

## Early field planning using optimisation and considering uncertainties Study case: Offshore deepwater field in Brazil

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### ABSTRACT

During early phases of oil field development, field planners must decide upon the optimal number of wells and optimal field plateau rate, usually by performing sensitivity studies. These design choices are then “frozen” in subsequent development stages. However, they often end up being suboptimal when the field is built and produced and the uncertainty is reduced.

In this work, we employ non-linear numerical optimisation, latin hypercube sampling and the Schwartz & Smith oil price model to compute probability distributions of the optimal number of wells, plateau rate and project value. We also employ an analytical model to compute production profiles and project value and consider uncertainties in in-place oil volume, well productivity and oil price. Then, we study how do these distributions change from early field planning until when the field is abandoned, when uncertainties are reduced to a minimum. The variation in time of the in-place oil volume uncertainty is modelled with a random walk. The well productivity is a step function altered randomly after production startup. The actual oil price trajectory is picked randomly from possible trajectories computed with the Schwartz and Smith model.

The results show that the distributions of the optimal number of wells, plateau rate and project value depend greatly on the uncertainties in the input data. Field designs based on the average of the distributions during the early phase are profitable, but suboptimal. A potential upside of such designs is that they entail less capital investment and therefore less financial risk when compared against the optimal field design.

### 1. Introduction

Field development is a complex task which demands large sums of capital (CAPEX) and operational (OPEX) expenditures to produce hydrocarbons. The required number of wells and the field production schedule are parameters that field planners must decide upon at an early stage with limited information, having a large impact on the economic feasibility of the project (Nystad, 1985; Haldorsen, 1996). A high number of wells and a highly developed production system allows to recover and sell hydrocarbons earlier, thus minimising the effect of cash flow discounts, and so contributing to a higher project net present value (NPV). However, the high number of wells, production equipment and higher volumes of produced fluids imply higher drilling expenditures and more expensive topside facilities. Therefore, the project NPV is impacted negatively. This trade-off is illustrated in Fig. 1.

The optimal number of wells and production schedule depends mainly on the expected well performance, reservoir size and oil price. For example, if we believe the reservoir is large, more wells will be required to produce the reserves in a feasible time. If the well

performance is assumed to be high, fewer wells will be needed to recover reserves earlier, thus ensuring high profitability of the project, and vice-versa. It also depends on oil price over time. In general terms, the oil production will be stimulated if the oil price is high, and if the oil prices are low, the investments will be slowed down, thus cutting down excess equipment from the design.

At the start of field development planning, information related to reservoir characteristics and well performance is scarce. Reservoir information at the early stages of development is vague and uncertain. As the field is developed and the reservoir is produced, more information is obtained, discovering the true reservoir characteristics.

However, decisions about the required number of wells, production schedule and topside facility size and capacity need to be taken at an early stage. While in some cases, it is possible to conduct exploration and appraisal campaigns to gather more information about the subsurface and hopefully reduce the uncertainty, it is often expensive, and the remaining uncertainty is still considerable. The uncertainty can

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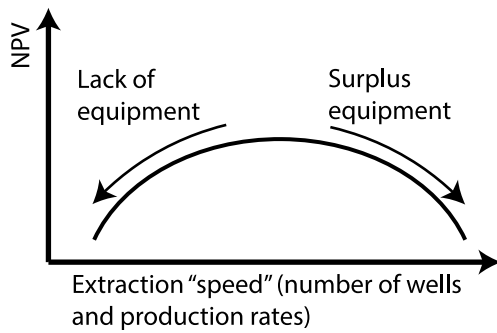


Fig. 1. Trade-off between NPV and extraction rate.

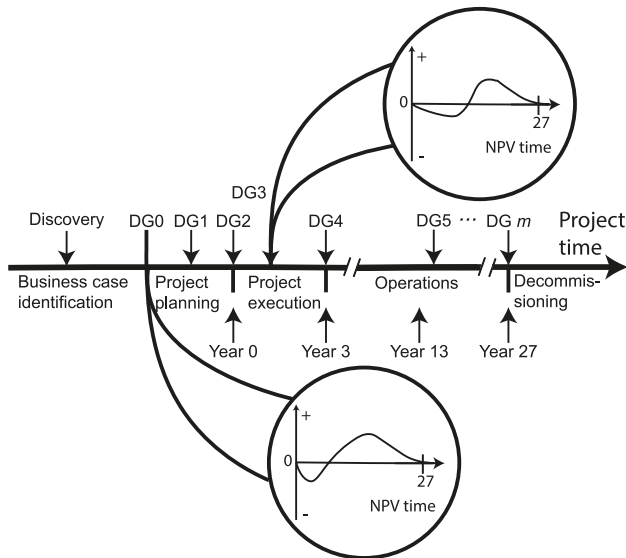


Fig. 2. General field development timeline.

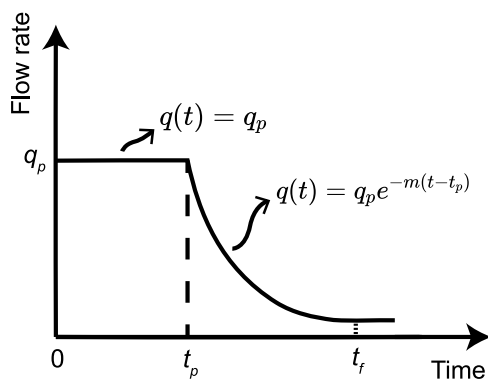


Fig. 3. Oil production profile model.

usually only be reduced to a minimum with production data, that is, after starting production itself.

There are several studies on the literature that help to understand the decision environment of field development, such as Lin (2008), Haugland et al. (1988) Jonsbråten (1998) and more recent ones (Isebor and Durlofsky, 2014; Rodrigues et al., 2016; Chen et al., 2017; Basilio et al., 2018; Melo and Valença, 2019; Rocha et al., 2019). To aid in this decision-making process, typically a model of the value chain is employed to determine best design parameters that yield highest economic profit (Hanea et al., 2019; McLachlan et al., 2019; Sauve

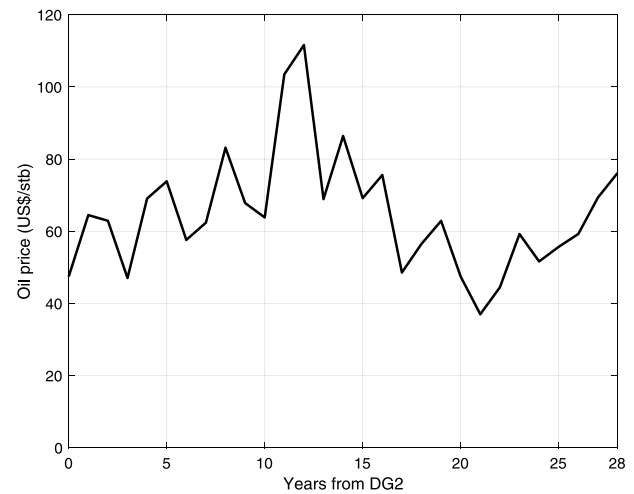


Fig. 4. Example of an oil price trajectory.

et al., 2019). Recently, Nunes et al. (2018) proposed a deterministic model to determine the optimal number of wells, applying it to a pre-salt field in Brazil. However, uncertainties were not considered.

