Aziz Sido

Parametric structural design of Catalan vaulting

Master's thesis in Civil and Environmental Engineering
Supervisor: Nils Erik Anders Rønnquist
Co-supervisor: Sverre Magnus Haakonsen
August 2021

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TITLE:

Parametric structural design of Catalan vaulting
Parametrisk strukturell modellering av den katalanske hvelvingen

BY:

Aziz Sido

SUMMARY:
This thesis investigates one parametric workflow using parametric tools to design and analyze Catalan arches. It is well-established that the Catalan vaulting technique has been used from history until today with some challenges in understanding the structural behavior. This thesis aims to determine the structural performance of such structures using parametric approaches. Specifically, it investigates the workflow to design Catalan arches and factors that can impact the structural performance under design. In this context, the Catalan vault is also known as a ``Masonry vault'' ``Gustavino Vault'', ``timbrel vault'', ``thin-tiled vault'', ``laminated vault'', ``flat vault'', ``layered vault and ``Catalan turn'' is one type of structure which is build of one or series of arches. Catalan arches depend on laying thin bricks(tiles) lengthwise alternated with a layer of mortar based on gypsum or lime. This technique in the building is called vaulting. Catalan vaulting is undergoing accelerated innovation on materials developments and technological approaches used for design and analysis. To better understand this technique, one comparison has been made between the results carried out from one experimental test done on Catalan arches in Portugal in 2014 with the results gotten from the model designed in this thesis using parametric tools. Parametric tools as Grasshopper, Karamba3D, and Rhino were used to design and analyze one parametric model. The results showed that many stages should be done in parametric design to get one model that simulates the Catalan arches. As establishing geometry, the support points, cross-sections, loads, rigidity of joints should be determined and discussed carefully before assembling the final model and starting analyzing to get structural results from the model.
I conclude that cross-section dimensions and properties of materials, the rigidity of joints, geometrical form, and arch dimensions are critical factors that must be considered in building Catalan arches in a parametric environment.

RESPONSIBLE TEACHER: Nils Erik Anders Rønnquist

SUPERVISOR(S): Sverre Magnus Haakonsen

CARRIED OUT AT: Department of Structural Engineering, NTNU, Trondheim

Preface

This master thesis was written as a concluding part of my Master of Science Degree at the Norwegian University of Science and Technology (NTNU), Department of structural engineering. I was engaged in the research and writing of this thesis from January 2021 to August 2021.

During my years at NTNU, I have been exposed to many structural design exercises and theoretical problems. The last semester before initiating my master thesis, I had two subjects related to parametric design and analysis of construction using parametric tools and a parametric design course that helps me use new tools in designing and analyzing structure. The desire to explore this field more could motivate me to write in conceptual structural design.

I want to begin by expressing my sincere gratitude to my advisors, Professor Anders Rønnquist, and special thanks to Ph.D. candidate Sverre Magnus Haakonsen for helping me formulate the research question and guiding me through this semester.

Indeed, I would like to express my appreciation and love to my parents and my siblings for their consistent and unlimited support throughout my studies and my life in general, who despite the long distances and difficult circumstances there, still supply me with more than enough power to pass the obstacles and build my future. Last but not least, my sense of gratitude to one and all, who directly or indirectly, have lent their hand in this venture.

Trondheim August, 2021

Aziz Sido

Abstract

This thesis investigates one parametric workflow using parametric tools to design and analyze Catalan arches. It is well-established that the Catalan vaulting technique has been used from history until today with some challenges in understanding the structural behavior.

This thesis aims to determine the structural performance of such structures using parametric approaches. Specifically, it investigates the workflow to design Catalan arches and factors that can impact the structural performance under design.

In this context, the Catalan vault is also known as a “Masonry vault” “Gustavino Vault”, “timbrel vault”, “thin-tiled vault”, “laminated vault”, “flat vault”, “layered vault and “Catalan turn” is one type of structure which is build of one or series of arches. Catalan arches depend on laying thin bricks(tiles) lengthwise alternated with a layer of mortar based on gypsum or lime. This technique in the building is called vaulting. Catalan vaulting is undergoing accelerated innovation on materials developments and technological approaches used for design and analysis.

To better understand this technique, one comparison has been made between the results carried out from one experimental test done on Catalan arches in Portugal in 2014 with the results gotten from the model designed in this thesis using parametric tools. Parametric tools as Grasshopper, Karamba3D, and Rhino were used to design and analyze one parametric model.

The results showed that many stages should be done in parametric design to get one model that simulates the Catalan arches. As establishing geometry, the support points, cross-sections, loads, rigidity of joints should be determined and discussed carefully before assembling the final model and starting analyzing to get structural results from the model.

I conclude that cross-section dimensions and properties of materials, the rigidity of joints, geometrical form, and arch dimensions are critical factors that must be considered in building Catalan arches in a parametric environment.

Sammendrag

Denne masteroppgaven undersøker en parametrisk arbeidsflyt ved hjelp av parametriske verktøy for å modellere og analysere katalanske hvelv. Det er vel etablert at den katalanske hvelveteknikken har blitt brukt fra historien til i dag med noen utfordringer med å forstå den strukturelle oppførselen.

Denne oppgaven tar sikte på å bestemme den strukturelle ytelsen til slike strukturer ved hjelp av parametriske tilnærminger. Spesielt undersøker den arbeidsflyten for å modellere katalanske hvelv og faktorer som kan påvirke den strukturelle ytelsen under design.

I denne sammenhengen er det katalanske hvelvet også kjent som et “Masonry vault” “Gustavino Vault”, “timbrel vault”, “thin-tiled vault”, “laminated vault”, “flat vault”, “layered vault and “Catalan turn” er en type struktur som er bygget av en eller serier buer. Katalanske buer er avhengig av å legge tynne murstein (fliser) på langs vekslet med et mørtellag basert på gips eller kalk. Denne teknikken i bygningen kalles hvelving. Katalansk hvelv gjennomgår en akselerert periode med innovasjon innen materialutvikling og teknologiske tilnærminger som brukes for modellering og analysen.

For bedre forståelse av denne teknikken, er det gjort en sammenligning mellom resultatene som er hentet fra en eksperimentell test utført på katalanske buer i Portugal i 2014 med resultatene som er hentet fra modellen har blitt modellert i denne oppgaven ved bruk av parametriske verktøy. Parametriske verktøy som Grasshopper, Karamba3D og Rhino ble brukt til å modellere og analysere én parametrisk modell.

Resultatene har vist at for å få en modell som simulerer de katalanske buene, bør mange trinn utføres i parametrisk modellering. Når det gjelder å etablere geometri, bør støttepunkter, tverrsnitt, belastninger, stivhet i ledd bestemmes og diskuteres nøye før den endelige modellen blir samlet og blir analysert for å få strukturelle resultater fra modellen.

Jeg konkluderer med at tverrsnittsdimensjoner og egenskaper for materialer, stivhet i ledd, geometrisk form, buedimensjoner og antall tynne murstein lag er kritiske faktorer som må tas i betraktning ved å bygge opp et katalansk hvelv i et parametrisk miljø.

Contents

1 Introduction

Catalan vaults have historically been resistant to seismic actions over a long time. Still, there is a need to upgrade the deficient masonry elements to meet the current design standards, which are strict about designing such structures with sufficient capacity [60]. In the absence of established standards or guidelines for the analysis of these systems, the numerical models, developed with available documentation about the structure, and engineering judgment offer a viable solution [22]. Newly some architects and structural engineers have started researches and practical experiences on Catalan vaults [131]. Still, none has focused on designing, analyzing, and comparing the results of experiments with the results of parametric modeling of the structures built on Catalan vaulting as a technique. In this thesis, I answer how to design and analyze Catalan vaults by using parametric modeling. This will be investigated and studied by comparing the results was carried out from one experiment that has been done in 2014 in Portugal by a group of researchers on one Catalan arch with my modeling by using parametric tools. This will give more understanding and confidence of the structural behavior of the structures built on Catalan vaulting.

Catalan vaulting has been used in history and still is used in our days around the world to build masonry vaults. In our time it is undergoing an accelerated period of innovation, and interest in its use is increasing. It requires new design approaches to simplify the complexity and new materials to reinforce the constructions built on this technique after understanding the structural behavior [69].

This technique is reducing costs because it does need any formwork or massive supports. Furthermore, it simplifies the complexity of construction shapes because, by this technique, they can be constructed with simple materials, simple processes, and simple technical methods which is beneficial for architects and structural engineers to design the optimized building components [94]. As well as, it is reducing carbon emissions because it has a lower embodied carbon impact than steel and concrete as building material [31, 69].

The architects were and still try to find a sophisticated system of building that attended to structural, aesthetic principles, acoustic, fire, and other safety of their designs [89]. Historical and new structures built on Catalan vaulting have all those features. Due to its undeniable structural and architectural features and constructive qualities. It is a solution with a high structural and architectural potential, satisfying, simultaneously, the comfort and sustainability requirements, as is evidenced by some studies. This aiming at design rules and structural assessment strategies, not only intending to estimating the safety of existing buildings but also to reinforce them and use them in new buildings [112].
But the absence of understanding the structural behavior and assessing such Catalan vaults using numerical models presents a significant challenge for the structural engineer [86]. Since the behavior of these structures is poorly understood, precautions must be taken to ensure structural safety due to the absence of established standards or guidelines for the analysis of these systems. So attempts to prove the safety of existing structures can also lead to discovering new structural forms that have not yet been invented.

The thesis is starting with a background chapter. The historical development of the Catalan vaulting technique and the most notable architects who have used this technique have been discussed; moreover, some most characteristics of this technique. The next chapter introduces relevant theory about Catalan vaulting from the beginning until using TNA (Thrust Network Analysis) as an approach. Then both old and new materials that have been used in Catalan vaulting will be discussed in this chapter. After that, in the results chapter, one group search in 2014 from Portugal has built, tested, and compared the behavior of eight prototypes of tiled arches, which were made over time on the Catalan vaulting until it collapses. There were built on two and three layers of tiles. It was testing for different types of reinforcement, layers, and properties of hydraulic mortar. Finally, the conclusion and some futuristic suggestions are presented.

2 Background

An arch is a curved structural form that carries loads around an opening, transferring them around the profile to abutments, jambs, or piers on either side and is structurally very stable in compression, and the loads are balanced through the form of an arch.

The arched structures generate projections in 3d space when the arches are extruded to form a vault. They generally run in a horizontal direction and are common in architecture from ancient Egyptian architecture to today.

A vault is a self-supporting arch form usually of stone, brick, tiles, concrete, timber, etc. It serving to cover a space with a ceiling or roof essentially. The arrangement of arches makes some of the vaults to be different from each other. They can be used for referring to a room or chamber used for storage underground or secure. The lower parts of all vaults have to withstand the outward pressure which the higher parts imposed. Commonly, the underground vaults are resisted by the 'fill' with surrounding and by thick supporting walls, supporting columns, buttresses, stiffening diaphragm beams, side anchors, or parallel walls if the vaults are overground [130].

The Catalan vault is also known as a "Masonry vault" "Gustavino Vault", "timbrel vault", "thin-tiled vault", "laminated vault", "flat vault", "layered vault"and "Catalan turn" consists of a single or series of arches built of plain bricks and makes a solid form with good strength in compression, low tensile strength as a shell structure. It has a very low thickness (in comparison with the other two dimensions) and rests on laying bricks lengthwise alternated with a layer of mortar based on gypsum or lime [12].

Tile vaults are masonry structures made with thin bricks (tiles), mortar and fast-setting cement or gypsum. The bricks are placed flat, building up to two, three or more courses. First and widely used in Catalonia, from which it derives its name. It is traditionally constructed without any formwork and by laying bricks lengthwise over a wood form or centering just to make a gentler curve (Figure1) [67, 111].

Figure 1: Catalan Vault

Source: [111]

The Catalan architect Josep Puig i Cadafalch (1867-1956) known as "Catalan Architecture" was the first one who promoted to use of the term "Catalan vaulting" [117].

Catalan vaulting was a masonry technique started in Catalonia, Spain, which used a fast-setting mortar and relied on structural form for strength. Some examples of this kind of constuction are Grand Central Terminal, Boston Public Library,the Dome of St John The Divine in New York City [106],the Santa Maria del Mar (Figure 2)and the Santa Maria del Pi(Figure 3), both churches in Barcelona, and the Cathedral of Gerona (the widest gothic nave in Europe) [125].

Figure 2: Sant Maria del Mar

Source: [125]

Figure 3: Santa Maria del Pi

Source: [125]

The Catalan Vault was rediscovered at the beginning of the 20th by the Modernist movement. One example is the crypt of the Colonia Guell (Figure4). The designer was Antoni Gaudí, and there was the hanging of ropes and weights from the ceiling. Other examples are the Aymerich Amat i Jover in Terrassa by Liuis Muncunill (1907-1908) which covers 11,000 m^2 (Figure 5) and the Celler Cooperatiu de Pinell de Brai by Cèsar Martinell [38, 109, 125].

Figure 4: Colonia Guell

Source: [125]

Figure 5: Roof and interior of Aymerich, Amat i Jover textile factory

Source: [109]

One of the distinguishing characteristics of this type of construction is that it does not need formwork for spanning large open spaces, and the resistance of the structures depends on their form [106]. A roman vault consists of a single layer of thick, wedge-shaped stones, while the Catalan vaults consist of more than one layer, making the resulting laminated shell strong as reinforced concrete [125].

Tabicada technique is one of the most famous techniques used in Catalan vaults in history and will be described in detail in the next section.

2.1 Tabicada technique

The Tabicada technique, known as bóveda tabicada, has been used as method to build a vaulted structure and is the result of the evolution of a traditional building technique belonging to the constructive Spanish tradition [45].This technique is one of the other techniques as tie-rods, or massive piers consisting of thin vaults [10, 21, 24].

Tabicada combines modernity (since many architects have adopted it in the modern era) with tradition (Since its origin starts in the ancient ages). Commonly the ratio between the brick and mortar layer thickness is no more than two. The material used in this technique can be regarded as the first type of composite material adopted in construction history [12].

The Arab-Islamic civilization was one of the first civilizations had used such as this construction. This has been evident in the Assyrians, the Sumerians, the Egyptian civilization, the Roman buildings. It was mainly composed of brick, mortar made of gypsum, and the absence of ribs during construction. It was then transmitted through Byzantium, first to Spain and then to Europe [103].

The authors Araguas and Philippe in their book (Comte d'Espie Manière de rendre Toutes sorte d'édifices incombustible), published in 1754, were the first ones who focused on the fire invulnerability of the tabicada technique [3].

The tabicada technique concerning Catalan vaults has been adopted by the most important architects in their masterpieces [12].

Bóveda tabicada were manufactured in America by the Catalan architect and builder Rafael Guastavino Moreno and by his son Rafael Guastavino Expósito. They obtained multiple patents; among them were for stratified laminar vaults and others for more sophisticated uses, such as acoustic bricks and sanitary uses. Guastavino father and son's work, in fact, includes vaults in churches, cathedrals, chapels, and public buildings in 70 years of working with an intensity of 30-60 buildings a year. One of the assignments Gustavino had received was Boston Public Library(Figure 6).

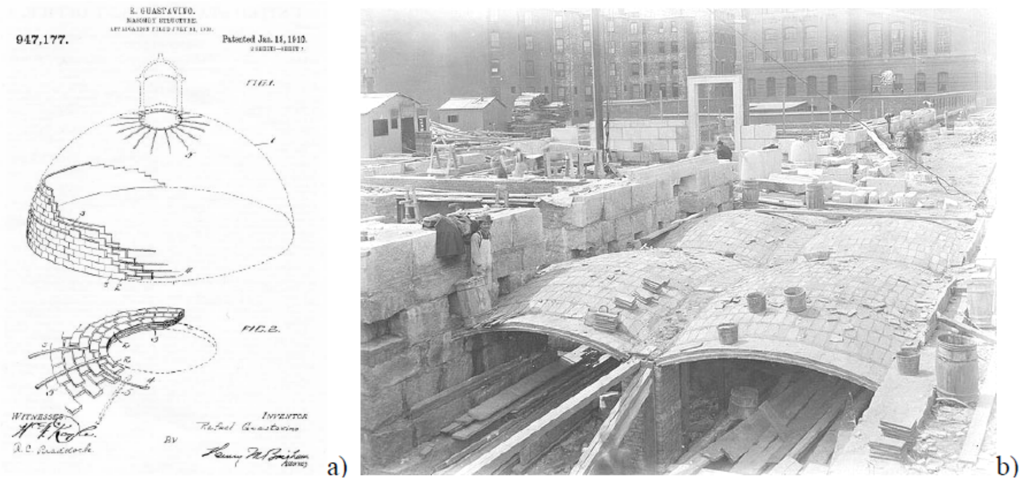


Figure 6: A)Paten of reinforced "boveda tabicada" of Gustavino's son(1910) ;b)Construction of Catalan Vaults of the Boston Public Library

Source: [12]

Tiles are arranged to break the joints of the adjacent layers. Doing it diagonally assures that they will break all joints between layers. In contrast to the conventional vaults, which depend on masonry holding by the force of gravity, the constructive Spanish tradition depends on the mortar's strength, which is very strong that tiles will ordinarily break or split before the mortar parts. The action of the vault approximates to that of a sheet of plywood whose wood laminations will separate before the adhesive bond gives way [22].

In the next section, the historical aspect to frame the Catalan vaults and their construction techniques will be discussed, along with the history of architecture.

2.2 Historical development of Catalan Vault and tile's applications

Catalan vaults were as Nubian vaults built by laying three to four layers in a herringbone pattern layer by layer and side by side with mortar as glue. What was interesting in Nubian vaults was both the binder and the laying method which was the foundation for Gusatvino's work with Catalan vaults [29].
Vaulting techniques were in use since the fifteenth and not only in Spain but also in Italy, France, Portugal, and Algeria. In each area, the vaulting method was given a name in the local language: voûte à la Roussillon or voüte plate in French, volta infolio in Italian, böveda tabicada in Spanish, voltes de barandat in Valencian, mad de pla in Catalan, aböbada de tijolo in Portuguese, and rhorfas in Arabic [24].
The first tile vault that Bergós dated in his text was from the Hospital of Santa María in Lleida, built-in 1352 [24].
In 1382 the tile vaulting was something revolutionary for builders of its high load capacity, very lightweight, miraculous thinness, economy (low-cost work), and speed of construction compared with traditional vaulting [4].
The traditional stone vaulting was built from a single layer of stones with the need for costly wooden formwork called centering after building the keystone in place(Figure 7) [89].

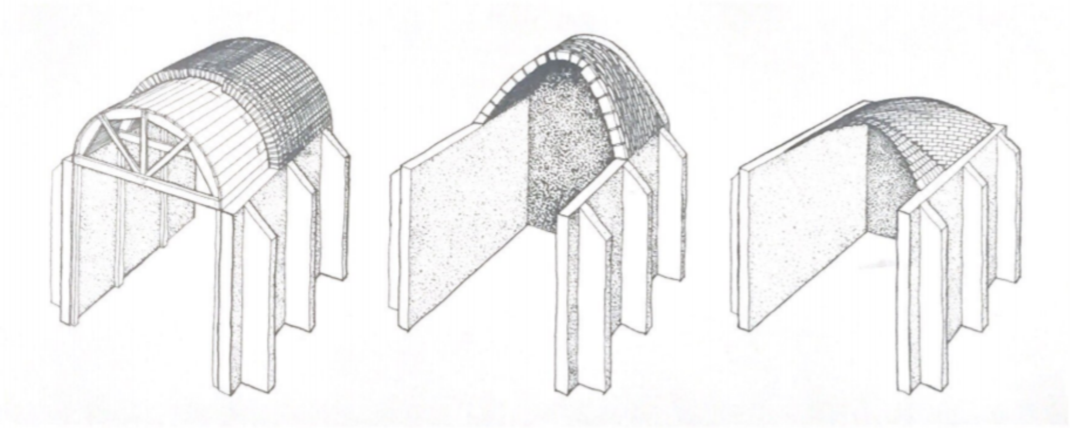
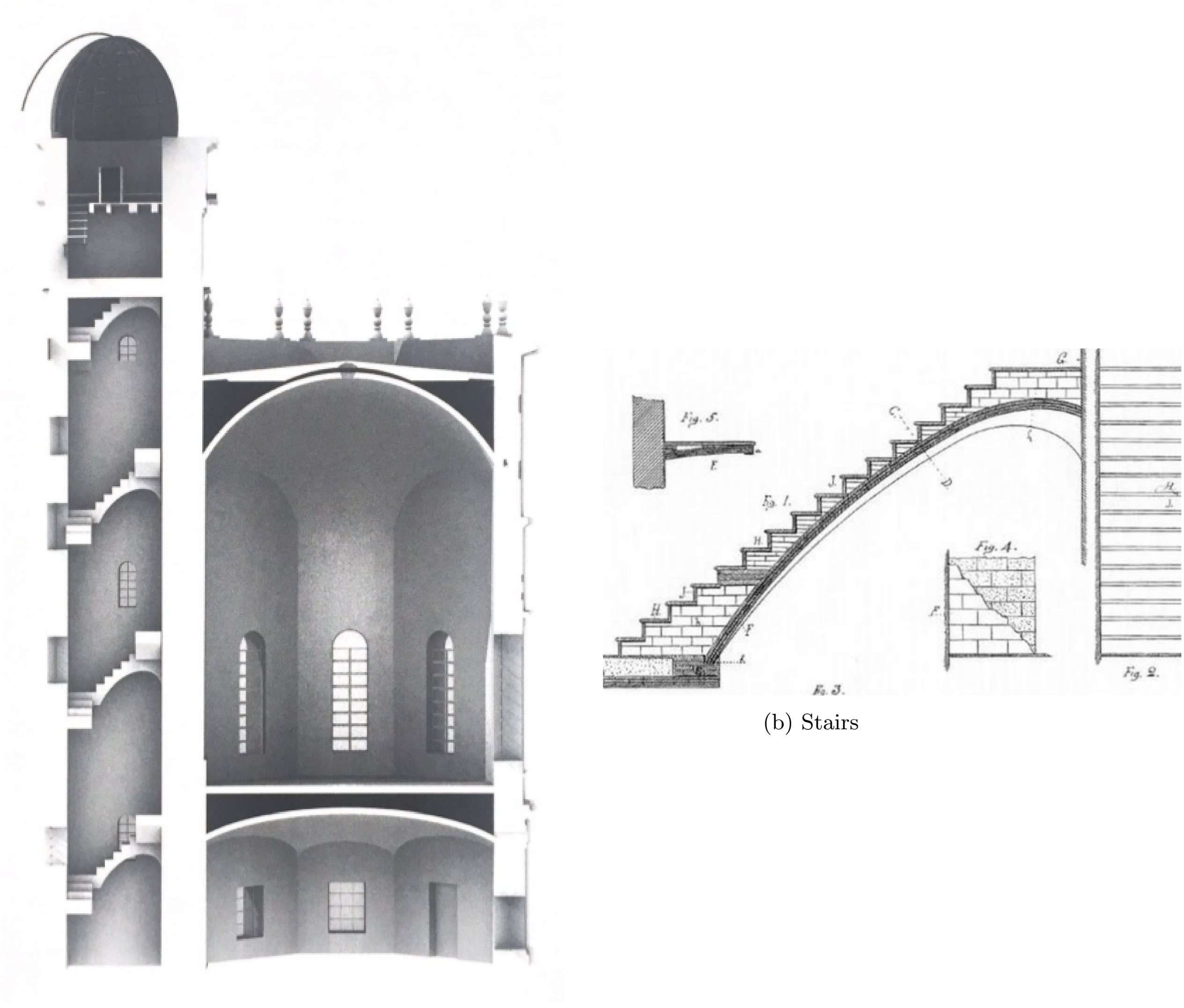


Figure 7: Three types of traditional masonry vaults: European vaults built on wooden centering, Middle eastern vaults built of pitched brick with no centering required, and Mediterranean tile vaults with no centering required

Source: [89]

Friar Domingo de Petrés in 1803 used tiles in two different places when he restored the Catedral Primada in Bogota'. The first place to build two large domes. the second one to build the staircase (Figure 8) [11, 120].



(b) Stairs

(a) Analytical section of Bogota' observatory showing the geometry of vaulted tile stair and section through two tile domes

Figure 8: The CatedralPrimada in Bogota'

Source: Friar Domingo de Petrés

In 1880 the high cost and minimal window openings and lack of expertise to build dozen masonry vaulted buildings made massive masonry spans did not meet the needs of architects [133].
Through the late nineteenth century was the time to determine the load capacity and fire-resisting ability of tile vaults, but the absence of standards for both construction techniques and materials made it determine which kind of diverse floor systems would offer the greatest safety and load capacity at the lowest cost [89].
In 1880 engineering Calculations for Guastavino vaulting were neither commonly used nor expected. The only method that was used was load testing(Figure 9) to demonstrate the safety and strength of tiles because it was common for construction practice to proceed in advance of structural theory [87].

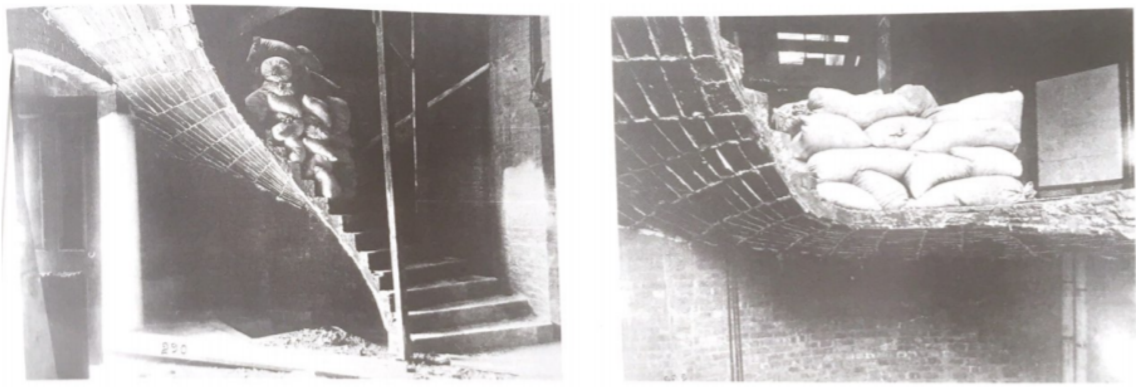


Figure 9: Load test on a Gustavino helicoidal tile stair, first Church of Christ Scientist New York City, ca 1900

Source: [89]

The earliest attempts to quantify the load capacities of flat tiles bonded with Mortar was in 1887 when Gustavino detailed a series of tests conducted on tile vaults in the department of tests and experiments in New York. He explained the essence of tile vaults as internal bonding together of the tiles and mortar, which make each vault tom function as a unified material that could take tension. and therefore exerted on the thrust on the supports. This explanation was based on his observation and emulation rather than through calculation as an engineer [45, 89].
Guastavino Fireproof Construction Company was incorporated in 1889. It could produce both glazed and unglazed tiles at a high level and, with the help of Charles McKim, could use these tiles to create interior surface finishes and at the same time serving as the structural support [105].
In this time the Gustavino fireproof construction company had a series of projects as Carnegie Hall in New York City, the famed Edison Electric Pearl Street power station in New York, and telephone building in Denver ,Guastavino was prospecting for materials to make lightweight clay tiles [20].
In the project Boston public library 1889, the tile vaulting technique could replace the iron beam floor system without delaying the construction of the building and the Guastavino company could install 400 square feet (37 square meters) of vaulting per day [82].

The Columbian exposition in Chicago in 1893 played a key role in spurring interest in the revival styles of the American Beaux-Arts architects. The bristol county courthouse in 1894 demonstrated the structural feats that could be accomplished with tile vaulting in building interiors and the ability of tile vaulting to be adapted to any style Beaux-Arts architects wanted [89].
Gustavino Sr had experimented tile manufacturing in 1890 and submitted a patent describing a manufacturing process for building tiles [132]. In the same time he began to explore the possibility of manufacturing custom-designed tiles.
An architectural critic in 1897 suggested using wooden domes; another suggested using a double dome built of interior plaster ceiling supported by an exterior structural dome of iron or steel. Gustavino dome could provide structural and design solutions and had the same function in the same structure [89].
One example of the complexity of these structural calculations was a spiral stair at the Cathedral of St. John the Divine (1899-1935), New York City. It was the largest dome ever built by the company [16].
The company used the steel bars between courses of the tiles for the first time At the St. John the Divine cathedral in 1892, and this allowed the architects to build soaring structures of Gothic cathedrals in a short time. At the same time, it updates the ancient construction technique to fit modern American building standards and demonstrated the possibilities for the tremendous economy of materials in thin-shell structures in the early twentieth century [24].
Using tile in the building made Gustavino be an essential person who brought color, texture, and light to building interiors. This was something new developing in tile capabilities because Guastavino company started to decorate the interiors of churches and residences. At the same time, it built structural vaulting systems in major public works projects as the City Hall subway station

(1904). The vaulted space has been called the “Mona Lisa of subway stations(Figure(10))” and “an underground cathedral” which achieved spatial, structural, and decorative effects, demonstrated the diverse ways in which tile vaulting could be employed [89].

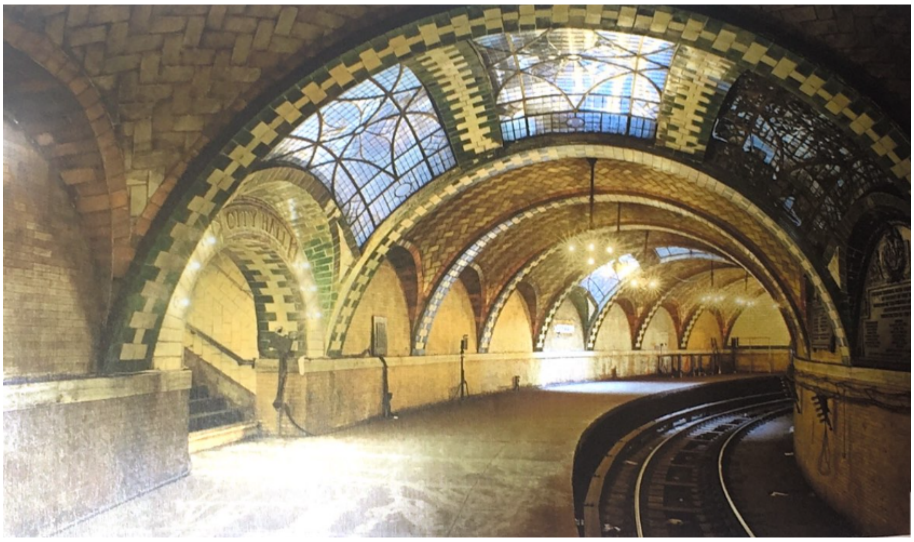


Figure 10: City Hall Subway Station,tile vaulting

Source: [89]

In 1903 the Williamsburg bridge showed the tile vaults cantilevered out into space with hidden steel truss (Figure 11) [35].

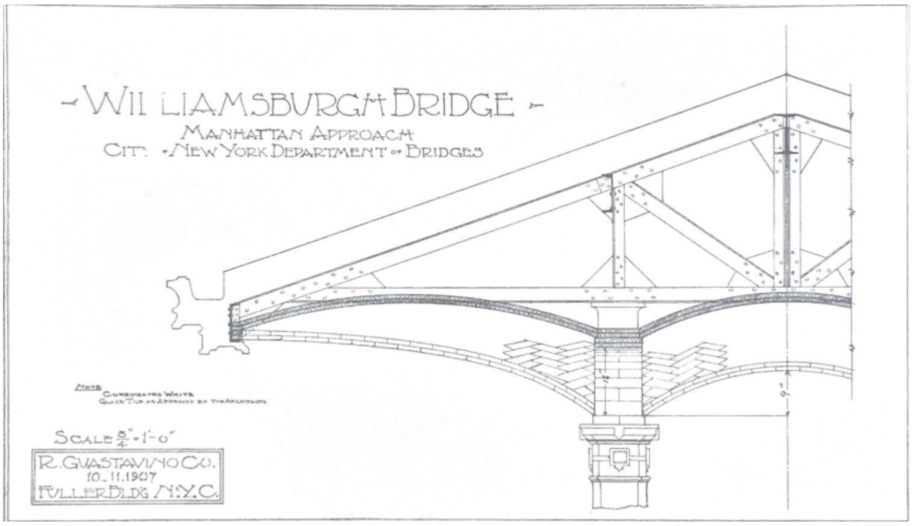


Figure 11: Sectional drawing of Williamsburg bridge approach showing steel trusses supporting a tile vault

Source: Gustavino Company for Henry Hornbostel,1907

The U.S. Army Corps of Engineers in 1905 requested the construction of tile vaults because of its low cost, which could be 40 percent less than It was built of steel and its resistance to fire and its impressive architectural space that results from the construction without additional cost for aesthetics or interior finishes [89].
In 1895 Gustavino Jr designed and constructed the dome of Grace Universalist Church in Lowell, Massachusetts, using the tile arch system. The thickness of tiles was twice as thin as an eggshell

by proportion, and the dome had a span of 21.3 meters in diameter. The dome was titled the thinnest dome ever built until that time without any supporting formwork, just minimal formwork to guide the geometry. Guastavino Jr provided a tensile band of steel to resist the outward thrust at the intersection of the buttressing barrel vaults and the dome [85, 88].
The engineers cited the 1897 test as proof that the fire-damaged vaults still had significant load capacity [121].
In the early twentieth century, the company had a lot of projects they were working on like Central Terminal and Pennsylvania Station (1910) in New York City, the major stations in Chicago (1911), Houston (1911), Rochester (1914), train stations in Quebec (1915), St. Paul (1923), Toronto (1927), and Buffalo Central Terminal (1929) in New York which was the largest station had been built by Structural Gustavino tile. The vaults spanned 90 feet (27.4 meters) and with (10 centimeters) thick [89].
After this time, It started using tile vaulting as expensive decorative finishes more than the inexpensive structural systems. However, the company produced tiles for long-span domes and vaulting in thin-shell tile. Creating thin concrete shells in the 1920s was the reason for the declining using of Guastavino's vaulting. The new architectural style emerged waves across the Atlantic began to replace the vaulted interiors and gothic spires of American Beaux-Art and Art Deco architecture. In contrast to the modernist myth of a new architecture divorced from the past, the sleek effect of Mies's pavilion relied on traditional construction techniques [59].
Even metal or timber domes cost less to build; Beaux-Arts designers had a different opinion because they did not have the same longevity and were not as satisfying for architects in terms of safety, design, or the emerging ideal of structural honesty [42].
Accounting to inflation, rising labor costs faster than the cost of materials caused the real cost of Guastavino vaulting to be double in thirty years. Guastavino Company closed permanently in 1962 [89].
The Guastavino company provided efficient structural solutions and tremendous versatility in adapting its tile vaulting to both architectural styles and structural engineer [89].
The company made solutions to architectural problems, gave the buildings a sense of decorative, architectural, and structural coherence [1].
of the thousands of vaulting systems the Gustavino company constructed, not a single vault has ever failed in service [89]. Market forces, wages, and materials manufacturer's powerful lobbying activities affected the materials market and passed bills that required the use of just such new materials. Thus the method of the Guastavinos gradually became expensive [29].
More rigid building codes and professional engineers rising prompted rectilinear framed structures of concrete and steel rather than curving shells of masonry. Building codes and the structural engineering profession also threatened future opportunities for the Guastavino Company because the company could not prove whether tile vaulting could standing against heavy wind or carrying snow loads. Vault constructions were considered outdated and off-putting. If lobbyists and politicians, architecture teachers, and star-architects had understood the value of the sustainable building, perhaps the construction method has survived into our time and beyond. Modernism demanded straight beams, flat roofs regardless of spans [89].

The medieval streets of Girona, the rural fields of Catalonia's Terra Alta, and the industrial complexes around Barcelona is all home to a wonderful array of vaulted buildings today [109].

2.3 Properties of Catalan Vaulting

The qualities of having vaulted structures are good light, strength, longevity, clear spans, and ease of construction and modification [109].

This technique has a lot of advantages that are difficult to find in another type. It is swift to build and does not need any wooden scaffolding, and good economical because less material is required. It is known for its ease of construction, high strength, lighter weight, and reduced size of supporting walls and buttresses in historical structures [119]. It proved to be very fire-proof and durable. The Santa Maria del Mar in Barcelona was one example during the Spanish civil war.

All floors, ceilings, arches, and stairs built of Catalan tiles were sound-insulating and resistant to floods, dampness, and the lodgement of pests such as roaches and rats [125].
Timbrel vaults form a solid mass and resistance to fire due to a good of the mortar (plaster) used [65]. Timbrel vaults were used for various construction elements as [83]:

- a) for staircases
- b)for floor systems by using portland cement which allowed to build roofs without any waterproofing methods.
- c)for covering the naves of churches.

The Catalan vaults knocked out all competing vault constructions, was thin and capable of bearing much higher loads that enable wider spans and gentler curves [125]. The fire resistance of Catalan vaults accorded great importance [12].
Tile vaulting uses mush material than conventional masonry. Still, it can be built much more quickly because the thin bricks are laid flat, with their narrow edges in contact and the total thickness of the vault is more minor ,in addition to the self-weight and corresponding horizontal thrust are reduced, which leads to lowering of the requirements for the size of the walls and buttresses needed to support the vaulting (Figure 12) [89, 111].

