Upstream Oil and Gas Technology 7 (2021) 100055

Contents lists available at ScienceDirect

Upstream Oil and Gas Technology

journal homepage: www.sciencedirect.com/journal/upstream-oil-and-gas-technology

Differential evolution for early-phase offshore oilfield design considering uncertainties in initial oil-in-place and well productivity

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ARTICLE INFO

Keywords:
Early-phase planning of offshore hydrocarbon fields
Decision support using optimization
Differential evolution optimization for hydrocarbon field planning
Quantification of uncertainty in field development

ABSTRACT

During the early phases of offshore oil field development, field planners must decide upon general design features such as the required number of wells and maximum oil processing capacity (field plateau rate), usually by performing sensitivity studies. These design choices are then locked in subsequent development stages and often prevent from achieving optimal field designs in later planning stages when more information is available and uncertainties are reduced.

In the present study, we propose using numerical optimization of net present value (NPV) to advice field planners when deciding on the appropriate number of wells, maximum oil processing capacity (plateau rate) in a Brazilian offshore oil field. Differential Evolution (DE) is employed for solving the optimization models. The uncertainties considered are well productivity and initial oil-in-place, handled by (1) using the mean of the distributions and (2) Monte Carlo simulation. A multi-objective optimization was also formulated and solved including ultimate recovery factor in addition to net present value.

The proposed method successfully computes probability distributions of optimal number of wells, plateau rate and NPV. If one wishes to compute the mean of such distributions only, for most cases it is adequate to run an optimization using the mean of the input values instead of performing Monte Carlo sampling. The multi-objective optimization allows to find field designs with high ultimate recovery factor and high NPV. In this case, the value of NPV is similar to the optimum NPV value when optimizing NPV only. The methods described could provide decision support to field planners in early stages of field development.

Introduction

The field development process is complex, demanding large sums of capital (CAPEX) and operational (OPEX) expenditures to produce hydrocarbons. In early stages of development, field planners must decide, with limited information, upon the approximate number of wells required, maximum capacities of topside facilities and field production schedule. These parameters have a large impact on the economic feasibility of the project [1,2]. A high number of wells and high processing capacities increase extraction rates, thus selling hydrocarbons earlier and minimizing the effect of cash flow discounts, which improves the net present value of the project. However, this strategy implies higher drilling expenditures and expensive topside facilities. Therefore, the project net present value (NPV) is reduced.

When starting a field development plan, reservoir characteristics and well performance are often highly uncertain. As the field is developed and the reservoir is produced, more information is obtained about reservoir characteristics. However, decisions about the required number of wells, production schedule and topside facility size and capacity are taken at early stages. While it is possible to conduct exploration and appraisal campaigns to gather more information about the subsurface in some cases, it is often expensive, and the remaining uncertainty may not change significantly or be still considerable. The uncertainty can usually only be reduced to a minimum after starting production. In posterior field planning phases field planners iterate on, refine and optimize other, more specific field design features such as well placement, production allocation per well, injection volumes, injector placement, and operating conditions of topside facilities. However, the optimization of specific field design features is constrained to the choices taken earlier on the more general field design features.

Optimization methods have been frequently used in the petroleum industry since the late 50s. We highlight some initial publications [3–6] and more recent publications [7–13]. Chen et al. [14] proposed a model

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https://doi.org/10.1016/j.upstre.2021.100055
Received 23 November 2020; Received in revised form 7 May 2021; Accepted 5 August 2021
Available online 23 September 2021

Nomenclature

NPV	Net Present value [USD]
$NPV_{p,with\ OPEX\ 1}$	Net present value of the revenue from oil sales minus rate-dependent OPEX [USD]
$CAPEX_{FPSO}$	cost of FPSO [USD]
$CAPEX_{WELLS}$	cost of well (construction and completion) [USD]
$CAPEX_{SUB}$	cost of subsea system (risers, flowlines, umbilicals, Xtrees, manifolds, installation, mooring of FPSO) [USD]
$R_{F,u}$	Ultimate recovery factor
N_w	Number of wells
J	Well productivity index [stb/year bar]
N	Initial oil in place [stb]
$q_{ppo,w}$	Well oil rate at initial time [stb/d]
$q_{p,f}$	Field plateau rate [stb/d]
q_{ppo}	Field oil rate at initial time [stb/d]
m	Rate decline constant [1/year]
i	Discounting annual rate [1/year]
T	Taxes
R	Royalties
Uptime	Number of operational days per year
t	Project time [years]

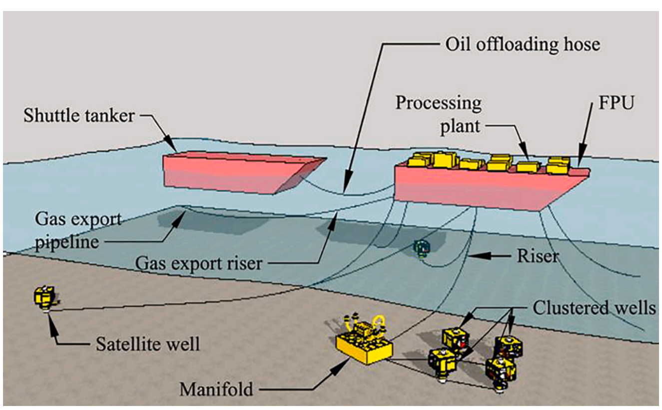

Fig. 1. Offshore oil field case study.

Table 1
Field characteristics.

Water depth	2000 m	**Platform system**	FPSO (spread mooring)
CO_2 concentration	30%	**Oil capacity**	150, 000 bpd
H_2S concentration	5–10 ppm	**Gas capacity**	7 million std m^3/d
Gas-oil ratio (GOR)	200 Sm^3/Sm^3	**Water injection**	200, 000 bpd
Recoverable reserves	560 million barrels	**Well system**	Vertical
°API	28	**Completion system**	Intelligent completion

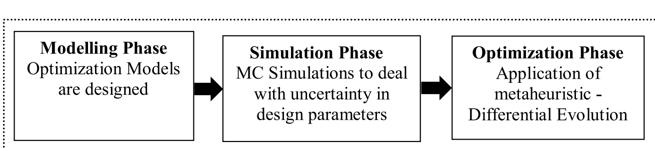

Fig. 2. General steps of the proposed method.

Table 2
Case studies considered.

	Objective function	Method	$q_{p,f}$	N_w
Case 1	NPV	Monte Carlo	Continuous	Discrete (1–20)
Case 2	$R_{F,u}$	Average inputs	Continuous	Continuous
Case 3	NPV+$R_{F,u}$	Average inputs	Continuous	Continuous

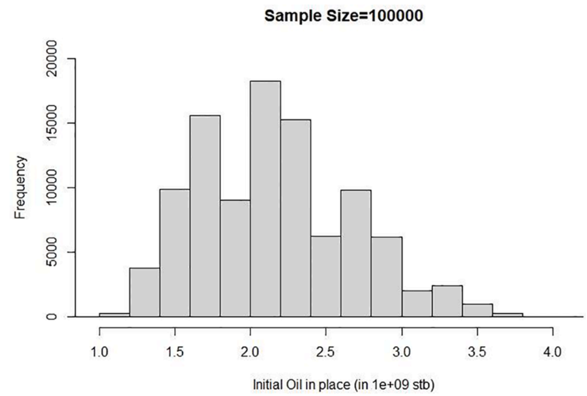

Fig. 3. Histogram of initial oil-in-place, *N*.

for maximizing the benefits while guaranteeing oil and gas field production, Almedallah and Walsh [15] proposed a hybrid k-means clustering and mixed-integer linear programming approach for optimizing the drilling path constraints; González et al. [16] proposed a decision support method to advice field planners during early-phase development, and formulated an optimized NPV as a mixed-integer linear problem using SOS2 models; and Hoffmann et al. [17] proposed a coupling strategy for maximum oil production at each time step of a small North Sea offshore field using a linear problem with SOS2 models.

Also, there are several studies to help understand the field development decision environment [18–20], along with more recent ones [21–24]. To aid in this decision-making process, typically a model of the value chain is employed to determine best design parameters that yield highest economic profit [25–27]. Recently, Nunes et al. [28] proposed a deterministic model to obtain the optimal number of wells and well plateau rate for a pre-salt field in Brazil, however, uncertainties were not considered.

Metaheuristics as an optimization tool in oil and gas fields

Metaheuristics are direct search methods, i.e. methods that do not require estimation on mathematical gradients of parameters. Metaheuristic numerical optimization techniques are usually inspired by some natural phenomena. Some popular metaheuristics include Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Differential Evolution (DE). These methods have been successfully applied to different domains, though not many instances are available to oil and gas fields. Literature reveals GA to be the most frequently used method applied to problems arising in different domains of oil and gas engineering. One of the initial applications of GA for oil and gas can be found in Mohaghegh et al. [29], where it was applied GA and neural networks (NN) for stimulation of gas storage wells. Later, Fang et al. [30] showed an application of GA for petrophysics; Sen et al. [31] showed the application of GA and simulated annealing (SA) for reservoir modeling, Bittencourt and Horne [32] implemented GA for scheduling in an oil field; Mohaghegh et al. [33] showed an application of hybrid neuro GA for hydraulic fracture treatment design and optimization; Fichter [34] implemented GA for portfolio optimization for the oil and gas industry; and Romero et al. [35] used evolutionary computation for improved reservoir characterization.

