of newspapers to buy from the publisher on a particular day to be sold to the customer. Let x be the number of newspapers bought in the morning, c be the buying price per paper, y be the number of newspapers sold, and q be the selling price per paper. The vendor can only carry a maximum of u newspaper each day. At the end of the day, any unsold newspaper can be returned to the publisher for a returned price of r in which r is less than c. The number of newspapers being returned to the publisher is denoted by w. Based on the experience of the newsvendor, demand for newspapers varies over days and is described by a random variable d.

Problems displayed above can be formulated as follows:

$$\begin{aligned} \min_x \quad & c(x) - L(x) \\ \text{s.t.} \quad & 0 \leq x \leq u \end{aligned}$$

where

$$L(x) = E_d\, Q(x, d)$$

and

$$\begin{aligned} Q(x, d) = \quad & \min_{y,w} \quad -qy - rw \\ \text{s.t.} \quad & y \leq d \\ & y + w \leq x \\ & y, w \geq 0 \end{aligned}$$

where E_d denotes the mathematical expectation (i.e., average) with respect to d. $L(x)$ represents the expected revenue on sales and returns while $Q(x, d)$ is the revenue on sales and returns if the demand is equal to d. The formulation showed above illustrates the two-stages form of stochastic programming where the amount of newspaper bought has to be decided before any information about the demand is known. When the information about the demand is given in the second stage, the profit can be computed.

Chapter 3

Base Model

The base model formulation in this thesis is based on the specialization project work of Bonti (2020), in which the objective function is to maximize the net present value (NPV). The decision variables of the formulation are the production profile and the drilling schedule of the field. One of the objectives of this chapter is to make a slight adjustment to the model formulated in the project. The other objective is to decide the most optimum tool-solver combination to solve the model as it will be utilized for further studies in the thesis. It is done by using each option to run the simulation several times using a fixed input or widely known as a deterministic simulation.

3.1 Base Model Formulation

The base model is formulated similar to the one in the specialization project with a slight improvement that includes the gas-oil ratio (GOR) and water cut (WC) curves as a function of recovery factor and the injection well factor for each reservoir. The GOR and WC curves are used to calculate the gas and water production rate. The more detailed formulation is discussed below:

3.1.1 Sets

Sets are an important aspect of optimization model as almost all variables and constraints are indexed by them. The following are sets that are defined in this model:

- $T = \{1, \ldots, 21\}$
  T is a set of years, which in this case is limited to 1 year of production startup and 20 years of production horizon.
- $Res = \{r1, r2\}$
  Res is a set of reservoirs, which in this case is limited to 2 reservoirs.

3.1.2 Parameters

Parameters act as inputs in the optimization model. The value, in general, is constant but can be changed for sensitivity study in the later stage. The following are parameters defined that are in this model:

- Scalar Parameters
  - $N_{w,f,year}$: Maximum number of producer wells drilled at the field each year
    value: 3
  - f_p: Factor represent the effect of adding more wells to an existing gathering network.
    value: 0.973
  - $days$: Number of operational days per year.
    value: 365
- Indexed Parameters
  - $N_r, \quad \forall r \in Res$
    Initial oil in place of reservoir-r.
    value: see Table 3.1
  - $q_{pp,well,r}, \quad \forall r \in Res$
    Maximum oil production potential for each well at reservoir-r.
    value: see Table 3.1
  - $N_{w,pd,r}, \quad \forall r \in Res$
    Number of pre-drilled producer wells at reservoir-r.
    value: see Table 3.1
  - $N_{w,max,r}, \quad \forall r \in Res$
    Maximum number of producer wells for each reservoir.
    value: see Table 3.1
  - $N_{w,injprod,r}, \quad \forall r \in Res$
    Ratio between the number of production and injection wells drilled at reservoir-r.
    value: see Table 3.1

Table 3.1: Parameters value for each reservoir

r	N_r ($\times 10^6$ Sm^3)	$q_{pp,well,r}$ (Sm^3/d)	$N_{w,r,pd}$	$N_{w,r,max}$	$N_{w,r,injprod}$
1	56.25	4592.47	3	6	0.50
2	39.25	2852.22	0	3	0.33

  - $Dimpot_{i,r}, \quad \forall i \in \{1,2,3,4,5\} \quad \forall r \in Res$
    Coefficients of the dimensionless oil production potential polynomial equation of reservoir-r.
    value: see Table 3.2
  - $gor_{i,r}, \quad \forall i \in \{1,2,3,4,5\} \quad \forall r \in Res$
    Coefficients of the gas-oil ratio polynomial equation of reservoir-r.
    value: see Table 3.3
  - $wc_{i,r}, \quad \forall i \in \{1,2,3,4,5\} \quad \forall r \in Res$
    Coefficients of the water cut polynomial equation of reservoir-r.
    value: see Table 3.4

Table 3.2: Dimensionless potential equation coefficients for each reservoir

r	$Dimpot_{1,r}$	$Dimpot_{2,r}$	$Dimpot_{3,r}$	$Dimpot_{4,r}$	$Dimpot_{5,r}$
1	-39.4	-13.9	6.3	12.7	-6.7
2	7067.6	-3761.5	553.0	21.8	-12.7

Table 3.3: Gas-oil ratio equation coefficients for each reservoir (in 10^3 Sm3/Sm3)

r	$gor_{1,r}$	$gor_{2,r}$	$gor_{3,r}$	$gor_{4,r}$	$gor_{5,r}$
1	556.88	-353.19	92.64	-5.27	0.32
2	11358.98	-3064.03	288.79	-5.49	0.15

Table 3.4: Water cut equation coefficients for each reservoir

r	$wc_{1,r}$	$wc_{2,r}$	$wc_{3,r}$	$wc_{4,r}$	$wc_{5,r}$
1	-111.57	119.26	-42.60	6.84	0.011
2	-178.94	95.27	-16.31	0.96	0.03

- Economic Parameters
  - D_1: Coefficient of DrillEx equation (in Million NOK/well)
    value: 500
  - $C_i, \quad \forall i \in \{1, 2, 3, 4, 5\}$
    Coefficients of CapEx equation
    value: see Table 3.5

Table 3.5: CapEx equation coefficients

i	C_i	Units
1	50	Million NOK/Wells
2	0.856	Million NOK/Sm^3/d
3	4.42E-2	Million NOK/Sm^3/d
4	2.35E-4	Million NOK/Sm^3/d
5	2150	Million NOK

  - $O_i, \quad \forall i \in \{1, 2, 3, 4, 5\}$
    Coefficients of OpEx equation
    value: see Table 3.6

Table 3.6: OpEx equation coefficients

i	O_i	Units
1	723.3	Million NOK
2	6.5	Million NOK/Wells
3	2.36E-2	Million NOK/Sm^3/d
4	1.85E-18	Million NOK/Sm^3/d
5	3.04E-21	Million NOK/Sm^3/d

  - i: Discount factor
    value: 0.12

- P_o: Oil price (in USD/bbl)
  value: 60
- vc: Volume conversion (in bbl/Sm^3)
  value: 6.29
- xr: Exchange rate to convert oil price (in NOK/USD)
  value: 8.5

3.1.3 Variables

A variable is a quantity that is assumed to be capable of varying in value during the optimization process. Unless stated otherwise, all variables in this model are within real values. The following are variables that are defined in this model:

- Independent Variables
  - $q_{o,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Oil production rate of reservoir-r at time-t (in Sm^3/d)
  - $N_{w,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Number of producer wells in reservoir-r at time-t
- Dependent Variables
  - $q_{o,f,t} \quad \forall t \in T$
    Oil production rate of the field at time-t (in Sm^3/d)
  - $q_{o,f,max}$: Oil production rate capacity of the field throughout the field life (in Sm^3/d)
  - $q_{pp,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Oil production potential of reservoir-r at time-t (in Sm^3/d)
  - $q_{g,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Gas production rate of reservoir-r at time-t (in 10^6 Sm^3/d)
  - $q_{g,f,max}$: Gas production rate capacity of the field throughout the field life (in 10^6 Sm^3/d)
  - $q_{w,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Water production rate of reservoir-r at time-t (in Sm^3/d)
  - $q_{w,f,max}$: Water production rate capacity of the field throughout the field life (in Sm^3/d)
  - $N_{w,f,t} \quad \forall t \in T$
    Number of wells in the field at time-t
  - $N_{p,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Cumulative oil production of reservoir-r at time-t (in Sm^3/d)
  - $N_{p,f,t} \quad \forall t \in T$
    Cumulative oil production of the field at time-t (in Sm^3/d)
  - $PV_{d,t} \quad \forall t \in T$
    Present value of CapEx at time-t (in Million NOK)
  - $PV_{c.t} \quad \forall t \in T$
    Present value of subsea CapEx at time-t (in Million NOK)
  - PV_{pre}: CapEx made prior the production start for the production facility (in Million NOK)
  - $PV_{o,t} \quad \forall t \in T$
    Present value of operational expenditure at time-t (in Million NOK)

- $PV_{r,t} \quad \forall t \in T$
  Present value of revenue at time-t (in Million NOK)
- $DCF_t \quad \forall t \in T$
  Discounted cash flow at time-t (in Million NOK)
- NPV: Net present value (in Million NOK)

3.1.4 Objective Function

The objective of the model is to maximize the net present value (NPV). It can be expressed as:

$$\text{Maximize} \quad NPV \tag{3.1}$$

3.1.5 Constraints

A constraint is defined as a condition of an optimization problem in which the solution must satisfy. In this formulation, the constraints mainly control the two main decision variables, the production profile and the drilling schedule. However, several constraints quantify the economic variables of the problem as the objective function is to maximize the NPV. The following are constraints that are defined in this formulation:

Equality Constraints

- At the initial condition, the cumulative oil production for both reservoirs equals to zero, while the number of producer wells equals to pre-drilled wells for each reservoir.

$$N_{p,r,0} = 0 \quad \forall r \in Res \tag{3.2}$$

$$N_{w,r,0} = N_{w,r,pd} \quad \forall r \in Res \tag{3.3}$$

- Cumulative production for each reservoir is calculated using the backward trapezoidal integration method. This constraint will be iterated for each reservoir and time step.

$$N_{p,r,t+1} = N_{p,r,t} + \frac{(q_{o,r,t+1} - q_{o,r,t}) \cdot days}{2}, \quad \forall r \in Res \quad \forall t \in T \setminus \{21\} \tag{3.4}$$

- Oil production potential is calculated using a 5th-degree polynomial equation derived from the actual plots. It includes a "pancake" factor which represents the effect of additional wells.

$$\begin{aligned} q_{pp,r,t} = N_{w,r,t} \cdot q_{pp,well,r} \cdot f_p^{(N_{w,r,t}-1)} \cdot \Bigg(& dimpot_{1,r} \left(\frac{N_{p,r,t}}{N_r}\right)^5 \\ & + dimpot_{2,r} \left(\frac{N_{p,r,t}}{N_r}\right)^4 + dimpot_{3,r} \left(\frac{N_{p,r,t}}{N_r}\right)^3 + dimpot_{4,r} \left(\frac{N_{p,r,t}}{N_r}\right)^2 \\ & + dimpot_{5,r} \left(\frac{N_{p,r,t}}{N_r}\right) + 1 \Bigg), \quad \forall r \in Res \quad \forall t \in T \end{aligned} \tag{3.5}$$

- Gas and water production rates are a function of oil production rate and each corresponding parameter (GOR for gas and WC for water).

$$\begin{aligned} GOR = gor_{1,r} \left(\frac{N_{p,r,t}}{N_r}\right)^5 + gor_{2,r} \left(\frac{N_{p,r,t}}{N_r}\right)^4 + gor_{3,r} \left(\frac{N_{p,r,t}}{N_r}\right)^3 \\ + gor_{4,r} \left(\frac{N_{p,r,t}}{N_r}\right)^2 + gor_{5,r} \left(\frac{N_{p,r,t}}{N_r}\right) + 1 \end{aligned}$$

$$q_{g,r,t} = q_{o,r,t} \cdot GOR, \quad \forall r \in Res \quad \forall t \in T \tag{3.6}$$

$$\begin{aligned} WC = wc_{1,r}\left(\frac{N_{p,r,t}}{N_r}\right)^5 + wc_{2,r}\left(\frac{N_{p,r,t}}{N_r}\right)^4 + wc_{3,r}\left(\frac{N_{p,r,t}}{N_r}\right)^3 \\ + wc_{4,r}\left(\frac{N_{p,r,t}}{N_r}\right)^2 + wc_{5,r}\left(\frac{N_{p,r,t}}{N_r}\right) + 1 \end{aligned}$$

$$q_{w,r,t} = q_{o,r,t} \cdot \frac{WC}{1-WC}, \quad \forall r \in Res \quad \forall t \in T \tag{3.7}$$

- Each variable that represents the field is defined as the summation of relevant variable of each reservoir.

$$\begin{aligned} q_{o,f,t} &= \sum_{r \in Res} q_{o,r,t} && \forall t \in T \\ q_{g,f,t} &= \sum_{r \in Res} q_{g,r,t} && \forall t \in T \\ q_{w,f,t} &= \sum_{r \in Res} q_{w,r,t} && \forall t \in T \\ N_{w,f,t} &= \sum_{r \in Res} N_{w,r,t} && \forall t \in T \\ N_{p,f,t} &= \sum_{r \in Res} N_{p,r,t} && \forall t \in T \end{aligned} \tag{3.8}$$

Inequality Constraints

- Production rate of each reservoir at any point in time should never exceed its production potential.

$$q_{o,r,t} \le q_{pp,r,t}, \quad \forall r \in Res \quad \forall t \in T \tag{3.9}$$

- The following constraints are related to the field and each reservoir number of producing wells. First, it must be guaranteed that the number of producing wells for each reservoir are non-decreasing.

$$N_{w,r,t+1} - N_{w,r,t} \ge 0, \quad \forall r \in Res \quad \forall t \in T \setminus \{21\} \tag{3.10}$$

  The next one is that each reservoir number of producing wells is not greater than the specified maximum number of producing wells.

$$N_{w,r,t} \le N_{w,r,max}, \quad \forall r \in Res \quad \forall t \in T \tag{3.11}$$

  The last one represents that at most, three new producer wells can be drilled in the field each year.

$$N_{w,f,t+1} - N_{w,f,t} \le N_{w,f,year}, \quad \forall t \in T \setminus \{21\} \tag{3.12}$$

- Cumulative production of each reservoir at certain time must be greater than or equal to the one from previous time step. Furthermore, it should not exceed its initial oil place.

$$\begin{aligned} N_{p,r,t} &\le N_{p,r,t+1} && \forall r \in Res \quad \forall t \in T \setminus \{21\} \\ N_{p,r,t} &\le N_r && \forall r \in Res \quad \forall t \in T \end{aligned} \tag{3.13}$$

- The following inequality constraints are used to determine the fluid production capacity by finding the maximum fluid production rate throughout the field life.

$$\begin{aligned} q_{o,f,t} &\le q_{o,f,max}, \quad \forall t \in T \\ q_{g,f,t} &\le q_{g,f,max}, \quad \forall t \in T \\ q_{w,f,t} &\le q_{w,f,max}, \quad \forall t \in T \end{aligned} \tag{3.14}$$

Economic Constraints

- As it is calculated before the production started at $t = 1$, the expenditure for the pre-drilled wells is not discounted and calculated as follows:

$$PV_{d,1} = \sum_{r \in Res} (N_{w,r,1} \cdot (1 + N_{w,injprod,r})) \cdot D_1 \tag{3.15}$$

The present value of DrillEx for each point of time since the beginning of field life is formulated as follows:

$$PV_{d,t+1} = \frac{\sum_{r \in Res} ((N_{w,r,t+1} - N_{w,r,t}) \cdot (1 + N_{w,injprod,r})) \cdot D_1}{(1+i)^t} \quad \forall t \in T \setminus \{21\} \tag{3.16}$$

- CapEx for subsea equipment is computed using similar equations as Equation 3.15 and Equation 3.16. The only difference is that the coefficient used in this formulation is for the cost of subsea equipment instead of the well itself.

$$\begin{aligned} PV_{c,1} &= \sum_{r \in Res} (N_{w,r,1} \cdot (1 + N_{w,injprod,r})) \cdot C_1 \\ PV_{c,t+1} &= \frac{\sum_{r \in Res} ((N_{w,r,t+1} - N_{w,r,t}) \cdot (1 + N_{w,injprod,r})) \cdot C_1}{(1+i)^t} \quad \forall t \in T \setminus \{21\} \end{aligned} \tag{3.17}$$

- CapEx for production facility is dependent on fluid production capacity and some other related parameters. It is calculated as follows:

$$PV_{pre} = q_{o,f,max} \cdot C_2 + q_{w,f,max} \cdot C_3 + q_{g,f,max} \cdot C_4 + C_5 \tag{3.18}$$

- Operational expenditure is a function of field fluid production rate and the number of wells at a certain point in time. Hence, the present value of it is defined as:

$$PV_{o,t} = \frac{O_1 + O_2 \cdot N_{w,f,t} + O_3 \cdot q_{o,f,t} + O_4 \cdot q_{w,f,t} + O_5 \cdot q_{g,f,t}}{(1+i)^t} \quad \forall t \in T \tag{3.19}$$

- Revenue for each year is calculated by multiplying the oil production in one year with the oil price, which in this case is kept fixed. On the other hand, gas is assumed to be reinjected and not sold. therefore gas sales are equal to zero.

$$PV_{r,t} = P_o \cdot xr \cdot vc \cdot \frac{N_{p,f,t} - N_{p,f,t-1}}{(1+i)^t} \quad \forall t \in T \tag{3.20}$$

- Discounted cash flow (DCF) is a function of the present value of revenue, DrillEx, CapEx, and OpEx.

$$\begin{aligned} DCF_0 &= -(PV_{pre} + PV_{d,0} + PV_{c,0}) \\ DCF_t &= PV_{r,t} - PV_{d,t} - PV_{c,t} - PV_{o,t} \quad \forall t \in T \end{aligned} \tag{3.21}$$

- Finally, the NPV is calculated using the following equation:

$$NPV = \sum_{t \in T0} DCF_t \tag{3.22}$$

3.2 Implementation in Pyomo and GAMS

The implementation of the base formulation in Pyomo is similar to the specialization project work of Bonti (2020), in which the models are built using two files structure, the model file and the data file. The formulation presented in the previous sections (i.e., sets, parameters, variables, objective function, and constraints) is declared in the .py extension file called as model file. It also contains a command to load the data file as well as a command to solve the model. With an extension of .dat, the data file stores the value of each declared parameters; thus, it is important to load the file before the model is solved. Another important thing to be defined is the solver in which the KNITRO solver from NEOS web server is chosen to solve the Pyomo model.

On the other hand, the implementation of the base formulation in GAMS is slightly different from Pyomo, where the model and data are included in one single .gms extension file. It also requires a different style in writing the line/code. One thing important in implementing the formulation in GAMS is that the type of problems and solvers have to be defined correctly. For instance, the problem formulated in this study is a mixed-integer non-linear programming (MINLP) problem and GAMS/LINDO solver is chosen as the solver. Thus, there should be a line specifying the type of problem to be MINLP and the solver name by typing "option minlp = LINDO" before the problem is solved. An incorrect statement would lead to error as some limitations, such as the use of discrete variables in common non-linear problem (NLP) or the use of power sign (**) in linear problem (LP), are applied.

3.3 Deterministic Simulations

Deterministic simulations usually refer to simulations of a certain formulation where parameter values fully determine the output. In contrast to stochastic simulations, where random variables are present due to uncertainties, all parameter values in the deterministic simulations are known.

In this chapter, the deterministic simulations are performed to compare two combinations of platform and solver. The comparison is made to select the most optimum platforms for further studies in the thesis. Using the most optimum option increases the effectiveness of the solving process and the accuracy of the results. The optimum level of the platforms is defined by two criteria, running time and results stability. Results stability is an important criterion as a different starting point could lead to an infeasible solution.

In the specialization project work of Bonti (2020), the model is formulated using Pyomo and solved using the KNITRO solver. While the running time on the project is relatively good, a comparison with other platforms and solvers is needed to make sure that the study is done using the best available option. As a comparison, the same model will be formulated using another platform called GAMS and solved using GAMS/LINDO solver. A good comparison will include a combination of GAMS and KNITRO solver as one of the options. Unfortunately, it is not possible to use the KNITRO solver in GAMS because the formulation exceeds the number of variables and constraint limitations on the community license.

The running time varies slightly in each run and thus, a minimum of three runs per case while running it in the same condition (e.g., same computer, all software but the platform is closed, etc.) is needed to calculate the average running time. A minimum of three runs is done to make sure that it is a fair comparison. The effect of the initial seed is also studied. Three initial conditions are used to study the variability of the solutions in each case. The 3 initial oil production rate are 5,000, 10,000, and 15,000 Sm^3/d. The NPV yields from each run will then be compared to the base NPV by calculating the percentage difference using equation as shown below:

$$\delta = \left| \frac{\nu_{original} - \nu_{initial\ i}}{\nu_{original}} \right| \times 100\% \tag{3.23}$$

3.4 Results and Discussions

3.4.1 Deterministic Simulation

As shown in Table 3.7, although the running time differences between the two options are not that significant, the GAMS-LINDO running time is lower for most samples. The main cause is that Pyomo-KNITRO uses NEOS web server to access the solver instead of having it built-in within the Pyomo package or installed in the computer. Thus, the running time is highly dependant on the connection between the server and computer in addition to the workload of the server itself. On the other hand, the GAMS/LINDO solver is available within GAMS and included in the community license used in the study. Hence, the running time of problem-solving in GAMS is solely dependant on the complexity of the formulation.

Table 3.7: Running time comparison between Pyomo-KNITRO and GAMS-LINDO

Platform-Solver	Running Time (s)			
	1	2	3	Average
Pyomo-KNITRO	5.44	11.98	8.63	8.68
GAMS-LINDO	6.74	6.72	6.47	6.64

The second comparison, which is based on the variability of the solutions, is shown in Table 3.8. Again, the GAMS-LINDO combination comes superior in which the average percentage difference figure is 0%. It indicates that no matter what initial condition is chosen, the solver always solves the problems and yields a similar NPV result with the base NPV value. On the contrary, the only time Pyomo-KNITRO gives in an identical figure is when the initial condition is defined as 10,000 Sm^3/d, which is close to the optimal field plateau rate shown in Figure 3.3. The instability of the KNITRO solver in this formulation could cause a significant problem once the solver is used for sensitivity analysis in which the value for some parameters changes from one scenario to another. Based on the two criteria, the GAMS-LINDO combination of platform and solver is chosen and will be used for further studies in this thesis.

Table 3.8: Variability of solutions comparison between Pyomo-KNITRO and GAMS-LINDO

Platform-Solver	NPV (Million NOK)			Average Perc.
	5000 Sm^3/d	10000 Sm^3/d	15000 Sm^3/d	Difference (%)
Pyomo-KNITRO	26179.20	26254.37	23772.00	3.25
GAMS-LINDO	26254.37	26254.37	26254.37	0.00

3.4.2 Base Model

Figure 3.1 shows the drilling schedule of the base model. While assuming that the wells have similar performance, the most optimal drilling schedule for the field is to drill wells on reservoir 1 first. While the maximum number of wells drilled per year is limited by 3, the field is only required to drill one more well from reservoir 1 in the second year to maintain the plateau production rate. In the following year, however, a total of 3 wells have to be drilled in order to maintain the desired plateau rate. From year 3, all available wells have been drilled and the number of wells is constant throughout the remaining field life.

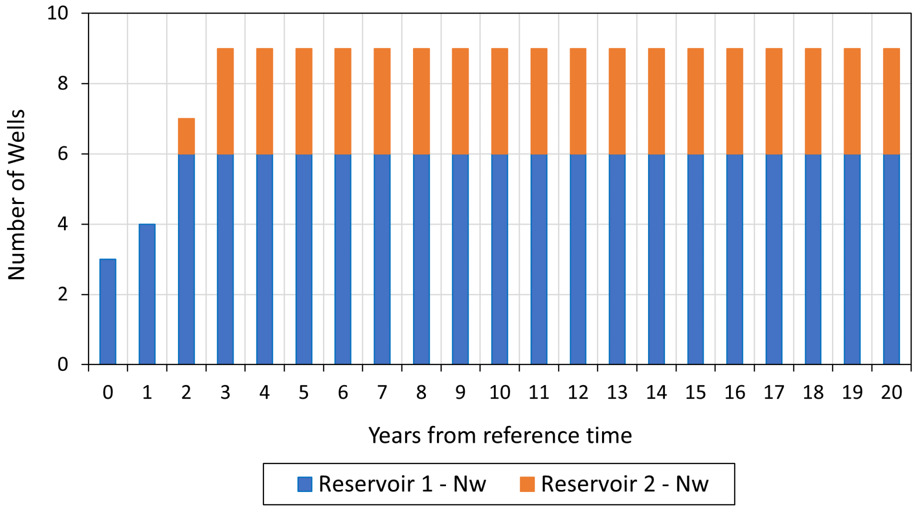


Figure 3.1: Optimal field drilling schedule of the base model

The solver opted to drill wells from reservoir 1 first as the value of the objective function (i.e., net present value) increases if more oil volumes are produced in the early years in which the discount factor is low. As the production potential of wells in reservoir 1 is higher than wells in reservoir 2, drilling more wells with greater potential would lead to higher revenue which resulting in higher NPV. The interesting point is that this figure is based on a relatively low maximum number of wells defined earlier. There is a possibility of a different approach to the drilling schedule if the limitation of the number of wells drilled is changed. The relaxation on the maximum number of producer wells will be one of the studies performed in the sensitivity analysis.

The production profile obtained from the base case is shown in Figure 3.2 and Figure 3.3. It can be seen that the constraint which stated that the oil production rate is controlled by production potential is active for both each reservoir production rate and the field production rate. An interesting point from the figure is that the field is producing oil in plateau mode for the first three years, with a plateau production rate of around 10,000 Sm^3/d, instead of decline mode from the beginning. When the potential of wells from reservoir 1 could not support the plateau rate anymore, wells on reservoir 2 start to produce (consistent with the drilling schedule for reservoir 2 shown in Figure 3.1). In other words, wells from both reservoirs are drilled to maintain the plateau rate of the field, especially in the early years. Ultimately, the potential of both reservoirs cannot maintain the plateau rate any longer, and the production profile starts to decline following the production potential curve until the end of the production horizon.

As discussed in the previous chapter, plateau mode (i.e., producing a constant rate lower than the production potential) is usually the most preferred mode to produce an oil and gas field. The main reason is that if the production rate is made equal to the production potential, particularly in the first year, the oil production capacity should also be increased. Increasing the rate in the early years increases the revenue and increases capital expenditure (CapEx), as more production requires bigger processing capacities. At some point, the increase in revenue does not compensate for the increase in cost. However, it is not always the case as there is a possibility of decline mode to be chosen if the cost of the production facility is not that high or in the case of extremely high oil price.