During early phases of oil field development, field planners decide upon the optimal number of wells and optimal field plateau rate, usually by performing sensitivity studies. These design choices are then “frozen” in subsequent development stages. However, they often end up being suboptimal when the field is built and produced and the uncertainty is reduced. Therefore, the main question we try to answer in the present study is: if we were designing the field at an early stage with uncertain information and using rigorous methods to quantify uncertainty, how much would it differ from the actual best design if we had perfect information (i.e. uncertainties reduced to none)? For this comparison, we simulate the design of the field at an early stage, considering uncertainties. Then, we simulate the design as we could predict future events and have more precise information about the field parameters.

In the present work, the objective is to find out the probability distribution of the optimal number of wells, plateau rate and project NPV. To this, we employ a non-linear numerical optimisation, latin hypercube sampling and the Schartz & Smith oil price model to compute probability distributions. The methodology is applied and discussed using a synthetic study case of a pre-salt field offshore Brazil, based on the one discussed by Nunes et al. (2018).

In the next section, the information about the synthetic case study is presented, and the optimisation model employed is described. Later, the uncertain parameters considered are described, and the methodology to quantify uncertainty is thoroughly explained. We present and discuss the results, and finally conclude about the model’s performance and its applicability, relevance and potential advantages for early field development planning.

## 2. Model description

An optimisation problem has been formulated to maximise NPV by varying number of wells and plateau rate. First, we will study a deterministic approach to this problem. Then, we present the modelling of uncertainties, dividing it into three types (in-place volume, well performance, and oil price).

Fig. 2 shows a general timeline for field development. Decision gates (DG) are milestones in the project that involve major decisions. Along the field life, uncertainties regarding reservoir volume decrease. Well performance uncertainties are eliminated after production starts. As the oil price is controlled by many external factors, oil price oscillations

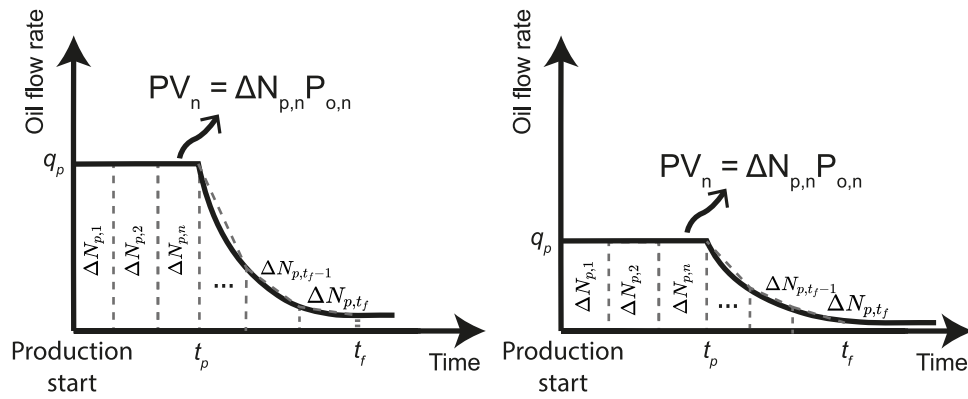


Fig. 5. Net present value of revenues.

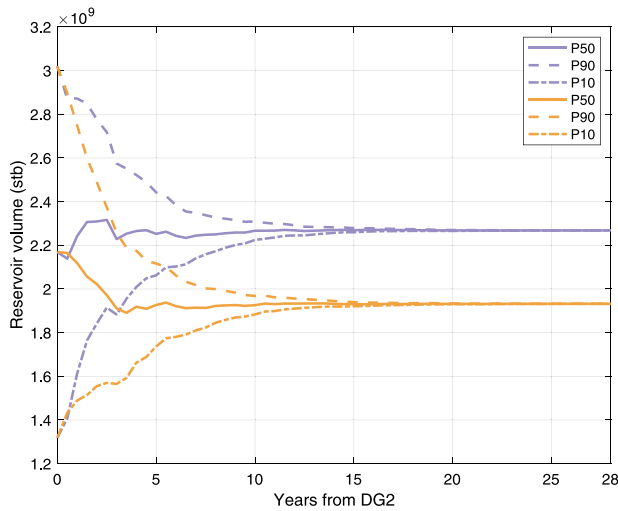


Fig. 6. Trajectories of in-place volumes.

happen spontaneously, so in general oil price uncertainties do not increase or decrease with time. We aim to simulate these uncertainties happening in the same manner.

As shown in Fig. 2, we analyse two time-spans. The decision gate timeline takes place on a day-to-day scenario, following the calendar. The studied DG can be before, during, and after the project's lifetime. Thus, we will reference it as field development project time. Nonetheless, the cash-flow analysis follows a simulated timeline to estimate the project NPV. Therefore, this timeline is contained in the project's lifetime. Thus, we will reference it as NPV time.

To quantify the value of precise information, we solve the same field development optimisation problem on different years of development. First, we solve the optimisation problem based on poor data, thus simulating the standard design approach, which always happens in decision gate year 0. After, we solve the same problem, now with data from the late field development (that is, having privileged information about the field outcomes), which happens on year 1 onwards. Then, we compare the resulting design of these scenarios. Fig. 2 also shows the years considered for the case study.

The optimisation problem is to maximise the NPV of the overall field development, optimising the number of wells and oil plateau rate. The objective function is shown in Eq. (1), where  $R$  means royalties,  $T$  are taxes, and  $NPV_R$  is the net present value of revenues (estimated using Eq. (2)).

$CAPEX_{FPSO}$ ,  $CAPEX_{WELLS}$  and  $CAPEX_{SUBSEA}$  are the capital expenditures (CAPEX) of wells, FPSO and subsea (shown in Eqs. (9)–(11)), where wells and subsea CAPEX are a function of  $N_w$ , and FPSO

CAPEX is a function of  $q_p$ . To derive these CAPEX equations, we assumed the following from Nunes et al. (2018):

- There are only 9 water injectors in the field, independent of the number of producers employed;
- The CO<sub>2</sub>, H<sub>2</sub>S, and sulphate removal unit factors are set to 1;
- The water depth is 2 000 m;
- Each well has a pipeline associated with it, with an average length of 6 000 m;
- Each subsea manifold can accommodate up to 4 producing wells;
- The cost of installation of flowlines is 2 000 USD/m.

The oil production profile is computed using the analytical expression in Eq. (3) and illustrated in Fig. 3, which is constant while the field production potential is above the field plateau rate and declines exponentially in time after the plateau period. The plateau duration and decline constant  $m$  are affected by the original oil-in-place (OOIP) and well performance. The discounted revenues ( $NPV_R$ ) are then computed using the production profile. Details of how to estimate  $NPV_R$  are presented in Appendix.

### Integer variable

$N_w$  Number of wells

### Continuous variables

$q_p$  Field plateau rate (stb/d)  
 $\Delta N_{p,t}$  Oil volume produced at period  $t$  (stb)

### Random variables

$J$  Well productivity index (stb/year bara)  
 $q_0$  Maximum oil well production rate (stb/d)  
 $P_o$  Oil price (USD/stb)  
 $N$  In-place volumes (stb)

