Global Asymptotic Tracking for Marine Surface Vehicles using Hybrid Feedback in the Presence of Parametric Uncertainties

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***Abstract*— In this paper, we propose a hybrid adaptive feedback control law for global asymptotic tracking control for marine surface vehicles in the presence of parametric uncertainties. The hybrid feedback is derived from a family of potential functions and employs a hysteretic switching mechanism that is independent of the vehicle velocities. The tracking references are constructed from a given parametrized loop and a speed assignment specifying the motion along the loop. Finally, we provide simulation results for a ship subject to parametric modeling uncertainties and unknown ocean currents.**

I. Introduction

It is well known that continuous-time systems whose state-space can be identified with a vector bundle on a compact manifold admit no point that can be globally asymptotically stabilized by continuous-time state feedback [1]. This is referred to as a topological obstruction to global asymptotic stability and follows from the fact that no compact manifold is contractible.

Topological obstructions to global asymptotic stability can be overcome by employing hybrid feedback with a properly defined switching logic [2]. In particular, hybrid feedback derived from a family of synergistic potential functions can be used to globally asymptotically stabilize compact sets using gradient descent and a hysteretic switching mechanism [3], [4]. Hybrid feedback has been employed to achieve global asymptotic stability of compact sets for planar orientation control [5], [6], reduced orientation control [7], spatial orientation control [8], [9], tracking for underwater vehicles [10] and on more general compact manifolds [11].

Although the problem of overcoming topological obstructions on compact manifolds using hybrid feedback has been extensively studied in the idealized case where all model parameters are assumed known, surprisingly little attention has been paid to the more practical case involving parametric modeling uncertainties. In [12], a global exponential tracking controller with integral action is derived for orientation control of a spatial rigid body subject to a matched and constant disturbance. Hybrid feedback using synergistic potential functions was extended to the case where the original control system is subject to matched uncertainties in [13].

The main contribution of this paper is the development of a hybrid adaptive feedback controller for global asymptotic tracking of a hybrid reference system for surface vehicles in the presence of parametric uncertainty. The hybrid reference system is constructed from a parametrized loop and a speed assignment for the motion along the loop. The main benefit of this formulation is that it decouples design of the path from the motion along the path. The proposed reference system can be considered as an adaptation of the maneuvering problem [14], [15] to our hybrid dynamical systems setting.

This paper is organized as follows. Section II presents kinematic and dynamic models of a surface vehicle on $\mathrm{SE}(2)$, before the hybrid reference system is presented. Then, the resulting error system is derived and the problem statement is given. In Section III, we construct a family of potential functions on $\mathrm{SE}(2)$ and derive a hybrid adaptive control law with hysteretic switching for global asymptotic tracking of surface vehicles subject to uncertainties. Then, Section IV presents simulations results verifying the theoretical developments, before Section V concludes the paper.

A. Notation and Preliminaries

We denote by $\mathbb{R}$ the field of real numbers, the Euclidean space of dimension n is denoted $\mathbb{R}^n$, and $\mathbb{R}^{n\times n}$ is the space of $n\times n$ matrices with real entries. The standard inner product on $\mathbb{R}^n$ is written $\langle x,y\rangle$ and the Euclidean norm $|x| = \langle x,x\rangle^{1/2}$. The entry of a matrix $a\in\mathbb{R}^{n\times n}$ corresponding to the ith row and jth column is denoted a_{ij}. For $\mathcal{S}\subset\mathcal{X} \coloneqq \mathcal{X}_1\times\mathcal{X}_2$, the projection of $\mathcal{S}$ onto $\mathcal{X}_1$ is defined by $\pi_{\mathcal{X}_1}(\mathcal{S}) \coloneqq \{x_1\in\mathcal{X}_1 : (x_1,x_2)\in\mathcal{S} \text{ for some } x_2\in\mathcal{X}_2\}$. The range (or equivalently the image) of a function $f:\mathbb{R}^m\to\mathbb{R}^n$ is defined as $\operatorname{rge} f = \{y\in\mathbb{R}^n : \exists x\in\mathbb{R}^m \text{ such that } y = f(x)\}$.

A matrix Lie group G is a closed subgroup of the general linear group $\mathrm{GL}(n) = \{g\in\mathbb{R}^{n\times n} : \det g\neq 0\}$. The identity element is denoted $e\in\mathrm{G}$. The Lie algebra of a matrix Lie group G is denoted $\mathfrak{g}$, and defined as $\mathfrak{g} \coloneqq \{a\in\mathbb{R}^{n\times n} : t\in\mathbb{R} \implies \exp(at)\in\mathrm{G}\}$, where $\exp:\mathbb{R}^{n\times n}\to\mathrm{GL}(n)$ is the matrix exponential. The Lie algebra $\mathfrak{g}$ is a real vector space with dimension equal to the dimension of G as a manifold. Therefore, there exists an isomorphism $(\cdot)^\wedge\colon\mathbb{R}^m\to\mathfrak{g}$ with inverse $(\cdot)^\vee\colon\mathfrak{g}\to\mathbb{R}^m$, where m denotes the dimension of G. With $g\in\mathrm{G},\xi\in\mathbb{R}^m$ and $\zeta\in\mathbb{R}^m$, we define the adjoint mappings

$$\begin{aligned} \mathrm{Ad}\colon \mathrm{G}\times\mathbb{R}^m\to\mathbb{R}^m, &\qquad \mathrm{Ad}_g\,\xi \coloneqq \big(g\widehat{\xi}g^{-1}\big)^\vee,\\ \mathrm{ad}\colon \mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^m, &\qquad \mathrm{ad}_\xi\,\zeta \coloneqq \big(\widehat{\xi}\widehat{\zeta}-\widehat{\zeta}\widehat{\xi}\big)^\vee. \end{aligned}$$

For each $\xi\in\mathbb{R}^m$, we define a left-invariant vector field $X_\xi(g) = g\widehat{\xi}$ on G with $g\in\mathrm{G}$. The Lie derivative of a continuously differentiable function $V\colon\mathrm{G}\to\mathbb{R}$ along the vector

