A Review on Modelling, Implementation, and Control of Snake Robots

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Abstract

This paper provides an overview of previous literature on snake robot locomotion. In particular, the paper considers previous research efforts related to modelling of snake robots, physical development of these mechanisms, and finally control design efforts for snake locomotion. The review shows that the majority of literature on snake robots so far has focused on locomotion over flat surfaces, but that there is a growing trend towards locomotion in environments that are more challenging, i.e. environments that are more in line with realistic applications of these mechanisms.

Keywords:
Snake robots, Review, Modelling, Implementation, Control

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1. Introduction

Inspired by biological snake locomotion, snake robots carry the potential of meeting the growing need for robotic mobility in unknown and challenging environments. These mechanisms typically consist of serially connected joint modules capable of bending in one or more planes. The many degrees of freedom of snake robots make them difficult to control, but provide traversability in irregular environments that surpasses the mobility of the more conventional wheeled, tracked and legged types of robots.

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Research on snake robots has been conducted for several decades. Early empirical and analytical studies of snake locomotion were reported already in the 1940s by Gray [1], and Hirose developed the world's first snake robot as early as 1972 [2]. In the last 20 years, the literature on snake robots has flourished enormously with numerous proposed approaches to modelling, development, and control of these mechanisms. In this paper, we attempt to provide an overview of these works. The main observation that we hope to convey with this review is that the majority of literature on snake robots so far has focused on locomotion over flat surfaces, but that there is a growing trend towards locomotion in environments that are more challenging, i.e. environments that are more in line with realistic applications of these mechanisms. Maintaining and strengthening this trend is, in the authors' opinion, imperative in order to realize the potential locomotion capabilities of snake robots in the future.

The review is structured according to the title of this paper. In particular, Section 2 presents previous research efforts related to modelling and analysis of snake robots, followed by previous research on physical development of these mechanisms in Section 3, and finally considering previous control design efforts for snake locomotion in Section 4. The paper ends with a discussion of the literature review in Section 5.

2. Modelling and Analysis of Snake Robot Locomotion

This section gives an overview of previous research efforts related to modelling and analysis of snake robot locomotion. The review is structured according to Table 1, which summarizes all papers referred to in this section. The table separates between works that consider snake locomotion from a planar (2D) perspective and works that also include three-dimensional aspects of the motion.

2.1. Biomechanical Studies of Biological Snakes

Research on snake robots is inspired by the robust motion capabilities of biological snakes. These amazing creatures are optimal in the sense that they have emerged through millions of years of evolution. Biomechanical studies of snakes are therefore relevant to research on snake robots.

One of the earliest analytical studies of snake locomotion was given by Gray in [1], where mathematical descriptions of the forces acting on a snake are proposed and used to derive properties of snake locomotion. One of

Table 1: Previous work on modelling and analysis of snake robot locomotion.

Biomechanical studies of biological snakes	
2D perspective	[1], [3], [4].
3D perspective	[2], [5].
Flat surface locomotion with sideslip constraints	
2D perspective	[2], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15].
3D perspective	[16], [17], [18].
Flat surface locomotion without sideslip constraints	
2D perspective	[19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [5].
3D perspective	[32], [33], [34], [35].
Robotic fish and eel-like mechanisms	
2D perspective	[36], [37].
3D perspective	[38], [39], [40], [41], [42], [43].
Locomotion in environments with obstacles	
2D perspective	[44], [45], [46], [47].
3D perspective	[48], [49], [50], [51], [52], [53].

Gray's conclusions was that forward motion of a planar snake requires the existence of external forces acting in the normal direction of the snake body.

Hirose [2] studied biological snakes and modelled the snake body as a continuous curve that could not move sideways (sideslip constraints). A well-known result by Hirose is the formulation of the *serpenoid curve*, which is a mathematical description of lateral undulation (the most common form of snake locomotion). Hirose discovered that a close approximation to the shape of a biological snake during lateral undulation is given by a planar curve whose curvature varies sinusoidally. The serpenoid curve is defined as

$$x(s) = \int\limits_0^s \cos\left(a\cos\left(b\sigma\right) + c\sigma\right) d\sigma, \quad y(s) = \int\limits_0^s \sin\left(a\cos\left(b\sigma\right) + c\sigma\right) d\sigma \tag{1}$$

where $(x(s), y(s))$ are the coordinates of the point along the curve at arc length s from the origin, and where a, b, and c are positive scalars. Hirose also investigated adaptive functions of biological snakes (i.e. sinus-lifting, the α-adaptive principle, and the l-adaptive principle) and proposed mathematical descriptions of how external factors, such as ground friction and temperature, affect the shape of a snake during locomotion. Furthermore, Hirose investigated locomotion efficiency inside a maze, i.e. when the snake touches a wall on each side.

An alternative description of lateral undulation, named the *serpentine curve*, was proposed by Ma in [4], where a mathematical model of the muscle characteristics of snakes is employed to derive the resulting form of the body shape during lateral undulation. Ma showed that snake locomotion according to the serpentine curve has a higher locomotive efficiency than locomotion according to the serpenoid curve. The locomotive efficiency during slip-free motion was defined as the ratio between the tangential and normal directon friction forces on the snake body.

Other interesting studies of snake locomotion include the work in [3], which considers the mechanism by which muscular activity of a snake produces curvature and propulsion. In particular, the muscular activity is studied as a snake interacts with pegs in order to push itself forward. A more recent study given in [5] experimentally investigates the frictional properties of snake skin. In particular, the study shows that the friction coefficient of a snake in the transversal direction of the body is larger than the friction coefficient in the tangential direction. This property is important during forward gliding motion. The study also shows that the weight distribution of a snake during lateral undulation is not uniform, but rather distributed so that the peaks of the body wave curve are slightly lifted from the ground. This is often referred to as sinus-lifting.

2.2. Modelling and Analysis of Flat Surface Locomotion with Sideslip Constraints

As noted in e.g. [1], each part of a biological snake conducting lateral undulation follows the path traced out by the head. This phenomenon is partially explained by the frictional anisotropy of snake skin studied in e.g. [5], but is also caused by irregularities on the surface that provide grip and enable the snake to glide forward without slipping sideways. To mimic this motion, many models of snake robots have been developed under the explicit assumption that the body cannot move sideways (sideslip constraints). This assumption introduces nonholonomic constraints [54] in the equations of motion of the robot. In practice, such conditions are usually achieved by installing passive wheels along the body of the snake robot.