The discounted cash flow (DCF) and NPV figures are presented in Figure 3.4. The NPV figure

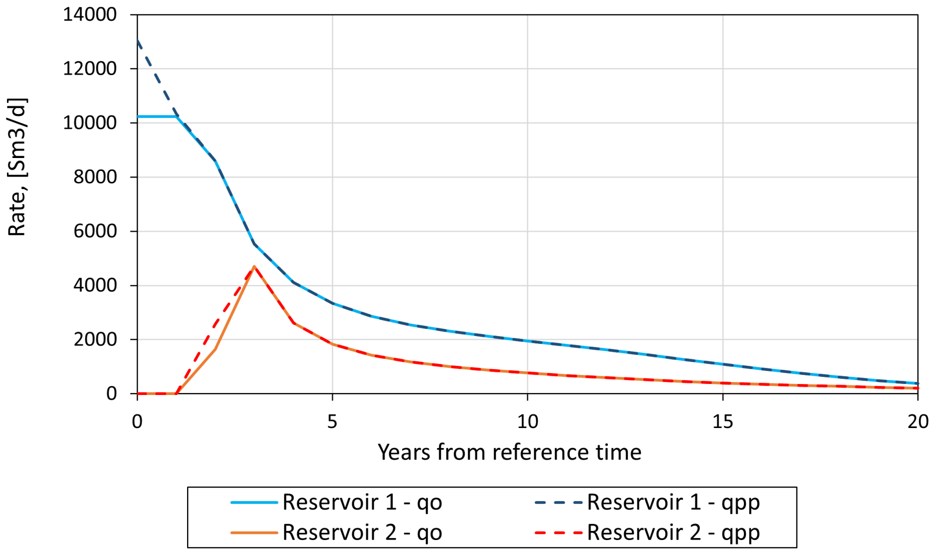


Figure 3.2: Optimal reservoir oil rate and production potential of the base model

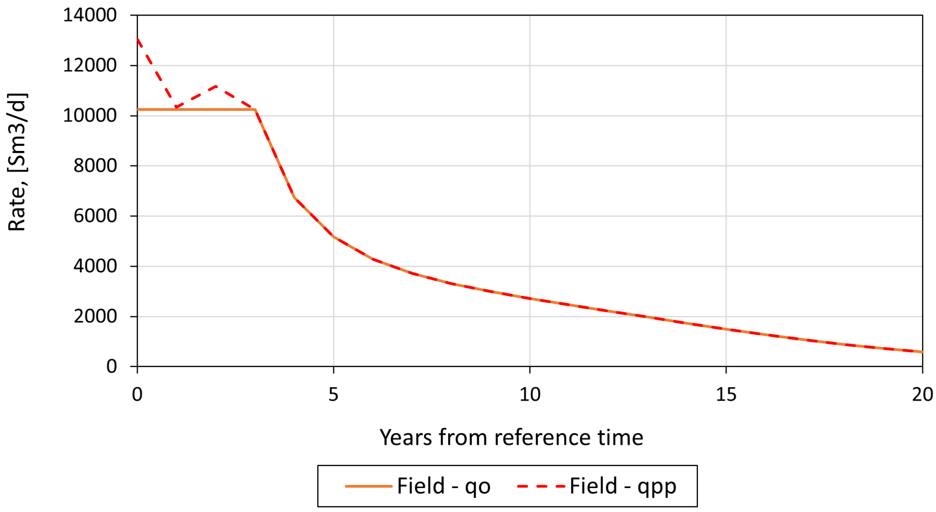


Figure 3.3: Optimal field oil rate and production potential of the base model

shows that the initial investment is returned within four years while keeping in mind that the time needed to prepare the production facility is assumed to be zero. The early years' values of DCF play an important role in ensuring that the investment is paid out quickly. By keeping the production at a plateau for several years and selecting the most optimum production capacity (i.e., production plateau rate), it generates enough revenue to maximize the NPV of the field.

As discussed in the formulation section, the oil price is kept fixed all the time. The consequence is that the DCF figure is consistently decreasing as the production is either plateau or declining. Furthermore, the effect of the discount factor is also getting more significant along with time. If the oil price fluctuates and can be predicted, the DCF figure might also fluctuate. That being said, the production profile and drilling schedule could be optimized in a slightly different strategy, such as drilling wells on both reservoirs simultaneously, not starting the production until the oil price is high enough, etc. Oil price trajectories would be one of the uncertainties in the sensitivity study in this thesis.

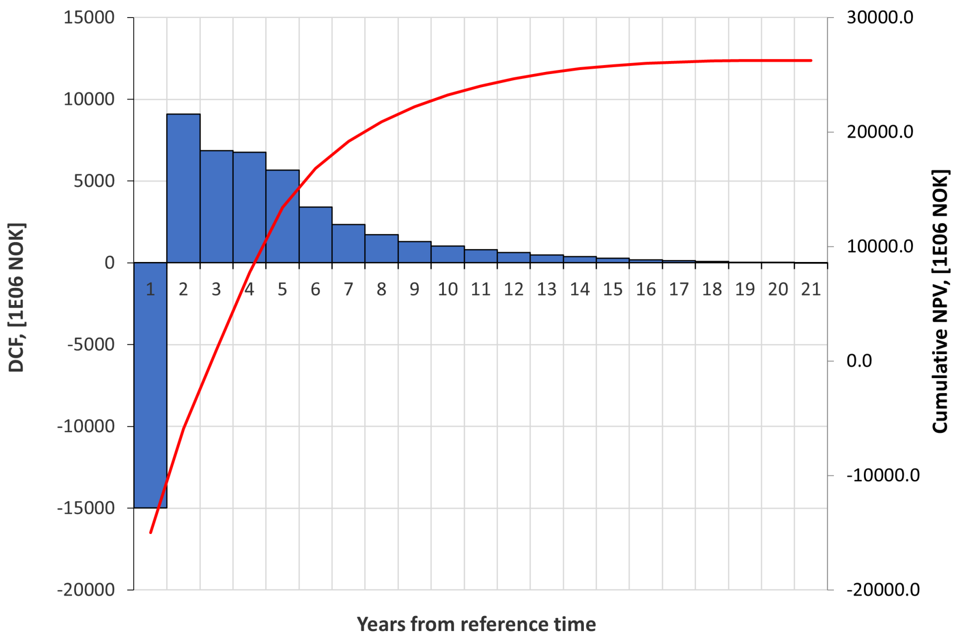


Figure 3.4: DCF and Cumulative NPV figure of the base model

Chapter 4

Uncertainty Analysis

The information used in field development and planning, especially in the early phase, is full of uncertainties in which most of them are originated from incomplete information and knowledge about the reservoir, production system, and economic data. Therefore, it is important to quantify the effect of uncertainty in early phases of field development to ensure the decisions (e.g., production system design, drilling schedule, etc.) made following the FDP study are suitable for the expected variability of uncertain data.

In this chapter, the effect of uncertainties in initial oil-in-place, well performance, development and operational costs, and oil price on the optimization results are analyzed. The main objective of this uncertainty analysis is to propose optimal designs of the field (i.e., production capacity and drilling schedule) considering these uncertainties. The optimization formulation was modified slightly to evaluate the effect of 1) relaxing the upper bound in the number of producer wells allowed and 2) considering flexibility in the drilling schedule. The first modification is done to analyze the possibility of improvement in net present value (NPV) distribution by increasing the maximum number of producer wells. On the other hand, the second one is a modification which applied to both the base case and the upper bound of the number of producer wells case. It is performed to further improve the optimal design from each case by having some wells that can be optional to be drilled or not, instead of a fixed one.

In order to produce the optimal designs, two methods of uncertainty analysis are used in this thesis. The first one is the Latin Hypercube Sampling (LHS) method, while the second is the Stochastic Programming (SP) method. At the end of this chapter, a comparison between these two methods in terms of their formulation complexity, required time to produce the optimal design, and accuracy of the results are also discussed.

4.1 Uncertainty Parameters

The optimization formulation was modified slightly to consider uncertain parameters. For some parameters, the base value was multiplied by a factor to represent uncertainty. The details of each uncertainty parameter are described below:

1. Initial oil-in-place (N_r)
   - As there are two reservoirs in the model, the uncertainty in the initial oil-in-place value

is assigned independently to both parameters in the formulation.

- The initial oil-in-place uncertainty in each reservoir is modeled with a log-normal distribution.
- Reservoir 1 in-place is distributed between 10 and 80 million Sm^3 while having a scale (i.e., $exp(\mu)$) and standard deviation (σ) of 4.00 and 0.43, respectively. On the other hand, the initial oil-in-place of reservoir 2 has a minimum value of 5 million Sm^3 and a maximum value of 60 million Sm^3. The scale and standard deviation of the distribution are 3.70 and 0.20, respectively.
- The in-place log-normal distribution of both reservoirs is shown in Figure 4.1a and Figure 4.1b.

2. Well performance factor ($F_{w,r}$)

   - This parameter multiplies the oil production potential. It represents the uncertainty in reservoir parameters that affect well productivity, e.g., the permeability of the rocks, properties of the fluids, etc.
   - Similar to the initial oil-in-place uncertainty, the well performance is also assigned independently to both reservoirs.
   - Well performance uncertainty for both reservoirs is uniformly distributed between 0.8 and 1.2.
   - The uniform distribution of well performance is shown in Figure 4.1c.

3. Cost factor (F_c)

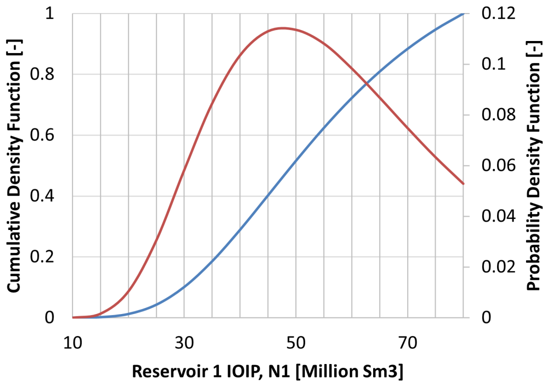
   - This parameter multiplies all cost items in the model (DrillEx, CapEx, OpEx), and it represents uncertainty on the cost model.
   - The cost factor is modeled with a normal distribution with a mean and standard deviation value of 1.0 and 0.2, respectively. The normal distribution is also limited between 0.6 and 1.4.
   - The normal distribution of the cost factor is shown in Figure 4.1d.

4. Oil price ($P_{o,t}$)

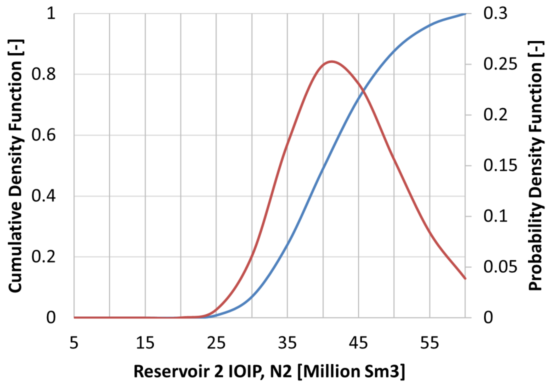
   - Instead of using a fixed oil price as formulated in the base model, the oil price is now variable in time. A certain number of trajectories were considered, each with pair points of oil price versus year, and in this study is defined as oil price trajectory.
   - The oil price trajectory is calculated using the Schwartz-Smith model (programmed by Ph.D. candidate Leonardo Sales).
   - An illustration of the oil price trajectory generated from the model is shown in Figure 4.1e.

4.2 Adjustments to the Optimization Formulation

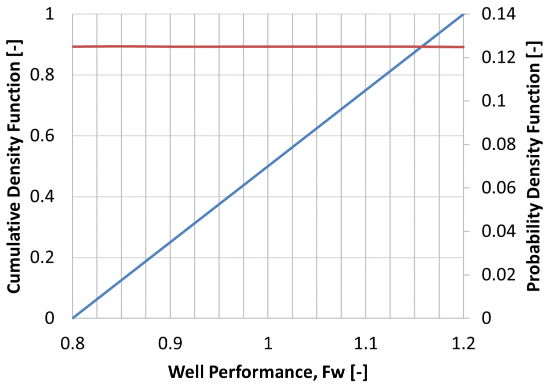
As several new parameters are introduced in the previous section, the following adjustments are needed to include them in the optimization formulation:



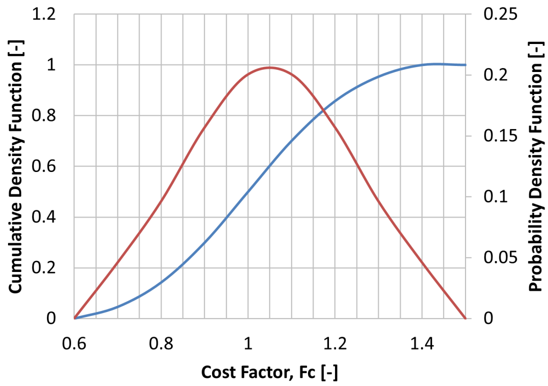
(a) Log-normal distribution of N_1



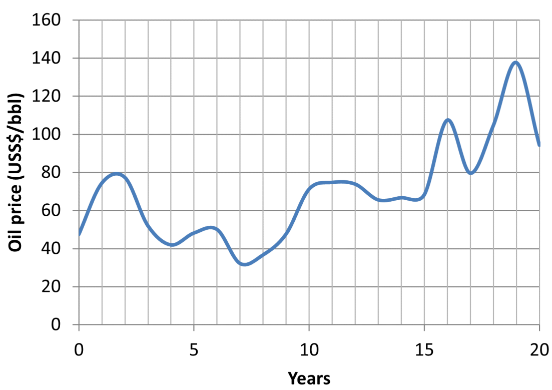
(b) Log-normal distribution of N_2



(c) Uniform distribution of F_w for both reservoirs



(d) Normal distribution of F_c



(e) Illustration of oil price trajectory

Figure 4.1: Distribution of the uncertainties considered in the study

- The oil production potential equation needs to be modified by including the well performance factor ($F_{w,r}$) as a multiplication factor:

$$
\begin{aligned}
q_{pp,r,t} = F_{w,r} \cdot N_{w,r,t} \cdot q_{pp,well,r} \cdot f_p^{(N_{w,r,t}-1)} \cdot \Bigg(dimpot_{1,r} \left(\frac{N_{p,r,t}}{N_r} \right)^5 \\
+ dimpot_{2,r} \left(\frac{N_{p,r,t}}{N_r} \right)^4 + dimpot_{3,r} \left(\frac{N_{p,r,t}}{N_r} \right)^3 + dimpot_{4,r} \left(\frac{N_{p,r,t}}{N_r} \right)^2 \\
+ dimpot_{5,r} \left(\frac{N_{p,r,t}}{N_r} \right) + 1 \Bigg), \quad \forall r \in Res \quad \forall t \in T
\end{aligned}
\tag{4.1}
$$

- The cost model also needs some adjustments as the cost factor (F_c) is introduced to accommodate the uncertainty in the cost of development and operation of the field:
  - Drilling Expenditure (DrillEx) equation:

$$
\begin{aligned}
PV_{d,1} &= F_c \cdot \sum_{r \in Res} (N_{w,r,1} \cdot (1 + N_{w,injprod,r})) \cdot D_1 \\
PV_{d,t+1} &= F_c \cdot \frac{\sum_{r \in Res} ((N_{w,r,t+1} - N_{w,r,t}) \cdot (1 + N_{w,injprod,r})) \cdot D_1}{(1+i)^t} \qquad \forall t \in T \setminus \{21\}
\end{aligned}
\tag{4.2}
$$

  - Capital Expenditure (CapEx) equation:

$$
\begin{aligned}
PV_{c,1} &= F_c \cdot \sum_{r \in Res} (N_{w,r,1} \cdot (1 + N_{w,injprod,r})) \cdot C_1 \\
PV_{c,t+1} &= F_c \cdot \frac{\sum_{r \in Res} ((N_{w,r,t+1} - N_{w,r,t}) \cdot (1 + N_{w,injprod,r})) \cdot C_1}{(1+i)^t} \qquad \forall t \in T \setminus \{21\}
\end{aligned}
\tag{4.3}
$$

  - CapEx for production facility (Pre-CapEx) equation:

$$
PV_{pre} = F_c \cdot (q_{o,f,max} \cdot C_2 + q_{w,f,max} \cdot C_3 + q_{g,f,max} \cdot C_4 + C_5) \tag{4.4}
$$

  - Operational Expenditure (OpEx) equation:

$$
PV_{o,t} = F_c \cdot \frac{O_1 + O_2 \cdot N_{w,f,t} + O_3 \cdot q_{o,f,t} + O_4 \cdot q_{w,f,t} + O_5 \cdot q_{g,f,t}}{(1+i)^t} \quad \forall t \in T \tag{4.5}
$$

- The revenue equation requires a slight modification in which the oil price is now formulated as a trajectory instead of a fixed value:

$$
PV_{r,t} = P_{o,t} \cdot xr \cdot vc \cdot \frac{N_{p,f,t} - N_{p,f,t-1}}{(1+i)^t} \quad \forall t \in T \tag{4.6}
$$

4.3 Base Uncertainty Analysis using LHS

In this section, the uncertainty analysis utilizes Latin Hypercube Sampling (LHS) as the method to sample the uncertainties from their distributions. As discussed in the previous chapter and based on results from previous work, LHS is one of the most preferred sampling methods as it could rebuild the distributions with fewer iterations than Monte Carlo Simulation (MCS). More detailed descriptions about the study are presented in the following subsections:

4.3.1 Sampling

In the specialization project work of Bonti (2020), an uncertainty analysis to study the effect of oil price trajectory was performed. The main objective of the analysis was to determine the minimum number of trajectories needed to achieve convergence on the distribution of the optimal parameters computed. The study shows that using 400 samples of oil price trajectories is sufficient such that the results are considerably similar to those obtained using a much higher number of trajectories. Assuming that it also applies to the other uncertainty parameters, 400 is chosen as the number of samples generated from each distribution of uncertainties in this study.

As there are hundreds of samples/cases in the uncertainty analysis, the five-number summary (i.e., minimum value, maximum value, average value, first quartile, and third quartile) and some parameters that are related to the NPV (i.e., P90, P50, and P10) will be calculated, and the definition of these parameters are provided as follows:

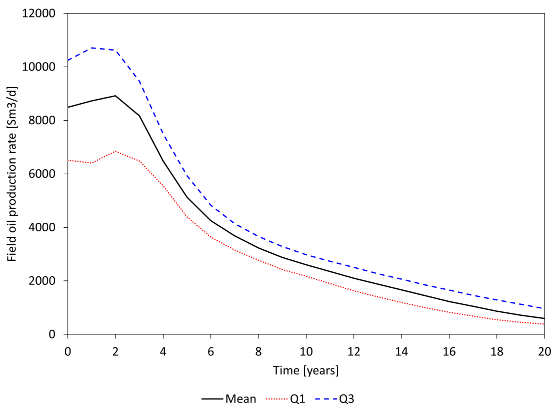
- min : minimum value of the data
- max : maximum value of the data
- Q1 : 25% of the data is below this number
- mean : average of the data
- Q3 : 75% of the data is below this number
- P90 : 90% of the data exceed this number
- P50 : 50% of the data exceed this number
- P10 : 10% of the data exceed this number

The LHS is done in Python. Since nonphysical samples had to be discarded (e.g., negative initial oil-in-place or negative cost factor), the LHS process is programmed from scratch. The general workflow of the function is first to determine the cumulative distribution function (CDF) of the minimum and maximum value as a function of the mean and standard deviation of the distribution. Then, the length of each interval is calculated by dividing the differences between CDF of the minimum and maximum value by the number of samples. By knowing the length of the interval, the sampling could be performed by looping through each interval. The final step is to randomize the generated samples to maintain the LHS method's desired randomness.

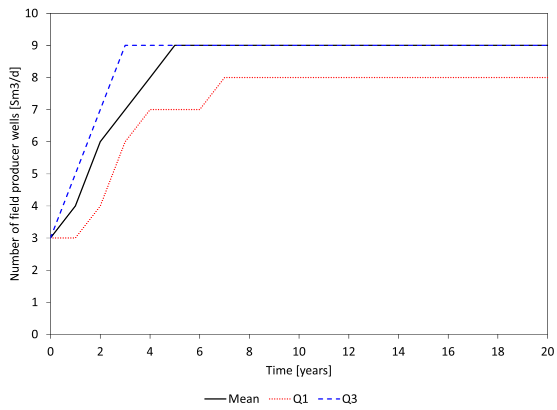
4.3.2 Methodology

This study aims to determine a good field design (or a method to determine a good field design) appropriate for the system variability caused by the multiple uncertainties considered. With this goal, several optimizations using varying input must be performed. Then, the distribution of the design parameters is studied to propose the ultimate optimal design based on the mean of the results or other relevant statistical parameters.

Multiple simulations of the uncertainty analysis model are required to evaluate the optimal design variability. The first step is to run several optimizations with varying (uncertain) input. The uncertain input values come from the LHS method. Probability distributions of the results (optimal NPV, optimal drilling and production schedule, optimal fluid production capacity) are then calculated. This study represents the ideal case that, when there is uncertainty, it will always be possible to design the system optimally to attain maximum NPV. Therefore it represents a reference against which all other designs should be compared against. The first simulation is renamed as "Base simulation" for the comparison purpose of this is shown in Figure 4.2.



(a) Field oil production profile



(b) Field drilling schedule

Figure 4.2: Illustration of five-number summary calculation on the decision variables

The field design will be selected using these results. For example, a proposed design could be to set oil, gas, and water processing capacities and drilling schedule equal to the expected value, as seen in Figure 4.2. The second set of optimizations are then conducted to test the performance of a given field design considering the variability in the input (uncertainties). The samples for this second study are identical to the samples used in the first study. In this second set of optimizations, the production schedule is set as a variable, but the drilling schedule, oil, gas, and water capacities are considered fixed. The results of this second set of optimizations are then analyzed, and probability distributions are computed. Results are compared against the expected NPV of the first step of simulations. As these results consider that all design parameters can be modified to maximize NPV, they represent the ideal case.

The overall goal of this section is to test several selection criteria to determine the best field design. As discussed before, the five-number summary is calculated for each decision variable, and there is always a possibility of changing the design. For instance, the design could be changed by increasing the production capacity from the mean value to the third quartile value or choosing the maximum value drilling schedule instead of the mean one. The most optimal design is selected based on the lowest average percentage error between the NPV parameters of the base simulation and proposed designs.

4.3.3 Implementation in GAMS

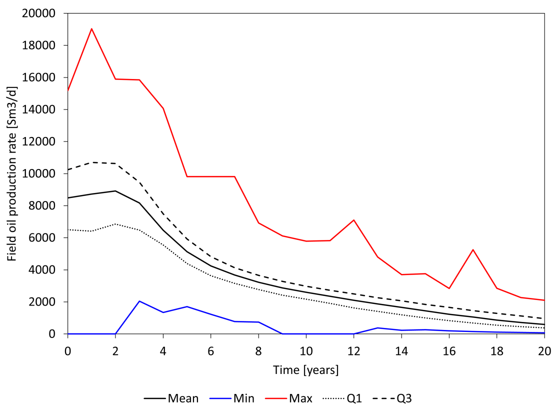
The uncertainty analysis using LHS requires several additional excel files. The files contain the uncertainty parameters data, which need to be loaded to the model file using data exchange in GAMS called gdxxrw. The data is stored in new dedicated parameters defined solely to store the uncertainty parameters for all 400 scenarios. It will also be used to export the results from GAMS to excel after the simulation of the model has been performed.

The uncertainty analysis, in which 400 scenarios are simulated on a single run, utilizes Gather-Update-Solve-Scatter (GUSS) from GAMS. It enables several features which found to be useful for the uncertainty analysis formulation. As mentioned above, the uncertainty parameters for all scenarios are stored in dedicated parameters outside the formulation. GUSS could instantly assign those parameters to the parameters from the formulation and it repeats for each scenario. Another important feature from GUSS is that each scenario is guaranteed to be run using an identical starting point to make sure that the results are valid. As the previous study shows, different starting points could lead to different results. One last important feature from GUSS is that the results could be filtered, and GAMS will store only the desired one. he limitation of using GUSS is that several solvers could not be utilized due to internal communication issues within GAMS. For that reason, LINDO is the only solver available to solve both the mixed-integer non-linear programming (MINLP) model and the GUSS feature for uncertainty analysis. The GAMS implementation of the base uncertainty analysis is provided in Appendix A.

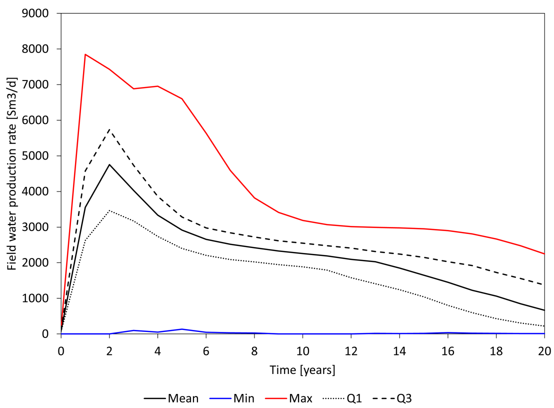
4.3.4 Results and Discussion

Evaluation of base simulation

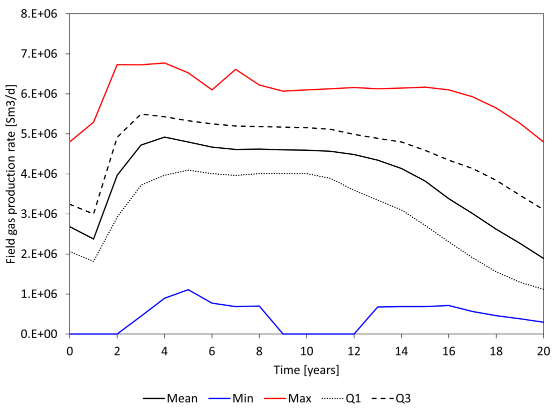
Figure 4.3 shows the distribution of the field fluids production profile after 400 simulations are performed. In each case, the processing capacity of the floating production storage and offloading (FPSO) of oil, gas, and water are set to the maximum values of each curve. Figure 4.4 presents the distribution of the optimal drilling schedule in each field.



(a) Field oil production profile

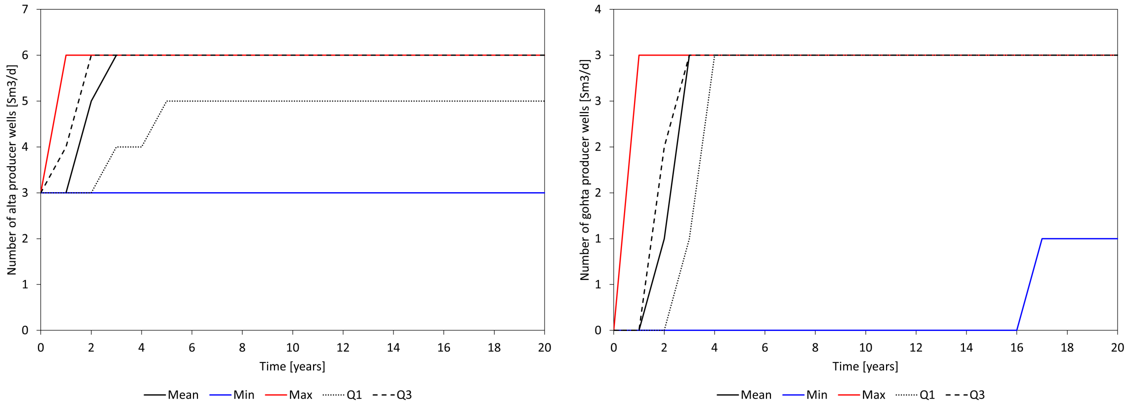


(b) Field water production profile



(c) Field gas production profile

Figure 4.3: Distributions of the field fluids production profile



(a) Reservoir 1 drilling schedule

(b) Reservoir 2 drilling schedule

Figure 4.4: Distributions of the field drilling schedule

The ultimate field design will be defined considering the following:

- Regarding processing capacities, choosing a field design based on the minimum or maximum oil, gas, and water production will most likely yield field designs that are very costly or unable to process high rates and therefore, yield a very low NPV.
- Regarding the drilling schedule: A design based on the maximum and third quartile of the drilling schedule is not feasible because the maximum number of producer wells drilled per year is set to 3 in the model, and those two drilling schedules violate this constraint. Although a design based on the minimum of the drilling schedule is feasible, it is also not considered since a total of 4 wells is assumed to be insufficient to yield a higher NPV figure than the expected value design.