### Parameters

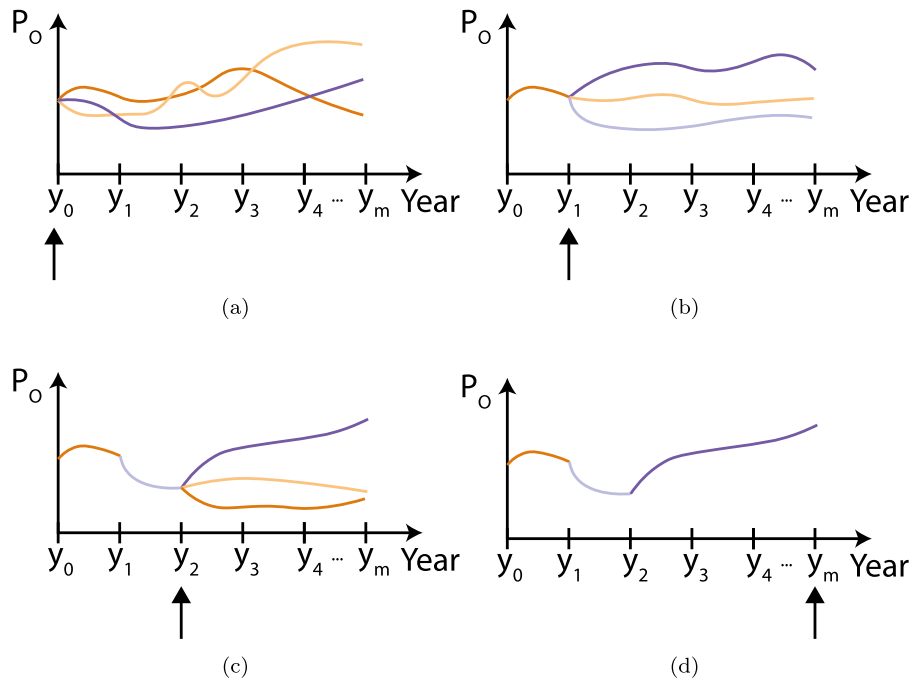
$R$  Royalties (%)  
 $T$  Tax (%)  
 $U$  Uptime (day)  
 $i$  Interest rate (%)  
 $t_f$  Production end (year)

### Mathematical model

$$\max NPV = (1 - R)(1 - T) \cdot NPV_R - CAPEX_{FPSO} - CAPEX_{WELLS} - CAPEX_{SUBSEA} \quad (1)$$

where

$$NPV_R = \sum_0^{t_f} \frac{P_{o,t} \Delta N_{p,t}}{(1 + i)^t} \quad (2)$$

Fig. 7. Oil price uncertainty over time, with  $p = 3$ .

$$q(t) = \begin{cases} q_p & t \leq t_p \\ q_p e^{-m(t-t_p)} & t > t_p \end{cases} \quad (3)$$

$$q_0 = N_w \cdot q_{0,w} \quad (4)$$

$$t_p = \frac{1}{m} \left( \frac{q_0}{q_p} - 1 \right) \quad (5)$$

$$m = A \cdot N_w \cdot J \quad (6)$$

$$A = \frac{a_1}{N} \quad (7)$$

$$q_p = q_0 \cdot N_w - m \cdot \sum_{t=0}^{t=t_p} \Delta N_{p,t} \quad (8)$$

$$CAPEX_{WELLS} = a_2 + a_3 \cdot N_w \quad (9)$$

$$CAPEX_{FPSO} = a_4 + a_5 \cdot q_p \quad (10)$$

$$CAPEX_{SUBSEA} = a_6 + a_7 \cdot \text{round} \left( \frac{N_w}{4} \right) + a_8 \cdot N_w \quad (11)$$

Subject to

$$q_p \leq q_0 \quad (12)$$

$$\sum_{t=0}^{t=t_f} \Delta N_{p,t} \leq N \quad (13)$$

$$N_w, q_p, J, q_0, N, \Delta N_{p,t}, P_o \geq 0 \quad (14)$$

where  $a_1 = 9.76 \times 10^2$  bara/stb well;  $a_2 = 1.35 \times 10^9$  USD;  $a_3 = 1.50 \times 10^8$  USD/well;  $a_4 = 1.07 \times 10^9$  USD;  $a_5 = 2.51 \times 10^3$  USD/bpd;  $a_6 = 4.93 \times 10^8$  USD;  $a_7 = 3.20 \times 10^7$  USD/well;  $a_8 = 9.20 \times 10^7$  USD/well.

There are three main constraints in the model. First, the total well production rate at the plateau must be lower than the maximum field production rate Eq. (12). Second, the oil volume produced must be less than the in-place volumes Eq. (13). Third, all variables must be nonnegative Eq. (14).

### 3. Uncertainty description and handling

One of the uncertainties considered here is related to in-place volumes ( $N$ ). To describe this uncertainty, we parametrised a log-normal distribution with  $\mu = 2.17 \times 10^9$  bbl and  $\sigma = 2.67 \times 10^{17}$  bbl. This PDF represents the in-place volume uncertainties existing at the beginning of the field planning phase, and we use it as *a priori* distribution for the following years. It is assumed to be log-normal because this is the typical probability distribution of  $N$  when estimating reserves (Ian and Noeth, 2004).

The well performance ( $J$ ) and maximum well production ( $q_0$ ) uncertainties are represented by a factor  $F$ , where we attribute a uniform distribution between 0.4–1.6 to it, if information about well performance is not known yet (because the field has not started production). At the moment the first well is drilled, well performance information is known, and it will either assume a low productivity or a high productivity well.

Lastly, we consider that the oil price varies in project time in a random manner according to the Schwartz and Smith (2000) price model, using the discretised version presented on Jafarizadeh and Bratvold (2012) and taking the oil price parameters from the work of Thomas and Bratvold (2015). These parameters were computed tuning the distribution to historic price data. The price model is a two-factor model that allows mean-reversion in short-term prices and uncertainty in the equilibrium level to which prices revert. Fig. 4 shows an example of an oil price trajectory generated by this model. A set of oil price trajectories defines an oil price ensemble.

#### 3.1. Sampling of uncertain variables

The production profile is affected by the uncertainties in in-place volume, well productivity, and plateau rate (the last two through  $F$ ). To quantify the effect these random parameters have on the optimisation results,  $n$  samples of  $N$  and  $F$  (each) are drawn using latin hypercube sampling (LHS) and inserted in the optimisation problem. A Monte Carlo Simulation was initially used instead of LHS, but LHS was found to be more computationally efficient (10 times fewer samples were required to achieve the same results).

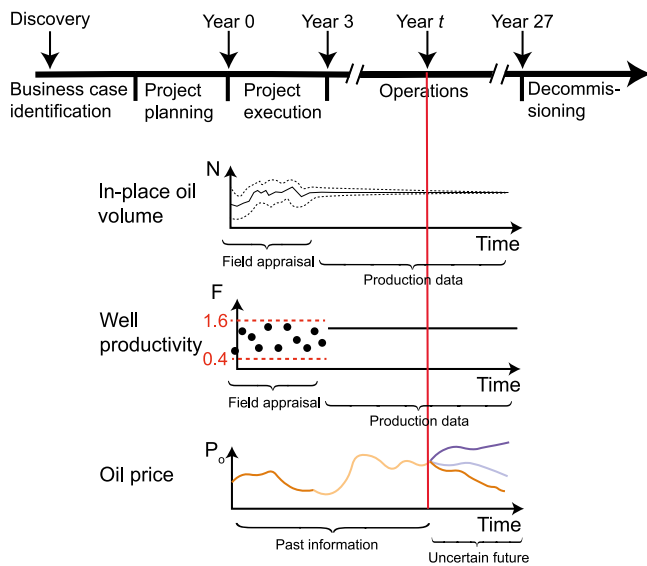


Fig. 8. Uncertainties over time.

For the oil price uncertainties, the Schwartz & Smith model generates trajectories similar to the one shown in Fig. 4. Based on these trajectories, we obtain the discounted revenue NPV by multiplying the oil produced in that year times the average oil price in that year Eq. (2), as illustrated in Fig. 5 for two examples of possible production curves. We do this for several trajectories, in order to consider several scenarios of  $NPV_R$ .