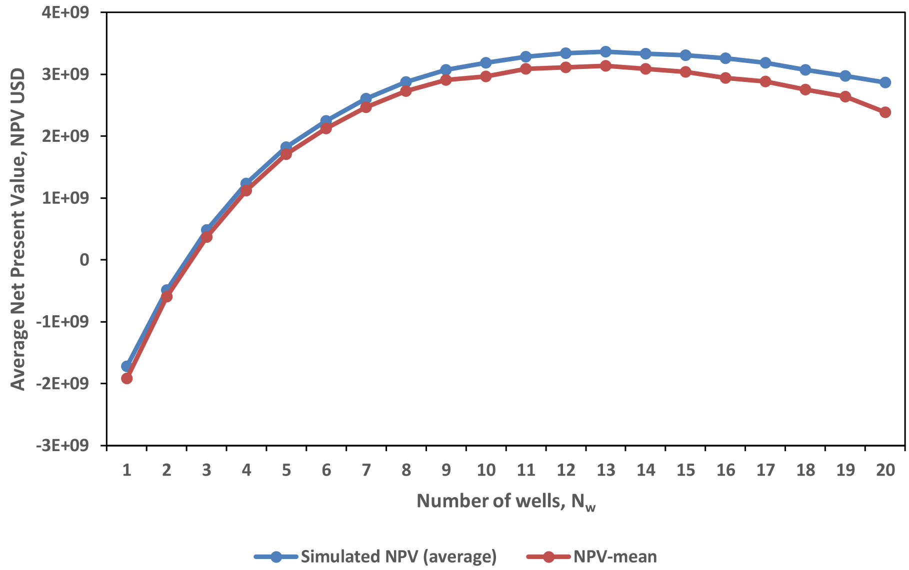

Fig. 4. Optimal NPV vs number of wells (N_w) calculated using (1) the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

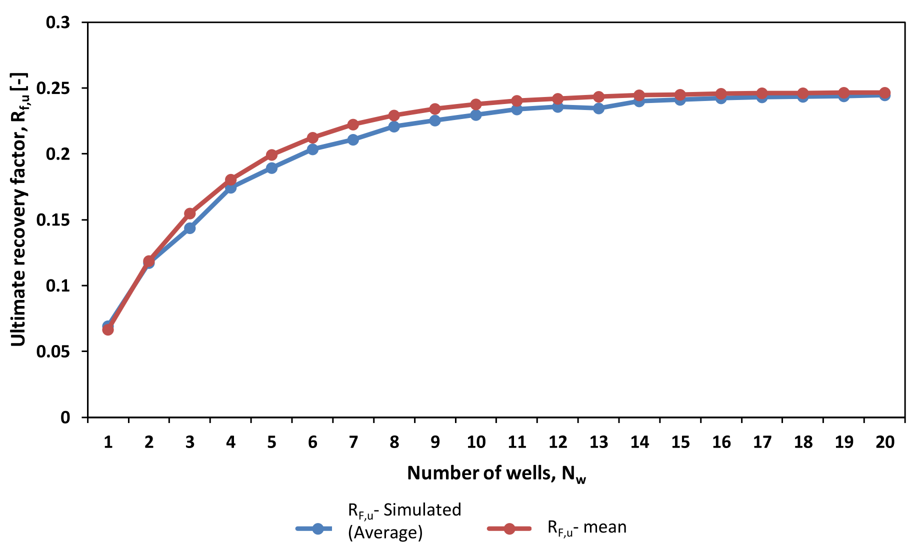

Fig. 5. Values of ultimate recovery factor ($R_{F,u}$) vs number of wells (N_w) calculated when optimizing NPV using: (1) the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Yeten et al. [36] combined GA with hill climber and neural network approach for multilateral well placement problems. Cullick et al. [37] combined Tabu search and scatter search with LP and NN. They used finite difference-based reservoir simulation for nonlinear production profile with a surface pipeline network economics-based model. Monte Carlo simulations were introduced in the model for dealing with uncertainty induced in reservoir volume, fluid quality, deliverability, and costs. Artus et al. [38] used GA for nonconventional well deployment, Bouzarkouna et al. [39] applied covariance matrix adaption evolution strategy for optimal placements of wells. Likewise, Carosio et al. [40] implemented DE, Dossari and Nasrabadi [41] implemented imperialist competitive algorithm and Chen et al. [42] implemented cat swarm optimization for optimal well placement. Al-Mudhafer and Shahed [43] suggested the application of GA for increasing oil recovery and NPV. Yang et al. [44] determined optimal NPV for mature reservoirs through DE and mesh adaptive direct search (MADS) algorithm.

Problem definition

During the early phases of field planning, the main features of the field must be decided upon, and the selection criteria is often to pick designs that provide maximum profit. Medium-to-large oil fields are typically produced with a constant rate initially, until they can not sustain that rate after which they enter into a phase of decline. The processing capacity (field plateau rate) and number of wells are usually decided on the basis of maximizing economic indicators such as the NPV, which is the sum of the project cash flow (revenues *minus* expenses) through the life of the field discounted in time.

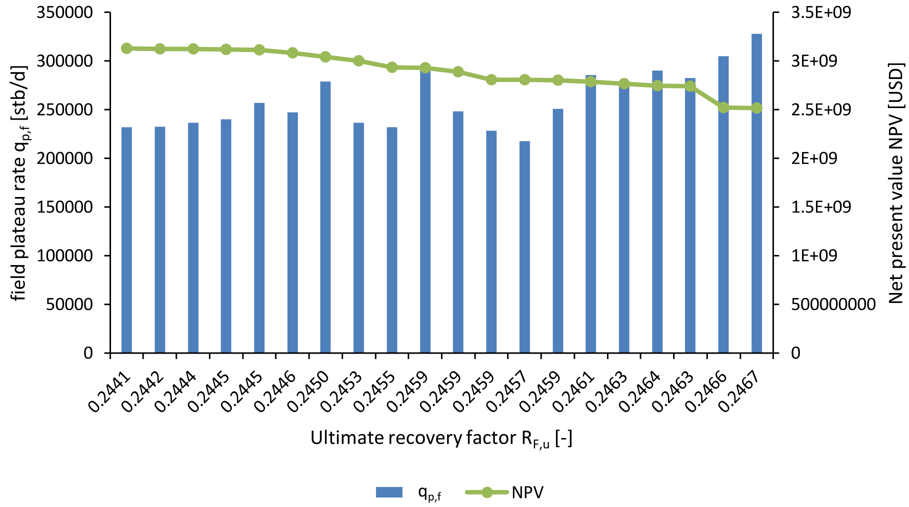

Fig. 6. Values of NPV and field plateau rate ($q_{p,f}$) calculated when optimizing NPV and ultimate recovery factor ($R_{F,u}$) versus ultimate recovery factor. Values are computed using the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N).

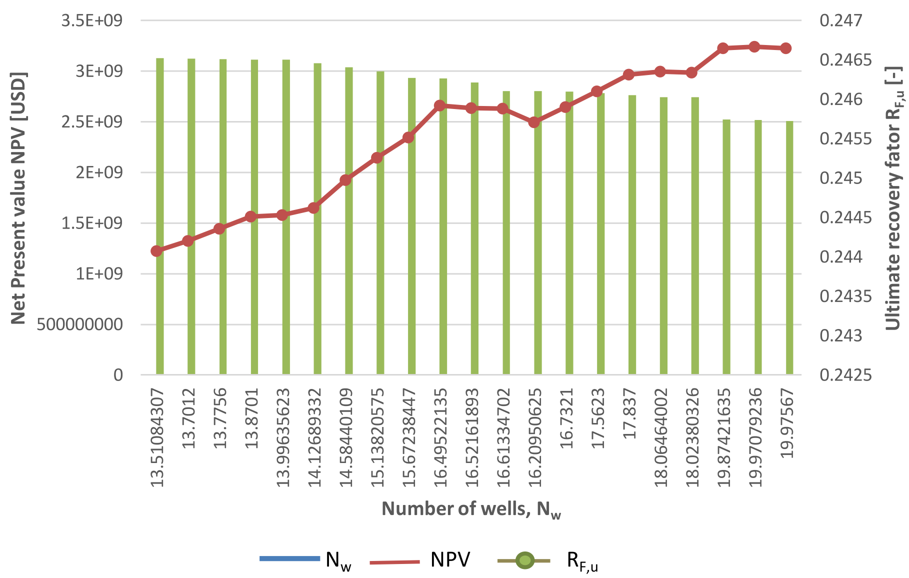

Fig. 7. Values of net present value (NPV) and number of wells (N_w) calculated when optimizing net present value (NPV) and ultimate recovery factor ($R_{F,u}$) versus ultimate recovery factor. Values are computed using the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N).