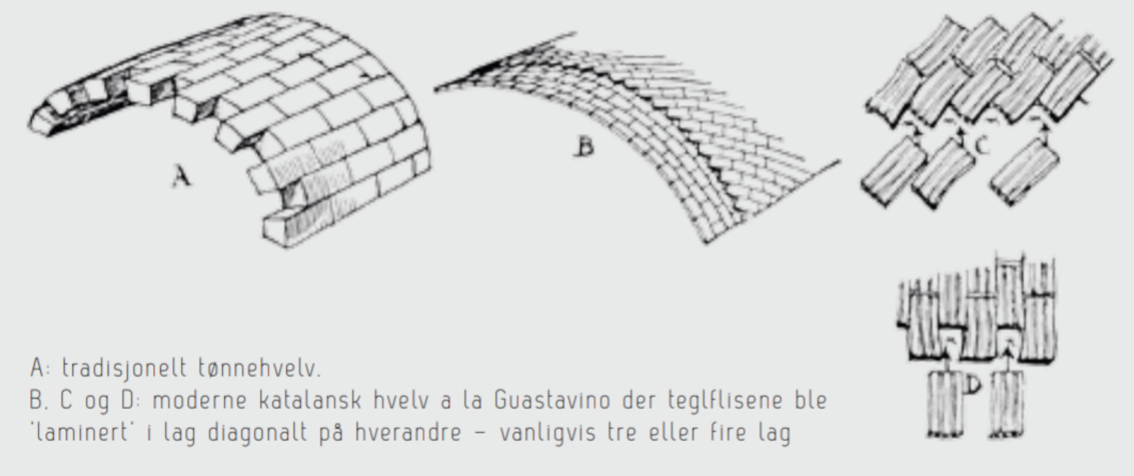


Figure 12: Traditional barrel vault versus modern Catalan vault

Source: [29]

Here are some of the advantages of Catalan vaults as a structural element [54]:

- The tensile resistance also allows a certain amount of bending resistance.
- The possibility of perforating a vault without collapse, which has been cited since Espie as a characteristic of the cohesive structure.
- They can be constructed without centering, using only light auxiliary elements to control the form.
- Catalan vaults present some resistance to bending, which permits the passage of light loads during construction.
- The introductory remarks about the 'masonry' material as high compressive strength, low tensile strength, and no sliding also apply to timbrel vaults .

Guastavino summarized some advantages of the Catalan arches which attributed to the reduction in the number of the joints [56]:

- The vertical joints are protected from cracking by the overlapping of joints.
- There are fewer vertical joints.
- There is the capacity to resist bending moments .

In one layer arch (mechanical arch), the joints between the bricks will work as voussoir in a traditional gravity arch. While in the double-layered arch (cohesive arch) with mortar between them and overlapping joints will work as a coherent structure capable of resisting bending moments(Figure 13) [45].

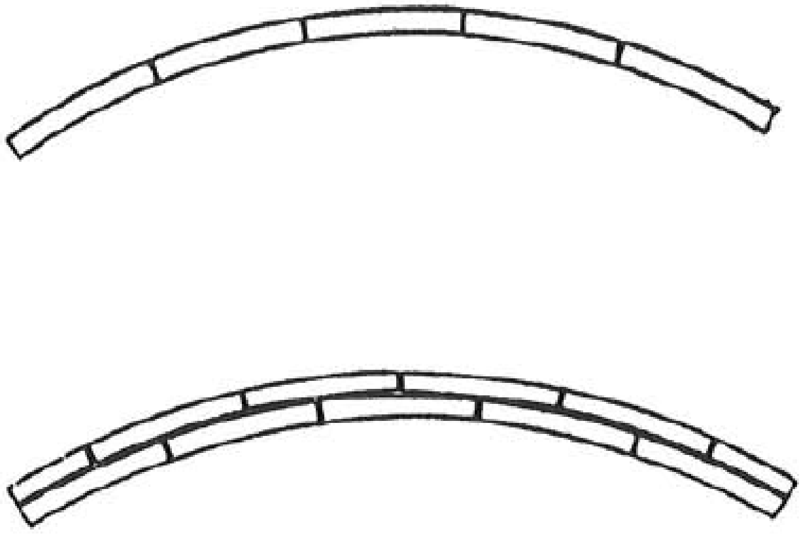


Figure 13: Comparison between a mechanical arch and a cohesive arch

Source: [45]

Some of the basic characteristics that generally distinguish the vaults from the dome are as shown in (Table 1) [111]:

Dome	Vault
A dome is an element of architecture that resembles that hollow upper part of a sphere.	A vault is a ceiling of brick, concrete, stone, timber, etc., the built-in principle of arch.
An arch rotated around its central axis.	An arch extended along its axis.
A masonry dome produces a thrust downward and outward.	It cannot be lighted except at the ends without being structurally weakened.
Columns or piers can support domes.	It must be buttressed along its entire length by heavy walls.

Table 1: Differences between dome and vault

There were two ways that distinguished Guastavino stair from a traditional Spanish vaulted stair. Firstly, Gustavino specified a piece of angle iron in two locations as a secondary means of reinforcing the stair.
Secondly, He specified the use of Portland cement, which, combined with the careful attention paid to "breaking" the joints, was thought to create greater "cohesion" or tensile capacity in the

material (Figure 14) [89].

Figure 14: Tile vaulted staircase of Baker Hall, Carnegie Mellon University, Pittsburg, 1914

Source: Courtesy of Michael Freeman

2.4 Notable architects in timbrel vaulting

Catalan architecture had an important place in masonry architecture thanks to the genius of Antoni Gaudi and other builders and architects as Rafael Guastavino, Josep Puig I Cadafalch, César Martinell, Lluis Mttncunill, Lluis Domenech I Montaner, and the masons of the Catalan tradition schools [40]. They created unimagined forms of terracotta tiles and their work is closely connected with Catalonian Modernismo which is an aesthetic, social, and political movement that matched with and has close bonds to Art Nouveau.

Guastavino's father and son made substantial contributions to tile vault further than any previous builder regarding material, structure, and construction [91]. Both Rafael Guastavino Sr. (1842-1908) and Jr. (1872-1950) had a significant impact across the United States in the most impressive masonry structures in history. Guastavino vaults are superior to reinforced shells because of their absence of formwork and minimal reinforcing steel [88]. Guastavino company could show that tile vaulting combined both decorative and structural functions [89].

2.4.1 Rafael Guastavino Moreno

The last work for Gustavino Sr in Spain was La Massa Theater in Vilassar de Dalt, with a 56-foot span built of unreinforced masonry only 4 inches thick. This is very thin, but it is possible because of the double-curvature of the masonry shell, which allows for compressive load paths to be transferred to the supports in multiple directions.

Rafael Sr or Gustavino Sr was born in March 1842 in Valencia became a carpenter in Valencia and witnessed a major period of urban renewal, demolition, and new construction [56]. One of the most famous who had an imprint in the Catalan vault is the Spanish architect and contractor Rafael Guastavino Sr(1842 - 1908), born in Valencia in 1885, who received a patent for a ”Tile Arch System” in the United States. He got a patent for a technique with self-supporting arches and vaults where several layers (normally 3 or 4) with thin terracotta tiles are locked together in a herringbone pattern using a special Portland cement mortar with strong cohesion, which was an

alternative for the lime mortar for obtaining higher and quicker strength [46, 90].
In the mid-nineteenth century and in the contrast to the architects emerging from beaux-arts institutions in France who concentrated on theoretical knowledge was Guastavino's focus on the practical, hands-on aspects of building. This and besides the scale of his project and the speed of construction make him be a designer of the buildings more than an engineer for his projects [89].
Tile vaulting had been used for decades in Catalonia, but the significant construction innovation was to use iron columns with traditional tile vaulting in the mid of nineteenth century. For Gustavino this innovation was to use Portland cement in place of the lime mortar of traditional tile vaulting [57, 98].
Guastavino Sr built many significant houses in Barcelona, including the Camilo Julia house, which was an important precedent for modernism or Catalan Art Nouveau [116]. In early 1881, he made a rather sudden decision to leave Spain for the United States [114].
The architectural competition to design the progress club (german Jewish social club) built in 1883 at East Fifty-ninth street was the first success for Guastavino when he won it [64].
In 1889 began Guastavino building of Boston public library with tile vaulting. This project influenced the scope and direction of his later work in the united states.
The first major success for Guastavino was at Boston public library when contracted by the leading firm of McKim Mead and White in 1889 (Figure 15).

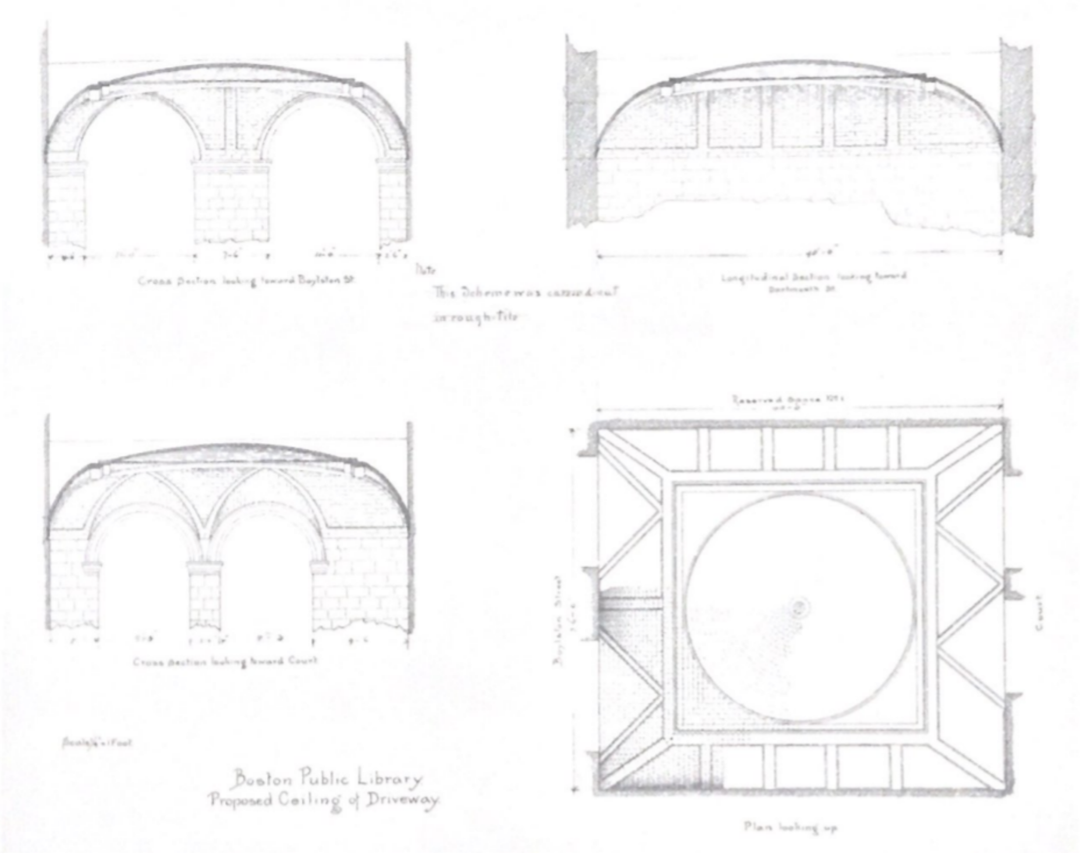


Figure 15: Gustavino drawing of vault design for driveway ceiling 1889

Source: Boston public library

His career ended with his death on February 2, 1908, after winning many titles such as inventor from newspapers Asheville and "architect of international fame" [89].
He got the first patent for the floor system of his tile vaulting which absorbs the sound or noise either over or under this floor [24].

2.4.2 Rafael Guastavino Expósito

Rafael Guastavino Expósito or Rafael Guastavino y Esposito (1872-1950) owned the patents for vault constructions and had his own company for building such constructions in Boston. New york city, for example, landmarks in Beaux-Arts with, in addition to McKim, Mead and White, Richard Morris Hunt, Cass Gilbert, Carrère Hastings. The new world was built with the help of the old [29].

This method of reinforcing the thickness of the vault made Guastavino Jr(1872-1950) receive a patent in 1910 [34].

2.4.3 The Comte D'Espie

The Comte D'Espie is the owner of the monolithic no-thrust theory and the author of the book (Manner of constructing all sorts of fireproof buildings, or treatise on the construction of vaults, made with brick and plaster, called flat vaults, and of a roof of brick, without wood, named comble briqueté) [65].

2.4.4 Antoni Gaudí

Antoni Gaudí was a world-famous Catalan architect (1852-1926). He climatically optimized his buildings, seeking ingenious ventilation and natural lighting solutions, using bricks and tiles sufficiently [2]. Although Gaudi was not famous for his writing on this technique, he used it continuously, including in innovative sculptural applications(Figure16)[65].

Figure 16: Examples of Gaudí works with timbrel vaults: a) Provisional School of the Sagrada familia, Barcelona, 1909. b) Chimneys and stairs exit on the terrace of Casa Milà, Barcelona,1906-12

Source: [44],[84]

Gaudí used both hanging models and graphical methods as design tools, used unusual geometrical forms for some of his vaults, and designed tree-forms of equilibrium for their support. His work embraces all aspects of architecture as layout, ornamentation, and stability. The structural design for him was an integral part of architectural design from its initial stages. It is not a matter of checking the stability of a certain design; it is a matter of calculating, from the start, using stable shapes [53].
He climatically optimized his buildings, seeking ingenious solutions for ventilation and natural lighting, using bricks and tiles sufficiently [75]

2.4.5 Lluís Domènech i Montaner

Lluís Domènech i Montaner (1850 –1923) born in Barcelona and was a Spanish architect who was highly influential on Modernisme català, the Catalan Art Nouveau/Jugendstil movement. He was also a Catalan politician. He initially studied physics and natural sciences, but soon switched to architecture played an important role in defining the Modernisme arquitectonic in Catalonia [93].

2.4.6 Eladio Dieste

Eladio Dieste (1917-2000) was born in Artigas Department I Uruguay. An excellent innovation was his Gaussian vault and a thin-shell structure for roofs in single-thickness brick which derives its stiffness and strength from a double curvature catenary arch form that resists buckling failure [1]. He was also one who brought architecture and structural engineering into proximity, notably when undertaking humble commissions. His buildings were often roofed with thin shell vaults constructed of brick and ceramic tiles. These forms were cheaper than reinforced concrete and didn't require ribs and beams [27].

2.4.7 Joan Bergós i Massó

Joan Bergós i Massó (1894 -1974) born in Barcelona He studied architecture at the Escuela Técnica Superior de Arquitectura in Barcelona and painting at the Francesc Galí School of Art. He was a professor of wood arts at the Escuela Superior de Bellos Oficios in Barcelona; of Industrial Constructions at the Institute of Applied Electricity and Mechanics; of Rural Constructions in the Higher School of Agriculture; and Reinforced Concrete and Masonry, at the Barcelona School of Work, where he was dean of the Artistic Trades section [39]. In Barcelona he met Antoni Gaudí in 1914, with whom he maintained a great friendship [43].

3 Theory

This section will cover the theory related to the research question. It will cover the following areas:

- Structural behavior of Catalan vaulting structures through history and the approaches used to analyze and design them.
- Materials used in the technique and new materials used to strengthen the structures to work more efficiently.
- First timbrel vaulted building in England was designed, developed, and built by one group of researchers.

3.1 Structural behavior of Catalan vaulting structures

Even today the conventional engineering can not understand or describe the structural behavior of Guastavino structures [16].
Although these structures have been used for a few centuries in Spain, they lacked confidence in their structural behavior [54]. Assessing the safety of Guastavino structures remains a challenge today for engineers, but new methods of equilibrium calculations, new technology, and new materials can help to discover the load paths [88].
For example, the new freeform tile vaults, with their complexity, require the development of approaches to both construction processes and new materials [69].
The mechanical behavior of the structural elements is essential to evaluate the actual stress levels, define the safety factor, and correctly design any structure after getting a mechanical model which can represent the material behavior.
The main goal of calculations did on timbrel vautl, was to demonstrate safe equilibrium solutions under all load cases and ensure that the force lines will not exit the masonry [88].The similarity between tile and masonry vaulting is that the safety depends on the geometrical form and the strength of the material [51].

The ancient builders distinguished the thrust of the vault with the necessary buttress to resist it. There was no distinction in the structural behavior of timbrel vaults from conventional brick or stone vaults. The only difference was that they required fewer buttresses due to the lighter weight, decreasing thrust [54].
The tile vault is symmetrical over the diagonal, double-curved where the individual tile follow the curvature of the vault-like a taut skin as above a tambourine. The double curvature ensure the balancing of the horizontal forces. Even Eladio Dieste's self-supporting "Ceramica armada" and all straight vaults had to absorb the horizontal forces in "irrelevant" additional structures outside the vault itself [29]. The high structural performance came from the layered arrangement of brick interposed to the mortar. The hydraulic binder is introducing an additional force which gives the system a special feature [12].
Centering is not crucial because the tiles can be stuck to the edge walls or finished arches or stable localities[56].
The modern theory finds that Guastavino vaults are strong in compression and weak in tension but behave like all traditional masonry. The difference between them is that the tile vaults have a lower horizontal thrust on the supporters of their lightweight [82]. Catalan vaulting had the same problem as brick or stone vaulting had, for example, cracks and hinges [54].

The most relevant text on the construction and mechanics of timbrel vaulting is from Fray Lorenzo de San Nicolas in the architectural treatise published in Madrid in 1639. Fray Lorenzo was an architect and built many timbrel vaults, described the construction of the basic types of vaulting (groined, barrel, hemispherical, cloister, etc.) in stone, in brick with radial joints, and in tiles. He considered those three methods are suitable. Regardless of the material used in construction, it has to be provided with lateral support to carry the thrust to the buttresses. He pointed two things

to support the horizontal thrust of soil and resist asymmetrical overloads and moving loads. The first one is to fill the haunches for the first third of the vault height. The second one is supporting transversal walls or lengüetas for the second third on the vault height.
His suggestions of the buttress were two: a continuous wall or a wall with counter-forts. He wrote that the thrust exerted on the walls in Catalan vaulting was reduced by using thin bricks arranged sidewise than it could happen if it was stone vaulting. The reason for such particular behavior is attributed to its lightweight and the monolithic character of the structure [68].
Before the eighteenth century, there was no distinction between the structural behavior of timbrel vaults, brick or stone vaults. After that, the timbrel vaults were considered to work entirely differently from conventional stone or brick vaulting. They were supposed to be monolithic and to exert no thrust. At this time in Spain, these constructions were known as “impossible to calculate”. As a new method at that time in France, the Duke of Belle Isle built a series of timbrel vaults in his castle, using the bricklayers of Perpignan [54].
In 1895 Gustavino’s company constructed Grace Universalist Church dome in a series of concentric rings without any formwork. The company opened some openings in the lower dome to have natural light and used flying buttresses after Guastavino’s calculations of the trajectory of the outward thrust of the dome. These flying buttresses had a function to channel the outward thrust of the vault to the vertical walls and down the ground (Figure 17) [89].

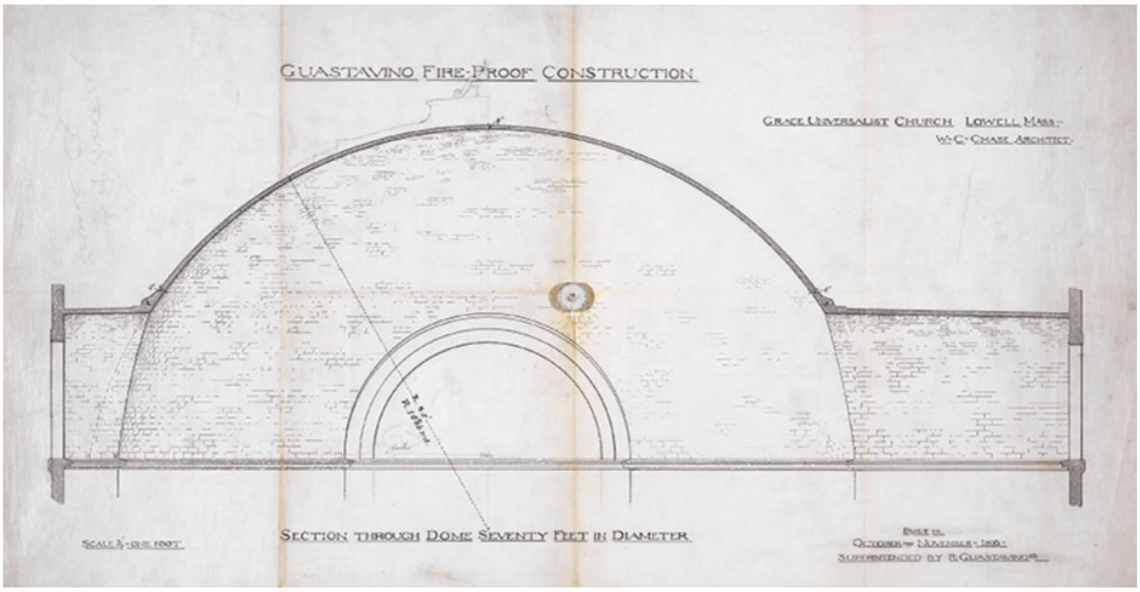


Figure 17: Grace Universalist Church by Rafael Gustavino Jr, Lowell, Massachusetts,1895

Source: Avery Library

The Uruguayan engineer Eladio Dieste said: The resistant virtues of the structures we seek to are stable, not because of the awkward accumulation of material but of their form. There is nothing more noble and elegant from an intellectual viewpoint than this: to resist through the form [1].

3.1.1 The first scientific experiments in France

The first scientific experiment on timbrel vaults was in France on three of them with a span of 4 m (rise of 0.4 m), spanning between wrought iron I-beams (of 47 cm depth) with a span of 6.25 m, covering a total area of 72 sq m. The test was carried out until failure occurred under a load of 1,250 kg/m^2. In another test on a timbrel vault spanning 3.75 m (again with a rising: span ratio of 1: 10), the vault carried a load of 2,700 kg/m^2 without failing [54].

3.1.2 Rafael Gustavino's theory of cohesive construction

Rafael Guastavino Sr was the first one who tried to formulate a theory that explained the structural behavior of timbrel vaulting in a scientific form. His theory is known as the theory of “cohesive construction”. He divided the masonry constructions into two different types [45]:

- The first one is ”Mechanical Construction,” or construction by gravity. It builds on the resistance of any solid to the action of gravity when opposed by another solid. These opposing forces lead to the equilibrium of the total mass without considering the cohesive power of the material set between the solids [47].the force of gravity keeping bricks in position by the compression exerted on the exclusively vertical joints (Figure18). It consists of a single layer of thick, wedge-shaped stones and it relies mainly on gravity [26].

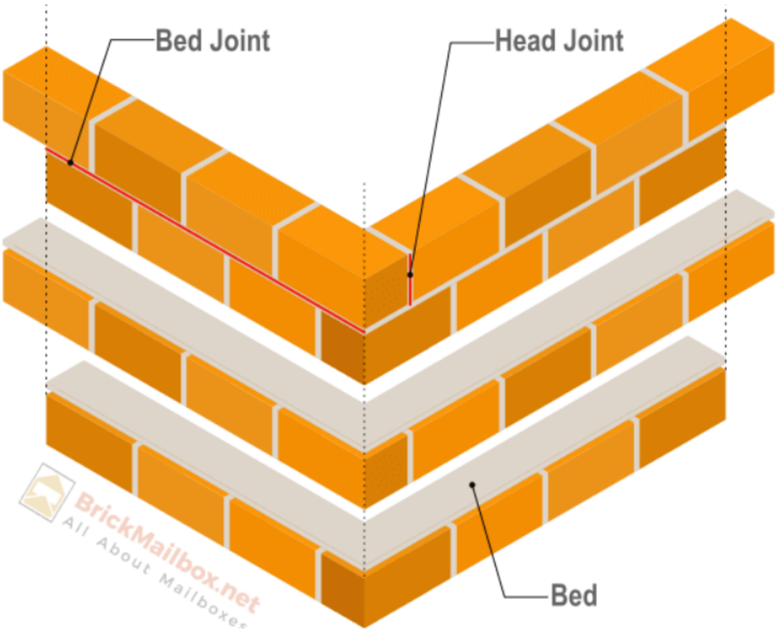


Figure 18: Brick masonry

Source: [128]

- The second one is ”Cohesive Construction,” or construction by assimilation, which has for a basis the properties of cohesion of several materials and by a transformation more or less rapid, resemble Nature's work in making conglomerates. Gustavino considered that any building constructed in material with good mortar adhesion, including Roman concrete, brick, timbrel vaulting falls in this category as cohesive construction. The cohesive character does not change the essential behavior of timbrel vault structures, but the research to develop the cohesion led to an unprecedented perfection of timbrel vault construction. In contrast to the gravity or mechanical system, the cohesive system makes possible the curvature of the vault by varying the joint angles [12].
  The cohesive system introduces an additional force that makes the component materials so cohesive that they can not be separated without destroying the entire structure [26]. Cohesiveness is the main structural characteristic of timbrel vaults, and that is structural monolithism. It is the force between the various layers of thin bricks and mortar [47].

3.1.3 Load tests

Everything was achieved by relying on intuition and practice without computers or engineering calculations [125].

The first systematic tests (structural load tests and fire tests) on timbrel specimens were by Guastavino in 1887(19). The table(2) shows the results he got in 1901 of his tests and could used it to verify the safety of his vaults by comparing the working stresses with the material failure stress [45].

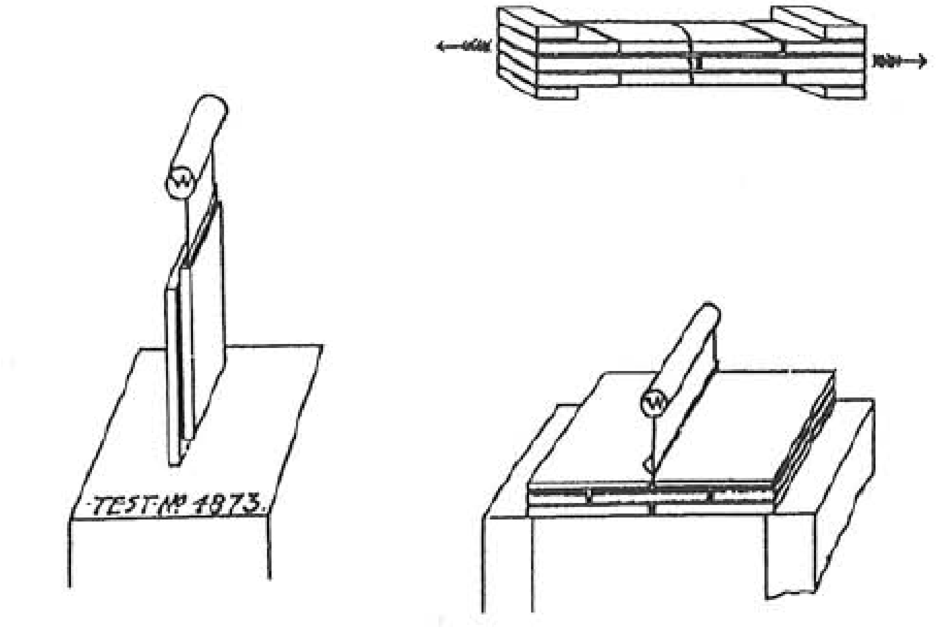


Figure 19: Specimens for the strength tests (tension, bending, shear)made by Gustavino

Source: [[45]]

Strength	N/MM^2
Compression	14.60
Tension	2.00
Shear	0.90

Table 2: Mean strength of timbrel specimens

3.1.4 The thrust of timbrel vaults and domes

For Gustavino the criterion dictating safety is not the strength of the material but the stability of the system. Safety can be achieved by giving a sufficient thickness. The formula Gustavino used to obtain the thrust of a flat arch or barrel vault is (20):

$$A(S_{br})=\frac{WI}{8f}$$

Figure 20: Gustavino's formula

Source: [Avery Library, university of Columbia]

where A = cross-sectional area of the vault at the crown per unit length; Sbr = breaking stress in compression; W = total load (self-weight plus fill and live load) acting on the vault per unit of length; I = span of the vault; f= rise of the vault. He considered that the weight of the dome is one-half the weight of the corresponding barrel vault and therefore, the thrust would behalf. In fact, the weight is different and also changes the position of the centers of gravity.

3.1.5 Graphical analysis

Graphical analysis is also based on a theory of equilibrium and includes a scaled drawing representing the dome in section and plan. The arch generated by the dome section is then divided into segments, and the forces working on each segment are represented by scaled lines connected in a force polygon. The stresses in the dome are derived from scaling lines in the force polygon and converting them into forces [28]. Gustavino claimed that his building method could support the bending moments of structural loads. Still, in fact, timbrel vaults are like other types of unreinforced masonry that can only support axial loads in compression [54].

These axial loads made the structural design of these vaults entirely fitted to analysis with graphic statics. The principle of the structural solution, which uses graphical statics, depends on the path of the forces. For example, if the resulting line of the force lies entirely within the masonry, then the structure will stand under the applied loads otherwise will collapse [109].

While Guastavino's father made his calculations and designed them by intuition, Gustavino junior calculated the forces in his vaulted structures using compressive equilibrium solutions defined by graphic statics, which demonstrated the formation of the vault based on the flow of forces(Figure 21) [1].

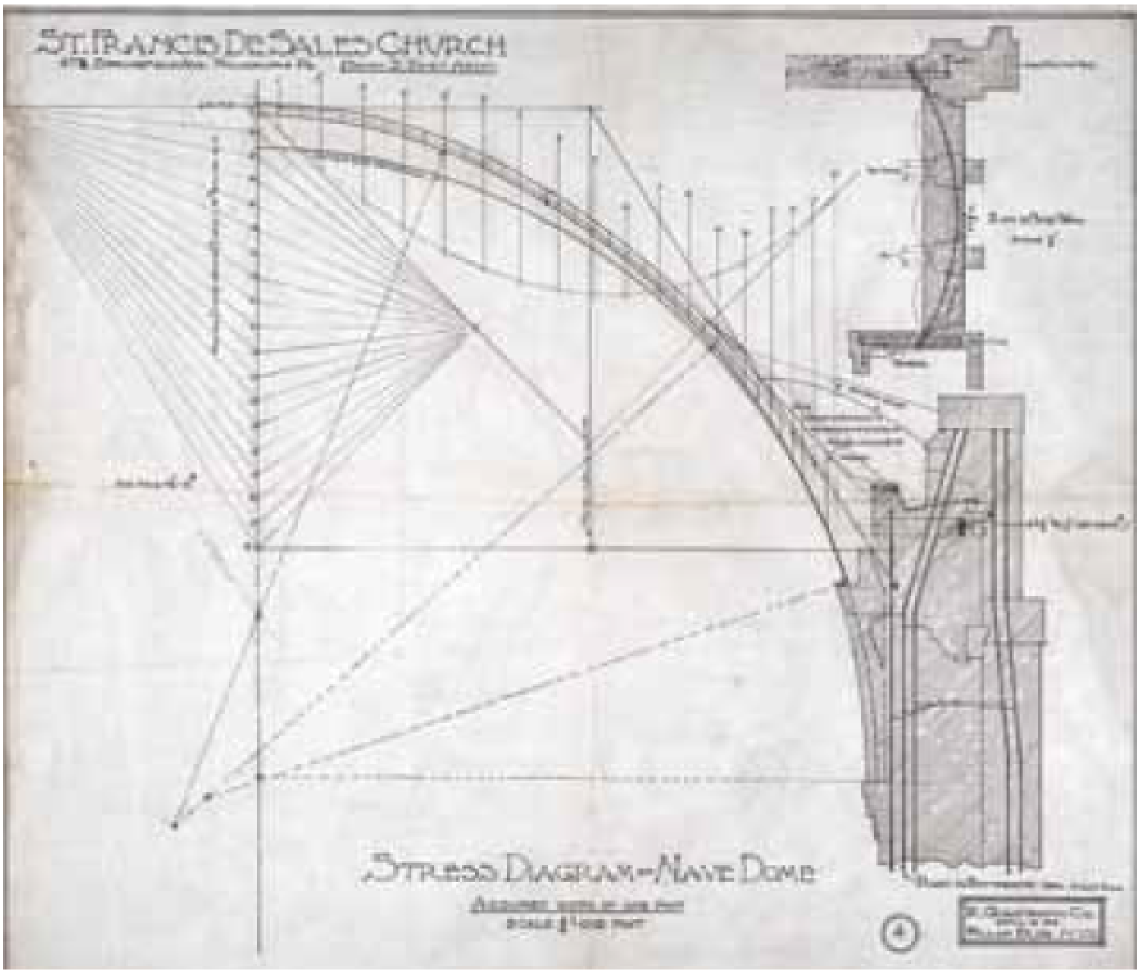


Figure 21: Graphic statics used by Guastavino Jr. to calculate the compressive forces in the dome of St. Francis de Sales Church in Philadelphia, 1909

Source: Avery Library

Guastavino Jr. recognized that the graphical analysis method could influence form, and he began to shape his structures in response to load paths (Figure 22). Since the dome would have tension forces near the base, Guastavino designed the geometry to ensure that the thrust would remain in compression. By this method, he was able to extend tile vaulting beyond anything previously built in masonry [89]. Gustavino Jr knew the crucial of the design of masonry structures. It was not the resistance of the material but the geometry of the structure. This was clear from his calculation of masonry arches using the usual and correct equilibrium approach employing simple formulae or graphical analysis (Figure 22) [54].

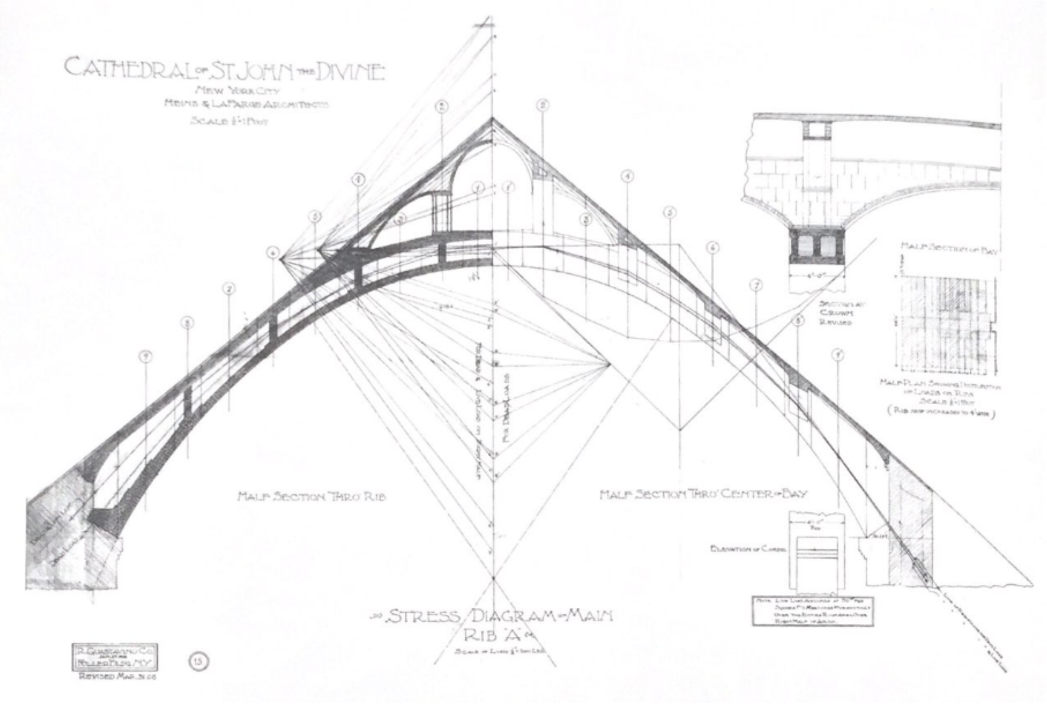


Figure 22: Section through vaulted choir roof with graphical analysis of arch forces

Source: [89]

3.1.6 Membrane theory analysis

The membrane theory, dating from the second half of the nineteenth century, was popularised in Europe in the 1930s, principally through the theoretical and practical work of the German engineer Franz Dischinger, using it to design thin shells of reinforced concrete.

Timbrel domes are thin shells, and to calculating the internal forces, both architects and engineers are using membrane analysis today. This analysis is an equilibrium analysis where all the internal forces are contained within the middle surface of the dome [50]. Theory assumes that no bending occurs and all forces are carried along the center-line surface of the dome as a mixture of hoop forces and meridional forces during construction and under asymmetrical live loading [50, 88].

The simple formulae of it were given by [110], and then it is developed by Schwedler in 1866 as an analytical method for trussed domes.

Eddy, in 1878 proposed the first graphic method(Figure 23) to the analysis of domes of revolution of any form [36].