Based on the discussion above, the alternative designs selected for this study are:

- Design method A: Expected value design. Mean value for all production capacities and the drilling schedule.
- Design method B: Third quartile value for all production capacities. Mean value for the drilling schedule.
- Design method C: First quartile value for all production capacities and the drilling schedule.
- Design method D: Mean value for oil and water production capacities as well as the drilling schedule. Third quartile value for gas production capacity.

Evaluation of design based on the expected value: Design method A

The drilling schedule of the expected value design is shown in Table 4.1, while the oil, water, and gas production capacities are displayed below:

- Oil production capacity ($q_{o,f,max}$): 8915.93 Sm^3/d
- Water production capacity ($q_{w,f,max}$): 4752.47 Sm^3/d
- Gas production capacity ($q_{g,f,max}$): 4918875.76 Sm^3/d

Table 4.1: Drilling schedule of the expected value design

Design A	Number of producer wells ($N_{w,t}$)							
	1	2	3	4	5	6	...	21
Reservoir 1	3	3	5	6	6	6	...	6
Reservoir 2	0	0	1	3	3	3	...	3

The second set of simulations is triggered with uncertain parameters and the field design suggested. The production schedule is allowed to vary (optimization variable). Table 4.2 shows the NPV results and runtime of both the base and expected value design. The run time of the proposed design, in which the production capacities and drilling schedule are used as parameters, is shorter than the base simulation. The significance difference makes sense as the production profile is the only decision variable left in the formulation.

Table 4.2: Comparison between the base and expected value design

Design	NPV (Million NOK) / Perc. Difference (%)						Average (%)	Run time (s)
	P90		P50		P10			
Base	5521.6	-	19225.3	-	37810.3	-	-	2710.9
Design A	3560.5	35.5	17816.4	7.3	35703.8	5.6	16.1	66.06

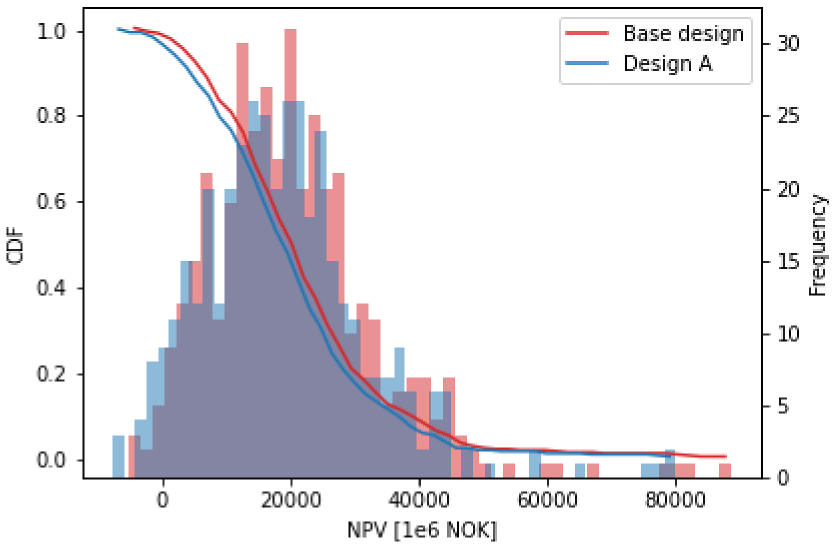


Figure 4.5: CDF curve comparison between the base simulation and expected value design

The P90, P10, and P50 for this second set of simulations are lower than in the first set of simulations. This is somewhat expected, as the only variable that is optimized is the oil production profile. As a preliminary explanation, since the data is only based on one proposed design, it might be because of the design selected at the moment. For instance, if the production capacity is set to 8915 Sm^3/d, and the well productivity is poor, then it could happen that the processing facilities are much more expensive than what they should be to increase NPV. If, on the other hand, the well productivity is very good, then the processing facilities might be too small and do not allow to increase the field rate and ultimately the NPV. On the other note, the absolute differences of the distributions are similar throughout the distributions, as shown in Figure 4.5. The percentage difference of P90 could be higher than the other two NPV parameters due to its low value and thus causing a high percentage.

Evaluation of alternative design: Design method B C and D

The average percentage difference of the expected value design is 16.1%. Although it is not a poor percentage, the figure could be increased by adjusting the design parameters. The parameters, however, are limited to the five-number summary calculated from the base simulation results. The alternative designs proposed in this study are shown in Table 4.3 and Table 4.4.

Table 4.3: Production capacity of the alternative designs

Design	Oil Capacity [Sm^3/d]	Water Capacity [Sm^3/d]	Gas Capacity [Sm^3/d]
Design B	10705.6	5734.2	5494727.9
Design C	6850.8	3462.9	4094426.3
Design D	8915.9	4752.5	5494727.9

Table 4.4: Drilling schedule of the alternative designs

Design	Reservoir	Number of producer wells ($N_{w,t}$)								
		1	2	3	4	5	6	7	...	21
Design B	Reservoir 1	3	3	5	6	6	6	6	...	6
	Reservoir 2	0	0	1	3	3	3	3	...	3
Design C	Reservoir 1	3	3	3	4	4	5	5	...	5
	Reservoir 2	0	0	0	1	3	3	3	...	3
Design D	Reservoir 1	3	3	5	6	6	6	6	...	6
	Reservoir 2	0	0	1	3	3	3	3	...	3

Based on the results shown in Table 4.5, design D is superior in almost all parameters with respect to designs B and C. However, the improvement from the expected value is still moderate, with only a 1% difference in the average percentage difference value. Although the P50 and P10 percentage differences are considerably decreased, the P90 figure is still similar to the previous one. In the previous discussion, it has been proposed that one of the most probable reasons for the extremely high P90 percentage differences is the selected design parameters. However, after proposing three alternative designs, the figure remains similar to or even worse than the expected value design. Thus, the only viable explanation for these extreme differences is the low value of the P90 figure. As per its definition, P90 is the value for which 90% of the population has equal or higher values. Hence, it usually gives a conservative NPV in the lower part of the range. As a result, all of the proposed designs are not able to reach a relatively similar P90 value to the base simulation even though with a relatively similar absolute difference to the P50 and P10 figures.

Table 4.5: Comparison between the base simulation and several proposed designs

Design	NPV (Million NOK) / Perc. Difference (%)						Average (%)
	P90		P50		P10		
Base	5521.6	-	19225.3	-	37810.3	-	-
Design A	3560.5	35.5	17816.4	7.3	35703.8	5.6	16.1
Design B	2139.6	61.3	17478.21	9.1	36522.4	3.4	24.6
Design C	3490.5	36.8	16686.6	13.2	32401.8	14.3	21.4
Design D	3507.5	36.5	18129.1	5.7	36583.4	3.3	15.1

Regardless of the P90 percentage difference, design D is selected as the most optimal design of the field as its average percentage difference is the lowest amongst the proposed designs. Figure 4.6 shows the inverse CDF curve of NPV figure for both base simulation and design D. It can be seen that the curve of design D is getting closer to the base simulation curve apart from the lower NPV.

In the subsequent subsection, flexibility in the drilling schedule is introduced and considered to design the field. Design method D will be used as a base in these studies.

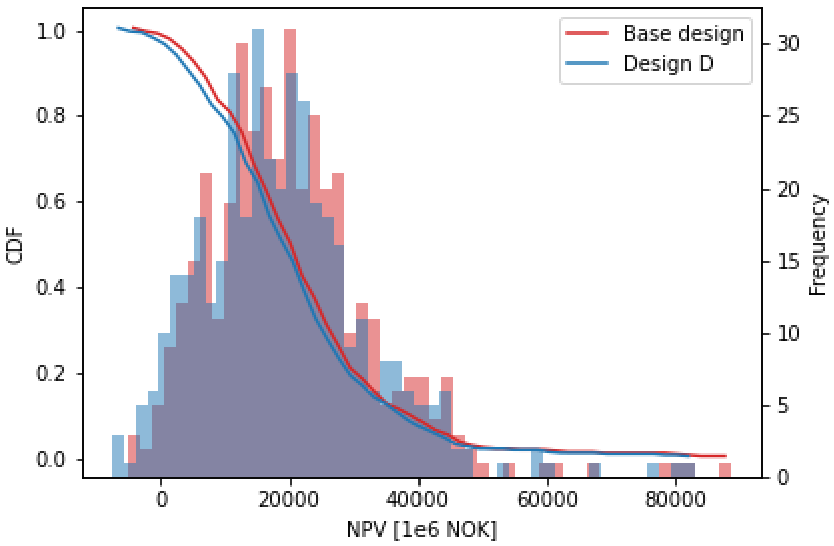


Figure 4.6: CDF curve comparison between the base simulation and design D

4.3.5 Flexibility in the Drilling Schedule

Methodology

Instead of using a fixed value of drilling schedule, the following study considers the flexibility in the field design, i.e., there will be a fixed drilling schedule and a number of "optional" producer wells that could be drilled or not, depending on the uncertainty that "materializes". Optimization will be rerun using variability in the input (uncertainty parameters), but now, the drilling schedule of the optional wells and the production schedule will be the optimization variables. The formulation requires some modifications to accommodate the flexibility as discussed below:

- **Parameters**
  - $N_{w,r,t,min} \quad \forall r \in Res \quad \forall t \in T$
    Minimum number of producer wells drilled in reservoir-r at time-t.
    value: Defined by first quartile or minimum value of the base simulation results.
  - $N_{w,r,t,max} \quad \forall r \in Res \quad \forall t \in T$
    Maximum number of producer wells drilled in reservoir-r at time-t.
    value: Defined by third quartile of the base simulation results.
- **Variables**
  - $N_{w,r,t,add} \quad \forall r \in Res \quad \forall t \in T$
    Additional Number of producer wells drilled in reservoir-r at time-t
- **Constraints**
  Equation 3.3, Equation 3.10, Equation 3.11, and Equation 4.3 are removed or adjusted to the following constraints:
  - The number of producer wells ($N_{w,r,t}$) is now a function of the minimum number of producer wells ($N_{w,r,t,min}$) and the additional number of producer wells ($N_{w,r,t,add}$):
  $$N_{w,r,t} = N_{w,r,t,min} + N_{w,r,t,add} \quad \forall r \in Res \quad \forall t \in T \tag{4.7}$$
  - The number of producer wells in reservoir-r at time-t is bounded by the specified maximum number of producer wells:
  $$N_{w,r,t} \leq N_{w,r,t,max} \quad \forall r \in Res \quad \forall t \in T \tag{4.8}$$
  - The additional number of producer wells is bounded by the differences between the maximum and the minimum number of producer wells:
  $$\sum_{t \in T} N_{w,r,t,add} \leq \sum_{t \in T} (N_{w,r,t,max} - N_{w,r,t,min}) \tag{4.9}$$
  - Capital Expenditure (CapEx) must be computed using the maximum number of producer wells, instead of the current number of producer wells, as the X-mass tree must be prepared beforehand:
  $$\begin{aligned} PV_{c,1} &= F_c \cdot \sum_{r \in Res} (N_{w,r,1,max} \cdot (1 + N_{w,injprod,r})) \cdot C_1 \\ PV_{c,t+1} &= F_c \cdot \frac{\sum_{r \in Res} ((N_{w,r,t+1,max} - N_{w,r,t,max}) \cdot (1 + N_{w,injprod,r})) \cdot C_1}{(1+i)^t} \qquad \forall t \in T \setminus \{21\} \end{aligned} \tag{4.10}$$

The GAMS implementation of the study on the flexibility in the drilling schedule is provided in Appendix B.

As all of the proposed designs fail to yield a relatively good percentage difference value with a fixed drilling schedule, the flexibility of the drilling schedule is expected to improve the figure, especially the P90 one significantly. The study is performed using the most optimal design from the previous section to understand the significance of relaxing the drilling schedule to the NPV figures and the running time.

Since the formulation has been slightly adjusted, especially the one related to the number of producer wells, the third quartile curve of the drilling schedule could be considered for the maximum number of producer wells parameter ($N_{w,r,t,max}$). In addition, the minimum curve could also be utilized as the minimum number of producer wells parameter ($N_{w,r,t,min}$). The other two proposed designs are described as follows:

- Flexible A: First quartile curve as the minimum number of producer wells and third quartile curve as the maximum number of producer wells. The total number of optional wells in this design is 28 wells.
- Flexible B: Minimum curve as the minimum number of producer wells and third quartile curve as the maximum number of producer wells. The total number of optional wells in this design is 110 wells. The optional wells data for each reservoir are shown in Table 4.6.

Table 4.6: Drilling schedule of the alternative flexible designs

Design	Reservoir	Number of producer wells ($N_{w,t}$)									
		1	2	3	4	5	6	...	17	18	...	21
Flexible A	Reservoir 1	0	1	3	3	2	1	...	1	1	...	1
	Reservoir 2	0	0	2	2	2	2	...	2	2	...	2
Flexible B	Reservoir 1	0	1	3	3	3	3	...	3	3	...	3
	Reservoir 2	0	0	2	3	3	3	...	3	2	...	2

Results and Discussions

The result of the study is shown in Table 4.7. In general, the flexibility in the drilling schedule successfully decreased all the percentage differences by a fine margin. As expected, the P90 percentage differences for both flexible designs are significantly lower than the design D from the base case. This could be due to the fact that the field now has an option whether to drill more wells or wait for another year considering a particular situation (i.e., oil price, cost, etc.). For instance, in a condition where the oil price is high, the field could opt to drill more wells to increase the production potential of the field, leading to higher revenue. On the contrary, design D of the base case must follow the fixed drilling schedule and missed the opportunity to yield more revenue by drilling additional wells.

Table 4.7: Comparison between the fixed and flexible drilling schedule designs

Design	NPV (Million NOK) / Perc. Difference (%)						Average (%)	Run time (s)
	P90		P50		P10			
Design D	3507.5	36.5	18129.1	5.7	36583.4	3.3	15.1	88.61
Flexible A	4273.6	22.6	18354.5	4.5	36705.7	2.9	10.0	773.92
Flexible B	4378.5	20.7	18439.2	4.1	36713.6	2.9	9.2	1873.07

The running times of both flexible designs are longer than the design method D with a fixed drilling schedule. This is somewhat expected as the optimization problem now has more decision variables to consider. Despite the fact that flexible B design has more optional wells for the solver to choose from, the average percentage differences of both designs are quite similar. It indicates that the optimal drilling schedule is mainly between Q1 and Q3 instead of min and Q3. Referring back to Table 4.2, the running time of the base simulation is 2710.9 seconds. Although its average percentage difference is the lowest amongst all designs, the flexible B running time is only about 30% faster than running the base case simulation. It indicates that the formulation is still complex. Oppositely, the flexible A running time is approximately 70% faster than the best design. It shows that the formulation has been simplified enough while at the same time producing a pretty decent average percentage difference value. Hence, it can be concluded that flexible A design is the most optimal design considering both the similarity to the base simulation and the running time of the formulation. The optimal design parameters are shown in Table 4.8.

Table 4.8: Flexible A: the most optimal design

Design Parameters	Value [Sm^3/d] or [-]
Oil Capacity	8915.9 - Mean
Water Capacity	4752.5 - Mean
Gas Capacity	5494727.9 - Q3
Drilling Schedule	Between Q1 and Q3

4.4 Study on the Upper Bound of the Number of Producer Wells

In this section, the maximum number of wells drilled in each reservoir is relaxed by increasing it to 20. It is done to understand whether the original formulation in which the maximum drilled wells of reservoir 1 and 2 are 6 and 3, respectively, is adequate to optimize the field or if it is possible to unlock additional value by increasing this number. Another slight modification includes the uncertainty in both water and gas productivity by adding a multiplication factor to the equation of gas and water production rates. More detailed descriptions about the study are presented in the following subsections:

4.4.1 Sampling

As the formulation of this section is slightly more complex than the previous one (includes two additional uncertain variables and increases the upper bound in the number of wells), it is crucial to reduce the running time. With that objective, a study is conducted to determine the number of samples to achieve convergence in the results. The study is carried with the model described in section 4.3, not with the new model.

It is evaluated by calculating several parameters such as the Q1, Q3, expected value, etc., of NPV and production profile of three number of samples (100, 200, and 300). They are then compared against the values computed using the highest number of samples (400). A number of samples is considered sufficient if the difference between its results and the results of the highest number of samples is equal or less than 5%. Some of the results, which are a function of time, needs to be analyzed using the mean absolute percentage difference (MAPE) calculated using the following equation:

$$MAPE = \frac{1}{n} \cdot \sum \left| \frac{\nu - \nu_{approx}}{\nu} \right| \times 100\% \tag{4.11}$$

Based on the results displayed in Figure 4.7, the differences between all results are visibly insignificant. Furthermore, all of the average MAPE for the production profiles showed in Table 4.9 are below the defined threshold of 5 percent. It indicates that using even 100 samples could produce an equally similar production profile distribution to the results where 400 samples are used. However, the results still need to be complemented by the NPV distribution percentage differences.

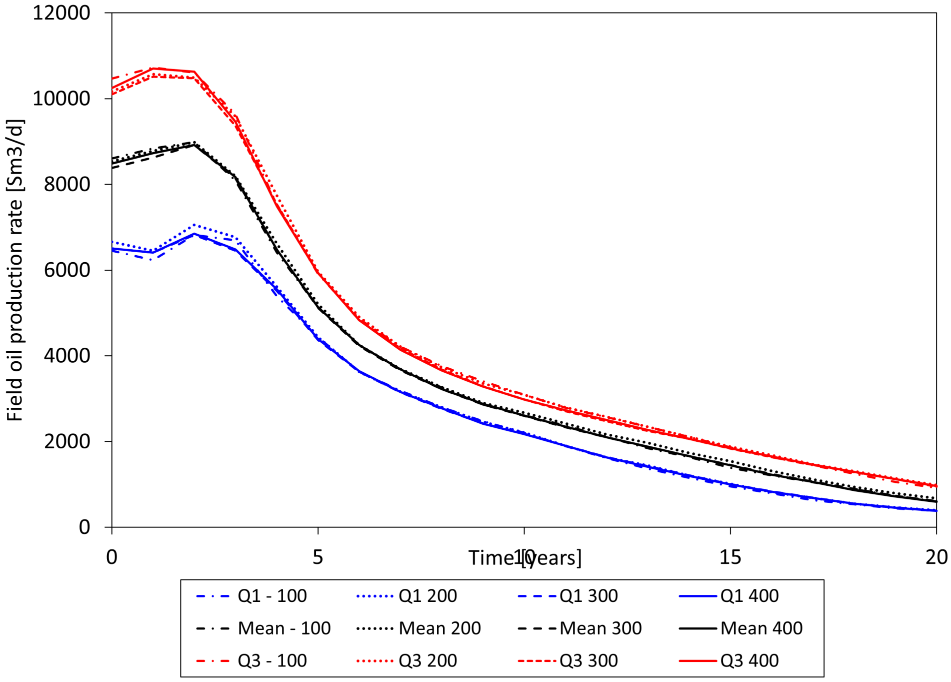


Figure 4.7: Comparison of oil production profile distributions using different number of samples

Table 4.9: Effect of number of samples on the production profile distribution

Number of samples	MAPE (%)			Average (%)
	Q1	Mean	Q3	
100	1.11	2.17	2.13	1.80
200	3.63	1.39	1.86	2.30
300	0.75	0.62	0.72	0.72

Table 4.10 presented the NPV distribution percentage differences between the base results to the one using fewer samples. Aside from the P90 figure, which is higher than the others due to its low value (i.e., low denominator causes high percentage), the percentage difference of both 200 and 300 samples are all below the threshold. Considering both the production profile and NPV distribution, 200 is selected as the most optimum number of samples for the second uncertainty analysis. However, it is important to make sure that these 200 samples use oil price trajectories that, on average, are "similar" (or provide a similar statistical variability) to the case using the highest number of samples (400 oil trajectories). Figure 4.8 shows the P90, P50, and P10 of the oil price trajectories calculated for every year for the 200 and 400 samples.

Table 4.10: Effect of number of samples on the NPV distribution

Number of samples	Percentage difference (%)		
	P90	P50	P10
100	43.7	7.1	6.1
200	8.9	4.9	0.1
300	17.7	0.0	1.6

Based on the comparison shown in Figure 4.8 and Table 4.11, the oil price trajectories used for the base results and the one using 200 samples are similar. This supports the observation made in Table 4.9 and Table 4.10, in which, for 200 samples, all MAPE values are below 5%. Thus, it can be concluded that a similar accuracy in the results is guaranteed even when 200 samples are used. The same sampling procedure, which utilizes the functions defined in Python, is applied to generate 200 samples for the new uncertainties introduced in the following subsection.

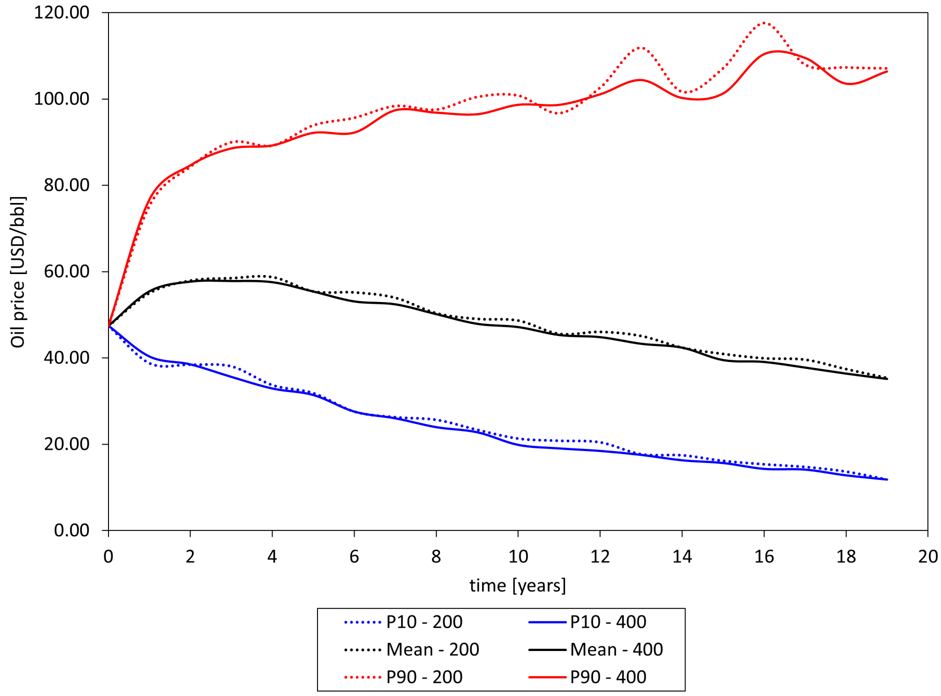


Figure 4.8: Comparison of oil price trajectories distributions between 200 and 400 samples

Table 4.11: Validation of oil price trajectories

Parameters	MAPE (%)
P10	4.16
P50	1.95
P90	2.39

4.4.2 Methodology

The tasks conducted previously in section 4.3 will be repeated using the same methodology as follows:

1. To evaluate the effect of uncertainty, several optimizations will be executed with varying input (generated with LHS).
2. Several design methods will be employed to determine the field design, and the performance of these designs will be evaluated by running multiple optimizations (varying production schedule only) with uncertain input.
3. Then, flexibility in the drilling schedule will be introduced by allowing the possibility to drill optional wells.

However, as mentioned before, one of the objectives of this study is to understand the effect of allowing more wells to be drilled during the field life. Hence, a comparison will also be made between the base simulation results of the two approaches. For comparison purposes, the first uncertainty analysis is renamed the "Original" case, while the second renamed the "Increased Max N_w" case.

There are two additional uncertainties added to the formulation, and the following adjustments are required to accommodate those changes:

- Parameters
  - $N_{w,max,r}, \quad \forall r \in Res$
    Maximum number of well for each reservoir is increased to 20 for both reservoir.
  - F_{gas}: New parameter to represent the effect of uncertainty on the gas productivity.
    value: follows the same distribution as shown in Figure 4.1c.
  - F_{water}: New parameter to represent the effect of uncertainty on the water productivity.
    value: follows the same distribution as shown in Figure 4.1c.
- Constraints
  Equation 3.6 and Equation 3.7 are adjusted to the following constraints:
  - Gas production rate equation is adjusted by adding the gas productivity multiplication factor:
    $$q_{g,r,t} = F_{gas} \cdot q_{o,r,t} \cdot GOR, \quad \forall r \in Res \quad \forall t \in T \tag{4.12}$$
  - Similar to the gas equation, the water production rate equation is also adjusted by adding the water productivity multiplication factor:
    $$q_{w,r,t} = F_{water} \cdot q_{o,r,t} \cdot \frac{WC}{1 - WC}, \quad \forall r \in Res \quad \forall t \in T \tag{4.13}$$

4.4.3 Implementation in GAMS

The exact implementation as presented in subsection 4.3.3 is applied to perform this study.

4.4.4 Results and Discussion

Comparison between two base simulations

The NPV CDF curves of both cases are shown in Figure 4.9, and it can be seen that the increment in the maximum number of producer wells has considerable effects on the NPV figures. This is validated by the calculation presented in Table 4.12, in which the percentage difference of P90, P50, and P10 are around 5% mark.

As discussed before, the samples generated for the increased max N_w case is 200, which is half the number used in the original case. Nevertheless, its running time is still longer than the original case. The maximum number of producer wells controls the room of freedom of drilling schedule, which is one of the most important decision variables. Thus, it is understandable that increasing it has a significant effect on the complexity of the formulation. However, the difference of average run time per case is quite significant as the increased max N_w case run time is more than doubled the original case.

