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Simon Indrøy Risanger

Electricity market design and production planning

Improving economic efficiency and supporting the integration of renewables

Doctoral thesis

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Norwegian University of Science and Technology
Thesis for the Degree of
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Dept. of Industrial Economics and Technology
Management

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Abstract

This thesis investigates novel market designs and production planning. Its overarching goal is to improve the economic efficiency of electricity markets and support the integration of renewable electricity production. Hence, the thesis considers problems faced by multiple stakeholders, from regulators to individual producers, using a variety of methods, where equilibrium models and stochastic optimization are most prominent. Paper I proposes "flexible electricity bidding zones," an innovative market design that changes zonal configuration according to congestion patterns in the power system. This results in cost savings and a design that is more robust to large-scale integration of renewables. Paper II examines oligopolistic wholesale electricity markets. It investigates inverse equilibrium models as a method to investigate market conditions. Paper III demonstrates how complete markets for risk, which allows market participants to hedge both locational and energy price risk, promote efficient investments. Specifically, it shows that extending the contract duration of financial transmission rights lower costs of capital for geographically remote projects, which large-scale renewables usually are. Finally, paper IV considers the cost of neglecting the co-movement between prices and inflows when establishing operational policies in hydro-dominated systems. It demonstrates modest cost savings and provides general insights on behaviors, like how a policy that considers co-movement values future water more.

Contents

Chapter 1

Introduction

The Intergovernmental Panel on Climate Change (IPCC) states that "rapid and far-reaching transitions in energy, land, urban and infrastructure (including transport and buildings), and industrial systems" would be required to limit global warming to 1.5°C (IPCC, 2018). Following the 2015 Paris agreement, with a goal to limit global warming to 2°C, preferably 1.5°C, committed countries plan their strategies to reduce greenhouse gas emissions. The energy sector, which includes electricity, heat, and transport, was responsible for 73.2% of greenhouse gas emissions in 2016 (Ritchie and Roser, 2017) and is thus a major contributor to global warming. To reach climate targets, IPCC recommends to increase renewables, phase out carbon-emitting production, and improve energy efficiency. Developments in renewable technologies indicate that a transition to a zero-carbon power system may be feasible. For example, the International Energy Agency (2020) includes a swift expansion in renewables for all scenarios in its 2020 World Energy Outlook. Maturing technologies and supportive policies promote access to cheap capital for investors, argues the report. It also highlights the developments in solar photovoltaics (PV), which after a sharp decrease in cost over the last decade offers some of the least-cost electricity ever, while installed wind capacity continues to grow.

Despite promising trajectories for renewable production capacity, its implementation is not without challenges. From an operational perspective, both wind and solar have the notorious drawback of variable production. As a result, large-scale production uncertainty increases and exposes stakeholders to new challenges, like operation and planning of renewable production, distributed generation, assessment of available transfer capabilities, reliability evaluation, state estimation, risk analysis, interaction with electricity markets, and operation and planning of distribution and transmission systems (Soroudi and Amraee, 2013). These challenges come on top of overarching uncertainties like technological developments, competitor behavior, regulation, and energy policies (Möst and Keles, 2010). In short, participants in the power sector are exposed to a wide array of uncertainties ranging from daily operational decisions to long-term investments.

This thesis consists of four papers that aim to improve the economic efficiency of electricity markets and support the integration of renewables by alleviating associated risks. It considers challenges faced by several stakeholders, from regulators and system operators, investors in renewable production, and producers, considering both system and operations perspectives. Paper I proposes a novel market design termed "flexible electricity bidding zones," which introduces more flexibility to existing zonal electricity markets so they can facilitate a larger share of intermittent renewable production and reduce costs. Paper II keeps a system perspective, but considers the market power challenges that electricity markets are exposed to. It investigates inverse equilibrium analysis, a data-driven method that fits observations to model structures, which regulators can use to assess electricity markets. Both papers I and II consider the wholesale electricity market, but liberalized electricity markets can also trade risk. As paper III demonstrates, these markets are crucial to secure project finance for investments. In particular, paper III shows that increasing the contract duration of financial transmission rights (FTRs) improves social surplus and encourages investments in geographically isolated assets, which large-scale renewables usually are. Finally, paper IV takes a hydropower producer's perspective and investigates the impact of considering the correlation between prices and inflows when establishing an operational policy. Hydropower allows large-scale storage that can counteract intermittent renewable resources. Improved production planning improves reservoir management and the efficiency of power systems.

The diverse set of papers aspires to reflect the various challenges an energy transition introduces. They therefore consider different stakeholders and decision levels. But the papers also reflect the diversity in an energy transition through varying degrees of disruption in their ideas. Most notably, paper I argues for a novel market design that would introduce a major systematic change, while paper III shows how a modest change in the contract period of an existing risk instruments can benefit project finance of renewables. The papers also relate in terms of methods.

Papers I and IV use stochastic optimization, a modeling framework for decision-making under uncertainty, while paper II uses equilibrium modeling. Paper III combines the two and solves a stochastic equilibrium model.

In the remainder of the thesis, Chapter 2 provides a background on the applications considered. It links applications to methods and foreshadows some relevant findings in the papers. Chapter 3 describes the papers and outlines their contributions. With this in mind, Chapter 4 provides a reflection on the papers and proposes directions for future research. Chapter 5 concludes the thesis. The full papers are in the appendix.

Chapter 2

Background

This chapter aims to put the papers of this thesis into context and provide background information on the applications and methods. All the papers consider some aspect of electricity markets, but they use a wide variety of methods motivated by the different research questions. Paper I employs Reverse Search (Avis and Fukuda, 1996) and Algorithm X (Knuth, 2000) from computer science, in addition to deterministic and two-stage stochastic optimization. Equilibrium modeling and inverse optimization are the focus of paper II, while paper III uses a decomposition technique, introduced by Mays et al. (2019), to solve a two-stage stochastic equilibrium model. Finally, paper IV formulates a novel price process and solves a multistage stochastic problem using stochastic dual dynamic programming (SDDP) (Pereira and Pinto, 1991). The papers, included in the appendix, present, explain, and discuss the methods, so this will not be repeated here. Instead, this chapter focuses on connecting the applications to the methods.

2.1 Wholesale electricity markets

Generation, transmission, distribution, and retail supply constitute the main elements of electricity supply. Historically, these tasks were vertically integrated as monopolies. But in the 1990s, several countries undertook efforts to liberalize sectors that did not benefit from natural monopoly effects (Al-Sunaidy and Green, 2006; Joskow, 2008). Their aim was to improve market efficiency and investments decisions by encouraging competition. Among the changes were the introduction of wholesale electricity markets. These markets facilitate the trades between generators, resellers and large consumers.

Electricity as a commodity has some particular features that warrants complex market designs (Cramton, 2017). A power system ensures the critical requirement that production must equal demand at all times. But it also introduces transmission constraints, which may prevent the most cost-efficient dispatches because they are physically infeasible. Moreover, because electricity is practically considered an essential service (Tully, 2006), reliability and security of supply are key considerations in addition to efficient markets. In Europe, three markets, day-ahead, intraday, and balancing, generally constitute wholesale electricity markets. These are separated by time to delivery, where the day-ahead market clears the day before delivery. The intraday market is a real-time market that closes shortly before delivery, while the balancing market corrects any imbalances between supply and demand in real-time.

2.1.1 Market designs

Efficient wholesale electricity markets should provide proper mechanisms for congestion management and electricity pricing. The former ensures that planned production and consumption comply with the physical constraints of the grid. Congestion is the situation where a corridor reaches its maximum capacity. Electricity pricing, on the other hand, is fundamental to ensure efficient short-term operations and long-term investment signals. Electricity market designs address both challenges. At present, liberalized electricity markets either follow nodal pricing schemes, proposed by Schweppe et al. (1988), or zonal based electricity pools (Weibelzahl, 2017). U.S. independent system operators use the former, while most European system operators apply the latter. In a nodal pricing design, a system operator performs an economic dispatch of the generation assets. An economic dispatch is an optimization problem that dispatches generators with the objective to maximize social surplus. Nodal pricing considers the full grid and thus provides locational marginal prices that reflect the price of injection and withdrawal at nodes. In contrast, zonal markets pool together nodes to form a zone with a uniform price. They either ignore or approximate the physical constraints within a zone, and approximate transfer capacities between zones.

Both nodal and zonal market designs have their benefits and limitations. The former is typically considered the theoretically ideal because the locational marginal prices provide both short- and long-term efficiency in models without uncertainty

(Holmberg and Lazarczyk, 2015). Zonal markets provide fewer prices and follow a democratic principle of equal electricity prices regardless of location (Stoft, 1997). Proponents of this scheme argue that these benefits, together with what they deem a simpler bidding process that increases liquidity, counteract an economic loss compared to a nodal pricing design (Bjørndal and Jörnsten, 2007). The loss is caused by zonal markets' requirement to ignore or approximate grid constraints within zones. Their day-ahead markets may therefore clear into production schedules that are physically infeasible, which the system operator must redispatch to a feasible schedule. Note that these are the main market design paradigms of liberalized wholesale markets, but different markets have adopted different variations of them.

An energy transition introduces some challenges for zonal markets. Energy traditionally followed predictable paths from large fossil fuel based generators to load hubs. This simplifies planning. A low-carbon power system, by contrast, requires production from distributed and intermittent renewables sources. This, in turn, creates a changing and uncertain congestion pattern that provides more intrazonal congestion and hence increased redispatch costs. In case studies of the economic consequences of large-scale renewable integration in European zonal markets, both Neuhoff et al. (2013) and Aravena and Papavasiliou (2017) demonstrate an efficiency loss and challenges related to the current scheme.

Efforts have been made to increase the efficiency of zonal markets. Flow-based market coupling, which approximates the grid within zones rather than ignores it, was implemented in the Central Western European markets in May 2015 and further expansions are planned (Van den Bergh et al., 2016). Although this is an improvement over the traditional approach that ignores intrazonal congestion, it approximates the grid and thus reduces rather than eliminates the challenges. The theoretical properties and superior performance by nodal markets over zonal markets in case studies (e.g., Leuthold et al., 2008; Neuhoff et al., 2013; Aravena and Papavasiliou, 2017), raise the question of whether to replace the zonal design with nodal pricing. Still, such a transition is vulnerable to political pushback and it is also a concern to what extent results from models translate to actual operations.