During the field development years, there are some capital expenditures, such as designing, manufacturing and installing the offshore structure, the topside facilities, well drilling costs, and manufacturing and installation of subsea equipment. The revenue influx starts after most of the wells are drilled, completed and tied-in to the processing facilities.

Increasing the number of wells will often allow to produce a higher field plateau rate or a longer plateau duration, thus increasing the revenue during the early years when the discounting effect is less pronounced. Increasing the plateau rate also increases revenue during the early years. However, a high number of wells and plateau rate also entails higher drilling costs and capital expenditures due to larger processing facilities. Thus, an optimum tradeoff is to be determined between the number of wells, plateau rate and the NPV.

Reservoir size and well productivity are parameters of high uncertainty during the early phase of field development. The forecast of the field production profile depends strongly on the assumed values of reservoir size and well productivity, thus affecting the field revenue stream, the NPV and ultimately, the number of wells and plateau rate.

National authorities typically encourage companies to recover a minimum amount of oil and gas from the field before abandonment. This amount varies depending on the characteristics of the field, and it is set in agreement between the operator and the entity that regulates hydrocarbon exploitation in the country. This is to avoid companies producing the “easy” oil and abandoning the field, leaving considerable reserves behind that are more costly to produce. Therefore, the ultimate recovery factor is typically a constraint that must be fulfilled by the field development plan. However, it often leads to more costly field designs

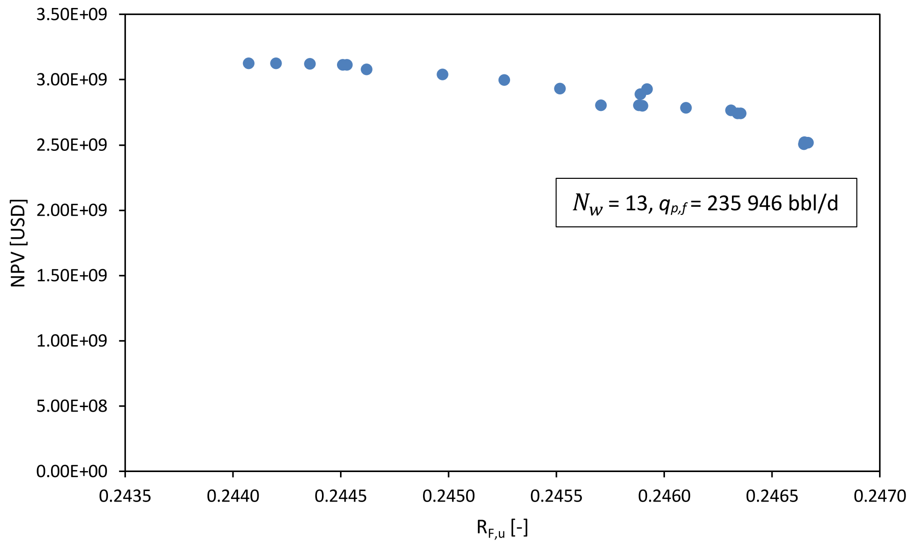

Fig. 8. Pareto front of optimal NPV and ultimate recovery factor ($R_{F,u}$) obtained with the multi-objective optimization. Values are computed using the mean of the input distributions of productivity index (J), well oil rate at the initial time ($q_{ppo,w}$) and initial oil-in-place (N).

that do not fulfill maximum NPV.

In the present study, the focus is to determine the oil production capacity (plateau rate) and number of wells to maximize net present value and ultimate recovery factor using numerical optimization. The effect of uncertainties in the size of the reservoir and well productivity is handled by (1) using the mean of the distributions and (2) Monte Carlo simulation.

The focus of our work is on proposing a method to determine the best possible general field design features, considering the existing uncertainties at early phase. For this, we use numerical optimization and probabilistic analyses. This does not mean that we aim to obtain an optimized field design, as this should also consider all subsequent field design phases, but rather that we use numerical optimization to determine a good general design, that hopefully enables achieving good (or optimal) specific designs at a later stage.

Case study

The case study used in this article is a Brazilian deep offshore oil field, illustrated in Fig. 1. The main field characteristics are shown in Table 1, while more information can be found in Nunes et. al [28]. In their work, they derived an analytical expression of NPV by:

- Assuming that the production rate in time of the field is constant and then follows an exponential decline. That is, during the plateau period, wells are choked to keep production constant. Afterwards, during the decline period, the bottom-hole pressure is kept constant. The build-up period is short or non-existent and can therefore be safely neglected;
- Performing a continuous discounting of the revenue;
- Using a constant oil price;
- Considering all drilling costs are executed at year zero. This represents a worst-case scenario, as, in reality, drilling costs will be spread throrought several years, and discounted in time, depending on the drilling schedule.

We employ the same analytical expressions of NPV, capital expenditures (CAPEX), plateau duration (t_P) and input data presented by Nunes et al. [28]. The following modifications and additions to their model were made:

- Derived an analytical expression of ultimate recovery factor by integrating the oil rate over the field lifetime and dividing by initial oil-in-place (N);
- Renamed the well rate decline factor (b) with the letter "m" and made it dependent on initial oil-in-place (N), well productivity (J) and number of wells (N_w). This was achieved by assuming the reservoir as under-saturated and produced by natural depletion, and that reservoir pressure never drops below the bubble point pressure during the production lifetime. Moreover, wells are identical and standalone;
- Nunes et al. [28] do not provide values of initial oil-in-place and well productivity. Therefore, we had to make some assumptions to back-calculate these parameters. Some details are provided in Appendix A;
- An OPEX analytical expression was derived and included in the model. Operational expenditures consist of a constant yearly value, and a yearly value depending on a linear relationship with field rate and number of wells.

It is assumed that all investments are made at the beginning and that the production starts immediately (thus discounting CAPEX is not necessary). The analytical equation for NPV is presented in Eq. (1):

$$NPV = (1-R)\cdot(1-Tax)\cdot\left(NPV_{p,with\ OPEX\ 1} - OPEX_2\right) - CAPEX_{FPSO} - CAPEX_{WELLS} - CAPEX_{SUB} \quad (1)$$

Where $NPV_{p,with\ OPEX\ 1}$, is the net present value of the revenue, deducting the rate-dependent part of OPEX:

$$NPV_{p,with\ OPEX\ 1} = (uptime\cdot P_o - 400)\cdot q_{p,f}\cdot \left[\frac{m + i - m\cdot e^{-\left(\frac{q_{ppo}}{q_{p,f}}-1\right)\cdot\frac{i}{m}} - i\cdot e^{-(m+i)\cdot t + \left(\frac{q_{ppo}}{q_{p,f}}-1\right)}}{i\cdot(m+i)}\right] \quad (2)$$

The non rate-dependent part of OPEX:

$$OPEX_2 = (N_w \cdot 700000 + 80E6) \cdot \frac{(1 - e^{-it})}{i} \tag{3}$$

Where:

$q_{p,f}$ is the plateau rate of the field

Uptime is the number of operational days per year (here assumed equal to 352)

The rate decline constant is $m = \frac{976}{N} \cdot N_w \cdot J$ [1/year]

The well productivity index is $J = F \cdot 29\,200$ stb/year bar

Initial oil-in-place is $N = 2.19$ E09 stb

The oil price is $P_o = 52\ USD/stb$

Discounting annual rate is $i = 0.09$ [1/year] t = 25 years

The field oil rate at initial time is $q_{ppo} = q_{ppo,w} \cdot N_w$

The well oil rate at initial time is $q_{ppo,w} = F \cdot 20\,000\ stb/d$

The royalties are R = 0.1

The tax is T = 0.35

The F is uniformly distributed between 0.4 and 1.6, representing the uncertainty in well productivity. $CAPEX_{FPSO}$, $CAPEX_{WELLS}$, $CAPEX_{SUB}$ are the capital expenditures of the offshore structure and topside facilities, the drilling expenditures, and the cost of the subsea system, respectively. These are empirical equations presented by Nunes et al. [28] that depend on the field plateau rate and the number of wells, and are given in Appendix B.

The ultimate recovery factor is given by

$$R_{f,u} = \frac{uptime}{m \cdot N}\left[q_{ppo} - q_{p,f} \cdot e^{\left(-m \cdot t + \left(\frac{q_{ppo}}{q_{p,f}} - 1\right)\right)}\right] \tag{4}$$

Eqs. (1) and (3) are the objectives to be maximized.