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field X_ξ can be written as $\langle\!\langle \nabla V(g), X_\xi(g)\rangle\!\rangle = \langle \mathrm{d}V(g), \xi\rangle$, where $\langle\!\langle a, b\rangle\!\rangle := \operatorname{tr}(a^\mathsf{T} b)$ is the Frobenius inner product and

$$\nabla V(a) = \begin{pmatrix} \frac{\partial V}{\partial a_{11}} & \cdots & \frac{\partial V}{\partial a_{1j}} \\ \vdots & \ddots & \vdots \\ \frac{\partial V}{\partial a_{i1}} & \cdots & \frac{\partial V}{\partial a_{ij}} \end{pmatrix}.$$

In this work, we consider the matrix Lie groups $\mathrm{SO}(2) = \{R \in \mathbb{R}^{2\times 2} : R^\mathsf{T} R = RR^\mathsf{T} = I, \det R = 1\}$ and $\mathrm{SE}(2) = \mathbb{R}^2 \rtimes \mathrm{SO}(2)$, where $\rtimes$ denotes the semi-direct product [16]. The associated Lie algebras are denoted $\mathfrak{so}(2)$ and $\mathfrak{se}(2)$, respectively.

II. Trajectory Tracking for Surface Vehicles

This section begins by presenting kinematic and dynamic models of a surface vehicle on $\mathrm{SE}(2)$. Then, given an r-times continuously differentiable loop $\gamma(s)$ on $\mathrm{SE}(2)$ and a speed assignment for $\dot{s}$, we derive a hybrid reference system generating continuous configuration, velocity and acceleration references. Finally, we define error coordinates on $\mathrm{SE}(2)$ in order to derive the error dynamics and present the problem formulation.

A. Modeling

The configuration of a surface vehicle can be identified with the matrix Lie group $\mathrm{SE}(2) = \mathbb{R}^2 \rtimes \mathrm{SO}(2)$. An element $g = (p, R) \in \mathrm{SE}(2)$ contains the position $p = (x, y) \in \mathbb{R}^2$ and orientation $R \in \mathrm{SO}(2)$ of a vehicle-fixed frame with respect to an inertial frame. Elements in $\mathrm{SE}(2)$ admit a homogeneous matrix representation through the injective homomorphism $\mathrm{SE}(2) \to \mathrm{GL}(3)$ defined by [16]

$$g := \begin{pmatrix} R & p \\ 0 & 1 \end{pmatrix} \in \mathbb{R}^{3\times 3}. \tag{1}$$

Denoting the vehicle-fixed linear and angular velocity by $v \in \mathbb{R}^2$ and $\omega \in \mathbb{R}$, respectively, define the vehicle-fixed velocity as $\nu := (v, \omega) \in \mathbb{R}^3$. An element $\nu \in \mathbb{R}^3$ maps to $\mathfrak{se}(2)$ through the isomorphism $(\cdot)^\wedge : \mathbb{R}^3 \to \mathfrak{se}(2)$ defined by

$$\widehat{\nu} := \begin{pmatrix} S\omega & v \\ 0 & 0 \end{pmatrix} \in \mathbb{R}^{3\times 3}, \quad S := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{2}$$

By assuming that the dynamics related to the ocean current (and other unmodeled dynamics) are captured by a slowly varying bias, $b \in \mathbb{R}^3$, given in the inertial frame, the equations of motion can be stated as

$$\dot{g} = g\widehat{\nu}, \tag{3a}$$

$$M[\dot{\nu} + \nabla^M_\nu \nu] = d(\nu) + g_0^\mathsf{T} b + \tau, \tag{3b}$$

where $M \in \mathbb{R}^{3\times 3}$ is the inertia matrix, including hydrodynamic added mass, $M\nabla^M_\nu \nu$ describes internal forces arising from curvature effects, the function $d : \mathbb{R}^3 \to \mathbb{R}^3$ describes dissipative forces, $g_0 = (0, R) \in \mathrm{SE}(2)$, and $\tau \in \mathbb{R}^3$ is the control force. Moreover, the bilinear map $\nabla^M : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$ induced by the inertia matrix M is given by [17]

$$\nabla^M_\nu \eta = \tfrac{1}{2} \operatorname{ad}_\nu \eta - \tfrac{1}{2} M^{-1}[\operatorname{ad}_\nu^\mathsf{T} M\eta + \operatorname{ad}_\eta^\mathsf{T} M\nu],$$

using the homogenous matrix representation

$$\operatorname{ad}_\nu = \begin{pmatrix} S\omega & -Sv \\ 0 & 0 \end{pmatrix} \in \mathbb{R}^{3\times 3}. \tag{4}$$

B. Hybrid Reference System

Definition 1. *Let $\mathcal{I} = [0, 1] \subset \mathbb{R}$. The parametric C^r-path $\gamma : \mathcal{I} \to \mathrm{SE}(2)$ defined by*

$$\gamma(s) := (\gamma_1(s), \gamma_2(s)), \quad \gamma_1(s) \in \mathbb{R}^2, \gamma_2(s) \in \mathrm{SO}(2), \tag{5}$$

is a C^r-loop if it satisfies

$$\gamma^{(k)}(0) = \gamma^{(k)}(1), \tag{6}$$

for all $0 \le k \le r$.

Note that $\operatorname{rge}\gamma$ is compact for any C^r-loop γ. Now, the motion along the loop can be controlled through a speed assignment for $\dot{s}$. Assuming $|\gamma_1'(s)| \neq 0$ for all $s \in \mathcal{I}$, we may set [14]

$$\dot{s} = \varrho(s, u_d) := \frac{u_d}{|\gamma_1'(s)|}, \tag{7}$$

where u_d is a commanded input speed obtained from the following second-order low-pass filter

$$\ddot{u}_d = p(u_d, \dot{u}_d, \mu) := \omega_n^2 \mu - 2\zeta_f \omega_n \dot{u}_d - \omega_n^2 u_d, \tag{8}$$

with $\mu \in [0, c]$, $c > 0$ and $\omega_n, \zeta_f > 0$. Note that (8) and $\mu \in [0, c]$ ensures that u_d and $\dot{u}_d$ take values in compact sets.