Except for hybrid pricing, an approach proposed by Bjørndal et al. (2014, 2018) where some countries have nodal pricing and others zonal markets, most research on market design considers the benefits and limitations of the two schemes rather than new designs. Paper I of this thesis aims to change this by proposing a novel electricity market design called "flexible electricity bidding zones." This design changes zones according to the most efficient dispatch. As a result, it considers congestion to a larger degree, but maintains the simpler bidding process and few prices of zonal markets.

2.1.2 Market clearing

An associated aspect of market designs is the market clearing. In nodal pricing designs, a system operator performs an economic dispatch that maximizes surplus based on production and transmission constraints. Zonal markets, on the other hand, facilitate an auction where producers and consumers make bids about their preferred quantities and prices. The intersection between supply and demand decides the zonal price and accepted bids. Both clearings are made the day before delivery and are therefore called day-ahead markets. Because market circumstances may change at delivery, for example due to equipment failure or forecast errors in renewable production and demand, the system operators ensure a feasible dispatch through redispatch actions.

The disparity between the day-ahead clearing and redispatch incurs costs to the system. Redispatch actions require response on short notice, which limits the set of available production assets and usually warrants higher costs because flexible peakers, like natural gas fired plants, generally have higher operating costs than baseload. Renewable production is also difficult to forecast in the day-ahead market. Its increased presence in the energy mix enhances differences between day-ahead and delivery and thus increases redispatch costs. As a response, researchers have suggested stochastic market clearings, which clears the day-ahead while considering expected cost over several redispatch scenarios (see, e.g., Wong and Fuller, 2007; Pritchard et al., 2010; Morales et al., 2014; Morales and Pineda, 2017; Kazempour et al., 2018; Bjørndal et al., 2018; Zakeri et al., 2019). A stochastic clearing provides lower expected costs for a given set of uncertain parameters, compared to a deterministic clearing (Bjørndal et al., 2018). Intuitively, this is because the system operator can decide a dispatch that is robust to a larger set of outcomes rather than perform a myopic optimization toward a single forecast. But this feature also causes the main criticism. A stochastic clearing scheme cannot guarantee short-term revenue adequacy and cost recovery. It only ensures them in expectation, although some works, like Morales et al. (2014), Morales and Pineda (2017), and Zakeri et al.

(2019), try to alter the scheme to satisfy the properties. Revenue adequacy ensures that the system operator does not incur losses, while cost recovery makes sure that generators and transmission operators always achieve non-negative profits. Both are therefore fundamental properties for an electricity market.

Paper I of this thesis uses stochastic clearing as the low-cost benchmark. It shows that a zonal stochastic clearing achieves costs similar to a nodal stochastic clearing for most zonal configurations. The reason is that both consider the physical constraints of the grid in the redispatch problem. More importantly, Paper I demonstrates that in a setting with decoupled day-ahead and redispatch markets, flexible electricity bidding zones achieves costs just slightly higher than the nodal stochastic clearing. This is a valuable insight because unlike a stochastic clearing, it can guarantee short-term revenue adequacy and cost recovery. To make these conclusions, paper I introduces a framework that can enumerate all zonal configurations of a power system.

2.2 Oligopolistic electricity markets

While the previous section considers market design, this section investigates the behavior of the participants in the markets. This is important because transmission constraints and high investment costs in production assets create barriers of entry and reduce access to electricity markets. Consequently, electricity markets usually have a limited number of large producers that can impose price-making behavior. Liberalized electricity markets are therefore often characterized as oligopolies (Newbery and Greve, 2017). That is, a market form where a small group of suppliers dominate the market. Market power abuse is thus a serious concern for regulators.

Electricity has several features that provide market power to generators (Joskow, 2008). Limited transmission capacity can lead to congestion and thus limit the area of competition. For example, generators at high-demand areas can actively try to congest import corridors and thus become price-makers within this area. Electricity has low elasticity of demand that generators can exploit to increase prices. Moreover, no cheap large-scale storage technology is available so consumers cannot easily keep storage. Empirical studies have uncovered extensive market power in electricity markets, where the studies on England and Wales (Wolfram, 1999; Sweeting, 2007) and the California electricity crisis (Joskow and Kohn, 2002; Borenstein et al., 2002) are most prominent. Nevertheless, Joskow (2008) summarizes that investigations on New Zealand, Chile, Colombia, the PJM Interconnection, Texas, Alberta, Brazil and some areas of continental Europe have identified various market power issues.

The economic dispatch problem, which the previous section introduced, assumes perfect competition. All market participants are price-takers and bid their marginal costs. The resulting dispatch is equivalent to a benevolent system operator that minimizes the costs while considering production and transmission constraints. In an oligopoly, this is no longer a valid assumption. Producers are price-makers and can maximize their own surplus rather than the system's. Market outcomes are therefore Nash equilibria, a condition that indicates that no participant can gain anything by changing their own strategy.

Equilibrium models aim to find equilibria among decision-makers with individual objectives. Because these models can represent oligopolies, they have been used extensively to investigate electricity markets (Gabriel et al., 2013). Both paper II and III of this thesis consider equilibrium models. Paper II investigates the potential of inverse equilibrium modeling, a data-driven method that combines inverse optimization and equilibrium models. Inverse optimization aims to fit parameters of an optimization problem according to observations of the decision-variables (Ahuja and Orlin, 2001). Similarly, an inverse equilibrium problem aspires to fit parameters of an equilibrium problem according to observations of equilibria. This provides insight on whether an equilibrium model fits the data and it has predictive power.

Paper III applies a stochastic equilibrium problem. Recall that stochastic optimization is a method for analyzing decisions under uncertainty. Specifically, paper III considers risk-averse producers that invest in generation and trade financial instruments to hedge energy price and locational risk with consumers. These decisions are made under the consideration of several possible future scenarios of how the market unfolds. Some scenarios have worse consequences for the market participants than others. Risk-averse agents put more emphasis on minimizing the impact of worst-case outcomes. Paper III incorporates this by using conditional value at risk (CVaR) as a risk measure. Using CVaR, the modeler can put all emphasis on a certain percent of worst outcomes and keep a convex optimization problem (Rockafellar and Uryasev, 2000). As a result, the market participants find equilibria for installed capacities and financial contracts considering risk-adjusted expected costs and contract payouts.

2.3 Financial transmission rights

The wholesale electricity markets that this thesis has covered so far are physical markets that sell energy. But financial markets where producers and consumers can trade financial instruments also exist. Financial instruments can protect a project's downside and provide predictable floor revenues. This, in turn, provides better credit ratings (Prabhu et al., 2017), which reduce the cost of capital. These markets have become important to secure project finance in liberalized electricity markets, as illustrated by merchant investments in gas-fired power plants in the United States (Eberhardt and Szymanski, 2015) or power purchase agreements (PPAs) for renewable generation (Bartlett, 2019; Kobus et al., 2021). Corporations and financial institutions have increasingly replaced utilities as counterparties for offtake contracts (Bartlett, 2019). Unlike utilities, these actors require settlements at liquid hubs rather than project locations. In liberalized U.S. electricity markets, which follow the nodal pricing scheme, this introduces a challenge. Producers receive prices according to the location where they inject energy, but must settle energy price hedges against a hub price different from the one they are exposed to. As a result, they experience a locational risk.

A financial transmission right (FTR), introduced by Hogan (1992), is a financial contract that entitles its holder to the difference in locational marginal prices between two locations. It ensures access to transmission for market participants without interfering with the economic dispatch, which physical transmission rights may. However, as for example Benjamin (2010) outlines, FTRs have multiple purposes in electricity markets. Because FTRs pay the price difference between two locations, they serve as hedges against locational risks and congestion. System operators that coordinate electricity markets accrue merchandising surplus, known as congestion rents, from buying and selling electricity at different prices in the system. As nonprofit entities, the system operators must allocate this revenue back to the grid owners. FTRs are a means to do so. Grid investors, usually ratepayers represented by load serving entities, receive FTRs that they can choose to keep for payouts or sell in an auction for proceeds. An advantage with this arrangement is that FTR holders are exposed to less counterparty risk than similar financial instruments, like a contract for differences, because congestion rents guarantee payouts. The condition where congestion rents are sufficient to cover FTR payouts is called revenue adequacy. For FTR auctions in liberalized U.S. markets, a simultaneous feasibility test ensures that revenue adequacy holds (Alderete, 2013). The final benefit of FTRs is that they provide price signals for market participants.

Even though the FTR is an established instrument to hedge locational risk, there is no evidence from industry that it supports project finance (Eberhardt and Szymanski, 2017). Paper III of this thesis demonstrates that by altering FTRs to longer contract periods that cover a project's lifetime, they reduce the cost of capital and encourage surplus-maximizing investments. This result is contingent on a risk market that also provides hedges for energy price risks. Producers at locations other than the hub combine financial instruments for locational and energy price risk, and thus hedge both risks. Paper III can make these findings because it considers the risk-adjusted expected returns. Better hedges reduce the negative consequence of unfortunate scenarios, and hence improve the risk-adjusted expected return. Risk trading strategies therefore influence the risk-adjusted expected revenue stream that investors use to determine investments. Protection against a project's downside reduces the risk premium demanded by investors and hence influences the cost of capital.

2.4 Hydropower planning

This chapter has so far kept a system perspective and will now change its focus to the decisions of individual producers. The wholesale electricity market is, after all, a construct where individual producers and consumers exchange energy. Following the liberalization of wholesale markets, the producers' objective became to maximize their profits (Wolfgang et al., 2009). With respect to the energy transition, hydropower producers are in a beneficial position because they provide renewable large-scale storage and flexibility that can balance intermittent renewable production (Gullberg, 2013; Egging and Tomasgard, 2018). These advantages also generate complex decision problems. Hydropower producers must allocate resources optimally both short and long term, where the latter introduces a planning horizon of several years into the future (Gjelsvik et al., 2010).

Medium- to long-term hydropower planning is therefore a sequential decision problem under uncertainty. Hydropower producers evaluate present production against opportunities several months or even years in the future. They face uncertain prices and inflow to reservoirs, and must determine whether to accept present prices or store water in anticipation of better prices in the future. Unlike the two-stage stochastic optimization problems in papers I and III, where all uncertainty realizes in the second stage, hydropower planning is a multistage stochastic problem where new information realizes at every stage. The problem can be formulated as a Markov decision process, a framework that can represent sequential decision problems under uncertainty (Powell, 2014).