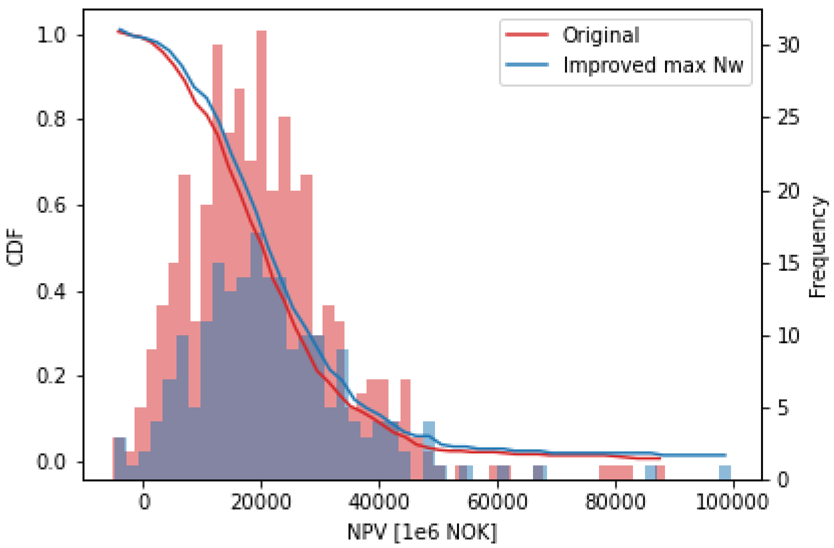


Figure 4.9: CDF curve comparison between the original formulation and increased max N_w case

Table 4.12: Comparison between the original case and increased max N_w case

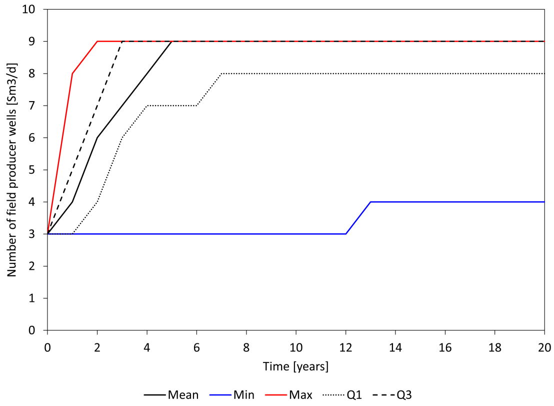
Case	NPV (Million NOK) / Perc. Difference (%)						Run time (%)	Average run time/case (s)
	P90		P50		P10			
Original	5521.6	-	19225.3	-	37810.3	-	2710.9	6.7
Increased Max N_w	5957.7	7.9	19927.2	3.7	39814.3	5.3	3507.4	17.5

Although the improvement in NPV distribution is considerable, the additional complexity and increased running time seem to indicate that the modified formulation has limited value. Moreover, Figure 4.10 shows that the expected value of the number of producer wells at the end of field life is 11, similar to the upper bound used in the original formulation, 9. Hence, it can be inferred that the original formulation has utilized an adequate maximum number of producer wells as its parameter value. However, in a case where the maximum number of producer wells has not been determined, this adjustment is an important analysis which could optimize both the results and run time of the formulation.

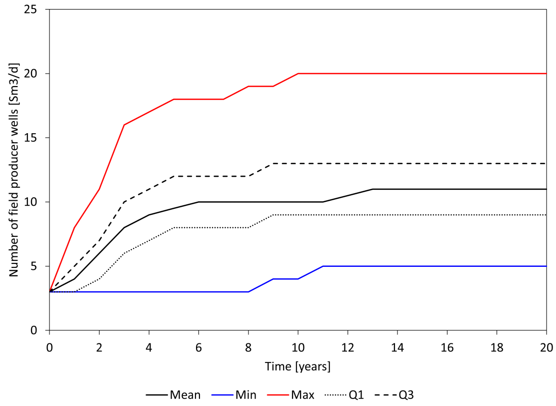
Evaluation of base simulation

Figure 4.11 shows the distribution of the field fluids production profile after 200 simulations are performed on the increased max N_w case, while Figure 4.12 presents the distribution of the optimal drilling schedule in each field. It can be seen that most of the cases fall in a narrow band around the mean (Q1, Q3), while the maximum and minimum values are far apart. Therefore, similar to what was done earlier in section 4.3, the design method to consider will include the expected value, Q1, and Q3, instead of the maximum and minimum, because these extreme cases could lead to poor NPV due to excessive costs or inability to process the production.

On the other hand, the design method for the drilling schedule will not follow a similar approach to the previous section, where the expected value and Q1 curve are the only two chosen curves. The design method will include the Q3 curve as the drilling schedule does not violate the constraint which controls the maximum number of producer wells drilled per year. However, the study is still not considering the minimum and maximum as the former curve might lead to NPV value

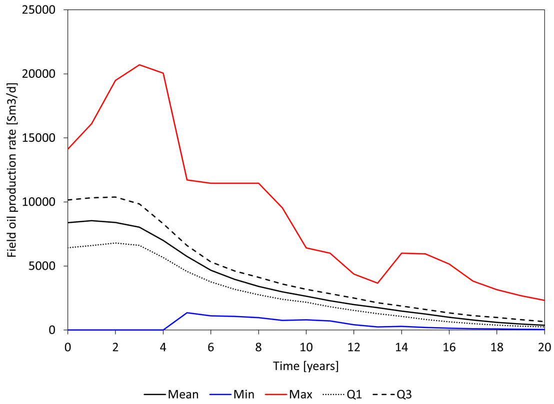


(a) Original case drilling schedule

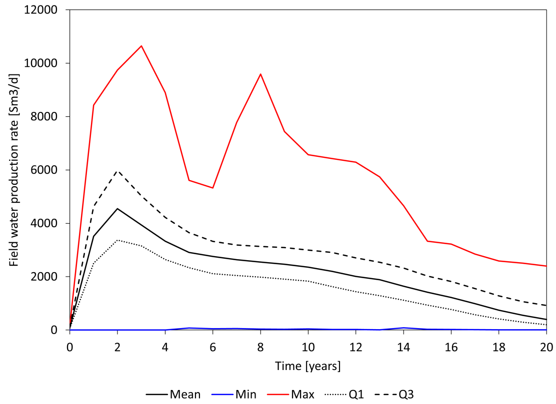


(b) Increased max N_w case drilling schedule

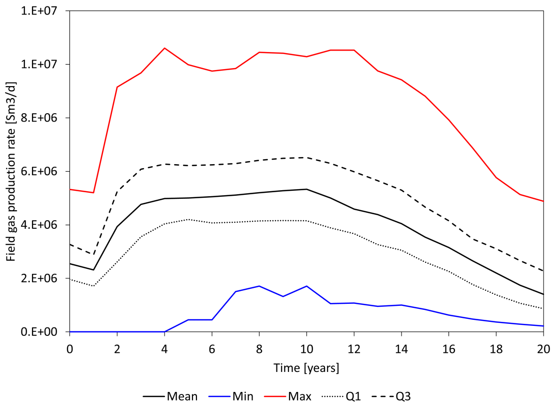
Figure 4.10: Distributions of the field drilling schedule for both cases



(a) Field oil production profile



(b) Field water production profile

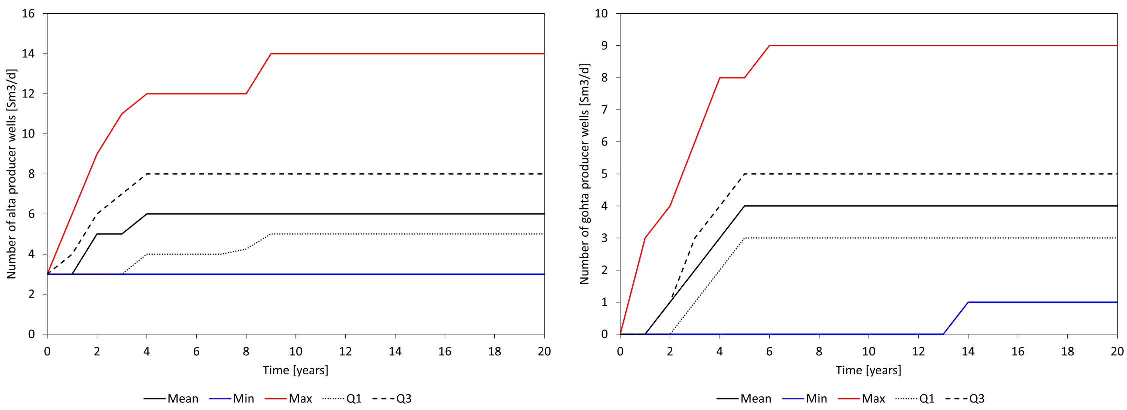


(c) Field gas production profile

Figure 4.11: Distributions of the field fluids production profile of the increased max N_w case

which definitely lower than the expected value design, while the latter curve violates the maximum number of producer wells drilled per year constraint.

Based on the discussion above, other than the expected value design, the alternative designs selected for this study are:



(a) Reservoir 1 drilling schedule

(b) Reservoir 2 drilling schedule

Figure 4.12: Distributions of the field drilling schedule of the increased max N_w case

- Design A: Expected value design. Mean value for all production capacities and the drilling schedule.
- Design B: Third quartile value for all production capacities and the drilling schedule. Design B for the case where the number of producer wells is increased is different from the one in section 4.3 as the third quartile value for the drilling schedule is now available to be chosen.
- Design C: First quartile value for all production capacities and the drilling schedule.
- Design D: Mean value for oil and water production capacities as well as the drilling schedule. Third quartile value for gas production capacity.

Evaluation of design based on the expected value - Design method A

The drilling schedule of the expected value design of the increased max N_w case is shown in Table 4.13, while the oil, water, and gas production capacities are displayed below:

- Oil production capacity ($q_{o,f,max}$): 8812.6 Sm^3/d
- Water production capacity ($q_{w,f,max}$): 4344.2 Sm^3/d
- Gas production capacity ($q_{g,f,max}$): 5316119.6 Sm^3/d

Table 4.13: Drilling schedule of the expected value design of the increased max N_w case

Design A	Number of producer wells ($N_{w,t}$)							
	1	2	3	4	5	6	7	...	21
Reservoir 1	3	3	5	5	6	6	6	...	6
Reservoir 2	0	0	1	2	3	4	4	...	4

Table 4.14 shows the NPV results and run time of both the base simulation and expected value design of the increased max N_w case. As expected, the run time of the expected value design is faster than the base design. In fact, it is also faster than the run time of the expected value design shown in Table 4.2. However, the difference is mainly because the number of samples used

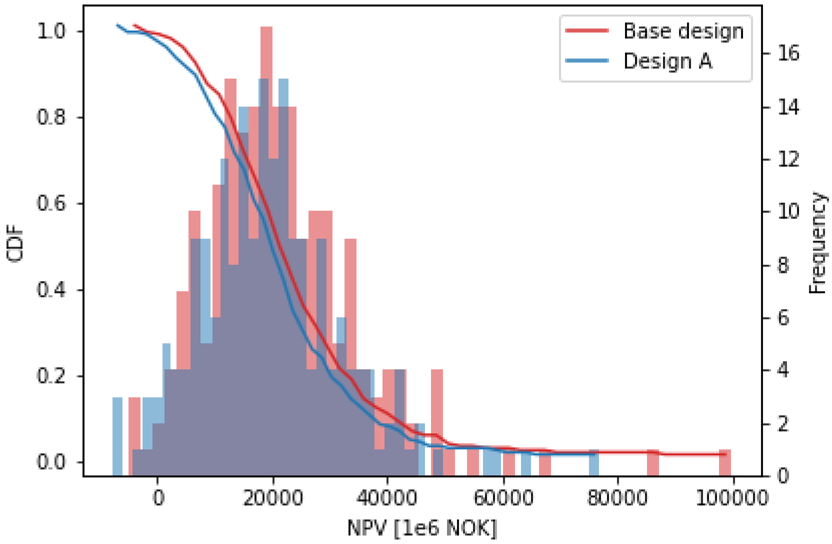


Figure 4.13: CDF curve comparison between the base simulation and expected value design of the increased max N_w case

in the second study is halved of the first one. The average run time per case is still similar as both formulations are equally simple (i.e., production profile as a sole decision variable).

As can be seen in Table 4.2, the most significant difference between the base simulation and the field design based on the expected value is on the P90 NPV value. This also occurred in section 4.4. Based on the P50 and P10 parameters, the expected value design yields a reasonably similar result to the base design. However, Figure 4.13 shows that it is possible to further improve the design as the absolute difference between the two curves are still visible.

Table 4.14: Comparison between the base simulation and expected value design of the increased max N_w case

Design	NPV (Million NOK) / Perc. Difference (%)						Average (%)	Run time (s)
	P90		P50		P10			
Base	5957.7	-	19927.2	-	39814.3	-	-	3507.4
Design A	4668.5	21.6	18969.7	4.8	36150.9	9.2	11.9	36.8

Evaluation of alternative designs - Design method B C D

With an 11.9% average percentage difference in the NPV P90, P50, and P10 values, it seems the design method A, considering an upper bound of 20 in the number of producers, is better than design method A when an upper bound of 9 in the number of producers is considered (section 4.3). However, several alternative designs will still be explored in order to determine the most optimal design of the increased max N_w case. The designs proposed for the second study are shown in Table 4.15 and Table 4.16.

Table 4.15: Production capacity of the alternative designs of the increased max N_w case

Design	Oil Capacity [Sm^3/d]	Water Capacity [Sm^3/d]	Gas Capacity [Sm^3/d]
Design B	10604.2	5835.9	6588391.3
Design C	7082.1	3292.3	4148609.9
Design D	8812.6	4344.2	6588391.3

Table 4.16: Drilling schedule of the alternative designs of the increased max N_w case

Design	Reservoir	Number of producer wells ($N_{w,t}$)									
		1	2	3	4	5	6	7	8	...	21
Design B	Reservoir 1	3	5	6	7	7	7	8	8	...	8
	Reservoir 2	0	0	2	4	5	5	5	5	...	5
Design C	Reservoir 1	3	3	3	3	4	4	4	5	...	5
	Reservoir 2	0	0	0	1	2	3	3	3	...	3
Design D	Reservoir 1	3	3	5	5	6	6	6	6	...	6
	Reservoir 2	0	0	1	2	3	4	4	4	...	4

The P90, P50, and P10 values of NPV of all these design methods and the base case are shown in Table 4.17. Based on these results, the expected value design is still better than all the alternative designs regarding the average percentage difference. The second best is design method D, although the difference in the P90 value of NPV is considerable. Thus, the expected value design is selected to be the most optimal design for the increase N_w case and will be used in the flexibility in the drilling schedule analysis.

Another side note which is interesting to be looked at is that design B, which uses the third quartile curves to design the field, is superior in terms of the P10 percentage difference. However, the P90 value of NPV is poor compared to the base case. On the other hand, design C is the best design when comparing the P90 percentage difference while simultaneously also the most inferior design in terms of the P10 percentage difference. It can be concluded that designs based on higher capacities (e.g., Q3 for oil, gas, and water processing capacity) and a higher number of producer wells will do well in P10 NPV estimates but will do poorly in P90 NPV estimates. On the other hand, designs based on low capacities (e.g., Q1 for oil, gas, and water processing capacity) and a lower number of producer wells will do well in P90 NPV estimates, but poor in P10 NPV estimates.

Table 4.17: Comparison between the base simulation and several proposed designs of the increased max N_w case

Design	NPV (Million NOK) / Perc. Difference (%)						Average (%)
	P90		P50		P10		
Base	5957.7	-	19927.2	-	39814.3	-	-
Design A	4668.5	21.6	18969.7	4.8	36150.9	9.2	11.9
Design B	2875.4	51.7	19143.9	3.9	37012.7	7.0	20.9
Design C	5113.7	14.2	17966.8	9.8	32987.7	17.1	13.7
Design D	4320.0	26.6	16002.7	4.2	35856.2	7.8	12.9

4.4.5 Flexibility in the Drilling Schedule

Methodology

The same methodology as presented in subsection 4.3.5 is applied to perform this study. The two proposed alternative designs are described as follows:

- Flexible A: First quartile curve as the minimum number of producer wells and Third quartile curve as the maximum number of producer wells. The total number of optional wells in this design is 101 wells.
- Flexible B: Minimum curve as the minimum number of producer wells and Third quartile curve as the maximum number of producer wells. The total number of optional wells in this design is 177 wells. The optional wells data for each reservoir are shown in Table 4.18.

Table 4.18: Drilling schedule of the alternative flexible designs of the increased max N_w case

Design	Reservoir	Number of producer wells ($N_{w,t}$)												
		1	2	3	4	5	6	7	8	...	15	16	...	21
Flexible A	Reservoir 1	0	2	3	4	3	3	4	3	...	3	3	...	3
	Reservoir 2	0	0	2	3	3	2	2	2	...	2	2	...	2
Flexible B	Reservoir 1	0	2	3	4	4	4	5	5	...	5	5	...	5
	Reservoir 2	0	0	2	4	5	5	5	5	...	5	4	...	4

Results and discussion

Table 4.19: Comparison between the optimal design and flexible drilling schedule designs of the increased max N_w case

Design	NPV (Million NOK) / Perc. Difference (%)						Average (%)	Run time (s)
	P90		P50		P10			
Design A	4668.5	21.6	18969.7	4.8	36150.9	9.2	11.9	36.8
Flexible A	5589.1	6.2	19801.4	0.6	36965.9	7.2	4.7	1243.2
Flexible B	5612.5	5.8	19801.4	0.6	36965.9	7.2	4.5	2939.2

The result of the study is shown in Table 4.19. The study yields similar results to the original case, where the percentage differences are significantly reduced. The P90 percentage difference is the parameter affected the most by the increment of the maximum number of producer wells as the figure is nearly 5% from around 21.6%. The adjustment gives the solver more cases to explore and thus increasing the accuracy of the results. In consequence, the formulation requires more time to be solved.

However, an odd thing happens in the second study in which both flexible A and B designs yield somewhat identical results (except for the P90 value) with a significant difference in running time. One thing that could cause this is that most of the optimal drilling schedule falls in the range between the first and third quartile curves. Although flexible B design considers the minimum curve as the lower bound of the drilling schedule, the optimal drilling schedule is still the same as for flexible A. However, considering that flexible B requires considerably more time to be solved, flexible A is selected as the best design method for the increased max N_w case.

The most optimal design of the second case is shown in Table 4.20. Referring to Table 4.8, it can be seen that both the original and the increased max N_w cases' most optimal design are similar in terms of value for the production capacities and range of distributions for the drilling schedule.

Table 4.20: Flexible A: the most optimal design of the increased max N_w case

Design Parameters	Value [Sm^3/d] or [-]
Oil Capacity	8812.6 - Mean
Water Capacity	4344.2 - Mean
Gas Capacity	5316119.6 - Mean
Drilling Schedule	Between Q1 and Q3

4.5 Stochastic Programming Method

In this section, the stochastic programming (SP) method is applied to perform optimization considering uncertainty. Similar to the previous study, the objective of this analysis is to produce the most optimal design of the field by considering all the specified uncertainties. Moreover, a comparison will be made between the SP method and the methods studied earlier to compare their formulation complexity, running time, and accuracy of results. The comparison is only made to the first LHS study in which the maximum number of producer wells for each reservoir has not been increased to 20 described in section 4.3. More detailed descriptions about the study are presented in the following subsections:

4.5.1 Spearman's Rank Coefficient Correlation

Since stochastic optimization embeds uncertain parameters in the optimization, it usually results in more complex formulations and time-consuming runs. Therefore, it is of interest to reduce the number of uncertain parameters to a minimum, and consider those that have first-order effects. To do this, an analysis will be performed on the results of the first study (section 4.3). The correlation between the uncertainties of the first study and the objective function (i.e., NPV) will be assessed by calculating a parameter called Spearman's Rank Coefficient Correlation (SRCC). As the results from the first study have been obtained, it is possible to calculate the SRCC parameter for each uncertainty parameter. It is done to verify the worthiness of including a certain parameter in the SP study. Considering only uncertainties that are correlated enough to the objective function will increase the accuracy of the results and, at the same time, reduce the complexity of the formulation.

The SRCC parameter calculation requires the set of the uncertain parameter and the NPV results with an equal number of samples. Since the uncertainty in oil price utilized trajectories instead of a particular value of multiplication factor like the others, it is unfortunately not possible to calculate the SRCC parameter. Thus, the calculation is only be made for the five parameters: uncertainty in initial oil in place of both reservoirs (N_1 and N_2), uncertainty in well performance ($F_{w,1}$ and $F_{w,2}$), and uncertainty in cost model (F_c). The calculation is performed using Python and its pre-defined function called spermanr of the scipy.stat module.

The result of the calculation is shown in Table 4.21. The SRCC value indicates the correlation between the variables, with a value that varies between -1 and 1, with both values imply an exact monotonic relationship and 0 implying no correlation. As the SRCC between the well performance of reservoir 2 ($F_{w,2}$) and the NPV is close to 0, it can be assumed that there is a very low to none correlation between the two variables. Therefore, the uncertainty in the well performance of reservoir 2 will be ignored in the stochastic optimization. Although the figures for the other four parameters are pretty low, they can still be considered as a correlation with the objective function. When comparing the magnitude of SRCC among each other, it can be concluded that the initial oil in place of reservoir 1 and the cost factor have the most impact on the NPV, followed by the initial oil in place of reservoir 2 and the well performance of reservoir 1.

Table 4.21: SRCC calculation of the uncertainty parameters

Parameters	SRCC
N_1	0.349
N_2	0.147
$F_{w,1}$	0.133
$F_{w,2}$	0.06
F_c	-0.302

4.5.2 Sampling

The SP method requires discretizing the probability distributions of the uncertain parameters instead of using samples like the earlier sections. The method of bracket mean is used to discretize the cumulative probability distributions of the uncertain parameters. An illustration of the method is shown in Figure 4.14. The distribution curve is discretized into three intervals, with each specified length implying the probability. Then, a sample is generated from each interval by drawing a line from the middle value of each interval y-axis (e.g., 0.15 from the first interval, 0.5 from the second one, etc.) to the curve and then to the x-axis, which yields the value. Finally, the value is assigned with each corresponding probability which in this case are 30 with 0.3, 45 with 0.4, and 65 with 0.3.

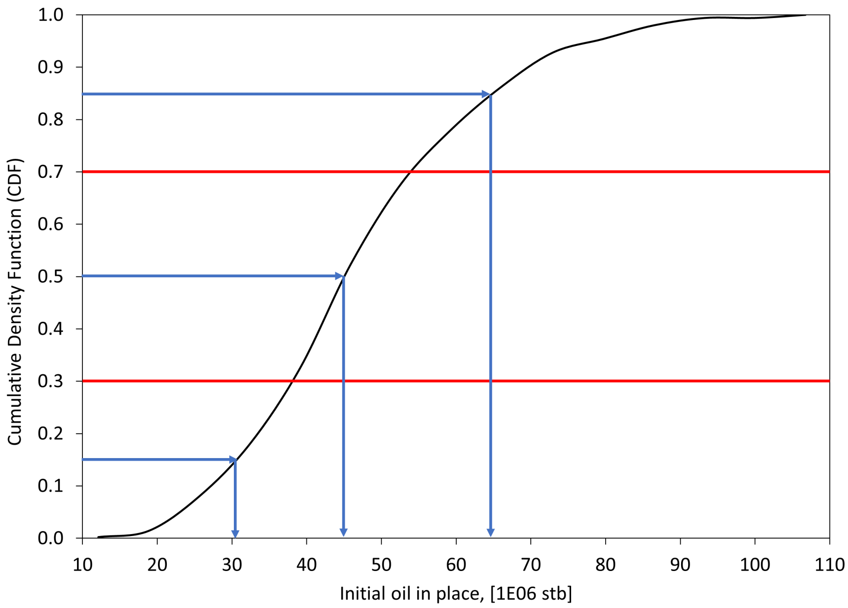


Figure 4.14: Illustration of bracket mean sampling method

Assuming that the number of bins (n) is the same for all parameters, this means that the stochastic optimization will evaluate n^x cases with x implying the number of uncertainty parameters involved in the formulation. For the purpose of simplifying the formulation as well as keeping the accuracy of the results, three is selected as the number of bins generated from each distribution. As an illustration, the formulation used in the study has five uncertainties, with $F_{w,2}$ being left out due to its uncorrelated behavior to the NPV, and if three samples are generated from each distribution, $3^5 = 243$ samples will be explored during the simulation. It is more than half of the number of samples used in the first uncertainty analysis study, and it has been proven that 200 is enough to yield a result with identical accuracy. The discrete version of the distributions of the uncertain parameters is shown in Figure 4.1, using three bins (probabilities) which are 0.3, 0.4, 0.3.

Regarding uncertainty in the oil price, It was decided to use a constant oil price in time instead of using trajectories. Moreover, it was assumed that the oil price is uniformly distributed between 30 and 90 USD/Sm^3. This distribution was discretized in 3 bins, similar to the other uncertain parameters. The discretized version of all uncertain parameters included in the SP study is shown in Table 4.22.

Table 4.22: Uncertainty samples generated for random variable in SP formulation

Parameters	Value		
	0.3	0.4	0.3
N_1 (in Million Sm^3)	31	49	68
N_2 (in Million Sm^3)	33	40	49
$F_{w,1}$	0.86	1.0	1.3
F_c	0.81	1.0	1.18
P_o (in USD/Sm^3)	40	60	81

4.5.3 Methodology

The model is formulated based on the two-stages stochastic programming form. In this approach, some decision variables are used in the first stage, while in the second stage, the remaining decision variable and uncertainties are considered. In this specific formulation, the design (i.e., production capacities and drilling schedule) are defined as the first stage variable, while the production profile and the others are defined as the second stage variable. Consequently, all equations that are not related to the design are defined as the second stage equations.

An important thing to point out is that the stochastic optimization proposed in this section is well aligned with the approach taken in the first uncertainty analysis using the LHS method, in which the design is decided after the first simulation. Then the simulations are re-run to determine the production profile and NPV distributions. The only difference is that instead of running the optimization multiple times, the stochastic programming approach could produce the optimal design with one single run. However, the output of the SP method is the optimal design parameters only, not the expected NPV of the project nor the statistic distribution of production profile. To find these and compare against the results of the first study, it is necessary, as it was done earlier, to run several optimizations (optimizing production profile only) using as input samples of the uncertain input (e.g., generated with LHS). Thus, the NPV distribution yielded from the simulation is now equivalent and could be compared to the NPV distributions of the first study.

The following formulation of the stochastic programming model is used to determine the most optimum design of the field:

First stage

All variables defined as first stage variables are fixed for all 243 scenarios as they are defined as the optimal design of the field.

- **Variables**
  - $q_{o,f,max}$: Oil production rate capacity of the field throughout the field life (in Sm^3/d)
  - $q_{w,f,max}$: Water production rate capacity of the field throughout the field life (in Sm^3/d)
  - $q_{g,f,max}$: Gas production rate capacity of the field throughout the field life (in Sm^3/d)
  - $N_{w,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Fixed number of producer wells drilled in reservoir-r at time-t
- **Constraints**
  Equations that are related to the drilling schedule are assigned in the first stage. On the

other hand, the production capacities constraints need to be defined in the second stage as it controls the production profile, which is categorized as the second stage variable.

$$N_{w,r,0} = N_{w,r,pd} \quad \forall r \in Res \tag{4.14}$$

$$N_{w,r,t+1} - N_{w,r,t} \geq 0, \quad \forall r \in Res \quad \forall t \in T \setminus \{21\} \tag{4.15}$$

$$N_{w,r,t} \leq N_{w,r,max}, \quad \forall r \in Res \quad \forall t \in T \tag{4.16}$$

$$\sum_{r \in Res} (N_{w,r,t+1} - N_{w,r,t}) \leq N_{w,f,year}, \quad \forall t \in T \setminus \{21\} \tag{4.17}$$

Second stage

Different from the first stage, all variables defined as second stage variables are varied for all 243 cases depending on the uncertainty parameters and the fixed design parameters.