Now, define $r := (s, u_d, a_d) \in \mathcal{R}$ and the compact exogenous state space

$$\mathcal{R} := \mathcal{I} \times \Omega_1 \times \Omega_2, \tag{9}$$

where $\Omega_1, \Omega_2 \subset \mathbb{R}^3$. Using the Lie group structure of $\mathrm{SE}(2)$ leads to the following hybrid reference system

$$\mathscr{E} : \begin{cases} \left.\begin{aligned} \dot{s} &= \varrho(s, u_d) \\ \dot{u}_d &= a_d \\ \dot{a}_d &= p(u_d, a_d, \mu) \\ \mu &\in [0, c] \end{aligned}\right\} & r \in \mathcal{R} \\ s^+ = 0 & r \in \mathcal{R}_\mathcal{D} \\ g_d = \gamma(s) & \\ \nu_d = \kappa(s)\varrho(s, u_d) & \\ \alpha_d = f_d(s, u_d, a_d) & \end{cases} \tag{10}$$

where $\kappa(s) = (\gamma(s)^{-1}\gamma'(s))^\vee$ is the desired tangent vector expressed in the desired frame, $\mathcal{R}_\mathcal{D} = \{1\} \times \Omega_1 \times \Omega_2$ is the jump set and the mapping $f_d : \mathcal{I} \times \Omega_1 \times \Omega_2 \to \mathbb{R}^3$ is given by

$$f_d(\cdot) = \kappa(s)\left(\frac{\partial \varrho}{\partial s}\varrho(s, u_d) + \frac{\partial \varrho}{\partial u_d} a_d\right) + \kappa'(s)\varrho(s, u_d). \tag{11}$$

Observe that $\mathscr{E}$ is a hybrid system with input $\mu \in [0, c]$ and output $y := (g_d, \nu_d, \alpha_d) = (\gamma(s), \kappa(s)\varrho(s, u_d), f_d(s, u_d, a_d))$, where $g_d \in \operatorname{rge}\gamma$, $\nu_d \in \mathbb{R}^3$ and $\alpha_d \in \mathbb{R}^3$ are desired configuration, velocity and acceleration references, respectively.

C. Error System and Problem Statement

The error dynamics are obtained by considering the continuous and invertible transformation $(g, \nu, r) \mapsto (g_e, \nu_e, r)$, using the natural (and left-invariant) error on $\mathrm{SE}(2)$ defined in homogeneous coordinates by [17]

$$g_e := g_d^{-1} g, \tag{12}$$

$$\nu_e := \nu - \operatorname{Ad}_{g_e^{-1}} \nu_d, \tag{13}$$

where the configuration error in homogeneous coordinates is

$$g_e = \begin{pmatrix} R_d^\mathsf{T} R & R_d^\mathsf{T}(p - p_d) \\ 0 & 1 \end{pmatrix}. \tag{14}$$

Observe that g_e expresses the configuration of the vehicle-fixed frame with respect to the desired vehicle-fixed frame. We can relate g_e to a position error $p_e = R_d^\mathsf{T}(p - p_d) \in \mathbb{R}^2$ and an orientation error $R_e = R_d^\mathsf{T} R \in \mathrm{SO}(2)$. Moreover, the term $\nu_r := \mathrm{Ad}_{g_e^{-1}} \nu_d$ can be interpreted as ν_d expressed in the vehicle-fixed frame, and it can be shown that

$$\dot{\nu}_r = \mathrm{Ad}_{g_e^{-1}} \alpha_d - \mathrm{ad}_{\nu_e} \mathrm{Ad}_{g_e^{-1}} \nu_d, \tag{15}$$

$$\mathrm{Ad}_{g_e^{-1}} = \begin{pmatrix} R_e^\mathsf{T} & R_e^\mathsf{T} S p_e \\ 0 & 1 \end{pmatrix}. \tag{16}$$

Using the reference system $\mathscr{E}$, the following error system is obtained

$$\mathscr{N}: \begin{cases} \left.\begin{aligned} \dot{g}_e &= g_e \widehat{\nu}_e \\ \dot{\nu}_e &= f_e(g_e, \nu_e, r, \tau) \\ \dot{s} &= \varrho(s, u_d) \\ \dot{u}_d &= a_d \\ \dot{a}_d &= p(u_d, a_d, \mu) \\ \mu &\in [0, c] \end{aligned}\right\} & (g_e, \nu_e, r) \in \mathcal{Z}, \\ s^+ = 0 & (g_e, \nu_e, r) \in \mathcal{Z}_\mathcal{D}, \end{cases} \tag{17}$$

where $\mathcal{Z} := \mathrm{SE}(2) \times \mathbb{R}^3 \times \mathcal{R}$, $\mathcal{Z}_\mathcal{D} := \mathrm{SE}(2) \times \mathbb{R}^3 \times \mathcal{R}_\mathcal{D}$ and

$$\begin{aligned} f_e(\cdot) &= M^{-1}(\tau - M\nabla_\nu^M \nu + d(\nu) + g_0^\mathsf{T} b) \\ &- \mathrm{Ad}_{g_e^{-1}} f_d(s, u_d, a_d) + \mathrm{ad}_{\nu_e} \mathrm{Ad}_{g_e^{-1}} \kappa(s) \varrho(s, u_d). \end{aligned} \tag{18}$$

Problem Statement: For a given C^2-loop $\gamma(s)$ satisfying $\gamma'(s) \neq 0$ for all $s \in \mathcal{I}$, and the speed assignment (7) for $\dot{s}$, ensure uniform global asymptotic stability of the compact set

$$\mathcal{A} = \{(g_e, \nu_e, r) \in \mathcal{Z} : g_e = e, \nu_e = 0, r \in \mathcal{R}\}, \tag{19}$$

for the system $\mathscr{N}$ under parametric uncertainties.