Multistage stochastic problems are usually exposed to the curse of dimensionality, and hydropower planning is no exception. The state and action spaces are so large that an exact solution becomes intractable in real-world applications. In hydropower planning, both states (the amount of water in reservoirs) and actions (production and spillage decisions) are continuous variables that create infinite combinations of states and actions. Even if a problem discretizes them, it is still computationally intractable to solve at a representative granularity. Decision-makers therefore try to approximate multistage stochastic problems to become tractable to solve while remaining representative of the full problem. This is for example the idea behind methods in approximate dynamic programming (Powell, 2011) and reinforcement learning (Sutton and Barto, 2018). In hydropower applications, the industry standard approach is to formulate a function that approximates the expected value in the future. Stochastic dual dynamic programming (SDDP), introduced by Pereira and Pinto (1991), formulates this as a piece-wise linear function from the Benders cuts of subproblems at each stage. The subproblems represent the decisions producers make based on what revenues they can earn now versus the expected revenues in the future. The approximated expected future value function allows continuous states and actions in the problem. See Pereira and Pinto (1991) or Gjelsvik et al. (2010) for technical details on SDDP.

Hydropower production has no fuel cost, which leads producers to calculate water values that represent the marginal value of an additional unit under an optimal production schedule. When reservoirs approach their maximum capacity, the water value decreases because producers have abundant supply and may risk spillage. Conversely, low reservoir levels indicate limited supply that increase the water value. A hydro-dominated system's reservoir level therefore influences electricity prices. Reservoir levels are in turn affected by inflows. Consequently, inflow to reservoirs influences electricity prices in hydro-dominated systems. Despite this intuitive relationship, the industry standard is to assume independent price and inflow processes when establishing an operational policy (Gjelsvik et al., 2010).

Paper IV of this thesis investigates the cost of assuming independent price and inflow processes when establishing an operational policy. It introduces a price model that includes the co-movements between inflows to reservoirs and electricity prices. Using Markov chain SDDP (Löhndorf and Shapiro, 2019) on a case study with industry data, it finds 0.17% to 0.30% reduction in expected revenues from assuming independent price and inflow processes. These findings are valuable for producers because the indicate that the theoretical differences result in modest additional costs in practice. The paper also identifies theoretical insights, like how a producer that considers co-movements values current water more in the future, and is hence more likely to postpone production and keep higher reservoir levels.

Chapter 3

Contributions

This chapter presents the four papers that constitute the main body this thesis. In addition to a summary, it also describes the papers' scientific contribution and my personal contribution to each paper. The appendix contains the full papers. This chapter also includes an overview of additional scientific contributions that are not part of the thesis.

3.1 Papers

3.1.1 Paper I: Flexible electricity bidding zones

Authors: Simon Risanger, Steffen J. Bakker, Stein-Erik Fleten, and Asgeir Tomasgard

Submitted to an international peer-reviewed journal.

Nodal pricing and zonal markets are the main paradigms for electricity market design in liberalized wholesale electricity markets. Markets based on the zonal design neglect or approximate transmission constraints within zones. This makes them vulnerable to costly redispatch actions when intrazonal congestion occurs. Studies show that large-scale integration of geographically decentralized and intermittent renewable production, which is is necessary to achieve a low-carbon power system, will increase costs. As a response, this paper proposes an alternative market design called "flexible electricity bidding zones." In contrast to existing zonal markets, this scheme changes zonal configuration according to the most efficient dispatch. Consequently, it considers a changing congestion pattern because it can choose configurations where bottlenecks appear between zonal boundaries. Because flexible electricity bidding zones consider the total cost of both a day-ahead clearing and redispatch, it is comparable to a stochastic market clearing. This scheme clears the day-ahead market while considering the expected cost of different redispatch scenarios. The main criticism against stochastic clearing is that it cannot guarantee short-term revenue adequacy and cost recovery. As a result, a producer may be asked to produce at a price lower than a marginal cost or a system operator may not be able to cover its costs. An implementation of flexible electricity bidding zones on a decoupled day-ahead clearing and redispatch provides slightly higher costs than a stochastic clearing but ensures short-term revenue adequacy and cost recovery. Moreover, a transition from a zonal design to a flexible scheme warrants less structural change than to introduce nodal pricing.

This paper contributes to multiple aspects of wholesale electricity market design. Most importantly, it introduces a novel design that decreases costs and is more adapt to the advent of large-scale renewable integration than existing zonal markets. It also keeps the benefits of a simple bidding process and few prices. The comparison and connection to stochastic market clearing combine two research topics that have developed in isolation of each other. By introducing a framework that identifies all zonal configurations, the paper also contributes to research on zonal selection. Existing literature either uses heuristics or solve directly for the best zones, which does not allow the flexibility of models or the ability to investigate all configurations.

My contribution to this paper includes the conceptualization and formulation of the research question. I formulated the mathematical framework, implemented it, made the case study, and acquired input data to perform the experiments. Afterwards, I took the lead on analysis and prepared the original draft. I facilitated the subsequent iterations between co-authors and revised the manuscript.

3.1.2 Paper II: Inverse Equilibrium Analysis of Oligopolistic Electricity Markets

Authors: Simon Risanger, Stein-Erik Fleten, and Steven A. Gabriel

Published as Risanger, S., S.-E. Fleten and S. A. Gabriel (2020). Inverse equilibrium analysis of oligopolistic electricity markets. *IEEE Transactions on Power Systems 35*(6), 4159-4166. doi: 10.1109/TPWRS.2020.2993070.

Wholesale electricity markets are usually modelled as an oligopoly due to features like transmission constraints, high investment costs, and limited amount of large producers. Researchers therefore frequently use equilibrium models, which can represent an oligopoly, to investigate electricity markets. This paper investigates inverse equilibrium models, a method that combines equilibrium models and inverse optimization. While inverse optimization fits parameters to observable decisions, inverse equilibrium models fit parameters to observable equilibria. Inverse equilibrium modeling is a data-driven method that can assess whether a market structure fits observations and it has predictive power. This paper introduces a novel methodology that exploits Karush-Kuhn-Tucker conditions when it formulates inverse equilibrium problems. Complementary problems expressed by Karush-Kuhn-Tucker conditions are widely used in the power system modeling community, and these models can transform into inverse models with little additional modeling effort. The paper illustrates this on established Nash-Cournot games between price-making producers. It also demonstrates and discusses how inverse equilibrium models provide generally inconsistent estimation. Econometric approaches are often better suited for this purpose.

The main contribution of this paper is to demonstrate how to formulate inverse equilibrium models from relaxed stationarity conditions from Karush-Kuhn-Tucker conditions. It illustrates how to transform existing complementarity models in the power system literature to inverse equilibrium models. Through two case studies, the paper demonstrates the advantages and caveats of inverse equilibrium models. For instance, how the data-driven method can assess the fit of model structures, but is an inconsistent estimator. In a similar vein, the paper discusses the similarities and differences between inverse equilibrium models and related machine learning and econometric approaches.

My contribution to this paper includes the conceptualization and formulation of the research question. I formulated the models, implemented them, made the case study, and performed the experiments. Afterwards, I took the lead on analysis and prepared the original draft. I facilitated the subsequent iterations between co-authors and revised the manuscript.

3.1.3 Paper III: Congestion risk, transmission rights, and investment equilibria in electricity markets

Authors: Simon Risanger and Jacob Mays

Submitted to an international peer-reviewed journal.

Investors in production assets depend on financial instruments to hedge against uncertain revenue streams from volatile wholesale electricity prices. A risk trading strategy provides predictable revenue streams and better service of debt, which means that projects receive better credit ratings and lower costs of capital. Liberalized U.S. electricity markets have locational marginal prices where producers receive a price according to where they inject energy. Financial instruments are increasingly offered by corporations and financial institutions, who want to settle contracts at liquid hubs. Producers are therefore exposed to locational risks. Financial transmission rights (FTRs) are contracts that pay the price difference between two locations. Despite FTRs' ability to hedge locational risk, industry reports no evidence that they support project finance. This paper uses a stochastic equilibrium model where risk-averse producers invest in installed capacity and trade financial instruments to investigate this phenomenon. It shows that combining energy price hedges with FTRs over the project's lifetime, in contrast to the current maximum duration of three years, encourages surplus-maximizing investments. Producers are thereby protected against both types of risk. Producers outside the hub use FTRs extensively and consequently receive lower cost of capital. Large-scale renewables tend to be geographically remote, and proper management of locational risk is important to encourage investments in these assets.

This paper contributes with a framework that can investigate the impact of FTRs and other energy price hedges on generation investments in an electricity market with network constraints. Using this framework, we contribute with practical policy insights. Incomplete risk markets, including instruments for locational risk, lead to suboptimal investments. An FTR-specific remedy is to provide contracts over longer duration, preferably over the project's lifetime, so producers achieve improved hedges and lower cost of capital. The management of locational risk is especially important for renewable projects because they tend to be geographically remote and increasingly secure energy price hedges, like power purchase agreements (PPAs), on liquid hubs.

My contribution to this paper includes the conceptualization and formulation of the research question together with Jacob Mays. I formulated the models, implemented them, and performed the experiments. Afterwards, I analyzed the results together with Jacob Mays and prepared the original draft. I facilitated the subsequent revisions of the manuscript with the co-author.

3.1.4 Paper IV: Co-movements between forward prices and resource availability in hydro-dominated electricity markets

Authors: Andreas Kleiven, Simon Risanger, and Stein-Erik Fleten

Submitted to an international peer-reviewed journal.