- **Variables**
  - $q_{o,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Oil production rate of reservoir-r at time-t (in Sm^3/d)
  - $q_{pp,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Oil production potential of reservoir-r at time-t (in Sm^3/d)
  - $q_{g,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Gas production rate of reservoir-r at time-t (in 10^6 Sm^3/d)
  - $q_{w,r,t} \quad \forall r \in Res \quad \forall t \in T$
    Water production rate of reservoir-r at time-t (in Sm^3/d)
  - $PV_{dc,t} \quad \forall t \in T$
    Present value of DrillEx and CapEx combination at time-t (in Million NOK).
  - PV_{pre}: CapEx made prior the production start for the production facility (in Million NOK)
  - $PV_{o,t} \quad \forall t \in T$
    Present value of operational expenditure at time-t (in Million NOK)
  - $PV_{r,t} \quad \forall t \in T$
    Present value of revenue at time-t (in Million NOK)
  - NPV: Net present value (in Million NOK)
- **Objective Function**
  The objective of the model is also formulated in the second stage since it depends on variables that are defined in the same stage. It is expressed as:

$$\text{Maximize} \quad NPV \tag{4.18}$$

- **Constraints**
  All equality equations corresponding to the field (e.g., $q_{o,f,t}$, $N_{p,f,t}$, etc.) are removed to reduce the complexity of the formulation. Consequently, adjustments in some constraints are required to calculate the field corresponding variable within the equation themselves. However, several constraints are still identical to the base model formulation.

Equality Constraints

– At the initial condition, the cumulative oil production for both reservoirs equal to zero.

$$N_{p,r,0} = 0 \quad \forall r \in Res \tag{4.19}$$

– Cumulative production for each reservoir is calculated using the backward trapezoidal integration method. This constraint will be iterated for each reservoir and time step.

$$N_{p,r,t+1} = N_{p,r,t} + \frac{(q_{o,r,t+1} - q_{o,r,t}) \cdot days}{2}, \quad \forall r \in Res \quad \forall t \in T \setminus \{21\} \tag{4.20}$$

– The well performance factor in reservoir 2 is removed from the formulation. However, instead of removing it completely from the equation, the factor, $F_{w,2}$ is equal to 1.

$$\begin{aligned} q_{pp,r,t} = F_{w,r} \cdot N_{w,r,t} \cdot q_{pp,well,r} \cdot f_p^{(N_{w,r,t}-1)} \cdot \Bigg(& dimpot_{1,r} \left(\frac{N_{p,r,t}}{N_r}\right)^5 \\ + dimpot_{2,r} \left(\frac{N_{p,r,t}}{N_r}\right)^4 + dimpot_{3,r} & \left(\frac{N_{p,r,t}}{N_r}\right)^3 + dimpot_{4,r} \left(\frac{N_{p,r,t}}{N_r}\right)^2 \\ + dimpot_{5,r} \left(\frac{N_{p,r,t}}{N_r}\right) + 1\Bigg), & \quad \forall r \in Res \quad \forall t \in T \end{aligned} \tag{4.21}$$

– Gas and water production rates are a function of the oil production rate and each corresponding parameter (GOR for gas and WC for water). The gas and water productivity factor are also not included in the formulation. Hence, the constraints are similar to the base model.

$$\begin{aligned} GOR = gor_{1,r} \left(\frac{N_{p,r,t}}{N_r}\right)^5 + gor_{2,r} \left(\frac{N_{p,r,t}}{N_r}\right)^4 + gor_{3,r} \left(\frac{N_{p,r,t}}{N_r}\right)^3 \\ + gor_{4,r} \left(\frac{N_{p,r,t}}{N_r}\right)^2 + gor_{5,r} \left(\frac{N_{p,r,t}}{N_r}\right) + 1 \end{aligned}$$

$$q_{g,r,t} = q_{o,r,t} \cdot GOR, \quad \forall r \in Res \quad \forall t \in T \tag{4.22}$$

$$\begin{aligned} WC = wc_{1,r} \left(\frac{N_{p,r,t}}{N_r}\right)^5 + wc_{2,r} \left(\frac{N_{p,r,t}}{N_r}\right)^4 + wc_{3,r} \left(\frac{N_{p,r,t}}{N_r}\right)^3 \\ + wc_{4,r} \left(\frac{N_{p,r,t}}{N_r}\right)^2 + wc_{5,r} \left(\frac{N_{p,r,t}}{N_r}\right) + 1 \end{aligned}$$

$$q_{w,r,t} = q_{o,r,t} \cdot \frac{WC}{1 - WC}, \quad \forall r \in Res \quad \forall t \in T \tag{4.23}$$

Inequality Constraints

– The production rate of each reservoir at any point in time should never exceed its production potential.

$$q_{o,r,t} \leq q_{pp,r,t}, \quad \forall r \in Res \quad \forall t \in T \tag{4.24}$$

– As mentioned before, the field production rates are calculated within the equations. All three equations below use the capacities defined in the first stage in which the values are fixed for all scenarios to control each fluid production rate.

$$\begin{aligned} \sum_{r \in Res} q_{o,r,t} \leq q_{o,f,max}, \quad \forall t \in T \\ \sum_{r \in Res} q_{g,r,t} \leq q_{g,f,max}, \quad \forall t \in T \\ \sum_{r \in Res} q_{w,r,t} \leq q_{w,f,max}, \quad \forall t \in T \end{aligned} \tag{4.25}$$

Economic Constraints

– The drilling expenditure (DrillEx) and capital expenditure (CapEx) variables are combined as they both are calculated using a similar equation with a different coefficient. Combining them reduces the number of variables in the formulation and its complexity at the same time.

$$PV_{dc,1} = F_c \cdot \sum_{r \in Res} (N_{w,r,1} \cdot (1 + N_{w,injprod,r})) \cdot (D_1 + C_1)$$

$$PV_{dc,t+1} = F_c \cdot \frac{\sum_{r \in Res} ((N_{w,r,t+1} - N_{w,r,t}) \cdot (1 + N_{w,injprod,r})) \cdot (D_1 + C_1)}{(1+i)^t} \quad \forall t \in T \setminus \{21\} \tag{4.26}$$

– As it is calculated using the production capacities which are fixed for all scenarios, pre-CapEx in this formulation is only a function of the cost multiplication factor.

$$PV_{pre} = F_c \cdot (q_{o,f,max} \cdot C_2 + q_{w,f,max} \cdot C_3 + q_{g,f,max} \cdot C_4 + C_5) \tag{4.27}$$

– Operational expenditure in this formulation is a function of field fluid production rate and the cost multiplication factor as the drilling schedule is a fixed parameter defined in the first stage.

$$\begin{aligned} PV_{o,t} = F_c \cdot \frac{1}{(1+i)^t}(O_1 + O_2 \cdot \sum_{r \in Res} N_{w,r,t} + O_3 \cdot \sum_{r \in Res} q_{o,r,t} \\ + O_4 \cdot \sum_{r \in Res} q_{w,r,t} + O_5 \cdot \sum_{r \in Res} q_{g,r,t}) \quad \forall t \in T \end{aligned} \tag{4.28}$$

– Revenue for each year is calculated by multiplying the oil production in one year with the oil price, which in this case is one of the random variables.

$$PV_{r,t} = P_o \cdot xr \cdot vc \cdot \frac{\sum_{r \in Res} (N_{p,r,t} - N_{p,r,t-1})}{(1+i)^t} \quad \forall t \in T \tag{4.29}$$

– Instead of calculating the discounted cash flow and store the value in another variable, this formulation calculates the NPV directly to eliminate one more variable and slightly simplify the complexity of the model.

$$NPV = -PV_{pre} + \sum_{t \in T} (PV_{r,t+1} - PV_{o,t+1} - PV_{dc,t}) \tag{4.30}$$

4.5.4 Implementation in GAMS

Implementing the stochastic programming model in GAMS requires only one .gms extension file, similar to the base formulation, as the LHS samples are no longer required. It will utilize the Extended Mathematical Programming (EMP) feature in GAMS to formulate the SP model. In the EMP section, the random variable (i.e., uncertainty parameter) needs to be defined using the 'randvar' keyword along with the distribution type and the value. In this case, a discrete type of distribution is selected as the samples are generated using the bracket mean sampling method, which is performed without GAMS.

Another thing to be defined in the EMP section is the decision variables and equations defined for each stage. For a two-stage stochastic model, the only stage required to be defined is the second stage as the remaining decision variables and equations are assumed to be defined for the first stage. As the uncertainty parameters are resolved in the second stage, they are also defined in the second stage along with the decision variables. The GUSS feature is also applied to update the random variables for each scenario and store the desired results. The GAMS implementation of the stochastic programming model is provided in Appendix C.

4.5.5 Results

Table 4.23: Optimal production capacities comparison between LHS and SP method

Method	Production Capacities [Sm^3/d]		
	Oil	Water	Gas
LHS - Design D	8915.9	4752.5	5494.7E3
SP - 243 cases	9799.6	5688.3	5693.6E3

Table 4.24: Drilling schedule of the alternative designs

Design	Reservoir	Number of producer wells ($N_{w,t}$)					
		1	2	3	4	...	21
LHS - Design D	Reservoir 1	3	3	5	6	...	6
	Reservoir 2	0	0	1	3	...	3
SP - 243 cases	Reservoir 1	3	4	5	6	...	5
	Reservoir 2	0	1	3	3	...	3

The optimal design produced by the stochastic programming formulation, as well as the comparison with the optimal design from the LHS study, are shown in Table 4.23 and Table 4.24. It can be seen that all production capacities of the design proposed by the SP model are greater than the one from the first study. It also can be observed that the design proposed by the SP model opts to drill more producer wells in the early years than the LHS study. Unlike the LHS study in which the design is selected based on the distribution curves, the SP model has more flexibility in choosing the design.

The performance of the field design obtained with SP under uncertain conditions was evaluated using LHS. Several optimizations were triggered considering the production profile as the decision variable while the design (i.e., drilling schedule and oil, water, and gas production capacity) is kept fixed. Table 4.25 shows P90, P50, and P10 values of base simulation, design D (optimal design without considering optional well), flexible A (optimal design considering optional well), and the SP model with 243 cases (3 samples from each uncertain parameters) along with their corresponding running time.

Table 4.25: Results and runtime comparison between the LHS method and SP model

Method	NPV (Million NOK) / Perc. Difference (%)						Average	Run time
	P90		P50		P10		(%)	(s)
Base	5521.6	-	19225.3	-	37810.3	-	-	-
LHS - Design D	3507.5	36.5	18129.1	5.7	36583.4	3.3	15.1	3030.9
LHS - Flexible A	4273.6	22.6	18354.5	4.5	36705.7	2.9	10.0	5677.9
SP - 243 cases	3819.7	30.8	18253.2	5.1	36803.8	2.7	12.8	68147.5

4.5.6 Comparison Between Field Design Using the LHS Method and Stochastic Optimization

Based on the results presented in Table 4.25, the optimal design proposed by the SP model yields lower average difference in all NPV parameters. It seems to indicate the following three things:

1. The bracket mean sampling method accurately generated the samples from the distribution curves in a case where the probabilities are similar (i.e., 0.3 - 0.4 - 0.3).
2. The stochastic programming model successfully produces a better optimal design than the LHS study. The flexibility in selecting the design parameters plays an important role in this matter as the chance of finding a more optimal design is drastically increased.
3. Using a constant oil price in time and assuming that the oil price is uniformly distributed between 30-90 seems to capture appropriately the variability of oil price in the case of trajectories varying in time.

However, the difference between the two methods is not that significant, especially on the P50 and P10 figures. Thus, it is safe to say that the first approach in which the design is selected based on evaluating the expected value and the other five-number summary parameters (min, max, Q1, and Q3) of the results (drilling schedule and production capacity) are accurate enough.

The total running time of the LHS model is the summation of the running time of the base design (i.e., 2710.9 seconds) and four proposed designs, assuming that the proposed design (e.g., expected value design, design B, C, and D) has an average run time of 80 seconds. The figure is not accurate enough as there is some time that is not accounted for, such as the time needed to compute the distributions of the results, comparing the results of all proposed designs, etc. Nevertheless, it will not surpass the running time of the SP model, in which almost 19 hours is required to produce the optimal design. The significance in time difference is expected since the SP model explored every feasible set of design instead of limited by the distribution curves as the LHS model. The improvement in the NPV figure does seem like a modest payback considering the time required to yield the optimal design. Hence, it could be stated that a methodology to determine the best field design using LHS and simulation-based optimization is appropriate, and it has a lower running time than the SP model.

Table 4.25 also shows that the flexible A design (the most optimal design from the first study when optional producer wells are considered) yields a better P90, P50, and P10 percentage difference than the SP model. It is somewhat anticipated as the SP model has a fixed drilling schedule for all 243 cases. In order to include the flexibility in the drilling schedule, the model has to be modified, and similar to the study using LHS, the complexity of the model and running time are expected to be significantly increased. However, time constraints and limitations in the tools utilized in the study hindered the possibility to evaluate the flexibility in the drilling schedule using the SP model.

Even though the SP model running time is significantly longer than the LHS model, it only needs one simulation to produce the optimal design. As mentioned before, the SP model removes some of the processes required to find the optimal design, such as selecting several designs, re-running the simulation, and comparing the results of each design. As a consequence, it limits the possibility of human error during each process as all calculations are performed by the tool and solving engine selected to solve the model. Hence, the LHS model is more preferred in regards to the procedure since the accuracy of the results is guaranteed to be better than or at least similar to the LHS model.

One important thing to note is that the SP model requires a measured sampling process to generate accurate samples that well-represent each uncertainty parameter distribution. The sampling must be done carefully since it is only limited to 3 samples per parameter instead of 400 samples as the LHS model. The 0.3-0.4-0.3 set of probability selected for this study is proven to be a good approach. However, the variability in the probabilities has not been studied in this thesis. It could be something interesting to analyze in the future to understand the effect of probabilities of the uncertainties on the accuracy of the SP model results.

Chapter 5

Model Improvement

As the base model is still formulated with some simplifications, it will be further improved to make it more realistic. The improvement is based on the works by Alkindira (2020), which involves several modifications, such as including the distinct well performance in the production performance model and pipeline length in the cost model. Alkindira (2020) approximated the non-linear functions in the formulations using piece-wise linear models. The main objective of this section is to develop a non-linear model comparable to the PWL (i.e., Linear) model presented by Alkindira (2020) and compare their computational performance and results.

5.1 Improved Model Formulation

The inclusion of distinct well performance and pipeline length requires the introduction of binary variables to represent the status of well (i.e., drilled or not drilled) in each reservoir and the selected scenario. The status is used to compute the production potential and the required pipeline length. A more detailed formulation of the improved model is specified as follows:

5.1.1 Sets

Besides the same two sets from the base model, there are several new sets defined in the new formulation to accommodate the improvement:

- $sr1 = \{s1, \ldots, s42\}$
  $sr1$ is a set of scenarios for reservoir 1. Reservoir 1 has six wells, so the total number of scenarios is $2^6 = 64$ (status indicator to the power of the number of wells). However, for this study, only scenarios in which at least three wells are producing are considered.
- $sr2 = \{s43, \ldots, s50\}$
  $sr2$ is a set of scenario for reservoir 2. Reservoir 2 has three wells, so the total number of scenarios is $2^3 = 8$.
- $wr1 = \{w1, \ldots, w6\}$
  $wr1$ is a set of wells in reservoir 1, which in this study is limited to 6 wells.
- $wr2 = \{w7, \ldots, w9\}$
  $wr1$ is a set of wells in reservoir 2, which in this study is limited to 3 wells.

5.1.2 Parameters

Parameters defined for the improved model are similar to the one defined in subsection 3.1.2. However, two parameters related to the production potential, such as the "pancake" factor (f_p) and maximum oil production potential for each well in reservoir-r ($q_{pp,well,r}$), are removed. The ratio between the number of production and injection wells ($N_{w,injprod,r}$) was set to zero as the reference model did not include this term in the formulation. In addition to the remaining parameters, two tables are declared, by utilizing GAMS formatting, to define the distinct well performance and pipeline length data, one for each reservoir. The tables and one new parameter are defined as follows:

- C_{pl}: Coefficient of capital expenditure (CapEx) equation to calculate the cost of the pipeline.
- $z_{1,s,w}, \quad \forall s \in sr1 \quad \forall w \in wr1$
  Represents the data of each producer wells' status in reservoir 1 for all scenarios. For instance, $z_{1,s2,w2} = 1$ is shown in Table 5.1, indicated by a black circle.
- $z_{1,s,qppo}, \quad \forall s \in sr1$
  Represents the data of each scenarios' maximum production potential in reservoir 1. For instance, $z_{1,s20,qppo} = 14333.6$ is shown in Table 5.1, indicated by a blue circle.
- $z_{1,s,pipe}, \quad \forall s \in sr1$
  Represents the data of each scenarios' required pipeline length in reservoir 1. For instance, $z_{1,s1,pipe} = 15.0$ is shown in Table 5.1, indicated by a red circle.
- $z_{1,s,*}, \quad \forall s \in sr1$
  Represents the data for all scenarios of producer wells in reservoir 1. It is a combination of the previous three parameters. An illustration of the data is presented in Table 5.1.
  comments: The * sign represents three different things, such as the status of the producer wells, the production potential, and the pipeline length of each scenario.
- $z_{2,s,w}, \quad \forall s \in sr2 \quad \forall w \in wr2$
  Represents the data of each producer wells' status in reservoir 2 for all scenarios. For instance, $z_{2,s43,w9} = 0$ is shown in Table 5.2, indicated by a black circle.
- $z_{2,s,qppo}, \quad \forall s \in sr2$
  Represents the data of each scenarios' maximum production potential in reservoir 2. For instance, $z_{2,s47,qppo} = 6549.2$ is shown in Table 5.2, indicated by a blue circle.
- $z_{2,s,pipe}, \quad \forall s \in sr2$
  Represents the data of each scenarios' required pipeline length in reservoir 2. For instance, $z_{2,s50,pipe} = 17.0$ is shown in Table 5.2, indicated by a red circle.
- $z_{2,s,*}, \quad \forall s \in sr2$
  Represents the data for all scenarios of producer wells in reservoir 2. It is a combination of the previous three parameters. An illustration of the data is presented in Table 5.2.
  comments: The * sign represents three different things, such as the status of the producer wells, the production potential, and the pipeline length of each scenario.

5.1.3 Variables

The base formulation utilizes two types of variables: continuous and discrete variables. The former represents measurable amounts in the model, such as the production profile, while the latter

Table 5.1: Scenario data of producer wells in reservoir 1

Scenario (s)	Well status						qppo	pipe
	w1	w2	w3	w4	w5	w6	[Sm^3/d]	[m]
s1	1	1	1	0	0	0	12386.6	15.0
s2	1	1	0	1	0	0	12581.3	8.5
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
s20	0	0	0	1	1	1	14333.6	15.8
s21	0	0	1	1	1	1	18750.0	22.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
s35	1	1	1	1	0	0	16714.5	15.0
s36	1	1	1	1	1	0	21479.9	17.7
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
s41	0	1	1	1	1	1	22713.5	24.6
s42	1	1	1	1	1	1	24010.6	27.7

Table 5.2: Scenario data of producer wells in reservoir 2

Scenario (s)	Well status			qppo	pipe
	w7	w8	w9	[Sm^3/d]	[m]
s43	0	0	0	0	0
s44	1	0	0	4273.9	17.0
⋮	⋮	⋮	⋮	⋮	⋮
s47	1	1	0	6549.2	17.0
⋮	⋮	⋮	⋮	⋮	⋮
s50	1	1	1	8114.0	17.0

represents counts in the model, such as the drilling schedule. Those two variable types are not enough to formulate the distinct well performance as the status of well is neither a count nor a measurable amount. Hence, as previously mentioned, several binary variables are introduced in the improved formulation to represent both the status of each producer wells and the selected scenario. The binary variables and other new variables are defined as follows:

- Binary Variables
  - $zw_{1,w,t}$, $\quad \forall w \in wr1 \quad \forall t \in T$
    Represents the status of producer well-w in reservoir 1 at time-t
    comments: $zw_{1,w,t}$ is equal to 1 when the producer well in reservoir 1 has been drilled. On the other hand, it is equal to 0 when the producer well has not been drilled.
  - $zw_{2,w,t}$, $\quad \forall w \in wr2 \quad \forall t \in T$

Represents the status of producer well-w in reservoir 2 at time-t
comments: $zw_{2,w,t}$ is equal to 1 when the producer well in reservoir 2 has been drilled. On the other hand, it is equal to 0 when the producer well has not been drilled.

  - $x_{s,t}, \quad \forall s \in sr1 \quad \forall t \in T$
    Represents the status of scenario-s in reservoir 1 at time-t
    comments: $x_{s,i}$ is equal to 1 when scenario-s is activated at time-t. As an illustration and referring to Table 5.1, $x_{s1,1} = 1$ means that $zw_{1,w1,1}$, $zw_{1,w2,1}$, and $zw_{1,w3,1}$ are all equal to 1 and indicates that those wells are drilled in the first year. On the other hand, $zw_{1,w4,1}$, $zw_{1,w5,1}$, and $zw_{1,w6,1}$ are all equal to 0 showing that those wells are not activated yet.
  - $y_{s,t}, \quad \forall s \in sr2 \quad \forall t \in T$
    Represents the status of scenario-s in reservoir 2 at time-t
    comments: $y_{s,i}$ is equal to 1 when scenario-s is activated at time-t. As an illustration and referring to Table 5.2, $y_{s50,1} = 1$ means that $zw_{2,w7,1}$ $zw_{1,w8,1}$, and $zw_{1,w9,1}$ are all equal to 1 and indicates that those wells are drilled in the first year.

- Dependent Variables
  - $qppo_{r,t}, \quad \forall r \in Res \quad \forall t \in T$
    Represents the maximum oil production potential of reservoir-r at time-t (in Sm^3/d)
  - $pipe_{r,t}, \quad \forall r \in Res \quad \forall t \in T$
    Represents the pipeline length installed for wells in reservoir-r at time-t.
  - $pipe_{max}$
    Represents the total pipeline length that must be installed throughout the field life.

5.1.4 Objective Function

The objective function defined in the improved formulation is the same as the objective function defined in subsection 3.1.4.

5.1.5 Constraints

The improved formulation uses similar constraints presented in subsection 3.1.5 except for Equation 3.4, Equation 3.5, Equation 3.10, Equation 3.11, and Equation 3.17, in which they are either adjusted to accommodate the improvement or completely removed from the formulation. In addition to that, several new constraints are introduced, mainly related to the scenarios of producer wells. The adjusted and new constraints are defined as follows:

New Constraints

- The scenario is selected using the lookup table method, an array of data that maps input values to output values. In this formulation, the constraint is used to help other constraints selecting a specific scenario at each timestep that optimizes the objective function the most. The lookup table constraint utilized the "Big M" method, which is a functionality provided by GAMS. It is defined as follows:

$$\begin{aligned} \sum_{w \in wr1} a_{w,t} \geq m \cdot x_{s,t}, \quad \forall s \in sr1 \quad \forall t \in T \\ \sum_{w \in wr2} b_{w,t} \geq n \cdot y_{s,t}, \quad \forall s \in sr2 \quad \forall t \in T \end{aligned} \tag{5.1}$$

where:

$$a_{w,t} = \begin{cases} zw_{1,w,t}, & \text{if } z_{1,s,w} = 1 \\ 1 - zw_{1,w,t}, & \text{if } z_{1,s,w} = 0 \end{cases}$$

$$b_{w,t} = \begin{cases} zw_{2,w,t}, & \text{if } z_{2,s,w} = 1 \\ 1 - zw_{2,w,t}, & \text{if } z_{2,s,w} = 0 \end{cases}$$

and m and n are the numbers of producer wells in reservoir 1 and 2, respectively. $z_{1,s,w}$ and $z_{2,s,w}$ are taken from table Table 5.1 and Table 5.2, respectively.

An illustration is provided below for both active and inactive scenarios to provide a better understanding of the lookup table constraint:

1. Active scenario
   In a case where scenario 1 ($s1$) is observed and active, the constraint is defined as:

$$zw_{1,w1,t} + zw_{1,w2,t} + zw_{1,w3,t} + (1 - zw_{1,w4,t}) + (1 - zw_{1,w5,t}) + (1 - zw_{1,w6,t}) \geq 6 \cdot 1$$
$$zw_{1,w1,t} + zw_{1,w2,t} + zw_{1,w3,t} - zw_{1,w4,t} - zw_{1,w5,t} - zw_{1,w6,t} \geq 6 \cdot 1 - 3$$

   It has been decided before that if a well is active, the value of zw is equal to 1, and on the other hand, an inactive well is indicated by $zw = 0$. In scenario 1, the first 3 wells are active while the rest are inactive. It makes the left-hand side to be equal to 3, and at the same time, the right-hand side also equal to 3. Thus, an active scenario is indicated by a condition in which the left-hand side of the equation is equal to the value on the right-hand side.

2. Inactive scenario
   On the other hand, in a condition in which scenario 1 ($s1$) is observed, but scenario 2 ($s2$) is the active one, the constraint is then defined as:

$$zw_{1,w1,t} + zw_{1,w2,t} + zw_{1,w3,t} + (1 - zw_{1,w4,t}) + (1 - zw_{1,w5,t}) + (1 - zw_{1,w6,t}) \geq 6 \cdot 0$$
$$zw_{1,w1,t} + zw_{1,w2,t} + zw_{1,w3,t} - zw_{1,w4,t} - zw_{1,w5,t} - zw_{1,w6,t} \geq 6 \cdot 0 - 3$$

   Well 1, 2, and 4 are the active wells, and the sum of the left-hand side value is now equal to 1. On the other hand, the right-hand side is equal to -3. Hence, an inactive scenario is indicated by a condition in which the left-hand side of the equation is greater than the value on the right-hand side.