III. Hybrid Control Design

In this section we present a hybrid adaptive control law for global asymptotic tracking of the error system defined in the previous section subject to parametric modeling uncertainty. In order to overcome topological obstructions to global asymptotic stability, the hybrid feedback control laws are derived from potential functions using a hysteretic switching mechanism. By using three potential functions we obtain improved transient performance by encoding smaller proportional gains into the global controllers relative to the local controller.

A. Potential Functions

Define $\rho_1 : \mathrm{SO}(2) \to (0, 2\pi]$, $\rho_2 : \mathrm{SO}(2) \to [-2\pi, 0)$ and $\rho_3 : \mathrm{SO}(2) \to (-\pi, \pi]$ by

$$\rho_1(R) := \begin{cases} (\log R)^\vee, & \text{if } (\log R)^\vee \in (0, \pi] \\ (\log R)^\vee + 2\pi, & \text{if } (\log R)^\vee \in (-\pi, 0], \end{cases} \tag{20a}$$

$$\rho_2(R) := \begin{cases} (\log R)^\vee, & \text{if } (\log R)^\vee \in (-\pi, 0) \\ (\log R)^\vee - 2\pi, & \text{if } (\log R)^\vee \in [0, \pi], \end{cases} \tag{20b}$$

$$\rho_3(R) := (\log R)^\vee, \tag{20c}$$

where $(\log R)^\vee = \mathrm{atan2}(R_{21}, R_{11})$ is the principal logarithm of $R \in \mathrm{SO}(2)$.

Now, for each $q \in \mathcal{Q} = \{1, 2, 3\}$, we define the potential functions $V_q : \mathrm{SE}(2) \to \mathbb{R}_{\geq 0}$ by

$$V_q(g) := \tfrac{1}{2} k_q \rho_q(R)^2 + o_q + \tfrac{1}{2} p^\mathsf{T} K p, \tag{21}$$

where $K = K^\mathsf{T} > 0$, $k_1 = k_2 = k > 0$, $k_3 > 0$, $o_1 = o_2 = o$ and $o_3 = 0$. We define the flow and jump sets by

$$\mathcal{C} := \bigcup_{q \in \mathcal{Q}} \mathcal{C}_q \times \{q\}, \tag{22a}$$

$$\mathcal{D} := \bigcup_{k \in \{1,2,3,4\}} \mathcal{D}_k, \tag{22b}$$

where

$$\mathcal{C}_1 := \{g \in \mathrm{SE}(2) : \delta \leq \rho_1(R) \leq \pi + \epsilon\}, \tag{23a}$$

$$\mathcal{C}_2 := \{g \in \mathrm{SE}(2) : -\delta \geq \rho_2(R) \geq -\pi - \epsilon\}, \tag{23b}$$

$$\mathcal{C}_3 := \{g \in \mathrm{SE}(2) : |\rho_3(R)| \leq \delta + \epsilon\}. \tag{23c}$$

and

$$\mathcal{D}_1 := \{g \in \mathrm{SE}(2) : \rho_1(R) \geq \pi + \epsilon\} \times \{1\}, \tag{24a}$$

$$\mathcal{D}_2 := \{g \in \mathrm{SE}(2) : \rho_2(R) \leq -\pi - \epsilon\} \times \{2\}, \tag{24b}$$

$$\mathcal{D}_3 := \{g \in \mathrm{SE}(2) : |\rho_3(R)| \leq \delta\} \times \{1, 2\}, \tag{24c}$$

$$\mathcal{D}_4 := \{g \in \mathrm{SE}(2) : |\rho_3(R)| \geq \delta + \epsilon\} \times \{3\}, \tag{24d}$$

where $\epsilon > 0$ is the hysteresis width and $\delta > 0$ determines the switching point between the local and global control laws. These quantities must satisfy $\delta + 2\epsilon < \pi$, which ensures that the jump sets are non-empty and non-overlapping. Finally, we define the jump map $q^+ = \mathcal{G}_1(g, q)$ by

$$\mathcal{G}_1(g, q) := \begin{cases} 3 - q, & \text{if } (g, q) \in \mathcal{D}_1 \cup \mathcal{D}_2 \\ 3, & \text{if } (g, q) \in \mathcal{D}_3 \\ \arg\min_{q \in \{1,2\}} V_q(g), & \text{if } (g, q) \in \mathcal{D}_4 \end{cases} \tag{25}$$

The following lemma provides conditions on the gains and offsets in (21) ensuring that V is non-increasing across jumps.

Lemma 1. *If $k_3 \geq k$, $\delta + 2\epsilon < \pi$ and*

$$\tfrac{1}{2}\delta^2(k_3 - k) \leq o \leq \tfrac{1}{2}(\delta + \epsilon)^2(k_3 - k), \tag{26}$$

then for all $(g, q) \in \mathcal{D}$, it holds that $V_z(g) - V_q(g) \leq 0$ for every $z \in \mathcal{G}_1(g, q)$.

B. Adaptive Tracking on SE(2)

To ensure global asymptotic tracking in the presence of parametric model uncertainty, we redefine the velocity error as

$$\nu_s := \nu - \nu_m, \tag{27}$$

where the modified reference velocity $\nu_m \in \mathbb{R}^3$ satisfies

$$\Lambda[\dot{\nu}_m + \nabla_\nu^\Lambda \nu_m] = \Lambda[\dot{\nu}_r + \nabla_\nu^\Lambda \nu_r] - \mathrm{d}V_q(g_e) - \vartheta_q(\nu_m - \nu_r).$$

From (27) it is clear that $\nu_m - \nu_r = 0$ implies $\nu_s = \nu_e$. Hence, the velocity tracking control objective $\nu_e = 0$ can be restated as $(\nu_s, \zeta) = 0$, where $\zeta := \nu_m - \nu_r$. If the model parameters in (3) are known, it can be shown that the hybrid control law defined by

$$\widetilde{\mathscr{C}}:\begin{cases}\dot{\zeta}=-\nabla_{\nu}^{\Lambda}\zeta-\Lambda^{-1}\big(\mathrm{d}V_q(g_e)+\vartheta_q(\zeta)\big), & (g_e,q)\in\mathcal{C}\\ q^+\in\mathcal{G}_1(g_e,q), & (g_e,q)\in\mathcal{D}\\ \tau=M[\dot{\nu}_m+\nabla_{\nu}^{M}\nu_m]-d(\nu)-g_0^{\mathsf{T}}b & \\ \qquad-\varphi_q(\nu_s)-\mathrm{d}V_q(g_e), &\end{cases} \tag{28}$$

globally asymptotically stabilizes the compact set $\mathcal{A}$ for the error system $\mathscr{N}$. Observe that the feedback control law (28) comprises a proportional action $\mathrm{d}V$, a derivative action φ and a feedforward term $\tau_{ff}=M[\dot{\nu}_m+\nabla_{\nu}^{M}\nu_m]-d(\nu)-g_0^{\mathsf{T}}b$.