In liberalized wholesale electricity markets, hydropower producers calculate a water value based on current and estimated future revenues. They must evaluate whether to produce now or store water in aspiration of better prices in the future, while considering uncertain inflow to reservoirs. If hydropower production dominates a system, the amount of water in the system's reservoirs determines the supply, which again affects the water values. Yet the industry standard is to neglect this relationship and assume independent price and inflow processes when establishing operational policies. This paper implements the state-of-the-art stochastic dual dynamic programming method for hydropower planning and trains policies on a novel price process that considers co-movements in prices and inflow. The multistage model reinforces the results of a simpler two-stage setting. Producers that consider co-movements expect low prices during high-inflow situations, which make them value current water more in the future. They therefore prefer slightly higher reservoirs and are more prone to postpone production, and risk more spillage. Producers that ignore the correlation undervalues current water. On data for a Norwegian hydropower producer, the paper finds 0.17% to 0.30% reduction in expected revenues for a producer that establish an operational policy without considering the co-movement in price and inflow. The results suggest that, despite theoretical differences, the current industry practice only incurs modest extra costs in practice. Although slim in relative terms, the savings can accumulate to large absolute values for sizable hydropower plants.

This paper contributes with a novel price process that considers the co-movement between prices and inflows. The process includes both local and system hydrological states, which influence each other and provide insight about the system supply and thereby prices. The paper's application is of significant industry interest. It examines the impact of assuming independent price and inflow processes both analytically and on a realistic case study on industry data. The paper provides evidence that current industry practice of assuming independent prices and inflows when establishing an operational policy only incurs modest additional costs, despite theoretical differences. In addition, the paper also outlines general insights through a two-stage example. Thus we extract general conclusions about the difference in policies that consider co-movements in price and inflow.

My contribution to this paper was to formulate, implement, pre-process, and perform experiments on the hydropower planning problem. This includes making the Markov chain from Monte Carlo simulations from the price process and implement stochastic dual dynamic programming to train policies. Andreas Kleiven was responsible for the price process and two-stage example. Both prepared the original draft and contributed to subsequent reviews and edits. We also verified and discussed each other's work and contributed to analysis.

This paper will also be included in Andreas Kleiven's PhD dissertation.

3.2 Additional contributions

In addition to the papers presented in Section 3.1, I also performed research on COVID-19 response that is not part of this thesis. Together with co-authors from the UT Austin COVID-19 Modeling Consortium, we investigated how to select pharmacies and United States Postal Service (USPS) facilities to ensure access to COVID-19 tests. The research was published in the following papers:

- Risanger, S., B. Singh, D. Morton, L. A. Meyers (2021). Selecting pharmacies for COVID-19 testing to ensure access. *Health Care Management Science*. doi: 10.1007/s10729-020-09538-w.

- Bismark, S., S. Risanger, D. Morton, M. Pignone, L. A. Meyers (2021). Expanding access to COVID-19 tests through US Postal Service facilities. *Medical decision making 41*(1), 3-8. doi: 10.1177/0272989X20969690.

Chapter 4

Reflections and further research

The papers in this thesis consider the objectives of improving electricity market efficiency and supporting the integration of renewables. Yet, they make contributions to both methods and policy in a diverse range of topics. The variety stems from an ambition to combine both disruptive research, like a novel market design in paper I or method in paper II, and incremental advances with potential for short-term practical impact, like investigating FTRs in paper III or operational policies for hydropower producers in paper IV. This chapter provides reflections on the papers, including their merits and limitations. It also provides suggestions for further research.

Paper I introduces the concept of flexible electricity zones. The intuition behind the idea is solid; several existing studies have investigated the benefits of selecting optimal zones. The paper raises the question of why these optimal zones needs to be fixed. Different system states will naturally have different optimal zones. In theory, flexible zones have only upsides from a cost perspective. If a fixed zone is indeed the best, the flexible zones would just take this form. Although the theoretical and system benefits are clear, we need to assess whether this holds in practice. In what manner does flexible zones impact the different stakeholders and how will they respond? Producers have years of experience on how to operate profitably under the current market scheme. Even though a scheme with flexible zones can still have the same auction structure, market participants need to reassess their bidding procedure. Political pushback is also a topic of practical concern. Advocates of zonal markets argue for the democratic principle of equal prices for all consumers within a zone. Flexible zones could increase the frequency where two locations close to one another experience different prices. Another practical concern is operations. System operators need to find feasible ways to assess multiple zonal configurations, clear the market, and organize cross-border capacity with neighboring zones. In other words, a structural change like a new market design impacts multiple stakeholders in various respects. The concerns mentioned here are just some among many. Further research on the practical implementation of flexible zones is necessary.

Being a novel market design, flexible electricity bidding zones provide multiple avenues for further research. Some are computational, like how to identify zonal configuration effectively in real-world power systems. The enumeration approach presented in the paper is not scalable to large systems. Moreover, how can system operators efficiently combine market clearing and zonal selection? Economic factors also warrant further investigation. Examples are how prone flexible zones are to abuse of market power, the cost-benefit allocation among stakeholders, and long-term price signals for investments. Even though the twelve-node case study in the paper shows promising results, more realistic ones are necessary. Both in terms of modeling detail and system size.

A main question surrounding flexible electricity bidding zones is that if a system has to undergo major structural changes regardless, why not go for nodal pricing? Although nodal pricing is the academic gold standard for congestion management, practical and political concerns surround its practical implementation. After all, European markets are still hesitant, despite successful implementations in the United States. The viability of flexible electricity bidding zones then depends on whether countries decide to discard existing zonal markets, but do not want to move to nodal pricing. As discussed in paper I, flexible zones share some beneficial traits with the current zonal scheme. Still, nodal pricing has been applied and tried in practice, something flexible zones have not. Regulators may therefore associate more risk with flexible electricity bidding zones.

Paper II investigates inverse equilibrium models, a topic with limited literature. The paper serves more as an inquiry to the method than a promotion. Although the method has merits, like being data-driven, can fit model structures to observations, and has predictive power, it also has associated challenges. The method shares features with both machine learning and econometric approaches but does not excel in neither field. Its increased interpretability compared to machine learning is a promising ability, but its practical applicability is limited by equilibrium models. Constraint qualifications necessary to find unique equilibria make equilibrium models less representative of real-world applications. For instance, an equilibrium model, and hence an inverse equilibrium model, cannot consider unit commitment decisions or an AC representation of the grid. These are important characteristics of a power system. In general, the impact of inverse equilibrium modeling is limited by the ability of equilibrium models to represent actual market conditions.

Considering the challenges associated with the inverse equilbrium modeling, paper II aims to be an impartial guide to the method for the power systems community. Further research should focus on either improving prediction or estimating parameters, not accomplishing both at the same time. Inverse optimization is considering how to include noisy observations (e.g., Aswani et al., 2018; Thai and Bayen, 2018; Aswani, 2019), which can serve as inspiration to improve the estimation ability of inverse equilibrium models.

Paper III has potential for short-term practical impact. It verifies statements from industry that calls for long-term protection against locational risks. A slight modification of FTRs may accomplish this and introduce more efficient investments. Practical challenges are also associated with this approach. Notably, all FTRs must satisfy the simultaneous feasibility test that guarantees revenue adequacy. This test limits the FTR supply by ensuring that congestion revenue covers all payouts. Potential long-term FTRs must also satisfy this condition along with FTRs of other contract durations. Paper III uses FTRs because they are established instruments to hedge price risks. Still, the important requirement for investors is to hedge locational risk, not necessarily the specific instrument.

Locational risk and its impact on generation investments have not received much academic scrutiny, which indicates potential for further research. The framework in Paper III provides a first step, but it also experienced computational challenges related to convergence. This may prevent investigation of more realistic case studies. Alternative approaches, for example a multi-agent system, must be considered. Other means to protect against locational risk is important from a policy perspective. This may include other types of financial contracts, like contract for differences, that are not constrained by the simultaneous feasibility test. Note, however, that the simultaneous feasibility test has advantages by ensuring FTR payouts through congestion rents. This reduces counterparty risk. Finally, it is also worthwhile to consider alternative allocation schemes. Generators could receive FTRs directly instead of participating in an auction. This can ensure that projects are protected against locational risk in the long term. However, such schemes must consider how they impact transmission financing.

Paper IV is operational and has the least barrier to implementation of the papers in this thesis. It mainly requires hydropower producers to alter their price modeling and pre-processing before establishing an operational policy. The work is connected to the Norwegian Research Centre for Hydropower Technology (HydroCen), which includes industry partners with interest for the results. Consequently, the paper includes a case study on industry data and detailed modeling similar to that in industry. Its findings are of direct relevance to all hydropower producers in hydro-dominated systems.

Future research from paper IV is to investigate cost savings over a larger set of case studies to get a general sense of the benefits of modeling co-movements. The price model is complex and requires a deliberate calibration procedure. This may discourage industry actors to incorporate it in their workflow. It is therefore worth investigating whether simpler models, like just using a correlation coefficient between prices and inflows, can reproduce similar results. This will reduce the barrier for industry adoption.

Chapter 5

Conclusion

A goal to limit global warming to below 1.5°C requires an unprecedented energy transition. Innovations in renewable technologies indicate that an energy transition in power systems is possible. At the same time, it is important to facilitate a transition within a system that maximizes social surplus. These objectives introduce a range of challenges to stakeholders. This thesis consists of four papers that consider some of them.

Paper I takes a system perspective and considers how wholesale electricity market designs can assist the integration of large-scale renewables. Zonal markets, that neglect or approximate congestion within zones, are not particularly suited to distributed and intermittent production. As a response, Paper I proposes a novel design called "flexible electricity bidding zones," which alters zonal configuration according to the most efficient dispatch. When it clears the day-ahead market separately from redispatch, flexible electricity bidding zones achieve just slightly higher costs than stochastic nodal clearing but guarantee short-term revenue adequacy and cost recovery.

Continuing a system perspective, paper II investigates inverse equilibrium models as a means to study oligopolistic electricity markets. This data-driven method fits observations of market outcomes to model structures. It shares traits with machine learning and econometric approaches. Still, its performance depends on whether a power system can be described by an equilibrium model and its parameter estimation is generally inconsistent.

Paper III investigates project finance and risk trading. In liberalized U.S. electricity markets, producers receive a price according to where they inject energy. Geographically remote producers, like large-scale renewables tend to be, are exposed to locational risk because energy price hedges usually settle at liquid hubs. These contracts are important to provide predictable revenue streams and secure funding. Paper III demonstrates how extending the duration of financial transmission rights to project lifetimes improve investment incentives when they are combined with energy price hedges. Complete risk markets promotes investments that increase the society's surplus.