### 3.2. Uncertainties over field development project time

Besides characterising and sampling uncertainties, we want to model their evolution in time at later field development stages. To this, we simulate how uncertainties in in-place volumes ( $N$ ), well performance ( $J, q_0$ ) and oil price ( $P_o$ ) change during the oil field life. In this section, we present each specific approach given to them.

#### Oil in-place uncertainty

Uncertainties in in-place volumes over field development project time are simulated using a Bayesian approach as presented by Lin (2008). An initial probability distribution function (PDF) for the reserve estimates must be given. This is not an issue, because initial oil-in-place estimates are generally available in early phases of field development created with exploration data, seismic and exploration wells. As we mentioned, we use a log-normal distribution with  $\mu_0 = 2.17 \times 10^9$  bbl and  $\sigma_0 = 5.17 \times 10^{08}$  bbl.

Then, the average  $\mu_i$  and  $\sigma_i$  are updated over field development project time, generating a random walk. This formulation allows for more significant oscillations in  $\mu$  in the beginning, and lower ones in the end. This is compatible with the real practice of reserves estimation, as wide estimation swings are more common in the beginning, because there are greater uncertainties at that moment.

Since there are many possibilities of in-place volume curves, we only use the resulting trajectories shown in Fig. 6. The upper P90 curve (solid purple) is a scenario where reservoir volume was underestimated at the beginning (it has increasing in-place volume). In contrast, the lower P90 curve (solid orange) represents a scenario where it was over-estimated (it has decreasing in-place volume). For example, on year 10 in the under-estimated scenario, we have a log-normal distribution for  $N$  with  $\mu_{10} = 2.27 \times 10^{09}$  bbl and  $\sigma_{10} = 1.34 \times 10^{08}$  bbl.

#### Well performance uncertainty

Regarding well performance, we assume that before production start, the data about well performance is highly uncertain. So, we multiply the well productivity index ( $J$ ) and the maximum well production

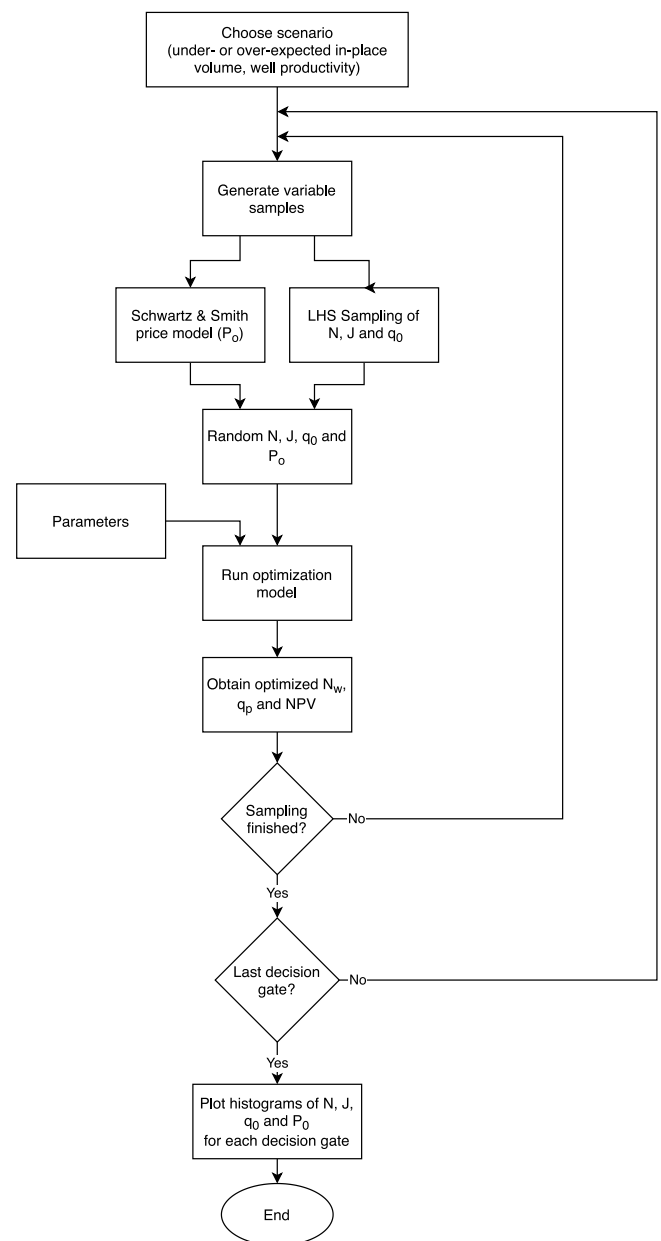


Fig. 9. Simplified view of the method.

( $q_0$ ) by a common factor  $F$ , uniformly distributed between 0.4 – 1.6 before production starts, and then apply LHS for  $F$ .

After production starts, well performance is known, then  $F$  is constant. If we consider a scenario of high well performance,  $F = 1.3$ , otherwise we have a low well performance scenario,  $F = 0.7$ . For example, considering a high well performance scenario, and that production starts at project year 3,  $F$  will assume representative values between 0.4 – 1.6 on project year 0 (due to LHS), and it will assume other representative values between 0.4 – 1.6 on year 2, but it will assume the value  $F = 1.2$  on year 3 onwards.

#### Oil price uncertainty

Fig. 7 illustrates how uncertainties in oil prices are simulated over field development project time. For year  $y_0 = 0$ , we generate  $p$  oil price trajectories until  $y_m$ , where  $y_m$  is the year that the field is abandoned. For the next year  $y_1$ , we randomly select one of the  $p$  oil price trajectories of  $y_0$  as the oil price trajectory that actually happened between  $y_0$  and  $y_1$ , and we generate more  $p$  oil price trajectories from

