Summary

A newly discovered reservoir needs to undergo a development study phase. Many development scenarios have to be evaluated to determine the scenario, which yields the maximum profit. A research has been carried out by a PhD student under the research center Subsea production and processing (SUBPRO) to develop an automated decision support methodology. The methodology includes integrating the production and economic elements of the field, mathematical optimization, and uncertainty analysis to decide upon the best strategy. González et al. (2020) and Angga (2019) had previously implemented this methodology on synthetic reservoirs consisting of simplified reservoir models having identical well performances. This thesis aims to develop the automated decision support methodology by taking into account the separate performances of each well in the reservoirs: oil production rates, Gas Oil Ratio (GOR), and Water Cut (WC) profiles.

There are three main stages of this work. The first stage was the problem formulation of Net Present Value (NPV) optimization as a Mixed Integer Linear Programming (MILP). The decision variables of the optimization were the production potential and drilling schedule. The optimization used production potential as the proxy model of the entire production system and cost proxy model to estimate the development cost. The cost proxy model was modeled as a linear function based on the available data. Since every well had distinct performance, the potentials oil rates were modeled as a non-linear function of cumulative oil productions and well status: active or inactive. Multidimensional Piecewise Linear (PWL) approximation was implemented to represent the non-linear behavior of the production potential imposing the Special Order Set (SOS)1/SOS2 constraints. The second stage was to develop a method to quantify the uncertainty of GOR and WC of the reservoir based on the individual well GOR and WC. Several optimization routines were formulated and performed to generate the new cumulative water and gas production curves by adjusting the well production schedule. Afterward, the NPV optimization routine was performed by employing these new curves to determine which curves have the highest and lowest NPV. The last stage is the uncertainty analysis study to quantify the uncertainties of the optimization result. Five parameters were considered: Initial Oil In Place (IOIP), water-gas profiles, cost figures, production potential, and oil price. The technique used in the study was a probability tree.

Based on the NPV optimization, changing the well status and drilling schedule throughout the production years had increased the NPV compared to the optimization with a fixed drilling schedule. Six extreme water and gas profiles were successfully generated to capture all the possible highest or lowest production of water and gas. From the NPV optimization, considering high production, both water and gas productions had lowered the NPV significantly from the base case. From the uncertainty analysis, the effect of the uncertainty was quantified in NPV, the optimal oil rates, and the optimal number of well.

Acknowledgement

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List of Abbreviations

1D	=	One dimension
2D	=	Two dimension
3D	=	Three dimension
6D	=	Six dimension
CapEx	=	Capital expenditure
CDF	=	Cumulative distributed function
DCF	=	Discounted cash flow
DrillEx	=	Drilling expenditure
EOR	=	Enhanced oil recovery
FPSO	=	Floating production storage and offloading
GOR	=	Gas oil ratio
IOIP	=	Initial oil in place
IP	=	Integer programming
IPR	=	Inflow performance relationship
LP	=	Linear programming
MILP	=	Mixed integer linear programming
NPV	=	Net present value
OpEx	=	Operational expenditure
PWL	=	Piecewise linear
QC	=	Quality control
SOS	=	Special order set
SUBPRO	=	Subsea production and processing
TPR	=	Tubing performance relationship
WC	=	Water cut

Chapter 1

Introduction

1.1 Background

A company recently found reservoirs located in the Barents sea. At present, the newly discovered field undergoes development study phase to decide upon the best exploitation plan. Initially, the development team needs to determine every possible development strategy subject to the challenges related to reservoir characteristics, economic, and regional constraints. As the study progresses, the strategies are narrowed down according to the stakeholder's preference. Generally, the aim is to obtain maximum financial benefits while preserving the environment.

Many key components are included in the planning, e.g., the production and well scheduling, well placement, topside facilities, and the types of offshore structure. Each of these critical units does not stand alone, but interrelated. Haldorsen (1996) described how increasing oil production rates could deliver two contradictive outcomes, i.e., higher revenue or higher capital expense. The field development study is accountable for finding the optimum equilibrium between these components. Furthermore, some elements in the oil and gas industry (e.g., oil price) are globally affected by social and political issues, making the situation relatively dynamic and rather uncertain throughout the field lifetime. It is essential to evaluate all of the development plannings while taking account of the associated uncertainties.