Finally, paper IV takes the perspective of hydropower producers, which provide necessary storage to balance intermittent renewables in an energy transition. A common industry assumption in hydro-dominated system is to ignore how inflow to reservoirs influences prices when establishing an operational policy. Hydropower producers therefore underestimate their water values. Paper IV introduces a novel price model that includes co-movements in prices and inflows. An operational policy that considers this co-movement will value current water more in the future. This leads to higher reservoir trajectories and an inclination to postpone investment. Despite theoretical differences, a case study on industry data indicates modest cost savings in practice.

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Papers

Paper I: Flexible electricity bidding zones

Authors: Simon Risanger, Steffen J. Bakker, Stein-Erik Fleten, and Asgeir Tomasgard

Submitted to an international peer-reviewed journal.

This paper is awaiting publication and is not included in NTNU Open

Paper II: Inverse Equilibrium Analysis of Oligopolistic Electricity Markets

Authors: Simon Risanger, Stein-Erik Fleten, and Steven A. Gabriel

Inverse Equilibrium Analysis of Oligopolistic Electricity Markets

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***Abstract*—Inverse equilibrium modeling fits parameters of an equilibrium model to observations. This allows investigation of whether market structures fit observed outcomes and it has predictive power. We introduce a methodology that leverages relaxed stationarity conditions from Karush-Kuhn-Tucker conditions to set up inverse equilibrium problems. This facilitates reframing of existing equilibrium approaches on power systems into inverse equilibrium programs. We illustrate the methodology on network-constrained and unconstrained Nash-Cournot games between price-making power generators. The inverse equilibrium problems in this paper reformulate into linear programming problems that are flexible and interpretable. Still, inverse equilibrium modeling provides generally inconsistent estimation and econometric approaches are better for this purpose.**

***Index Terms*—Inverse equilibrium, inverse optimization, equilibrium modeling, electricity markets.**

I. Introduction

DESPITE the liberalization of electricity markets, features such as a limited amount of large producers, high investment costs, and transmission constraints may cause price-making behavior, barriers of entry, and reduce access to markets. As a result, the markets are vulnerable to abuse of market power. Equilibrium models, which represent these oligopolistic tendencies, are therefore widely used to study electricity markets [1].

When we study actual energy markets, it is generally easy to observe the equilibrium outcomes, such as prices and flows. The theoretical development in inverse equilibrium modeling [2], [3] leverages this fact. The framework expands the theory of inverse optimization [4], which fit parameters of an optimization problem given observations of decision variables. As a result, we can use actual data to analyze markets and participant behavior to a greater extent.

Recent literature shows an increased interest from the power systems community in inverse optimization. Applications include the investigation of price response of consumers [5], [6], estimation of offer prices from rival producers [7], and investigation of the parameters of transmission constraints in electricity markets based on locational marginal prices [8]. Relevant work on inverse equilibrium models include [9] and [10], which use the variational inequality approach of [3] to estimate bid curves of competing firms that employ strategic bidding.

Expanding the literature cited above, we show how to use a Karush-Kuhn-Tucker (KKT) representation [11] to formulate inverse equilibrium models. This allows existing equilibrium models from KKT formulations to be rearranged into inverse problems. Although [2] also considers inverse nonlinear complementarity problems, their approach requires initial estimates of parameters. Our methodology follows the idea of [3] and [11], where they minimize relaxed optimality conditions. As a result, we can apply our observations directly and solve the inverse equilibrium problem as an optimization problem.

Considering the rich history of equilibrium modeling in the power system community, it is natural to assume that inverse equilibrium modeling can be a valuable tool. While this is true to some extent, the approach also has limitations. The goal of this paper is to highlight both strengths and weaknesses of inverse equilibrium to modelers who consider using this method. Our contributions are the following:

- We develop a method to fit objective function coefficients of participants in a power system by inverting an equilibrium model from KKT conditions.
- We explain how inverse equilibria relate to similar concepts in econometrics and machine learning.
- We invert a Nash-Cournot game of transmission-constrained and unconstrained electricity markets.
- We use examples to illustrate how inverse equilibrium fits models and describe its performance in the presence of noise.
- We discuss performance, implementation, and challenges of inverse equilibrium models, as illustrated

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by our examples.

The remainder of this paper is as follows: Section II outlines how inverse equilibrium modeling relates to econometrics and machine learning. Section III provides an introduction to equilibrium models from KKT conditions and explain how to utilize stationarity conditions to invert the problem. We apply the method on relevant examples in Section IV. Section V addresses implementation challenges, while Section VI concludes the paper.

II. Relationship to econometrics and machine learning

At first glance, inverse equilibrium modeling may seem like another addition to the literature on structural econometrics [12]. Several econometric studies exist on electricity markets, especially intending to expose market power (see e.g. [13], [14] and [15]). However, the major difference is that inverse equilibrium modeling is a completely data-driven method. As a result, we make no assumptions on the distribution of our observations. Rather, we try to fit an equilibrium model of a market structure and see whether it fits the data well or not. Econometric estimation, on the other hand, assumes that there is an underlying population, which our sample data should reasonably represent, and tries to estimate true parameters of the population. This gives greater explanatory power than inverse equilibrium modeling. The cost, however, is careful data collection and estimator formulations. For instance, the estimators require that data comply with certain attributes, a traditionally prominent example is the Gauss-Markov assumptions, to enjoy statistical properties like unbiasedness and consistency. Although state-of-the-art econometrics have non-parametric estimation methods and approaches to handle challenges such as heteroskedasticity, serial correlation and endogeneity, the field nevertheless require a set of assumptions on the data in order to infer from it. Estimations from inverse equilibrium modeling, which do not require these assumptions, do thereby not share these properties. We illustrate this by example in Section IV. For an example of structural estimation in power systems see [16], for an overview on econometric methods, see e.g. [17] or [18]. In addition, [3, Appendix 2] discusses the relationship between inverse equilibrium modeling and structural estimation, while [19] suggest poor accuracy from estimation by first-order conditions within a conjectural variations framework of an oligopolistic electricity market.

Inverse equilibrium modeling relates more to a machine learning philosophy, which values prediction over explanation, than econometrics. However, inverse equilibrium modeling adds more structure than a pure machine learning predictor. Most notably, inverse equilibrium modeling has a strong prior. We believe that a certain equilibrium market structure is the basis for the observations and want to see whether or not this is correct. If an inverse model is a good fit to the data, we can insert the fitted parameters in the original problem to obtain good predictive power [3]. Although this is a nice feature, we limit this paper to only consider formulating and solving inverse equilibrium problems, and refer the interested reader to [10] and [20] that use inverse optimization for prediction.

From the discussion, we see that inverse equilibrium complements existing econometrics and machine learning methods. We emphasize that inverse equilibrium modeling is generally an inconsistent estimator. Even if we get interpretable fitted parameters, such as costs or willingness-to-pay, we cannot conclude with confidence that they represent those of an underlying market. They are merely a good fit. If the goal of a study is to estimate true market parameters, econometric approaches should be used. That being said, inverse equilibrium modeling has several advantages:

- We require no assumptions on the input data.
- Inverse equilibrium modeling is flexible, and one can easily add or remove constraints and alter the problem.
- The problem often rearranges into a tractable linear programming problem.
- One can obtain estimates for other values than objective function coefficients, for instance coefficients of transmission constraints as shown in [8].
- By using the KKT approach of this paper, it is simple to invert mixed complementarity models.
- Inverse equilibrium models have more structure than pure machine learning predictors, which increases interpretability.

III. Inverse equilibrium modeling

A. Equilibrium models

We consider a set of decision-makers, $\mathcal{P} = \{1, \ldots, |\mathcal{P}|\}$, where each player $p \in \mathcal{P}$ has an optimization problem illustrated by (1). Functions f_p, g_{pi}, and h_{pj} may be different or similar for the different decision-makers. Moreover, θ, ϕ and ψ denote the parameters of the respective functions. Notice that the objective (1a) is dependent on $x_{-p} = (x_k)_{k \in \mathcal{P} \setminus p}$, which denotes the decisions of the other players, in addition to its own decision variable vector x_p. The problem can be restricted by inequality constraints $i \in \mathcal{I}$ and equality constraints $j \in \mathcal{J}$. Because restrictions (1b) and (1c) do not depend on x_{-p}, they are internal constraints for player p. Finally, we note that λ_{pi} and ν_{pj} represent the dual variables of constraints (1b) and (1c), respectively.

$$\begin{aligned} \min_{x_p} \quad & f_p(x_p, x_{-p}; \theta_p, \theta_{-p}) && && \text{(1a)} \\ \text{s.t.} \quad & g_{pi}(x_p; \phi_p) \leq 0, \quad && (\lambda_{pi}) \quad i \in \mathcal{I} && \text{(1b)} \\ & h_{pj}(x_p; \psi_p) = 0, \quad && (\nu_{pj}) \quad j \in \mathcal{J} && \text{(1c)} \end{aligned}$$

The decision-makers cannot optimize their own problem without considering the responses of the other players. Solving all $p \in \mathcal{P}$ problems simultaneously leads to an equilibrium problem. Both variational inequalities (VIs) and mixed complementarity problems (MCPs) are paradigms to model the simultaneous solution of these player-specific problems. VIs are based on considering the variational principle related to non-negative directional derivatives for feasible directions (to minimization problems). MCPs rely on the KKT conditions and involve both primal and dual variables, which has a modeling advantage in some cases [1]. We only consider MCPs in the remainder of this paper.