A constraint was imposed to ensure that the plateau rate of the field is never higher than the maximum field rate at initial time:

$$q_{p,f} \le q_{ppo,w} \cdot N_w \tag{5}$$

The expression presented for NPV of the revenue in Eq. (2) is still valid even if some wells are drilled after production start and as long as all wells are drilled before the end of plateau. This is because, in the plateau period, one can always adjust the production of each well with wellhead chokes to produce the desired rates.

Methodology

The proposed methodology can be described in three phases, as shown in Fig. 2:

Modeling phase

Three cases are considered in this study to quality control the values obtained and to separate the effects of multiple variables, as seen in Table 2:

(1) In the first case, maximizing NPV is taken as the objective function and the ultimate recovery factor, along with the corresponding values for optimal number of wells and plateau rate, are recorded. This is the only case where Monte Carlo simulation is used, as it is shown further that using the average of the inputs is sufficient.
(2) In the second case, maximizing ultimate recovery factor is taken as objective and the corresponding values for NPV achieved, along with optimal plateau rate and number of wells, are recorded.
(3) In the third case, both NPV and $R_{F,u}$ are taken as objective functions to be maximized simultaneously and the corresponding values for the number of wells and plateau rate are recorded.

For the first case, plateau rate ($q_{p,f}$) is treated as a continuous variable and number of wells (N_w) is treated as a discrete variable for which the values were varied from 1 to 20 and the optimum plateau rate is recorded, along with the corresponding value for $R_{F,u}$, for the maximum value of NPV.

In Cases 2 and 3, we treated both decision variables ($q_{p,f}$ and N_w) as continuous variables within the given range and recorded the results for maximum value of NPV. Also, the calculations were repeated for all the sample values.

In the optimization, the number of wells was handled as a continuous variable instead of an integer. The authors consider that, because it is only one variable, optimization results can be rounded and still provide valuable information while simplifying considerably its numerical implementation in the numerical algorithm.

Simulation phase

Three model parameters J, $q_{ppo,w}$ and N have inherent uncertainty which had to be dealt with before optimizing the models. We assume that the data about well performance is highly uncertain. So, we multiply the well productivity index J and the maximum well production $q_{ppo,w}$ by a common factor F, uniformly distributed between 0.4 and 1.6. The initial oil-in-place, N is distributed according to the log-normal truncated cumulative distribution function given in Fig. 3, with a mean of $2.16 \cdot 10^9$ stb (determined through inverse sampling using Monte Carlo simulations). This distribution represents the in-place volume uncertainties existing at the beginning of the field planning phase. It is assumed to be log-normal because this is the typical probability distribution of N when estimating reserves [45]. The authors would like to highlight that this approach to handle subsurface uncertainties is rather simplistic and does not allow to study more complex situations with e.g. permeability and porosity areal variability. Unfortunately these limitations are due to the analytical model we employed. However, we believe this model could still be useful for early phases of field development, where data and models are scarce and inaccurate. For later stages of field development, when more detailed models and information is available, it is important to perform a more complex uncertainty analysis on subsurface parameters.

We sampled 30 random realizations for each variable and arranged them in triplets. This was used as an input for the numerical optimization. Also, we repeated the numerical experiments by taking the mean values of the given distributions. Here, F is given to be uniformly distributed between 0.4 and 1.6.

Optimization phase

This is the final phase for solving the problem. Differential Evolution (DE) [46] is used as the optimization tool for the present study. Here, for each triplet obtained during the MC simulation phase, DE is executed 30 times and the best value is recorded. Similarly, DE is used for evaluating the objective function values when the mean values for J, $q_{ppo,w}$ and N are used. The population size of DE is kept as 50 and stopping criterion is taken as number of function evaluations (equal to 25,000 in this study). Mutation rate and crossover rate are kept as 0.5 each.

Working of DE: The field oil rate at initial time is taken as $q_{ppo} = q_{ppo,w} \cdot N_w$. Where N_w is varied from 1 to 20. In Case 1, plateau rate, $q_{p,f}$ is treated as continuous variable, which is to be determined. Now $q_{p,f}$ is initialized or generated between upper and lower bound. The upper bound is q_{ppo}, as given in the function, and the lower bound is set to 1. Then, $q_{p,f}$ is generated as

$$q_{p,f} = 1 + \left(q_{ppo} - 1\right) \cdot rand(0, 1) \tag{6}$$

For each $q_{p,f}$ the objective function is calculated after which $q_{p,f}$ undergoes the mutation and crossover phase for which the parameters F and C_r are kept as 0.5 each. After performing mutation and crossover, a new $q_{p,f}$ is obtained, say $q_{p,f}{}'$. Now, for each $q_{p,f}{}'$ the objective function is calculated and the maximum value is recorded. Finally, in the selection phase the tournament selection is performed between the objective function values obtained through $q_{p,f}$ and $q_{p,f}{}'$ and the one having the highest objective function value is selected for the next generation. The process go on until the stopping criteria, fixed as the maximum numbers of function evaluations in the present study, are satisfied.

For Case 2, N_w and $q_{p,f}$ both are treated as continous variables. Here N_w is generated between the upper and lower bound i.e. between 1 and 20. The $q_{p,f}$ is generated as in Case 1 and all the process is kept the same as in Case 1. The same is done for Case 3 as well.

Parameter tuning: DE has two parameters: scaling factor F and crossover rate C_r which are generally varied between 0.9 to 0.1 and 0.1 to 0.8, respectively. In the present study a fine tuning of both F and C_r was done and the best performance was recorded for 0.5 for both the parameters.

Results

Case 1 - Maximizing net present value

Tables C.1 and C.2 in Appendix C give the results obtained when the objective function is maximizing NPV. N_w is treated as a discrete input parameter, and the values for $q_{p,f}$ and $R_{F,u}$ are recorded along with other associated parameters. Results are shown when using the mean of the distributions of J, $q_{ppo,w}$ and N and the 30 random samples (hereby referred to as "simulated") .

From these tables the maximum value for NPV is obtained to be $3.13 \cdot 10^9$ and $3.37 \cdot 10^9$ USD and the corresponding $q_{p,f}$ and $R_{F,u}$ are calculated to be 233,194 and 239,128 stb/d, 0.243 and 0.234, respectively for mean and simulated values of J, $q_{ppo,w}$ and N_w. The optimal number of wells is 13.

Graphical results for calculations when using the mean values and simulated values of J, $q_{ppo,w}$ and N are shown in Figs. 4, 5 and Figs. C.1, C.2, C.3, and C.4, in Appendix C. The graphical results indicate the similarity between the results obtained through the simulated values of J, $q_{ppo,w}$ and N as well as the results obtained while using the mean values.

Due to the similarity between the results using the mean values and the average of the simulated values, the runs performed in subsequent sections of the study are using the means only. In early field development, decisions about the design features of the field are often made based on the mode of the distributions.

Table C.3 in Appendix C provides the results when both $1 < N_w < 20$ and $0 < q_{p,f} < q_{ppo,w}$ are taken as continuous variables varying between the specified ranges. The results are calculated using the mean values of J, q_{ppo} and N. The table presents the results of 10 independent runs of DE, while each run is executed 30 times. Finally, the mean of all 10 simulations is recorded. The value of NPV is calculated as $3.13 \cdot 10^9$ USD with N_w as 13. The corresponding values of $q_{p,f}$ and $R_{F,u}$ are evaluated as 232,812 stb/d and 0.243, respectively.

Case 2 - Maximizing ultimate recovery factor ($R_{F,u}$)

As a second case, the ultimate recovery factor, $R_{F,u}$, is taken as the function to be maximized and the corresponding values of NPV and $q_{p,f}$ are recorded while (1) varying the values of N_w from 1 to 20 by considering it as a discrete variable and (2) by considering N_w as a continuous variable along with $q_{p,f}$. Results are presented in Tables C.4 and C.5 in Appendix C.

It was observed that the best value for $R_{F,u}$ was obtained as 0.246 with $q_{p,f}$ calculated as 399 750 stb/d for $N_w = 20$. However, there was a modest decrease in NPV when compared against the case optimizing NPV only ($2.41 \cdot 10^9$ USD vs $3.13 \cdot 10^9$ USD, a 23% decrease). A similar result was observed when both $q_{p,f}$ and N_w were treated as continuous variables. Here also, the best value of $R_{F,u}$ (0.247) was obtained with $q_{p,f}$ = 398 591 stb/d and was obtained for $N_w = 19.9$ while NPV was calculated as $2.42 \cdot 10^9$ USD. Therefore, we see a tradeoff between $R_{F,u}$ and NPV. These are expected results as $R_{F,u}$ and $q_{p,f}$ are directly proportional to the number of wells (N_w), while NPV is concave with respect of N_w.