Consider now the case where the model parameters are unknown. If the dissipative forces $d(\nu)$ are linear in the unknown parameters, it follows that

$$M[\dot{\nu}_m+\nabla_{\nu}^{M}\nu_m]-d(\nu)-g_0^{\mathsf{T}}b=\Phi(g_e,\zeta,\nu_s,r)\theta, \tag{29}$$

where $\Phi\colon \mathrm{SE}(2)\times\mathbb{R}^3\times\mathbb{R}^3\times\mathcal{R}\to\mathbb{R}^{3\times l}$ is a known matrix-valued function of available data, and $\theta\in\mathbb{R}^l$ is a vector of unknown model parameters. Assume that the parameters are upper and lower bounded by the constants $\overline{\theta}$ and $\underline{\theta}$, respectively, i.e. that the parameters are contained in the convex set

$$\mathcal{P}:=\{\theta\in\mathbb{R}^l:\underline{\theta}\leq\theta\leq\overline{\theta}\}. \tag{30}$$

Define the extended tangent cone to $\mathcal{P}$ by

$$\begin{aligned}T_{\mathbb{R},\mathcal{P}}(\theta)&:=T_{\mathbb{R},[\underline{\theta}_1,\overline{\theta}_1]}(\theta_1)\times T_{\mathbb{R},[\underline{\theta}_2,\overline{\theta}_2]}(\theta_2)\times\cdots\\&\qquad\times T_{\mathbb{R},[\underline{\theta}_l,\overline{\theta}_l]}(\theta_l),\end{aligned} \tag{31}$$

where the extended tangent cone to each interval is given by

$$T_{\mathbb{R},[\underline{\theta}_i,\overline{\theta}_i]}(\theta_i):=\begin{cases}[0,\infty) & \text{if } \theta_i\leq\underline{\theta}_i\\(-\infty,\infty) & \text{if } \theta_i\in(\underline{\theta}_i,\overline{\theta}_i)\\(-\infty,0] & \text{if } \theta_i\geq\overline{\theta}_i\end{cases} \tag{32}$$

Let $\theta_a\in\mathbb{R}^l$ denote the estimate of θ and define the convex set

$$\mathcal{P}_\epsilon:=\{\theta_a\in\mathbb{R}^l:\underline{\theta}-\epsilon\leq\theta_a\leq\overline{\theta}+\epsilon\}, \tag{33}$$

where $\epsilon=(\epsilon_1,\ldots,\epsilon_l)\in\mathbb{R}^l$, defines boundary layers of length $\epsilon_i>0$ around each interval in (30). The goal is to enforce $\theta_a\in\mathcal{P}_\epsilon$ through the adaptive update law. To this end, define the projection operator $\mathrm{Proj}\colon\mathbb{R}^l\times\mathcal{P}_\epsilon\to\mathbb{R}^l$ by [18]

$$\mathrm{Proj}(\chi,\theta_a):=\begin{cases}\chi, & \text{if } \chi\in T_{\mathbb{R},\Omega}(\theta_a)\\(1-h(\theta_a))\chi & \text{if } \chi\notin T_{\mathbb{R},\Omega}(\theta_a)\end{cases} \tag{34}$$

where the components of $h(\theta_a)$ are given by

$$h_i(\theta_{a,i})=\begin{cases}0 & \text{if } \theta_{a,i}\in(\underline{\theta}_i,\overline{\theta}_i)\\\min\{1,\frac{\underline{\theta}_i-\theta_{a,i}}{\epsilon_i}\} & \text{if } \theta_{a,i}\leq\underline{\theta}_i\\\min\{1,\frac{\theta_{a,i}-\overline{\theta}_i}{\epsilon_i}\} & \text{if } \theta_{a,i}\geq\overline{\theta}_i\end{cases} \tag{35}$$

Lemma 2. *The projection operator* (34) *satisfies [18]*

(P1) *The mapping* $\mathrm{Proj}\colon\mathbb{R}^l\times\mathcal{P}_\epsilon\to\mathbb{R}^l$ *is Lipschitz continuous in* χ *and* θ_a.

(P2) *The differential equation*

$$\dot{\theta}_a=\mathrm{Proj}(\chi,\theta_a),\quad \theta_a(t_0)\in\mathcal{P}_\epsilon, \tag{36}$$

satisfies $\theta_a\in\mathcal{P}_\epsilon$ *for all* $t\geq t_0$.

(P3) *Let* $\theta_e=\theta-\theta_a$ *denote the estimation error, then*

$$-\langle\theta_e,\Gamma^{-1}\mathrm{Proj}(\chi,\theta_a)\rangle\leq-\langle\theta_e,\Gamma^{-1}\chi\rangle, \tag{37}$$

for all $\theta_a\in\mathcal{P}_\epsilon$ *and* $\theta\in\mathcal{P}$.

