Understanding sound radiation from surface vibrations moving at subsonic speeds

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Extended abstract

The interaction between a vibrating surface and an adjacent fluid is a common topic in many subfields of acoustics, for example building acoustics, noise control, and acoustic non-destructive testing. In the most fundamental formulation, the specified normal velocity v_y of the vibrating surface at $y = 0$ and the resulting fluid pressure p can be expressed as

$$v_y(x, 0, t) = v_{y0}\, \mathrm{e}^{\mathrm{i}(k_x x - \omega t)}, \tag{1a}$$

$$p(x, y, t) = p_0\, \mathrm{e}^{\mathrm{i}(k_x x + k_y y - \omega t)}. \tag{1b}$$

Here, the fluid wavenumber y-component k_y can be calculated from the surface wavenumber k_x and the fluid wavenumber $k_f = \omega / c_f$ as

$$k_y = \sqrt{k_f^2 - k_x^2} = k_f \sqrt{1 - (k_x/k_f)^2}. \tag{2}$$

With c_f being the fluid sound speed and $c_v = \omega/k_x$ being the surface vibration speed, this can also be expressed as $k_y = k_f\sqrt{1 - (c_f/c_v)^2}$.

Classic treatments of this problem (see e.g. [1]) find two solution domains depending on the surface vibration speed c_v. In the *supersonic* domain ($c_v > c_f$), the wavenumber k_y is real-valued, and (1b) expresses a plane fluid wave radiating away from the surface. In the *subsonic* domain ($c_v > c_f$), however, k_y is imaginary-valued, so that (1b) expresses an evanescent, non-radiating fluid wave that decays exponentially away from the surface.

However, multiple articles have shown radiating fluid waves to exist even in the subsonic domain, specifically for Rayleigh waves [2] and Lamb waves [3–5]. These results follow by numerically solving the complex dispersion equations for the investigated type of wave. While this mathematical approach is standard and correct, it does not explain *why* this subsonic radiation can occur. One brief qualitative explanation is provided in [2], where the subsonic radiation is related to the attenuation of the surface vibration. This attenuation is caused by the loss of the power radiated into the fluid by the pressure wave.

In our work, we investigate subsonic radiation more closely through a simple physical model. We take the formulation in (1) and (2) and generalise it to an attenuated

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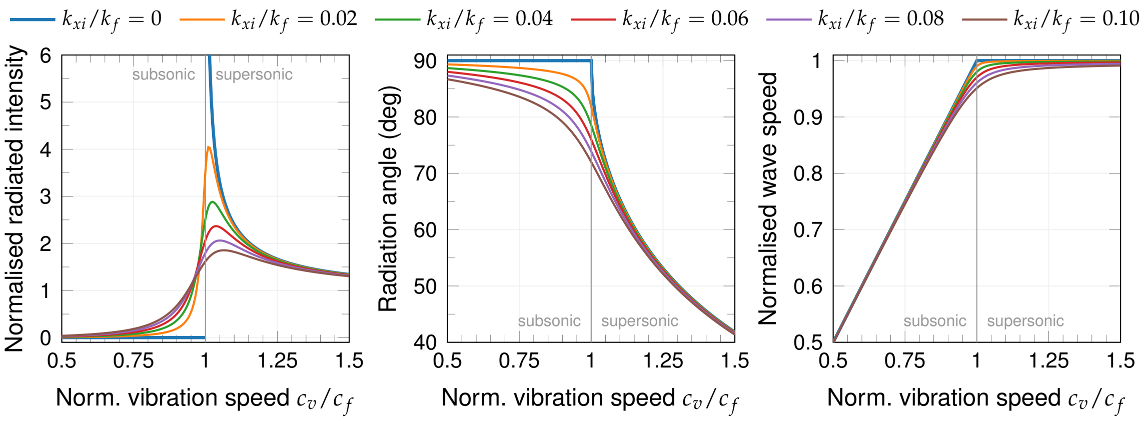

Figure 1: Properties of the wave radiated from an attenuated surface vibration. Left: Normalised radiated intensity $I_{y0}/(v_{y0}^2 Z_f/2) = (k_{yr}/k_f)/|k_y/k_f|^2$. Middle: Radiation angle $\theta = \arctan(k_{xr}/k_{yr})$. Right: Normalised fluid wave speed $c/c_f = k_f/\sqrt{k_{xr}^2 + k_{yr}^2}$.

surface vibration using a complex surface wavenumber $k_x = k_{xr} + \mathrm{i}k_{xi}$, where the real part relates to the surface vibration speed as $k_{xr} = \omega/c_v$ and the imaginary part provides an exponential attenuation $\mathrm{e}^{-k_{xi}x}$.