We assume that problem (1) for all $p \in \mathcal{P}$ satisfies a constraint qualification that makes the KKT conditions necessary. The KKT conditions are sufficient, for example, when f_p is convex (concave for a maximization problem) while g_{pi} and h_{pj} are affine. A solution that satisfies the KKT conditions (when these conditions are sufficient) is thus an optimal solution of (1). Likewise, a solution that simultaneously satisfies the KKT conditions for all $p \in \mathcal{P}$, as shown in (2), is an equilibrium solution.

$$\begin{aligned} & \nabla_{x_p} f_p(x_p, x_{-p}; \theta_p, \theta_{-p}) + \sum_{i \in I} \lambda_{pi} \nabla_{x_p} g_{pi}(x_p; \phi_p) \\ & + \sum_{j \in J} \nu_{pj} \nabla_{x_p} h_{pj}(x_p; \psi_p) = 0, \quad p \in \mathcal{P} \end{aligned} \qquad \text{(2a)}$$

$$\begin{aligned} g_{pi}(x_p; \phi_p) &\leq 0, \quad i \in \mathcal{I}, p \in \mathcal{P} && \text{(2b)} \\ h_{pj}(x_p; \psi_p) &= 0, \quad j \in \mathcal{J}, p \in \mathcal{P} && \text{(2c)} \\ \lambda_{pi} &\geq 0, \quad i \in \mathcal{I}, p \in \mathcal{P} && \text{(2d)} \\ \lambda_{pi} g_{pi}(x_p; \phi_p) &= 0, \quad i \in \mathcal{I}, p \in \mathcal{P} && \text{(2e)} \end{aligned}$$

B. Inverse equilibrium models

Problem (2) assumes that parameters, θ, ϕ and ψ, are fixed and seeks a solution satisfying all the conditions. By contrast, inverse equilibrium modeling is the reverse-engineering direction to this. Namely, given an equilibrium solution, it seeks to find the parameters θ, ϕ and ψ that best fit the observed solution. The equilibrium outcomes, represented by the decision variables $x_1, \ldots, x_{|\mathcal{P}|}$ become fixed observations, and thus parameters, $\tilde{x}_1, \ldots, \tilde{x}_{|\mathcal{P}|}$, in the inverse problem. Our method is similar to [11], which applies KKT relaxations to convex optimization problems.

We allow stationarity conditions (2a) to be relaxed, while constraints (2b) to (2e) must hold. A deviation from (2a) results in near-equilibrium solutions, but outcomes are still feasible when (2b) to (2e) hold. We can thus relax the stationarity condition by deviation ϵ_p, as shown in (3), to create a near-equilibrium solution. This allow the inverse model to consider observations that are not necessarily optimal strategies for its assumed model. Note that the deviations are not independent because we relax the stationarity condition, which includes decision variables of the other problems.

$$\begin{aligned} \nabla_{x_p} f_p(x_p, x_{-p}; \theta_p, \theta_{-p}) - \sum_{i \in I} \lambda_{pi} \nabla_{x_p} g_{pi}(x_p; \phi_p) \\ - \sum_{j \in J} \nu_{pj} \nabla_{x_p} h_{pj}(x_p; \psi_p) = \epsilon_p \end{aligned} \qquad \text{(3)}$$

We assume that observations come from rational players, and thus are optimal decisions in the actual market. The inverse equilibrium problem (4) therefore seeks to minimize the vector norm of these deviations, $\|\boldsymbol{\epsilon}\|$ where $\boldsymbol{\epsilon} = \{\epsilon_p : p \in \mathcal{P}\}$. This fits the parameters in a manner where the observations are as optimal as possible for the assumed model. Recall that observations $\tilde{x}_1, \ldots, \tilde{x}_{|\mathcal{P}|}$ are parameters in the inverse problem. The dual variables, λ_{ki} and ν_{kj}, become parameters if they are observable. A notable example is prices, which are dual variables of market-clearing constraints and observable at the power exchange. If unobservable, the dual variables continue to be decision variables, which we assume for the remainder of the paper. The parameters we want to fit, for instance cost coefficients, slopes or intercepts of inverse demand functions, also become decision variables.

$$\min_{\boldsymbol{\epsilon}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{\phi}} \quad \|\boldsymbol{\epsilon}\| \qquad \text{(4a)}$$

$$\begin{aligned} \text{s.t.} \quad & \nabla_{x_p} f_p(\tilde{x}_p, \tilde{x}_{-p}; \theta_p, \theta_{-p}) \\ & - \sum_{i \in I} \lambda_{pi} \nabla_{x_p} g_{pi}(\tilde{x}_p; \phi_p) \\ & - \sum_{j \in J} \nu_{pj} \nabla_{x_p} h_{pj}(\tilde{x}_p; \psi_p) = \epsilon_p, \quad p \in \mathcal{P} \end{aligned} \qquad \text{(4b)}$$

Constraints (2b) to (2e)

Depending on the number of variables that are observable and how many parameters we try to fit, there may be several optimal solutions for (4). With respect to interpretability, we want the solution space as small as possible. We can achieve this by adding constraints, getting observations for variables and fitting fewer parameters. Several different observations also increase the probability of having marginal observations, i.e. observations that reveals some limit of the variables. This reduces scale invariance, which is the situation where the fitted parameters has a range of optimal solutions.

We therefore introduce $h \in \mathcal{H} = \{1, \ldots, |\mathcal{H}|\}$ as index for different observations. For instance, the electricity market outcomes for multiple hours or days. We introduce observations $\tilde{x}_{1h}, \ldots, \tilde{x}_{|\mathcal{P}|h}$ into the inverse equilibrium problem and minimize the deviation at each observation, ϵ_{ph}, constrained to (4b) and (2b) to (2e) for all observations.

The inverse equilibrium problem has several convenient computational properties compared to ordinary equilibrium problems. Complementarity constraints of equilibrium problems are non-convex, and thus computationally challenging for large instances. When decision variables become fixed observations, they cease being variables. If an observed variable is part of a bilinear term, the term becomes linear. If one wants to fit parameters in an inequality constraint, i.e. ϕ, complementarity conditions can arise because we multiply ϕ with the dual variable λ in constraint (2e). However, this is not an issue if we do not need to estimate ϕ or if we have observations of its corresponding dual variable $\tilde{\lambda}$.

Objective function (4a) minimizes the distance from the objective and can be represented by any norm. An L_1-norm (the sum of absolute values) or L_∞-norm (the single largest magnitude in a vector) linearizes the inverse equilibrium objective. For the examples in Section IV, we use the L_1-norm. If the constraints are affine, then (4) becomes a linear programming problem. Consequently, we are able to solve much larger instances of inverse equilibrium problems than equilibrium problems.

C. Pre-process data to reduce problem size

Although we can solve the inverse equilibrium problem in its original form (4), pre-processing data reduces problem size and decreases the risk of numerical complications. Take for instance restriction (2e):

$$\lambda_{pi} g_{pi}(x_p; \phi_p) = 0.$$

Given an observation $\tilde{x}_p$ and we know ϕ, then we know the value of $g_{pi}(\tilde{x}_p; \phi)$, which now becomes a parameter in the problem. If $g_{pi}(\tilde{x}_p; \phi) = 0$, we can omit restriction (2e), because we know it is satisfied. Similarly, if $g_{pi}(\tilde{x}_p; \phi) \neq 0$, we can set $\lambda_{pi} = 0$ instead of the numerically more complicated (2e). In addition, non-negativity constraint (2d) becomes redundant.

IV. Illustrative examples of inverse equilibrium models

To illustrate the computational aspects of solving inverse equilibrium problems, we introduce two Nash-Cournot games where strategic generators use market power to maximize profits. Throughout the section, we use the PATH solver [21] in GAMS to solve the equilibrium problems, while we implement the inverse equilibrium problems, which become linear programming problems, in the Pyomo package for Python and solve with the Gurobi solver.

A. Generic Nash-Cournot game

1) Model formulation: First we consider a generic Nash-Cournot game between $p \in \mathcal{P}$ price-making generators with finite capacity. They supply a price-sensitive load without any transmission constraints. Generation is denoted x_p, and has a marginal cost c_p, as described by optimization problem (5). Each generator tries to maximize its profits, given by objective function (5a). A linear inverse demand function with slope $a \geq 0$ and intercept $b \geq 0$ determines the price. We include ξ as a demand shock that increases or decreases the demand intercept. In actual application, there is significant uncertainty regarding ξ. We include it merely to generate different observations for the case study. A generator cannot exceed its maximum generation capacity Q_p^{max}, as enforced by (5b), and generation is non-negative. Finally, μ_p denotes the dual variable of the maximum generation restriction.

$$\max_{x_p} \quad -c_p x_p + \left(b + \xi - a \sum_{k \in \mathcal{P}} x_k\right) x_p \tag{5a}$$

$$\text{s.t.} \quad x_p \leq Q_p^{max} \qquad (\mu_p) \tag{5b}$$

$$x_p \geq 0 \tag{5c}$$

We formulate the KKT conditions of (5) as described in Section III-A. The objective (5a) is concave and constraints (5b) and (5c) are affine, so the KKT conditions (6) are necessary and sufficient to represent a global optimum of (5). The market equilibrium is the set of $x_1, \ldots, x_{|\mathcal{P}|}$ that satisfy (6) for all players, where the perp operator $\perp$ signifies that the product of the constraints on both sides of the operator must equal zero.

$$0 \leq c_p - b - \xi + a\left(x_p + \sum_{k \in \mathcal{P}} x_k\right) + \mu_p \quad \perp \quad x_p \geq 0 \tag{6a}$$

$$0 \leq -x_p + Q_p^{max} \quad \perp \quad \mu_p \geq 0 \tag{6b}$$

We apply the option to deviate by ϵ_h from the stationarity condition (6a), as explained in Section III-B, and use several observations $h \in \mathcal{H}$. Each observation differs by realizations of the demand shock ξ_h. Equation set (7) becomes the inverse equilibrium problem, where the objective function (7a) is to minimize the distance to an equilibrium point considering all observations.

$$\min_{\boldsymbol{c},a,b,\boldsymbol{\mu},\boldsymbol{\epsilon}} \quad \|\boldsymbol{\epsilon}\| \tag{7a}$$

$$\text{s.t.} \quad \Big(c_p - b - \xi_h + a\Big(x_{ph} + \sum_{k\in\mathcal{P}} x_{kh}\Big) + \mu_{ph} + \epsilon_{ph}\Big)x_{ph} = 0, \quad p\in\mathcal{P}, h\in\mathcal{H} \tag{7b}$$

$$0 \leq c_p - b - \xi_h + a\Big(x_{ph} + \sum_{k\in\mathcal{P}} x_{kh}\Big) + \mu_{ph} + \epsilon_{ph}, \quad p\in\mathcal{P}, h\in\mathcal{H} \tag{7c}$$

$$(-x_{ph} + Q_p^{max})\mu_{ph} = 0, \quad p\in\mathcal{P}, h\in\mathcal{H} \tag{7d}$$

$$\mu_{ph} \geq 0, \quad p\in\mathcal{P}, h\in\mathcal{H} \tag{7e}$$

2) Illustrative case study: To illustrate the inverse Nash-Cournot game, we create a case study where we consider three price-making electricity generators. All have a maximum production of $Q_p^{max} = 5000MWh$ and their marginal costs are $c_1 = 50.0€/MWh$ and $c_2 = c_3 = 60.0€/MWh$. Their collective consumers are represented by a linear inverse demand function with slope $a = 0.01€/MWh^2$ and intercept $b = 200.0€/MWh$. We insert these values into the equilibrium problem (6) and solve. The Nash-Cournot equilibrium is $x_1 = 4250MWh$ and $x_2 = x_3 = 3250MWh$ when the demand shock $\xi = 0$.