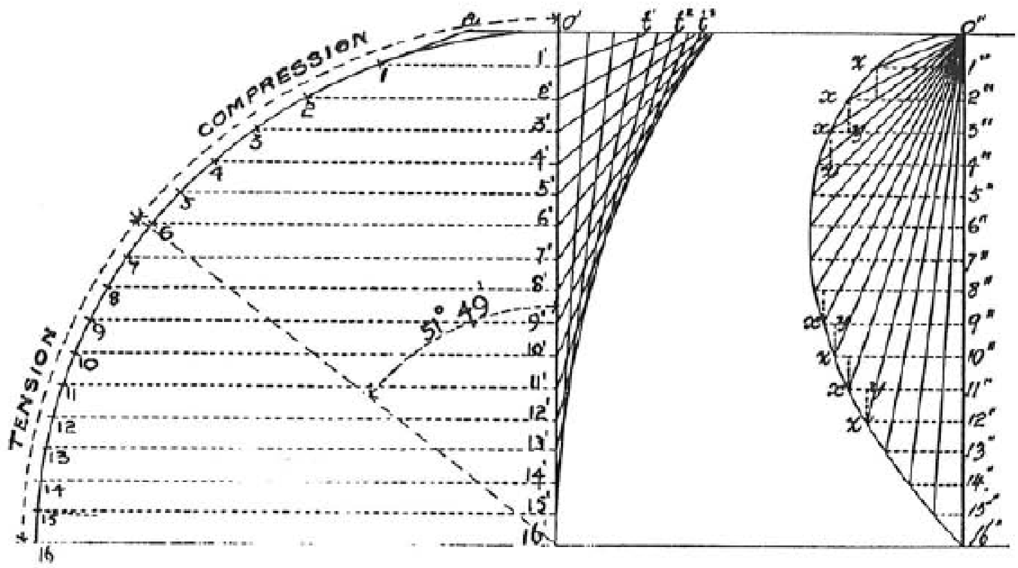


Figure 23: Graphical method for the membrane analysis of metal or masnory domes

Source: [[36]]

Guastavino Jr wanted to calculate and place iron reinforcement after his probably reading of Dunn's contribution. In the domes, there are two critical places: At the oculus when there is a lantern and at the base(Figure 24).

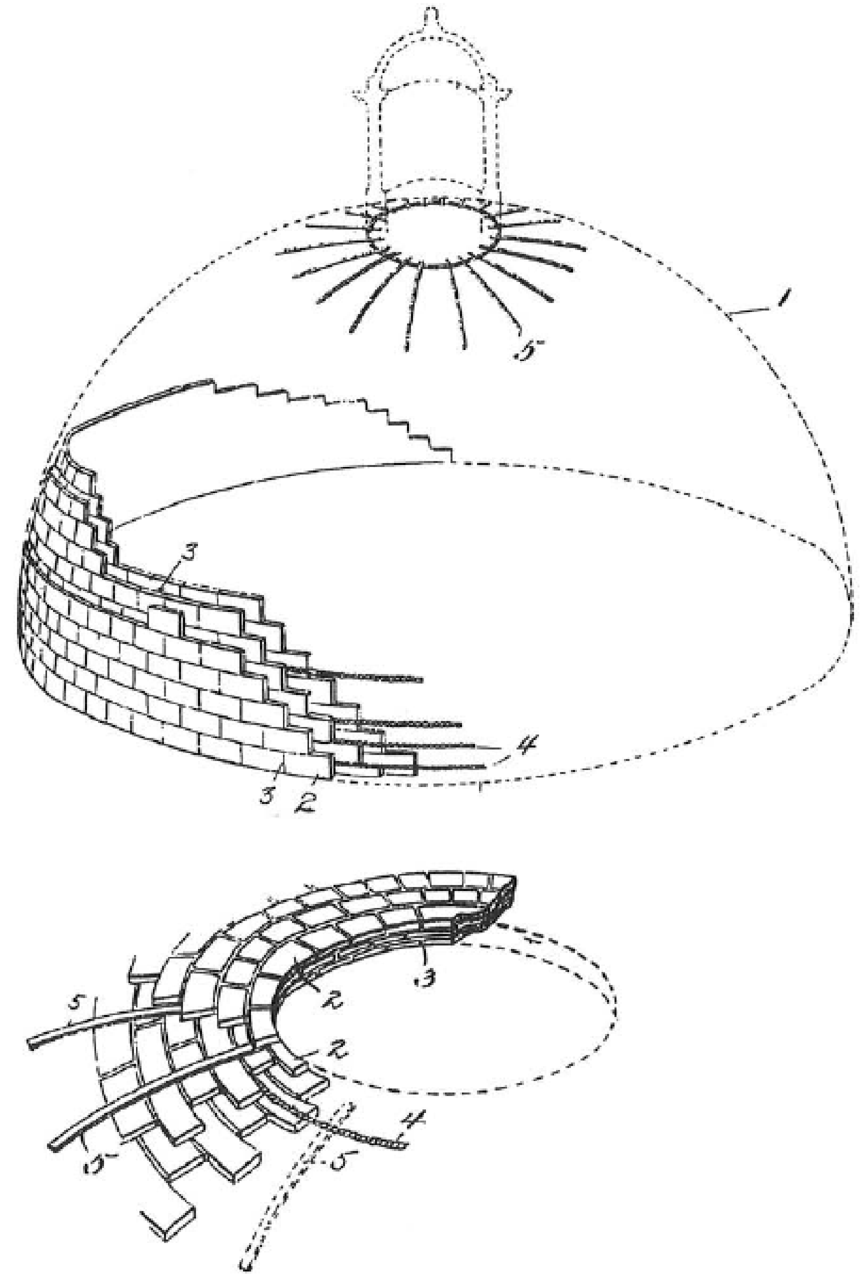


Figure 24: Placement of metal reinforcing in timbrel domes

Source: Gustavino,Jr.,Patent,1910

He used the method on many domes where St. John the Divine dome (Figure 25) was one of them and with reinforcement.

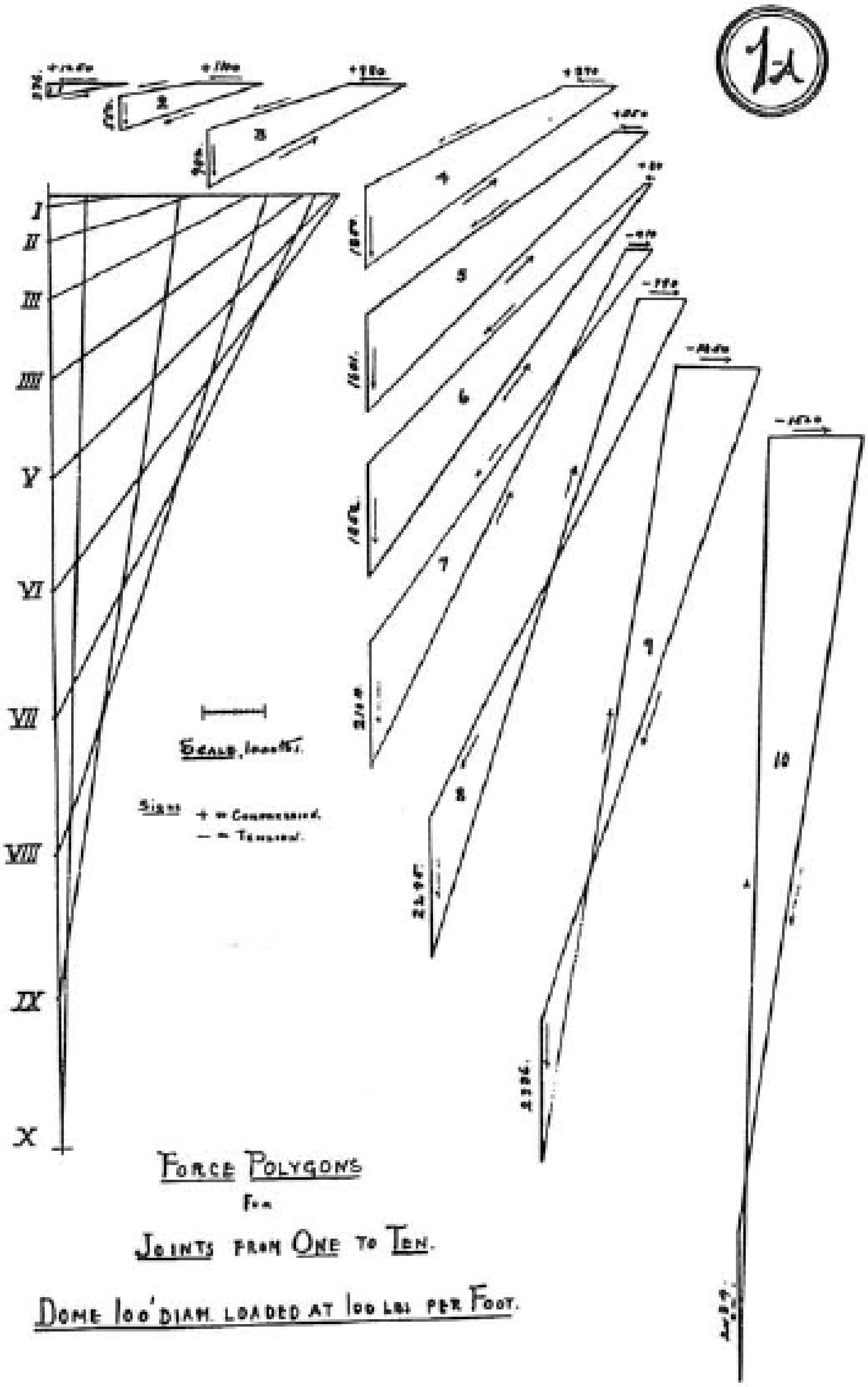


Figure 25: Graphical analysis of a thin dome with a span of 100 ft, force polygons

Source: Avery Library, Columbia University

The domes are working by compression and tension and not on the material. It is important to know where the tension occurs to place some supporting such as metal rings(hoops) that vary the direction of the force. This is associated strictly with the geometrical form of the vaulting. The approximate formulas (26, 27) used in calculating and dimensioning the counter-thrusts (buttresses, tension ties, or hoops) (Figure 28) are sufficient in designing of masonry dome [45].

$$N'_{\phi} = -aq\frac{1}{1+\cos\phi}$$

Figure 26: Meridional forces

Source: [28]

$$N'_{\theta} = aq\left(\frac{1}{1+\cos\phi} + \cos\phi\right)$$

Figure 27: Hoop forces

Source: [28]

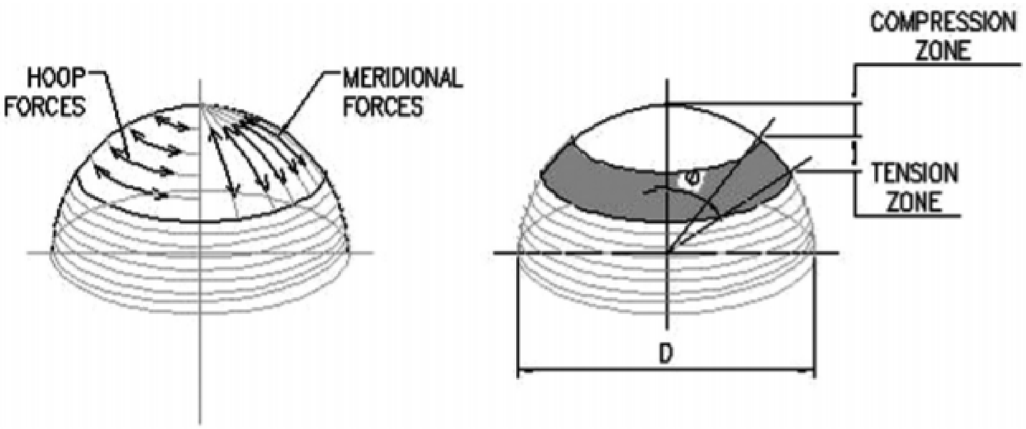


Figure 28: The sketches above were used to analyze the dome using meridian and hoop stresses

Source: [28]

The method for designing domes without tension was by determining the level of tension zones if they were reinforced. From that level to down, the thrust would remain constant. The upper section was a spherical shell, and from tension zones, the dome could be traced from the force diagram to give the form of a dome without any tension. Due to the lack of tension strength, the reinforced did not need, and the tension hoop rings could easily be calculated at the base. Gustavino used this method in his dome design(Figure 29), and this approach is better than the complete catenary approach (for example, Gaudi) [54].

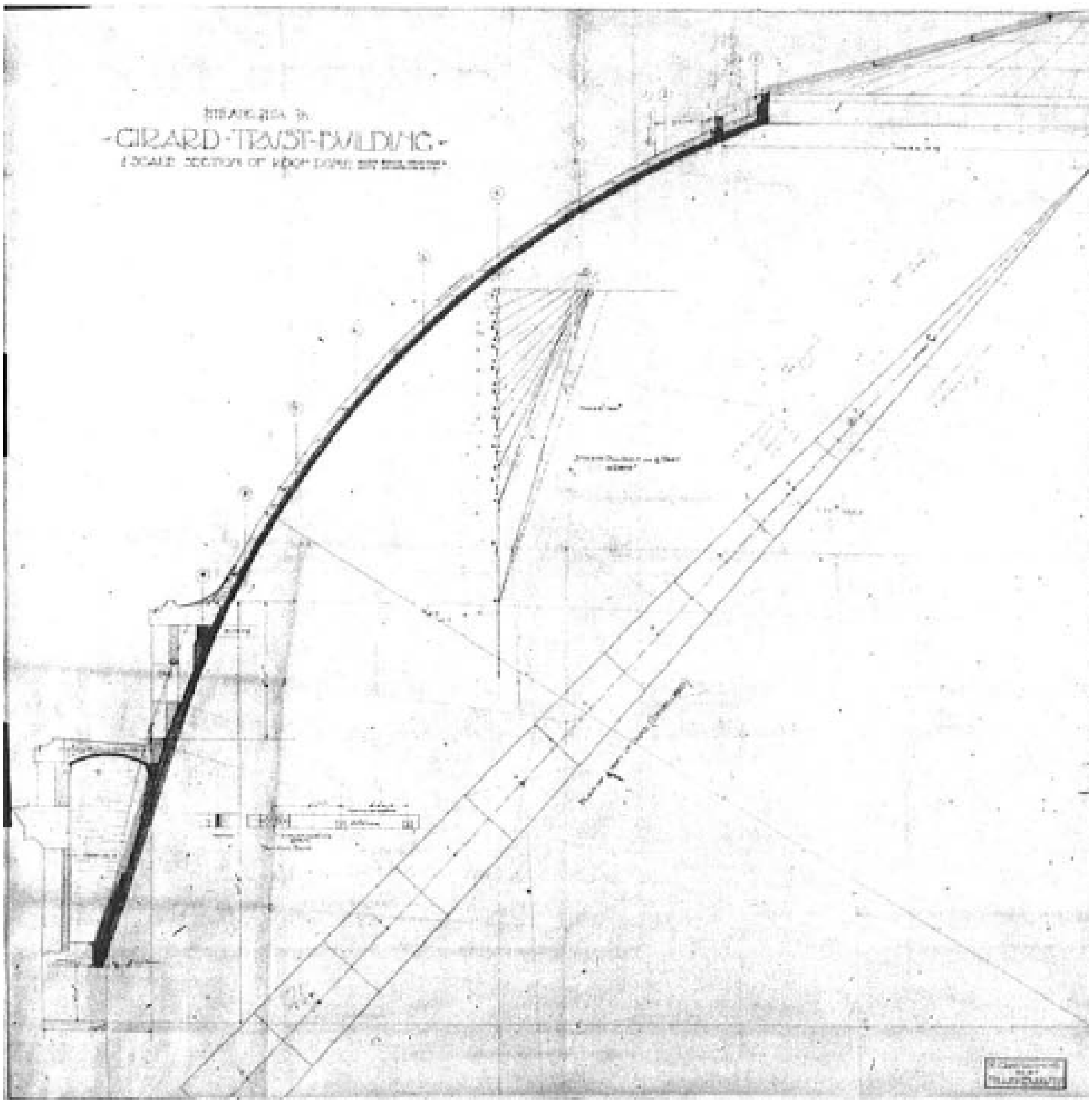


Figure 29: Design of tension-free timbrel dome.Note the change of curvature below the point of zero-stress,the horizontal component of the thrust remaining constant below, see the force polygon. Dome of the Girard Trust Building 1905-1907;101 ft span(31m)

Source: Avery Library, Columbia University

3.1.7 Elastic analysis

The second half of the nineteenth century was the time for Elastic theory, which incorporated the earlier concepts of Espie (changing monolithic behavior for continuity, homogeneity, isotropic materials, etc.).

Elastic analysis was considered one of the best approaches for masonry arches. There were many problems with these arches as discontinuity, heterogeneity, difficulty in obtaining the elastic constants, the movements during construction, the cracking, etc. Therefore, engineers were conscious of the dubious character of elastic assumptions applied to masonry arches [122], but the force of elastic ideas was an exciting thing to overcome any resistance. Elastic concepts of continuity, tension, and bending strength respond well with Espie's monolithism and Guastavino's cohesion. The only fundamental difference is that elastic arches do thrust. For this reason, made both Bergós, Gaudi some calculations to take into account the bending strength of timbrel arches [127].

José Doménech Estapá was the first one who considered the necessity of considering the resistance

to bending moments. He claimed that the success of the thin timbrel vaults came from its capacity to resist bending moments that could cancel the horizontal thrust. In other words, the mechanical secret to the timbrel vaults is not in limiting the calculation of the compressive strength of the materials used but in taking advantage of the tensile resistance, and transverse strength [33].
Domenech made a lucid analysis of timbrel arches by application uniform load in which the line of thrusts is a parabola. He remarked that if the directrix of the arch coincides with the line of thrusts (i.e., the arches are exactly parabolic), then there would only be compression. He could find the bending moments and shear forces for a given line of thrust [33].

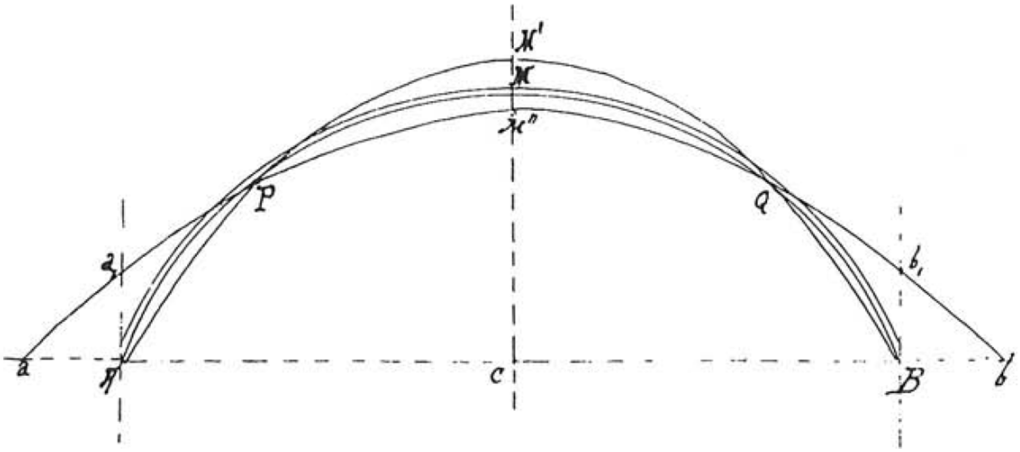


Figure 30: Possible positions of the thrust line in a timbrel arch

Source: [33]

But Domenech committed an error by his method, which identified the structure only as of the vault, and he forgot the fill over the supports, and the transverse diaphragms that support the vault [33].

The method of graphic mechanics applied to brick arches and, in a particular way, to timbrel arches. The cohesion, the rigidity of timbrel vaults significantly lowers the thrust and allows vaults to be built in implausible Forms. He highlighted the necessity for tests allowing the calculation of the “ coefficients used in calculations to evaluate the bending resistance and the transverse forces in the timbrel vaults” [71].

Jaime Bayo was the first one in Spain to propose using proper elastic analysis for timbrel vaults. For him, the timbrel vaults thrust, but this thrust corresponds to metal arches (two-hinged). He tried to find “the funicular of the elastic forces”, which is the line of thrust and complies with elastic deformations compatibility conditions. He recommended adjusting the thickness (number of bricklayers) according to the bending stresses. He noticed that the thrust could be calculated as if it were made of voussoirs in the flat vaults, working only in compression, but if they are higher, it is precise to adapt the form to the line of thrust. He proposed a method to design timbrel vaults of any shape as follow :
It is desired to construct equilibrated vaults, or of equal resistance that responds to the design suggested by the imagination of an artist, one should proceed as shown in(Figure 31). After determining the funicular of the elastic forces, the thickness of the vault is given concerning the value of the bending moments [80].

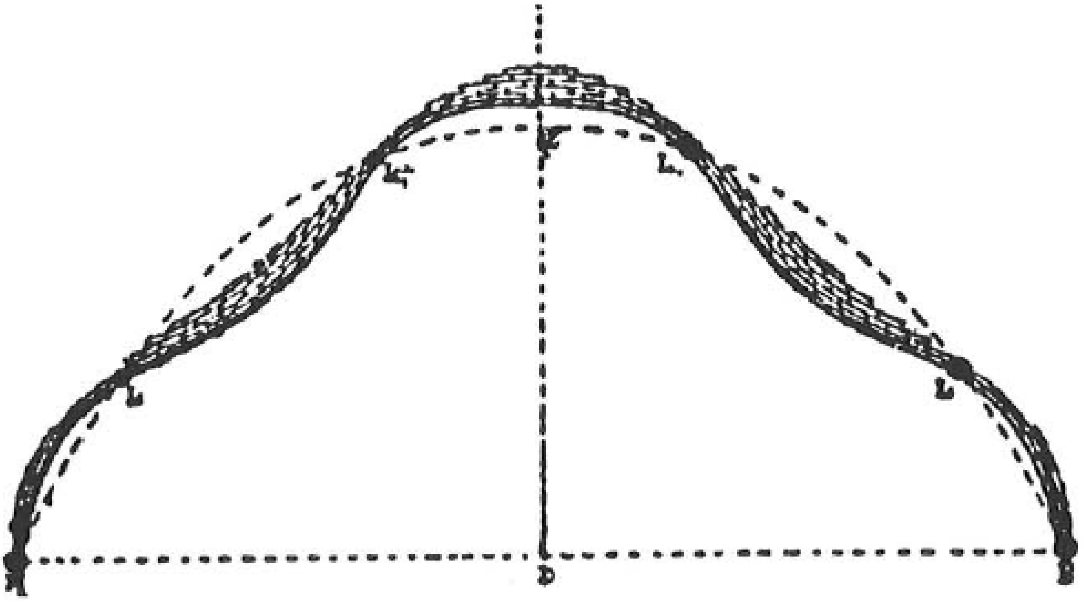


Figure 31: Timbrel vault of a peculiar form (impossible to construct in reality)where the thickness is prescribed according to the magnitude of the bending moments resulting from an elastic analysis

Source: [80]

Planning the elastic equilibrium equations for a spatial structure such as a vaulted staircase was not easy. Esteve Terradas, the great Spanish engineer and mathematician, was the first to try an elastic analysis and examined well-known elastic problems as buckling of a more complex timbrel vault: the vault of a staircase but failed. The failure of Terradas served to reinforce the idea of the impossibility of calculating the forces in timbrel vaults [118].

3.1.8 Calculations in practice

The architects or engineers made simple calculations to decide the dimensions of the principal elements as the thickness of the vaults and sizing of the systems to resist the thrust.

Luis Moya Bosch in 1957 was the last great builder of timbrel vaults, recognized the insufficiency of calculations due to the lack of data on the elastic constants of timbrel vaults. Therefore he later made or directed another to make equilibrium calculations based on the line of thrust to design and build his astonishing vaults [81].

Bosch expressed in favor of membrane analysis but for practical cases. He introduced an ingenious system (inspired, no doubt, by nineteenth-century manuals on the theory of vaulting) to calculate the thrust of timbrel vaulting. His system was by cutting the vault into a series of arches and imagined the existence of virtual crossing ribs which support a series of parallel arches between the ribs. This was an equilibrium method that sought to find one possible compression state within the masonry [113].

Joan Bergós I Massó devoted several decades to studying the mechanical properties of masonry walls and timbrel vaults. He tested timbrel arches of various sizes (up to 3.2 m in span), working to explain the application of elastic theory. But in fact, he used graphic methods of thrust lines, that is, equilibrium methods [72]. Angel Pereda Bacigalupi in 1951 supposed two-hinged arches on rigid supports and calculated them with the typical equations for elastic arches. In that time, the vaults were often built with tension ties to take the thrust. The problem was the deformation of this tie was not considered in the calculation, even though it would lead to significant bending moments. He understood that an elastic calculation could not play any role in the flexural resistance of timbrel vaulting. He looked for the thickness so that the line of elastic thrusts was contained within the middle third of the section. To achieve that, Pereda reduced the admissible tensile stress, showed a better knowledge of the material properties than his earlier predecessors working with elastic calculations [100].

3.1.9 The use of Finite Element Methods

The finite element method (FEM) is one method that has been applied to the analysis of timbrel vaults.

In traditional masonry structures, the stresses are very low because safety depends on stability and strength. Finite element methods depend on minimizing the strain energy by invoking assumptions about the material behavior. It is difficult to demonstrate the safety of these thin masonry shells by this method. The calculation methods used by Gustavino are based on static equilibrium and not on the stress distributions in the hyperstatic structure [88].

However Gulli has used the method to carry out elastic calculations. This method is like traditional elastic calculations, consider the masonry as a continuum with specific elastic properties which demand assumptions about the support conditions [47, 48]. These assumptions about the supports and the material, together with static equilibrium, form equations that give a unique solution. Even the use of FEM programs allows a non-linear analysis, but this brings some challenges as :

- The system of equations or construction model is susceptible to small changes in the support conditions. For example, a small settlement or rotation of one of the supports, invisible to the eye, will give a large variation in the system of internal forces (the analyst can use a FEM program to verify this point).
- The load history of the structure.
- Timbrel construction is far from a continuum and is frequently cracked.
- The formation of cracks in unexpected locations, etc. This means that both an elastic analysis or the FEM method have no assistance in understanding the structural behavior of the timbrel vault or masonry structure.

3.1.10 Thrust Network Analysis (TNA)

A new interactive equilibrium method or new form-finding approach was used as prototype at ETH Zurich in two hands-on workshops in Sydney and Melbourne, Australia with using tile vaulting to build three-dimensional networks of structural ribs and infills or “patches” between them (Figure32).

Figure 32: Workshop at MADA, Melbourne, 2013

Source: [69]

Two steps in the design process to acheive the final design: by an abstraction of the structural action to obtain the equilibrium of the ribs alone using a rough and straightforward form diagram subdivision and the following refinement of the "low-poly" designs together with the addition of the infills.
The design of the ribs in the first workshop examined "undulating strips of hexagonal units" [15]. In contrast, the second structure was based on a stretched, quadrilateral grid, in which each segment was straight in the plan.

The new method called Thrust Network Analysis to design and analyze free-form tile vault is developing a masonry vault. This method was a theoretical basis for developing the computational form-finding tool RhinoVault [69].

Inspired by the prototype built at the ETH by the BRG in 2011, The new equilibrium method developed in 2013 on graphic statics was two-dimensional thrust-line analysis [13] and used on the cross-sections of the doubly curved, parabolic vaults to define their final shape [107]. Besides, 3D equilibrium verification was carried out using Thrust Network Analysis [14, 108]. The architects of Map13 Barcelona (Marta Domènech, David López López, and Mariana Palumbo) [70] used the form-finding tool RhinoVault to design the first human-scale, free-form tile vault at the International Festival of Architecture Eme3 in Barcelona in 2013 "Brick-topia" (Figure 33).

Figure 33: Brick-topia

Source: [69]

This showed the relevance of the newly developed form-finding computational tool to build projects of tile vaults (Figure34).

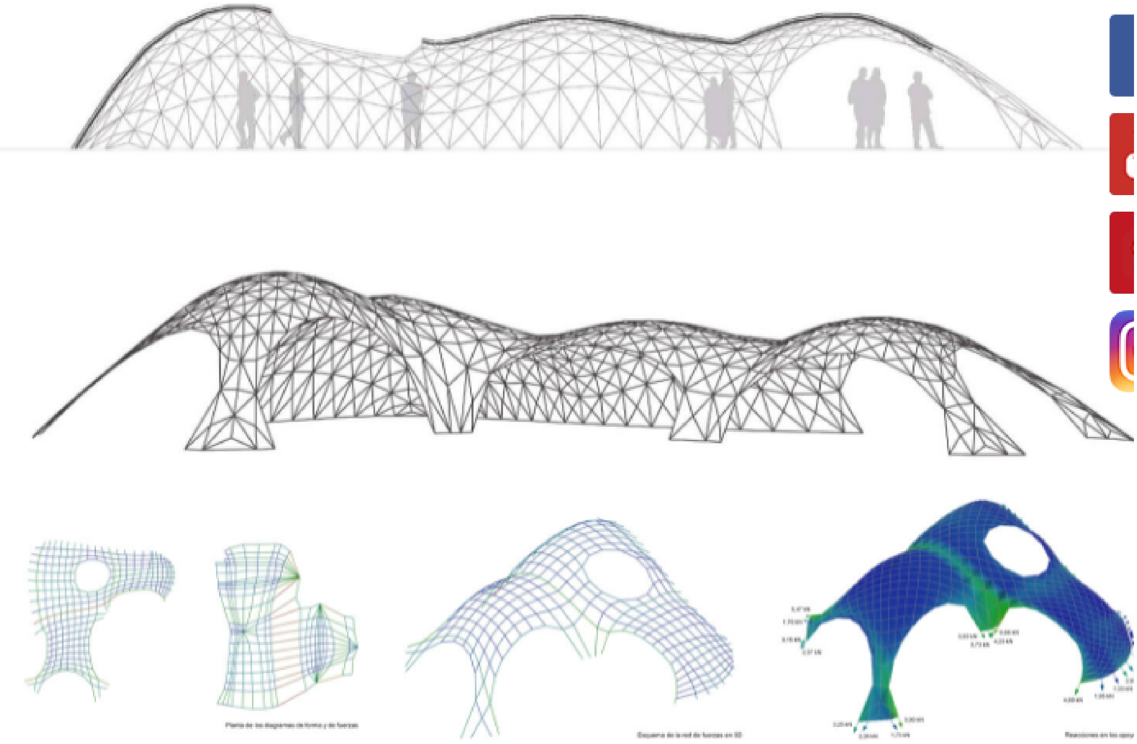


Figure 34: Brick-topia under analyzing

Source: [19]

Free-form vaults build on tile vaulting rely on the fully three-dimensional structural action to stand in compression [69]. Any structure was designed on any of follows methods are safe and will not fall. -The equilibrium analysis of the old theory of vaults is, therefore, perfectly correct and lies within the scope of Lirnit Analysis [126]. -The simplified formulas of Guastavino, the graphic analysis, Gaudi's use of hanging models, and membrane analysis of compression states by Guastavino, Jr., are all correct. -Elastic analysis in compression, like that of Pereda, cited above, is also correct. -The traditional proportional rules for the design of vaults and buttresses (like those of Fray Lorenzo) are also essentially correct [55].

3.2 Materials

Common to all arches and vault constructions is that they need extra measures to absorb the horizontal forces. Stag, strap, chain, drill weight/counterweight was used. Sometimes visible struts are massive counterweights and friction structures; other times are invisible steel stays in the ground, stair pillars, etc. But almost always, the ceilings appeared beautiful, with visible stones, bricks, and "terracotta" tiles, and concrete [29].

Usually, two layers of brick tiles are used in Catalan vaulting (about 10 cm in total thickness, including the mortar between the layers); even brick of only one layer can be found (5 cm thickness). The slenderness ratio is typically 100, this ratio relates to the radius of curvature to the span, but there are even thinner vaults. The largest span of 33 m was the dome over the crossing in St. John the Divine, New York, with [103].

Many projects or prototypes have been built using compressed, stabilized earth tiles, tiles made of stone, or even ice tiles.

The material includes in the composition of this kind of vault was easy to make and affordable [111].

A structural engineer confronts difficulties for the stability and safety of the masonry arches or vaults. According to a modern understanding of structural behavior and material strength, a structural engineer is often confronted with withstanding to choose between reinforcing the structure or trying to make sense of the behavior and anticipated strength of the system on a more fundamental level [60].

Conventional retrofitting techniques are one of the solutions to strengthen and adequately protect them [18, 30]. Traditional retrofit methods also combined with innovative materials can be helpful in achieving seismic performance by interventions that respect the structural system and, at the same time, remain completely removable is often hardly possible [115].

There are some conventional retrofitting techniques as single or double-sided jacketing with the cast in situ reinforced concrete, reinforced grouted injections (3-4), crack stitching ties, external or internal post-tensioning with steel ties has the ability to adequate the increment in strength and stiffness. These were some of the ways and benefits, but there were some drawbacks as it is short-lived, labor-intensive, conservation and restoration requirements [18, 26, 30].

The wight of Materials used in the construction of the St. John the Divine Cathedral was ten times less than it was at the dome of the Basilica di Santa Maria del Fiore (1436) in Florence and the use of steel bars in areas with tension forces illustrated the understanding of structural behavior.This method of reinforcing the thickness of the vault make Guastavino Jr receive a patent in 1910 [34].

Gustavino's company built one final decorative layer of tile from below which allowed the creation of more complex tile patterns while the pattern could be resolved in three-dimensional forms. The masons often worked from the top of the vault down to the supports as following:

- The first layer was putting the fast-setting plaster of pairs and the goal of this step was to minimize the formwork.
- One layer of tile on top with adding portland-cement based mortar
- Third layer of finish tile from below glued with portland cement mortar(cement mortar).
- The joints finished form below with an extruded portland cement mortar(cement mortar). After defining the form of the vault with the initial structural layers of tile (Figure 35) the additional layers could be applied on the top and that was depending on the strength of the vault [89].

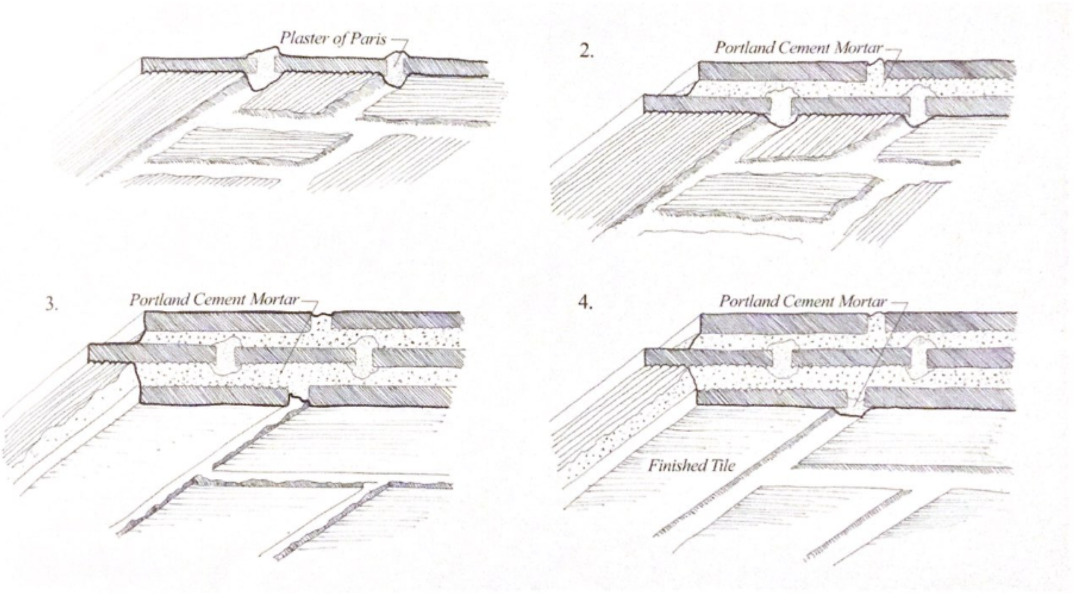


Figure 35: Four stages of mature Gustavino tile construction procedure

Source: [89]

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The company faced structural difficulty at the Custom House(1904) project in New York, which was with spanning(26 meters by 40 meters) because there was no clear concept for supporting the dome. They solved this by inventing a double-tile dome with an elliptical steel compression ring at the top and a steel tension ring at the bottom [74].

The Guastavino company integrated its vaulting as a part of the building's structural system into the steel framing so that the loads were shared between the two systems [96].
Guastavino used iron hoops to manage the flow of the thrusts within the masonry. He also used many other materials, such as flying buttresses, dwarf vaults, and massive cornices(Figure 36).