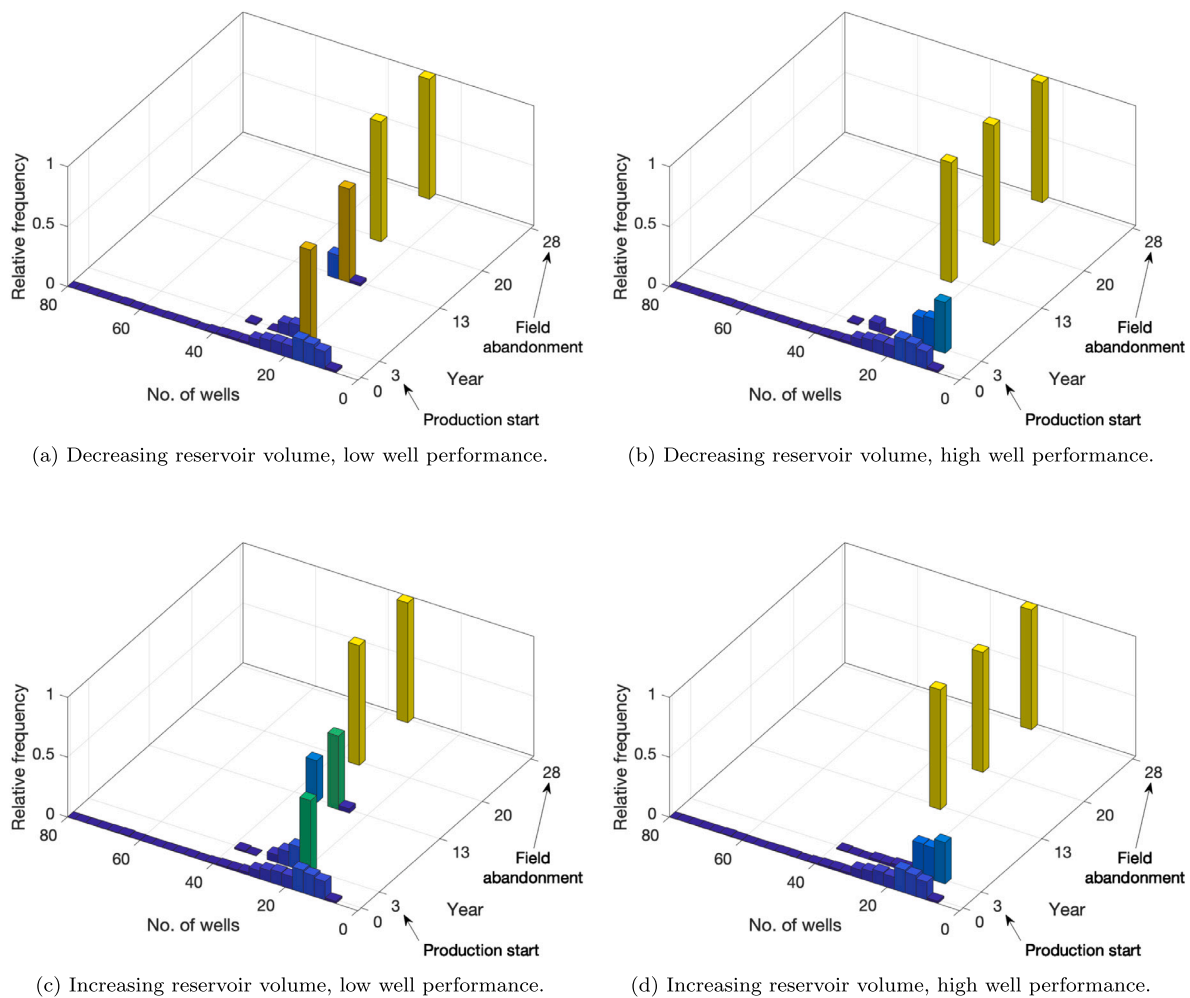


Fig. 10. Histograms of number of wells.

$y_1$  to  $y_n$ . After, we select one of the  $p$  oil price trajectories from  $y_1$  to represent the oil price trajectory that happened between  $y_1$  and  $y_2$ , and so on, until we reach  $y_m$ , where a single trajectory is selected, and none generated.

Fig. 8 shows how uncertainties evolve over field development project time, having year  $t$  as reference. In that year, the in-place oil volume has a log-normal distribution with parameters  $\mu_t$  and  $\sigma_t$ , a well performance factor  $F_t$ . Also, it has a fixed oil price trajectory before year  $t$  and  $p$  samples of Schartz & Smith oil trajectories after year  $t$ . The brackets show the behaviour of the random variable along field development project time.

#### 4. Overall view

The proposed method is summarised in Fig. 9. Samples for three input variables ( $N, J, F$ ) are generated based on LHS and Schwartz & Smith price model. These samples are employed in the optimisation model proposed here. The output variables ( $N_w, q_p$ , and  $NPV$ ) are the main result of the optimisation, being summarised by histograms.

For each scenario, we study a set of years counting from the beginning of project execution (0, 6, 13, 17, and 28). For each year studied, the random variables will be sampled and combined in order to optimise the problem. The oil price ensemble, oil reserves and well performance distribution curves evolve through the years, as we can observe in Fig. 8.

The oil price ensemble will have a single trajectory before the year considered. After the selected year, there exists  $p$  different oil price

paths, therefore we have a sample of  $p$  oil price trajectories for each decision gate iteration. From these  $p$  oil price trajectories, we randomly select one trajectory to be the assigned path.

The in-place volume distribution parameters  $\mu_t, \sigma_t$ , for each year  $t$ , are employed in LHS. For each one of the  $p$  oil price trajectories,  $n$  samples of oil reserves are drawn.

The well performance distribution will vary before and after production start. Before production start, the  $F$  factor will be drawn from a uniform distribution, totalling  $n$  samples for each one of the  $n$  in-place volume LHS samples. After production start,  $F$  will assume a constant value, either 0.7 if we are in a low well performance scenario, or 1.3 otherwise.

Having set values for these random variables, we can run and optimise the deterministic model. As we optimise the problems, we log  $N_w, q_p$ , and  $NPV$  outputs corresponding to each sample, and plot histograms of it, as shown in Figs. 10, 11, and 12. This is the end of the algorithm.

#### 5. Case study

##### 5.1. Study set-up

The case study presented here is based on the data provided by Nunes et al. (2018) paper, which presents a Brazilian pre-salt offshore oil field in 2 000 m water depth. The reservoir will be produced at a constant rate and then decline for a production horizon of 25 years. The NPV of the project is computed considering discounted revenue

**Table 1**  
Scenarios studied here.

	In-place volume (Fig. 6)	Well performance	Oil price ensembles
Scenario 1	Under-estimated (purple curve)	High well performance (+40%)	Random
Scenario 2	Under-estimated (purple curve)	Low well performance (-40%)	Random
Scenario 3	Over-estimated (orange curve)	High well performance (+40%)	Random
Scenario 4	Over-estimated (orange curve)	Low well performance (-40%)	Random

**Table 2**  
Increasing in-place volumes, high well performance.

	Base	Perfect information	Gap
$NPV$	$11.7 \times 10^9$	$13.1 \times 10^9$	11.4%
$N_w$	20	29	45.0%
$q_p$	380 518	754 000	98.2%
$CAPEX_{WELLS}$	$4.35 \times 10^9$	$5.70 \times 10^9$	31.0%
$CAPEX_{FPSO}$	$2.02 \times 10^9$	$2.96 \times 10^9$	46.4%
$CAPEX_{SUB}$	$2.49 \times 10^9$	$3.38 \times 10^9$	35.8%

**Table 3**  
Increasing in-place volumes, low well performance.

	Base	Perfect information	Gap
$NPV$	$7.41 \times 10^9$	$8.54 \times 10^9$	15.2%
$N_w$	19	33	73.7%
$q_p$	266 000	462 000	73.7%
$CAPEX_{WELLS}$	$4.20 \times 10^9$	$6.30 \times 10^9$	50.0%
$CAPEX_{FPSO}$	$1.73 \times 10^9$	$2.23 \times 10^9$	28.4%
$CAPEX_{SUB}$	$2.40 \times 10^9$	$3.78 \times 10^9$	57.6%

from oil sales, and capital expenditures (all executed at production start) due to wells, topside facilities, and subsea components. All wells are assumed identical and enter into production at the same time. The model's source code is available in Sales et al. (2020).

The main differences from Nunes et al. (2018) are:

- Nunes et al. (2018) uses a constant decline factor  $m$  for the decline period which does not depend on  $N_w$ . This is unrealistic as, if there are more wells draining from the reservoir, the constant  $m$  should be larger (it should decline quicker, although with a longer plateau). We have addressed this deficiency by making  $m$  dependent on  $N_w$ .
- The decline factor  $m$  presented by Nunes et al. (2018) should also depend on  $N$ . However, no information about  $N$  is provided in the work by Nunes. To express this dependency, and back-calculate the  $N$  of the case of Nunes, we assume that the reservoir studied is an undersaturated oil reservoir and that its bubble point pressure is very low and never reached during its producing horizon. Using this assumption,  $m = N_w J a_1 / N$ , where  $a_1$  is a constant that depends on oil compressibility ( $c_o$ ), oil formation volume factor ( $B_o, B_{o,i}$ ), connate and formation compressibility ( $c_w, c_f$ ), and oil and water saturation ( $S_o$  and  $S_w$ ):

$$a_1 = \frac{B_o}{[B_{o,i} \cdot (c_o + \frac{c_w S_w + c_f}{S_o})]} \quad (15)$$

where  $B_o = B_{o,i} = 1.127$ ,  $c_o = 9.5 \times 10^{-4} \text{ bar}^{-1}$ ,  $c_w = 4.0 \times 10^{-5} \text{ bar}^{-1}$ ,  $c_f = 4.0 \times 10^{-5} \text{ bar}^{-1}$ ,  $S_o = 0.7$  and  $S_w = 0.3$ ). Therefore,  $a_1 = 976$ .