Several works attack the motion control problem of wheeled snake robots with tools from differential geometry. Early approaches of such form are presented in [6, 7], which model the kinematics of wheeled snake robots and analyse the relationship between body shape changes and the resulting displacement of the robot. These works also assess the controllability of such mechanisms. Similar approaches are considered in [8, 9], where also the dynamics of wheeled snake robots is considered, and where system symmetries are utilized to arrive at reduced forms of the model. At a purely kinematic level, the connection between the body velocity $\boldsymbol{\xi}$ of a wheeled snake robot and its shape variables $\boldsymbol{r}$ is written in [8, 9] as

$$\boldsymbol{\xi} = -\boldsymbol{A}(\boldsymbol{r})\dot{\boldsymbol{r}} \tag{2}$$

where $\boldsymbol{A}(\boldsymbol{r})$ is a matrix denoted as the *local connection*. Modelling and controllability analysis of the kinematics of a three-linked wheeled snake robot is also considered in [10]. Furthermore, the concept of a body velocity integral is introduced in [11] in order to easily approximate the net displacement of a snake robot during a gait. The method requires that the system coordinates are properly chosen.

A model of the 2D dynamics of a wheeled snake robot is developed in [12] from Lagrange's equations of motion, and in [13] from first principles. In [12], the model of the snake robot is written in the convenient form

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{p}} + \boldsymbol{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{p}} + \boldsymbol{D}(\boldsymbol{\theta})\dot{\boldsymbol{p}} = \boldsymbol{F}^T\boldsymbol{E}\boldsymbol{u} \tag{3}$$

where $\boldsymbol{\theta}$ and $\boldsymbol{p}$ are the link orientations and the position of the robot, respectively, and where $\boldsymbol{u}$ is a vector of joint torques. The works in [14, 15] present models of the 2D kinematics and dynamics of snake robots, respectively, where some, but not all of the links are wheeled. The wheel-less links correspond to links that are lifted from the ground. Lifting some of the wheeled links is sometimes desirable from a control perspective to make the motion of the robot less constrained. A model of the 3D kinematics of a snake robot that describe the lifting of the links more accurately is presented in [16]. Furthermore, the work in [17] present a model of the 3D dynamics of a snake robot consisting of a grounded base part and a lifted head part (for manipulation purposes), where some, but not all, of the links in the base part are wheeled.

Continuum models of snake robot dynamics, where the snake is treated as a continuous curve that cannot move sideways, are presented in [2, 18]. The model in [2] is planar, while the model in [18] considers the 3D dynamics of the continuous snake robot.

2.3. Modelling and Analysis of Flat Surface Locomotion without Sideslip Constraints

In addition to the many models of snake robots with sideslip constraints, there are also many models that do not enforce such constraints, but instead only assume that the links exhibit *anisotropic* ground friction properties similar to biological snakes. With anisotropic ground friction properties, the friction coefficients describing the friction force in the tangential and normal direction of a link, respectively, are different. Models based on such ground friction properties are generally more complex to analyse than models based on sideslip constraints since there is no longer a direct connection between the body shape changes and the resulting displacement of the robot.

The work in [19] employs the Newton-Euler formulation to develop a 2D model of the dynamics of a snake robot with anisotropic ground friction properties. The ground friction model include both static and dynamic Coulomb ground friction forces. The model of the robot is formulated in two ways, where the first form gives the propulsion of the robot and the joint torques based on knowledge of the body shape changes, whereas the second form gives the propulsion and body shape changes of the robot based on knowledge of the joint torques. The model is extended in [20] to also describe snake locomotion on a slope.

Another model of planar wheel-less snake robot dynamics is developed in [21] from first principles. The model can be written as

$$\boldsymbol{A}\begin{bmatrix}\ddot{\psi}\\ \ddot{\boldsymbol{p}}\end{bmatrix}+\boldsymbol{B}\begin{bmatrix}\dot{\psi}\\ \dot{\boldsymbol{p}}\end{bmatrix}+\boldsymbol{C}\dot{\boldsymbol{\phi}}=\boldsymbol{0} \tag{4}$$

$$\ddot{\boldsymbol{\phi}}+\boldsymbol{D}\left(\boldsymbol{E}\dot{\boldsymbol{\theta}}^{2}+\boldsymbol{F}\dot{\boldsymbol{\theta}}+\boldsymbol{G}\dot{\boldsymbol{\omega}}\right)=\boldsymbol{H}\boldsymbol{u} \tag{5}$$

where $\dot{\psi}$ is a measure of the angular momentum of the robot, $\boldsymbol{p}$ is the position of the robot, $\boldsymbol{\phi}$ is a vector of joint angles, $\boldsymbol{\theta}$ is a vector of link orientations, $\boldsymbol{u}$ is a vector of joint torques, and $\boldsymbol{A}$, $\boldsymbol{B}$, $\ldots$, $\boldsymbol{H}$ are state-dependent system matrices. A nice feature of this model is that the shape motion of the robot in (5) is decoupled from the overall locomotion of the robot in (4). Simulations with this model are carried out in [21] to derive properties of snake robot dynamics. The model from [21] is employed in [22] to study the controllability of the joints of a snake robot under the assumption that one joint is passive. However, the analysis does not consider the position of the robot. A partial feedback linearization of the model from [21] is proposed by the authors in [23] in order to reduce the complexity of the model. The transformed model is employed to study the controllability of snake robot locomotion, and to derive properties related to the motion of a snake robot during lateral undulation. The authors also propose a simplified model of snake locomotion in [24], where the body shape changes of the snake robot are modelled as purely linear displacements of the links. This simplified model is employed in [25] to derive properties of the locomotion velocity during lateral undulation.

Models of planar snake robot dynamics with anisotropic viscous ground friction are presented in [26, 27, 28]. The work in [28] exploits symmetries in the system (cyclic coordinates) to transform the model to a reduced form where the shape dynamics is decoupled from the displacement dynamics of the snake robot, and investigates general requirements for the propulsion of a three-linked snake robot. A friction model that includes both viscous and Coulomb friction forces is proposed and analysed in [29].