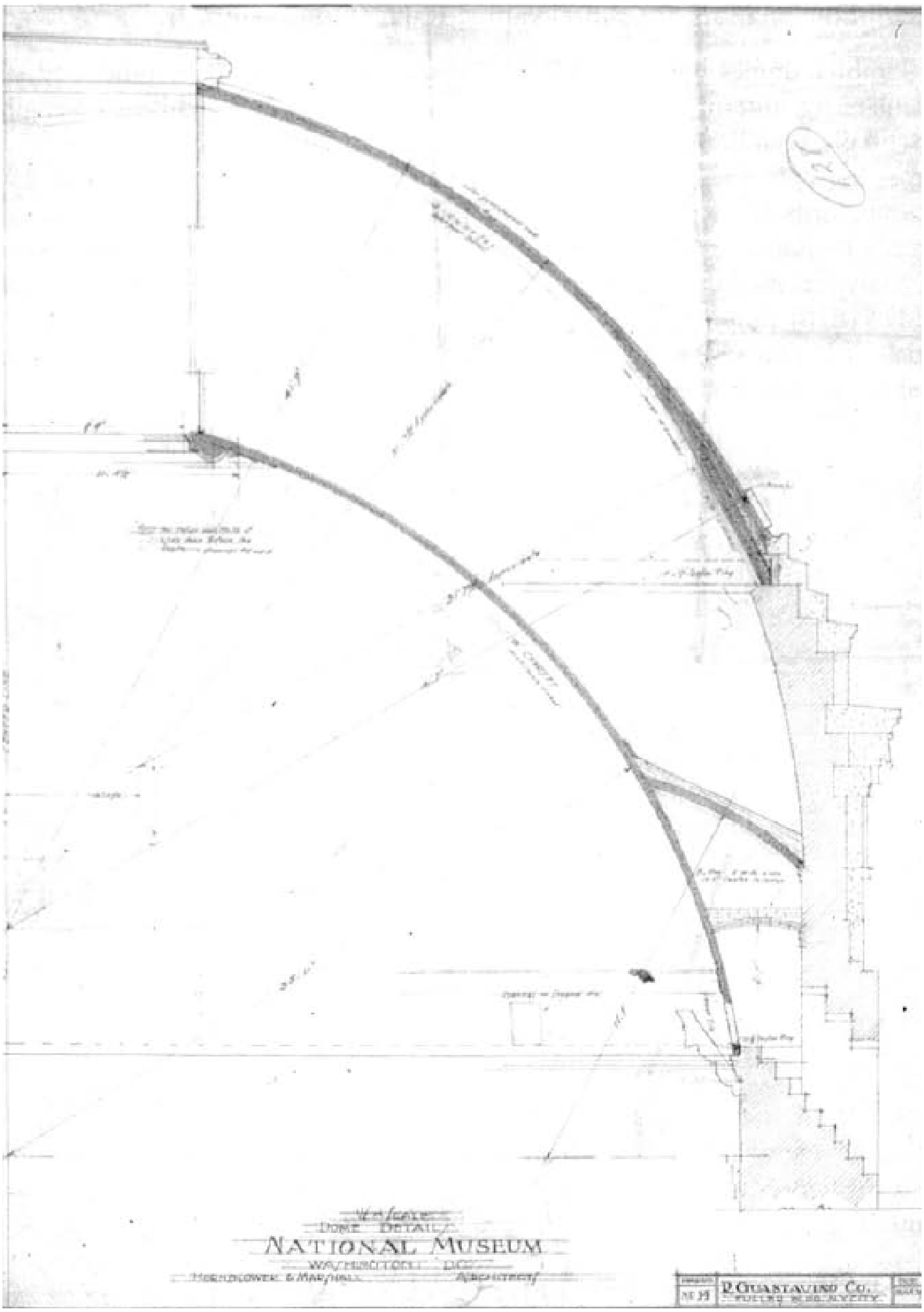


Figure 36: Double timbrel dome in the national museum of Washington,1906. Note the variations of curvature of both shells to avoid tensions and the different devices to resist the thrust of the domes including metallic rings, dwarf,flying buttresses and heavy stone cornices

Source: Avery library, Columbia university

The wooden planks at the top are not centering in the timbrel dome (Figure 37) but serve only as a guide to control the geometry during the construction [54].

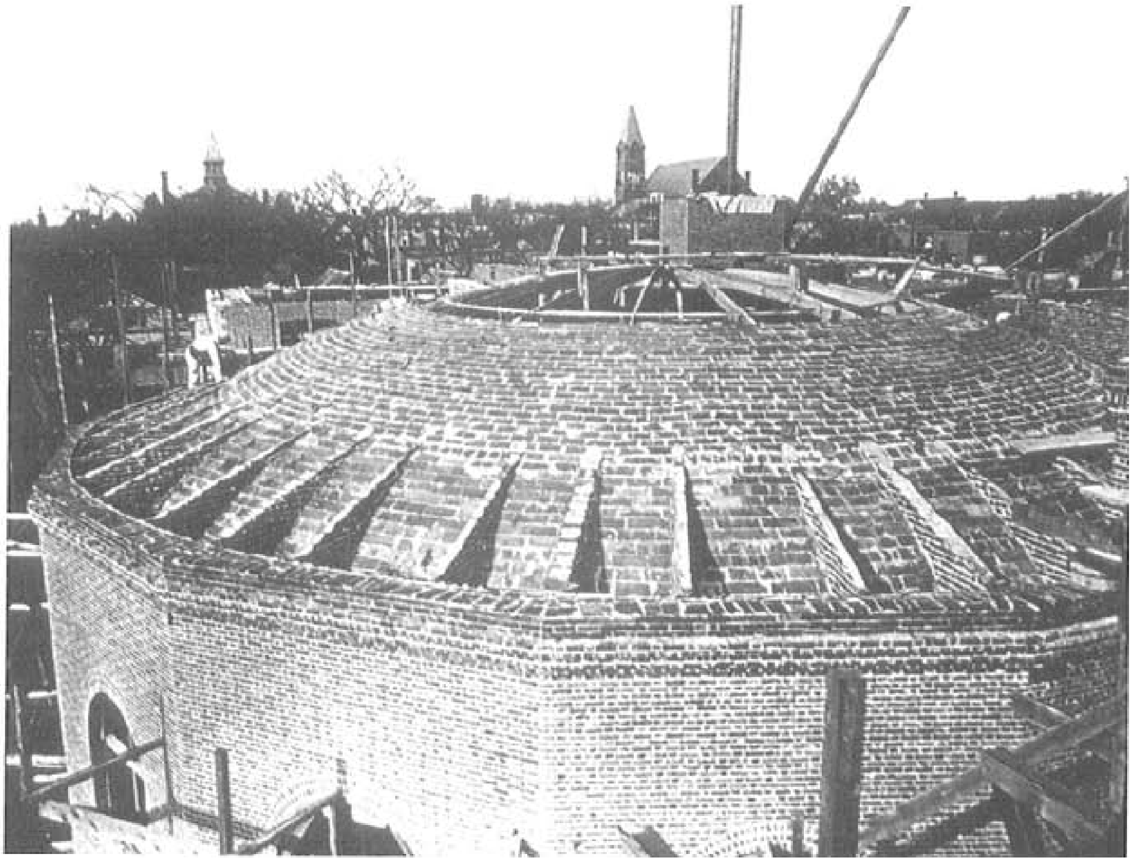
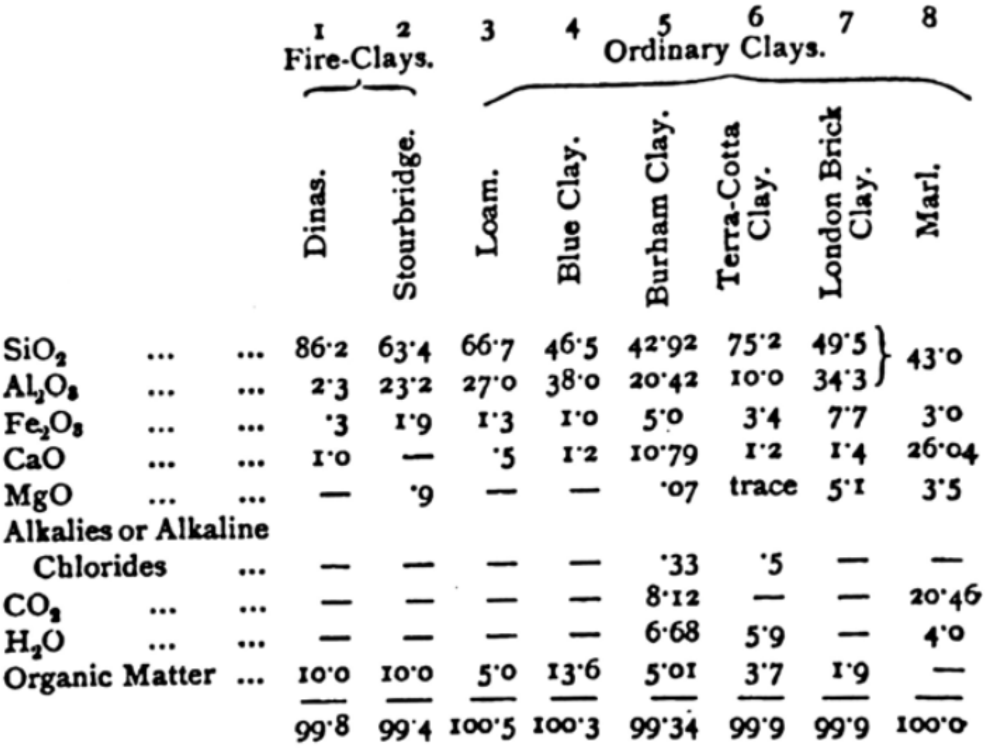


Figure 37: East Boston High School

Source: Courtesy Avery Library,Columbia University

3.2.1 Thin bricks(tiles)

Bricks are an artificial kind of stone made of burnt or baked argillaceous or clayey earth, and the quality depends upon (a) The chemical properties of the earth. (b) The preparation of the earth. (c) The different degrees of burning and baking. There are many different kinds of clays, or silicate of alumina, with other chemical structures. Mitchell provides a thumb-rule for good brick earth or clay, "alumina, one fifth; Silica, three-fifths; iron, lime, magnesia, manganese, soda, and potash forming the remaining fifth" [76]. The table shows the chemical composition for different clays, and proportions are expressed in percentage.



	1	2	3	4	5	6	7	8
	Fire-Clays.		Ordinary Clays.					
	Dinas.	Stourbridge.	Loam.	Blue Clay.	Burham Clay.	Terra-Cotta Clay.	London Brick Clay.	Marl.
SiO_2 … …	86·2	63·4	66·7	46·5	42·92	75·2	49·5	43·0
Al_2O_3 … …	2·3	23·2	27·0	38·0	20·42	10·0	34·3	
Fe_2O_3 … …	·3	1·9	1·3	1·0	5·0	3·4	7·7	3·0
CaO … …	1·0	—	·5	1·2	10·79	1·2	1·4	26·04
MgO … …	—	·9	—	—	·07	trace	5·1	3·5
Alkalies or Alkaline Chlorides …	—	—	—	—	·33	·5	—	—
CO_2 … …	—	—	—	—	8·12	—	—	20·46
H_2O … …	—	—	—	—	6·68	5·9	—	4·0
Organic Matter …	10·0	10·0	5·0	13·6	5·01	3·7	1·9	—
	99·8	99·4	100·5	100·3	99·34	99·9	99·9	100·0

Figure 38: The chemical composition of different types of brick clay

Source: [76]

Catalan vaulting's distinction stems from their construction. The thin bricks (tiles) are laid flatly to form one or more layers which constitute the surface. The first layer of tiles is joined with fast-setting plaster, which sets so quickly that tiles are held in place instantaneously and without any costly wooden formwork or any hage supports [89].
This technique in the ceiling was with layers of thinner, lighter bricks to create a ceiling not only light but also solid [111].
The tiles are arranged in arches or successive rings to complete the vault. During construction, the bricks are supported by the adhesion of the fast-setting mortar to the completed courses or the bordering walls. In some cases, for large vaults or a high-quality finish, some materials can be used as guides to control the geometry of the vault [103].
The adhesion of the bricks is performed by inclining the brick courses to form arches or rings [78].
The main advantage of tiles was the structured layout of the nodes, which gives more flexibility to generate a straightforward mesh [29].

The decorative tile, the last layer, could be any Guastavino's tiles either as an acoustical product or a glazed ceramic tile of various colors.
The ceramic finishes had a lot of benefits: Easier cleaning, improving sanitation, reflecting sounds, creating noisy spaces, and allowing new aesthetic possibilities [111].

This combination between the function of load-bearing masonry and an acoustical was unprecedented [124]. Some examples of decorative Guastavino vaulting is:
The Della Robbia Room of the Vanderbilt Hotel in New York City (1912; for Warren and Wetmore), the Holy Trinity Roman Catholic Church in New York (1911; for Joseph H. McGuire), and the Forsyth Dental Institute in Boston (1913; for Edward T. P. Graham) [89].
Gustavino's company got a patent for producing two types of tiles "Rumford tile" which was ceramic. The second type was "Akoustolith title" which was nonceramic [92].

3.2.2 Mortar

Mortar consists of inert siliceous (sandy) material combined with cement and water in proportions and used in masonry for joining tiles, stones, bricks, or concrete blocks [79]. The main ingredients of mortar are [79]:

- Water
- Binder Generally are based on one of these categories:
  - Hydraulic cement, which reacts chemically with water at average site temperatures.
  - lime-silica mixtures, which react only in the appearance of high-pressure steam.
  - lime-pozzolan mixtures set slowly at ambient temperatures, or pure lime, which sets slowly in the air by carbonation.
- Aggregates The aggregates are often sand which is rock particles of different sizes from 10 mm in diameter down to 75m.
- Admixtures, examples are plasticizers, superplasticizers, accelerators, retarders.
- Pigments

The differences between mortar and plaster lie in the capacity of plasters to take better finish, which depend on the type of sand used in the mix [17]. Contemporary engineering found that using fast-setting plaster provide sufficient cohesion to help the weight of several tiles and at the same time proving the idea of monolithic behavior [24].

Builders have to use very slowly a fast-setting mortar or gypsum plaster to allow the brick to hold itself after being tapped into place [111]. plaster of Paris (gypsum mortar) is one type of mortar which sets quickly enough that the vault's interior does not require any support from below under construction [88].

Rafael Gustavino substituted the traditional mortar with rapidly hardening portland cement of the bricks with thin tiles. In this way, he could build some vaults which were 3 to 5 times wider than the typical size of a traditional timbrel arch was [12].

The first layer of tile vaults was built by the quick adhesion of mortars like gypsum or fast-setting cement. After the first layer was located, then it could work as permanent formwork for other layers [24].

The use of Portland cement-based mortar between the layers was to minimize the use of plaster of pairs (gypsum mortar) which is susceptible to water damage and lower strength. Manufacturing of tile vaults and the use of flanged tile provided for a cleaner and more durable mortar joint on the exposed surface of the vault and limited the use of plaster of Paris (gypsum mortar) in construction [89].

3.2.3 Catalan vaulting with reinforced concrete

By Guastavino Sr's definition of cohesive construction, the massive concrete arches (without reinforcing) are also cohesive. Still, he discards them due to high cost and problems with the irregular setting of the concrete. He did not like the steel frame in the late nineteenth century and described it as "a human skeleton only enveloped with the skin, without artistic life, and soul [45, 97].

Eladio Dieste could combine the traditional technique of "bóveda tabicada" with a thin layer of reinforced concrete to allow crossing big spans getting extreme shapes. The church in Atlándida was one example, and the second one was Cuba national art school(Figure 39) [125].

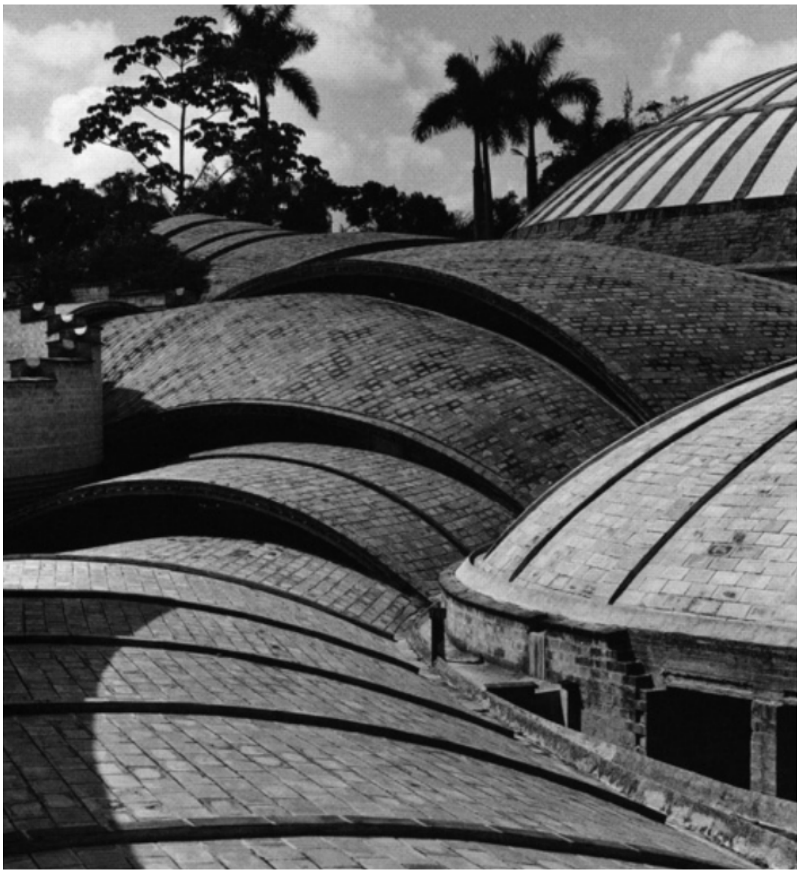
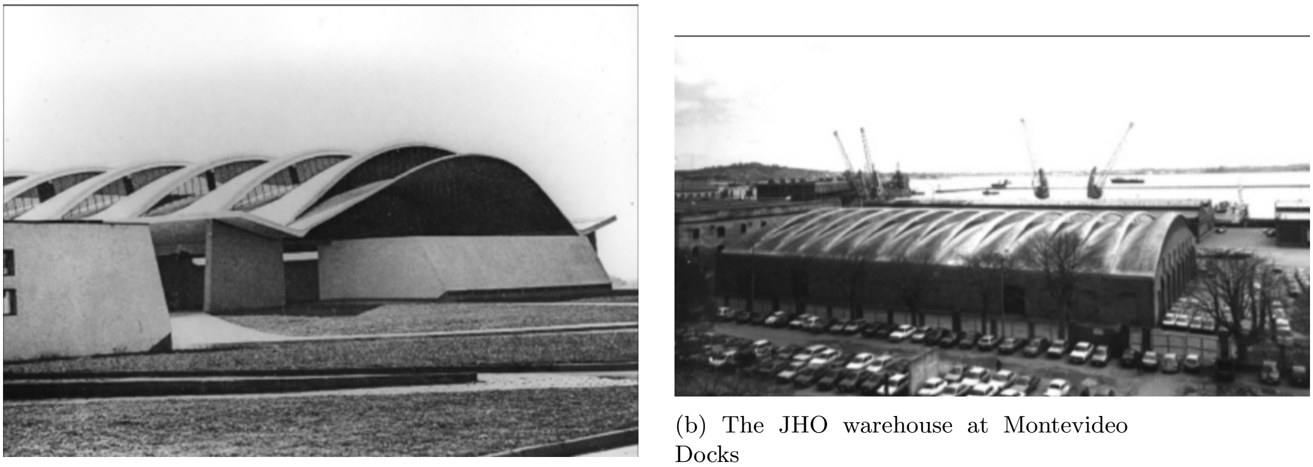


Figure 39: Cuba national art school

Source: [125]

Dieste used brick as the basic material. He bent it to take advantage of its resistance to compression and developed two types of vaults: double curvature Gaussian vaults, with and without skylights(40a), and self-supporting vaults(Figure 40b) [12].



(a) Gaussian Vaults, Porto Alegere, Brazil

(b) The JHO warehouse at Montevideo Docks

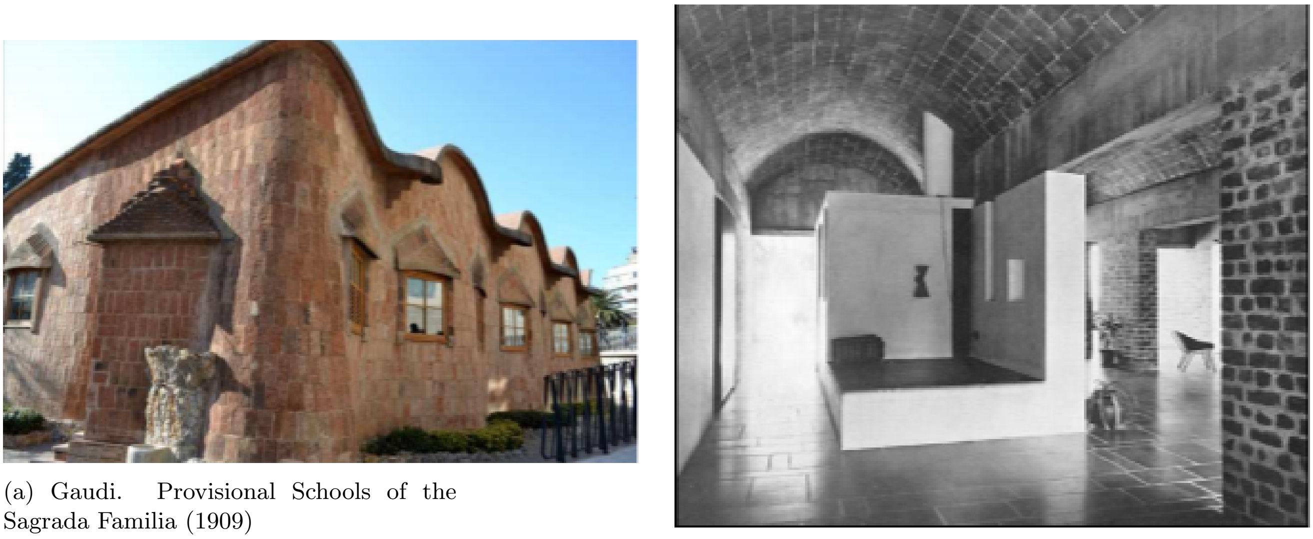
Figure 40: Eladio Deste's construction

Source: [[99]]

Antoni Gaudi found his manifesto in the Provisional Schools of the Sagrada Familia (1909), where there was no distinction between supporting and supported ones and manufactured to the design criteria of the tabicada technique. Everything responds to one of the sinusoidal profiles of the conoid and the tabicada technique.

This was adopted in the wall and the roofing. He called this form for conoid. Layers of bricks were obtained with dimensions 15×150×300 mm linked with gypsum paste and lime mortar and cement in the first and second layers(Figure 41a).

Le Corbusier designed one system between the Catalan vault and concretion, one used by the Romans. This system consisted of two independent layers of large format bricks. The relationship quantity binder-size brick and the concept of structural cohesiveness aren't respected as it was with Gustavino. This way was by filling the first layer extrados with lightweight concrete mixed with a mesh of iron rods, and the resistant section includes the area of the abutment filled with a porous material. The Catalan vault became a casing but with reduced resistance (Figure 41b) [103].



(a) Gaudi. Provisional Schools of the Sagrada Familia (1909)

(b) Villa Sarabhai (India, 1951)

Figure 41: Gaudi's vs. Le Corbusier's system

Source: [[12]]

3.2.4 Aerated Autoclaved Concrete (ACC)

AAC (Aerated Autoclaved Concrete) is lightweight cellular ceramic with a rough surface well-suited to the application of mortar; and good insulating and acoustical properties.

ACC was invented in the 1920s in Sweden, but it's only introduced by an American company called TruStone. It reaching as large blocks but can be cut on a band saw into tiles for vaulting.

The ceramic tiles and fast-setting gypsum mortar are the hallmarks of timbrel vaults. This technique allows thin structural spans to be built without any supporting formwork by replacing traditional brick of "terracotta" with tiles made from AAC [109]. There are many advantages to mixing traditional timbrel vaulting with Aerated autoclaved concrete (ACC) for its lightweight(Figure 42).

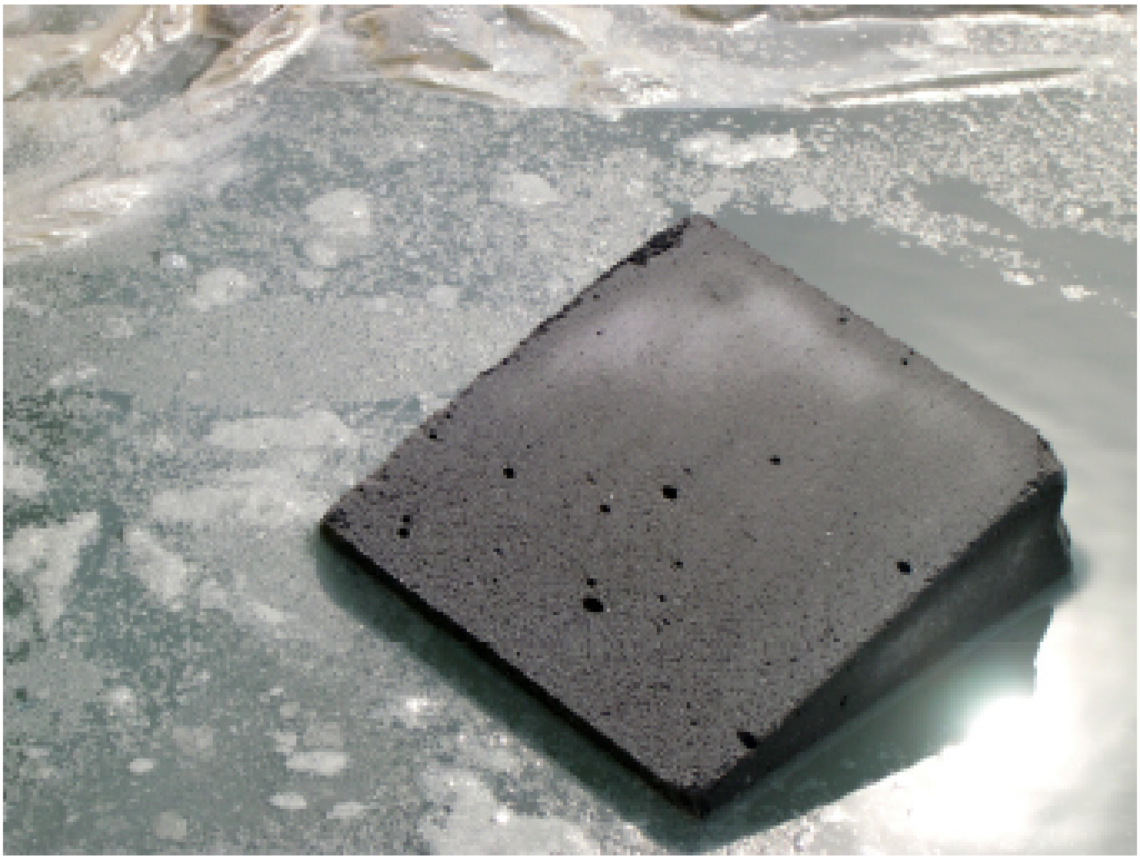


Figure 42: AAC block floating in water

Source: [109]

In addition to formal, material, and economic possibilities that can be gained through AAC. Some other qualities of this product are its R-value, fire resistance, easing of cutting and assembly, and ready-to-finish surfaces [73].

The ingredients that create the material AAC are the same as concrete, with a small amount of powdered aluminum mixed seconds before the batch is poured into a mold. The goal of aluminum is to bind the chemicals in the mixture by releasing hydrogen gas which imparts macroscopic bubbles into the “cake”.Then the green cake must be left to rise for 45 minutes, after which it is run through a wire slice to produce the desired block size. The final stage is cutting material and moving it into an autoclave for cooking for a few hours under high temperature and pressure.
What distinguishes AAC brick from “terracotta” brick is that it can be shaped with conventional tools, which means it is easy to cut it for tight curvature and complex joints simple. Bonding AAC with quick setting gypsum plaster is difficult than doing it with a brick of “terracotta”, but the result is stronger. They taste three types Gypsum Moulding 1 plaster, Hydrocal, and Hydrostone for mortars in the laboratory and found that hydrocal gave the best results because it was stronger in compression and easier to compare the two types. Even this type was more expensive, but its faster setting times make it a good choice.
AAC allows the economy of masonry spans to be reconsidered. (Table 3) showing the construction cost comparison for the Pines Calyx domes.

	Material cost	Build hours	Labor cost	Total cost	£/sf
Robus tiles (UK)	£26,000	£1,850	£28,000	£54,000	£25
True Stone AAC	£2,000	1,040	£16,000	£30,000	£14

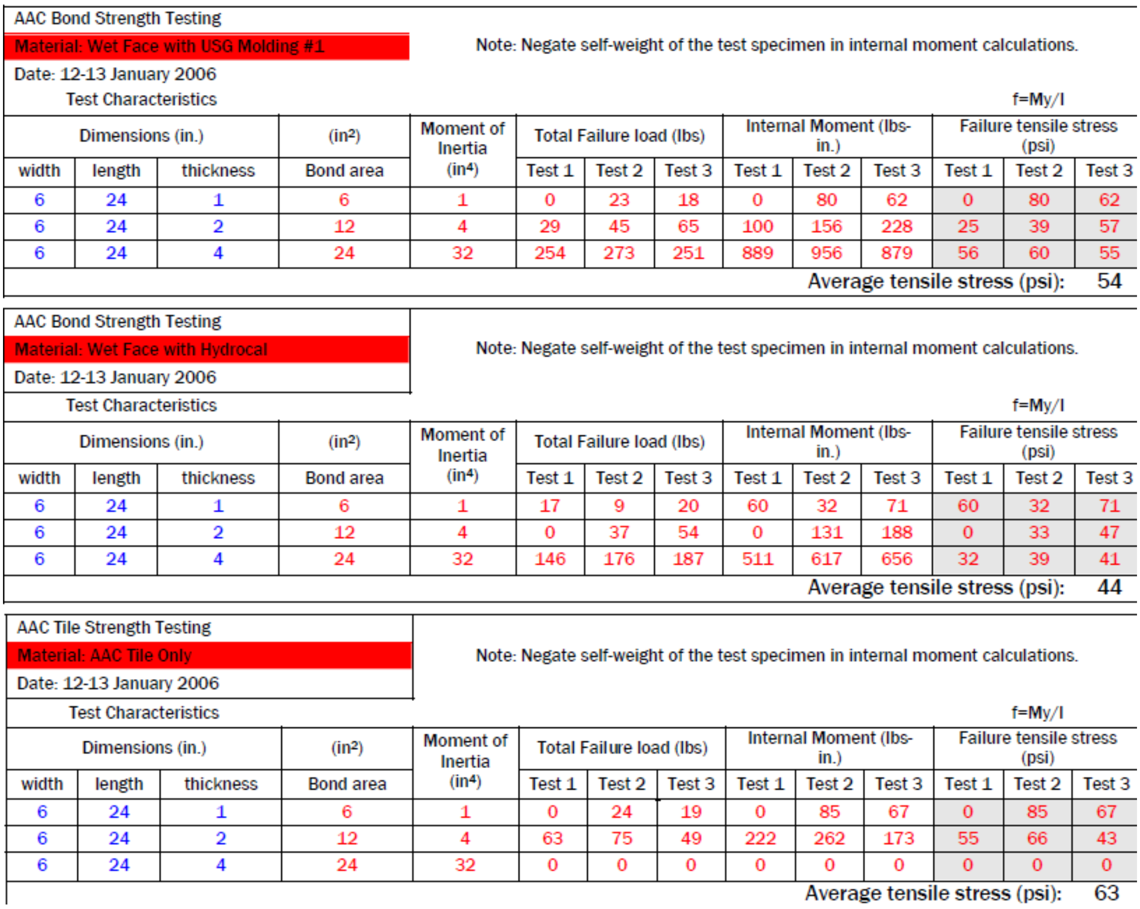
Table 3: Construction cost comparison for the Pines Calyx domes

The lightweight and the tensile capacity of the mortar joint allowing more freedom of assembly and making a wide variety of forms available to the designer and the craftsman, for example, to build cantilevered vaults (Figure 43).

Figure 43: Cantilevered barrel vault

Source: [109]

For parts where tensile capacity is wanted, combining AAC with tension bands made of high-strength steel strapping is possible. This made it possible to create an 8’ span beam in AAC using epoxy as mortar and strapping to constrain the tension(Figure 45). A fresh AAC—mortar joint (using Hydrocal) can only take about 10psi in direct tension. Nevertheless, this is enough strength to hold a 5-gallon bucket of water with a 1”x4” joint. The group tested for tensile strength for Moulding 1 plaster, hydrocal gypsum cement, and the AAC tiles with no bond. (Table 44) showing the average tensile stress for each of them.



AAC Bond Strength Testing

Material: Wet Face with USG Molding #1

Date: 12-13 January 2006

Note: Negate self-weight of the test specimen in internal moment calculations.

Test Characteristics

f=My/I

Dimensions (in.)			(in^2)	Moment of Inertia (in^4)	Total Failure load (lbs)			Internal Moment (lbs-in.)			Failure tensile stress (psi)		
width	length	thickness	Bond area		Test 1	Test 2	Test 3	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3
6	24	1	6	1	0	23	18	0	80	62	0	80	62
6	24	2	12	4	29	45	65	100	156	228	25	39	57
6	24	4	24	32	254	273	251	889	956	879	56	60	55
										Average tensile stress (psi):			54

AAC Bond Strength Testing

Material: Wet Face with Hydrocal

Date: 12-13 January 2006

Note: Negate self-weight of the test specimen in internal moment calculations.

Test Characteristics

f=My/I

Dimensions (in.)			(in^2)	Moment of Inertia (in^4)	Total Failure load (lbs)			Internal Moment (lbs-in.)			Failure tensile stress (psi)		
width	length	thickness	Bond area		Test 1	Test 2	Test 3	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3
6	24	1	6	1	17	9	20	60	32	71	60	32	71
6	24	2	12	4	0	37	54	0	131	188	0	33	47
6	24	4	24	32	146	176	187	511	617	656	32	39	41
										Average tensile stress (psi):			44

AAC Tile Strength Testing

Material: AAC Tile Only

Date: 12-13 January 2006

Note: Negate self-weight of the test specimen in internal moment calculations.

Test Characteristics

f=My/I

Dimensions (in.)			(in^2)	Moment of Inertia (in^4)	Total Failure load (lbs)			Internal Moment (lbs-in.)			Failure tensile stress (psi)		
width	length	thickness	Bond area		Test 1	Test 2	Test 3	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3
6	24	1	6	1	0	24	19	0	85	67	0	85	67
6	24	2	12	4	63	75	49	222	262	173	55	66	43
6	24	4	24	32	0	0	0	0	0	0	0	0	0
										Average tensile stress (psi):			63

Figure 44: Average tensile stress

Source: [109]

Figure 45: Hybrid masonry beam

Source: [109]

The result of the test was very good and the dome was stiff where the deflection was less than

0.05 before yielding at a load of over 900 pounds. At 1,000 pounds this dome, which was only an inch and a quarter thick, collapsed (Figure 47). The features of load and failure were exactly as one would expect for a typical masonry structure, meaning that solid ceramic and foamed ceramic materials perform similarly at the scale of a building (Fgiure 46).

(a) Dome under loading

(b) Dome failure

Figure 46: Dome loading and failure

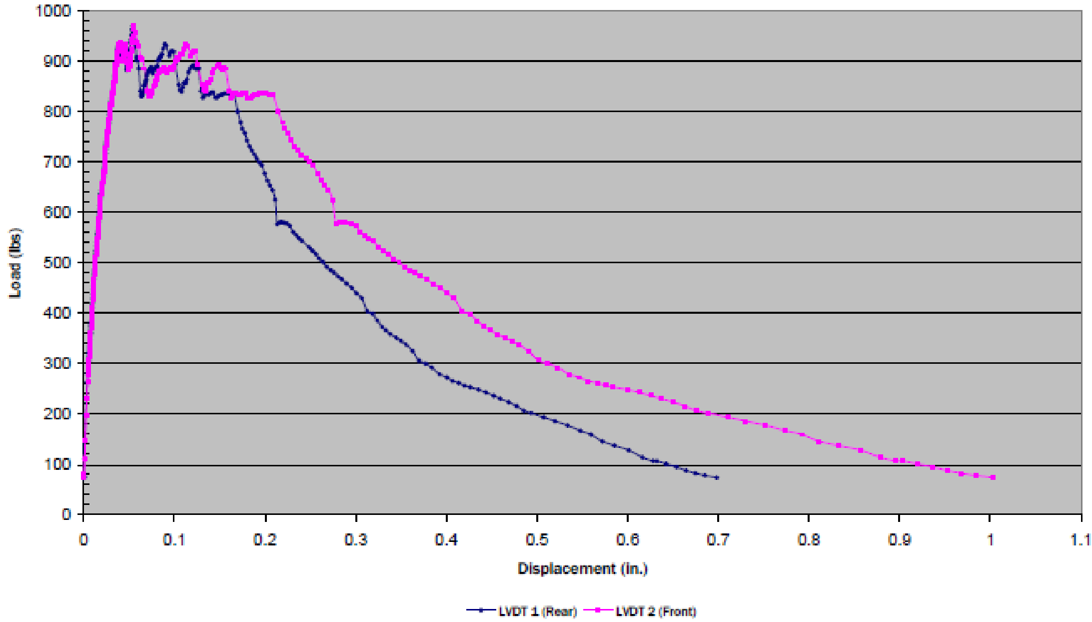


Figure 47: Load vs displacement curves for AAC dome failure

Source: [109]

3.2.5 Glass Fiber Reinforced Polymer(GFRP) mesh

There is an innovative technique that can improve the mechanical performance of tile vaulting by using composite materials. This method consists of several layers of thin bricks, which are placed alternated and coatings of hydraulic mortar and inserting of hybrid grids with high mechanical properties in between layers and the outer surface of the vaults (Figure 48) [22].