Define $x:=(g_e,q,\nu_s,r,\zeta,\theta_a)\in\mathcal{X}$ and the extended state space

$$\mathcal{X}:=\mathrm{SE}(2)\times\mathcal{Q}\times\mathbb{R}^3\times\mathcal{R}\times\mathbb{R}^3\times\mathcal{P}_\epsilon. \tag{38}$$

The control objective is to ensure global stability of the set

$$\mathcal{A}_1=\{x\in\mathcal{X}:g_e=e,\nu_s=0,\zeta=0,q=3,\theta_a=\theta\}, \tag{39}$$

and ensuring that every solution to $\mathscr{H}$ converges to

$$\begin{aligned}\mathcal{A}_2=\{x\in\mathcal{X}:g_e=e,\nu_s=0,\zeta=0,q=3,\\\Phi(e,0,0,r)\theta_e=0\}.\end{aligned} \tag{40}$$

Using the projection operator defined in (34), for each $q\in\mathcal{Q}$, an adaptive version of (28) is given by

$$\mathscr{C}:\begin{cases}\left.\begin{aligned}\dot{\zeta}&=-\nabla_{\nu}^{\Lambda}\zeta-\Lambda^{-1}\big(\mathrm{d}V_q(g_e)+\vartheta_q(\zeta)\big)\\\dot{\theta}_a&=\mathrm{Proj}(-\Gamma\Phi(g_e,\zeta,\nu_s,r)^{\mathsf{T}}\nu_s,\theta_a)\end{aligned}\right\} & (g_e,q)\in\mathcal{C}\\q^+\in\mathcal{G}_1(g_e,q) & (g_e,q)\in\mathcal{D}\\\tau=\Phi(g_e,\zeta,\nu_s,r)\theta_a-\mathrm{d}V_q(g_e)-\varphi_q(\nu_s),&\end{cases} \tag{41}$$

where $\varphi\colon\mathbb{R}^3\times\mathcal{Q}\to\mathbb{R}^3$ is such that $\varphi_q(\nu_s)^{\mathsf{T}}\nu_s>0$, for each $q\in\mathcal{Q}$, and for all $\nu_s\neq0$. The adaptive hybrid control law (41) leads to the hybrid closed-loop system

$$\mathscr{H}:\begin{cases}\left.\begin{aligned}\dot{g}_e&=g_e(\nu_s+\zeta)^\wedge\\\dot{\nu}_s&=\tilde{f}(x)\\\dot{s}&=\varrho(s,u_d)\\\dot{u}_d&=a_d\\\dot{a}_d&=p(u_d,a_d,\mu)\\\mu&\in[0,c]\\\dot{\zeta}&=-\nabla_{\nu}^{\Lambda}\zeta-\Lambda^{-1}\big(\mathrm{d}V_q(g_e)+\vartheta_q(\zeta)\big)\\\dot{\theta}_a&=\mathrm{Proj}(-\Gamma\Phi(g_e,\zeta,\nu_s,r)^{\mathsf{T}}\nu_s,\theta_a)\end{aligned}\right\} & x\in\widetilde{\mathcal{C}}\\(s^+,q^+)\in\mathcal{G}(g_e,q,s) & x\in\widetilde{\mathcal{D}},\end{cases} \tag{42}$$

where the extended jump map, flow set, and jump set are defined by

$$\begin{aligned}\mathcal{G}(g_e,q,s)&:=(0,q)\cup(s,\mathcal{G}_1(g_e,q))\\&=\begin{cases}(s,\mathcal{G}_1(g_e,q)), & (g_e,q,s)\in\mathcal{D}\times(\mathcal{I}\backslash\{1\})\\\{(s,\mathcal{G}_1(g_e,q)),(q,0)\} & (g_e,q,s)\in\mathcal{D}\times\{1\}\\(0,q), & (g_e,q,s)\in(\mathcal{C}\backslash\mathcal{D})\times\{1\}\end{cases}\end{aligned} \tag{43}$$

and

$$\begin{aligned}\tilde{f}(x)&:=-M^{-1}\Phi(g_e,\zeta,\nu_s,r)\theta_e-\nabla_{\nu}^{M}\nu_s\\&\qquad-M^{-1}(\mathrm{d}V_q(g_e)+\varphi_q(\nu_s)).\end{aligned} \tag{44}$$

We note that the closed-loop system $\mathscr{H}$ satisfies the hybrid basic conditions [19, Lemma 2.21].

Theorem 1. *The set* $\mathcal{A}_1$ *is uniformly globally stable for the hybrid system* $\mathscr{H}$, *and every solution to* $\mathscr{H}$ *converges to* $\mathcal{A}_2$.

Proof. Consider the continuously differentiable function

$$\begin{aligned}W(g_e,q,\nu_s,\zeta,\theta_a) = V_q(g_e) + \frac{1}{2}\langle \nu_s, M\nu_s\rangle \\ + \frac{1}{2}\langle \zeta, \Lambda\zeta\rangle + \frac{1}{2}\langle \theta_e, \Gamma^{-1}\theta_e\rangle.\end{aligned} \tag{45}$$

Differentiating W along flows of $\mathscr{H}$ yields

$$\begin{aligned}&\langle \mathrm{d}V_q(g_e), \nu_e\rangle + \langle \zeta, -\mathrm{d}V_q(g_e) - \vartheta_q(\zeta)\rangle \\ &+ \langle \nu_s, -\Phi\theta_e - M\nabla_\nu^M \nu_s - \mathrm{d}V_q(g_e) - \varphi_q(\nu_s)\rangle \\ &- \langle \theta_e, \Gamma^{-1}\mathrm{Proj}(-\Gamma\Phi^\mathsf{T}\nu_s, \theta_a)\rangle,\end{aligned} \tag{46}$$

which simplifies to

$$\begin{aligned}&- \langle \nu_s, \varphi_q(\nu_s)\rangle - \langle \zeta, \vartheta_q(\zeta)\rangle \\ &\quad - \langle \theta_e, \Gamma^{-1}\mathrm{Proj}(-\Gamma\Phi^\mathsf{T}\nu_s, \theta_a) + \Phi^\mathsf{T}\nu_s\rangle \\ &\le -\langle \nu_s, \varphi_q(\nu_s)\rangle - \langle \zeta, \vartheta_q(\zeta)\rangle \\ &\le 0,\end{aligned} \tag{47}$$

where the first inequality follows from (P3) in Lemma 2. For any $(g_e, q, \nu_s, r, \zeta, \theta_a) \in \widetilde{\mathcal{D}}$ and $(m, z) \in \mathcal{G}(g_e, q, s)$, the change in W across jumps is