A model that considers *isotropic* Coulomb ground friction forces (both static and dynamic friction) is presented in [30]. Isotropic ground friction is also assumed in [31], where a continuum approach along with energy arguments are employed to analyse planar snake locomotion under isotropic friction conditions. In [32], the frictional contact forces between a snake robot and a compliant surface are modelled. The dynamics of planar snake locomotion is described in terms of a continuum model in [5], where the snake is treated as a continuous curve influenced by Coulomb friction forces from the ground. The model is employed to study the effect of anisotropic ground friction properties on the propulsion of snakes.

The 3D dynamics of a snake robot during locomotion across flat surfaces is considered in [33, 34, 35]. The model in [33] is developed from the Newton-Euler formulation and includes both static and dynamic Coulomb ground friction forces. The model is employed to study sinus-lifting during lateral undulation. In [34], the 3D dynamics of a snake robot is modelled by use of standard equations of motion of robotic manipulators, which gives a complete model of the snake robot of the form

$$\boldsymbol{M}\left(\boldsymbol{q}\right)\ddot{\boldsymbol{q}} + \boldsymbol{C}\left(\boldsymbol{q}, \dot{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} + \boldsymbol{g}\left(\boldsymbol{q}\right) = \boldsymbol{u} + \boldsymbol{u}_{\text{ext}} \tag{6}$$

where $\boldsymbol{q}$ and $\boldsymbol{u}$ are the generalized coordinates and forces of the robot, respectively, and $\boldsymbol{u}_{\text{ext}}$ represents the effect of the external forces on the robot from the environment. A set of virtual links with zero mass are employed at the base of the manipulator to model the rotational and translational degrees of freedom of the robot. The work in [35] models snake robot dynamics by use of the framework of nonsmooth dynamics. The model, which represents a hybrid system, describes the normal direction contact forces from the ground and the Coulomb ground friction forces by use of set-valued force laws.

2.4. Modelling and Analysis of Robotic Fish and Eel-like Mechanisms

Research on robotic fish and eel-like mechanisms is relevant to research on snake robots since these mechanisms are very similar. A complete treatment of robotic underwater locomotion is beyond the scope of this review. However, a representative part of previous research related to modelling of such mechanisms is presented in the following.

A model of eel-like motion is developed in [36] based on tools from differential geometry that were also considered in some of the works concerning wheeled snake robots described above. However, the model does not place sideslip constraints on the robot. Instead, the eel-like mechanism is propelled by hydrodynamic forces modelled by a viscous friction model. The dynamics of eel-like motion is also considered in [37], where model reductions are proposed to allow the net motion of the robot to be described as a sum of geometric and dynamic phases over closed curves in the shape space, and in [38], where a continuum model is formulated based on beam theory, and in [39], where first principles are employed to model the dynamics of a swimming snake robot. The works in [40, 41, 42, 43] model the dynamics of a robotic fish influenced by lift and drag forces in an inviscous fluid. The controllability of the fish-like mechanism is also assessed in these works. A single actuated swimming robot is considered in [55], where a simulation model of the robot is presented along with a simulation study of its motion parameters.

2.5. Modelling and Analysis of Locomotion in Environments with Obstacles

In [48, 49], the kinematics of snake robots is modelled in terms of a continuous backbone curve that captures the macroscopic geometry of the robot. Gaits for the backbone curve, which determine the shape of the snake robot, are specified with respect to environment constraints and the desired locomotion trajectory of the robot. The approach is original in that the problem of locomotion in cluttered environments is attacked at a purely kinematic level. The work by Chirikjian and Burdick is extended in [50], where a continuum kinematics model is presented that explicitly handles the case of backbone curves that can be bent, but not twisted. This condition is in line with most physical snake robots, which are generally able to bend, but not twist their body. The kinematic constraints imposed on a snake robot due to external obstacles are modelled in [44, 51]. These works also analyse how obstacles around a snake robot affect its degrees of freedom.

The only known works that consider the dynamics of snake robots in environments with obstacles (i.e. where obstacle contact forces are considered) are presented in [45, 52, 46, 53, 47]. In [45], a dynamic simulation software called *WorkingModel* is used to simulate a planar snake robot interacting with circular obstacles. Contact forces are calculated from a spring-damper approximation. A similar approach is employed in [52], where the simulation software *Open Dynamics Engine* (ODE) is used to model a snake robot interacting with various forms of obstacles. The work in [46] uses the multi-body dynamics simulation software *Autolev* to study the motion of a snake robot during contact with a single peg, where the contact with the peg is modelled as a spring-damper system. The works in [45, 52, 46] do not provide the equations underlying the dynamics of the snake robot due to the use of general-purpose simulation software. On the other hand, the models proposed in [53] and [47], respectively, are, to our best knowledge, the only works which explicitly present the equations of motion underlying the obstacle interaction dynamics of a snake robot. The model in [53] is formulated within the framework of nonsmooth dynamics. A timestepping method is used to simulate the dynamics of the robot, which means that the system equations are discretized with a time step determined by a fixed error criterion, and trajectories of the system are approximated without tracking events (i.e. obstacle impacts). The model in [47], on the other hand, is based on tracking discrete events and is formulated within the hybrid modelling framework described in [56]. Obstacle interaction is modelled in [47] by introducing a unilateral velocity constraint on each contacted link of the snake robot, and the complete model is written in the form

$$\begin{array}{rclcl} \dot{\boldsymbol{x}} & = & \boldsymbol{F}(\boldsymbol{x},\boldsymbol{u}) & \text{for all} & \boldsymbol{x} \in \boldsymbol{C} \\ \boldsymbol{x}^{+} & = & \boldsymbol{G}(\boldsymbol{x}) & \text{for all} & \boldsymbol{x} \in \boldsymbol{D} \end{array} \tag{7}$$

where the interaction of the robot with its environment determines if the state vector $\boldsymbol{x}$ evolves continuously according to the flow map $\boldsymbol{F}$ or if it jumps to a new value $\boldsymbol{x}^{+}$ according to the jump map $\boldsymbol{G}(\boldsymbol{x})$.