We solve equilibrium problem (6) a hundred times to produce observations $\tilde{x}_{1h}$, $\tilde{x}_{2h}$, and $\tilde{x}_{3h}$. Each observation has a different demand shock ξ_h selected at random from a normal distribution with mean of 0 and standard deviation $20€/MWh$. We thus have $|\mathcal{H}| = 100$ different observations.

The inverse generic Nash-Cournot game (7) takes observations $\tilde{x}_{1h}$, $\tilde{x}_{2h}$, and $\tilde{x}_{3h}$ as parameters and solves for c_1, c_2, c_3, a, b, $\boldsymbol{\mu}$, and $\boldsymbol{\epsilon}$. We assume that the demand shocks $\tilde{\xi}_h$ are known and thus parameters as well. Note that this is not a realistic assumption, but prevents noise in the example, which is a topic we consider in Section IV-A4.

The objective value of (7a) becomes $8 \cdot 10^{-5}$, so sufficiently small to indicate that the model fits the data. Slope a is correctly fitted to $0.01€/MWh^2$, but some deviation occurs for $b = 150.0€/MWh$, $c_1 = 0.0$, and $c_2 = c_3 = 10.0€/MWh$. All deviations are fitted exactly $50.0€/MWh$ less than the original value, so we have a case of scale invariance. Whenever we are dealing with a market, we can use price observations $\tilde{\lambda}_h$. We introduce the relationship that the inverse demand function determines price, as shown in (8), as a scaling constraint.

$$\tilde{\lambda}_h = b + \tilde{\xi}_h - a\sum_{k\in\mathcal{P}} \tilde{x}_{kh}, \quad h\in\mathcal{H} \tag{8}$$

When we include (8) to the inverse problem (7), we obtain the same objective value, but parameters fit exactly to the true value. Hence, we show that if data coincide with the inverse equilibrium model, it fits perfectly.

3) Fit inverse equilibrium models to other market structures: The inverse equilibrium approach fits data to models. To illustrate, we fit data from a competitive equilibria to the inverse Cournot model (7). We use 100 observations from when a social planner coordinates all decisions. Table I outlines the results.

TABLE I
RESULTS OF FITTING PERFECT COMPETITION DATA TO INVERSE COURNOT MODEL.

	True	Without (8)	With (8)
Deviation, ε [$€/MWh$]	0	208.1	1113.2
Intercept [$€/MWh$], b	200.0	143.6	200.0
Slope, a [$€/MWh^2$]	0.01	0.0067	0.01
Cost gen. 1, c_1 [$€/MWh$]	50.0	0.0	0.0
Cost gen. 2, c_2 [$€/MWh$]	60.0	20.3	17.6
Cost gen. 3, c_3 [$€/MWh$]	60.0	20.3	14.2

In contrast to the previous example, we observe a non-zero deviation. The inverse model does not manage to fit parameters such that the observations become an equilibrium of (7). In other words, the players deviate from their optimal Cournot strategy and a Cournot model is not a good representation of the data.

Table I also shows that the price relationship (8) increases the deviation ϵ and thus changes the solution space. It is therefore no longer a scaling constraint. We also note that the fitted parameters do not resemble the true parameters. This example illustrates the strength of inverse equilibrium modeling to test different market structures. It also emphasizes caution towards considering the fitted parameters as true estimations.

4) Performance under noise: In general, we cannot prove that inverse equilibrium modeling, as inverse optimization in its canonical form, yield consistent estimators. That is, as the number of observations increase, the fitted parameters will not converge to a true value. If the goal is to estimate parameters, consistency is an important feature. For this reason, we cannot recommend inverse equilibrium as an estimator.

To display the caveat of using inverse equilibrium as an estimator, we solve the generic Nash-Cournot game for $|\mathcal{H}| = 10$, 100, 500, and 1000 observations with a known random demand shock. We then add a normally distributed noise with mean 0 and standard deviation $200MWh$ to the output of Generator 3. If production with noise exceeds its production limit, we simply set it to Q_3^{max}.

A consistent estimator would be able to reduce the noise as the number of observations increase and converge to the true value. Table II shows that this is not the case for the inverse equilibrium model. In fact, the fitted parameters show no significant trend and adhere to the randomness of the noise. The total deviation ϵ in

Table II shows a steady increase because it gets more terms that deviate. Theoretically, we can observe this from objective (7a) of the inverse equilibrium model. We only minimize the deviation from optimum and have no noise correcting term. With a noise correcting term, the problem becomes non-convex (see [22]) and thus computationally hard to solve.

TABLE II
PERFORMANCE OF INVERSE COURNOT MODEL WHEN GENERATOR 3 HAS NOISE THAT FOLLOW DISTRIBUTION $\mathcal{N}(0, 200MWh)$.

Observations, $\lvert\mathcal{H}\rvert$	10	100	500	1000
ε [€/MWh]	4.85	61.88	294.55	611.33
b [€/MWh]	198.88	200.78	199.21	198.78
a [€/MWh]	0.0099	0.010	0.0099	0.0098
c_1 [€/MWh]	50.64	50.33	50.89	51.57
c_2 [€/MWh]	60.43	60.38	60.81	61.47
c_3 [€/MWh]	60.44	60.87	60.68	61.64

B. Nash-Cournot equilibrium in power systems

1) Model formulation: To illustrate the inverse equilibrium method for power systems, we use the model formulation of [23] that neglects the presence of arbitrageurs. See [23] for the assumptions that provide a unique equilibrium solution. We want to fit demand and supply function parameters to observations. The inverse demand function (9) sets the price λ_i at a particular bus $i \in \mathcal{N}$, where $\mathcal{N}$ is the set of nodes, with respect to total quantity q_i, slope a_i and intercept b_i. Equation (10) denotes the linear marginal cost for a producer p, where x_p is its generation.

$$f_i^{-1}(q_i) = \lambda_i(q_i) = b_i - a_i q_i \tag{9}$$

$$MC_p(x_p) = d_p + c_p x_p \tag{10}$$

A profit-maximizing producer p decides its sales to a particular node s_{pi} and its generation x_p according to problem (11). The objective (11a) is to maximize profits, given by the difference between revenue and cost. The cost of using the transmission network, w_i, is a parameter in problem (11), but we define it later as the dual variable of the market-clearing condition (15). Constraint (11b) enforces a maximum limit on x_p, while restriction (11c) ensures that sales are equal to generation.

$$\max_{s_{pi}, x_p} \quad \sum_{i \in \mathcal{N}} (b_i - a_i \sum_{k \in \mathcal{P}} s_{ki} - w_i) s_{pi} - (d_p + c_i x_p - w_{p(i)}) x_p \tag{11a}$$

$$\text{s.t.} \quad x_p - Q_p^{max} \leq 0, \quad (\alpha_p) \tag{11b}$$

$$\sum_{i \in \mathcal{N}} s_{pi} - x_p = 0, \quad (\beta_p) \tag{11c}$$

$$s_{pi} \geq 0, \quad x_p \geq 0 \tag{11d}$$

The KKT conditions of the producer problem (11) become (12). Notation $p(i)$ denotes the mapping from producer p to node i, i.e. the location of the generator.

$$0 \leq -b_i + a_i(s_{pi} + \sum_{k \in \mathcal{P}} s_{ki}) + w_i + \beta_p \perp s_{pi} \geq 0, \quad i \in \mathcal{N} \tag{12a}$$

$$0 \leq d_p + 2c_p x_p - w_{p(i)} + \alpha_p - \beta_p \perp x_p \geq 0 \tag{12b}$$

$$0 \leq -x_p + Q_p^{max} \perp \alpha_p \geq 0 \tag{12c}$$

$$\sum_{i \in \mathcal{N}} s_{pi} - x_p = 0, \quad \beta_p \in \mathbb{R} \tag{12d}$$

A system operator oversees energy flow while maximizing revenue from grid use, as shown in problem (13), where y_i is net energy injection at node i. Constraints (13b) and (13c) guarantee flows within the minimum and maximum limits of line $l \in \mathcal{L}$, where $\mathcal{L}$ is the set of lines. A PTDF matrix determines the flows in the system, where element $PTDF_{li}$ gives the ratio of flow on line l caused by power injections at node i. Although the system operator has an optimization problem, the net injection y_i is in fact determined by sales and production by the producers, as we show later in the market-clearing condition (15). Consequently, the system operator does not act strategically.

$$\max_{y_i} \quad \sum_{i \in \mathcal{N}} w_i y_i \tag{13a}$$

$$\text{s.t.} \quad -F_l^{cap} - \sum_{i \in \mathcal{N}} PTDF_{li} y_i \leq 0, \quad (\gamma_l^-) \quad l \in \mathcal{L} \tag{13b}$$

$$\sum_{i \in \mathcal{N}} PTDF_{li} y_i - F_l^{cap} \leq 0, \quad (\gamma_l^+) \quad l \in \mathcal{L} \tag{13c}$$

The KKT conditions of the system operator problem (13) are (14):

$$w_i + \sum_{l \in \mathcal{L}} PTDF_{li}(\gamma_l^- - \gamma_l^+) = 0, \quad y_i \in \mathbb{R} \quad i \in \mathcal{N} \tag{14a}$$

$$0 \leq F_l^{cap} + \sum_{i \in \mathcal{N}} PTDF_{li} y_i \perp \gamma_l^- \geq 0, \quad l \in \mathcal{L} \tag{14b}$$

$$0 \leq F_l^{cap} - \sum_{i \in \mathcal{N}} PTDF_{li} y_i \perp \gamma_l^+ \geq 0, \quad l \in \mathcal{L} \tag{14c}$$

Finally, the market-clearing condition (15) states that the net injection for each node must be equal to the difference between sales to the node and its internal production.

$$\sum_{p \in \mathcal{P}} s_{pi} - x_{p(i)} = y_i, \quad w_i \in \mathbb{R} \quad i \in \mathcal{N} \tag{15}$$

Both the producer and system operator problems are concave with affine constraints, so the KKT conditions are necessary and sufficient to represent the global optimum. The equilibrium problem is to find the set of variables that satisfy (12) for all the players, (14), and (15).