- There is only one scenario selected for each timestep, and it is controlled by the following constraints:

$$\begin{aligned} \sum_{s \in sr1} x_{s,t} = 1, \quad \forall t \in T \\ \sum_{s \in sr2} y_{s,t} = 1, \quad \forall t \in T \end{aligned} \tag{5.2}$$

- The maximum production potential of wells in reservoir-r at a certain timestep-t is a function of the selected scenario yielded from the lookup table constraint. The constraint is defined as follows:

$$\begin{aligned} qppo_{r1,t} = \sum_{s \in sr1} \left(z_{1,qppo} \cdot x_{s,t} \right), \quad \forall t \in T \\ qppo_{r2,t} = \sum_{s \in sr2} \left(z_{2,qppo} \cdot y_{s,t} \right), \quad \forall t \in T \end{aligned} \tag{5.3}$$

- The following two constraints are related to the producer wells in each reservoir. First, a well that has been drilled must stay active throughout the field life. Thus, the well status variable is controlled by the following constraints:

$$\begin{aligned} zw_{1,w,t} \leq zw_{1,w,t+1}, \quad \forall w \in wr1 \quad \forall t \in T \setminus \{21\} \\ zw_{2,w,t} \leq zw_{2,w,t+1}, \quad \forall w \in wr2 \quad \forall t \in T \setminus \{21\} \end{aligned} \tag{5.4}$$

The second one stated that the number of producer wells is now a dependent variable as it is a function of well statuses which is defined as follows:

$$
\begin{aligned}
N_{w,r1,t} &= \sum_{w \in wr1} zw_{1,w,t}, \quad \forall t \in T \\
N_{w,r2,t} &= \sum_{w \in wr2} zw_{2,w,t}, \quad \forall t \in T
\end{aligned}
\tag{5.5}
$$

- The following constraints are related to the length of pipeline installed for wells in each reservoir. First, pipeline length for each reservoir is calculated using a similar structure to Equation 5.3:

$$
\begin{aligned}
pipe_{r1,t} &= \sum_{s \in sr1} \left(z_{1,pipe} \cdot x_{s,t}\right), \quad \forall t \in T \\
pipe_{r2,t} &= \sum_{s \in sr2} \left(z_{2,pipe} \cdot y_{s,t}\right), \quad \forall t \in T
\end{aligned}
\tag{5.6}
$$

Similar to the well in which it cannot be undrilled, the pipeline length cannot be reduced:

$$
pipe_{r,t} \leq pipe_{r,t+1}, \quad \forall r \in Res \quad \forall t \in T \setminus \{21\} \tag{5.7}
$$

The last one is used to determine the total pipeline length required throughout the field. This figure will be used to calculate the capital expenditure (CapEx) of subsea equipment.

$$
\sum_{r \in Res} pipe_{r,t} \leq pipe_{max}, \quad \forall t \in T \tag{5.8}
$$

Adjusted Constraints

- To make the model comparable with the case from Alkindira (2020), the method to calculate the cumulative oil production was modified. Instead of using the backward trapezoidal integration approach similar to Equation 3.4, the cumulative oil production of the improved model is calculated using the forward rectangular approach. The equation assumes that the oil production rate is identical for the whole year, thus reflecting a less accurate approach than the previous one.

$$
N_{p,r,t+1} = N_{p,r,t} + q_{o,r,t} \cdot days, \quad \forall r \in Res \quad \forall t \in T \setminus \{21\} \tag{5.9}
$$

- Since the identical well assumption is removed and distinct well performance is applied, Equation 3.5 is adjusted by removing the well-related parameters, $N_{w,r,t}$, $q_{pp,well,r}$, and f_p, and changing it to be maximum oil production potential of the reservoir ($q_{ppo,r,t}$) instead. However, the dimensionless production potential curve is assumed to be identical for each scenario, and thus, the oil production potential is calculated as follows:

$$
\begin{aligned}
q_{pp,r,t} = q_{ppo,r,t} \cdot \Bigg(& dimpot_{1,r} \left(\frac{N_{p,r,t}}{N_r}\right)^5 \\
& + dimpot_{2,r} \left(\frac{N_{p,r,t}}{N_r}\right)^4 + dimpot_{3,r} \left(\frac{N_{p,r,t}}{N_r}\right)^3 + dimpot_{4,r} \left(\frac{N_{p,r,t}}{N_r}\right)^2 \\
& + dimpot_{5,r} \left(\frac{N_{p,r,t}}{N_r}\right) + 1 \Bigg), \quad \forall r \in Res \quad \forall t \in T
\end{aligned}
\tag{5.10}
$$

- All required pipeline is assumed to be installed in the first year, even though the producer wells around it have not been drilled. To accommodate the inclusion of the pipeline length in the cost model, The first year of Equation 3.17 must be slightly modified, while the rest of the timesteps remain similar.

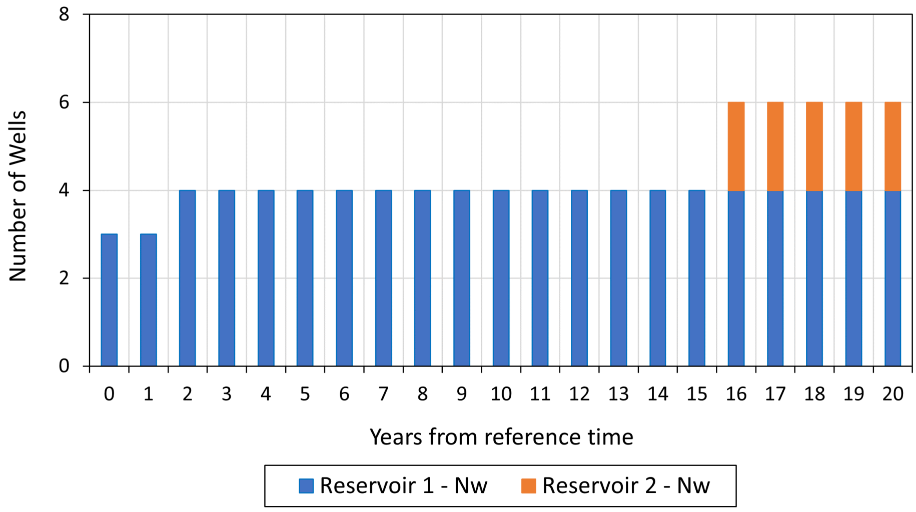
$$
\begin{aligned}
PV_{c,1} &= \sum_{r \in Res} \left(N_{w,r,1} \cdot (1 + N_{w,injprod,r})\right) \cdot C_1 + pipe_{max} \cdot C_{pl} \\
PV_{c,t+1} &= \frac{\sum_{r \in Res} \left((N_{w,r,t+1} - N_{w,r,t}) \cdot (1 + N_{w,injprod,r})\right) \cdot C_1}{(1+i)^t} \qquad \forall t \in T \setminus \{21\}
\end{aligned}
\tag{5.11}
$$

5.2 Implementation in GAMS

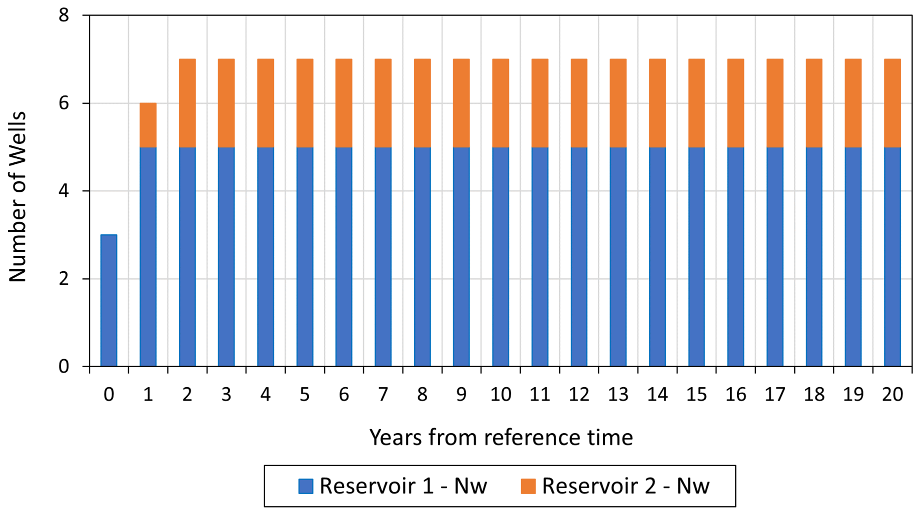
The exact implementation as presented in the second paragraph of section 3.2 is applied to perform this study. The GAMS implementation of the improved model is provided in Appendix D.

5.3 Solver Selection

There are several solvers available to perform the optimization, for example, GAMS/LINDO and DICOPT. DICOPT is also capable of solving a mixed-integer non-linear programming (MINLP) model. The computational performance of the GAMS/LINDO solver was not satisfactory. The inclusion of lookup table constraints made it difficult for GAMS/LINDO solver to find the most optimum solution for the formulation. In this section, both solver results will be compared to choose the most optimal solver for the improved formulation.



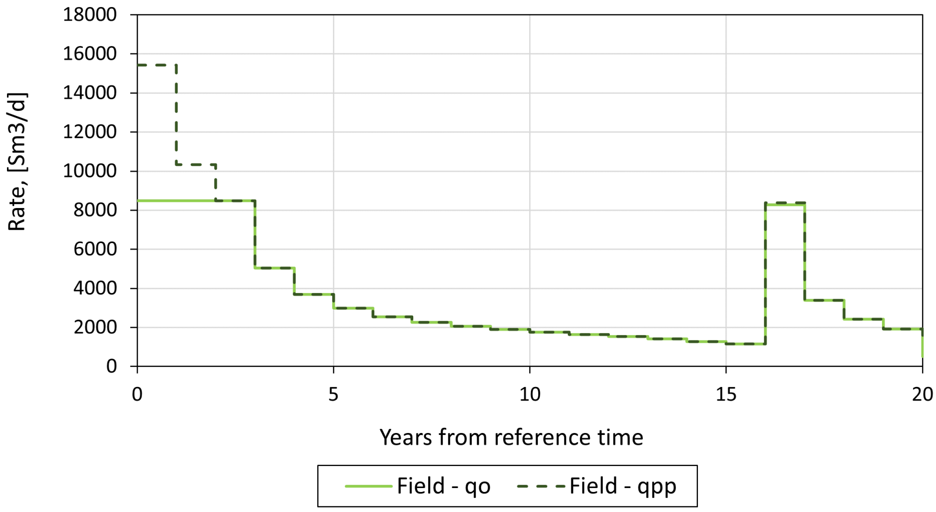
(a) GAMS/LINDO solver



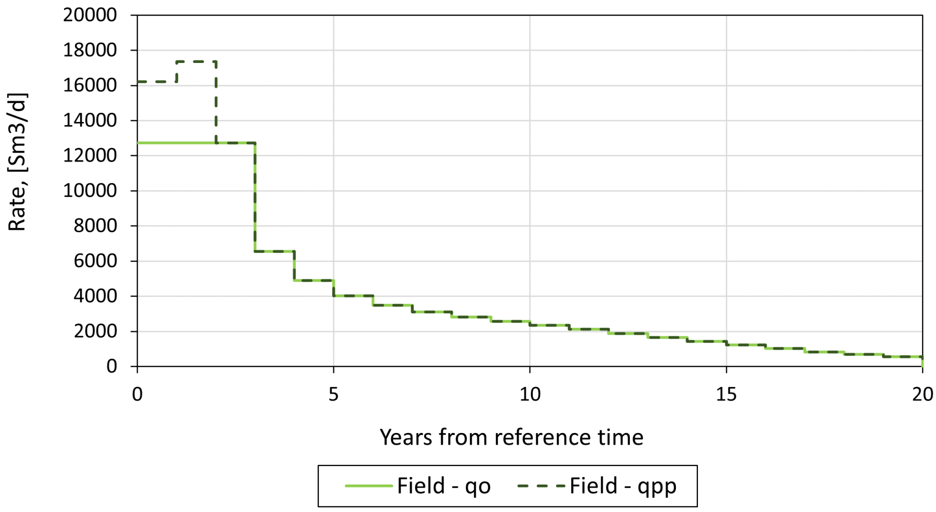
(b) DICOPT solver

Figure 5.1: Optimal drilling schedule of the improved model using two different solving engines

Figure 5.1 shows that GAMS/LINDO solver opt not to drill producer wells in reservoir 2 until the 16th year of the field life. Moreover, it chooses to only drill four producer wells from a maximum of 6 in reservoir 1. On the other hand, the drilling schedule from the DICOPT solver seems to follow a more realistic pattern (i.e., referring to the result from the base model in subsection 3.4.2) in which all producer wells are drilled in the early years. It is done as an effort to maximize the production in the early years where the discount factor effect is still insignificant.



(a) GAMS/LINDO solver



(b) DICOPT solver

Figure 5.2: Optimal field production profile of the improved model using two different solving engines

As a consequence of the delay in the drilling schedule from the GAMS/LINDO solver, its optimal production profile has two "peaks", the plateau rate from reservoir 1 and when wells are drilled in reservoir 2 at year 16, as shown in Figure 5.2. Subsequently, the oil production capacity (i.e., maximum oil production rate) needed for the field is relatively low as the production from the two reservoirs is done separately. By separately, it means that the production from reservoir 2 is started as soon as the potential from reservoir 1 is low enough to ensure that the total production rate does not surpass the plateau rate/capacity. In contrast, the DICOPT solver yields a result in which the production from the two reservoirs is done simultaneously. As a result, the production capacity is higher than the one from the GAMS/LINDO solver. The difference in both the production profile approach and production capacity will surely affect the discounted cash flow (DCF) and cumulative net present value (NPV) figure.

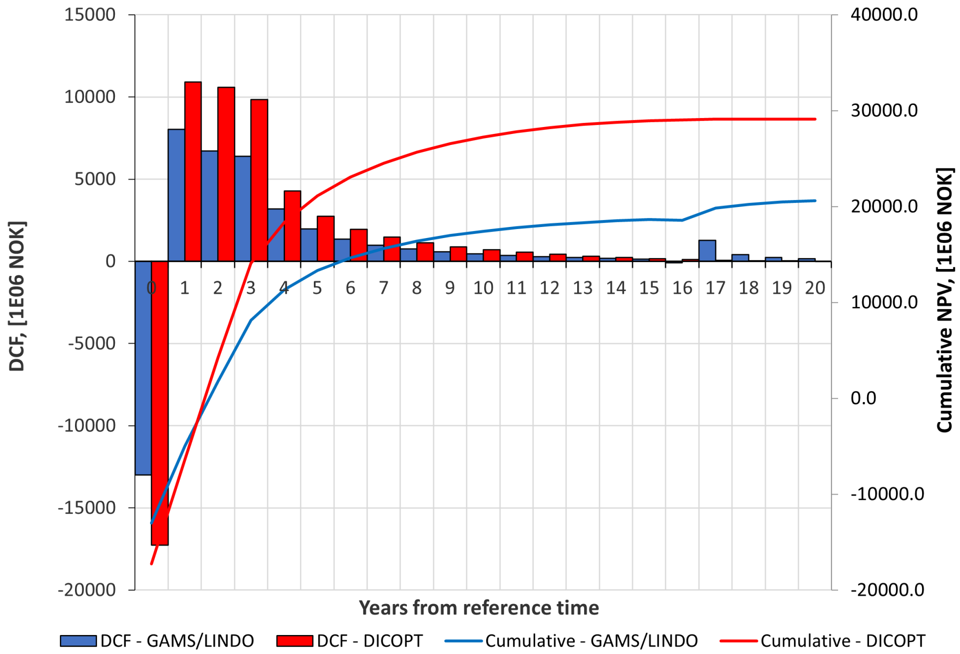


Figure 5.3: DCF and cumulative NPV of the improved model using two different solving engines

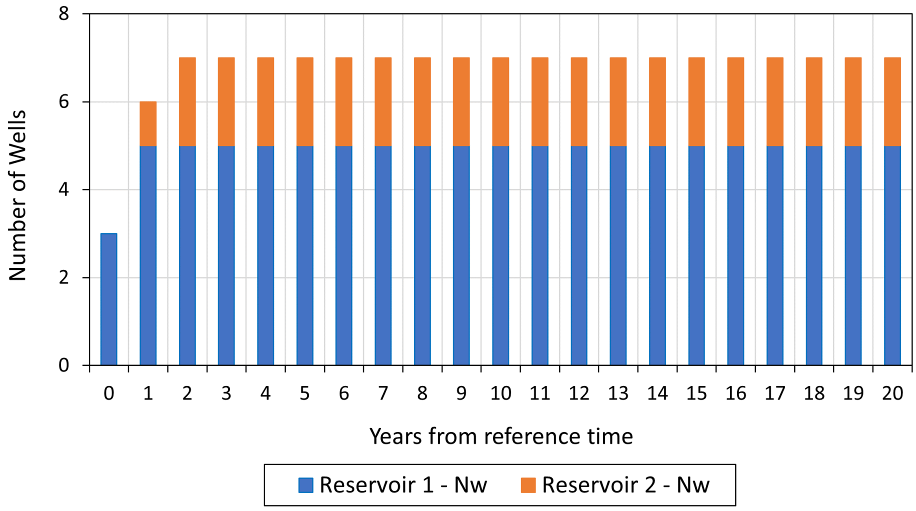
The production capacity difference from the two solvers is quite significant, causing an equally significant difference in the capital expenditure of the production facility (i.e., pre-CapEx, DCF at year 0), as can be seen in Figure 5.3. Although the result from the DICOPT solver requires greater capital and higher oil production than the GAMS/LINDO results, especially in the early years, it is proven to be a more preferred approach as its return on capital (ROC) is faster by three years than the result from GAMS/LINDO solver. Both low production capacity and the delay in drilling for wells in reservoir 2 turns out to be invaluable as the DCF figure at year 16 is insignificant to the NPV of the project. The difference in NPV figures is significant as the NPV resulted from the GAMS/LINDO solver is 20609.6 Million NOK, around 9000 Million NOK less than the NPV yielded by the DICOPT solver. However, the solutions of both solvers converged properly, which makes the difference is due to the difficulty of the GAMS/LINDO solver to find the most optimal scenario using the lookup table constraints defined in this formulation.

Table 5.3: Running time comparison between the two solvers

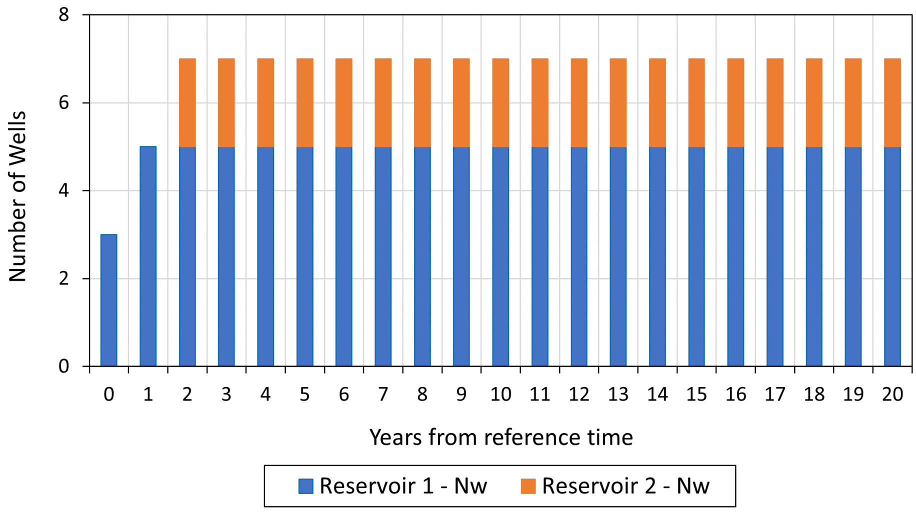
Solver	Run time (s)
GAMS/LINDO	357.05
DICOPT	1.94

Other than its performance in solving the improved formulation, the two solvers are also compared based on the running time. Table 5.3 shows that the DICOPT solver is significantly superior to the GAMS/LINDO solver in which the former only requires less than 2 seconds to solve the problem, whereas the latter runs for more than 6 minutes to find the optimal solution. Considering the fact that DICOPT solves the problem faster while at the same time finding a better optimal solution for the formulation, change in the solving engine is essential in order to yield the most optimal result as wells as a comparable one with the reference model (linear model from the work of Alkindira (2020)).

5.4 Comparison against the Piecewise Linearization (PWL) Model of Alkindira (2020)



(a) Non-linear Model



(b) Linear Model

Figure 5.4: Optimal drilling schedule comparison between the improved non-linear model and the reference linear model

The comparison between the optimal drilling schedule of the two models is shown in Figure 5.4. The drilling schedules of both models are quite similar. Both models were built by assuming that the minimum number of producer wells in reservoir 1 at any time is three. Therefore, well combinations that give less than three wells in reservoir 1 were excluded from the analyzes. As a consequence, both models opt to drill three producer wells at the initial year of the production horizon in reservoir 1. The only difference is that the non-linear model chose to drill one producer well in reservoir 2 at the second timestep, while the linear model waited until the third timestep to drill two producer wells.

Table 5.4: Active wells comparison between the improved non-linear model and the reference linear model

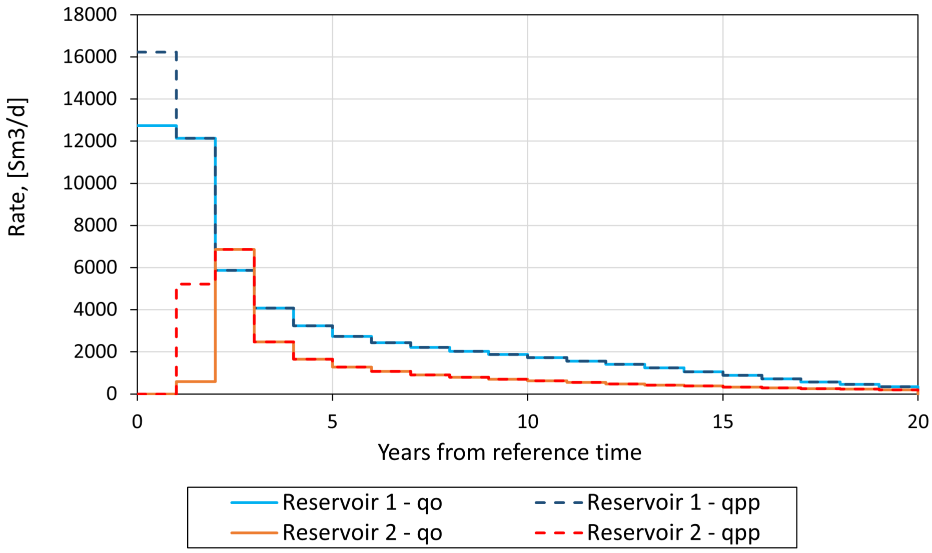
Year(s)	Model	
	Non-linear	Linear
0	3 4 5	3 4 5
1	2 3 4 5 6 9	2 3 4 5 6
2	2 3 4 5 6 8 9	2 3 4 5 6 8 9
⋮	⋮	⋮
20	2 3 4 5 6 8 9	2 3 4 5 6 8 9

Besides the total number of producer wells being similar, the optimal well combination was almost identical between the non-linear and linear models, as shown in Table 5.4. For example, the startup well combination obtained with the non-linear and the linear model is wells 3,4,5, which gives a maximum oil potential of 16219.3 Sm^3/d. This combination is the one that provides the highest production potential value among all other potential combinations (19) that also have three wells. However, the non-linear combination obtains that it is better to drill well 9 a year earlier than the PWL, i.e., in year 1 instead of in year 2.

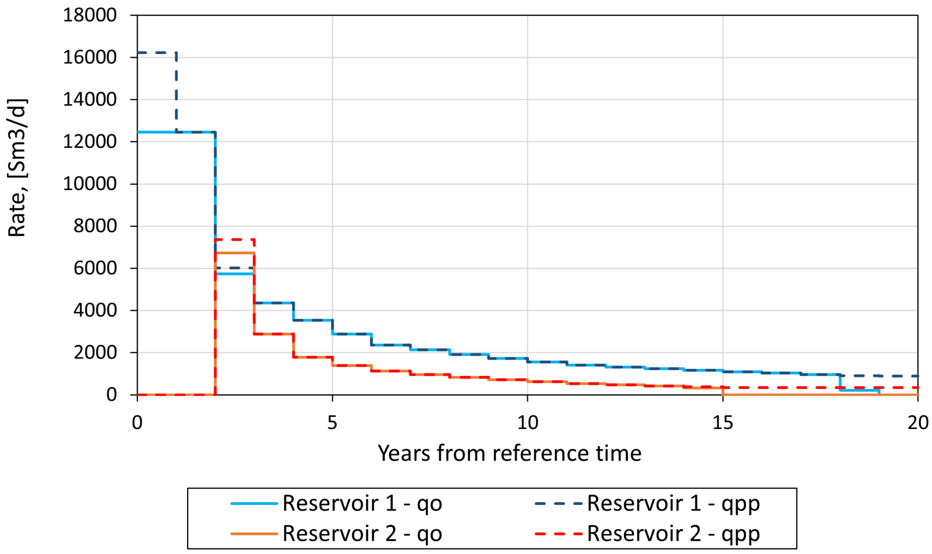
Both models' optimal production profiles of each reservoir are shown in Figure 5.5. In general, the profiles are almost identical to each other, in which plateau mode is chosen as their production strategy, except for the early and later years. In the second timestep, as the non-linear model opt to drill one well from reservoir 2, the production in reservoir 2 started earlier than the linear model. As for the later years, the linear model chose to stop the production on reservoir 1 and 2 around the 19-th and 16-th timestep, respectively. This might have been done to reduce economic loss, as in these years, the operational expenditure exceeds the revenue generated from the low production in the later years. This is not the case for the non-linear model due to several possibilities of reason such as 1) difference in calculation of gas and water production rates (which affects the operational expenditure) and 2) production potential difference (which affects the production profile and cumulative production). Nevertheless, the difference in the later stage of the production profile, in most cases, is insignificant to the NPV calculation.

Figure 5.6 presents the optimal field production profile of both non-linear and linear models. As a consequence of drilling more wells, the field plateau rate of the non-linear model turns out to be slightly greater than the one from the linear model. However, the non-linear model is not consistently producing oil at a greater rate than the linear model. There are also times when the production rate of the linear model is greater than the production rate of the non-linear model. Nevertheless, the differences between them are quite modest, concluding that the production profiles are similar to each other.

The discussions made above regarding the similarities in both the drilling schedule and production profile of both models are supported by the DCF and NPV figures presented in Figure 5.7. There are some visible differences in DCF figures due to the slight difference in production profiles. However, it turns out to be trivial as the cumulative NPV curves of both models are close to indistinguishable from each other from the first year. In the end, however, the NPV of the non-linear model is slightly higher than the NPV of the linear model with a difference of 0.5%, which



(a) Non-linear Model



(b) Linear Model

Figure 5.5: Optimal production profile of each reservoir comparison between the improved non-linear model and the reference linear model

still could be considered as a negligible difference between the figures.