3. Implementation of Physical Snake Robots

In this section, we give an overview of previous literature that considers implementation of physical snake robots. The locomotion of these mechanisms in unknown and cluttered environments relies heavily on the ability to sense the interaction with their surroundings. We have therefore chosen to separate the works that consider snake robtos *with* contact force sensors from the works that do *not* include such sensor capabilities in the robot design. With this partitioning, the section clearly illustrates that previous research on environment sensing for snake robots is limited. The referred works are summarized in Table 2, which separates between snake robots with passive wheels, which are advantageous during motion across flat surfaces, snake robots without such passive wheels, and snake robots equipped with active propulsion.

3.1. Snake Robots without Contact Force Sensors

Hirose developed the world's first snake robot as early as 1972 [2]. The robot, which is shown in Fig. 1, was equipped with passive wheels to realize the anisotropic ground friction property that enables forward locomotion on flat surfaces.

Several other snake robots with passive wheels have been proposed over the years, such as the robots presented in [57], [58], [59], [60], [61] (see Fig. 2), [62] (see Fig. 3), [63], [64] (see Fig. 4), [65], [66], [67], [68], and [69]. Some of the robots can only display planar motion, while other robots can

Table 2: Previous work on implementation of physical snake robots.

Snake Robots without Contact Force Sensors	
With passive wheels	[57], [58], [59], [60], [61], [62], [63], [64], [65], [66], [67], [68], [69].
Without passive wheels	[70], [71], [72], [73], [74], [75], [76], [21], [77], [78], [79], [80], [81], [82], [83].
With active propulsion	[84], [85], [86], [87], [88], [89], [90], [91], [92].

Snake Robots with Contact Force Sensors	
With passive wheels	[2], [93].
Without passive wheels	[94], [95], [96], [97], [98].
With active propulsion	[86]

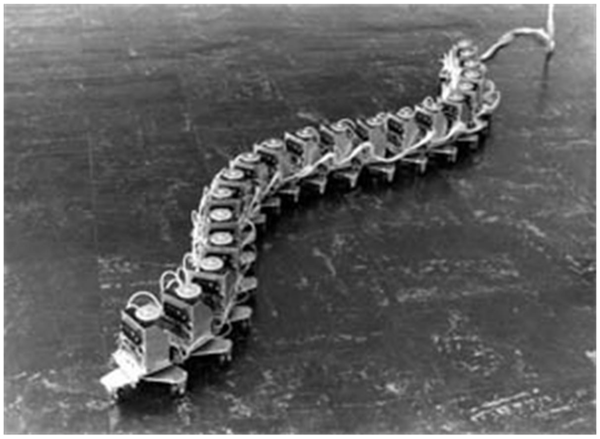

Figure 1: The snake robot *ACM III*, which was the world's first snake robot developed by Prof. Shigeo Hirose in 1972. Courtesy of Tokyo Institute of Technology.

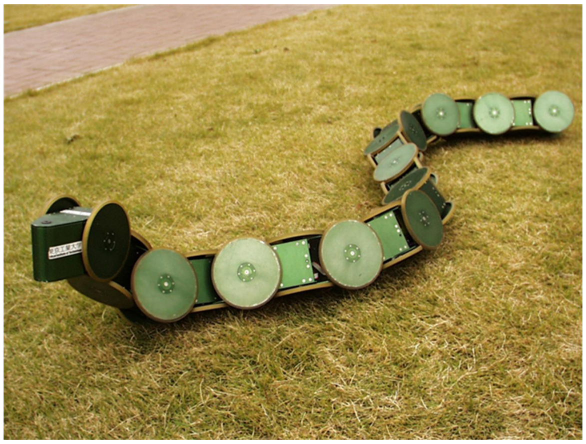

Figure 2: The snake robot *ACM R3* developed at Tokyo Institute of Technology. The robot is covered with passive wheels. Courtesy of Tokyo Institute of Technology.

move their links both horizontally and vertically. Some robots have shielded joint modules that enable motion in environments with e.g. mud and dust, and even motion under water (see [64] and Fig. 4), while other robots have modules with exposed electronic components which only allow them to move in clean lab environments. A common feature of these mechanisms, however, is that they are generally only able to move across relatively flat surfaces since passive wheels do not move very well in a cluttered environment. Such mechanisms are therefore suitable for motion control experiments on relatively flat surfaces, but not for practical applications of snake robots in more challenging environments.

Snake robots without passive wheels, i.e. robots that basically consist of straight links interconnected by motorized joints, are presented in [70], [71], [72], [73], [74], [75], [76], [21], [77], [78], [79], [80], [81], [82], and [83]. Despite its lack of wheels, the snake robot in [21] maintains an anisotropic ground friction property since the underside of each link has edges, or grooves, that run parallel to the link. This robot can therefore move forward by lateral undulation through purely planar motion. Robots whose ground friction properties are isotropic, on the other hand, can move forward during lateral undulation by resorting to sinus-lifting, i.e. by slightly lifting the peaks of the body wave curve from the ground, as demonstrated in [76, 83]. However, snake robots with isotropic friction are mostly used for studying gaits

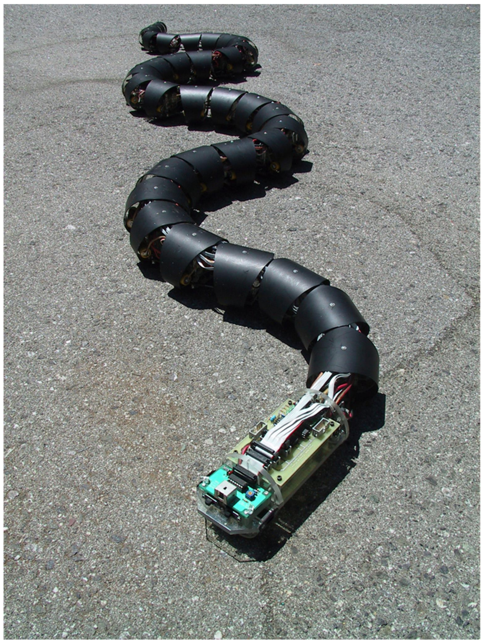

Figure 3: The snake robot *S5* developed by Dr. Gavin Miller. The robot has passive wheels on its underside. Courtesy of Dr. Gavin Miller.