Case 3 – Multi-objective optimization

Results in the previous model indicated a tradeoff between $R_{F,u}$ and NPV. This shows that a compromise solution is needed that will maximize both simultaneously. This led us to the third model which is multi-objective and maximizes both NPV and $R_{F,u}$ simultaneously. The problem thus becomes:

Maximize

$$NPV = (1 - R) \cdot (1 - Tax) \cdot \left(NPV_{p,with\ OPEX\ 1} - OPEX_2\right) - CAPEX_{FPSO} - CAPEX_{WELLS} - CAPEX_{SUB} \tag{5a}$$

and

$$R_{f,u} = \frac{uptime}{m \cdot N} \left[q_{ppo} - q_{p,f} \cdot e^{\left(-m \cdot t + \left(\frac{q_{ppo}}{q_{p,f}} - 1 \right) \right)} \right] \tag{6a}$$

Subject to the constraints:

$$q_{p,f} \le q_{ppo,w} \cdot N_w \tag{7}$$

Where both $q_{p,f}$ and N_w were treated as continuous variables. The results of the optimization are given in Table C.6 in Appendix C. Figs. 6 and 7, and Figs. C.5, C.6 present plots depicting optimal NPV, optimal field plateau rate, optimal number of wells and optimal ultimate recovery factor. The relationship is proportional between number of wells and ultimate recovery factor, and between field plateau rate and ultimate recovery factor. The relationship is inversely proportional between NPV and ultimate recovery factor, between NPV and number of wells, and between NPV and field plateau rate.

Fig. 8 presents a Pareto front of optimal NPV and ultimate recovery factor ($R_{F,u}$) obtained with the multi-objective optimization. The plot shows that it is possible to obtain field designs with high NPV and with high ultimate recovery factor.

Comparison of results with Nunes et al. [28]

In the present study, the analytical model suggested by Nunes et al. [28] is treated as an optimization problem, incorporating inherent uncertainties in the model parameters to depict a more realistic scenario.

Nunes et al. [28] considered the base case with $N_w = 10$ and the corresponding NPV and NPV_p are calculated as 3.66 and 16.12, in billion dollars, respectively. In the present study, NPV and NPV_p are calculated approximately as 3.72, 16.4 for mean and 3.48, 15.9 for simulated values in billion dollars, which is in good agreement with the values of Nunes et al. [28]. Furthermore, the authors calculated the optimal number of wells to be 13, with NPV equal to 4.86 billion dollars. In the present study, the optimal number of wells is also 13. However, the corresponding NPV is obtained approximately as 3.89 and 3.70 (billion US dollars) for mean and for simulated values, respectively. An increase in the number of wells to 20 does not provide an improvement in the value of NPV, but it does in the value of ultimate recovery factor.

Conclusions

A numerical optimization model was employed to compute the number of wells, plateau rate and NPV on a Brazilian offshore oil field. Well productivity and initial oil-in-place uncertainties were considered, and handled by (1) using the mean of the distributions and (2) Monte Carlo simulation. The proposed method successfully computes probability distributions of optimal number of wells, field plateau rate and NPV. These distributions can provide decision support to field planners in early stages of field development. Using uniform distributions to estimate J and $q_{ppow,w}$ is a limited approach to capture the diversity of subsurface effects that could result from a real complex field. However, the authors believe that it is appropriate considering the model employed and the application (early field planning).

As seen in Case 1, it is adequate to run an optimization using the average of the input values instead of performing Monte Carlo sampling. In Case 2, it is observed a tradeoff between NPV and ultimate recovery factor. As seen in Case 3, the multi-objective optimization allows to find field designs with high ultimate recovery factor and an NPV very close to the optimum found when considering NPV optimization only.

In case 3, it is also possible to observe the proportional relationship between the number of wells and ultimate recovery factor, and between field plateau rate and ultimate recovery factor. The relationship is inversely proportional between NPV and ultimate recovery factor, between NPV and number of wells, and between NPV and field plateau rate.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This article has been written under the Norwegian Center for Research-based Innovation on Subsea Production and Processing (SUBPRO). The authors greatly acknowledge the financial support by the Research Council of Norway, as well to the industrial partners involved in this project.

Appendix A. Assumptions and estimation of the original oil-in-place and productivity index from the work of Nunes et al. [28]

If the reservoir is under-saturated and produced with N_w identical standalone wells, the production decline constant m is a function of the number of wells, well productivity index (J), and initial oil-in-place (N) as indicated in Eq. (A.1):

$$m = \frac{N_w \cdot J \cdot c}{N} \tag{A.1}$$

Where c is a constant that depends on oil compressibility (c_o), oil formation volume factor (current B_o and initial $B_{o,i}$), connate water (c_w) and formation compressibility (c_f), oil saturation (S_o) and connate water saturation (S_w):

$$c = \frac{B_o}{\left[B_{o,i} \cdot \left(c_o + \frac{c_w \cdot S_w + c_f}{S_o}\right)\right]} \tag{A.2}$$

Assuming that B_o is constant and equal to $B_{o,i}$, and using the values presented in Table A.1. Substituting in Eq. (A.2), this gives $c = 976$ *bar*.

To estimate well productivity (J) from the data provided by Nunes et al. [28], initial reservoir pressure (p_{Ri}) is assumed to be 350 bara and it is assumed that the minimum flowing bottom-hole pressure achievable ($p_{wf,min}$) is 100 bara. This gives that the maximum rate of each well at initial conditions is:

$$q_{ppo,w} = J \cdot \left(p_{Ri} - p_{wf,min}\right) \tag{A.3}$$

Substituting these values and $q_{ppo,w} = 20\,000$ *stb/d* gives $J = 80$ stb/d/bar. Finally, using the values of Nunes, $b = 0.13$ [1/year] ($b = m$), $N_w = 10$, and clearing N from Eq. (A.1). then

$$N = \frac{976}{0.13} \cdot 10 \cdot 80 \cdot 365 = 2.19E09 \; stb \tag{A.4}$$

Table A.1
Values of oil compressibility (co), connate water (cw), connate water saturation (Sw), formation compressibility (cf) and oil saturation (So) used in the model.

Values of oil compressibility (c_o)	9.50E-04	[1/bar]
Connate water (c_w)	4.00E-05	[1/bar]
Connate water saturation (S_w)	0.3	
Formation compressibility (c_f)	4.00E-05	[1/bar]
Oil saturation (S_o)	0.7	

Appendix B. Auxiliary equations

Equations to estimate capital expenditures (CAPEX) where taken from the work of Nunes et al. [28] and customized as follows:

- As the number of water injector wells is more dependent of the reservoir drainage pattern and reservoir geometry than other reservoir characteristics or the number of producer wells, we assume there are only 9 water injectors in the field, independent of the number of producers employed.
- The CO_2, H_2S, and sulfate removal unit factors are set to 1.
- The water depth is 2000 m.
- Each well has a pipeline associated with it, with an average length of 6 000 m.
- Each subsea manifold can accommodate up to 4 producing wells.
- The cost of installation of flowlines is 2 000 USD/m.

The final equations obtained are given in Eq. (B.1) (drilling and well costs), Eq. (B.2) (topside and offshore structure) and Eq. (B.3) (subsea system).

$$CAPEX_{wells} = N_w \cdot 150 \cdot 10^6 + 1.35 \cdot 10^9 \tag{B.1}$$

$$CAPEX_{fpso} = q_{p,f} \cdot 2.51 \cdot 10^3 + 1.07 \cdot 10^9 \tag{B.2}$$

$$CAPEX_{SUB} = N_w \cdot 9.20 \cdot 10^7 + 4.93 \cdot 10^8 + 32 \cdot 10^6 \cdot round\left(\frac{N_w}{4}\right) \tag{B.3}$$

In these equations, q_{pf} must be in stb/d and the output is in USD.

Appendix C. Results in table format and additional plots

Table C.1
Values of optimal NPV, discounted value of revenue (NPV_p), field plateau rate ($q_{p,f}$) and ultimate recovery factor ($R_{F,u}$) obtained when maximizing net present value NPV while varying the number of wells from 1 to 20. Values are computed using the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) and using 30 random samples (simulated).