The objective function value and the average running time of both models are shown in Table 5.5. The running time has been evaluated four times under a pretty similar condition (i.e., same computer and all software but the tool is closed). While the result of both the non-linear and linear models are similar to each other, the required running time to solve a deterministic simulation of the non-linear model is significantly lower than the running time of the linear model. The low running time of the non-linear model could be due to the fact that the production potential curve is represented with a polynomial equation and a multiplier representing the performance of a well combination instead of PWL. In the PWL model, two interpolations are performed. One to find the production potential of the system when all wells are producing at a given reservoir cumulative

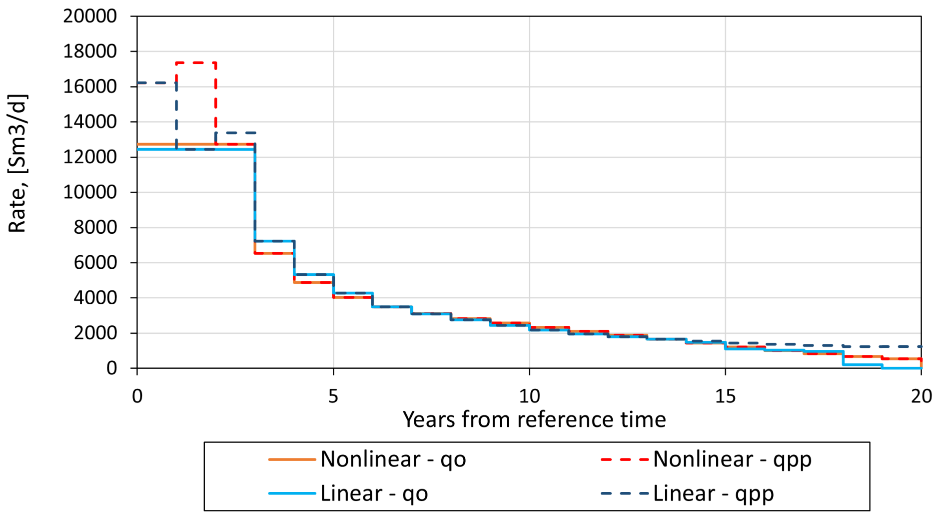


Figure 5.6: Optimal field production profile comparison between the improved non-linear model and the reference linear model

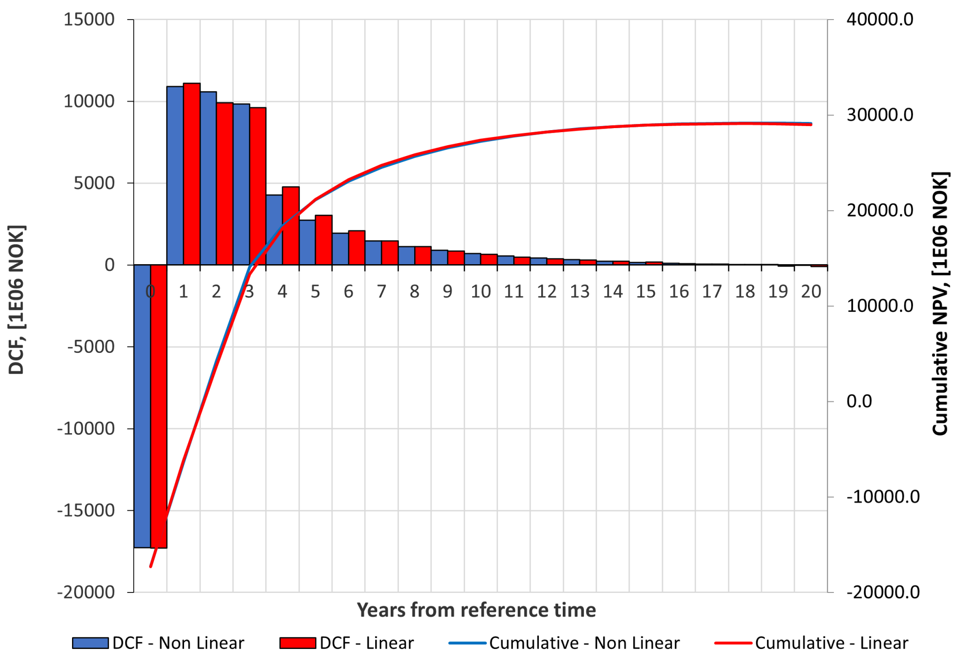


Figure 5.7: DCF and cumulative NPV comparison between the improved non-linear model and the reference linear model

production and a second one to scale this value to account for the presence of a different well combination. Another disadvantage of using PWL approximation is that the formulation becomes more complex as the number of interpolants (i.e., the number of breakpoints used to approximate the non-linear function by piece-wise linearization) increases.

It can be concluded, based on the study and analyses conducted, that the non-linear model allows to obtain results that are practically identical to the ones obtained with the PWL model and in significantly less time. Therefore, the non-linear model seems to be a superior modeling and optimization approach to the PWL formulation. However, this conclusion is based only on one

Table 5.5: NPV and Running time comparison between the improved non-linear model and the reference linear model

Model	NPV (Million NOK)	Run time (s)
Non-linear	29119.6	1.8
Linear	28968.5	21544.5

study case. Further evaluation is required using other cases and field data to generalize this conclusion.

Chapter 6

Conclusion and Further Work

6.1 Conclusion

The non-linear formulation developed in the specialization project of Bonti (2020) was successfully extended by including gas and water production rate in addition to the injection well factor. The formulation was then defined as the base formulation in which the objective function was to maximize the net present value (NPV) while optimizing the production profile and the drilling schedule of the field as well as the processing capacities of oil, gas, and water of the facilities. The base formulation was then simulated, and the main observations of the results obtained are:

- The optimal drilling schedule depends on the magnitude of the production potential, and the optimizer usually prioritizes reservoirs/wells with high production potential. For the base case studied, wells in reservoir 1 have higher potential than those in reservoir 2. Thus, the most optimal drilling schedule of the base formulation is to drill wells in reservoir 1 first.
- The production mode selected by the solver is plateau mode as in most cases, it is a typical design choice to produce standalone oil and gas fields with dedicated processing facilities. As a consequence, the wells, particularly in the early years, are drilled to maintain the plateau rate.
- The base formulation employed a fixed oil price in time. The oil price trajectory (behavior in time) affects the optimal solution.

Deterministic simulation on the base formulation had been performed using two different platform-solvers (Platform-Solver), Pyomo-KNITRO (run in a web server) and GAMS-LINDO. Conclusions derived from the results are:

- Running time of Pyomo was longer than GAMS. The running time is highly dependent on the internet connection between the computer and the NEOS web server.
- GAMS-LINDO yielded the same results for all initial conditions tested. However, the results using Pyomo-KNITRO depends on the initial conditions.

Based on those findings, GAMS-LINDO was chosen as a platform-solver combination utilized to perform the uncertainty study.

The effect of uncertain parameters such as initial oil in place, cost factor, well performance, and oil price was evaluated using simulation-based optimization and Latin Hypercube Sampling (LHS) method. The LHS method is used to generate 400 samples from each uncertain parameter. These samples are used as input to the uncertainty analysis model.

Several field design methods using the results of the LHS simulations were tested, i.e., drilling schedule and production capacity were selected using several criteria. The performance of the resulting design was then tested by repeating the LHS study (but this time optimizing the production profile only) and compared against the original results of the LHS. The analysis using the LHS method also evaluated the effect of 1) relaxing the upper bound in the number of producer wells allowed and 2) considering flexibility in the drilling schedule by having wells that can be "optional" to be drilled or not. Observation on the obtained field designs shows that:

- Design Method D (i.e., mean value for oil and water production capacities as well as the drilling schedule and third quartile value for gas production capacity) was the most optimal design method when the drilling schedule was assumed to be fixed throughout the field life. The percentage difference of the NPV results to the base simulation results was the lowest, with 15.1%.
- When flexibility in the drilling schedule was applied, flexible design method A (i.e., first quartile curve as the minimum number of producer wells and third quartile curve as the maximum number of producer wells) was selected as the most optimal design method. The approach was successfully decreased the percentage difference to 10.0% while at the same time keeping the complexity of the formulation to be minimum.
- When relaxing the upper bound in the number of producer wells to 20, instead of 9, the expected value of the number of producer wells is 11. This could indicate that the current maximum of 9 is adequate but that it might be beneficial for some cases to consider a few additional producer wells.
- Design method A (i.e., expected value design in which the mean value are used for all production capacities and the drilling schedule) was the most optimal design for the increased max N_w case when the drilling schedule was assumed to be fixed throughout the field life. The percentage difference of the NPV results to the base simulation results was the lowest, with 11.9%.
- Flexible design method A (i.e., first quartile curve as the minimum number of producer wells and third quartile curve as the maximum number of producer wells) was also selected as the most optimal design method for the case with a higher allowed number of producers. The percentage difference of the NPV results was significantly improved to 4.7%.
- While the design methods for both the base uncertainty analysis model and the increased max N_w case are different (i.e., design method D for the base model and expected value design for the second model), the value of the fluid production capacities and the range of distributions for the flexible drilling schedule were similar to each other.

Stochastic programming (SP) was also employed to determine the optimal field design considering uncertainties. The SP model was built to be a comparison for the base uncertainty model in terms of its accuracy of the results, running time, and complexity. The main observations of the results obtained are:

- The study on the Spearman's Rank Coefficient Correlation (SRCC) showed that well performance of reservoir 2 ($F_{w,2}$) had a very low correlation to the objective function and was removed from the uncertain parameters of the SP model.

- The SP model produced a better percentage difference of NPV figures than design method D (i.e., the most optimal design of the base uncertainty model), with 12.8%. The improvement, however, was very modest if compared to the difference in the running time of both methods. The difference was mainly due to the fact that the SP model explored any possible design while the base model only evaluated four alternative designs.

- SP model requires one single simulation to produce the optimal design. It could limit the possibility of human error during each procedure needed to yield the optimal design using the base uncertainty model.

- Despite having higher complexity in the procedure, it could be concluded that using the LHS method and simulation-based optimization to produce a good field design provides good accuracy and requires shorter running time than the SP model.

The base formulation was successfully improved by including the distinct well performance in the production performance model and pipeline length in the cost model. A comparison with the PWL model developed by Alkindira (2020) was made, and the following findings are acquired:

- The drilling schedule of both the non-linear model and the PWL model were quite similar. However, the non-linear opted to drill one producer well in reservoir 2 at the second timestep while the PWL model chose to wait for another one year to drill two producer wells in reservoir 2. The active wells at every timestep were also almost identical. The only difference is that in the non-linear model, well 9 is drilled 1-year earlier than in the PWL model.

- Both models chose the plateau mode as their production strategy. Furthermore, their production profiles were also almost similar to each other. However, the field plateau rate of the non-linear model is slightly greater than the plateau rate of the PWL model due to the fact that the former chose to drill one more producer wells in the second timestep.

- The NPV of the non-linear model is 0.5% higher than the NPV of the PWL model, with 29119.6 Million NOK compared to 28968.5 Million NOK.

- Despite the similarities in every aspect mentioned earlier, the non-linear model required only 1.8 seconds to be solved, a significant improvement to the PWL model in which nearly 6 hours was needed to find the optimal solution. Based on the findings above, it could be concluded that the non-linear model was a superior optimization approach to the PWL model.

6.2 Further Work

The following are works that the writer finds interesting and important to be studied but could not be performed in this thesis due to time and resource limitations:

- Evaluate the current non-linear model using other cases in order to generalize the conclusion made in this thesis.

- Perform the uncertainty analysis using the improved non-linear model (distinct well performance).

- Study on another objective functions such as the CO2 emissions and oil/gas recovery to the formulation. It will be more realistic if the formulation includes several objective functions and introduce a new parameter to adjust the weight of each objective function.

- Further study the stochastic programming model such as 1) evaluate the effect of flexibility in the number of producer wells using the SP model 2) study the effect of probabilities of uncertain parameters on the accuracy of the SP model results and 3) study on the effect of the number of samples from each uncertainty on the accuracy of the results.

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Appendix A

GAMS Code for the Base Uncertainty Model

```
$Title Multi reservoir MINLP model.

Sets
    i /t1*t21/
    k /N1, N2, Fc, Fw1, Fw2/
    res /r1*r2/;

*Defining parameters for the model
Scalars

    R /0/
    Tax /0/
    a_drillex /5E8/
    a_capex /50E6/
    b_capex /8.56E5/
    c_capex /4.42E4/
    d_capex /2.35E2/
    e_capex /2.15E9/
    a_opex /723.3E6/
    b_opex /6.5E6/
    c_opex /2.36E4/
    d_opex /1.855E-12/
    e_opex /3.04E-15/
    disc /0.12/
    Nwf_year /3/
    fp /0.973/
    days /365/
    Np_init /0.0/
    Nw_init /3/
    xr /8.5/
    vc /6.29/
    Fc /1.0/
*    qoMax /9799.6/
*    qgMax /5693604.2261/
*    qwMax /5688.3126/;

Parameter
    Nw_max(res)
        /r1 6,
```

```
        r2  3/
    qpp_well(res)
        /r1 4592.475,
        r2  2852.217/
    Nw_injprod(res)
        /r1 0.5,
        r2 0.33/
    a_dimpot(res)
        /r1 -39.413,
        r2 7067.6/
    b_dimpot(res)
        /r1 -13.968,
        r2 -3761.5/
    c_dimpot(res)
        /r1 6.2734,
        r2 553.01/
    d_dimpot(res)
        /r1 12.684,
        r2 21.825/
    e_dimpot(res)
        /r1 -6.7185,
        r2 -12.684/
    a_gor(res)
        /r1 556885.71,
        r2 11358985.64/
    b_gor(res)
        /r1 -353194.93,
        r2 -3064031.89/
    c_gor(res)
        /r1 92643.10,
        r2 288797.17/
    d_gor(res)
        /r1 -5277.34,
        r2 -5494.71/
    e_gor(res)
        /r1 316.34,
        r2 150.00/
    a_wc(res)
        /r1 -111.57,
        r2 -178.94/
    b_wc(res)
        /r1 119.26,
        r2  95.27/
    c_wc(res)
        /r1 -42.60,
        r2  -16.31/
    d_wc(res)
        /r1 6.84,
        r2 0.96/
    e_wc(res)
        /r1 0.011,
        r2 0.03/
    N(res)
        /r1 56.25e6
        r2 39.25e6/
    Fw(res)
        /r1 1.0
        r2 1.0/
    Po(i)
        /t1    60
        t2    60
        t3    60
        t4    60
```

```
        t5    60
        t6    60
        t7    60
        t8    60
        t9    60
        t10    60
        t11    60
        t12    60
        t13    60
        t14    60
        t15    60
        t16    60
        t17    60
        t18    60
        t19    60
        t20    60/;

*Table
*    Nw_fx(res,i)
*        t1  t2  t3  t4  t5  t6  t7  t8  t9  t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21
*    r1  3   4   5   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6
*    r2  0   1   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3;


Parameter val(i);
val(i) = ord(i)-1;

*Defining variables
Variables
    DCF(i)
    NPV;

Positive variables
    qo(res,i)
    qo_f(i)
    qoMax
    qwMax
    qgMax
    qg(res,i)
    qg_f(i)
    qw(res,i)
    qw_f(i)
    Np(res,i)
    Np_f(i)
    Gp(res,i)
    Gp_f(i)
    Wp(res,i)
    Wp_f(i)
    qpp(res,i)
    PV_d(i)
    PV_c(i)
    PV_pre
    PV_o(i)
    PV_r(i);

Integer variables
    Nw(res,i)
    Nw_f(i);


Nw.up(res,i) = Nw_max(res);
*Nw.fx(res,i) = Nw_fx(res,i);
```

```
*Declaring equations
Equations
    a1
    a2
    a4
    a5
    a6
    a7
    a8
    a9
    a10
    a11
    a12
    a13
    a14
    a15
    a16
    a17
    a18
    a19
    a20
    a21
    a22
    a23
    a24
    a25
    a26
    a27
    a28
    a29
    a30
    a31
    a32
    a33
    a34
    obj;


*OBJECTIVE FUNCTION

obj.. NPV =e= (1-R)*(1-Tax)*NPV;

a1(res).. Np(res,'t1') =e= Np_init;
a2.. Nw_f('t1') =l= Nw_init;
a4(i).. Nw_f(i) =e= sum(res, Nw(res,i));
a5(res,i+1).. (qo(res,i+1) + qo(res,i))*0.5*days =e= Np(res,i+1) - Np(res,i);
a6(res,i).. qg(res,i) =e= qo(res,i) * (a_gor(res) * (Np(res,i)/N(res))**4 + b_gor(res)
                            * (Np(res,i)/N(res))**3 + c_gor(res) * (Np(res,i)/N(res))**2
                            + d_gor(res) * (Np(res,i)/N(res)) + e_gor(res));
a7(res).. Gp(res,'t1') =e= 0;
a8(res).. Wp(res,'t1') =e= 0;
a9(res,i+1).. (qg(res,i+1) + qg(res,i))*0.5*days =e= Gp(res,i+1) - Gp(res,i);
a10(res,i).. qw(res,i) =e= qo(res,i) * (a_wc(res) * (Np(res,i)/N(res))**4 + b_wc(res)
                            * (Np(res,i)/N(res))**3 + c_wc(res) * (Np(res,i)/N(res))**2
                            + d_wc(res) * (Np(res,i)/N(res)) + e_wc(res))/(1-(a_wc(res)
                            * (Np(res,i)/N(res))**4 + b_wc(res) * (Np(res,i)/N(res))**3
                            + c_wc(res) * (Np(res,i)/N(res))**2 + d_wc(res)
                            * (Np(res,i)/N(res)) + e_wc(res)));
a11(res,i+1).. (qw(res,i+1) + qw(res,i))*0.5*days =e= Wp(res,i+1) - Wp(res,i);
a12(i).. qo_f(i) =e= sum(res, qo(res,i));
a13(i).. qg_f(i) =e= sum(res, qg(res,i));
a14(i).. qw_f(i) =e= sum(res, qw(res,i));
a15(i).. Np_f(i) =e= sum(res, Np(res,i));
```

```
a16(i).. Gp_f(i) =e= sum(res, Gp(res,i));
a17(i).. Wp_f(i) =e= sum(res, Wp(res,i));
a18(res,i).. qpp(res,i) =e= Fw(res) * Nw(res,i)*qpp_well(res) * fp**(Nw(res,i)-1) *
                            (a_dimpot(res) * (Np(res,i)/N(res))**5 + b_dimpot(res)
                            * (Np(res,i)/N(res))**4 + c_dimpot(res) * (Np(res,i)/N(res))**3
                            + d_dimpot(res) * (Np(res,i)/N(res))**2 + e_dimpot(res)
                            * (Np(res,i)/N(res)) + 1);
a19(res,i).. qo(res,i) =l= qpp(res,i);
a20(res,i+1).. Nw(res,i+1) =g= Nw(res,i);
a21(i+1).. Nw_f(i+1) - Nw_f(i) =l= Nwf_year;
a22(i).. qo_f(i) =l= qoMax;
a23(i).. qg_f(i) =l= qgMax;
a24(i).. qw_f(i) =l= qwMax;
a25.. PV_d('t1') =e= Fc * sum(res, Nw(res,'t1')*(1+Nw_injprod(res))) * a_drillex
                        /(1 + disc)**val('t1');
a26(i+1).. PV_d(i+1) =e= Fc * sum(res, (Nw(res,i+1) - Nw(res,i))*(1+Nw_injprod(res)))
                        * a_drillex/(1 + disc)**val(i+1);
a27.. PV_c('t1') =e= Fc * sum(res, Nw(res,'t1')*(1+Nw_injprod(res)))*a_capex/(1+disc)**val('t1');
a28(i+1).. PV_c(i+1) =e= Fc * sum(res, (Nw(res,i+1) - Nw(res,i))*(1+Nw_injprod(res)))
                        * a_capex/(1 + disc)**val(i+1);
a29.. PV_pre =e= Fc * (qoMax * b_capex + qwMax * c_capex + qgMax * d_capex + e_capex);
a30(i+1).. PV_o(i+1) =e= Fc * (a_opex + b_opex * Nw_f(i) + c_opex * qo_f(i) + d_opex
                        * qw_f(i) + e_opex * qg_f(i))/(1 + disc)**val(i+1);
a31(i+1).. PV_r(i+1) =e= (Po(i) * (Np_f(i+1) - Np_f(i)) * xr * vc)/(1 + disc)**val(i+1);
a32.. DCF('t1') =e= -(PV_pre + PV_d('t1') + PV_c('t1'));
a33(i+1).. DCF(i+1) =e= (PV_r(i+1) - PV_o(i+1) - PV_c(i+1) - PV_d(i+1));
a34.. NPV =e= sum(i,DCF(i))/1E6;
*Generating model
Model mod3 /all/;

option minlp = lindo;

set s /s1*s400/

$call gdxxrw.exe LHS.xlsx par=LHS rng=Sheet1!A1:F401
Parameter LHS(s,k);
$gdxin LHS.gdx
$load LHS
$gdxin

Parameter N_s(s,res), Fc_s(s), Fw_s(s,res);
N_s(s,'r1') = LHS(s,'N1')*1e6;
N_s(s,'r2') = LHS(s,'N2')*1e6;
Fc_s(s) = LHS(s, 'Fc');
Fw_s(s,'r1') = LHS(s, 'Fw1');
Fw_s(s,'r2') = LHS(s, 'Fw2');

$call gdxxrw.exe OilPriceTrajectory.xlsx par=oilprice rng=Sheet2!B2:W402
Parameter oilprice(s,i);
$gdxin OilPriceTrajectory.gdx
$load oilprice
$gdxin

Parameter qoresult(s,res,i);
Parameter qppresult(s,res,i);
Parameter qgresult(s,res,i);
Parameter qwresult(s,res,i);
Parameter Nwresult(s,res,i);
Parameter Npresult(s,res,i);
Parameter Gpresult(s,res,i);
Parameter Wpresult(s,res,i);
Parameter NPVresult(s);
```

```
Set mattrib / system.GUSSModelAttributes /;

Parameter
   srep(s, mattrib)                  'model attibutes like modelstat etc'
   o(*)/ SkipBaseCase 0
   RestartType 1/;

Set dict /  s.scenario.''
            o.  opt     .srep
            Po. param   .oilprice
            N.  param   .N_s
            Fc. param   .Fc_s
            Fw. param   .Fw_s
            qo. level   .qoresult
            qpp. level  .qppresult
            qg. level   .qgresult
            qw. level   .qwresult
            Nw. level   .Nwresult
            Np. level   .Npresult
            Gp. level   .Gpresult
            Wp. level   .Wpresult
            NPV. level   .NPVresult/;


Solve mod3 maximizing NPV using minlp scenario dict;

execute_unload "results.gdx" qoresult qppresult qgresult qwresult Nwresult Npresult NPVresult

*=== Now write to variable levels to Excel file from GDX
*=== Since we do not specify a sheet, data is placed in first sheet
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=qoresult rng=qo!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=qgresult rng=qg!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=qwresult rng=qw!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=Nwresult rng=nw!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=NPVresult rng=npv!'
```

Appendix B

GAMS Code for the Uncertainty Model Considering Flexibility in the Drilling Schedule

```
$Title Multi reservoir MINLP model.

Sets
    i /t1*t21/
    k /N1, N2, Fc, Fw1, Fw2/
    res /r1*r2/;

*Defining parameters for the model
Scalars

    R /0/
    Tax /0/
    a_drillex /5E5/
    a_capex /50E3/
    b_capex /8.56E2/
    c_capex /4.42E1/
    d_capex /2.35E2/
    e_capex /2.15E6/
    a_opex /723.3E3/
    b_opex /6.5E3/
    c_opex /2.36E1/
    d_opex /1.855E-15/
    e_opex /3.04E-15/
    disc /0.12/
    Nwf_year /3/
    fp /0.973/
    days /365/
    Np_init /0.0/
    Nw_init /3/
    xr /8.5/
    vc /6.29/
    Fc /1.0/
    qoMax /8915.93/
    qgMax /5494.73/
    qwMax /4752.47/;
```

```
Parameter
    qpp_well(res)
        /r1 4592.475,
        r2  2852.217/
    Nw_injprod(res)
        /r1 0.5,
        r2 0.33/
    a_dimpot(res)
        /r1 -39.413,
        r2 7067.6/
    b_dimpot(res)
        /r1 -13.968,
        r2 -3761.5/
    c_dimpot(res)
        /r1 6.2734,
        r2 553.01/
    d_dimpot(res)
        /r1 12.684,
        r2 21.825/
    e_dimpot(res)
        /r1 -6.7185,
        r2 -12.684/
    a_gor(res)
        /r1 556885.71,
        r2 11358985.64/
    b_gor(res)
        /r1 -353194.93,
        r2 -3064031.89/
    c_gor(res)
        /r1 92643.10,
        r2 288797.17/
    d_gor(res)
        /r1 -5277.34,
        r2 -5494.71/
    e_gor(res)
        /r1 316.34,
        r2 150.00/
    a_wc(res)
        /r1 -111.57,
        r2 -178.94/
    b_wc(res)
        /r1 119.26,
        r2  95.27/
    c_wc(res)
        /r1 -42.60,
        r2  -16.31/
    d_wc(res)
        /r1 6.84,
        r2 0.96/
    e_wc(res)
        /r1 0.011,
        r2 0.03/
    N(res)
        /r1 56.25e6
        r2 39.25e6/
    Fw(res)
        /r1 1.0
        r2 1.0/
    Po(i)
        /t1    60
        t2    60
        t3    60
```

```
        t4    60
        t5    60
        t6    60
        t7    60
        t8    60
        t9    60
        t10    60
        t11    60
        t12    60
        t13    60
        t14    60
        t15    60
        t16    60
        t17    60
        t18    60
        t19    60
        t20    60/;

Table
    Nw_min(res,i)
        t1  t2  t3  t4  t5  t6  t7  t8  t9  t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21
    r1  3   3   3   4   4   5   5   5   5   5   5   5   5   5   5   5   5   5   5   5   5
    r2  0   0   0   1   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3;

Table
    Nw_max(res,i)
        t1  t2  t3  t4  t5  t6  t7  t8  t9  t10 t11 t12 t13 t14 t15 t16 t17 t18 t19 t20 t21
    r1  3   4   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6   6
    r2  0   0   2   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3;


Parameter val(i);
val(i) = ord(i)-1;

*Defining variables
Variables
    DCF(i)
    NPV;

Positive variables
    qo(res,i)
    qo_f(i)
*qoMax
*qwMax
*qgMax
    qg(res,i)
    qw(res,i)
    Np(res,i)
    Np_f(i)
    qpp(res,i)
    PV_d(i)
    PV_c(i)
    PV_pre
    PV_o(i)
    PV_r(i);

Integer variables
    Nw(res,i)
    Nw_add(res,i)
    Nw_f(i);

*Declaring equations
Equations
```

```
    a1
    a2
    a3
    a4
    a5
    a6
    a7
    a8
    a9
    a10
    a11
    a12
    a13
    a14
    a15
    a16
    a17
    a18
    a19
    a20
    a21
    a22
    a23
    a24
    a25
    a26
    a300
    obj;

*OBJECTIVE FUNCTION

obj.. NPV =e= (1-R)*(1-Tax)*NPV;

a1(res).. Np(res,'t1') =e= Np_init;
a2(res,i).. Nw(res,i) =e= Nw_min(res,i) + Nw_add(res,i);
a3(res).. sum(i,Nw_add(res,i)) =l= sum(i,Nw_max(res,i)-Nw_min(res,i));
a300(res,i).. Nw(res,i) =l= Nw_max(res,i);
a4(i).. Nw_f(i) =e= sum(res, Nw(res,i));
a5(res,i+1).. (qo(res,i+1) + qo(res,i))*0.5*days =e= Np(res,i+1) - Np(res,i);
a6(res,i).. qg(res,i) =e= qo(res,i) * (a_gor(res) * (Np(res,i)/N(res))**4 + b_gor(res)
                            * (Np(res,i)/N(res))**3 + c_gor(res) * (Np(res,i)/N(res))**2
                            + d_gor(res) * (Np(res,i)/N(res)) + e_gor(res))/1E3;
a7(res,i).. qw(res,i) =e= qo(res,i) * (a_wc(res) * (Np(res,i)/N(res))**4 + b_wc(res)
                            * (Np(res,i)/N(res))**3 + c_wc(res) * (Np(res,i)/N(res))**2
                            + d_wc(res) * (Np(res,i)/N(res)) + e_wc(res))/(1-(a_wc(res)
                            * (Np(res,i)/N(res))**4 + b_wc(res) * (Np(res,i)/N(res))**3
                            + c_wc(res) * (Np(res,i)/N(res))**2 + d_wc(res)
                            * (Np(res,i)/N(res)) + e_wc(res)));
a8(i).. qo_f(i) =e= sum(res, qo(res,i));
a9(i).. Np_f(i) =e= sum(res, Np(res,i));
a10(res,i).. qpp(res,i) =e= Fw(res) * Nw(res,i)*qpp_well(res) * fp**(Nw(res,i)-1) *
                            (a_dimpot(res) * (Np(res,i)/N(res))**5 + b_dimpot(res)
                            * (Np(res,i)/N(res))**4 + c_dimpot(res) * (Np(res,i)/N(res))**3
                            + d_dimpot(res) * (Np(res,i)/N(res))**2
                            + e_dimpot(res) * (Np(res,i)/N(res)) + 1);
a11(res,i).. qo(res,i) =l= qpp(res,i);
a12(res,i+1).. Nw(res,i+1) =g= Nw(res,i);
a13(i+1).. Nw_f(i+1) - Nw_f(i) =l= Nwf_year;
a14(i).. qo_f(i) =l= qoMax;
a15(i).. sum(res, qg(res,i)) =l= qgMax;
a16(i).. sum(res, qw(res,i)) =l= qwMax;
a17.. PV_d('t1') =e= Fc * sum(res, Nw(res,'t1')*(1+Nw_injprod(res))) * a_drillex
                        /(1 + disc)**val('t1');
```

```
a18(i+1).. PV_d(i+1) =e= Fc * sum(res, (Nw(res,i+1) - Nw(res,i))*(1+Nw_injprod(res)))
                        * a_drillex/(1 + disc)**val(i+1);
a19.. PV_c('t1') =e= Fc * sum(res, Nw_max(res,'t1')*(1+Nw_injprod(res)))
                        * a_capex/(1+disc)**val('t1');
a20(i+1).. PV_c(i+1) =e= Fc * sum(res, (Nw_max(res,i+1) - Nw_max(res,i))
                        *(1+Nw_injprod(res))) * a_capex/(1 + disc)**val(i+1);
a21.. PV_pre =e= Fc * (qoMax * b_capex + qwMax * c_capex + qgMax * d_capex + e_capex);
a22(i+1).. PV_o(i+1) =e= Fc * (a_opex + b_opex * Nw_f(i) + c_opex * qo_f(i)
                        + d_opex * sum(res, qw(res,i)) + e_opex
                        * sum(res, qg(res,i)))/(1 + disc)**val(i+1);
a23(i+1).. PV_r(i+1) =e= (Po(i) * (Np_f(i+1) - Np_f(i)) * xr * vc)/(1 + disc)**val(i+1)/1E3;
a24.. DCF('t1') =e= -(PV_pre + PV_d('t1') + PV_c('t1'));
a25(i+1).. DCF(i+1) =e= (PV_r(i+1) - PV_o(i+1) - PV_c(i+1) - PV_d(i+1));
a26.. NPV =e= sum(i,DCF(i))/1E3;

*Generating model
Model mod3 /all/;

option minlp = lindo;

set s /s1*s400/

$call gdxxrw.exe LHS.xlsx par=LHS rng=Sheet1!A1:F401
Parameter LHS(s,k);
$gdxin LHS.gdx
$load LHS
$gdxin

Parameter N_s(s,res), Fc_s(s), Fw_s(s,res);
N_s(s,'r1') = LHS(s,'N1')*1e6;
N_s(s,'r2') = LHS(s,'N2')*1e6;
Fc_s(s) = LHS(s, 'Fc');
Fw_s(s,'r1') = LHS(s, 'Fw1');
Fw_s(s,'r2') = LHS(s, 'Fw2');

$call gdxxrw.exe OilPriceTrajectory.xlsx par=oilprice rng=Sheet2!B2:W402
Parameter oilprice(s,i);
$gdxin OilPriceTrajectory.gdx
$load oilprice
$gdxin

Parameter qoresult(s,res,i);
Parameter qppresult(s,res,i);
Parameter qgresult(s,res,i);
Parameter qwresult(s,res,i);
Parameter Nwresult(s,res,i);
Parameter Npresult(s,res,i);
Parameter NPVresult(s);

Set mattrib / system.GUSSModelAttributes /;

Parameter
   srep(s, mattrib)                  'model attibutes like modelstat etc'
   o(*)/ SkipBaseCase 0
   RestartType 2/;

Set dict /  s.scenario.''
            o.  opt     .srep
            Po. param   .oilprice
            N.  param   .N_s
            Fc. param   .Fc_s
            Fw. param   .Fw_s
```

```
            qo. level    .qoresult
            qpp. level   .qppresult
            qg. level    .qgresult
            qw. level    .qwresult
            Nw. level    .Nwresult
            Np. level    .Npresult
            NPV. level    .NPVresult/;


Solve mod3 maximizing NPV using MINLP scenario dict;

execute_unload "results.gdx" qoresult qppresult qgresult qwresult Nwresult Npresult NPVresult

*=== Now write to variable levels to Excel file from GDX
*=== Since we do not specify a sheet, data is placed in first sheet
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=qoresult rng=qo!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=qgresult rng=qg!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=qwresult rng=qw!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=Nwresult rng=nw!'
execute 'gdxxrw.exe results.gdx o=results1.xlsx par=NPVresult rng=npv!'
```