- The decline factor  $m$  also depends on well productivity,  $J$  (a more productive well will experience a sharper decline and viceversa, although with a longer plateau). To back-calculate well productivity, initial reservoir pressure ( $p_i$ ) is assumed to be 350 bara and

**Table 4**  
Scenario 3: Decreasing in-place volumes, high well performance.

	Base	Perfect information	Gap
$NPV$	$9.86 \times 10^9$	$10.72 \times 10^9$	8.7%
$N_w$	19	26	36.8%
$q_p$	372 451	675 948	81.5%
$CAPEX_{WELLS}$	$4.20 \times 10^9$	$5.25 \times 10^9$	25.0%
$CAPEX_{FPSO}$	$2.00 \times 10^9$	$2.76 \times 10^9$	38.1%
$CAPEX_{SUB}$	$2.40 \times 10^9$	$3.11 \times 10^9$	29.5%

**Table 5**  
Decreasing in-place volumes, low well performance.

	Base	Perfect information	Gap
$NPV$	$6.47 \times 10^9$	$6.89 \times 10^9$	6.3%
$N_w$	20	29	45.0%
$q_p$	280 000	406 000	45.0%
$CAPEX_{WELLS}$	$4.35 \times 10^9$	$5.70 \times 10^9$	31.0%
$CAPEX_{FPSO}$	$1.77 \times 10^9$	$2.09 \times 10^9$	17.9%
$CAPEX_{SUB}$	$2.49 \times 10^9$	$3.38 \times 10^9$	35.8%

$q_{p,w} = 20\ 000 \text{ stb/d}$ . It is also assumed that the minimum flowing bottom-hole pressure ( $p_{w,f}$ ) achievable is 100 bara. This gives:

$$q_{p,w} = J(p_i - p_{w,f}) = 20\ 000 \text{ stb/d} \quad (16)$$

therefore,  $J = 80 \text{ stb}/(\text{year bara})$  considering  $m = 0.13$  and  $N_w = 10$ ,  $N = N_w \cdot J \cdot a_1 / m = 2.19 \times 10^9 \text{ stb}$ .

We tuned the sampling size required to make the average results converge for each random variable. We tested from 5 samples of  $N$  and  $J$ , and 10 samples of oil trajectories, which took on average 13 min to solve, to 20 samples of  $N$  and  $J$ , and 110 samples of oil trajectories, which took around 21 h on the same computer. The smallest sampling size where the results successfully converged are  $n = 15$  samples of  $N$  and  $J$ , and 90 samples of oil trajectories, taking around 18 h. Therefore, this is the sampling size we employ here.

For year 0, 6, 13, and 17, we consider  $p = 90$  oil trajectories  $\times n = 15$  oil reserves samples  $\times n = 15$  well performance samples. For year 28, we have a single sample, as  $N$  and  $F$  are deterministic, and there is only one oil trajectory for  $P_o$ . This means we have in total 20 251 problems to be optimised.

The problem was implemented in Microsoft Excel Visual Basic for Applications (VBA) and solved using the Solver built-in module. The solver employed was the Generalised Reduced Gradient (GRG). The computer has an Intel i9-9900 processor and 32 GB RAM. We observed the solver output codes and noticed that less than 10% of the integer solutions converged within the tolerance of 1%, while the others fully converged. This means that the solver successfully obtained locally optimum solutions for most problems.

As mentioned previously in Sections 3 and 3.2, we studied four scenarios, shown in Table 1. The four scenarios are a combination of under- and over-estimated in-place volumes of  $N$  (compared to the average of the early distribution) and high and low well performance ( $J, q_0$ ). We generate the oil price ensemble randomly, as presented in Section 3.2, but we use the same generated ensembles for all scenarios to establish a clear comparison between them.

## 5.2. Simulation results

Figs. 10–12 show histograms of the samples obtained for each output variable (number of wells, plateau rate, and NPV), with normalised frequency. The second variable on the plots is the year of the information employed to optimise the field development design. Warmer colours represent higher relative frequency.

We notice that the histograms at year 0 are very similar, since the initial values and initial uncertainty are the same. As the information

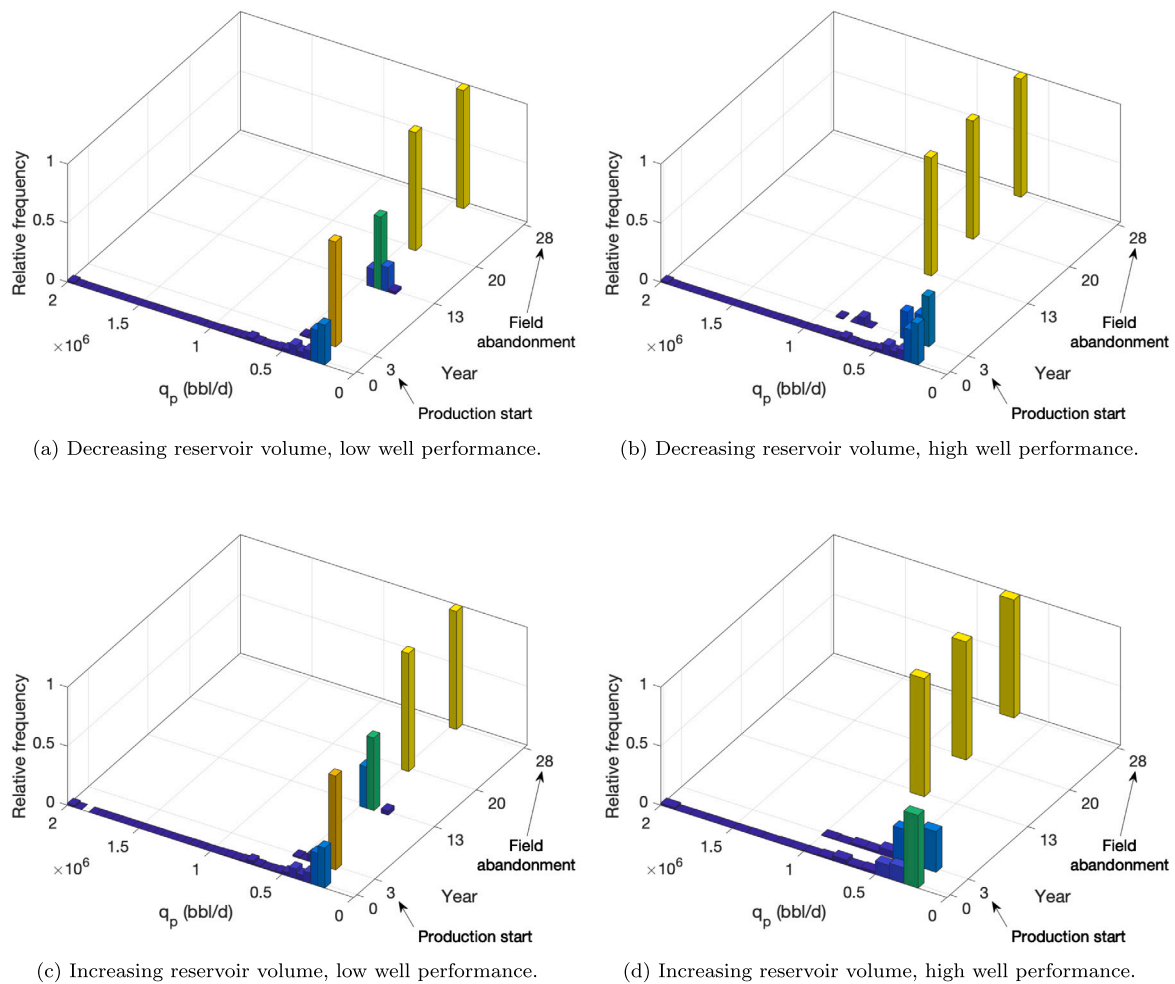


Fig. 11. Histograms of plateau rate.

about the field increases along the years, uncertainty decreases. This is why the histograms obtained decrease in variability over the years. For all variables of a given scenario, we observe that the variables converge. However, if we compare the results between scenarios, we notice that very different designs are obtained, as the variables converge to different values.