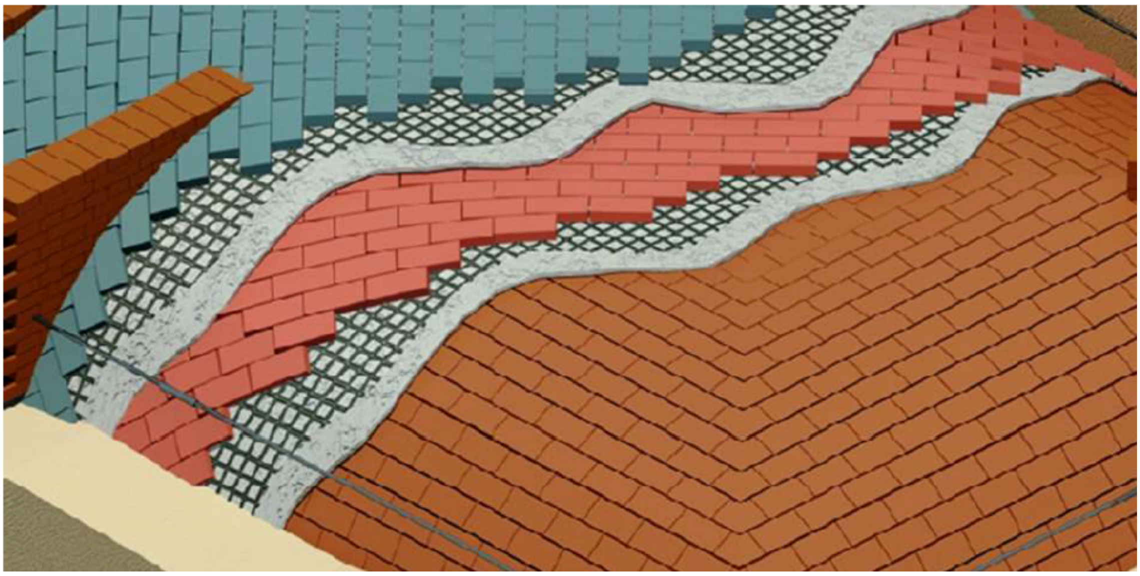


Figure 48: Reinforced Tabicada Technique

Source: [26]

Many researchers have developed analytical predictions of the ultimate strength of FRP (Fiber Reinforced Polymer) strengthened masonry arches. A lot of analysis methods combine aspects of mechanical failure and component failure; some of these were modeled as discrete block models, another focused on the sectional analysis of the strengthened arch. But in the experiment, the research variables were: strengthening scheme, number of arch rings and mortar type [22].

This kind of vault is strong on coherence and weak on tensing. Therefore this method will improve more tensile strength of the vaulted structures. One testing with arch specimens is tested under a monotonic vertical compression load applied at the keystone. The mechanical behavior of the specimens and the axial stress-axial strain relationships are considered under testing [22].

The mortar works as the thick blanket around and amongst the tiles. This gives reinforced element to the vault and carries substantial tensile stresses, which is the greatest mechanical shortcomings in tile vaulting. Using composite strips or fabrics into the mortar bed joints and the outer surface of the vaults represent a natural evolution increasing the load-carrying capacity by improving extra tensile strength to those curved structures. This technique consists of an outer layer of GFRP (Glass Fiber Reinforced Polymer) mesh (Figure 49), which prevents the arch soffit from hinging, and will install this on the extrados surface. At the same time, the inner layers (i.e., those interposed between the layers of bricks) prevent both the extrados and the soffit of the arch from hinging. This method will not substitute tie-rods on the haunches or abutments. Still, it must be applied in conjunction with them to minimize the displacement capacity, especially for the unreinforced structure as tile vaults.

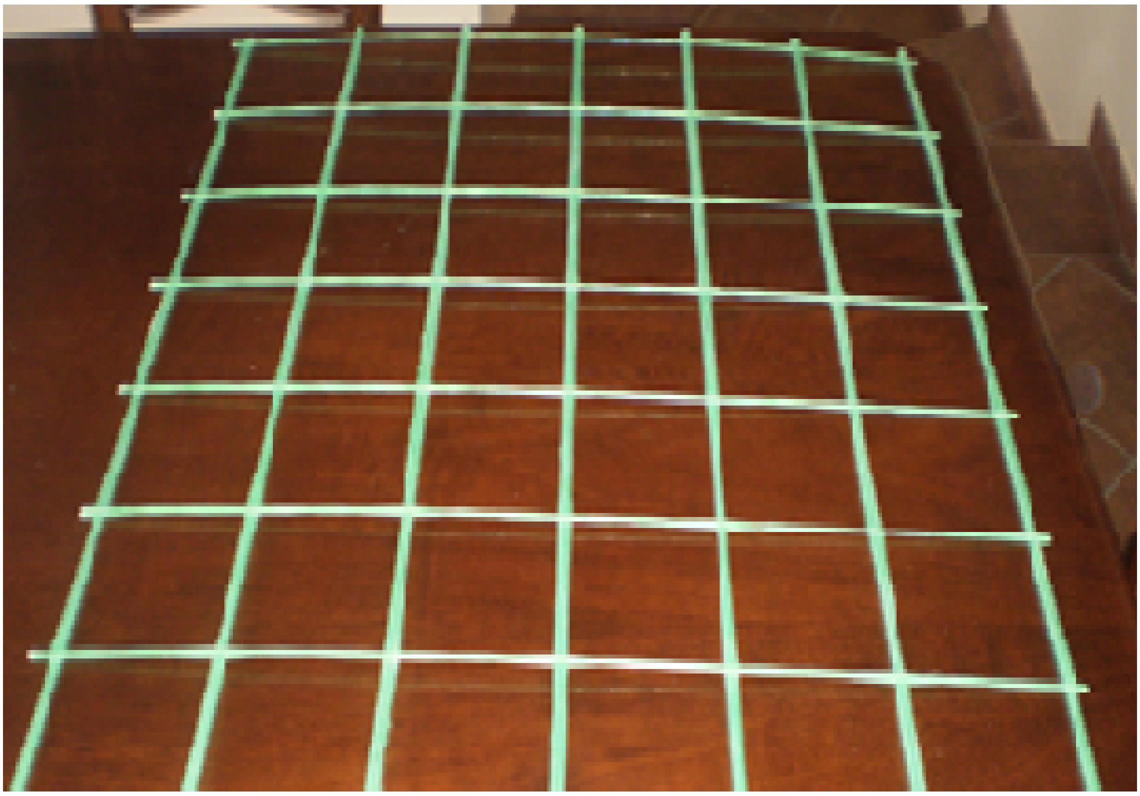


Figure 49: GFRP grid

Source: [26]

While the retrofitting methods as jacketing with the cast in situ reinforced concrete or shotcrete aimed to relieve the existing vault from any contribution, the strengthening system giving the vaults contribution as a structural part of the structural system and static equilibrium of one construction [22]. This Can improve the load capacity by using some interventions which do not change the structural behavior and remain removable at any time for example innovative materials can be helpful in this matter [41, 129].
Using this technique in practice is described in detail in the results chapter (5)

3.2.6 Block Research Group's System

The Block Research Group (BRG) in ETH Zurich in 2011 presented one falsework system to help them to define the shape of the free-form vault and resist the weight under construction. This falsework consisted of CNC-cut cardboard boxes (Figure 50). This could allow building in a safe, large and cheaper structure in a short time.
The ribs were built by changing the reciprocal diagrams and bringing forces to the wanted places in the shell [15].

Figure 50: First free-form tile vault. BRG, ETH Zurich, 2011

Source: [69]

Researchers introduced another scaffolding system that has three elements: scaffolding, cardboard, and thin steel rods(Figure 51).

Figure 51: Another system for falsewrok

Source: [69]

This makes it easier for masons to inspect the joint's quality in simplifying the falsework. By this system, just ribs needed a falsework whereas the infill could be built as in traditional way (Figure 52).

Figure 52: Workshop at MADA

Source: [69]

Some of these kinds of construction are the Earth Pavilion in London (2010) and Sussex Cellars in London (2015), which were build by using tile vaulting technique [52]. This system was mainly used in the Bowls project for seismic reasons and was used in the Earth Pavilion for reducing the thickness of the vault [104]. Ramage and Matthew DeJong have used reinforcement by applying a geogrid in between the layers of bricks [32]. This kind of material (polymeric grids, glass-fiber mesh, basalt-fiber mesh, etc.) has been commonly used for the reinforcement of existing masonry structures, including tile vaults, in the field of restoration. Water, snow, and ice were new materials in this field were tested by the BRG in January 2015 as a prototype which introduced formal and aesthetical possibilities in ice construction in the same time showed a new sustainable way to build a different kind of tile vaults in areas with low temperatures(Figure 53).

Figure 53: Ice tile vault

Source: [69]

The revival of tile vaulting depends on new materials produced locally and can reduce costs and be more sustainable. All those features were achieved in Mapungubwe National Park Interpretive Centre (2008) by using earth bricks instead of the typically fired clay and producing the compressed stabilized soil cement tiles [107, 108].
The Block Research Group had in Addis Ababa, Ethiopia, one project called SUDU (Sustainable Urban Dwelling Unit) 2010-2011where they build the floor of this project by combining two traditional ways to stabilize thin-tile vaults: first one by adding lightweight stiffening walls and the second one by adding compacted fill (Figure 54) [69].

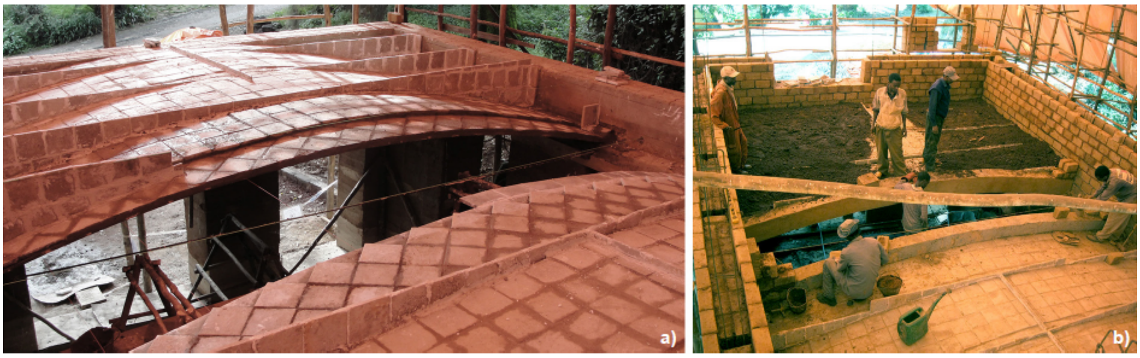


Figure 54: Tile vaults at SUDU project BRG, ETH Zurich, (2010-2011), a) tile vaults with lightweight stiffening walls, b) tile vaults with lightweight stiffening walls and compacted fill

Source: [69]

John Ochsendorf, Wanda Lau, and Michael Ramage of the MIT Masonry Research Group were involved in the structural design, analysis, and construction of the two 12m-span, 12mm-thick tile domes a tile-vaulted stair. They were awarded for their use of waste clay to make tiles and the excavations for the building's foundation to make rammed chalk walls.This was something sustainability and low embodied energy [105].

3.3 First timbrel-vaulted building in England

John Ochsendorf, Wanda Lau, and Michael H.Ramage worked together in 2005 to design and constructed the first timbrel-vaulted building in England. It was a conference center and known as Pines Calyx (Figure 55). The roof of this building constructed of three layers of ceramic tile with portland cement mortar. It take shape of two spherical domes with a thickness of 100mm, a rise of 1.33m at the center, and spans of 11.3m supported by concrete ring beams [109].

Figure 55: Pines Calyx

Source: [102]

In the middle of each dome was a circular oculus tapering upward to 1.6m in diameter. These openings were supported by steel struts inserted between the dome and the ring beam in the upper dome and by the wall of the upper dome in the lower opening (Figure 56).

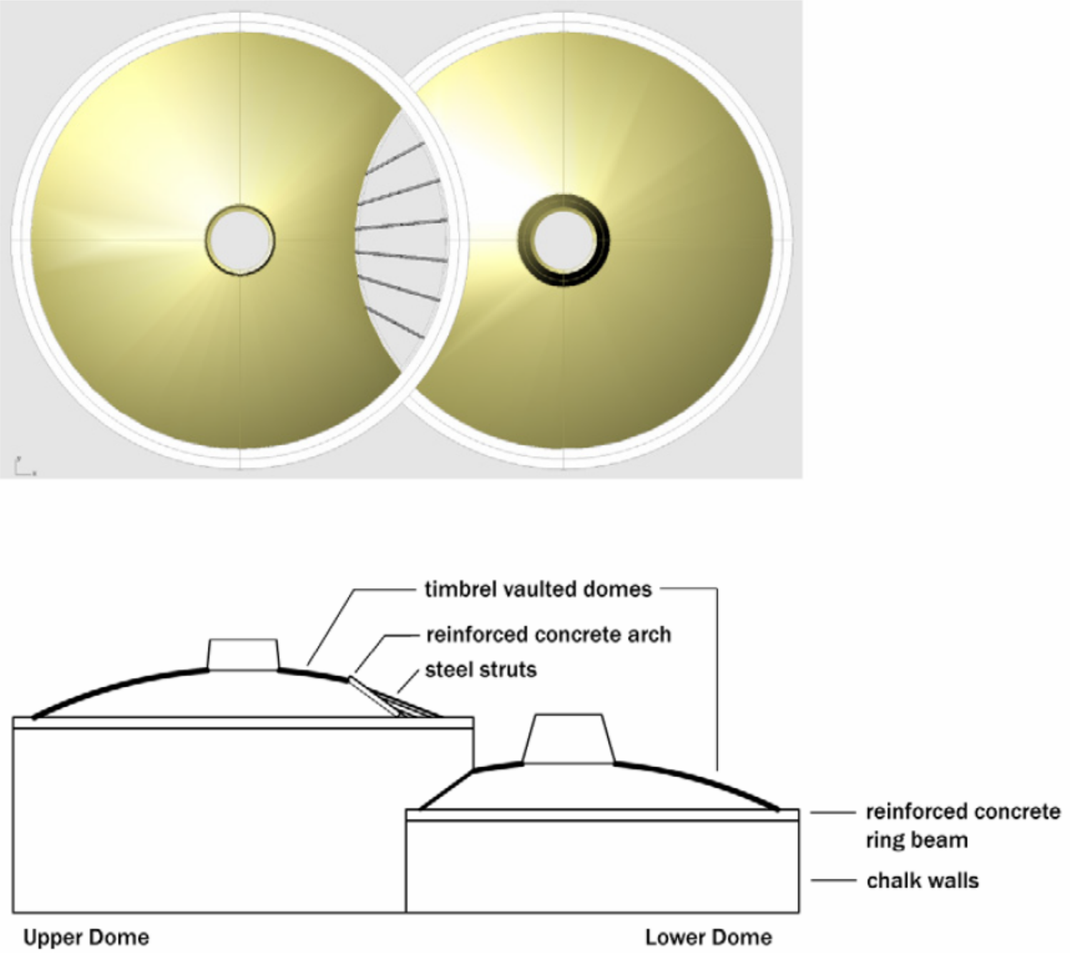


Figure 56: The Pines Calyx plan and section

Source: [109]

Timbrel vaults in this project built of unreinforced masonry with steel ties to take tension. The safety of the construction was by applying membrane theory and to include live loads on the domes it was applying graphic statics. The flexibility of the masonry design gave the designers a chance to do a lot of changes during construction. The tiles used were made by Robus Ceramics and had a size of 150mm x 300mm x 25mm. To reduce the weight of soffit tiles the clay was mixed with Floite.

Each dome was built on a reinforced concrete ring beam that resists its thrust, and the first course is level, circular, and coincident with the dome's sphere as designed. To make sure two wires, pulled tautly and set to the plywood geometric guides. The first wire was connected to a rotating collar at the top of the central pole, and the second one was connected to the base of the pole(Figure 57). The wires could be rotated in the space along the sphere. Once the first course is built all the way around the base of the dome, the soffit is built up in many courses. As soon as the first layer is built up, the second layer is applied at degree 45 to the soffit, the point of this to break all joints. It follows the same procedure in the last layer, and once the mortar is set, it can be walked on, greatly facilitating the construction. The course had to be measured first and set level using tiles cut down to suit the tighter curvature at the oculus. After that is often used 20mm of concrete to the entire structure to apply insulating and roofing material [109].

Figure 57: The second layer mortar bed

Source: [109]

This method gave more freedom during construction to change the shape and details of unbuilt elements than a method that requires complete structural formwork.
Similar techniques were used to design the spiral staircase by applying automated graphic statics to the basic form. The spiral stairs were constructed of three layers of ceramic tile with intermediate layers of Portland cement-placed mortar that provide a total structural thickness of 120mm.
AAC used in this project had a compressive strength of 4.41MPa, an R-value of about 15, and a density of 560 kg/m^3.

The completion of the Pines Calyx in St. Margaret's Bay, England, the ongoing development of the Mapungubwe National Park visitor's center in South Africa, covered than 3,000 m^2 and used 200,000 locally-made,pressed,soil soil-cement tiles (Figure 58) [108], and the design for Kirby Hollow Studio are some examples of renewed potential which demonstrate the economic efficiency and formal viability of timbrel vaulting in contemporary architecture [109].

Figure 58: the Mapungubwe National Park visitor's center in South Africa

Source: [77]

Some of the notices the research recommend to be aware of under construction [109] :

- The dome has 3 different ways to be deformed: vertical, horizontal, and radial.
- Keeping plaster dry and joints as tight as possible is important.
- Buckets, any tiles on the first layer that are not covered by the second layer, can be covered by tarpaulin, especially overnight.
- The dirty or defective tiles must be saved for use in the outer layers.
- Avoiding that large waves form on the soffit of the dome.
- It is worth knowing that plaster takes a little time to go off in humid and wet/damp conditions.
- The straight joints can happen over 2 courses; then, it must be bridged on the third one.
- Bonding should not be less than 50mm from the tile edge, and laying all tiles must happen with a concave side up so as not to trap air between layers.

4 Method

In this thesis, the following tools will be used to design and analyze the Catalan arches.

4.0.1 Rhinoceros

Rhino,Rhinoceros or Rhino3D is a three-dimensional computer graphics and computer-aided design (CAD) application software. This is developed by Robert McNeel Associate and presents an interface for defining complex geometry in 3D and provides a visual studio for the programs made in Grasshopper. All geometries in Rhino rely on (NURBS), which is the abbreviation for a non-uniform rational basis spline and is a mathematical model for representing curves and surfaces [5, 49]. This software provides good flexibility to designers who can either model the construction either in Rhino or import it from other programs.

4.0.2 Grasshopper

Grasshopper (GH) is a graphical algorithm editor or visual programming language (VPL) and, at the same time, one plug-in for Rhinoceros developed by David Rutten and Robert McNeel Associates. Grasshopper is an integrated parametric environment with robust and varied modeling. It can be used by creative professionals across various fields, including architecture, engineering, product design, and more. The geometry can be defined in two ways: it can be attained from Rhino and controlled parametrically in this program, or it can be built directly in Grasshopper. It is using to design algorithms which are step-by-step procedures designed to operate. This parametric environment includes some functional blocks called"components," which can do an action by connecting each to another by "wires" [37]. The built geometry can be adjusted by connecting the built components to number sliders. The whole built elements in Rhino viewport change accordingly as one or more outputs with any changes in one or more of the Grasshopper input parameters.It includes many standard components and gives the designers flexibility to make their own components by using any one of the programming languages like C# [123].

4.0.3 Karamba3D

In this thesis, Karamba will be adopted for evaluating the structural performance of Catalan vaults. Karamba3D is an interactive, parametric structural engineering tool integrated with Rhino and as a plugin for Grasshopper [62]. It is a parametric finite element analysis (FEA) software application and is developed by Clemens Preisinger in cooperation with Bollinger und Grohmann ZTGmbH in Vienna [61]. Assemble model is the key as a component in Karamba3D. This component will consist of inputs which are boundary conditions as the cross-sections of elements, the type of materials, supporting points, and the type of loads. After defining inputs for the model, the outputs will be assembling model. The structural analysis as the Finite Element-analysis (FEA) can be made in the same environment and structural properties can be shown in Karamba [95]. Finite Element Method (FEM) distributes any complex system into several subsystems where the behavior is "known" [63].

5 Results

One search has been done in Portugal in 2014 by experimenter who have built, tested, and compared the behavior of eight prototypes of tiled arches to collapse through the studying of different types of reinforcement, layers, and properties of hydraulic mortar showed a new and innovative technique which is combined the tradition of the constructive Spanish tradition of reinforcing tiled vaults and the modernity which is depending on the science of materials [22, 26]. This test was selected to provide a specific case for design and evaluation of a Catalan arch.

5.1 Materials characterization

Some samples have been taken from historic center of Palermo in Italia where structural elements was made in the same technique as “bòvedas tabicadas” .
The Laboratory of the Department of Civil, Environmental, Aerospace, Materials Engineering of the University of Palermo conducted tests on one masonry unit constituted by two layers, four masonry units constituted by three layers, and four masonry units constituted by four layers and (all units with a mean dimension of 320×320×115 mm) to analyze compression behavior of samples with two, three and four sheets of bricks as well as with appropriate petrographic and chemical investigations on mortar samples. After identifying macroscopic problems (such as embedded tubes, macro voids, and so forth). The water saw had been used to cut the samples. The way the tests have been performed was as follows: The external plaster was placed on two of the four three-layers samples. The two faces was in contact with the testing materials machine were spread with a thin layer of gypsum paste in order to straighten the loading surface. The mean amount of mortar was about 20-25% of the total thickness. The thickness of brick was ranging from 18 to 20 mm while the layer of mortar varies from 6 to 8 mm. The total thickness was 105 mm for 4 layers, 75 mm for 3 layers, and 45 mm for 2 layers.
The tests were conducted under displacement control with a speed of the crossbar of 0.5 mm/min except for the sample of 2 layers the speed of the crossbar set to 0.15 mm/min (Figure 59) [12].

Figure 59: Samples: a)before the test; b,c)during the test d)at the end of the test

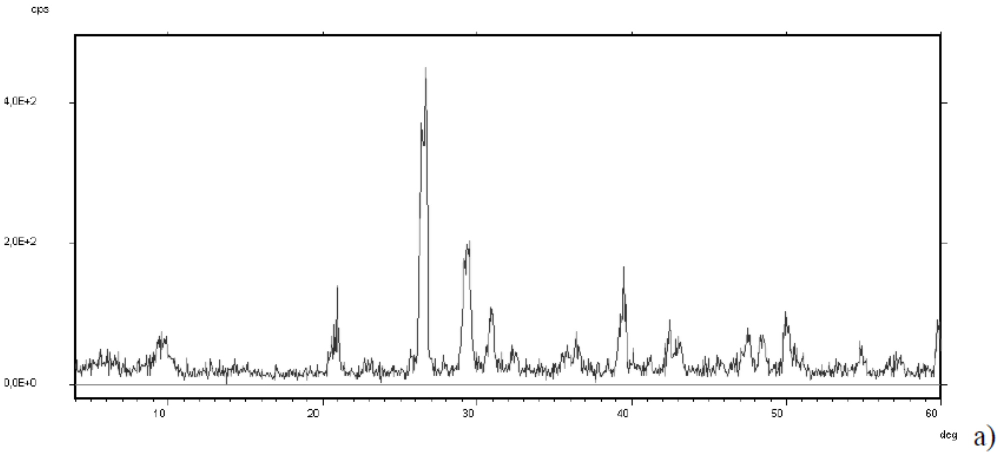
Source: [12]

The petrographic and chemical (XRD) analysis are raported in order to confirm the characteristics of the materials under investigation (Table 4) [12].

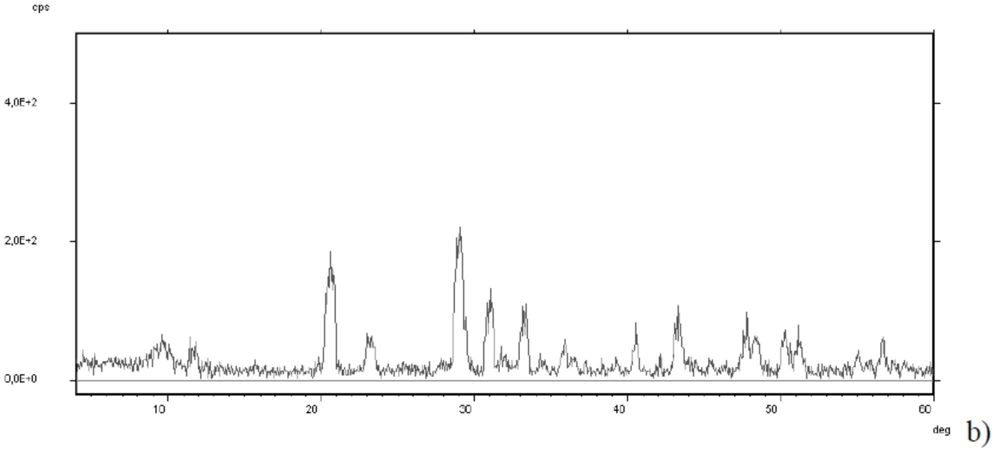
Level	Thickness [mm]	Aggregate/Binder/Pores ratio Temper/Matrix /Pores ratio	Description
A	10	(A/B/P): 65/25/10	Lime mortar: aggregates (0.1 d 2 mm) made mainly of quartz and in a minor fraction of micrite and spar (dm, = 5 mm). High sphericity, roundness, and sorting.
B	4	(A/B/P): 20/55/25	Mortar of gypsum: aggregates, poorly sorted (d ...– 2 mm), made of microcrystalline gypsum (main fraction) and subordinately fine grained selenitic gypsum.
C	20	(T/M/P): 25/70/5	Fictile material characterized by a medium matrix birefringence. The inclusions are mainly of silt and rarely of monocrystalline quartz (d„,,,„ = 0.8 mm).
D	4	(A/B/P): 20/55/25	Mortar of gypsum: aggregates (0.5-4:1 3 mm) made of gypsum both microcrystalline (main fraction)both selenitic. Macropores are frequent (d = 5 mm).
E	>20	(T/M/P): 25/70/5	Fictile material characterized by a medium birefringence of the matrix. Inclusions are mainly of silt (d –.= 0,02 mm)numerous monocrystalline quartz grains are present (0,5 <d <0,9 mm) somewhere in small clusters.

Table 4: The petrographic and chemical analysis (XRD)

The petrographic and chemical (XRD) analysis has been performed on two samples: one obtained from the external layer (plaster) and the other one from the internal layer (mortar). On the horizontal axis has been drawn Bragg's diffraction angle by a degree and on the vertical axis intensity in counts per second (c.p.s) for materials involved in the formation of plaster and mortar (figure 60) [12].



Quartz SiO_2 **Calcite** $CaCO_3$ **Dolomite** $CaMg(CO_3)_2$



Gypsum $CaSO_4 \cdot 2H_2O$

Figure 60: XRD results:a)plaster; b)mortar.

Source: [12]

The results reported for stress and strain (Figure 61)show the role of plaster as stiffening the material behavior without increasing the ultimate stress. At the same time, the behavior of 2 layers sample shows how the intrinsic presence of irregularities in the material plays a fundamental role in the overall material behavior [12].

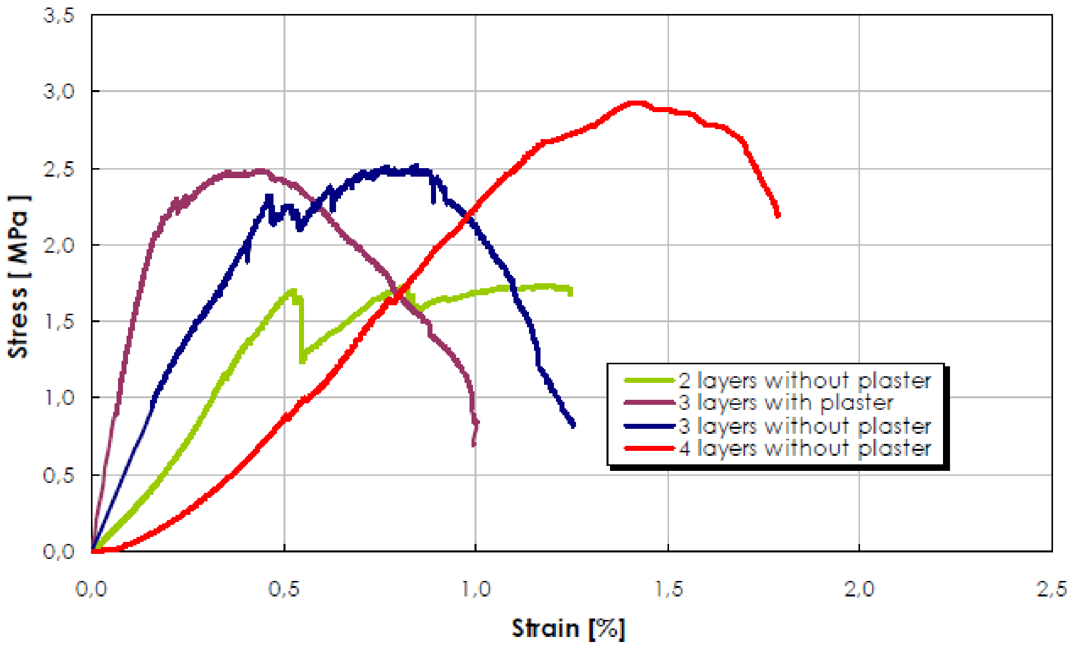


Figure 61: Stress-strain diagrams deduced by the compression tests

Source: [12]

5.1.1 Hydraulic mortar

In this experiment, researchers used two kinds of hydraulic lime mortar. For each mortar type, three specimens of size (160*40*40)mm were used for getting flexural strength tests and twelve specimens for compression tests.The static tests have been carried out on 8 masonry arches
It was two types of hydraulic lime mortar for masonry mortar: The the first type was to reproduce the quality and the mechanical behavior. The second type was a single-component professional repair mortar manufactured by CVR. Flexural and compressive strength tests were performed by ASTM C348 [6] and ASTM C349 [7]. The properties of the first kind of mortar was as follow:

- was a weak mortar and used for masonry construction.
- Ingredients were 1:2:0.15 by volume for lime:sand: cement.
- The flexural strength was 0.16 MPa.
- The compression strength was 0.58MPa.

The second kind of mortar was a single-component high-strength repair mortar manufactured by CVR and had 1.96 MPa as flexural strength and the compression strength was 5.25 MPa. For both tests, the coefficients of variation equal to 0.43 and 0.15 in strength tests, 0.10 and 0.06 for compression tests, respectively.

5.1.2 Solid flat tiles

The tiles or thin bricks called (rasillas) had a flat surface and rectangular forms and sizes (250*120*35)mm for the assemblage of the masonry arches. Both compression and bending tests was done on six samples each in the head joint direction (Figure18). Six samples were performed according to the ASTM C67 standard [8] to doing the compression and bending tests. The compressive strength to bed joints was 20.99MPa, and the coefficient of variation was 0.11. The bending tensile strength was 6.75 MPa, and the coefficient of variation was 0.27MPa.

5.1.3 GFRP grid

The GFRP grid of size (66*66) mm made up of AP (Alkali Resistant) glass fiber with a zirconium content equal to or greater than 16% pre-impregnated with thermosetting epoxy-vinyl ester resin. The mesh was tested, and the mechanical properties were determined according to ASTM standard D3039 [9].(Table 5) shows the mechanical properties of coupon specimens.

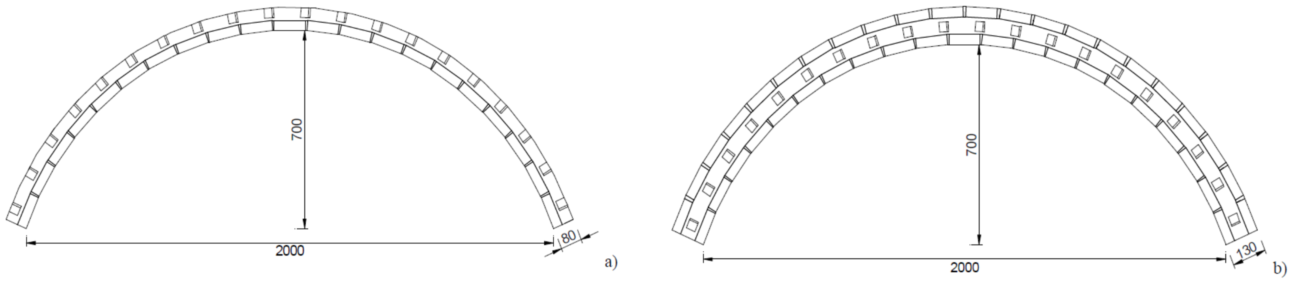
property	**FB mesh 66*66**
Thickness (mm)	2.800
Tensile strength (N/mm^2)	680
Elastic Modulus (N/mm^2)	39.800
Ultimate strain (%)	1.9

Table 5: Mechanical properties of the reinforcement

5.2 Experimental program

5.2.1 Test matrix

Experimentally there were sixteen specimens to build masonry arches, used and built with arranged in either two or three layers of the thickness 80mm and 130 mm this har been tested under monotonic vertical loads applied at the keystone span of this arch was 2000 mm and the height was 700 mm above the springing level (Figure 62).



(a) Double layer arch

(b) Triple layer arch

Figure 62: Masonry arches

(Table 6) shows all details about the arches. All specimens are identified by a three index code, in which the first indicates the number of arch rings (DR, double-ring arch; TR, triple-ring arch); the second code designates the type of intervention carried out (UT, “Tabicada” technique; IT, Reinforced tabicada technique with GFRP grid placed into the mortar bed joints; OT, Reinforced “Tabicada ”technique with GFRP grid placed into the mortar bed joints and on the outer surface of the arch), and the third index indicates the identification number of the specimen.

Specimen	Number of brick layer	Mortar type	Strenghtening schemes
DR.UT.01	Double-ring arch	Type 1	Tabicada technique
DR.UT.02		Type 2	
DR.IT.01		Type 1	Reinforced tabicada technique (GFRP grid into the mortar bed joints)
DR.IT.02		Type 2	
DR.OT.01		Type 2	Reiforced tabicada technique (GFRP grid placed both into the mortar bed joints and on the outer surface of the arch)
DR.OT.02		Type 2	
DR.OT.03		Type 2	
DR.OT.04		Type 2	
Specimen	Number of brick layer	Mortar type	Strenghtening schemes
TR.UT.01	Triple-ring arch	Type 1	Reinforced tabicada technique (GFRP grid into the mortar bed joints)
TR.UT.02		Type 2	
TR.UT.03		Type 2	
TR.IT.01		Type 1	
TR.IT.02		Type 2	
TR.IT.03		Type 2	Reinforced tabicada technique (GFRP grid placed both into the mortar bed joints and on the outer surface of the arch)
TR.OT.01		Type 2	
TR.OT.02		Type 2	
TR.OT.03		Type 2	

Table 6: Test matrix

Three index codes identify the test matrix and the specimens: the first one is for the number of brick layers. The second one is for the type of intervention(TU, tabicada technique; TG, Reinforced tabicada technique), and the third one is for identification of the specimen. Details about the model are given in (Table 7) [26].

Test	**Thickness (mm)**	**Mortar Type**	**Strengthening system**
S2-TU-01	80	Type A	Tabicada
S2-TU-02	80	Type B	Tabicada
S2-TG-01	80	Type A	Reinforced Tabicada
S2-TG-02	80	Type B	Reinforced Tabicada
S3-TU-01	130	Type A	Tabicada
S3-TG-01	130	Type A	Reinforced Tabicada
S3-TG-02	130	Type B	Reinforced Tabicada
S3-TG-03	130	Type B	Reinforced Tabicada

Table 7: Test matrix

First, the experiment started with putting the bottom brick layer. Then the GFRP grid was put on after following the recommendations for ACI 549.4R-13 provisions [25] (Figure 63). The grid had a length of 3000 mm and a width of 300 mm as dimensions of the arch.

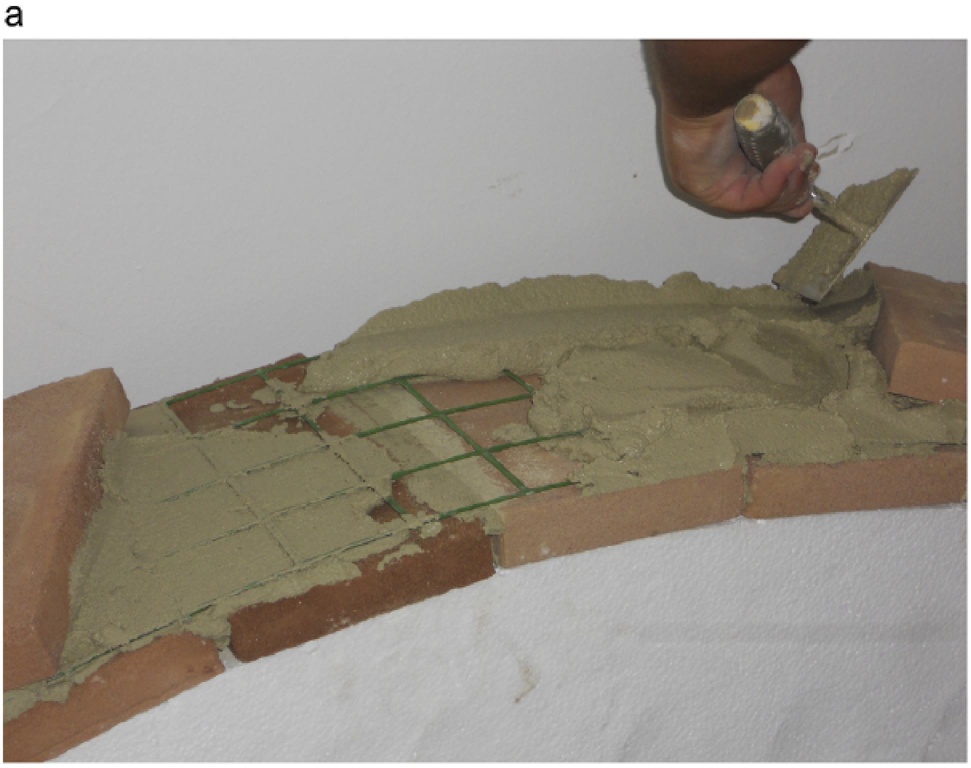


Figure 63: Application of GFRP grid

Source: [22]

The researchers used the ribbed roller to grout impregnation and allowed the excess matrix to be squeezed out. Lastly, the second and the third layer (in the case of the OT strengthening scheme) were applied in the same procedure (Figure 64).