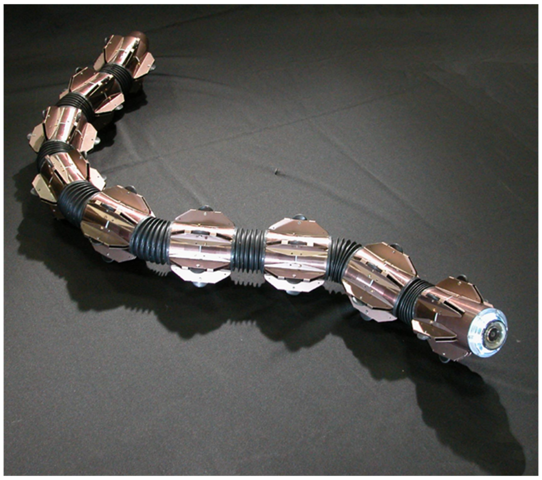

Figure 4: The snake robot *ACM R5* developed at Tokyo Institute of Technology. The robot is covered by passive wheels and can swim under water. Courtesy of Tokyo Institute of Technology.

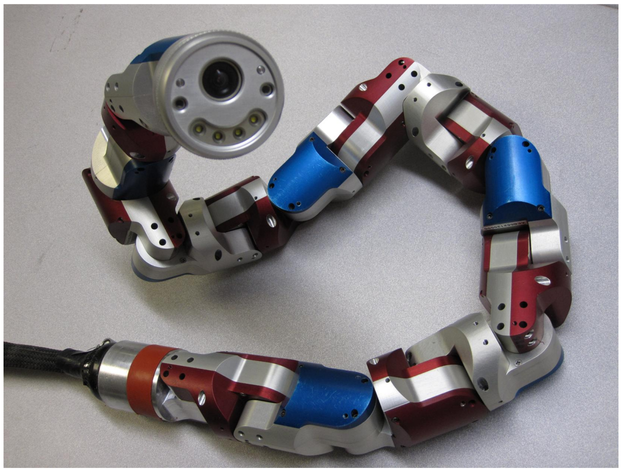

Figure 5: The snake robot *Uncle Sam* developed at Carnegie Mellon University. The robot has a strong and compact joint mechanism and can climb trees. Courtesy of Carnegie Mellon University.

other than lateral undulation, such as gaits based on sidewinding, inchworm motion, or lateral rolling. A notable feature of the works presented in [79] (see Fig. 5) and [81] (see Fig. 6) is the focus on development of small, light-weight, and strong joint actuation mechanisms, which are important for many future applications of snake robots.

There are also works that consider active propulsion along the body of a snake robot, for example by equipping each link with motorized wheels [84, 85, 86], or by installing tracks along the body of the snake robot [87, 88, 89, 90, 91], or by employing a screw drive mechanism [92]. The robots presented in [89] and [91] are shown in Fig. 7 and Fig. 8, respectively.

3.2. Snake Robots with Contact Force Sensors

Previous research on environment sensing for snake robots is limited. The wheeled snake robot developed by Hirose already in 1972 [2] was equipped with contact switches, which enabled the robot to demonstrate lateral inhibition with respect to external obstacles. A snake robot with active wheels, where each wheel axis is equipped with a 3-axial force sensor, is presented in [86]. The force sensor measures the translational forces on the wheel axis based on optical range measurements. [94] presents a wheel-less snake robot with contact switches and presents experimental results where the robot is

Figure 6: A snake robot with a miniature joint mechanism developed at Tokyo Institute of Technology. Courtesy of Tokyo Institute of Technology.

Figure 7: The *OmniTread* snake robot developed at the University of Michigan. The robot has pneumatic joints and is covered by motorized tracks. Courtesy of the University of Michigan.

Figure 8: A snake robot with a skin drive propulsion system developed at Carnegie Mellon University. A motor drives the outer skin backwards along the snake body in order to propel the robot forward. Courtesy of Carnegie Mellon University.

propelled forward by pushing against pegs that are detected by the contact switches. A snake robot with passive wheels and strain gauge sensors is proposed in [93], where the strain gauge sensors are shown to successfully measure the constraint forces on the wheels. Ideas related to environment sensing for snake robots are considered in [95], where the preliminary design of a capacitive contact sensor is proposed that can be wrapped around each module of a snake robot. Snake robots with joint modules covered by force sensors are proposed by the research group of the authors in [96, 97, 98]. The snake robots in [96, 97] have cylindrical joint modules and their respective force sensing systems are able to detect and, to some extent, assess the magnitude of external forces applied at certain areas of the joint modules. The snake robot in [98], on the other hand, has ball-shaped joint modules with force sensors mounted underneath the shell of each module (see Fig. 9). The outer surface of the robot is smooth, thereby allowing gliding motion in cluttered environments. To our best knowledge, this is the first reported snake robot that can measure the magnitude of external forces applied along its body.

4. Control of Snake Robots

This section gives an overview of previous research efforts related to control of snake robot locomotion. The review is structured according to Table 3, which summarizes all papers referred to in this section. The table separates

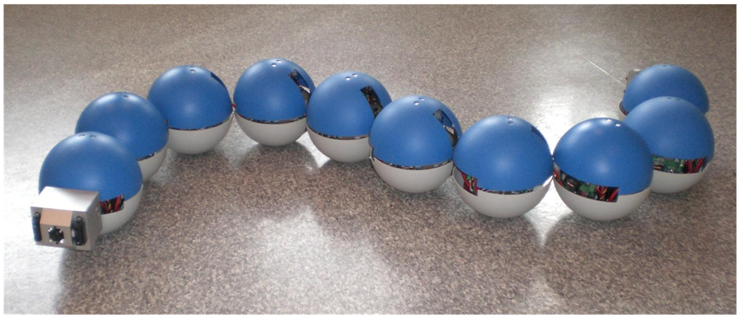

Figure 9: The snake robot *Kulko* developed at the Norwegian University of Science and Technology. Each joint module is covered by force sensors in order to measure contact forces from the environment.

between works that present gait patterns without explicitly controlling the position or heading of the snake robot and works that present gait patterns along with position and/or heading controllers.