We can also observe that for all output variables, including NPV, the expected values at year zero are lower compared to the expected values of the following years. This means that due to the high uncertainties involved, the expected optimised development at year 0 is under-designed.

Tables 2–5 show the expected number of wells and plateau rate for the base scenario (that is, locking the number of wells and plateau rate at year 0), the resulting NPV and CAPEX, and the scenario with perfect information (designing the field with perfect information). The gap column shows the difference of the base scenario and the scenario with perfect information. We observe that the NPV of the design with perfect information is between 5%–15% higher. However, this is a modest NPV increase for the oil field planning phase.

Besides, the 5%–15% NPV increase requires 25%–50% more capital expenditures, which may be a high-risk option to the decision-maker. So, although the base scenario is not ideal, it is proved to be a good option, even if there is only uncertain information available.

## 6. Conclusions

An MINLP combined with LHS and Schwartz & Smith sampling procedures was proposed to determine the probability distribution of

the optimal number of wells, plateau rate and project NPV of an oil field. The in-place oil volume, well productivity, maximum well plateau rate, and oil price were considered as uncertain variables. We simulated the oil field presented by Nunes et al. (2018), and then we computed the probability distribution of the optimal number of wells, plateau rate and NPV at early field design and at subsequent stages in the life of the field where uncertainty is reduced. A total of 20 251 problems were optimised.

The proposed method seems to be a robust approach to quantify uncertainties while computing optimal design and is therefore suitable to provide decision support in early field development. The authors are not aware of other studies that combine these uncertainties simultaneously as in this work.

The method successfully converged for all 20 251 problems that were optimised, and less than 10% of the integer solutions converged within the tolerance of 1%. The total running time was 18 h.

It was found that using LHS allowed to use less samples when compared to Monte Carlo, thus saving considerable simulation time. The computational time for executing the whole method is feasible and suitable for the time frame of early phase field development studies. However, the use of more efficient solvers than Excel's GRG solver could reduce the computational time required and guarantee global optimum solutions. Regardless, the proposed method requires low resources to obtain optimal solutions for the field design problem.

As knowledge about the field is improved and the actual oil price is known, the optimal design changes. Generally, the optimal field design at the beginning of development lacks investment compared to



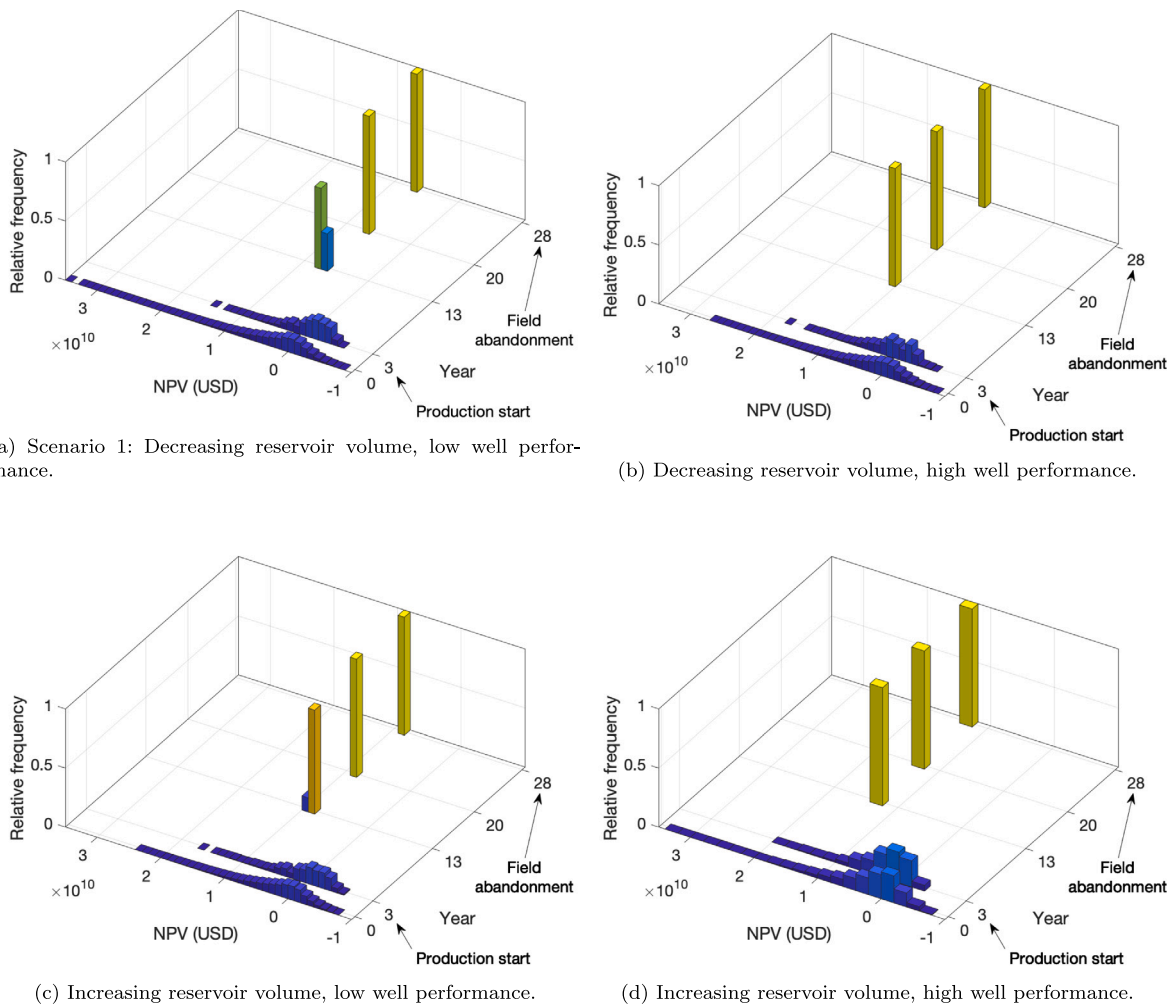


Fig. 12. Histograms of NPV.

the optimal field design with perfect information. The decision-makers should invest 30%–60% more in capital expenditures to obtain the optimal field design.

Although the designs are different, the value of perfect information is modest for the cases studied in this paper, around 3%–15% of the NPV. In a real scenario, we understand that this increase may not worth the risks of investing 30%–60% more in field development, so we conclude that the initial design is a conservative option.