Figure 64: Application of the second layer of flat tiles

Source: [22]

5.2.2 Test setup

The test was conducted by using a closed-loop load configuration without any external reaction. The load applied at keystone and generated by hydraulic jack with hand pump reacted against the steel frame (Figure 65). The horizontal reaction was supplied by a reaction frame, made with a steel C-shaped profile and flat bars. Inductive displacement transducers (LVDT) were used to measure radial displacements which were measured rather than vertical displacement of their dominating the hinge mechanism.

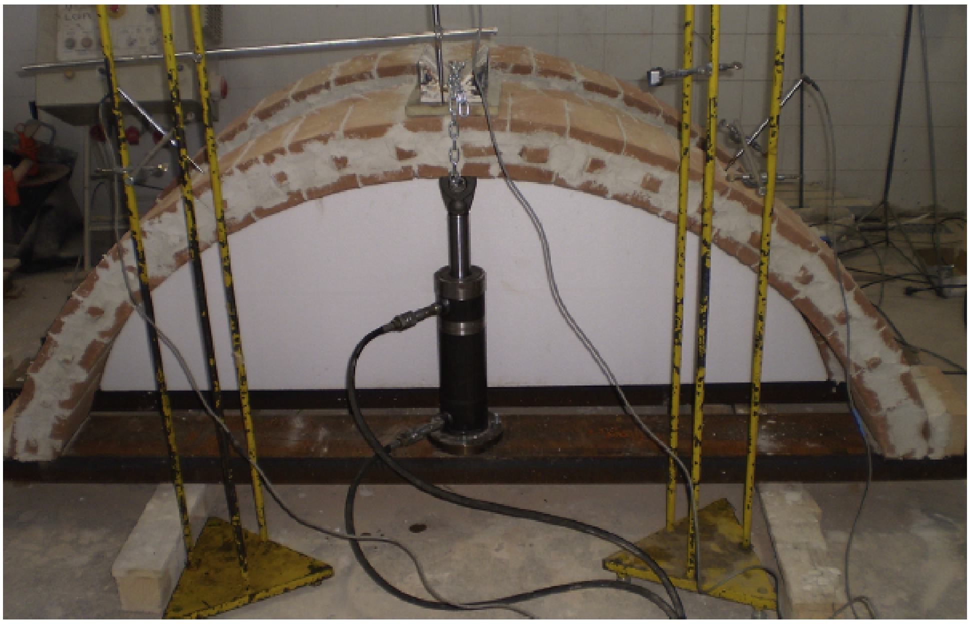


Figure 65: Test setup

Source: [22]

5.3 Test results

The results of the test are grouped by the number of arch rings (Table 8) [22].Read (5.2.1) to understand abbreviations in the table

Specimen	Ultimate load capacity (KN)	Load point deflection (mm)	Mode of failure
DR.UT.01	0.15	-	Mechanism
DR.UT.02	1.57	2.46	Mechanism
DR.IT.01	0.82	4.88	Ring separation + snap-through buckling
DR.IT.02	3.52	26.53	Ring separation + snap-through buckling
DR.OT.01	8.32	27.13	Ring separation + snap-through buckling
DR.OT.02	10.54	3.21	Ring separation + snap-through buckling
DR.OT.03	8.50	13.93	Ring separation + snap-through buckling
DR.OT.04	7.49	8.64	Ring separation + snap-through buckling
TR.UT.01	1.07	1.34	Mechanism
TR.UT.02	6.83	4.93	Mechanism
TR.UT.03	4.10	7.69	Mechanism
TR.IT.01	2.71	2.23	Shear sliding
TR.IT.02	8.46	17.60	Ring separation + snap-through buckling
TR.IT.03	12.99	24.79	Ring separation + snap-through buckling
TR.OT.01	16.66	14.04	Ring separation + snap-through buckling
TR.OT.02	17.83	13.06	Ring separation + snap-through buckling
TR.OT.03	13.19	9.90	Ring separation + snap-through buckling

Table 8: Comparsion among the experimental results

Source: [22]

5.3.1 Double-ring arches

In this case, was two modes of collapse: the first one was the arch displacement mechanism. The second one was the snap-through buckling. For two kinds of specimens DR.UT.01 and DR.UT.02, which were reinforced with the tabicada technique, the failure was dedicated to forming four

hinges and caused the arch displacement mechanism. The cohesive force established between layers against collapsing of the arch was different in each case of mortar and was equal to 0.15 KN for arch DR.UT.01 with weak mortar type 1, while it was 1.57 KN for arch DR.UT.02 with a single-component high strength repair mortar type 2.
All specimens were supported at the abutments and the force increased statically until failure and was applied at the crown through a hydraulic jack. the load and its displacements were recorded. Displacements were measured by LVDTs. the first layer of bricks was build and the GFRP grid was placed into the mortar bed joints and then come to other layers.

(Table 9) show the results for double layer arches the failure mechanisms of reinforced arches are :

- Sliding brick /mortar.
- Masonry crushing.
- Deboning of GFRP reinforcement.
- Tensile failure of GFRP grid

Test	Failure load (kN)	Load point deflection (mm)	Mode of failure
S2-TU-01	0.15	-	Mechanism
S2-TU-02	1.57	2.46	Mechanism
S2-TG-01	0.82	4.88	Sliding (GFRP – Mortar)
S2-TG-02	3.52	26.99	Sliding (GFRP – Mortar)

Table 9: Double layer arches: experimental results

The GFRP grid restrained collapsing mode of the four hinge mechanism by preventing the opening of the arch. This reinforcement allowed one side thrust to move outside of the arch thickness and on the other side the presence of additional cracks within the arch. The failure consisted of partial ring separation between successive courses that progressed as long the crack separated masonry sagged and became unstable (Figure 68).

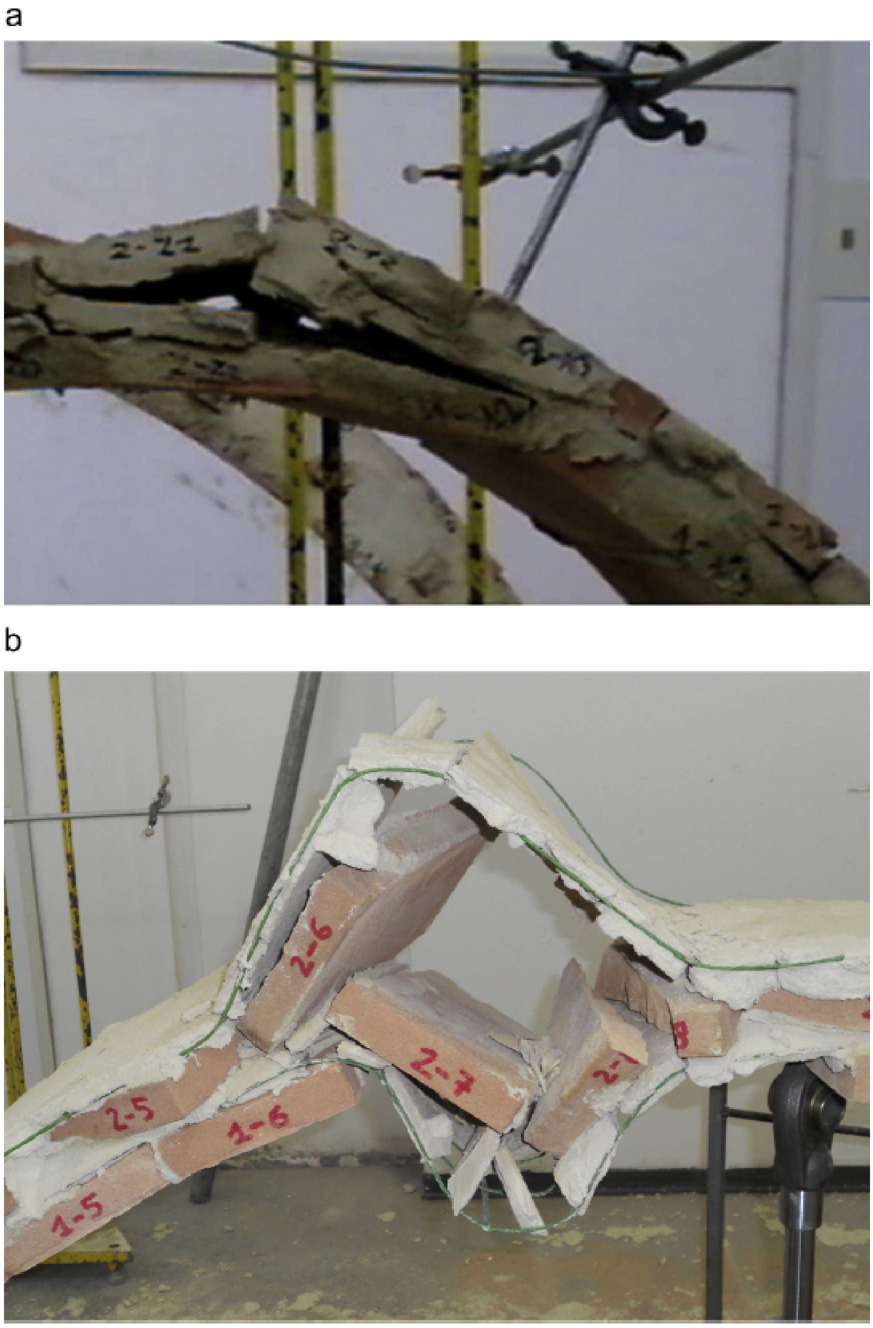


Figure 66: Ring separation and snap-through buckling:a) arch DR.IT.01; b) arch DR.OT.02

Source: [22]

Consequently, inserting of GFRP grid both into the mortar bed joints and the outer surface of the arch (OT arch) increases the load capacity between 376% and 569%. It was also growing in the case of IT arches (strengthened with GFRP grid placed into the mortar bed joints) by 448% from 0.15 to 0.82 kN for arch DR.IT.01, built with mortar type 1, and by 123% from 1.57 to 3.52 kN for arch DR.IT.02, built with mortar Type 2.

5.3.2 Triple-ring arches

The behavior of tripling arches was similar to the double-ring specimens because the failure mechanism was similar to the four-hinge mechanism with partial tile layer separation between successive courses near the loading point (Figure 67).

Figure 67: Arch displacement mechanism(arch TR.UT.01)

Source: [22]

The ultimate load was 1.07 KN for arch TR.UT.01 made with mortar type 1, whereas there was a significant increase in the two specimens made with mortar type 2. The failure load was 6.83 kN (arch TR.UT.02) and 4.10 kN (arch TR.UT.03).
According to all arches reinforced with the GFRP, there was local buckling of the tiles (snap-through buckling) near the point of the compression load (Figure 68), with one exception of arch TR.IT.01 which failed of shear sliding between brick and mortar at the arch abutment (Figure 69).

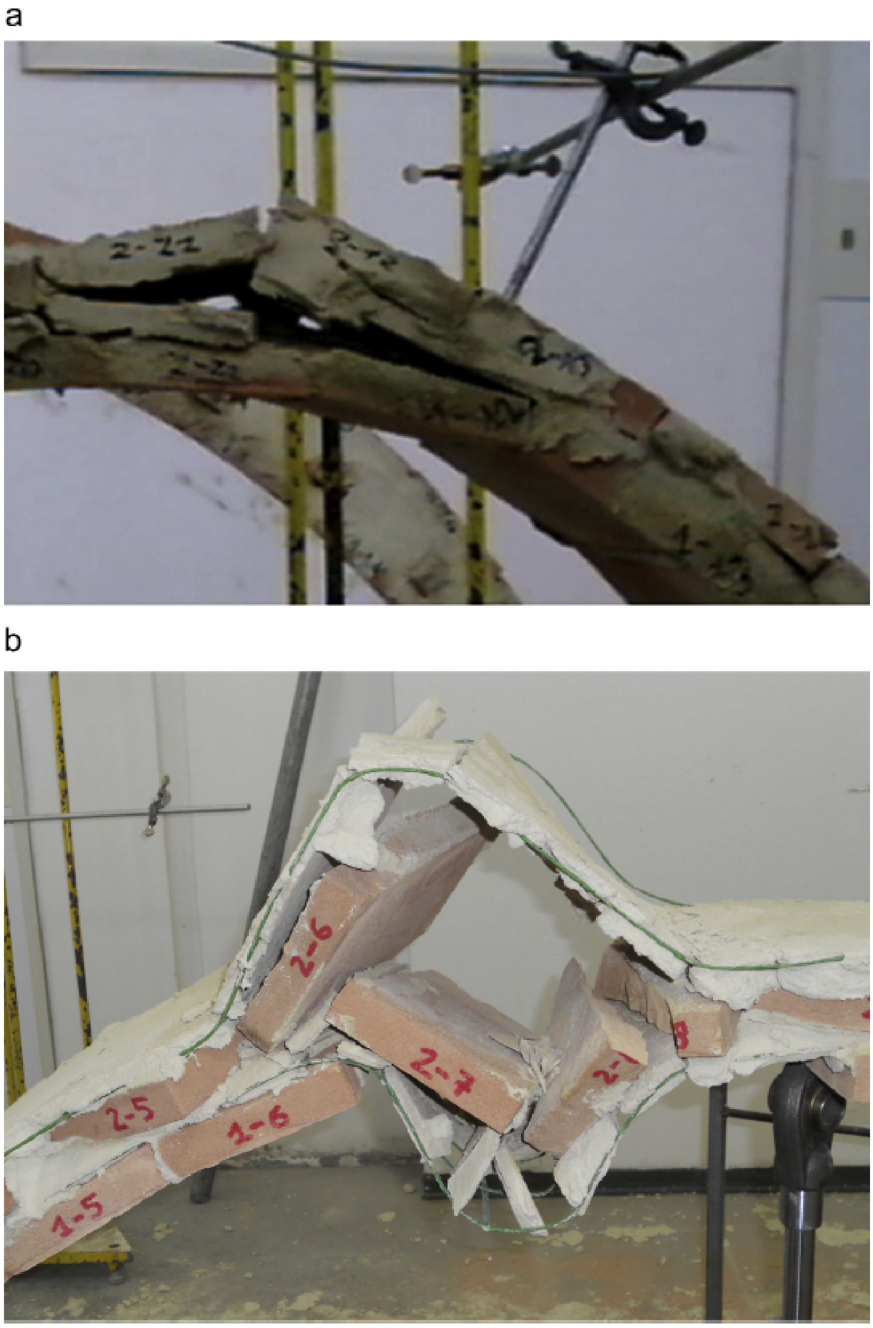


Figure 68: Ring separation and snap-through buckling:a) arch TR.IT.02; b) arch TR.OT.03

Source: [22]

Figure 69: Shear sliding mechanism for arch TR.IT.01

Source: [22]

Although all arches had the same failure mode, the specimens had different load capacities. The average capacity increment for the specimens reinforced with tabicada technique was 153% for arch TR.IT.01 built with mortar type 1 and between 55% and 138% for the two specimens (arches TR.IT.02 and TR.IT.03) built with mortar type 2, whereas the greatest capacity recorded on OT specimens (between 141% and 226%).

The second experiment was Test S3-TU-01 was carried out on an unreinforced arch which the results giving a 1.07 kN maximum load and failure due to a 4-hinge mechanism(Figure 70). The capacity of reinforced arches constructed with the ready-to-use mortar, tests S3-TG-02 and S3-TG-03, are 8.46 KN and 12.99 KN respectively with both experiencing sliding between GFRP grid and mortar with the subsequent destabilization of bricks as their failure mode. However, sliding between bricks and mortar at the base of the arch (abutments) was experienced at failure for test S3-TG-01 (made with lime aerial mortar).(Table 10) show the results for triple layer arches

Test	Failure load (kN)	Load point deflection (mm)	Mode of failure
S3-TU-01	1.07	-	Mechanism
S3-TG-01	2.71	2.36	Sliding (Brick – Mortar)
S3-TG-02	8.46	17.60	Sliding (GFRP – Mortar)
S2-TG-03	12.99	24.79	Sliding (GFRP – Mortar)

Table 10: Triple layer arches: experimental results

Figure 70: 4-hinges failure mechanisms

Source: [26]

5.4 Analysis of the results

1-The reinforcement effectively prevented the arches from collapsing with the four-hinge mechanism because it caused increasing the capacity to double-ring arches by 2-7 times and by 3 times in the triple-ring arches more than the original arch without reinforcing.
2-By using the GFRP grid, the problems caused by local buckling were reduced. Otherwise, the crack between joints could separate tiles and lead to collapse. The mortar type 2 had a significant role in providing an ultimate load capacity of 148% and 48% larger than the specimens built with mortar type 1. Still, even the mortar was weak. The load capacity could be increased by 145% and 29% with reinforcing with GFRP.
3-The load-deflection response (Figure 71) for IT and OT specimens provided a different behavior pattern. While in IT specimens, the load-deflection curves were bilinear with a long post-peak branch. The structure had a significant ductility behavior; in the OT arches, failure occurred suddenly. For both double- and triple-ring arches, the ultimate load achieved for a displacement is 2 times greater than the ones corresponding to the OT specimens. The only exceptional case observed with arch DR.OT.01 with the same post capacity behavior but undesirable variables as handwork can explain a larger deformation capacity (more than 25mm).

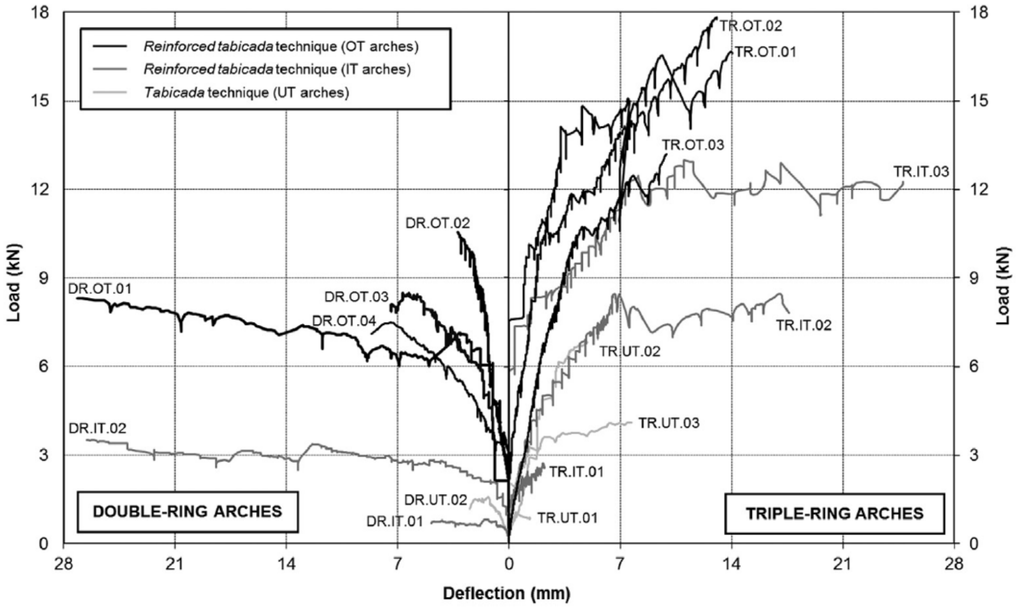


Figure 71: Load-deflection curves (measured at the location where load was applied)

Source: [22]

(Figure 72) shows the load vs displacement curves for the triple brick-layout arches (Series S3) [26].

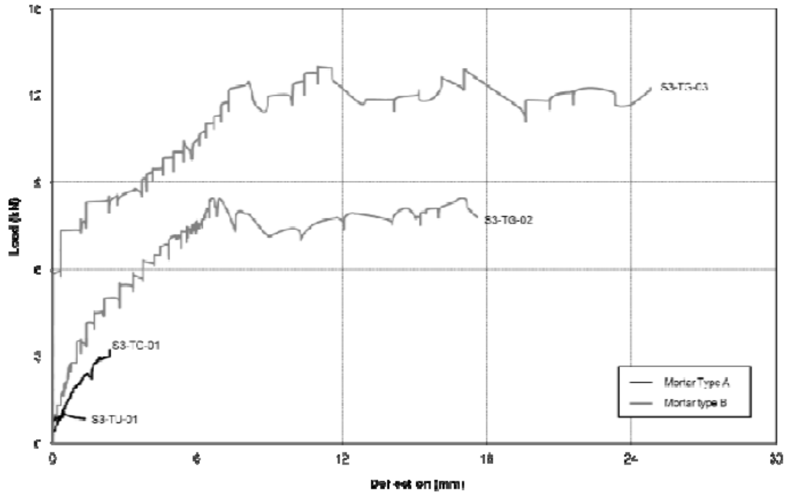


Figure 72: Load-deflection curves (measured at the location where load was applied)

Source: [26]

The strengthening of arches caused an increase in the load-carrying capacity and this was increased more with the addition of GFRP reinforcement. This is a technique that can be used for its reversibility of seismic upgrading intervention and non-invasiveness. The GFRP could easily be removed by simple mechanical action. There was a detachment of composite materials from the masonry surface regarding the bond properties. The surface of materials was as a plan. With regard to the bond properties, it can be noted that the detachment of composite materials from masonry surface, was often observed. However, the smooth surface conditions of the masonry in the tested arches should be noted. This surface can almost be considered a plane. In general, the presence of surface irregularities could improve the mechanical interlocking between the bricks, mortar, and GFRP grids [26].

5.5 Parametric analysis

In this chapter, the parametric modeling, analysis of Catalan arch will be explained by using the software tools described in (Chapter 4). The procedure was used as parametric work is displayed in the chart (Chart 73)

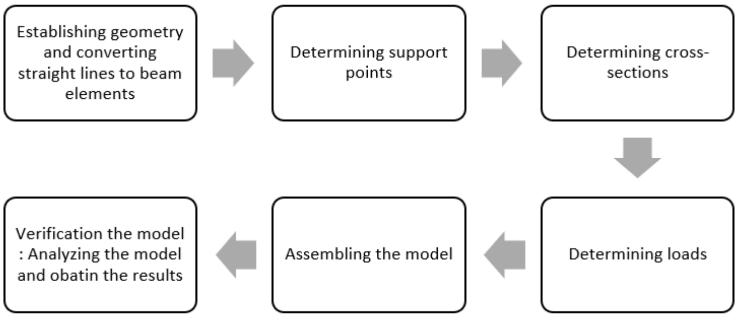


Figure 73: The process of parametric workflow

Technology development enables engineers to design parts that incorporate complexity in construction or, in other words, complex geometries. Designers are no longer restricted to the limitations of traditional machines and can create components with more significant design freedom [69].

The tools that had been used gave a lot of ease of work and flexibility in modifying the model under designing due to ease changing of any parameter in the different components.

Grasshopper was mainly used for modeling while Rhino was just the viewport. since karamba3D had many parameters, it make the work more accessible to change anything in the geometry rather than drawing it manually in Rhino. The following sections will represent the determination of the components used in the design.

5.5.1 Beam elements

The component (Line to beam) converted all done lines of segments to beam elements. Each beam element, in this case, represented one tile. The cross-sections of bricks just as one layer first were connected to this component.
The geometry started first with three points which could made an arch and were constructed to represent the coordinate points of the shape by using Number Slider components. (Figure 74) represents the geometry establishing of the arch.

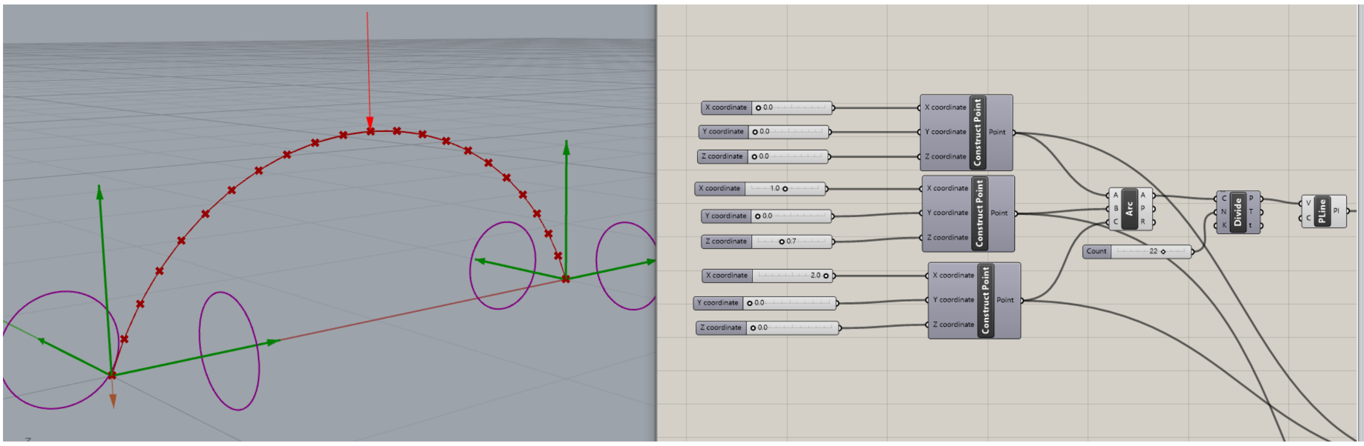
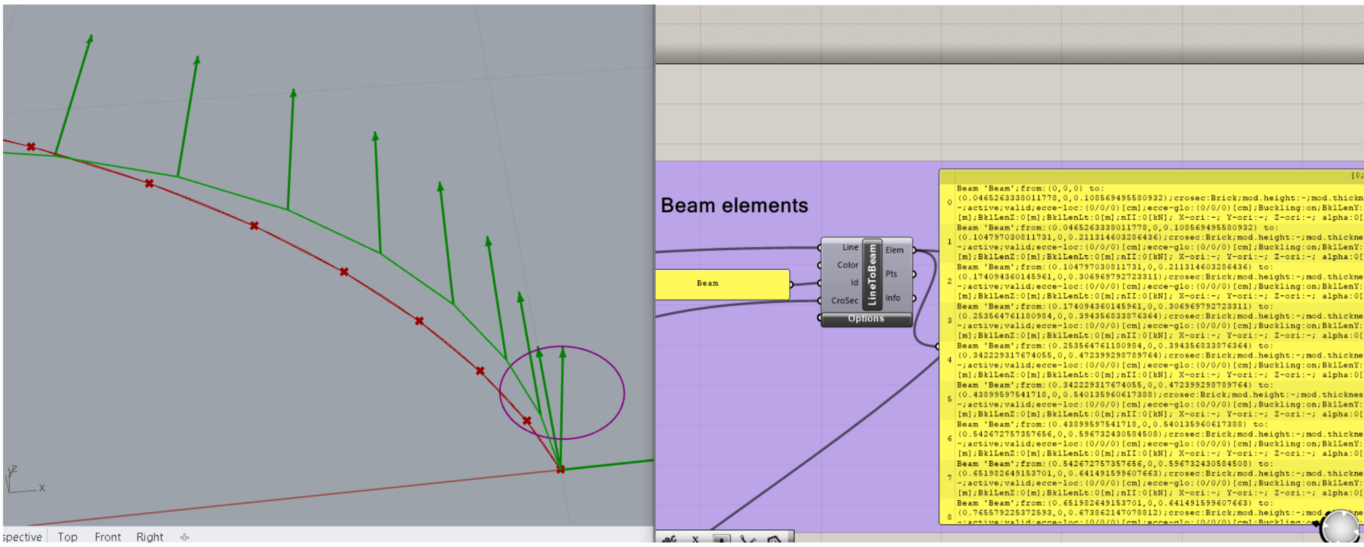


Figure 74: Establishing geometry

Then the arch was divided by (Number Slider) to get straight polylines that were exploded to many segments that made up one shape near the shape of the arch. More consecutive lines were decided in this component, more perfect the model becomes to have a shape as an arch. So the shape of segments could more represent the arch just if the number of segments increased. The number of beams was 21, as was the number of tiles used in the test. The component (Line To Beam) was used in the Karamba3D tool to convert the line to beam (Figure75).



```
0 Beam 'Beam';from:(0,0,0) to:
  (0.0465263338011778,0,0.108569495580932);crosec:Brick;mod.height:-;mod.thickn
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
1 Beam 'Beam';from:(0.0465263338011778,0,0.108569495580932) to:
  (0.104797030811731,0,0.211314603286436);crosec:Brick;mod.height:-;mod.thickne
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
2 Beam 'Beam';from:(0.104797030811731,0,0.211314603286436) to:
  (0.174094360145961,0,0.306969792723311);crosec:Brick;mod.height:-;mod.thickne
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
3 Beam 'Beam';from:(0.174094360145961,0,0.306969792723311) to:
  (0.253564761180984,0,0.394356833876364);crosec:Brick;mod.height:-;mod.thickne
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
4 Beam 'Beam';from:(0.253564761180984,0,0.394356833876364) to:
  (0.342229317674055,0,0.472399298789764);crosec:Brick;mod.height:-;mod.thickne
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
5 Beam 'Beam';from:(0.342229317674055,0,0.472399298789764) to:
  (0.438995975417l8,0,0.540135960617388);crosec:Brick;mod.height:-;mod.thicknes
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
6 Beam 'Beam';from:(0.43899597541718,0,0.540135960617388) to:
  (0.54267275735765 6,0,0.596732430584508);crosec:Brick;mod.height:-;mod.thickne
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
7 Beam 'Beam';from:(0.542672757357656,0,0.596732430584508) to:
  (0.651982649153701,0,0.641491599607663);crosec:Brick;mod.height:-;mod.thickne
  -;active;valid;ecce-loc:(0/0/0)[cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:
  [m];BklLenZ:0[m];BklLenLt:0[m];nII:0[kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[
8 Beam 'Beam';from:(0.651982649153701,0,0.641491599607663) to:
  (0.765579225372593,0,0.673862147078812);crosec:Brick;mod.height:-;mod.
```

Figure 75: Beam elements

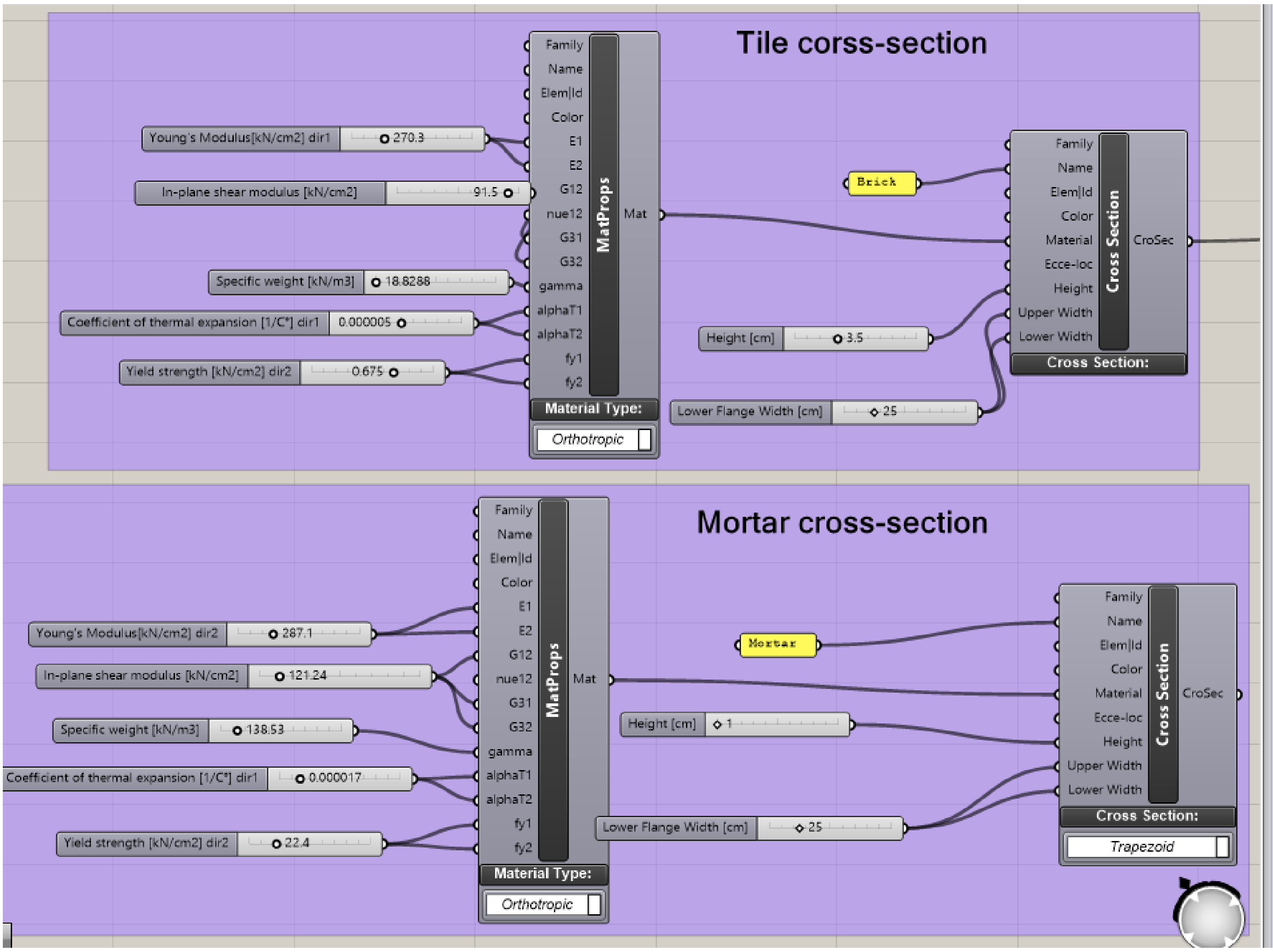
5.5.2 Cross section and properties of materials

The arch presented in the test was made out of thin bricks (tiles) layers with a lime mortar and GFRP grid between layers. The cross-sectional dimensions is described in details in (Chapter 5.1.1, Chapter 5.1.2, Chapter 5.1.3). The thin bricks(tiles) had a rectangular form over the whole arch with the same dimension, and the properties of materials used in the grasshopper script are shown in the (Table11).

Materials	Thin brick	Mortar type A	Mortar type B	Glass Fiber Reinforced Polymer (GFRP)	Unit
Young's modulus	270.1	287.1	504	3.980	kN/cm^2
In-plane shear modulus	91.5	121.24	211.91	357.89	kN/cm^2
Spesific weight	19.2	17	81	26.30	kN/cm^3
Coefficient of thermal expansion	0.000005	0.000017	0.0093	0.000017	1/C
Yiled strength	0.675	22.4	50	22.40	kN/cm^2

Table 11: Material properties Karamba3D

The material properties is not represented the materials in the test but they carried out from some research as [58, 101] on the same materials since they were not presented in the test report. Since the materials (Bricks-mortar- GFRP grid) was not among the standard materials in Karamba3D, then they were defined by inserting the properties of each material as self-made in the tool (Figure 76).