$$\begin{aligned}W(g_e, z, \nu_s, \zeta, \theta_a) - W(g_e, q, \nu_s, \zeta, \theta_a) \\ = V_z(g_e) - V_q(g_e),\end{aligned}$$

which is clearly equal to zero when $(g_e, q, s) \in (\mathcal{C}\backslash\mathcal{D}) \times \{1\}$, i.e. when $z = q$. Otherwise, it follows from Lemma 1 that $V_z(g_e) - V_q(g_e) \le 0$ for all $(q, z) \in \mathcal{Q} \times \pi_\mathcal{Q}(\mathcal{G}(g_e, q, s))$. Consequently, the growth of W along solutions to $\mathscr{H}$ is bounded by

$$u_c(x) = \begin{cases} -\langle \nu_s, \varphi_q(\nu_s)\rangle - \langle \zeta, \vartheta_q(\zeta)\rangle, & \text{if } x \in \widetilde{\mathcal{C}} \\ -\infty, & \text{otherwise}\end{cases} \tag{48}$$

$$u_d(x) = \begin{cases} 0, & \text{if } x \in \widetilde{\mathcal{D}} \\ -\infty, & \text{otherwise}\end{cases} \tag{49}$$

along flows and across jumps, respectively. Since W is positive definite on $\widetilde{\mathcal{C}} \cup \widetilde{\mathcal{D}}$ with respect to the compact set $\mathcal{A}_1$, and for any $m > 0$, the set $\{x \in \mathcal{X} : W(g_e, q, \nu_s, \zeta, \theta_a) \le m\}$ is compact, it follows that $\mathcal{A}_1$ is uniformly globally stable. Therefore, since W is continuous, $\mathscr{H}$ satisfies the hybrid basic conditions, and every maximal solution to $\mathscr{H}$ is complete, it follows from [20, Corollary 8.7 (b)] that each solution to $\mathscr{H}$ converges to the largest weakly invariant subset Ψ contained in

$$W^{-1}(r) \cap \left\{\overline{u_c^{-1}(0)} \cup \left(\overline{u_d^{-1}(0)} \cap G\big(\overline{u_d^{-1}(0)}\big)\right)\right\}, \tag{50}$$

for some $r \in \mathbb{R}$, where

$$\begin{aligned}\overline{u_c^{-1}(0)} &= \{x \in \mathcal{X} : \nu_s = 0, \zeta = 0, (g_e, q) \in \mathcal{C}\}, \\ \overline{u_d^{-1}(0)} &= \widetilde{\mathcal{D}}.\end{aligned} \tag{51}$$

The system $\mathscr{H}$ permits at most two consecutive jumps before a non-zero time of flow follows. Hence, $\Psi \subset W^{-1}(r) \cap \overline{u_c^{-1}(0)}$. It follows from (20), (21) and (22) that $\mathrm{d}V_q(g_e) = 0$ implies that $\log R = 0$, $q = 3$ and $p_e = 0$, which is equivalent to $(g_e, q) = (e, 3)$. Moreover, the closed-loop system (42) is such that $\zeta \equiv 0$ implies $\mathrm{d}V_q(g_e) \equiv 0$. Additionally, $\nu_s \equiv 0$ implies that $\Phi(g_e, \zeta, \nu_s, r)\theta_e \equiv 0$, which results in

$$\Psi \subset W^{-1}(r) \cap \overline{u_c^{-1}(0)} \subset W^{-1}(r) \cap \mathcal{A}_2 \subset \mathcal{A}_2.$$

Consequently, since every solution is complete and bounded, every solution converges to $\mathcal{A}_2$. □

IV. Simulations

This section presents simluation results using the C/S Inocean Cat I Drillship [21]. The desired path is given by the C^r-loop $\gamma(s) = (p_d(s), R_d(s)) \in \mathrm{SE}(2)$, where $p_d(s) = 5(\cos s, \sin 2s) \in \mathbb{R}^2$ and

$$R_d(s) = \frac{5}{|p_d'(s)|}\begin{pmatrix} -\sin s & -2\cos 2s \\ 2\cos 2s & -\sin s\end{pmatrix} \in \mathrm{SO}(2), \tag{52}$$

which assigns the desired ship heading as the tangent vector along the path. We consider an irrotational ocean current $V_c = U_c(\cos\beta, \sin\beta, 0)$, where $U_c = 0.05\,\mathrm{m/s}$ is the current speed and $\beta = \frac{\pi}{6}$ is the current direction. The current velocity in the body frame is denoted by $\nu_c = R^\mathsf{T} V_c$. By defining the relative velocity $\tilde{\nu} = \nu - \nu_c$, the simulation model is given by

$$\dot{g} = g\widehat{\tilde{\nu}}, \tag{53a}$$

$$M[\dot{\tilde{\nu}} + \nabla_{\tilde{\nu}}^M \tilde{\nu}] = d(\tilde{\nu}) + \tau, \tag{53b}$$

where

$$M = \begin{pmatrix} 138 & 0 & 0 \\ 0 & 233 & 1 \\ 0 & 1 & 65\end{pmatrix}, \tag{54}$$

$$\begin{aligned} d(\tilde{\nu}) = &-\underbrace{\begin{pmatrix} 5.3 & 0 & 0 \\ 0 & 10 & 7.3 \\ 0 & 0 & 15\end{pmatrix}}_{D}\tilde{\nu} \\ &-\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.9|\tilde{\nu}_2| + 0.8|\omega| & 0.8|\tilde{\nu}_2| + 3.5|\omega| \\ 0 & 0.21|\tilde{\nu}_2| - 0.08|\omega| & -0.08|\tilde{\nu}_2| + 10|\omega|\end{pmatrix}\tilde{\nu},\end{aligned} \tag{55}$$

while the control model is given by (3) with $d(\nu) = D\nu$. The resulting parameter vector is then

$$\theta = (138, 233, 1, 65, b, 5.3, 10, 7.3, 15) \in \mathbb{R}^{11}. \tag{56}$$