	NPV [1e09 USD]		NPV_p [1e09 USD]		$q_{p,f}$ [stb/d]		$R_{F,u}$ [-]	
N_w	NPV_{simul}	NPV_{mean}	$NPV_{p,\ simul}$	$NPV_{p,\ mean}$	$q_{p,f,\ simul}$	$q_{p,f,\ mean}$	$R_{F,u\,simul}$	$R_{F,u\,mean}$
1	-1.72	-1.9	2.4	2.18	20990.48	19885.43	0.069171	0.066564
2	-0.48	-0.59	5.1	5.01	41750.07	39683.53	0.117138	0.118977
3	0.48	0.37	6.7	7.16	62201.41	59146.21	0.143691	0.154829
4	1.24	1.1	9.1	8.93	82277.74	78324.14	0.174361	0.180638
5	1.83	1.7	10	10.4	101913.4	97555.52	0.18945	0.199226
6	2.25	2.1	12.3	11.7	121051.8	115730.1	0.203641	0.212599
7	2.60	2.5	13.1	12.8	139675	134309.2	0.211025	0.222234
8	2.87	2.7	14.0	13.7	157724.1	151867.8	0.221026	0.229167
9	3.07	2.90	15.4	14.5	175203.3	169076.8	0.225319	0.23416
10	3.19	3.0	15.7	15.1	192084	185826.7	0.229676	0.237608
11	3.28	3.08	16.5	15.8	208359.2	202420.2	0.233832	0.240346
12	3.34	3.11	17.1	16.3	224005.1	217733.9	0.235908	0.242157
13	3.37	3.13	17.5	16.9	239128.5	233194.4	0.234852	0.243553
14	3.33	3.09	18.3	17.3	253605.3	247647.6	0.239976	0.244521
15	3.31	3.0	18.5	17.7	267561.9	261746	0.241206	0.245219
16	3.26	2.9	18.9	17.9	280946.7	275677.1	0.242222	0.245691
17	3.19	2.9	19.2	18.4	293773.8	288780.7	0.243033	0.246085
18	3.07	2.8	19.5	18.7	306048.5	301237	0.243591	0.246346
19	2.98	2.60	19.6	19.0	317886.8	313343.6	0.244056	0.246534
20	2.87	2.40	20.0	19.1	329149.7	324917.4	0.244673	0.246527

Table C.2
Values of topside, drilling and subsea capital expenditures (CAPEX) and decline constant *m* obtained when maximizing NPV while varying the number of wells from 1 to 20. Values are computed using the mean of the input distributions of productivity index (*J*), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (*N*) and using 30 random samples (simulated).

	$CAPEX_{FPSO}$ [1e09 USD]		$CAPEX_{WELLS}$ [1e09 USD]		$CAPEX_{SUB}$ [1e09 USD]		*m* [1/year]	
N_w	$CAPEX_{FPSO,sim}$	$CAPEX_{FPSO,mean}$	$CAPEX_{WELLS,sim}$	$CAPEX_{WELLS,mean}$	$CAPEX_{SUB,sim}$	$CAPEX_{SUB,mean}$	m_{sim}	m_{mean}
1	1.1	1.11	1.50	1.50	0.585	0.585	0.013658	0.013142
2	1.21	1.17	1.65	1.65	0.709	0.709	0.027316	0.026285
3	1.26	1.21	1.80	1.80	0.801	0.801	0.040973	0.039427
4	1.3	1.26	1.95	1.95	0.893	0.893	0.054631	0.052569
5	1.33	1.31	2.10	2.10	0.985	0.985	0.068289	0.065712
6	1.37	1.36	2.25	2.25	1.110	1.110	0.081947	0.078854
7	1.41	1.4	2.40	2.40	1.200	1.200	0.095604	0.091996
8	1.46	1.45	2.55	2.55	1.290	1.290	0.109262	0.105139
9	1.51	1.49	2.70	2.70	1.380	1.390	0.12292	0.118281
10	1.56	1.54	2.85	2.85	1.510	1.510	0.136578	0.131424
11	1.60	1.57	3.00	3.00	1.600	1.600	0.150235	0.144566
12	1.64	1.6	3.15	3.15	1.690	1.690	0.163893	0.157708
13	1.68	1.65	3.30	3.30	1.780	1.790	0.177551	0.170851
14	1.71	1.69	3.45	3.45	1.910	1.910	0.191209	0.183993
15	1.75	1.72	3.60	3.60	2.000	2.000	0.204866	0.197135
16	1.79	1.78	3.75	3.75	2.090	2.090	0.218524	0.210278
17	1.80	1.79	3.90	3.90	2.180	2.190	0.232182	0.22342
18	1.84	1.82	4.05	4.05	2.310	2.310	0.24584	0.236562
19	1.87	1.85	4.20	4.20	2.400	2.400	0.259497	0.249705
20	1.89	1.87	4.35	4.35	2.490	2.490	0.273155	0.262847

Table C.3
Values of optimal NPV, discounted revenue (NPV_p), optimal number of wells (N_w), optimal field plateau rate ($q_{p,f}$), ultimate recovery factor ($R_{F,u}$), capital expenditures ($CAPEX_{FPSO}$, $CAPEX_{WELLS}$, $CAPEX_{SUB}$) and decline constant *m* when maximizing NPV and assuming q_{pf} and N_w are continuous variables. Values are computed using the mean of the input distributions of productivity index (*J*), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (*N*).

NPV [1e09 USD]	NPV_P [1e09 USD]	$q_{p,f}$ [stb/d]	N_w [-]	$R_{F,u}$ [-]	$CAPEX_{FPSO}$ [1e09 USD]	$CAPEX_{WELLS}$ [1e09 USD]	$CAPEX_{SUB}$ [1e09 USD]	*m* [1/year]
3.13	16.8	232629.4	12.92018	0.243463	1.65	3.29	1.78	0.169802
3.13	16.7	228057.7	12.65107	0.243134	1.64	3.25	1.75	0.166265
3.13	16.7	226544.2	12.56728	0.243026	1.63	3.24	1.75	0.165164
3.13	16.7	229436.6	12.69772	0.243194	1.6	3.25	1.76	0.166878
3.13	16.8	229827.8	12.77853	0.243293	1.64	3.27	1.76	0.16794
3.13	16.5	220453.2	12.14348	0.242432	1.62	3.17	1.71	0.159594
3.13	16.7	226262.7	12.5687	0.243027	1.63	3.24	1.75	0.165182
3.13	16.5	221068	12.21581	0.242549	1.64	3.18	1.71	0.160544
3.13	16.9	232812.8	13.03234	0.243485	1.65	3.29	1.78	0.170053
3.13	16.8	230729.2	12.80974	0.243341	1.66	3.27	1.77	0.16835

Table C.4
Optimal values of ultimate recovery factor ($R_{F,u}$), and associated values of NPV, optimal field plateau rate, discounted value of revenue, capital expenditures ($CAPEX_{FPSO}$, $CAPEX_{WELLS}$, $CAPEX_{SUB}$), decline constant *m* obtained when maximizing $R_{F,u}$ while varying the number of wells from 1 to 20. Values are computed using the mean of the input distributions of productivity index (*J*), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (*N*).

N_w	$R_{F,u}$ [-]	NPV [1e09 USD]	$q_{p,f}$ [stb/d]	NPV_P [1e09 USD]	$CAPEX_{FPSO}$ [1e09 USD]	$CAPEX_{WELLS}$ [1e09 USD]	*m* [1/year]	$CAPEX_{SUB}$ [1e09 USD]
1	0.069112	-1.8	19988.76	2.4E	1.11	1.5	0.013142	0.59
2	0.118981	-0.59	39986.1	5.01	1.17	1.65	0.026285	0.71
3	0.154838	0.37	59848.84	7.16	1.22	1.8	0.039427	0.8
4	0.180654	1.12	79695.06	8.94	1.27	1.95	0.052569	0.89
5	0.197094	1.7	99726.47	9.88	1.25	2.1	0.065712	0.99
6	0.212621	2.12	119408.3	11.7	1.37	2.25	0.078854	1.11
7	0.222216	2.46	139749.5	12.8	1.4	2.4	0.091996	1.2
8	0.228689	2.71	159891.7	13.4	1.39	2.55	0.105139	1.29
9	0.234186	2.9	179797.2	14.5	1.52	2.7	0.118281	1.38
10	0.237781	2.99	199901.5	15.2	1.57	2.85	0.131424	1.51
11	0.24037	3.07	219388.6	15.9	1.62	3	0.144566	1.6
12	0.242119	3.1	229974.9	16.2	1.55	3.15	0.157708	1.69
13	0.243575	3.11	239666.1	16.9	1.72	3.3	0.170851	1.78
14	0.244541	3.04	279541.2	17.4	1.77	3.45	0.183993	1.91
15	0.245236	2.99	299596.6	17.8	1.82	3.6	0.197135	2
16	0.245712	2.91	319491	18.0	1.73	3.75	0.210278	2.09
17	0.246098	2.82	339988.9	18.5	1.92	3.9	0.22342	2.18
18	0.246352	2.68	358762.7	18.8	1.87	4.05	0.236562	2.31
19	0.246544	2.55	379164	19.1	2.02	4.2	0.249705	2.4
20	0.246679	2.41	399750.8	19.4	2.07	4.35	0.262847	2.49

Table C.5
Optimal values of ultimate recovery factor ($R_{F,u}$), and associated values of NPV, optimal field plateau rate ($q_{p,f}$), the optimal number of wells (N_w), discounted value of revenue (NPV_P), capital expenditures ($CAPEX_{FPSO}$, $CAPEX_{WELLS}$, $CAPEX_{SUB}$), decline constant m obtained when maximizing $R_{F,u}$. Values are computed using the mean of the input distributions of productivity index (J), well oil rate at the initial time ($q_{ppo,w}$), and initial oil-in-place (N).