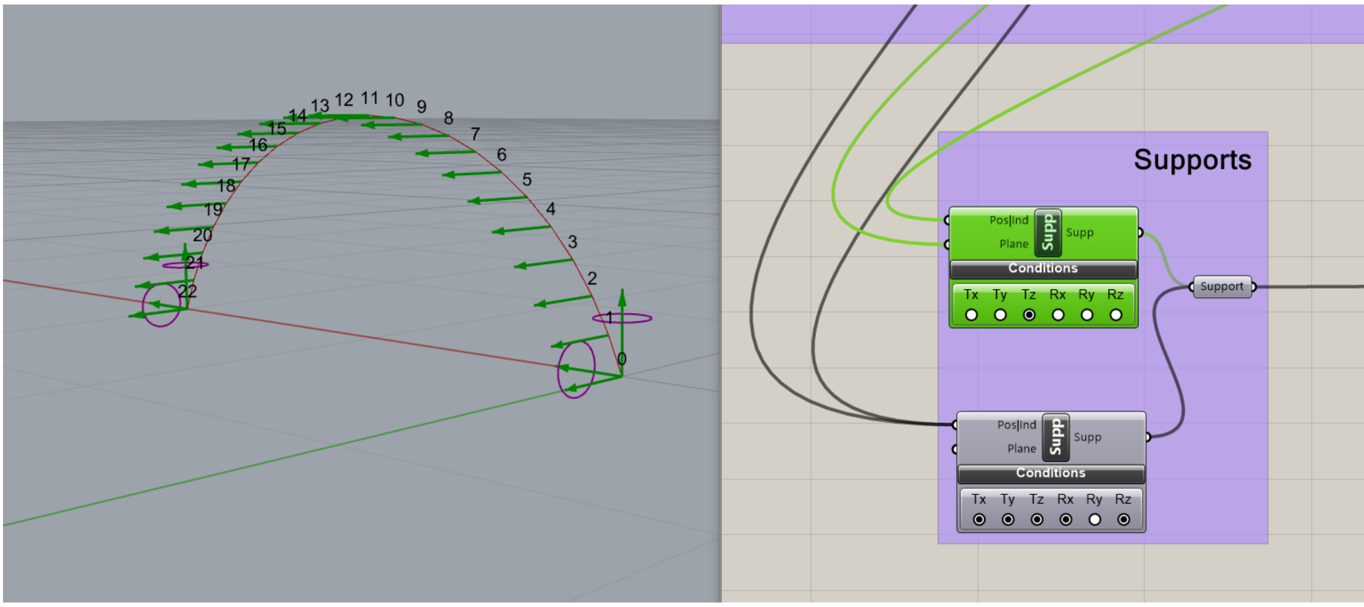
Young's Modulus[kN/cm2] dir1 270.3
In-plane shear modulus [kN/cm2] 91.5
Specific weight [kN/m3] 18.8288
Coefficient of thermal expansion [1/C°] dir1 0.000005
Yield strength [kN/cm2] dir2 0.675
Material Type: Orthotropic
Brick
Height [cm] 3.5
Lower Flange Width [cm] 25
Cross Section:

Young's Modulus[kN/cm2] dir2 287.1
In-plane shear modulus [kN/cm2] 121.24
Specific weight [kN/m3] 138.53
Coefficient of thermal expansion [1/C°] dir1 0.000017
Yield strength [kN/cm2] dir2 22.4
Material Type: Orthotropic
Mortar
Height [cm] 1
Lower Flange Width [cm] 25
Cross Section: Trapezoid

Figure 76: Cross-section components

5.5.3 Supports

The Support component in Karamba is defined with six degrees of freedom, three rotations, and three translations. Two types of supports are needed in the thesis. The first one is on the boundary bottom edges (just two points) of the arch. All translations (movements) and rotations on those points were restricted except the rotation of direction Y. The rotation of direction y will give the arch freedom to collapse under loads in the same mechanism as in the test. The second kind of supports was on the joints between beam elements. The number of joints expected for the points of boundary points was 21. The conditions of those supports were to have the freedom as translations and rotations in all directions except the movement in the direction Z to prevent the joints from moving out of the system (Figrue 77).



Tx ○ Ty ○ Tz ● Rx ○ Ry ○ Rz ○

Tx ● Ty ● Tz ● Rx ● Ry ○ Rz ●

Figure 77: Support conditions

5.5.4 Loads

The ultimate state (US) is a physical situation that involves either excessive deformations leading to collapse of the structure as a whole, as relevant, or deformations exceeding pre-agreed values. The state was used in the test, and the loads were given in this state. While the ultimate limit state (ULS) is an agreed computational condition uses for strength and stability of the structure under design loads. Any structural system can be considered to satisfy the ultimate limit state criterion if all factored bending, shear, and tensile or compressive stresses are below the factored resistances calculated for the section under consideration. Therefore it is essential to design the loads in the parametric environment by multiple them in one safety factor which is depend on the type of load [66].

Four types of loads affected the structure. The first one was the gravity load which represents the self-weight of the arch. The second type was the point load which meant the external load in the test. This load had different values in the experimental test. Therefore (number slider)component was used under designing to make it easy to change the value. The third type of load was for joint loads, representing the head joints between bricks and are full of mortar. The fourth type of load was for mortar in the bed joints (as one layer between two layers of bricks) as uniform loads over the bricks (beam elements). The last type of load was the GFRP grid which was also a uniform load over the bricks (Figure 78).

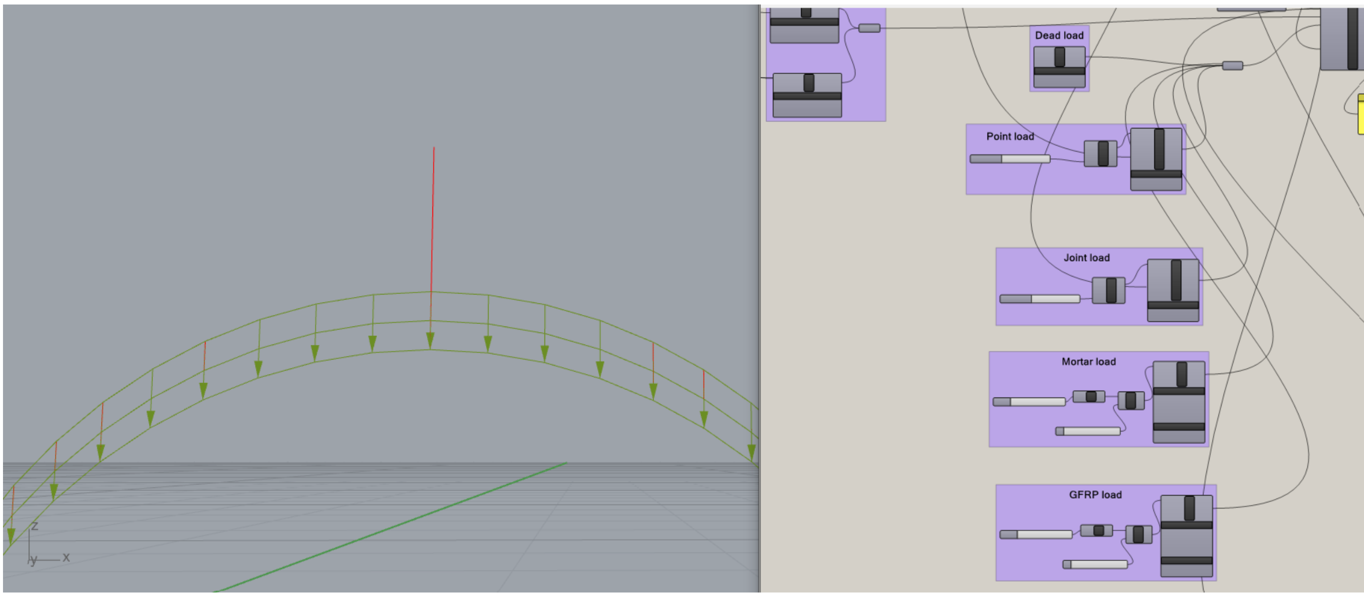


Figure 78: Loads

First it is important to calculate volum of the material, then by using equation (1) the mass of material will be known.

$$\rho = m/v - - - - - - - - - - - - - - - - (1)$$

- ρ :The density of material
- m :Mass of material
- v :the volum of material.

The weight of any material is the product of its density multiplied by the standard gravity. In designing the weight of material represents the force that the material can affect on the structure and that is dependent on material properties. After getting the value of mass from the equation (1) the force can be calculated by using equation (2)

$$P = m * g * 0.001 - - - - - - - - - - - - - - -(2)$$

- p : The force of material (Kn)
- m : The mass (Kg)
- g : Gravity (m/s^2)

5.5.5 Joints

The rigidity in the beam joints needs to be considered under parametric design, as they have a crucial influence on the behavior and strength of the Catalan arch. The reason is that the joints must be so rigid as possible with some flexibility to reach the safe state of the structure. It is crucial to treat them as hinges with rotational spring stiffness and give them the freedom to move and rotate but do not out of the plan. This stiffness of joint spring depends upon the properties of both mortar and brick [23]. Therefore. They were restricted in rotation in the direction Y (Figure 79).

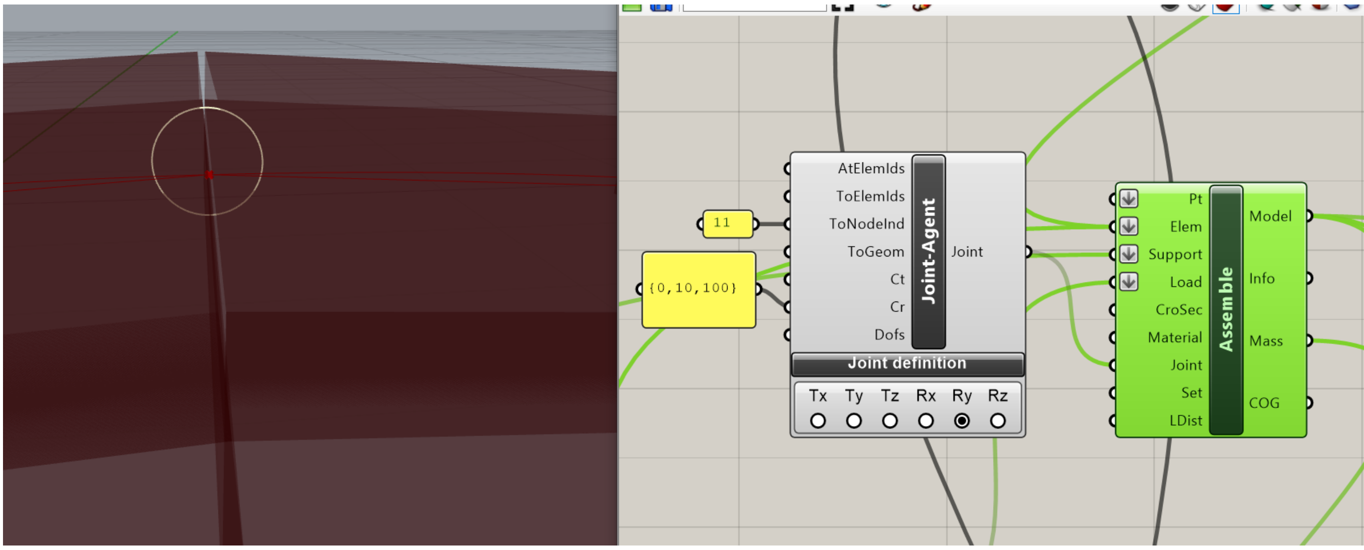


Figure 79: Joint rigity

5.5.6 Assemble model

The model was assembled with the Karamba component (Assemble Model), which needed several parameters such as beam elements, cross-sections, material, support points, and loads (Figure 80).

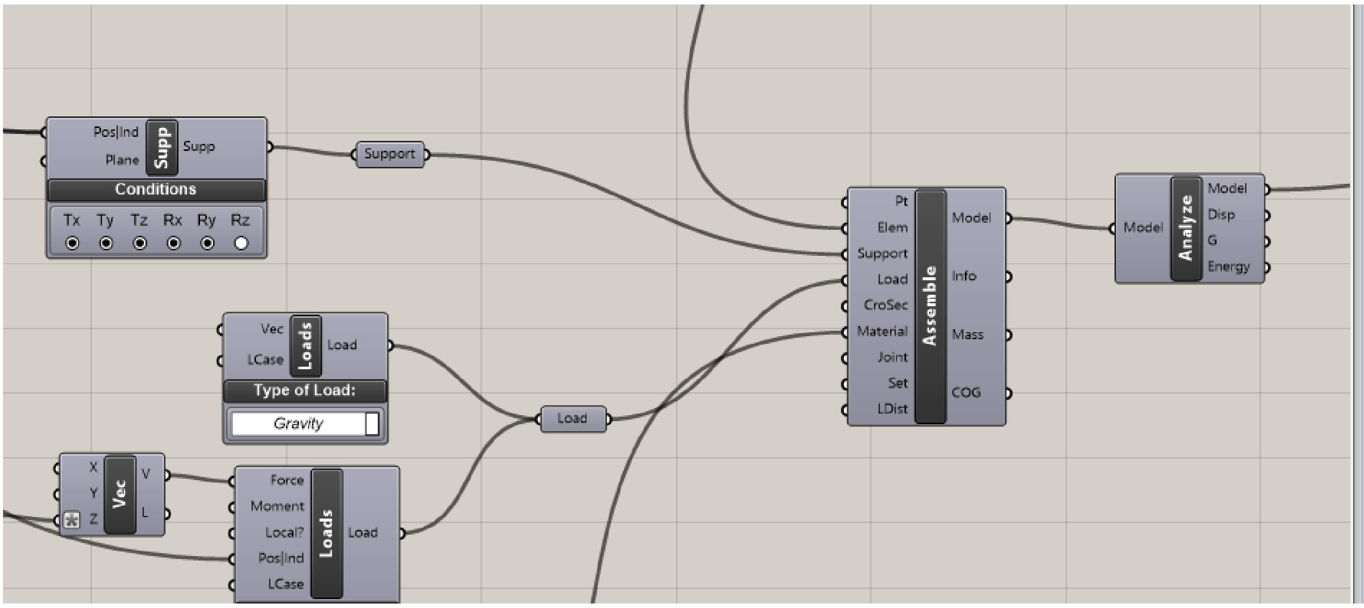
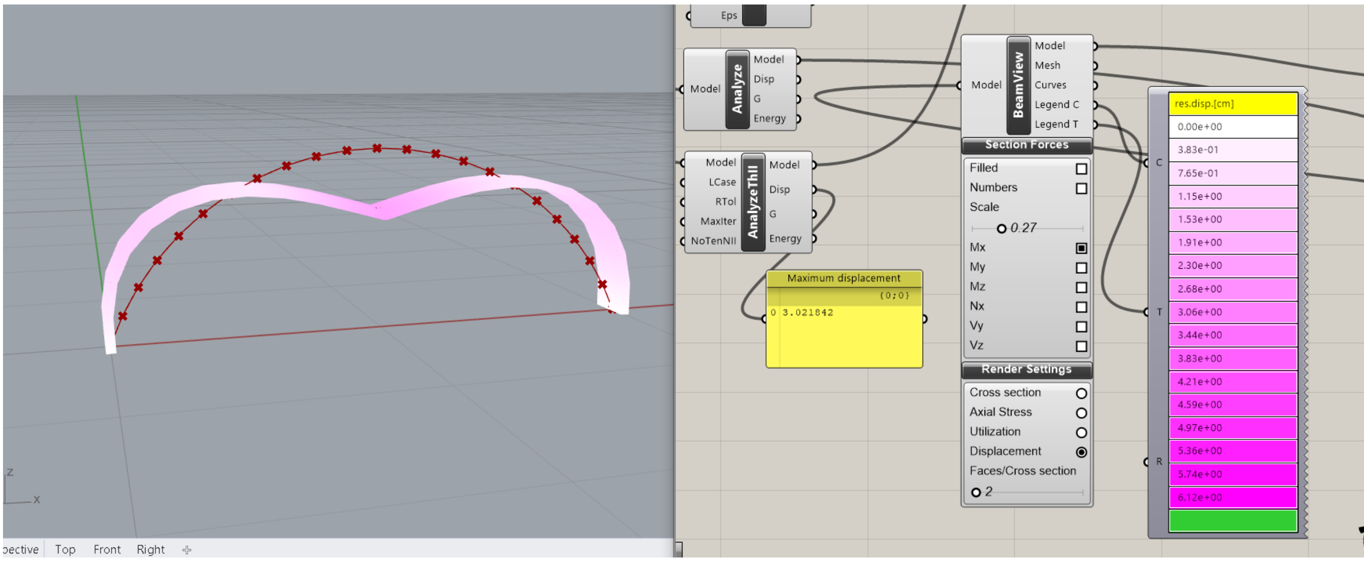


Figure 80: Assemble model

After the model was assembled, the analysis could start by connecting the component of (Assemble model) to (Analyze)component for using first order theory for small deflections and (AnalyzeThII) component for second order theory for small deflections. THis component (AnalyzeThII) would give more accurate results of analyzing because it will not affect the global shape of the model. Then the model could be shown by connecting the last component to (Beam view) component for visualization of the arch and cross-sectio,axial stresses, utilization and displacement.

5.5.7 Displacement

The maximum displacement and distribution of displacement are essential to design the Catalan arch. If the global shape deflects, the structure will be less resistant to compression forces and finally will buckle and collapse (Figure 81).



Maximum displacement
{0;0}
0 3.021842

Section Forces
Filled ☐
Numbers ☐
Scale 0.27
Mx ☒
My ☐
Mz ☐
Nx ☐
Vy ☐
Vz ☐

Render Settings
Cross section ○
Axial Stress ○
Utilization ○
Displacement ●
Faces/Cross section 2

res.disp.[cm]
0.00e+00
3.83e-01
7.65e-01
1.15e+00
1.53e+00
1.91e+00
2.30e+00
2.68e+00
3.06e+00
3.44e+00
3.83e+00
4.21e+00
4.59e+00
4.97e+00
5.36e+00
5.74e+00
6.12e+00

Figure 81: Displacements

Rhino port view showed displacements for all loads on the arch and all construction layers according to the first-order theory and second order theory for small deflections. In the same view, normal forces, shear forces, bending moments, maximum displacements, structure weight, and buckling length factors could be selected to be shown as a part of the analysis of the arch. The values of axial forces, shear forces, and bending moments could be shown in the (beam view) component.

5.5.8 Comparing the results

One layer arch: In the case of just one layer of tiles with the mortar in joints betweem them. The research group claims that the arch will collapse without any testing or explanation if it happens under material failure or structural instability due to loss of structural stiffness within the material's elastic limit. I all cases, this was in opposite of what the results of parametric design showed. One layer of tiles would collapse neither under the self-weight of tiles and mortar nor under the external load to build the arch. The arch could bear its self-weight and one external load of 8.67 kn with a maximum displacement of 5.69346 cm according to first order theory for small deflections and 9.04917 accordign to second order theory for small deflections until it collapsed after that (Figure 82).

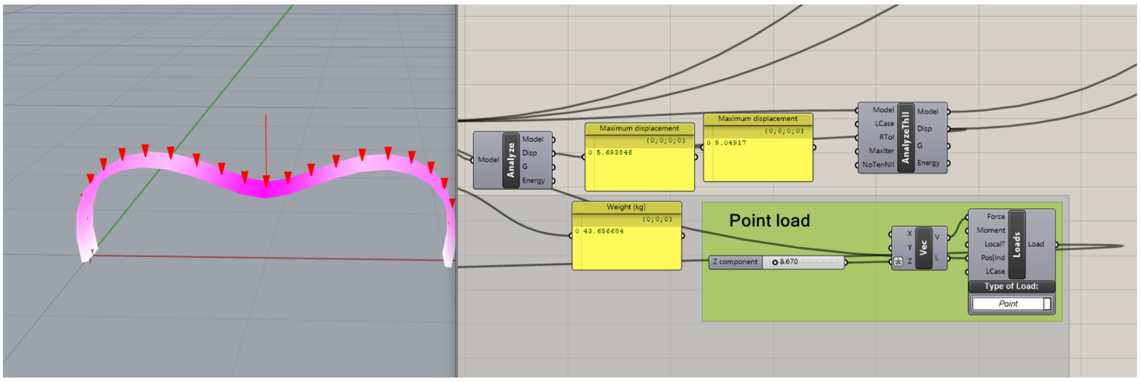


Figure 82: Buckling under one layer of tiles and mortar in joints between them

Double ring arches:
(Table 12)showing the results especially the displacements carried of the test and compared with the results the design model showed under the same conditions.

Test	Failure Load (kN)	Deflections from test (mm)	Deflections from model according to first order theory (mm)	Deflections from model according to second order theory(mm)	Weight of the structure in th model(Kg)
S2-TU-01	0.15	-	4.64	5.10	95.27
S2-TU-02	1.57	2.46	8.88	9.88	125.20
S2-TG-01	0.82	4.88	7.49	8.35	100.05
S2-TG-02	3.52	26.99	15.11	17.76	129.99

Table 12: Comparing the results between the test and the model for double ring arches

The model showed something important and interesting at the same time. The buckling load for the arch was constructed of two layers of bricks, and the mortar between them was bigger than the case with reinforcing of GFRP. The load capacity in the case without reinforcing GFRP was 20.9 kN (Figure 84)while it was reduced to 20 kN (Figure 83)in the case of GFRP reinforcing; after those loads, the arch started to buckle. GFRP has both reduced the capacity of Catalan vaulting and affected with more weight on the arch.

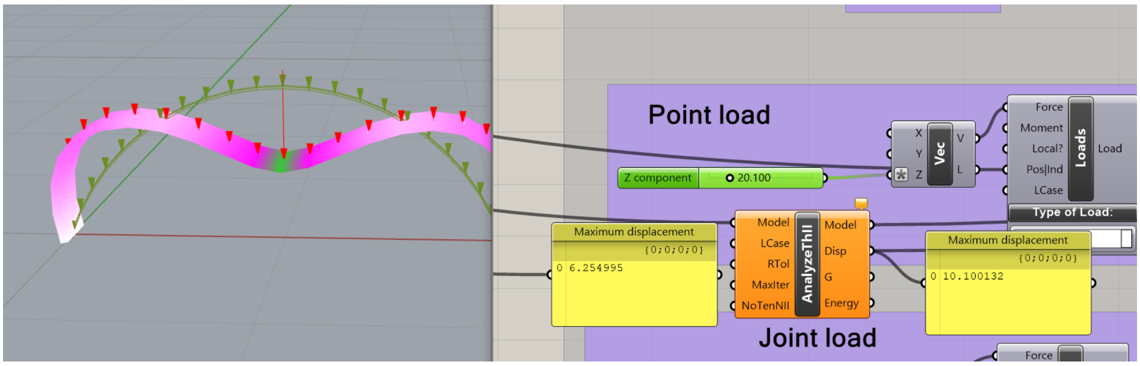


Figure 83: 2 layer bricks with GFRP

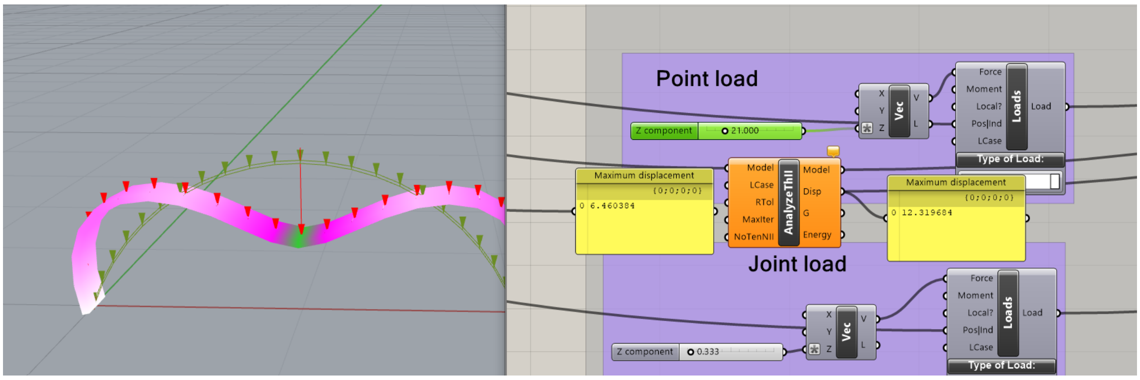


Figure 84: 2 layer bricks without GFRP

Triple layer arches:
(Table 13)showing the results especially the displacements carried of the test and compared with the results the design model showed under the same conditions.

Test	Failure Load (kN)	Deflections from test (mm)	Deflections from model according to first order theory (mm)	Deflections from model according to second order theory(mm)	Weight of the structure in th model(Kg)
S3-TU-01	1.07	-	5.00	5.37	138.92
S3-TG-01	2.71	2.36	8.84	9.69	143.71
S3-TG-02	8.46	17.60	19.97	22.93	173.64
S3-TG-03	12.99	24.79	28.83	34.79	173.64

Table 13: Comparing the results between the test and the model for triple ring arches

Comparing different types of arches under the same conditions: The model was used to build three types of arches. All are built on Catalan vaulting (by bricks and lime mortar type B) and without reinforcing. All arches, in this case, were had the same conditions and just different layers of bricks to check the load capacity of Catalan vaulting. The first arch was built on just one layer of bricks and mortar in joints(head joints) between them. Here the arch had load capacity to bear self weight of the structure and one external load of 8.6 kN to start to buckle after that (Figure 85). The second arch was built on two layers of bricks and mortar (in bed and head joints). Here the arch had load capacity to bear self weight of the structure and one external load of 20.9 kN to start to buckle after that (Figure 84). The third arch was built on three layers of Catalan vaulting. Here the arch had load capacity to bear the self-weight of the structure and one external load of 35.9 kN to start to buckle after that (Figure 86).

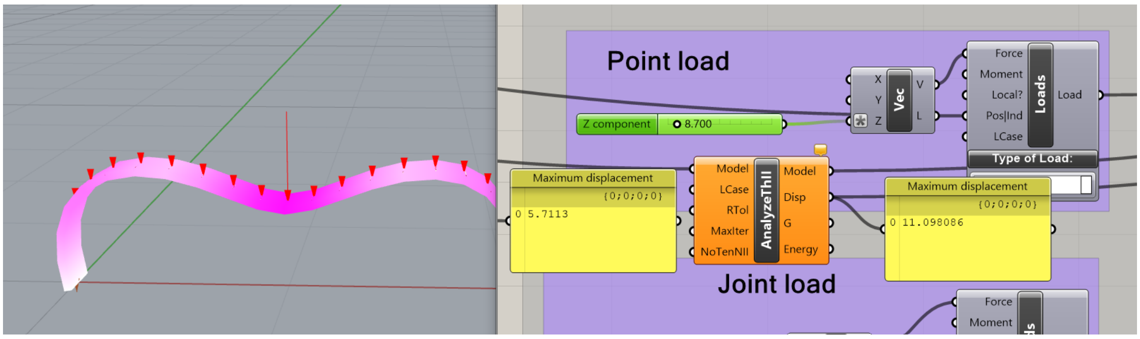
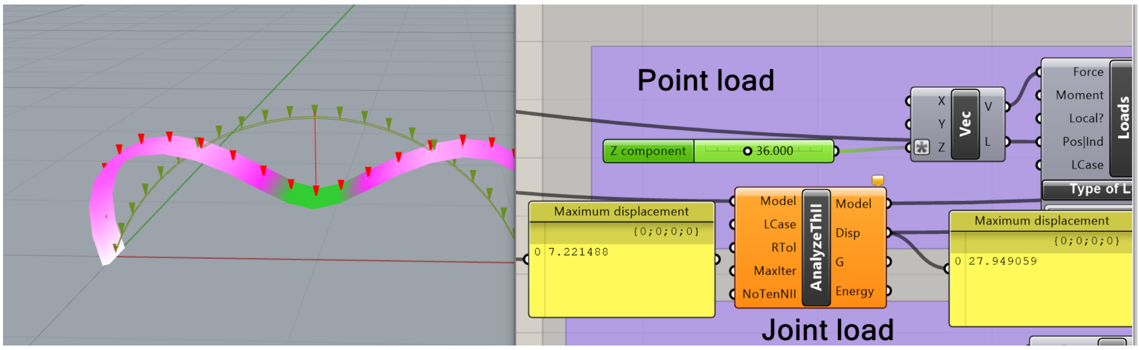


Figure 85: 1 layer brick without reinforcing



Z component 36.000

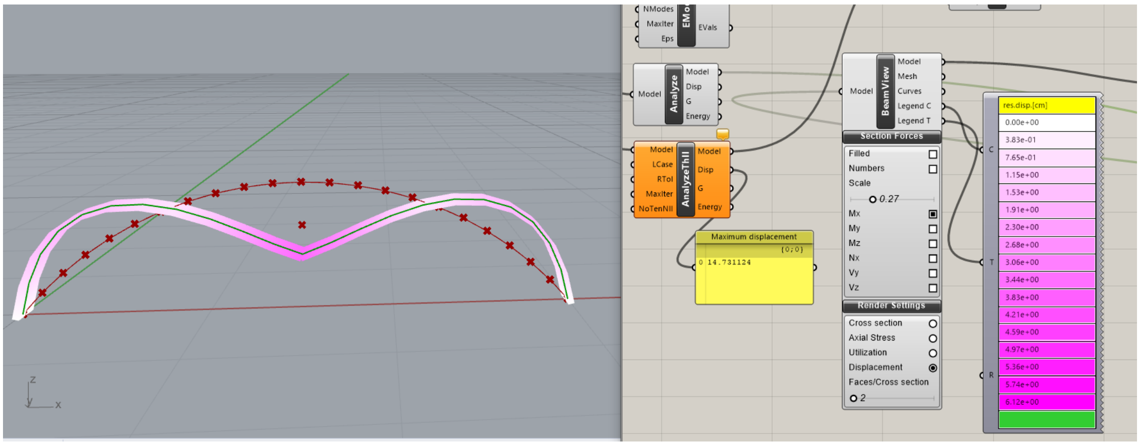
Maximum displacement
{0;0;0;0}
0 7.221488

Maximum displacement
{0;0;0;0}
0 27.949059

Figure 86: Buckling under 3 layer bricks without reinforcing

5.5.9 Discussion

Timbrel vaults are like other masonry structures that have little resistance to tension, can crack, trust, and are neither monolithic nor cohesive [54].
The strength of Catalan vaulting comes from the different layers of the bricks and lime mortar, which working as one unit to carry the loads. The load capacity was increased from 8.6 kN by one layer of Catalan vaulting to be 35.9 kN by using three layers of Catalan vaulting as the results showed from the model and were discussed in the previous section. Some parameters give more efficiency arch and optimization of the structural behavior of timbrel arch. The form and dimensions of the arch (span and height)must be considered under design. Grasshopper is a good tool to use and see how the structure will behave by easily changing those parameters. By changing the arch span from 2m as it was from the test to be 3m, the maximum displacement has been increased from 3.022mm to be 14.731, and the same thing (Hauge growing in displacement) will happen by changing the height of the arch. The arch will buckles and collapse if other parameters are constant as it was and changing the dimension (Figrue 87).



```
Maximum displacement
{0;0}
0 14.731124
```

Section Forces
Filled ☐
Numbers ☐
Scale 0.27
Mx ☒
My ☐
Mz ☐
Nx ☐
Vy ☐
Vz ☐

Render Settings
Cross section ○
Axial Stress ○
Utilization ○
Displacement ●
Faces/Cross section 2

Figure 87: Changing of span

Cross-section dimensions and properties of materials are other factors in structural behavior that must be carefully considered under design. Larger cross-sections give better load-bearing capacity, but at the same time, will push more weight on the structure. The same thing applies on number of layers and number of bricks uses to build the arch even they result in a stiffer structure. In any case, just one layer on bricks will collapse after time under self-weight and can not stand against any external load becuase the strength of the timbrel arch comes from double layers of materials that make the structure as strong as a reinforced concrete structures. By reducing one property of tile material as young's modulus from 270 kn/cm2 to 200 kn/cm2, the maximum displacement has been increased from 3.022 mm to be 5.285 mm. Therefore the capacity of the Catalan arch can be increased by using a material with suitable properties and lesser weight as GFRP grid to reinforce this kind of construction (Figure 88).

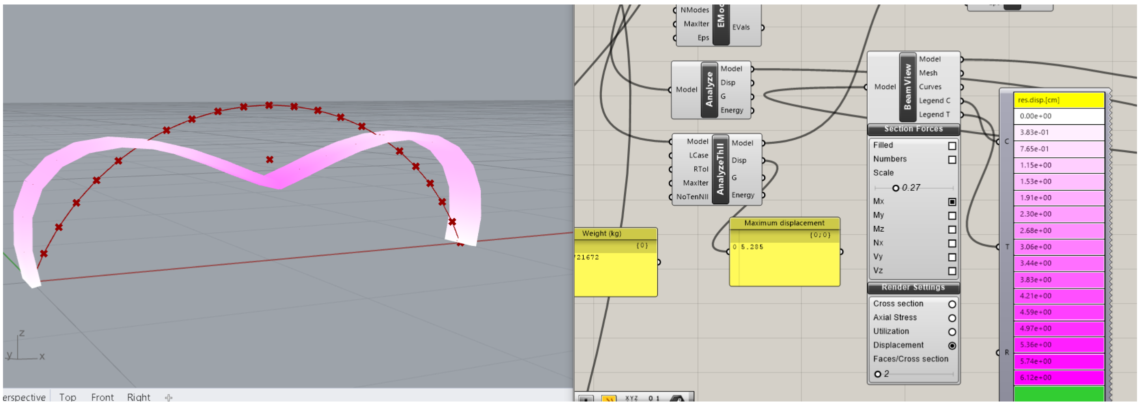


Figure 88: Changing of properties

The lime mortar has an important contribution to the system by making possible the curvature of the vault by changing the angle between joints. Therefore it was necessary to have hinges in the joints between elements and check their rigidity. By combining the lime mortar with composite materials will result in a high increase in tensile capacity.
The rigidity of joints is also one factor must be considered because the joint's rotational stiffness affects the failure mode. The failure mode can change from overall buckling to local buckling if the stiffness of joints is not enough. This will weaken the structural integrity by reducing the arch compression stress. With more negligible joint rotational stiffness, the lower utilization rate of the material strength will be.(Figure 89) is showing how the failure mode is changed and the maximum displacement is increased just of changing the place of hinge from one place to another on the arch.

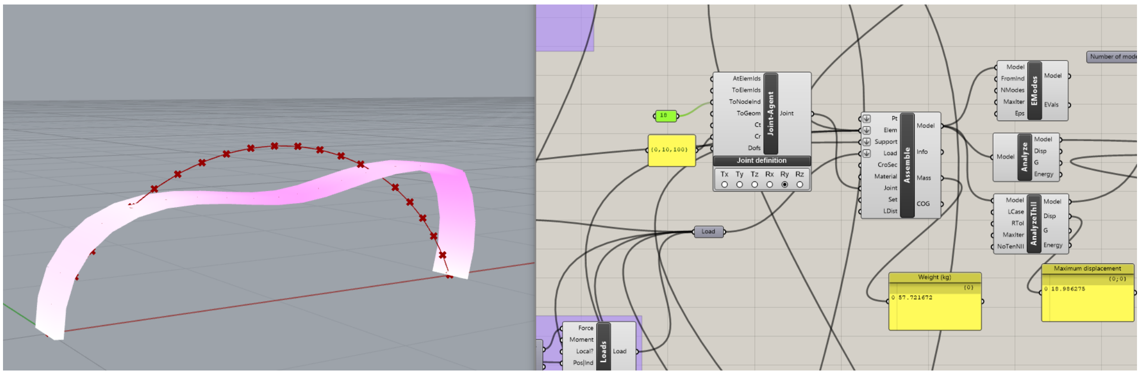


Figure 89: Changing of joint's stiffness

Moving the supports in another masonry vault will crack and collapse the structure. As opposed to the tile vaults, which will survive not because of the tension capacity of the materials but because the form is correct [89].

There are some observations regarding the design and optimization process of the Catalan arch in a parametric environment. Catalan arch in the test would behave like a shell structure, which means it is exposed to buckling. It can buckle even the stress in the system is below those needed to cause failure in the material. While bending is the state of stress, buckling is the state of instability. Karamba has a component (Buckling modes) that takes in the model as input and gives the buckling load factors as an output to buckling analysis of the model. The buckling load factors are shown as the lowest number for the particular buckling mode. If multiplied with the current normal force, it would lead to collapse due to global buckling. The buckling load factors as output must be over 1 to avoid the global buckling. The visualization of buckling modes shows where the buckling can occur; subsequently, the designer can strengthen the structure there (Figure 90).

```
21 16);crosec:Brick;mod.height:-;mod.thickness:-;active;valid;ecce-lo
   [cm];ecce-glo:(0/0/0)[cm];Buckling:on;BklLenY:0[m];BklLenZ:0[m];Bk
   [kN]; X-ori:-; Y-ori:-; Z-ori:-; alpha:0[deg];
```

```
{0;0}
0 1.473485
```

Figure 90: Buckling state

There is no component in Karamba3D that can represent the whole Catalan arch built of many layers, so it was important to build it in several stages. The arch in parametric design will not accurately represent the arch because it divides the arch into different segments, which can simulate the original shape of the arch. This will give different results (Figure91).

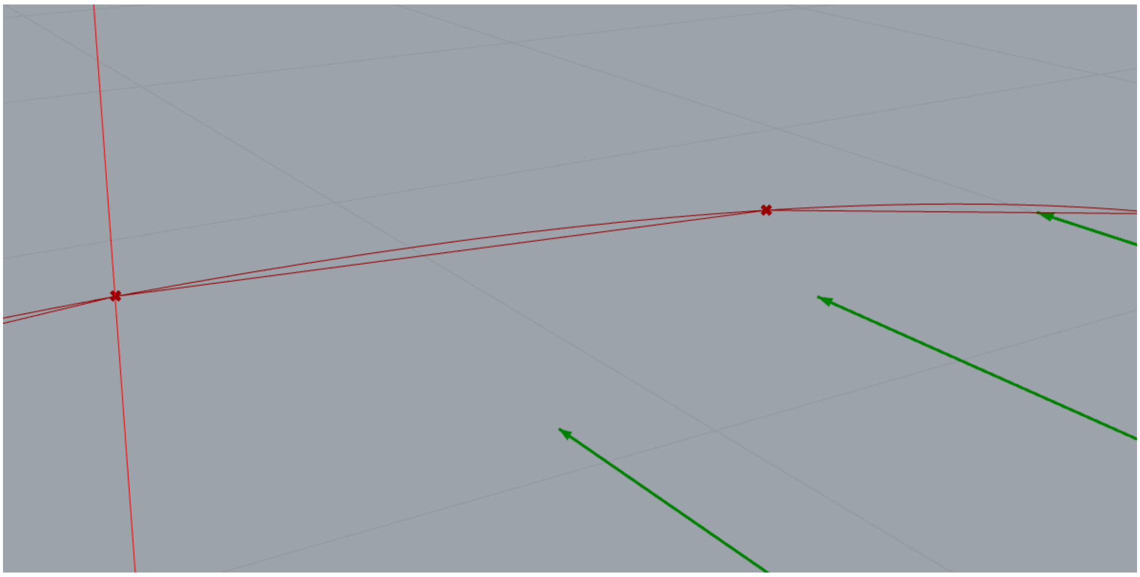


Figure 91: Difference between the actual and segments

The results obtained through this parametric workflow will not give the same values obtained in test results. The reason is that the absolute properties of the materials used in the test were not represented. The results were carried out first on one layer of each material build over each other in one model to show how adjusting of different parameters in Karamb3d affects the system, at the same time, provides an understanding of the structural behavior of Catalan arch under different type of loads and materials.