The desired speed reference is given by $\mu = 0.1\,\mathrm{m/s}$, while the parameters in (23) and (24) are $\epsilon = \frac{\pi}{18}$ and $\delta = \frac{\pi}{6}$. The control gains are given by $K = \mathrm{diag}(1, 1)$, $k = 0.05$, $k_3 = 0.4$, $\varphi_q(\nu_s) = \mathrm{diag}(0.5, 0.5, 1)\nu_s$, $\vartheta_q(\zeta) = \mathrm{diag}(0.5, 0.5, 1)\zeta$ and $\Lambda = I_3$. Moreover, the adaptation gain and the initial guess for $\theta \in \mathbb{R}^{11}$, as well as the bounds on θ are given by

$$\begin{aligned}\Gamma &= 50\,\mathrm{blkdiag}(25, 35, 0.5, 10, 0.01I_3, 2I_4), \\ \theta_{a,0} &= (70, 130, 0.5, 50, 0_{7\times 1}) \\ \underline{\theta} &= (0, 0, -20, 0, -0.1, -0.1, -0.05, 0_{4\times 1}), \\ \overline{\theta} &= (250, 250, 20, 70, 0.1, 0.1, 0.05, 10, 10, 10, 10),\end{aligned}$$

and the boundary layers $\epsilon_i = 0.3$ for all $i \in \{1, 2, \dots, 11\}$.

The system is initialized at $g_0 = (p_0, R_0)$ where $p_0 = (5, 0)$ and $R_0 = -I_2$ with $q_0 = 1$ and $\nu = 0$. Simulation results are shown in Figs. 1 to 6. Even though the control

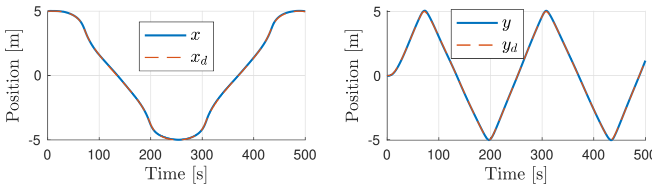

Fig. 1. The position (x, y) and desired position (x_d, y_d).

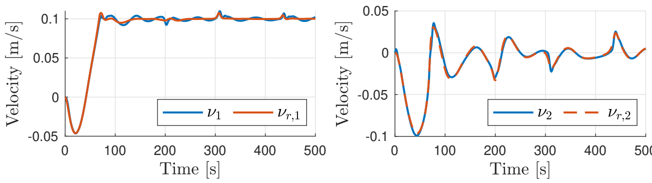

Fig. 2. The linear body velocities ν and the desired linear body velocities ν_r mapped to the body frame.

model does not accurately account for the dynamic effect of the ocean currents, it is clear from Figs. 1 to 5 that the vehicle tracks the reference with increasing accuracy. The increased tracking accuracy is due to the adaptation law, and is especially apparent in Fig. 5. Finally, we note that the discontinuity in τ_3 at approximately $t \simeq 70\,\mathrm{s}$ is a consequence of our choice of a higher proportional gain for the local controller.

V. Conclusion

In this paper, we proposed a hybrid adaptive feedback law for global asymptotic tracking for marine surface vehicles in the presence of parametric uncertainties. Furthermore, we formulated a hybrid reference system generating continuous and bounded configuration, velocity and acceleration references from an r-times continuously differentiable parametric loop and a given speed assignment.

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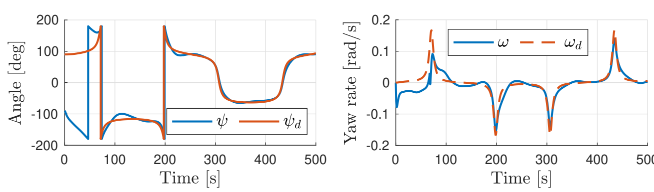

Fig. 3. The heading angle $\psi = (\log R)^\vee$, desired heading angle $\psi_d = (\log R_d)^\vee$, angular velocity ω and desired angular velocity ω_d.

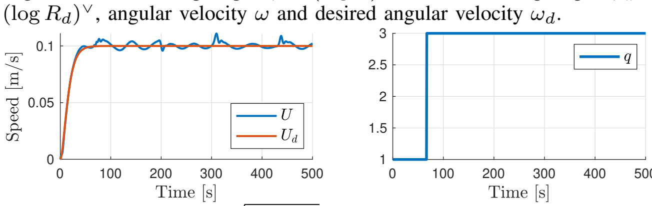

Fig. 4. The speed $U = \sqrt{\nu_1^2 + \nu_2^2}$, desired speed U_d and logic variable q.

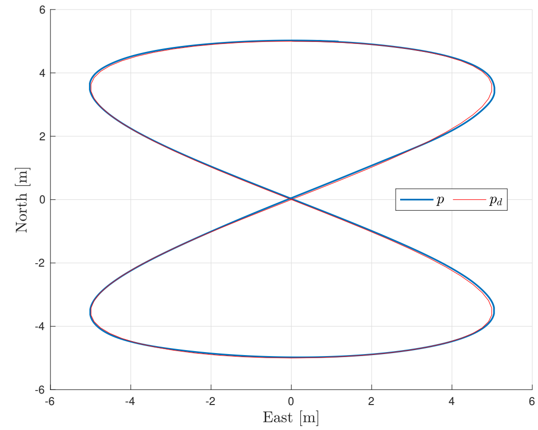

Fig. 5. North-East plot showing the position p and the desired position p_d.

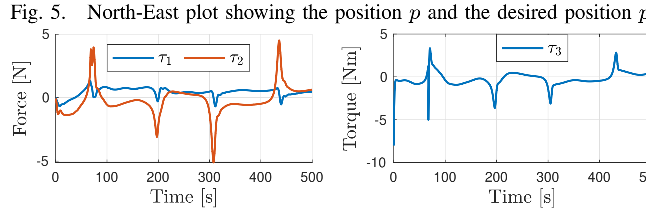

Fig. 6. The control forces and moment τ.

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