The complex surface wavenumber k_x makes $k_y = k_{yr} + \mathrm{i}k_{yi}$ complex as well. In fact, closer investigation of (2) reveals that *any* propagating ($k_{xr} > 0$) and attenuated ($k_{xi} > 0$) surface vibration leads to a radiating ($k_{yr} > 0$) fluid wave whose pressure increases exponentially with distance from the surface ($k_{yi} < 0$). (While the latter fact might at first seem troubling, [5] explains why this is the correct physical behaviour for the inhomogeneous fluid wave [2, 6] that an attenuated surface vibration radiates.)

We then investigate the properties of the radiated wave further. The left plot in Fig. 1 shows that if the surface vibration is not attenuated ($k_{xi} = 0$), the radiated time-averaged intensity I_y has a discontinuous singularity at the supersonic-subsonic transition and is zero in the subsonic domain. If the surface vibration is attenuated, however, I_y is smooth and continuous at the transition, showing power radiation in the entire subsonic domain. The middle plot in Fig. 1 shows that the fluid wave will only ever graze the surface ($\theta = 90°$) in the subsonic domain if the surface vibration is not attenuated. Finally, if the surface vibration is not attenuated, the right plot in Fig. 1 shows a sharp transition in the actual fluid wave speed c from the fluid sound speed c_f in the supersonic domain to the surface vibration speed c_v in the subsonic domain. For attenuated surface vibrations, this transition is smoothed due to the lower speed of the inhomogeneous radiated waves.

For a more physically realistic treatment, instead of simply imposing a particular attenuation of the surface vibration, we connect the radiated power to the power lost in the surface vibration. With a surface vibration power flow $P_x(x) = P_{x0}\,\mathrm{e}^{-2k_{xi}x}$, this implies $-\partial P_x/\partial x = 2k_{xi}P_x = I_y$ [5]. With I_y and potentially P_x depending on the variable k_{xr} and the unknown k_{xi}, valid propagation modes are represented by the roots of the function $f(k_{xr}, k_{xi}) = 2k_{xi} - I_{y0}(k_{xr}, k_{xi})/P_{x0}(k_{xr}, k_{xi})$ plotted in Fig. 2. The figure shows two such modes: One 'classic' non-radiating subsonic mode, and one radiating supersonic mode that extends into the subsonic domain, demonstrating very similar behaviour to the subsonic radiation shown for A_0 Lamb waves in [3–5].

To validate our results, we apply them to leaky A_0 Lamb waves on a 1 cm thick steel plate radiating into air on both sides. A number of perturbation methods already exist

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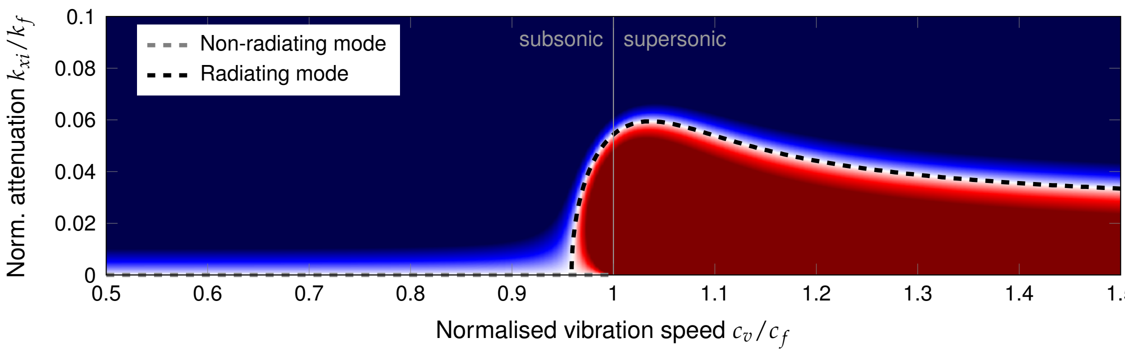

Figure 2: Diverging colour plot of the function $f(k_{xr}, k_{xi}) = 2k_{xi} - I_{y0}(k_{xr}, k_{xi})/P_{x0}(k_{xr}, k_{xi})$, with arbitrary values chosen for material constants and power flow. White colour and dashed lines indicate the function's roots ($f = 0$), which represent valid propagating modes.