$R_{F,u}$[-]	NPV [1e09 USD]	$q_{p,f}$ [stb/d]	N_w	NPV_P [1e09 USD]	$CAPEX_{FPSO}$ [1e09 USD]	$CAPEX_{SUB}$ [1e09 USD]	$CAPEX_{WELLS}$ [1e09 USD]	m [1/year]
0.246604	2.58	387794.8	19.32	19.2	1.95	2.43	4.25	0.254006
0.24666	2.3	393627.9	19.84	18.4	1.65	2.48	4.33	0.260791
0.246658	2.44	395294.3	19.82	19.3	2.06	2.48	4.32	0.260549
0.246621	2.48	388539.7	19.52	19.2	2.04	2.45	4.28	0.256638
0.246512	2.55	372498.2	18.80	18.5	1.69	2.38	4.17	0.247102
0.246653	2.54	390093.8	19.78	19.1	1.86	2.47	4.32	0.259956
0.246616	2.48	389706.4	19.49	19.2	2.05	2.45	4.27	0.256212
0.246674	2.42	398591.2	19.96	19.4	2.07	2.49	4.34	0.262372
0.246662	2.46	392630.7	19.86	19.3	2.03	2.48	4.33	0.26101

Table C.6
Results of multi-objective optimization of ultimate recovery factor ($R_{F,u}$) and NPV. The number of well (N_w) and field plateau rate ($q_{p,f}$) are treated as continuous variables. Values are computed using the mean of the input distributions of productivity index (J), well oil rate at the initial time ($q_{ppo,w}$), and initial oil-in-place (N).

$R_{F,u}$ [-]	NPV [USD]	$q_{p,f}$ [stb/d]	N_w
0.244072	3126299365	231999.3	13.51
0.2442	3125267324	232167	13.70
0.244356	3120631902	236450.2	13.77
0.24451	3115215635	240234.3	13.87
0.244528	3112926050	256691	13.99
0.24462	3079596181	247378.7	14.12
0.244973	3040210280	278937.2	14.58
0.245258	2998954300	236403.2	15.13
0.245517	2933576185	231949.4	15.67
0.245921	2929968976	290348.1	16.49
0.24589	2887604829	248044.3	16.52
0.245884	2805128868	228099.2	16.61
0.245707	2804210902	217367.7	16.20
0.2459	2800346768	250978.5	16.73
0.246101	2785681642	285628.8	17.56
0.246311	2765495241	277554.3	17.83
0.246354	2743832214	290269	18.06
0.246341	2741927825	282624.3	18.02
0.24665	2522596729	304992.9	19.87
0.246668	2517779650	327653.6	19.97
0.246649	2505462893	328565.4	19.97

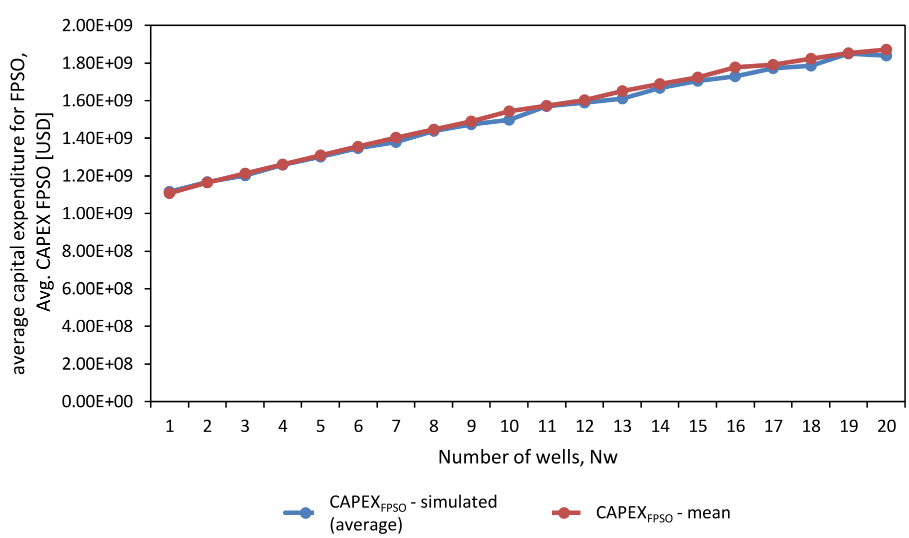

Fig. C.1. Values of capital expenditures of topside ($CAPEX_{FPSO}$) vs number of wells (N_w) calculated when optimizing NPV using (1) the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

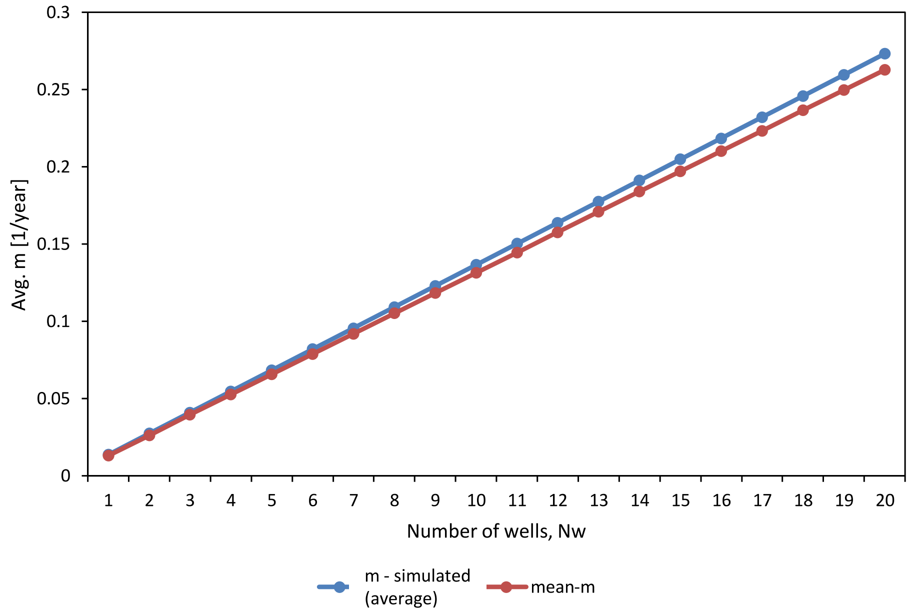

Fig. C.2. Values of decline constant m vs number of wells (N_w) calculated when optimizing NPV using (1) the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

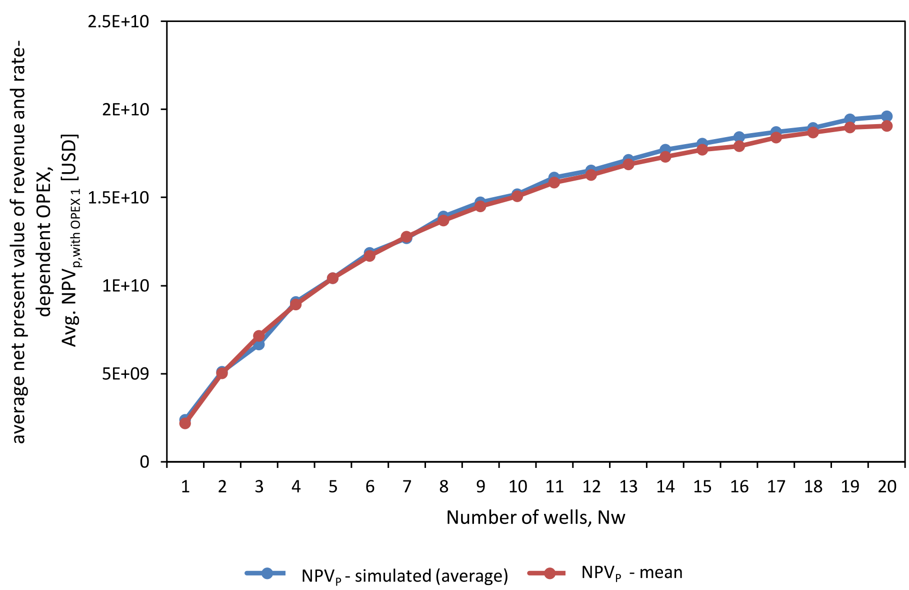

Fig. C.3. Values of discounted value of revenue (NPV_P) vs number of wells (N_w) calculated when optimizing NPV using (1) the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