Here, we presented only four scenarios for in-place oil volume and well performance. Further studies must be conducted to determine a more precise range for the value of perfect information and new insights about the problem. To this, other scenarios must be evaluated, varying the in-place volume ensembles and  $F$  after production start. Another suggestion is to study flexibility, that is, start the design with the best-known solution and improve it along time with options theory, and to study uncertainties in CAPEX costs. Finally, we applied the proposed method for a specific field only. More studies are required to guarantee the conclusions shown here to other fields.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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#### Appendix

##### $NPV_R$ Estimation

To estimate  $NPV_R$ , we need to integrate numerically

$$NPV_R = \int_0^{t_f} P_o(t)q(t)e^{-it} dt$$

where  $P_o(t)$  is the oil price,  $i$  is the interest rate, and  $q(t)$  is the flow rate at given time  $t$ . At first, let us consider  $P_o(t) = P_o$  and the field plateau rate as  $q_p$ , thus estimating the decline rate as Eq. (3),

$$q(t) = q_p e^{-m(t-t_p)}$$

where  $t_p$  is the plateau end period, calculated using the maximum well flow rate  $q_0$ , which is the sum of each maximum well flow rate  $q_{0,w}$  as seen in Eqs. (5) and (6),

$$q_0 = N_w \cdot q_{0,w}$$

$$t_p = \frac{1}{m} \left( \frac{q_0}{q_p} - 1 \right).$$

The exponential decline parameter  $m$  is shown in Eq. (6),

$$m = A \cdot N_w \cdot J$$

where  $J$  is the well productivity index,  $N_w$  the number of wells and  $A = a_1/N$  Eq. (7).

The production potential ( $q_{pp}$ ) curve can be employed to perform production planning while avoiding the use of coupled reservoir and production models (González et al., 2020). It is defined as

$$q_{pp}(t) = q_0 - m \cdot \sum_t \Delta N_{p,t}$$

where  $\Delta N_{p,t}$  is the oil volume produced at period  $t$ . The plateau production will end at  $t_p$  when  $q_{pp}(t_p) = q_p$ . So, we insert it as a constraint in the optimisation model Eq. (8),

$$q_p = q_0 - m \cdot \sum_{t=0}^{t=t_p} \Delta N_{p,t}$$

where  $\sum_{t=0}^{t=t_p} \Delta N_{p,t}$  is the oil volume produced until  $t_p$ , given by Eq. (5). Considering  $P_o = P_o(t)$ , we can integrate numerically  $NPV_R$  using Eq. (2).

## References

- Basilio, L., Noronha, C., Passos, M., Calaza, D., Nova, A.L.d., Daly, R., 2018. Integrated design computational model applied to O&G offshore field development. In: Offshore Technology Conference, <http://dx.doi.org/10.4043/28664-MS>.
- Chen, B., Fonseca, R.M., Leeuwenburgh, O., Reynolds, A.C., 2017. Minimizing the Risk in the robust life-cycle production optimization using stochastic simplex approximate gradient. *J. Petrol. Sci. Eng.* 153, 331–344. <https://doi.org/10.1016/j.petrol.2017.04.001>.
- González, D., Stanko, M., Hoffmann, A., 2020. Decision support method for early-phase design of offshore hydrocarbon fields using model-based optimization. *J. Petrol. Explor. Prod. Technol.* 10 (4), 1473–1495. <https://doi.org/10.1007/s13202-019-00817-z>.
- Haldorsen, H.H., 1996. Choosing between rocks, hard places and a lot more: the economic interface. In: Dore, A.G., Sinding-Larsen, R. (Eds.), *Norwegian Petroleum Society Special Publications*. In: Quantification and Prediction of Hydrocarbon Resources, vol. 6, Elsevier, pp. 291–312. [https://doi.org/10.1016/S0928-8937\(07\)80025-7](https://doi.org/10.1016/S0928-8937(07)80025-7).
- Hanea, R.G., Bjorlykke, O.P., Hashmi, Y., Feng, T., Fonseca, R.M., 2019. Robust Multi-Objective Field Development Optimization for the Mariner Asset. Society of Petroleum Engineers, <https://doi.org/10.2118/193883-MS>.
- Haugland, D., Halleford, A., Asheim, H., 1988. Models for petroleum field exploitation. *European J. Oper. Res.* 37 (1), 58–72. [https://doi.org/10.1016/0377-2217\(88\)90280-9](https://doi.org/10.1016/0377-2217(88)90280-9).
- Ian, L., Noeth, S., 2004. *Economics of Petroleum Production: Profit and Risk*, vol. 1, Multi-Science Publishing, Brentwood, United Kingdom.
- Isebor, O.J., Durlafsky, L.J., 2014. Biobjective optimization for general oil field development. *J. Petrol. Sci. Eng.* 119, 123–138. <https://doi.org/10.1016/j.petrol.2014.04.021>.
- Jafarizadeh, B., Bratvold, R., 2012. Two-factor oil-price model and real option valuation: an example of oilfield abandonment. *SPE Econ. Manage.* 4 (03), 158–170. <https://doi.org/10.2118/162862-PA>.
- Jonsbråten, T.W., 1998. *Optimization Models for Petroleum Field Exploration* (Ph.D. thesis). Norwegian School of Economics and Business Administration, Stavanger, Norway.
- Lin, J., 2008. *Exploring Flexible Strategies in Engineering Systems Using Screening Models: Applications to Offshore Petroleum Projects* (Ph.D. thesis). Massachusetts Institute of Technology, Cambridge, USA, URL <https://dspace.mit.edu/handle/1721.1/55173>.
- McLachlan, D.J., Isherwood, J., Peile, M., 2019. Field development: agile value optimisation. Offshore Technology Conference, <https://doi.org/10.4043/29607-MS>.
- Melo, R.T.d., Valença, C.J.G.M., 2019. Efforts and experiences, developing deepwater projects in Brazil. Offshore Technology Conference, <https://doi.org/10.4043/29871-MS>.
- Nunes, G.C., Silva, A.H.d., Esch, L.G., 2018. A cost reduction methodology for offshore projects. Offshore Technology Conference, Houston, USA, <https://doi.org/10.4043/28898-MS>.
- Nystad, A.N., 1985. *Petroleum Reservoir Management: a Reservoir Economic Approach*. Society of Petroleum Engineers, URL <https://www.onepetro.org/general/SPE-13833-MS>.
- Rocha, P.S., Oliveira Goulart, R.d., Kawathekar, S., Dotta, R., 2019. Atlanta field development - present and future. In: Offshore Technology Conference, <http://dx.doi.org/10.4043/29846-MS>.
- Rodrigues, H.W.L., Prata, B.A., Bonates, T.O., 2016. Integrated optimization model for location and sizing of offshore platforms and location of oil wells. *J. Petrol. Sci. Eng.* 145, 734–741. <https://doi.org/10.1016/j.petrol.2016.07.002>.
- Sales, L., Jäschke, J., Stanko, M., 2020. Early field planning using optimisation and considering uncertainties - Model. <https://doi.org/10.17632/8rnmvtrff.1>.
- Sauve, R., Lindvig, T., Stenhaug, M., Holyfield, S., 2019. Integrated field development: process and productivity. In: Offshore Technology Conference, <http://dx.doi.org/10.4043/29631-MS>.
- Schwartz, E., Smith, J.E., 2000. Short-term variations and long-term dynamics in commodity prices. *Manage. Sci.* 46 (7), 893–911. <https://doi.org/10.1287/mnsc.46.7.893.12034>.
- Thomas, P., Bratvold, R.B., 2015. *A Real Options Approach to the Gas Blowdown Decision*. Society of Petroleum Engineers, Houston, USA, <https://doi.org/10.2118/174868-MS>.