Note that the primary focus of the review is on control design efforts based on the gait pattern *lateral undulation*. This is the fastest and most common form of snake locomotion, and is considered in the majority of previous research on snake robots. Moreover, we consider lateral undulation to be most relevant to motion in cluttered and challenging environments, which is generally the main motivation behind research on snake robots. Lateral undulation is achieved by creating continuous body waves that are propagated backwards from head to tail. During this wave motion, the sides and underside of the snake robot push against the environment so that the robot is propelled forward. A well-known and common approach for achieving lateral undulation is to control the snake robot according to the serpenoid curve proposed by Hirose [2]. In particular, Hirose proposed that lateral undulation is realized by controlling each joint of the snake robot according to the sinusoidal reference

$$\phi_{i,\mathrm{ref}} = \alpha \sin\left(\omega t + (i-1)\,\delta\right) + \phi_o \tag{8}$$

where $\phi_{i,\mathrm{ref}}$ is the reference angle of the ith joint, α and ω are the amplitude and frequency, respectively, of the sinusoidal joint motion, δ determines the phase shift between the joints, and ϕ_o is a joint offset used to control the direction of the motion.

Remark 1. *Stability analysis of control laws for snake robots is challenging* *due to the complexity of existing models of these mechanisms. For this reason, applications of formal stability analysis tools in previous snake robot literature are very limited. Simulations and experimental investigations are instead the common approach in the literature for providing support of proposed control strategies.*

4.1. Controllers for Flat Surface Locomotion with Sideslip Constraints

A majority of previous control design efforts for snake robots has focused on locomotion where the links are subjected to nonholonomic constraints, i.e. where each link is constrained from moving sideways. A snake robot that uses solenoids for attachment to the environment is considered in [44] along with gaits for forward and turning motion of this mechanism. Tools from differential geometry are employed in [7, 9] to demonstrate that sinusoidal shape inputs to wheeled snake robots lead to propulsion.

A position and path following controller for a wheeled snake robot is proposed in [102]. The stability of the controller is proved using a Lyapunov function candidate of the form

$$V = \frac{1}{2}\dot{\boldsymbol{p}}^T \boldsymbol{M} \dot{\boldsymbol{p}} + \frac{1}{2}(\boldsymbol{p} - \boldsymbol{r})^T \boldsymbol{B} (\boldsymbol{p} - \boldsymbol{r}) \tag{9}$$

where $\boldsymbol{p}$ and $\boldsymbol{r}$ are the actual and the desired position of the robot, respectively. The work also considers approaches for preventing the snake robot from attaining a straight shape, which is singular with respect to propulsion. The works in [103, 104, 105] propose path following controllers for wheeled snake robots aimed at minimizing the lateral constraint forces on the wheels during lateral undulation. The controllers are based on a measure of dynamic manipulability, which describes the ability of the robot to generate propulsive force. One approach is based on specifying the desired acceleration of the head a_{head} as the weighted sum

$$\boldsymbol{a}_{\text{head}} = w_1 \boldsymbol{a}_{\text{path}} + w_2 \boldsymbol{a}_{\text{man}} \tag{10}$$

where w_1 and w_2 are design parameters, $\boldsymbol{a}_{\text{path}}$ is a desired head acceleration aimed at making the robot track a desired path, and $\boldsymbol{a}_{\text{man}}$ is a desired head acceleration aimed at maintaining high dynamic manipulability. A related approach is employed in [106], which proposes a gait pattern aimed at minimizing the lateral constraint forces on the wheels, and in [18], which formulates and solves an optimization problem in order to minimize the torque

Table 3: Previous work on control of snake robot locomotion.

Flat surface locomotion with sideslip constraints	
Without position or heading control	[44], [7], [9], [18], [99], [13], [100], [101].
With position and/or heading control	[102], [103], [104], [105], [106], [14], [16], [107], [15], [108], [17], [60], [109], [110], [10], [111], [112], [113], [114].
Flat surface locomotion without sideslip constraints	
Without position or heading control	[73], [74], [19], [33], [21], [115], [30], [116], [117], [118], [67], [34], [119], [49], [120], [70], [71], [76], [121], [122], [61], [123], [82].
With position and/or heading control	[28], [124], [23], [125].
Robotic fish and eel-like mechanisms	
Without position or heading control	[40], [126], [66].
With position and/or heading control	[127], [36], [41], [42], [43].
Locomotion in environments with obstacles	
Without position or heading control	[2], [128], [80], [129], [130], [69], [131], [132], [78], [133], [134].
With position and/or heading control	[45], [94], [47], [135], [46], [136].

input. The optimization problem is solved using a 3D continuum model of the snake robot.

In [14, 16, 107, 15], position and path following controllers are proposed for the case where some, but not all, of the snake robot links are wheeled. The wheel-less links correspond to links that are lifted from the ground, which give the system more degrees of freedom that can be utilized to follow a trajectory while simultaneously maintaining a high manipulability. Similar approaches are considered in [108, 17, 99], where also strategies for sinus-lifting during lateral undulation are proposed.

A gait based on a self-excitation principle is proposed in [13], where joint angle information determines the winding motion of a snake robot. Directional control during lateral undulation is considered in [60, 109]. The work in [110] proposes a position controller for a wheeled snake robot that takes ground friction forces into account. A similar approach is employed in [100], where deviations of the joint angles from their setpoints are used to modify the oscillatory joint motion, thereby enabling the snake robot to automatically adapt its motion to variations in the ground friction conditions. The works in [10, 111] propose position and path following controllers for three-linked and four-linked wheeled snake robots based on Lie bracket calculations and controllability analysis results. The concept of passive creeping is considered in [101], which involves adjusting the motion of a snake robot based on a measure of the dissipated energy, thereby achieving adaptation of the motion to different surface conditions. Local orbital stability of state trajectories during the motion is concluded based on recurrence plots.

A snake robot with *active* wheels is considered in [112, 113], where an optimization scheme is employed to make the robot follow the path that minimizes energy dissipation due to friction forces. Active wheels are also assumed in [114], where a path following controller for such snake robots is proposed on a kinematic level.