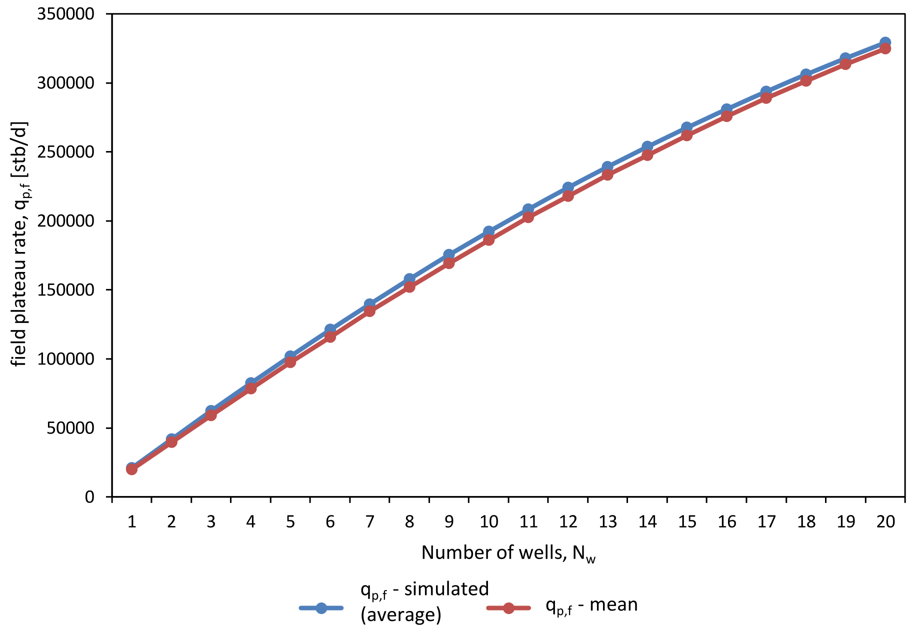

Fig. C.4. Values of optimal field plateau rate ($q_{p,f}$) vs number of wells (N_w) calculated when optimizing NPV using (1) the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

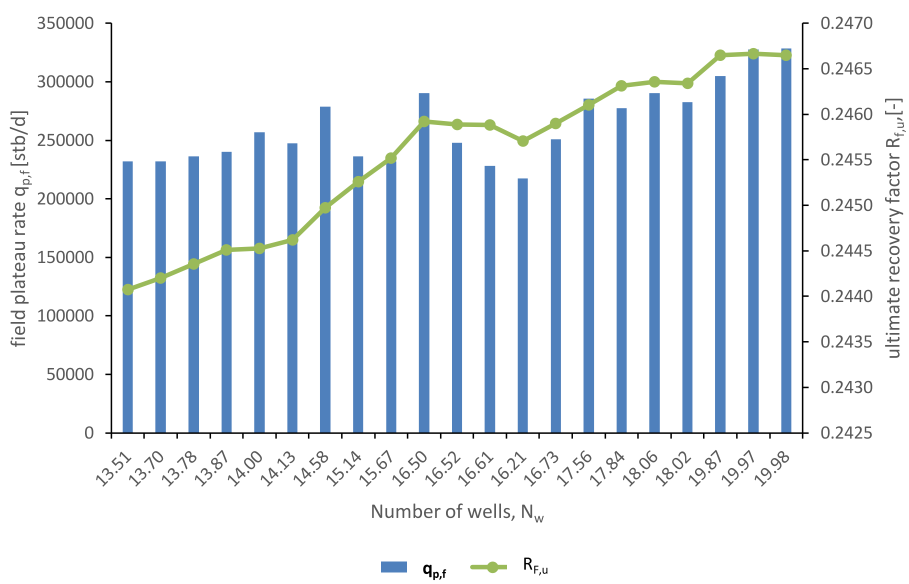

Fig. C.5. Values of optimal field plateau rate ($q_{p,f}$) and ultimate recovery factor ($R_{F,u}$) calculated when optimizing NPV versus number of wells (N_w). Values are computing using the mean of the input distributions of productivity index (J), well oil rate at the initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

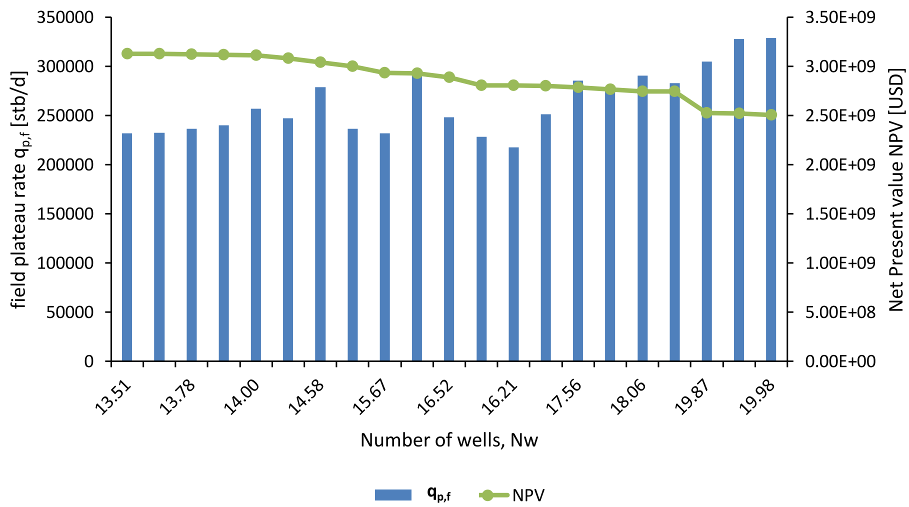

Fig. C.6. Values of optimal field plateau rate ($q_{p,f}$) and NPV calculated when optimizing NPV and ultimate recovery factor ($R_{F,u}$) versus number of wells (N_w). Values are computing using the mean of the input distributions of productivity index (J), well oil rate at initial time ($q_{ppo,w}$) and initial oil-in-place (N) (red line) and (2) 30 random samples (blue line, average). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

Appendix D. Algorithmic steps: differential evolution and multi-objective differential evolution (MODE)

Differential evolution (DE)

DE [46] works in two phases: Initialization and Evolution, defined below.

A. *Initialization:* this is the first step of DE, during which a uniformly distributed population set $S^G = \{X_j^G : j = 1, 2, \ldots, NP\}$ is generated. Here, NP denotes the population size for generation G. $X_j^G = \{x_{1,j}^G, x_{2,j}^G, \ldots., x_{D,j}^G\}$ generated as follows:

$$X_j^G = X_{low} + \left(X_{upp} - X_{low}\right)\cdot rand(0,1)$$

Where D is the dimension of the problem and X_{low}, X_{upp} indicates the lower and upper bounds respectively for the search space S^G and *rand (0,1)* denotes a uniformly generated random number between 0 and 1.

B. *Evolution:* in this phase, three operations: mutation, crossover, and selection are activated as follows:

i. *Mutation:* in this step a mutant vector V_j^G is obtained for each target vector X_j^G as

$$V_j^G = X_{r_1}^G + F\cdot\left(X_{r_2}^G - X_{r_3}^G\right)$$

Where F is the scaling factor varying between 0 and 1; $r_1, r_2, r_3 \in \{1, 2, \cdots, NP\}$ are vectors, randomly selected such that they are mutually different from each other and also from the index j.

ii. *Crossover:* during crossover, a new vector known as the trial vector denoted as $U_j^G = \{u_{1,j}^G, u_{2,j}^G, \ldots., u_{D,j}^G\}$ is generated with the help of target vector $X_j^G = \{x_{1,j}^G, x_{2,j}^G, \ldots., x_{D,j}^G\}$ and mutant vector $V_j^G = \{v_{1,j}^G, v_{2,j}^G, \ldots., v_{D,j}^G\}$. This is shown below

$$u_{i,j}^G = \begin{cases} v_{i,j}^G & if\ rand_j \le Cr \\ x_{i,j}^G & otherwise \end{cases}$$

Where i $\in \{1, 2, \ldots, \mathrm{D}\}$ and *Cr*, the crossover probability, $\in [0,1]$.

iii. *Selection:* during this process, candidate for the next generation is selected out of target vector and trial vector. The one with better fitness moves to the next generation. This is done as follows:

$$X_j^{G+1} = \begin{cases} U_j^G & if\ f\left(U_j^G\right) \ge f\left(X_j^G\right) \\ X_j^G & otherwise \end{cases}$$

The above three operations are repeated iteratively, until a predefined termination criterion is obtained. Fig. D.1 shows a diagram with the graphical representation of the steps described above.

Fig. D.1. Diagram describing the details of the DE method used in this work.

Multi-objective differential evolution (MODE)

The main difference between DE and MODE is in the selection phase. In DE a one-to-one selection is performed between the trial and the target vector i.e. only the target and trial vectors having the same index are compared with each other and the one with better fitness is carried forward to the next generation. MODE follows a non-dominated selection process, where the comparison between trial and target vector is according to the rules of dominance, according to which, a vector x dominates a vector y [47] if at least one of the following conditions are being satisfied

1. x is at least as good as y for all the objectives.
2. x is strictly better than y for at least one objective.

Thus, the trial vector U replaces the target vector X only if it dominates X.

$$X_j^{G+1} = \begin{cases} U_j{}^G & \text{if } f_{1,2,..n}\left(U_j^G\right) \text{ dominates } f_{1,2,\ldots n}\left(X_j^G\right) \\ X_j^G & \text{otherwise} \end{cases}$$

Here, X_j^G represents a set of non-dominated solutions. Finally, the non-dominated solutions are sorted using the naïve and slow approach suggested by Deb [47] to obtain the Pareto front.

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