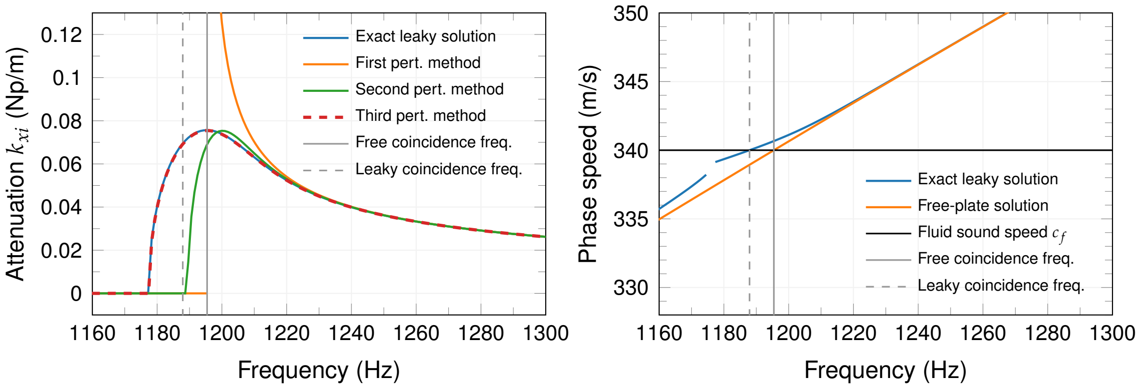

Figure 3: Comparison of exact leaky A_0 Lamb wave solutions for a 1 cm steel plate in air against approximate solutions. Left: Attenuation due to radiation. Right: Phase speed of leaky and free Lamb waves.

to predict the attenuation of leaky Lamb waves from free-plate solutions, as summarised in [7], but none of them can predict subsonic radiation. Our results can be used as an improved perturbation method to overcome this weakness. We base this perturbation on the dispersion relations $k_{xr}^{\text{free}}(\omega)$ and $k_{xr}^{\text{leaky}}(\omega)$ of free and leaky A_0 Lamb waves, respectively, in addition to the field equations for free Lamb waves. We then compare the results against the exact attenuation $k_{xi}^{\text{leaky}}(\omega)$.

The left plot in Fig. 3 shows our attenuation results. First, we calculate the attenuation based on the roots of $2k_{xi} - 2I_{y0}(k_{xr}^{\text{free}}, 0)/P_{x0}(k_{xr}^{\text{free}}, 0)$, i.e., as a simple perturbation to a non-attenuated surface vibration. This results in the same attenuation as existing perturbation methods, with no radiation in the subsonic domain. Second, we take the attenuation of the surface vibration into account, finding the roots of $2k_{xi} - 2I_{y0}(k_{xr}^{\text{free}}, k_{xi})/P_{x0}(k_{xr}^{\text{free}}, k_{xi})$. This results in the same qualitative behaviour as the exact solution, although the attenuation peak and cutoff frequencies are different. The reason for this difference is apparent from the phase speed of the free and leaky Lamb waves in the right plot in Fig. 3: Close to the supersonic-subsonic transition, the two diverge, leading to different coincidence frequencies. Third, we take this phase speed divergence into account by finding the roots of $2k_{xi} - 2I_{y0}(k_{xr}^{\text{leaky}}, k_{xi})/P_{x0}(k_{xr}^{\text{leaky}}, k_{xi})$. This shows a very good match with the exact solution, thus validating our results.

In summary, we have found that subsonic radiation occurs because any attenuated

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surface vibration will radiate power into the fluid. Our simple power flow model, which connects the power radiated into the fluid to the power lost in the surface vibration, shows that such subsonic radiation can occur in a small area of the subsonic domain. Furthermore, we have shown that this model can be used as a perturbation method for leaky Lamb waves that improves on existing ones. A full match with the exact attenuation, however, requires using the exact phase speed, which itself is part of the exact leaky solution.

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