Remark 2. *The works in e.g. [102, 103, 105, 14, 16, 15], which were described above, all employ a common approach for motion control in that the nonholonomic constraints on the links are used to establish an explicit connection between body shape changes and propulsion of the form*

$$\boldsymbol{A}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} - \boldsymbol{B}(\boldsymbol{\theta})\dot{\boldsymbol{p}} = \boldsymbol{0} \tag{11}$$

where $\boldsymbol{\theta}$ is a vector containing the orientation of each link and $\boldsymbol{p}$ is the position of the snake robot. This mathematical relationship allows the control *input to be specified directly in terms of the desired propulsion of the robot. Such approaches are, to our best knowledge, the only known approaches for motion control of wheeled snake robots which infer some formal and model-based conclusions on the motion of the robot.*

4.2. Controllers for Flat Surface Locomotion without Sideslip Constraints

The works in [73, 74] employ Fourier series to specify periodic functions representing the gait patterns of a wheel-less snake robot. The parameters of the Fourier series are determined using certain learning techniques. In [19], computer simulations are employed to study properties of lateral undulation related to the optimality of the motion. A control strategy for sinus-lifting during lateral undulation is proposed in [33] by solving a quadratic optimization problem. Snake robots influenced by anisotropic ground friction are considered in [21], where the gait parameters of the lateral undulation motion in (8) are optimized based on simulations. The work also proposes a forward velocity controller for wheel-less snake robots by introducing a new control input $\dot{\eta}$ defined as

$$\eta := \omega t, \quad \dot{\eta} = \dot{\omega} t + \omega. \tag{12}$$

By inserting (12) into (8) and also specifying $\dot{\eta}$ appropriately, the authors develop a BIBO stable mapping between $\dot{\eta}$ and the forward speed of the robot. The works in [115, 30] consider several elementary motions for planar snake robots and derive conditions for the feasibility of these motions, such as required actuator strength. In [28, 124], methods based on numerical optimal control are considered for determining optimal gaits during positional control of snake robots influenced by anisotropic viscous ground friction. The work in [116] proposes a general joint angle controller for planar snake robots influenced by anisotropic Coulomb ground friction and proves that the resulting translational and rotational velocity of the robot is bounded. Straight line path following control of planar snake robots is considered by the authors in [23, 125]. In these works, the heading $\overline{\theta}$ of the snake robot is controlled according to a heading reference of the form

$$\overline{\theta}_{\text{ref}} = -\arctan\left(\frac{p_y}{\Delta}\right) \tag{13}$$

where p_y is the distance to the desired path and $\Delta > 0$ is a design parameter referred to as the *look-ahead distance*. Exponential stability of this path following controller is proved in [23] by use of a Poincaré map, while cascaded systems theory is employed in [125] to prove that the proposed controller $\mathcal{K}$-exponentially stabilizes the snake robot to any desired straight path.

The following works consider other gait patterns than lateral undulation, and the gaits are carried out in open-loop without explicitly controlling the position and orientation of the snake robot. Gaits for sidewinding motion, which is a sideways rolling type of motion, are proposed in [117, 118, 67, 34, 119]. Inchworm locomotion gaits are proposed in [49, 120, 70, 71, 76, 121, 118, 122]. Lateral rolling, which is achieved by continuously forming the snake body into a vertical U-shape that tips over, is considered in [73, 76, 61, 123, 118]. Furthermore, gaits for loop forming motion are proposed in [70, 71, 82], where the head and tail of the snake robot are connected to turn the robot into a rolling wheel.

Remark 3. *To our best knowledge, the works by the authors in [23, 125] are the only works in the snake robot literature which present formal mathematical proofs regarding positional control of wheel-less snake robots.*

4.3. Controllers for Robotic Fish and Eel-like Mechanisms

A complete review of previous control efforts related to robotic underwater locomotion is beyond the scope of this paper. However, we consider the following works to be representative of previous research related to control of such mechanisms.

Eel-like motion is considered in [127, 36], where controllers for tracking straight and curved trajectories are proposed. The works in [40, 41, 42, 43] consider motion control of robotic fish. Lie bracket calculations based on the dynamics of the robotic fish are used to derive gaits for forward motion and various forms of turning motion. Algorithms for closed-loop heading and depth control are also considered. Open-loop gaits for a robotic fish are proposed in [126] based on curvature plots of the mechanical connection between the shape space motion and the overall displacement of the robot. A CPG-based control approach for a robotic fish is presented in [137]. A swimming snake robot is considered in [66], where a gradient-free optimization method is employed to adjust the gait parameters online, i.e. while the robot is moving, in order to maximize the forward veloctiy.

4.4. Controllers for Locomotion in Environments with Obstacles

Similar to a biological snake, a snake robot achieves locomotion in cluttered and unstructured environments by using external objects (or obstacles) as push points to aid the propulsion. The term *obstacle-aided locomotion* was introduced in [53] to fully embrace this form of motion. Only a few works in previous literature consider control strategies for obstacle-aided locomotion (i.e. locomotion in environments with obstacles).

To our best knowledge, the works in [2, 45, 94, 47] present the only control strategies where contact force sensing is employed in the feedback loop. In [2], a strategy for lateral inhibition is proposed that modifies the shape of a snake robot based on contact force sensing along the snake body in order to avoid obstacles. An inverse dynamics approach is proposed in [45], where an optimization problem is formulated and numerically solved in order to, for a given set of obstacle contacts, calculate the contact forces required to propel the robot in a desired direction. A strategy for calculating the actual torque inputs to the joints from the desired contacts was, however, not presented. A kinematic approach is proposed in [94], where a curve fitting procedure is used to determine the shape of the robot with respect to the detected obstacles. Subsequently, this shape is propagated backwards along the snake body under the assumption that this will push the robot forward. In [47], the authors propose a hybrid controller for obstacle-aided locomotion aimed at resolving situations where the snake robot is jammed between obstacles. Experimental results are presented in [135], where the proposed controller is shown to successfully propel a physical snake robot through various obstacle courses.

Sensing the environment of a snake robot must not necessarily involve contact force sensing since the environment can be indirectly sensed through the joint angle measurements and/or the actuator torques. This approach is considered in [46], where the joint torques of a snake robot are specified solely in terms of the measured joint angles to achieve motion through a winding corridor. In the case of strictly planar motion, the control strategy proposed in [46] suggests that the optimal bending torque at the ith joint along the snake robot, denoted by τ_i, is given by

$$\tau_i = K\left(v_d - v\right)\left(\phi_{i-1} - \phi_i\right) \tag{14}$$

where K is a controller gain, v_d and v are the desired and the actual forward velocity of the robot, respectively, and ϕ_i is the angle of the $i$the joint. A related approach is considered in [128], which presents a control strategy that uses motor current measurements to adjust the shape of a snake robot moving through an elastically deformable channel, and in [80], where the deviations of the joint angles from their setpoints are used to adapt the body shape of a snake robot moving inside pipe structures.