Designing was not implemented according to Eurocode because there is no component in Karamba3D have this feature. To do that the model should be exported to another parametric design as Etabs or Robot.

6 Conclusion

This thesis aimed to design and analyze the tile arches using parametric modeling. Based on the results carried out of the experimental test and the results of parametric design. The result indicates that the cross-section dimensions of materials and the properties, the rigidity of joints, geometrical form, and arch dimensions are critical factors that must be considered in the building of Catalan arches. One layer of bricks is not structurally sufficient for the Catalan arch to bear any external load, and it will collapse under self-weight after time. The strength of Catalan vaults comes up from the double layers of materials that make them behave as a reinforced concrete structure. It can be concluded that understanding the structural behavior of this kind of construction and the materials used to build them are essential factors that must be considered under designing and analyzing.

The above discussion highlights the importance of designing and analyzing one model in a parametric environment and comparing it with one experimental test. The failure mode was known from the test and was expected, but the results found in this thesis were unexpected findings. The arch could be built using just one layer of bricks and mortar (in head joints) and could carry both self-weight of the structure and one external load of 8.6 kN without any buckling. The strength of Catalan vaulting comes from the different layers which are working as one unit because by increasing the number of layers from one layer to three-layers, the design model showed an increase in the load capacity by 4.17 times under the same conditions. Glass fiber reinforced polymer (GFRP) had brought more weight on the structure and reduced the load capacity of the arch instead of carrying more loads. Therefore, it is important to design, analyze, and compare the test results and the model using parametric tools.

In addition, the results presented in this thesis constitute an essential step, even if not definitive, to open new perspectives in analyzing and designing Catalan vaults taking into account the recent technological tools and material developments which can improve the mechanical performance.

Using parametric tools also gives many benefits according to the time every component needs to work and the flexibility they have to be changed while the results appear simultaneously. While unknowing of material properties used in the test limited the generalizability of the results that have been found in this thesis, the approach in this thesis provides insight into how different parameters combined could change the behavior of the structure.

6.1 Further work

To better understand the implications of these results, future studies could address the effects and relationship between horizontal loads, uniform loads, and impact loads applied on one Catalan vaulting structure and see how the structural performance will be changed. The future developments are to perform a deep analysis of the interaction between mortar and bricks to analyze the vault, which is subjected to seismic actions. Trying new materials with suitable properties is another field to reinforcing the system and can be one case in the future. Designing and optimizing some Catalan vaults with some openings in the top can also be one case in the future by using the workflow of designing a model found in this thesis.

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Appendix

A The parametric workflow

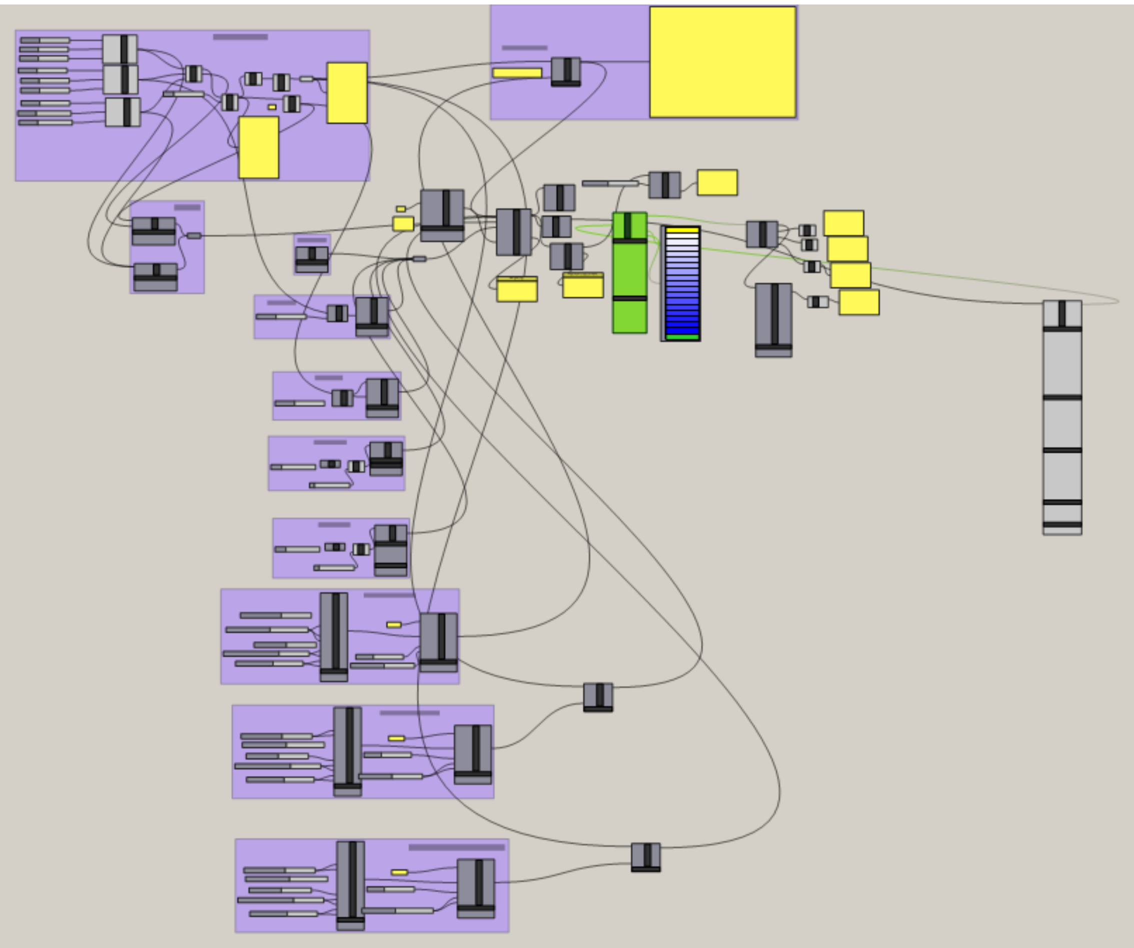
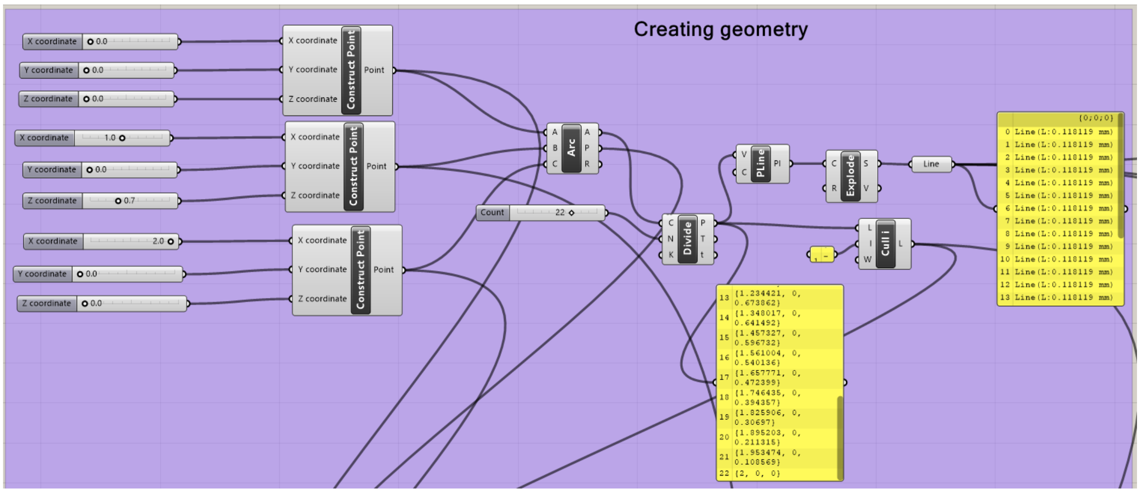


Figure 92: The entire components used in karamba3D



```
13 {1.234421, 0,
   0.673862}
14 {1.348017, 0,
   0.641492}
15 {1.457327, 0,
   0.596732}
16 {1.561004, 0,
   0.540136}
17 {1.657771, 0,
   0.472399}
18 {1.746435, 0,
   0.394357}
19 {1.825906, 0,
   0.30697}
20 {1.895203, 0,
   0.211315}
21 {1.953474, 0,
   0.108569}
22 {2, 0, 0}
```

```
                       {0;0;0}
 0 Line(L:0.118119 mm)
 1 Line(L:0.118119 mm)
 2 Line(L:0.118119 mm)
 3 Line(L:0.118119 mm)
 4 Line(L:0.118119 mm)
 5 Line(L:0.118119 mm)
 6 Line(L:0.118119 mm)
 7 Line(L:0.118119 mm)
 8 Line(L:0.118119 mm)
 9 Line(L:0.118119 mm)
10 Line(L:0.118119 mm)
11 Line(L:0.118119 mm)
12 Line(L:0.118119 mm)
13 Line(L:0.118119 mm)
```

Figure 93: Creating geometry

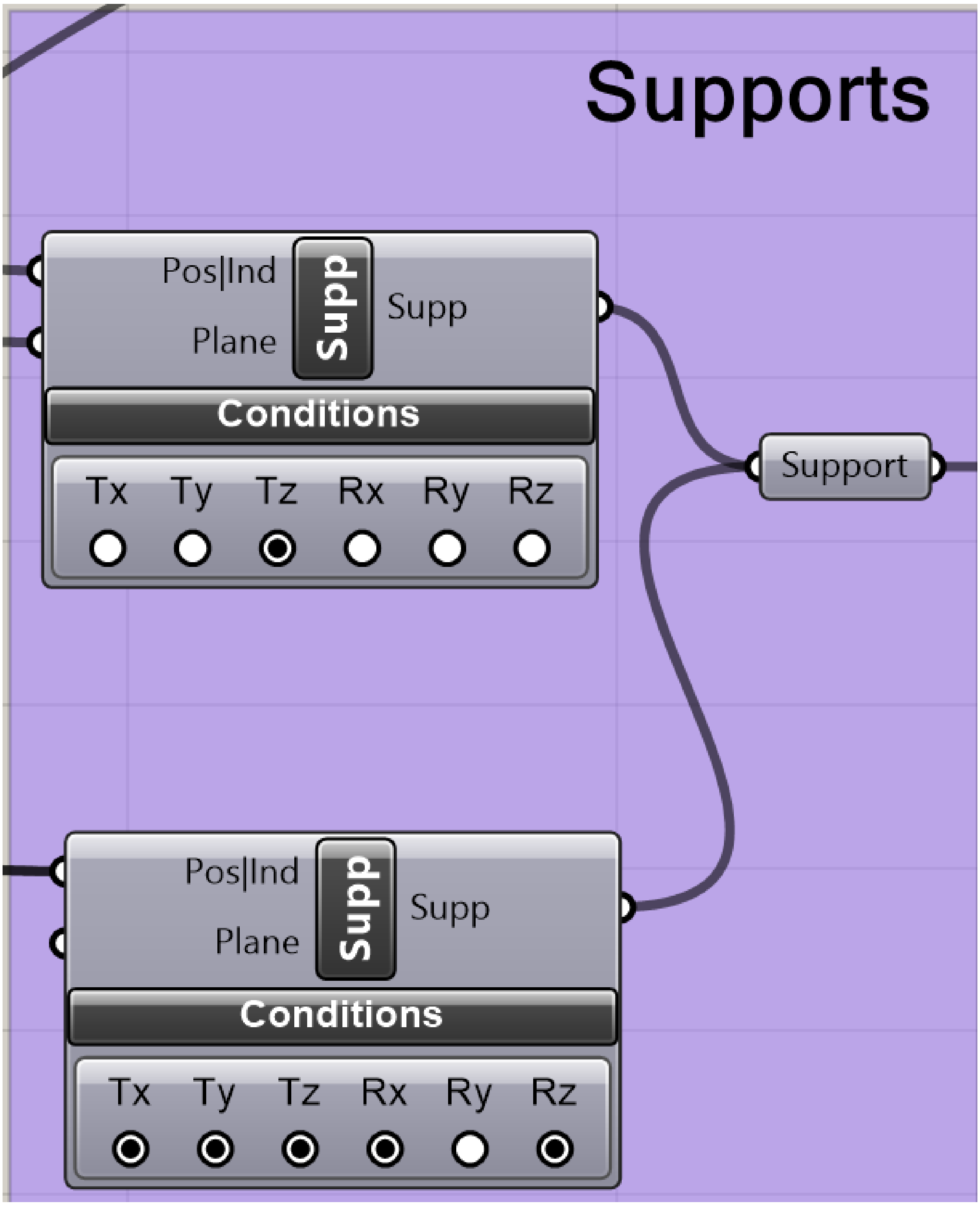
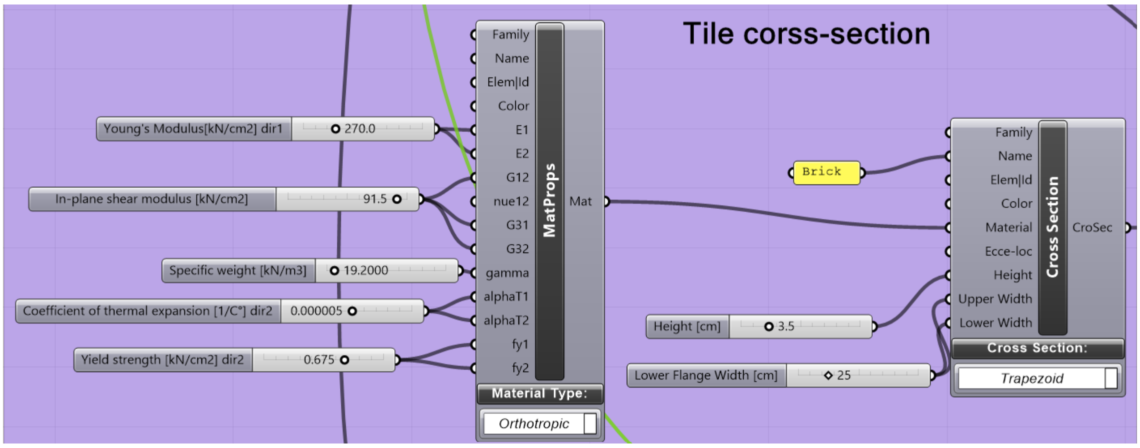


Figure 94: Determining supports



Young's Modulus[kN/cm2] dir1 270.0

In-plane shear modulus [kN/cm2] 91.5

Specific weight [kN/m3] 19.2000

Coefficient of thermal expansion [1/C°] dir2 0.000005

Yield strength [kN/cm2] dir2 0.675

Material Type: Orthotropic

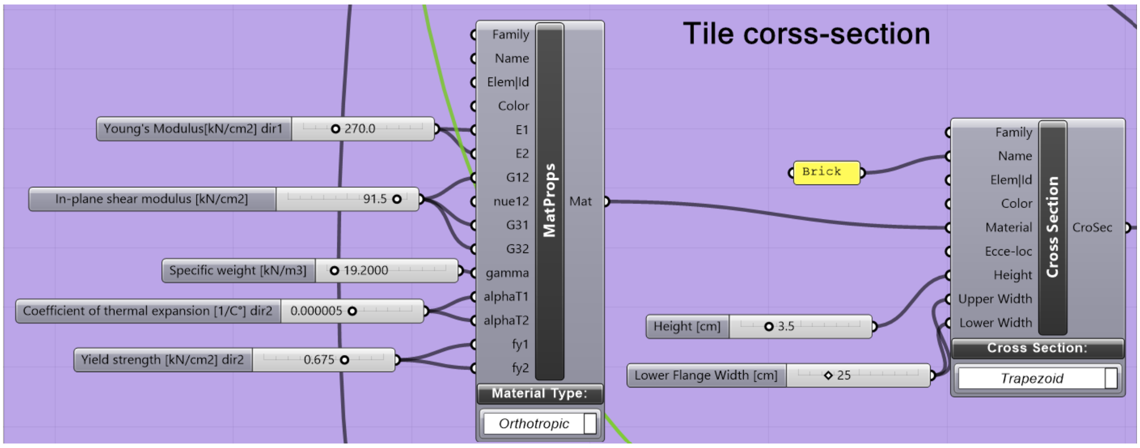
Brick

Height [cm] 3.5

Lower Flange Width [cm] 25

Cross Section: Trapezoid

Figure 95: Tile cross-section



Young's Modulus[kN/cm2] dir1 270.0

In-plane shear modulus [kN/cm2] 91.5

Specific weight [kN/m3] 19.2000

Coefficient of thermal expansion [1/C°] dir2 0.000005

Yield strength [kN/cm2] dir2 0.675

Material Type: Orthotropic

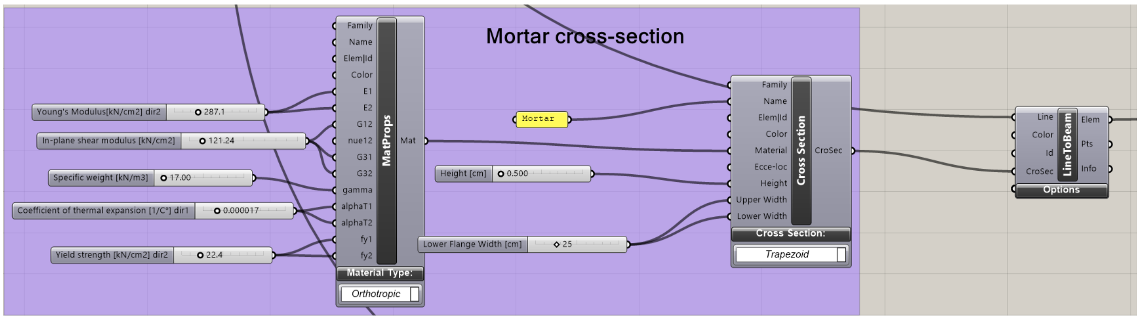
Brick

Height [cm] 3.5

Lower Flange Width [cm] 25

Cross Section: Trapezoid

Figure 96: Tile cross-section



Young's Modulus[kN/cm2] dir2 287.1

In-plane shear modulus [kN/cm2] 121.24

Specific weight [kN/m3] 17.00

Coefficient of thermal expansion [1/C°] dir1 0.000017

Yield strength [kN/cm2] dir2 22.4

Material Type: Orthotropic

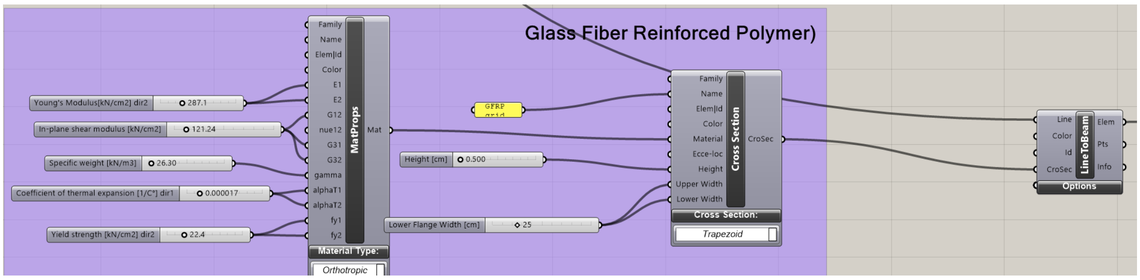
Mortar

Height [cm] 0.500

Lower Flange Width [cm] 25

Cross Section: Trapezoid

Figure 97: Mortar-cross section



Young's Modulus[kN/cm2] dir2 287.1

In-plane shear modulus [kN/cm2] 121.24

Specific weight [kN/m3] 26.30

Coefficient of thermal expansion [1/C°] dir1 0.000017

Yield strength [kN/cm2] dir2 22.4

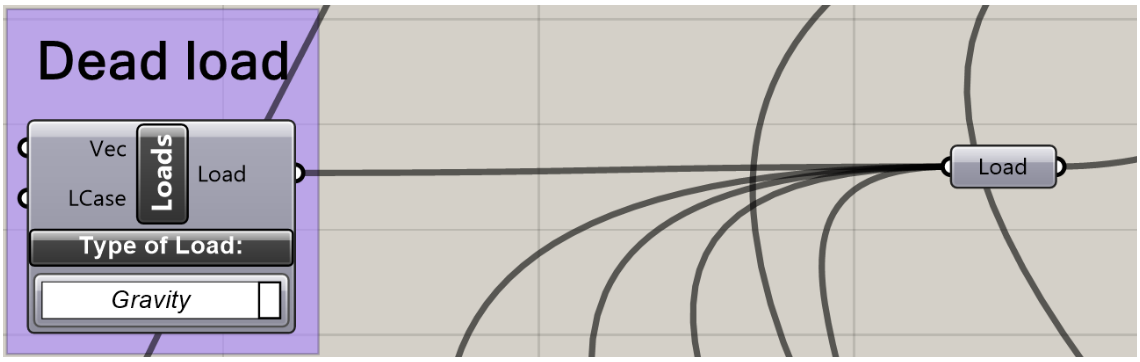
Material Type: Orthotropic

Height [cm] 0.500

Lower Flange Width [cm] 25

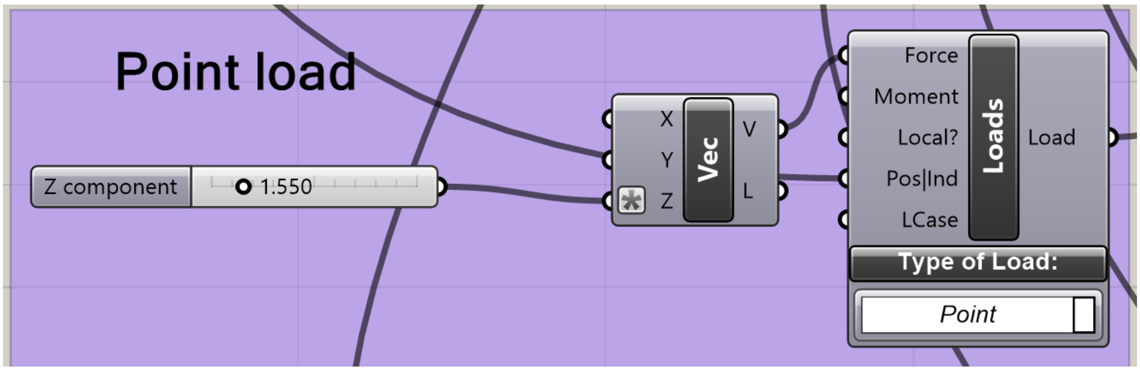
Cross Section: Trapezoid

Figure 98: Glass Fiber Reinforced Ploymer



Type of Load: Gravity

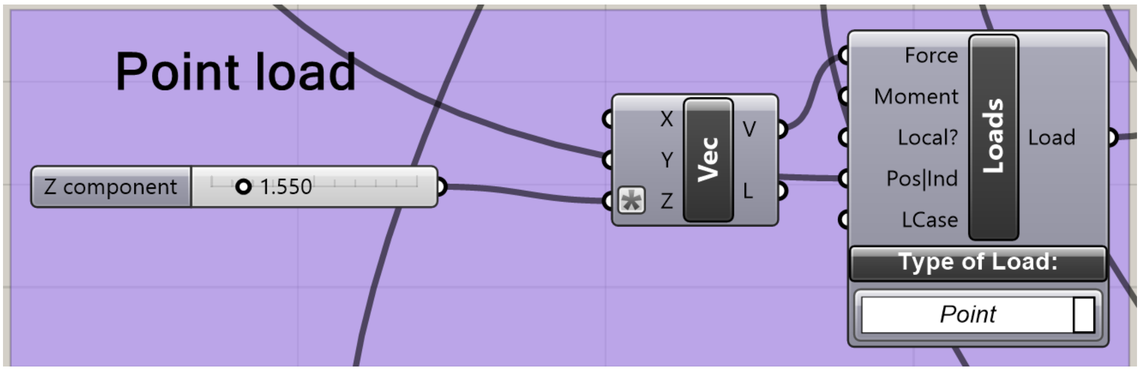
Figure 99: Dead load



Z component 1.550

Type of Load: Point

Figure 100: Point load



Z component 1.550

Type of Load: Point

Figure 101: Point load

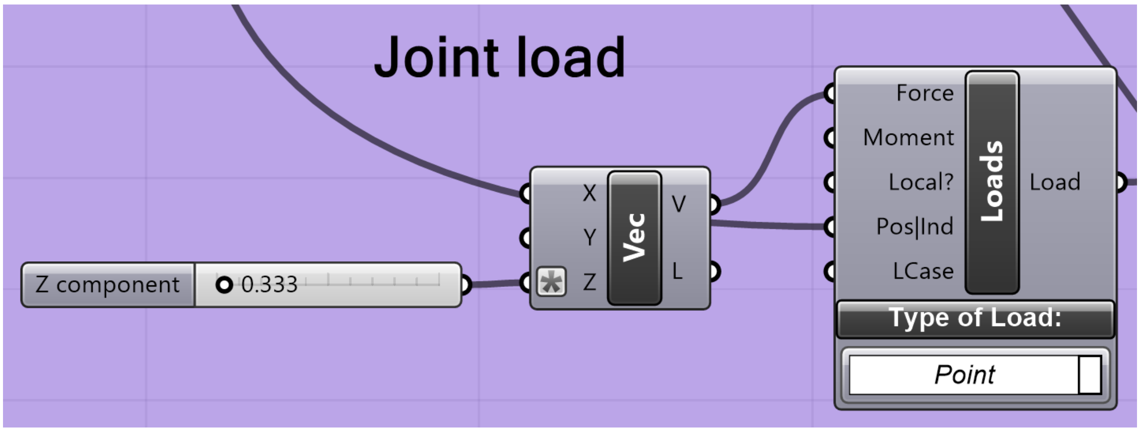


Figure 102: Joint load

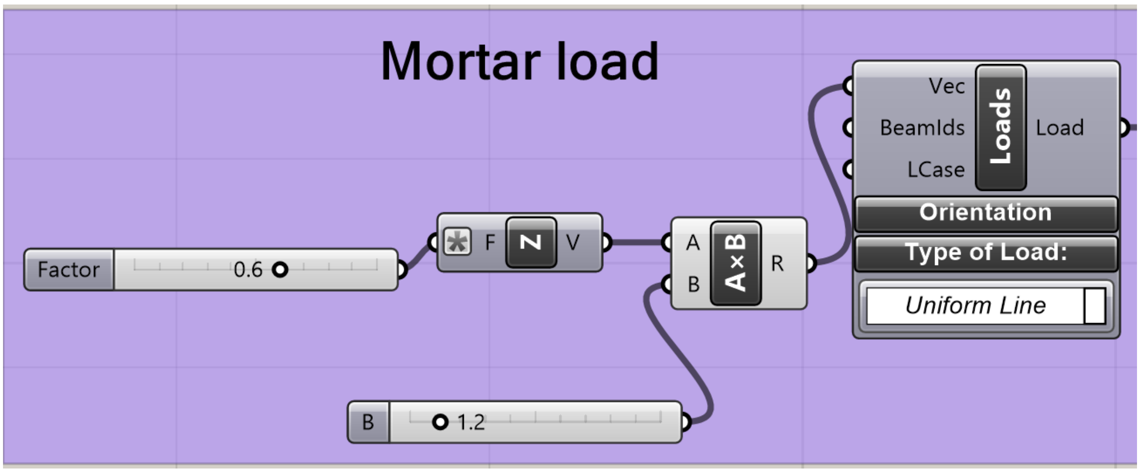


Figure 103: Mortar load

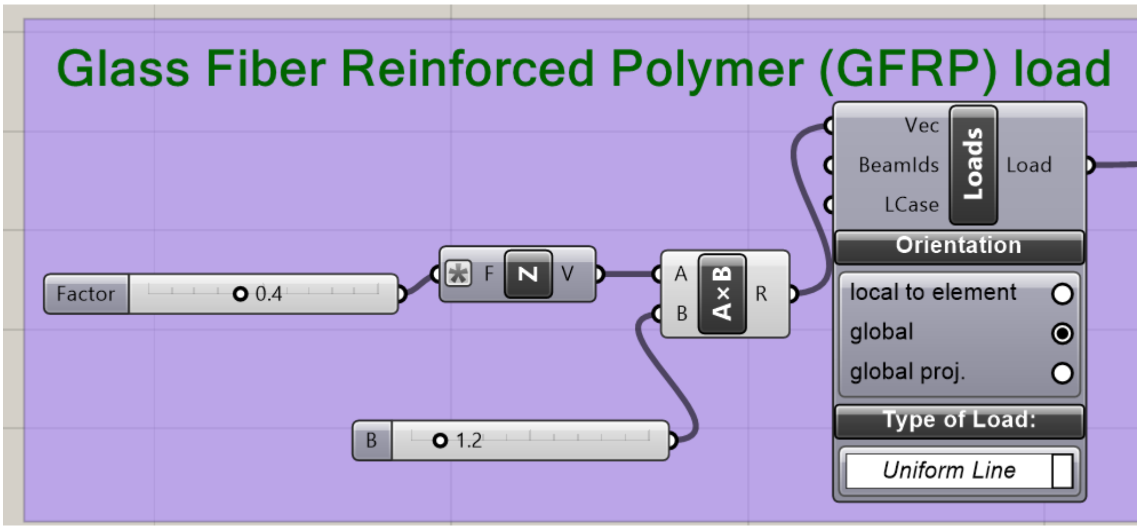


Figure 104: Glass Fiber Reinforced Polymer (GFRP) load

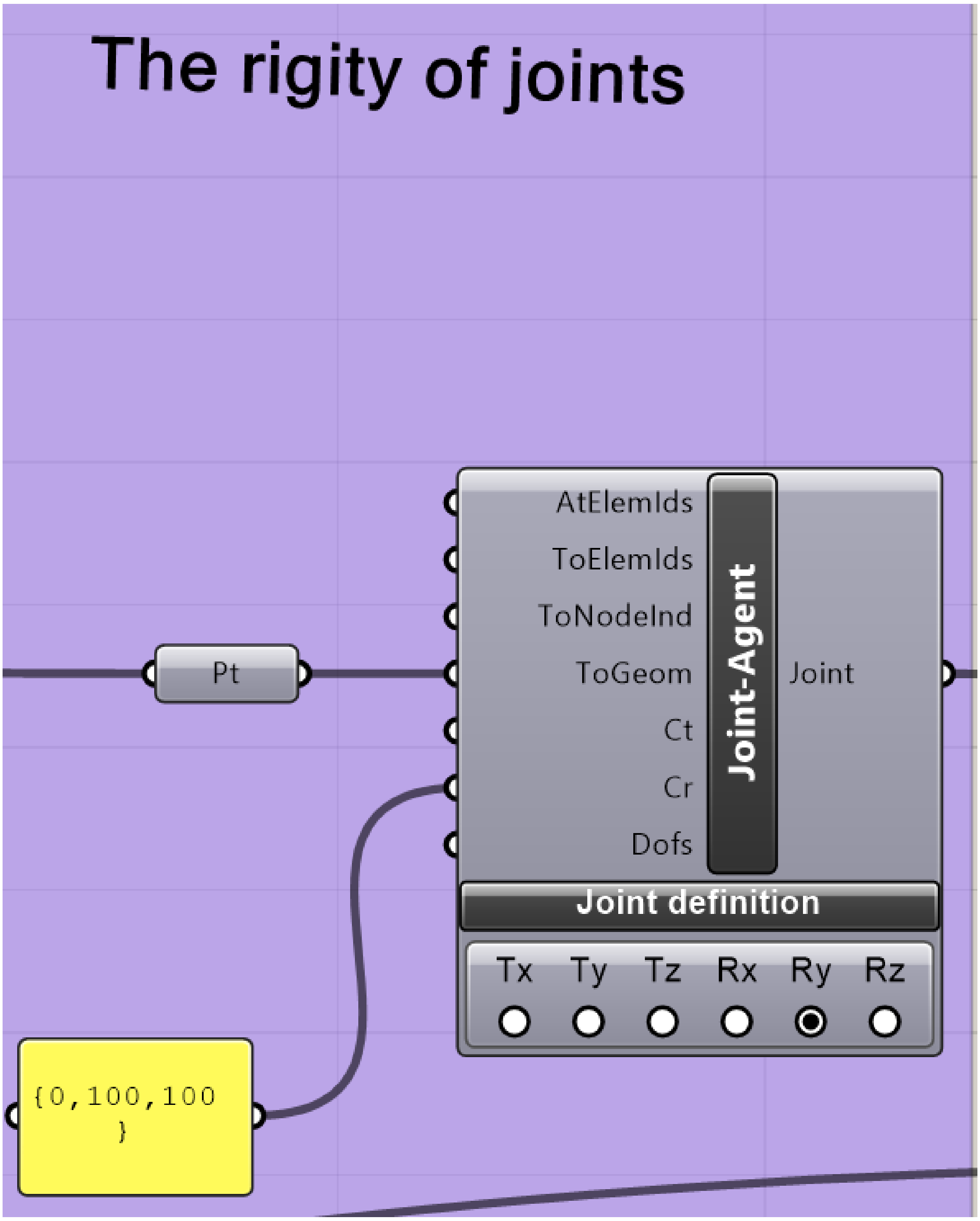
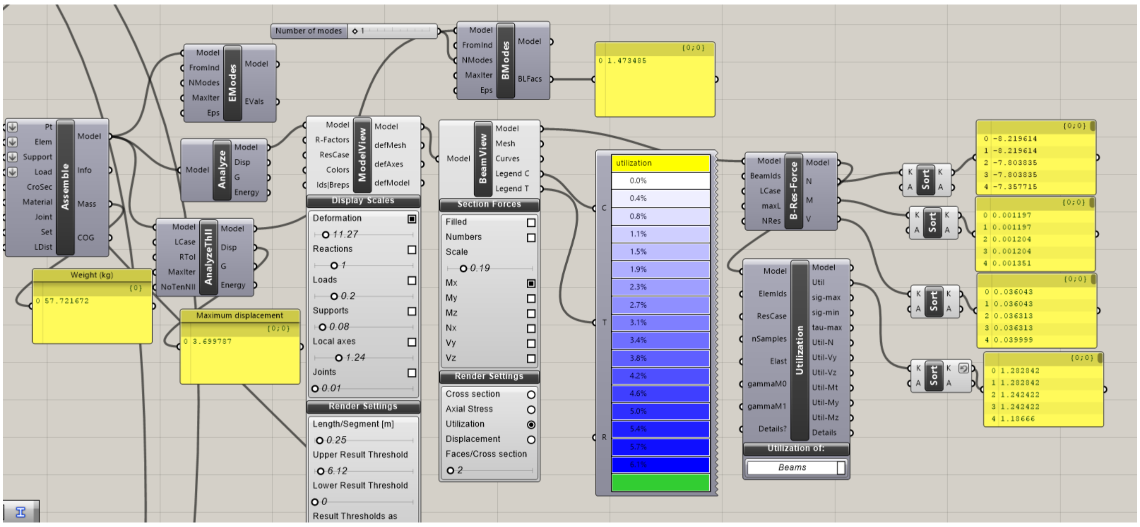


Figure 105: The rigity of joints



Number of modes ◇ 1

Weight (kg)
{0}
0 57.721672

Maximum displacement
{0;0}
0 3.699787

{0;0}
0 1.473485

Deformation ☒
11.27
Reactions ☐
1
Loads ☐
0.2
Supports ☐
0.08
Local axes ☐
1.24
Joints ☐
0.01

Length/Segment [m]
0.25
Upper Result Threshold
6.12
Lower Result Threshold
0
Result Thresholds as

Filled ☐
Numbers ☐
Scale
0.19
Mx ☒
My ☐
Mz ☐
Nx ☐
Vy ☐
Vz ☐

Cross section ○
Axial Stress ○
Utilization ●
Displacement ○
Faces/Cross section
2

Beams

{0;0}
0 -8.219614
1 -8.219614
2 -7.803835
3 -7.803835
4 -7.357715

{0;0}
0 0.001197
1 0.001197
2 0.001204
3 0.001204
4 0.001351

{0;0}
0 0.036043
1 0.036043
2 0.036313
3 0.036313
4 0.039999

{0;0}
0 1.282842
1 1.282842
2 1.242422
3 1.242422
4 1.18666

Figure 106: Assembling, analyzing the model

Aziz Sido

Parametric structural design of Catalan vaulting

NTNU
Norwegian University of
Science and Technology