We invert the equilibrium problem to (16) for multiple observations $h \in \mathcal{H}$ according to the method of Section III-B. Because the producer problem has two decision variables, sales, s_{pi}, and production, x_p, it has two stationarity conditions. Consequently, we introduce two sets of deviation variables, ϵ^s_{pih} and ϵ^x_{ph}, for s_{pih} and x_{ph}, respectively. In the example, we weigh the deviations equally.

$$\min_{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}^-,\boldsymbol{\gamma}^+,\boldsymbol{w},\boldsymbol{\epsilon}} \quad \|\boldsymbol{\epsilon}\| \tag{16a}$$

$$\text{s.t.} \tag{16b}$$

$$\Big(-b_i + a_i(s_{pih} + \sum_{k\in\mathcal{P}} s_{kih}) + w_i + \beta_{ph} + \epsilon^s_{pih}\Big)s_{pih} = 0, \quad p \in \mathcal{P}, i \in \mathcal{N}, h \in \mathcal{H} \tag{16c}$$

$$0 \le -b_i + a_i(s_{pih} + \sum_{k\in\mathcal{P}} s_{kih}) + w_i + \beta_{ph} + \epsilon^s_{pih}, \quad \forall p \in \mathcal{P}, i \in \mathcal{N}, h \in \mathcal{H} \tag{16d}$$

$$(d_p + 2c_p x_{ph} - w_{p(i)} + \alpha_{ph} - \beta_{ph} + \epsilon^x_{ph})x_{ph} = 0, \quad p \in \mathcal{P}, h \in \mathcal{H} \tag{16e}$$

$$0 \le d_p + 2c_p x_{ph} - w_{p(i)} + \alpha_{ph} - \beta_{ph} + \epsilon^x_{ph}, \quad p \in \mathcal{P}, h \in \mathcal{H} \tag{16f}$$

$$(-x_{ph} + Q^{max}_p)\alpha_{ph} = 0, \quad p \in \mathcal{P}, h \in \mathcal{H} \tag{16g}$$

$$w_{ih} + \sum_{l\in\mathcal{L}} PTDF_{l,i}(\gamma^-_{lh} - \gamma^+_{lh}) = 0, \quad i \in \mathcal{N}, h \in \mathcal{H} \tag{16h}$$

$$(F^{cap}_l + \sum_{i\in\mathcal{N}} PTDF_{l,i} y_{ih})\gamma^-_{lh} = 0, \quad l \in \mathcal{L}, h \in \mathcal{H} \tag{16i}$$

$$(F^{cap}_l - \sum_{i\in\mathcal{N}} PTDF_{l,i} y_{ih})\gamma^+_{lh} = 0, \quad l \in \mathcal{L}, h \in \mathcal{H} \tag{16j}$$

$$\alpha_{ph} \ge 0, \quad p \in \mathcal{P}, h \in \mathcal{H} \tag{16k}$$

$$\gamma^-_{lh}, \gamma^+_{lh} \ge 0, \quad l \in \mathcal{L}, h \in \mathcal{H} \tag{16l}$$

$$y_i, w_i \in \mathbb{R}, i \in \mathcal{N}, \quad \beta_p \in \mathbb{R}, p \in \mathcal{P} \tag{16m}$$

2) Illustrative case study: As a case study, we consider the the 6-bus system from [24], as shown in Figure 1. Network flows behave according to the PTDF matrix represented in Table III where we define bus 1 as the hub. The line from bus 1 to 6 has a capacity of $200MW$, bus 2 to 5 has $250MW$, while the rest are sufficiently high not to limit any flows. Buses 1, 2, and 4 contain price-making producers, while buses 3, 5, and 6 are price-taking consumers. Table IV outlines the intercept and slope of both producer marginal cost and inverse demand.

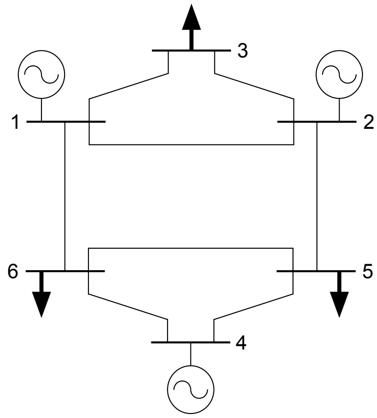

Fig. 1. Illustration of the 6 bus network from [24] used for equilibrium in power systems example.

TABLE III
PTDF MATRIX OF 6-BUS EXAMPLE.

Line/bus	1	2	3	4	5	6
(1,2)	0	-0.583	-0.292	-0.292	-0.333	-0.25
(1,3)	0	-0.292	-0.646	-0.146	-0.167	-0.125
(1,6)	0	-0.125	-0.063	-0.563	-0.5	-0.625
(2,3)	0	0.292	-0.354	0.146	0.167	0.125
(2,5)	0	0.125	0.063	-0.438	-0.5	-0.375
(4,5)	0	-0.042	-0.021	0.479	-0.167	0.125
(4,6)	0	0.042	0.021	0.521	0.167	-0.125
(5,6)	0	0.083	0.042	0.042	0.333	-0.25

TABLE IV
FITTED PARAMETERS FROM 6-BUS EXAMPLE. INTERCEPTS IN €$/MWh$ AND SLOPES IN €$/MWh^2$.

Bus	Fitted intercept	Fitted slope	True intercept	True slope
1	10.0	0.05	10.0	0.05
2	15.0	0.05	15.0	0.05
3	37.5	0.05	37.5	0.05
4	42.5	0.25	42.5	0.025
5	75.0	0.1	75.0	0.1
6	80.0	0.1	80.0	0.1

We obtain observations by solving the KKT conditions (12) for all the players, (14), and (15) as an equilibrium problem using the input data of the 6-bus example. To get different observations we apply both supply and demand shocks. We assume that all producers have fossil fuel generators with equal emission per unit energy and must pay a carbon price, λ^{CO_2}, for their emissions. We select carbon prices at random from a normal distribution with mean of 10€$/MWh$ and standard deviation 2€$/MWh$. The carbon price becomes an additional term in the marginal cost, $MC_p(x_p)$ from (10), of the producers. The demand shock ξ_h comes from a normal distribution with mean 0 and standard deviation $2€/MWh$. We assume a sufficiently high generation limit, $Q_p^{max} = 1000MWh$, as to not be binding for any of the observations.

We select 100 random carbon prices and demand shocks before solving the equilibrium model to generate observations. The objective value of (16a) becomes slightly above zero at 0.021. Thus we can conclude that the Nash-Cournot model fits the data. Moreover, we see from Table IV that the fitted parameters coincide with the actual variable. In contrast to the example in Section IV-A we have no scale invariance. The data provides sufficient marginal observations to scale the fitted parameters correctly.

V. Comments on implementation

As demonstrated in the examples of Section IV, existing equilibrium models can easily be recast as inverse equilibrium models. Although the fitted parameters in our examples provide good estimates of objective function parameters, we emphasize that the data was generated in a controlled environment. In real applications, the data will be noisy and the results more challenging to interpret. Inverse equilibrium modeling tries to fit a hypothesis, i.e. equilibrium structure, to data. A benefit of this approach is that the inverse equilibrium models are interpretable. While this limits generalization, it enables the modeler to use domain knowledge.

Data from real-world applications are subject to noise. Inverse equilibrium models are unlikely to enjoy as small deviations as our examples. This is expected, as it only shows that an equilibrium structure does not perfectly fit the data. An interesting feature is that we can try different equilibrium set-ups and observe what structure has the least deviation, and thus is the best fit for the data. Note that there may be several reasons for deviations; the model structure may not adhere to the observations, the observations can be noisy or there may be underlying dynamics or costs unobserved by the modeler.

The KKT approach in this paper benefits from the close relationship to existing MCP models applied to power systems. Consequently, the deviations are measured in costs per variable unit, which is less intuitive to interpret than just costs. The VI approach [3], on the other hand, measures deviations in the unit of the objective function. However, this requires a VI representation of the equilibrium problem.

As discussed in Section II, it is important to be cautious when investigating the fitted parameters. They are not representative of characteristics of an underlying population as in econometrics, they are merely the best fit to the data. Estimation of underlying market parameters is an important task for market monitors. For this purpose we recommend consistent estimators established in the econometric literature. If the reader is interested to try inverse equilibrium approaches in an estimation direction, we refer to [25], [26] and [22], which consider inverse optimization with noisy observations.

Inverse equilibrium modeling is a general approach that can be applied to any equilibrium problem. In this paper we use Cournot models because they are familiar to the power system modeling community. An alternative approach are conjectural variations models (see e.g. [27], [28] and [29]), which are more general. A challenge with inverting for instance the model in [27], is that even if the KKT conditions of the problem are necessary and sufficient, the inverse problem becomes non-convex in parameters. Hence, to make the inverse equilibrium problem convex, we need observations on a parameter in the bilinear term. For more information on estimation of conjectural variations models in power systems we refer to [30].

VI. Conclusion

Inverse equilibrium modeling is a data-driven method that fit parameters of an equilibrium model in order to minimize the deviation from an observation. This paper shows how to use Karush-Kuhn-Tucker (KKT) conditions to invert equilibrium problems. As shown in two applications, a constrained and an unconstrained Nash-Cournot game between power producers, this only requires a small deviation from the original equilibrium problem. Our methodology is thus easy to apply on existing equilibrium models applied to power systems, where working with KKT conditions is prominent. Inverse equilibrium models as shown in this paper can transform into linear programming problems. The method can investigate if data fit a model structure and it has predictive power. However, its estimation is generally inconsistent and econometric approaches are better for this purpose.

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Paper III: Congestion risk, transmission rights, and investment equilibria in electricity markets

Authors: Simon Risanger and Jacob Mays

Submitted to an international peer-reviewed journal.

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