The remaining works presented in the following consider controllers aimed at locomotion in environments that are not flat, but do not appear to involve sensing of the *interaction* between the snake robot and its environment. In [129], a fuzzy logic controller is employed to switch between various predefined gaits during motion in an obstacle environment. The goal of the motion controller is to avoid the obstacles. In [130], an algorithm is presented that takes contact constraints on a snake robot into account in order to compute the joint torques that produce the desired motion. The algorithm is applied to achieve climbing motion with a snake robot. A gait for climbing motion is also proposed in [69]. Range sensor measurements are used in [136] to centre a crawling snake robot between the walls of a corridor. The work in [131] analyses the efficiency of earthworm-like motion on compliant surfaces motivated by biomedical applications of worm robots. Moreover, various gaits aimed at motion in unstructured environments, including climbing gaits, are proposed in [132, 78, 133, 134].

5. Discussion of the Literature Review

Based on the presented literature review, we end this paper with an elaboration of what we consider to be the most significant research challenges that must be addressed before we will ever see useful snake robots outside the laboratory.

5.1. Research Challenges related to Modelling and Control of Snake Robots

Future applications of snake robots will generally require these mechanisms to move and operate in unknown and unstructured environments. To this end, the ability of the robot to *sense* its environment and *adapt* its body shape and movements accordingly is essential. However, to our best knowledge, non-planar locomotion in unstructured environments based on environment sensing and body shape adaptation has *not* yet been studied in the literature. Our primary claim is therefore that future applications of snake robots require significantly more research on adaptive behaviour during motion in unknown and cluttered environments.

The literature review shows that the majority of existing models of snake robots considers motion over flat surfaces. The main differences between these models concern assumptions regarding properties of the modelled snake robot (e.g. purely planar motion versus fully three-dimensional motion, links with nonholonomic constraints versus links with anisotropic friction properties, discrete links versus a continuum perspective, etc.). An important observation from the literature review is that models of snake robot locomotion in non-planar environments, i.e. in environments that are more in line with realistic applications of these mechanisms, are very limited. In fact, the models proposed in [53, 47] are, to our best knowledge, the only works which explicitly present the equations of motion underlying the motion of a snake robot in an environment with external objects. In addition, there are a few works that employ general-purpose simulation software to simulate locomotion in unstructured environments (see [45, 52, 46]). A drawback of this approach, however, is that the simulation software does not provide the equations underlying the dynamics of the snake robot, which makes controller design and analysis difficult. In the authors' opinion, more research is needed on simple and analysable models of snake robot locomotion in cluttered environments in order to increase our understanding of how these mechanisms should be controlled under such challenging conditions.

Future control design efforts for snake robot locomotion should go beyond pure heuristics and instead base the controllers on analysable mathematical models and well established control design techniques. As indicated by the literature review, applications of formal stability analysis tools in previous snake robot literature are very limited. Simulations and experimental investigations are instead the common approach in the literature for providing support of proposed control strategies. Unfortunately, model-based control design for snake robots is a major challenge. In particular, the dynamics of snake locomotion across flat surfaces is very complex due to the many degrees of freedom of the robot (see e.g. the models in [21, 33, 116]). When contact forces from an unstructured environment are included, the model becomes even more complex because the discrete nature of the contact forces turns the model of the robot into a hybrid system (see e.g. the models in [53, 47]). However, model-based control design can be achieved by pursuing simplified mathematical descriptions of the motion of snake robots that can be analysed from a control perspective. For instance, with a simple description of how the environment interaction affects the motion of a snake robot, it is possible to analytically derive the control action that, in a given environment, will propel the robot in a desired direction. The path following controller proposed by the authors in [125] is an example of how a simplified modelling approach can be employed to derive model-based control strategies for snake robots.

5.2. Research Challenges related to Development of Physical Snake Robots

Although research on snake robots is motivated by their potential ability to move in unstructured environments, the literature review shows that the majority of existing snake robots have been developed to move over flat surfaces. More research is therefore required on hardware solutions that will enable snake robots to move also in environments that are not flat. As explained in the previous subsection, snake robot locomotion in unknown and unstructured environments requires that the robot can *sense* its environment and *adapt* its body shape and movements accordingly. Measuring external contact forces on the snake robot is a natural approach for sensing the environment, but is quite challenging since the robot is articulated. The literature review shows that only a few works propose contact force sensing solutions for snake robots (see e.g. [2, 94, 98]).

The large majority of snake robots developed so far do not have a smooth exterior surface. However, in order to achieve forward gliding motion in irregular environments similar to the motion of biological snakes, then a sufficiently smooth exterior surface is essential since any irregularities along the body may potentially induce large obstructive friction forces on the robot. Obtaining a smooth surface combined with contact force sensing at articulated parts of the robot represents a significant design challenge. The snake robot presented in [98] illustrates a proposed solution to this challenge. The friction forces opposing the motion of a snake robot can also be limited by introducing active propulsion along the body. This approach is employed by the snake robot with active tracks presented in [89] and by the skin drive mechanism described in [91]. Whether active propulsion or simply a smooth body surface is the best solution for future applications of snake robots, is still an open question. However, a drawback of active propulsion along the body of a snake robot is that the mechanical complexity of the robot is significantly increased. In our opinion, the ideal solution is a snake robot with a passive and smooth tactile skin that can glide forward like a biological snake. Mechanism simplicity is important to the future use of snake robots since this increases the reliability and reduces the development cost of the robots.

In order to move in unstructured environments, a snake robot must generally be able to lift parts of its body. This means that there is some lower limit to the ratio between the strength of the actuators and the weight of the robot. Developing joint mechanisms for snake robots where this ratio is maximized is an important design challenge, and is addressed by the works in e.g. [79, 81].

Applications of snake robots outside the generally clean lab environments require the robots to operate despite of mud and dirt in their environment. Water resistance is also generally a great advantage. Dustproofing and waterproofing techniques for snake robots are therefore important design challenges that should be addressed. The works in [64, 85, 68] present snake robots that can operate under water.

In summary, there is a growing trend in the snake robot literature towards locomotion in environments that are more challenging than the flat surface conditions assumed in the majority of the literature so far. Maintaining and strengthening this trend is imperative in order to realize the potential locomotion capabilities of snake robots in the future.

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