Appendix C

GAMS Code for the Stochastic Programming Model

```
$Title Multi reservoir MINLP model.

Sets
    i /t1*t21/
    res /r1*r2/;

*Defining parameters for the model
Scalars

    R /0/
    Tax /0/
    a_drillex /5E8/
    a_capex /50E6/
    b_capex /8.56E5/
    c_capex /4.42E4/
    d_capex /2.35E2/
    e_capex /2.15E9/
    a_opex /723.3E6/
    b_opex /6.5E6/
    c_opex /2.36E4/
    d_opex /1.855E-12/
    e_opex /3.04E-15/
    disc /0.12/
    Nwf_year /3/
    fp /0.973/
    days /365/
    Np_init /0.0/
    Nw_init /3/
    xr /8.5/
    vc /6.29/
    Fc /1.0/
    Po /60/;

Parameter
    Nw_max(res)
        /r1 6,
        r2  3/
    qpp_well(res)
        /r1 4592.475,
```

```
        r2  2852.217/
    Nw_injprod(res)
        /r1 0.5,
        r2 0.33/
    a_dimpot(res)
        /r1 -39.413,
        r2 7067.6/
    b_dimpot(res)
        /r1 -13.968,
        r2 -3761.5/
    c_dimpot(res)
        /r1 6.2734,
        r2 553.01/
    d_dimpot(res)
        /r1 12.684,
        r2 21.825/
    e_dimpot(res)
        /r1 -6.7185,
        r2 -12.684/
    a_gor(res)
        /r1 556885.71,
        r2 11358985.64/
    b_gor(res)
        /r1 -353194.93,
        r2 -3064031.89/
    c_gor(res)
        /r1 92643.10,
        r2 288797.17/
    d_gor(res)
        /r1 -5277.34,
        r2 -5494.71/
    e_gor(res)
        /r1 316.34,
        r2 150.00/
    a_wc(res)
        /r1 -111.57,
        r2 -178.94/
    b_wc(res)
        /r1 119.26,
        r2  95.27/
    c_wc(res)
        /r1 -42.60,
        r2  -16.31/
    d_wc(res)
        /r1 6.84,
        r2 0.96/
    e_wc(res)
        /r1 0.011,
        r2 0.03/
    N(res)
        /r1 56.25e6
        r2 39.25e6/
    Fw(res)
        /r1 1.0
        r2 1.0/;

Parameter val(i);
val(i) = ord(i)-1;

*Defining variables
Variables
    NPV;
```

```
Positive variables
    qo(res,i)
    qoMax
    qwMax
    qgMax
    qg(res,i)
    qw(res,i)
    Np(res,i)
    qpp(res,i)
    PV_d(i)
    PV_pre
    PV_o(i)
    PV_r(i);

Integer variables
    Nw(res,i);

Nw.up(res,i) = Nw_max(res);
Nw.fx('r1','t1') = 3;
Np.fx(res,'t1') = Np_init;

*Declaring equations
Equations
    OilRate
    GasRate
    WaterRate
    OilPotential
    OilRate_Bound
    NumWells_Bound
    FieldNumWells_Bound
    OilCapacity
    GasCapacity
    WaterCapacity
    DrillExCapEx_1
    DrillExCapEx
    PreCapEx
    OpEx
    Revenue
    Objective;

*Objective Function
Objective.. NPV =e= -PV_pre/1E3 + sum(i,PV_r(i+1) - PV_o(i+1) - PV_d(i))/1E3;
*Constraints
OilRate(res,i+1).. (qo(res,i+1) + qo(res,i))*0.5*days =e= Np(res,i+1) - Np(res,i);
GasRate(res,i).. qg(res,i) =e= qo(res,i) * (a_gor(res) * (Np(res,i)/N(res))**4
                            + b_gor(res) * (Np(res,i)/N(res))**3 + c_gor(res)
                            * (Np(res,i)/N(res))**2 + d_gor(res) * (Np(res,i)/N(res))
                            + e_gor(res));
WaterRate(res,i).. qw(res,i) =e= qo(res,i) * (a_wc(res) * (Np(res,i)/N(res))**4
                            + b_wc(res) * (Np(res,i)/N(res))**3 + c_wc(res)
                            * (Np(res,i)/N(res))**2 + d_wc(res) * (Np(res,i)/N(res))
                            + e_wc(res))/(1-(a_wc(res) * (Np(res,i)/N(res))**4
                            + b_wc(res) * (Np(res,i)/N(res))**3 + c_wc(res)
                            * (Np(res,i)/N(res))**2 + d_wc(res) * (Np(res,i)/N(res))
                            + e_wc(res)));
OilPotential(res,i).. qpp(res,i) =e= Fw(res) * Nw(res,i)*qpp_well(res)
                            * fp**(Nw(res,i)-1) * (a_dimpot(res) * (Np(res,i)/N(res))**5
                            + b_dimpot(res) * (Np(res,i)/N(res))**4 + c_dimpot(res)
                            * (Np(res,i)/N(res))**3 + d_dimpot(res) * (Np(res,i)/N(res))**2
                            + e_dimpot(res) * (Np(res,i)/N(res)) + 1);
OilRate_Bound(res,i).. qo(res,i) =l= qpp(res,i);
NumWells_Bound(res,i+1).. Nw(res,i+1) =g= Nw(res,i);
FieldNumWells_Bound(i+1).. sum(res, Nw(res,i+1)-Nw(res,i)) =l= Nwf_year;
```

```
OilCapacity(i).. sum(res, qo(res,i)) =l= qoMax;
GasCapacity(i).. sum(res, qg(res,i)) =l= qgMax;
WaterCapacity(i).. sum(res, qw(res,i)) =l= qwMax;
DrillExCapEx_1.. PV_d('t1') =e= Fc * sum(res, Nw(res,'t1')*(1+Nw_injprod(res)))
                          * (a_drillex+a_capex)/(1E3*(1 + disc)**val('t1'));
DrillExCapEx(i+1).. PV_d(i+1) =e= Fc * sum(res, (Nw(res,i+1) - Nw(res,i))
                              * (1+Nw_injprod(res))) * (a_drillex+a_capex)
                              /(1E3*(1 + disc)**val(i+1));
PreCapEx.. PV_pre =e= Fc * (qoMax * b_capex + qwMax * c_capex + qgMax * d_capex
                        + e_capex)/1E3;
OpEx(i+1).. PV_o(i+1) =e= Fc * (a_opex + b_opex * sum(res, Nw(res,i)) + c_opex
                        * sum(res, qo(res,i)) + d_opex * sum(res, qw(res,i))
                        + e_opex * sum(res, qg(res,i)))/(1E3*(1 + disc)**val(i+1));
Revenue(i+1).. PV_r(i+1) =e= (Po * sum(res, Np(res,i+1) - Np(res,i)) * xr * vc)
                        /(1E3*(1 + disc)**val(i+1));

*Generating model
Model mod3 /all/;

*Choosing subsolvers for milp and nlp problems
option EMP = LINDO;
option minlp = lINDO;

File emp / '%emp.info%' /;
put emp '* problem %gams.i%'/;
$onput
randvar N('r1') discrete 0.3 31E6 0.4 49E6 0.3 68E6
randvar N('r2') discrete 0.3 33E6 0.4 40E6 0.3 49E6
randvar Fc discrete 0.3 0.81 0.4 1 0.3 1.18
randvar Po discrete 0.3 40 0.4 60 0.3 81
randvar Fw('r1') discrete 0.3 0.86 0.4 1 0.3 1.13
stage 2 qo qg qw Np qpp PV_d PV_pre PV_o PV_r NPV N Fc Fw Po
stage 2 OilRate GasRate WaterRate OilPotential OilRate_Bound OilCapacity GasCapacity
        WaterCapacity DrillExCapEx_1 DrillExCapEx PreCapEx OpEx Revenue
$offput
putclose emp;

Set s  "scenarios" /s1*s243/;
Parameter
    s_N(s,res)
    s_Fc(s)
    s_Fw(s,res)
    s_Po(s)
    s_Nw(s,res,i)
    s_qo(s,res,i)
    s_NPV(s)
    s_qoMax(s)
    s_qgMax(s)
    s_qwMax(s)
    s_rep(s,*) "scenario probability"   / #s.prob 0/;;

Set dict /  s       .scenario   .''
            N       .randvar    .s_N
            Fc      .randvar    .s_Fc
            Po      .randvar    .s_Po
            Fw      .randvar    .s_Fw
            qo      .level      .s_qo
            Nw      .level      .s_Nw
            NPV     .level      .s_NPV
            qoMax   .level      .s_qoMax
            qgMax   .level      .s_qgMax
            qwMax   .level      .s_qwMax
            ''   .opt      .s_rep/;
```

```
Solve mod3 maximizing NPV using EMP scenario dict;

display  s_NPV, s_rep, s_N, s_Fc, s_Po, s_Fw

execute_unload "results.gdx" s_NPV s_rep s_N s_Fc s_Po s_Fw s_Nw s_qoMax s_qgMax s_qwMax
```

Appendix D

GAMS Code for the Improved Formulation

```
$Title Multi reservoir MINLP model.

Sets
    i /t1*t21/
    res /r1*r2/
    sr1 /s1*s42/
    sr2 /n1*n8/
    wr1 /w1*w6/
    wr2 /w7*w9/;

*Defining parameters for the model
Scalars

    R /0/
    Tax /0/
    a_drillex /5E5/
    a_capex /50E3/
    pl_capex /25E3/
    b_capex /8.56E2/
    c_capex /4.42E1/
    d_capex /2.35E-1/
    e_capex /2.15E6/
    a_opex /723.3E3/
    b_opex /6.5E3/
    c_opex /2.36E1/
    d_opex /1.855E-15/
    e_opex /3.04E-18/
    disc /0.12/
    Nwf_year /3/
    fp /0.973/
    days /365/
    Nw_init /3/
    xr /8.5/
    vc /6.29/
    Fc /1.0/
    Po  /0.06/;

Parameter
    Nw_max(res)
```

```
        /r1 6,
        r2  3/
    Nw_injprod(res)
        /r1 0,
        r2 0/
    Nw_pd(res)
        /r1 3,
        r2 0/
    a_dimpot(res)
        /r1 -39.413,
        r2 7067.6/
    b_dimpot(res)
        /r1 -13.968,
        r2 -3761.5/
    c_dimpot(res)
        /r1 6.2734,
        r2 553.01/
    d_dimpot(res)
        /r1 12.684,
        r2 21.825/
    e_dimpot(res)
        /r1 -6.7185,
        r2 -12.684/
    a_gor(res)
        /r1 556885.71,
        r2 11358985.64/
    b_gor(res)
        /r1 -353194.93,
        r2 -3064031.89/
    c_gor(res)
        /r1 92643.10,
        r2 288797.17/
    d_gor(res)
        /r1 -5277.34,
        r2 -5494.71/
    e_gor(res)
        /r1 316.34,
        r2 150.00/
    a_wc(res)
        /r1 -111.57,
        r2 -178.94/
    b_wc(res)
        /r1 119.26,
        r2  95.27/
    c_wc(res)
        /r1 -42.60,
        r2  -16.31/
    d_wc(res)
        /r1 6.84,
        r2 0.96/
    e_wc(res)
        /r1 0.011,
        r2 0.03/
    N(res)
        /r1 56.25e6
        r2 39.25e6/
    Fw(res)
        /r1 1.0
        r2 1.0/;

Table
    z1(sr1,*)
        w1  w2  w3  w4  w5  w6  qppo1       pl1
```

```
    s1  1   1   1   0   0   0   12386.60318 15
    s2  1   1   0   1   0   0   12581.3048  8.5
    s3  1   1   0   0   1   0   12737.3061  11.2
    s4  1   1   0   0   0   1   10683.08899 19
    s5  1   0   1   1   0   0   14200.41828 15
    s6  1   0   1   0   1   0   13341.51113 17.7
    s7  1   0   1   0   0   1   11287.29402 25.5
    s8  1   0   0   1   1   0   14679.92228 11.2
    s9  1   0   0   1   0   1   12625.70517 19
    s10 1   0   0   0   1   1   11290.49405 21.2
    s11 0   1   1   1   0   0   15417.52842 11.9
    s12 0   1   1   0   1   0   15240.42695 14.6
    s13 0   1   1   0   0   1   13186.30984 22.4
    s14 0   1   0   1   1   0   15973.73306 8.1
    s15 0   1   0   1   0   1   13919.51594 15.9
    s16 0   1   0   0   1   1   13280.51062 18.1
    s17 0   0   1   1   1   0   16219.3351  12.3
    s18 0   0   1   1   0   1   14165.11799 20.1
    s19 0   0   1   0   1   1   11977.89977 19.7
    s20 0   0   0   1   1   1   14333.61939 15.8
    s21 0   0   1   1   1   1   18750.05618 22.3
    s22 0   1   0   1   1   1   18504.35413 18.1
    s23 0   1   1   0   1   1   17771.14803 24.6
    s24 0   1   1   1   0   1   18128.75101 22.4
    s25 0   1   1   1   1   0   20182.86812 14.6
    s26 1   0   0   1   1   1   17210.54336 21.2
    s27 1   0   1   0   1   1   15872.13221 27.7
    s28 1   0   1   1   0   1   16911.64087 25.5
    s29 1   0   1   1   1   0   18965.85798 17.7
    s30 1   1   0   0   1   1   15267.92718 21.2
    s31 1   1   0   1   0   1   15292.52738 19
    s32 1   1   0   1   1   0   17346.74449 11.2
    s33 1   1   1   0   0   1   15097.82576 25.5
    s34 1   1   1   0   1   0   17152.04287 17.7
    s35 1   1   1   1   0   0   16714.53923 15
    s36 1   1   1   1   1   0   21479.97892 17.7
    s37 1   1   1   1   0   1   19425.76181 25.5
    s38 1   1   1   0   1   1   19682.76395 27.7
    s39 1   1   0   1   1   1   19877.36557 21.2
    s40 1   0   1   1   1   1   21496.47906 27.7
    s41 0   1   1   1   1   1   22713.5892  24.6
    s42 1   1   1   1   1   1   24010.6     27.7;

Table
    z2(sr2,*)
        w7  w8  w9  qppo2       pl2
    n1  0   0   0   0           0
    n2  1   0   0   4273.936632 17
    n3  0   1   0   4915.397104 15.7
    n4  0   0   1   5222.198198 14
    n5  1   1   0   6549.266422 17
    n6  1   0   1   7120.101246 17
    n7  0   1   1   7359.496494 15.7
    n8  1   1   1   8114        17;

Parameter val(i);
val(i) = ord(i)-1;

*Defining variables
Variables
    DCF(i)
    NPV;
```

```
Positive variables
    qoMax
    qwMax
    qgMax
    plMax
    qppo(res,i)
    qpp(res,i)
    qo(res,i)
    qo_f(i)
    qg(res,i)
    qw(res,i)
    Np(res,i)
    Np_f(i)
    pipe(res,i)
    PV_d(i)
    PV_c(i)
    PV_pre
    PV_o(i)
    PV_r(i);

Integer variables
    Nw(res,i)
    Nw_f(i);

Binary variables
    zw1(wr1,i)
    zw2(wr2,i)
    x(sr1,i)
    y(sr2,i);

*Declaring equations
Equations
    OBJ

    LookupTable_1
    LookupTable_2

    NumWells_Init
    CumulativeProd_Init
    OilRate
    GasRate
    WaterRate
    MaximumPotential_1
    MaximumPotential_2
    OilPotential
    NumWells_1
    NumWells_2
    FieldNumWells
    FieldOilRate
    FieldCumulativeProd
    ActiveScenario_1
    ActiveScenario_2
    Pipeline_1
    Pipeline_2

    CumulativeProd_Bound1
    CumulativeProd_Bound2
    OilRate_Bound
    BinaryWells_Bound1
    BinaryWells_Bound2
    FieldNumWells_Bound
    Pipeline_Bound1
    OilCapacity
```

```
    GasCapacity
    WaterCapacity
    PipelineMax

    DrillEx_1
    DrillEx
    CapEx_1
    CapEx
    PreCapEx
    OpEx
    Revenue
    CashFlow_1
    CashFlow
    ;

*OBJECTIVE FUNCTION

OBJ.. NPV =e= (1-R)*(1-Tax)* sum(i,DCF(i))/1E3;;

*Lookup Table

LookupTable_1(sr1,i).. sum(wr1$(1=z1(sr1,wr1)), zw1(wr1,i)) + sum(wr1$(0=z1(sr1,wr1)),
1-zw1(wr1,i)) =g= card(wr1)*x(sr1,i);
LookupTable_2(sr2,i).. sum(wr2$(1=z2(sr2,wr2)), zw2(wr2,i)) + sum(wr2$(0=z2(sr2,wr2)),
1-zw2(wr2,i)) =g= card(wr2)*y(sr2,i);

*Equality Constraints

NumWells_Init(res).. Nw(res,'t1') =e= Nw_pd(res);
CumulativeProd_Init(res).. Np(res,'t1') =e= 0;

OilRate(res,i+1).. qo(res,i)*days =e= Np(res,i+1) - Np(res,i);
GasRate(res,i).. qg(res,i) =e= qo(res,i) * (a_gor(res) * (Np(res,i)/N(res))**4
                            + b_gor(res) * (Np(res,i)/N(res))**3 + c_gor(res)
                            * (Np(res,i)/N(res))**2 + d_gor(res) * (Np(res,i)/N(res))
                            + e_gor(res));
WaterRate(res,i).. qw(res,i) =e= qo(res,i) * (a_wc(res) * (Np(res,i)/N(res))**4
                            + b_wc(res) * (Np(res,i)/N(res))**3 + c_wc(res)
                            * (Np(res,i)/N(res))**2 + d_wc(res) * (Np(res,i)/N(res))
                            + e_wc(res))/(1-(a_wc(res) * (Np(res,i)/N(res))**4
                            + b_wc(res) * (Np(res,i)/N(res))**3 + c_wc(res)
                            * (Np(res,i)/N(res))**2 + d_wc(res) * (Np(res,i)/N(res))
                            + e_wc(res)));

MaximumPotential_1(i).. qppo('r1',i) =e= sum(sr1,z1(sr1,'qppo1')*x(sr1,i));
MaximumPotential_2(i).. qppo('r2',i) =e= sum(sr2,z2(sr2,'qppo2')*y(sr2,i));

OilPotential(res,i).. qpp(res,i) =e= Fw(res) * qppo(res,i) * (a_dimpot(res)
                                * (Np(res,i)/N(res))**5 + b_dimpot(res)
                                * (Np(res,i)/N(res))**4 + c_dimpot(res)
                                * (Np(res,i)/N(res))**3 + d_dimpot(res)
                                * (Np(res,i)/N(res))**2 + e_dimpot(res)
                                * (Np(res,i)/N(res)) + 1);

NumWells_1(i).. Nw('r1',i) =e= sum(wr1,zw1(wr1,i));
NumWells_2(i).. Nw('r2',i) =e= sum(wr2,zw2(wr2,i));
FieldNumWells(i).. Nw_f(i) =e= sum(res, Nw(res,i));
FieldOilRate(i).. qo_f(i) =e= sum(res, qo(res,i));
FieldCumulativeProd(i).. Np_f(i) =e= sum(res, Np(res,i));
ActiveScenario_1(i).. sum(sr1,x(sr1,i)) =e= 1;
ActiveScenario_2(i).. sum(sr2,y(sr2,i)) =e= 1;
Pipeline_1(i).. pipe('r1',i) =e= sum(sr1,z1(sr1,'pl1')*x(sr1,i));
Pipeline_2(i).. pipe('r2',i) =e= sum(sr2,z2(sr2,'pl2')*y(sr2,i));
```

```
*Inequality Constraints

CumulativeProd_Bound1(res,i+1).. Np(res,i) =l= Np(res,i+1);
CumulativeProd_Bound2(res,i).. Np(res,i) =l= N(res);
OilRate_Bound(res,i).. qo(res,i) =l= qpp(res,i);
BinaryWells_Bound1(wr1,i+1).. zw1(wr1,i+1) =g= zw1(wr1,i);
BinaryWells_Bound2(wr2,i+1).. zw2(wr2,i+1) =g= zw2(wr2,i);
FieldNumWells_Bound(i+1).. Nw_f(i+1) - Nw_f(i) =l= Nwf_year;
Pipeline_Bound1(res,i+1).. pipe(res,i) =l= pipe(res,i+1);

OilCapacity(i).. qo_f(i) =l= qoMax;
GasCapacity(i).. sum(res, qg(res,i)) =l= qgMax;
WaterCapacity(i).. sum(res, qw(res,i)) =l= qwMax;
PipelineMax(i).. sum(res,pipe(res,i))=l= plMax;

DrillEx_1.. PV_d('t1') =e= Fc * sum(res, Nw(res,'t1')*(1+Nw_injprod(res))) * a_drillex;
DrillEx(i+1).. PV_d(i+1) =e= Fc * sum(res, (Nw(res,i+1) - Nw(res,i))*(1+Nw_injprod(res)))
                            * a_drillex;
CapEx_1.. PV_c('t1') =e= Fc * (sum(res, Nw(res,'t1')*(1+Nw_injprod(res)))*a_capex + plMax
                            * pl_capex);
CapEx(i+1).. PV_c(i+1) =e= Fc * sum(res, (Nw(res,i+1) - Nw(res,i))*(1+Nw_injprod(res))) * a_capex;
PreCapEx.. PV_pre =e= Fc * (qoMax * b_capex + qwMax * c_capex + qgMax * d_capex + e_capex);
OpEx(i+1).. PV_o(i+1) =e= Fc * (a_opex + b_opex * Nw_f(i) + c_opex * qo_f(i)
                            + d_opex * sum(res, qw(res,i)) + e_opex * sum(res, qg(res,i)));
Revenue(i+1).. PV_r(i+1) =e= (Po * (Np_f(i+1) - Np_f(i)) * xr * vc);
CashFlow_1.. DCF('t1') =e= -(PV_pre + PV_d('t1') + PV_c('t1'))/(1 + disc)**val('t1');
CashFlow(i+1).. DCF(i+1) =e= (PV_r(i+1) - PV_o(i+1) - PV_c(i+1) - PV_d(i+1))/(1 + disc)**val(i+1);

*Generating model
Model mod3 /all/;

option minlp = dicopt;
Solve mod3 maximizing NPV using minlp;
```

NTNU

Norwegian University of
Science and Technology