Despite that, it is often impossible to evaluate all the scenarios, considering many possible combinations. It demands extensive inter-discipline human resources to build every aspect of the model, possibly leading to iterative modifications. Consequently, many of these scenarios have left unstudied due to time constraints (Hoffmann et al., 2019).

A research center called Subsea Production and Processing, SUBPRO (2020), has research related to an automated decision support method for early field development. In a project led by Diana Gonzalez, the best development strategy was determined by employing an integrated modeling, mathematical optimization, and uncertainty analysis. The procedure is portrayed in Figure 1.1. The methodology applies proxy models to reduce the computation time during the mathematical optimization while providing a presentable portrayal of the actual systems: integrated production system and cost figures. Furthermore, uncertainty analysis for different elements is carried out to measure its impacts on decision making.

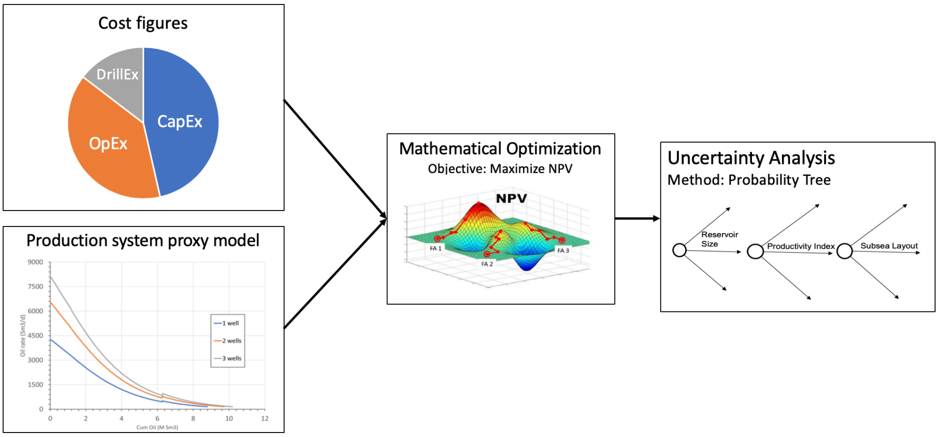


Figure 1.1: The Workflow of Automated Decision Support Methodology developed by SUBPRO (2020)

Several works had tested this methodology to optimize the Net Present Value (NPV) on synthetic fields consisting of a simplified reservoir model (tank) and production network. González et al. (2020) had utilized it on a synthetic reservoir based on the Wisting Field data. In this work, the optimization decision variables are the production profile and drilling schedule for a single reservoir. Another work by Angga (2019) used this approach on a more complex synthetic field, called 'Safari' with three separate reservoirs. The objective was to find the best configuration of the production schedule, drilling and injection schedule, and recovery mechanism, that gave the highest NPV and longest plateau duration.

In this work, the methodology was implemented and expanded in the new field discovered in the Barents Sea. It was expected to help in finding the optimum field development plan. Previously, the specialization project had generated the production system proxy model of this field (Alkindira, 2019). The production potential curves were derived from production data, instead of using models to compute production potential. The new production potential curves captured the changes in active wells by scaling the base case curves. The proxy model was validated against a more realistic reservoir model to mimic the complexity of the reservoir performance. This was demonstrated to be a good approximation. Thus, this thesis will continue the early development study shown by utilizing the production potential generated in Alkindira (2019).

1.2 Objective

This thesis's main objective is to improve the automated decision support methodology developed by SUBPRO. González et al. (2020) and Angga (2019) had optimized NPV by using production potentials that assumed all wells have identical performance. In this work, the methodology is improved by employing the potential production curves with different well performances during the optimization: production, Gas Oil Ratio (GOR), and Water Cut (WC) profiles. The work is segmented into several parts:

1. Generate the proxy model for cost figures.

2. Develop a mathematical optimization problem to maximize the NPV by configuring the drilling and production schedule. The production potential from Alkindira (2019) is employed in the optimization as the reservoir proxy model. The optimization would be formulated to predict the production profiles when well scheduling changes and the wells have different performance.

3. Perform NPV optimization by taking into account the possible changes in water and gas production profiles. This is to quantify the uncertainty in the producing water cut and GOR behavior of the reservoir based on the individual well GOR and WC profile.

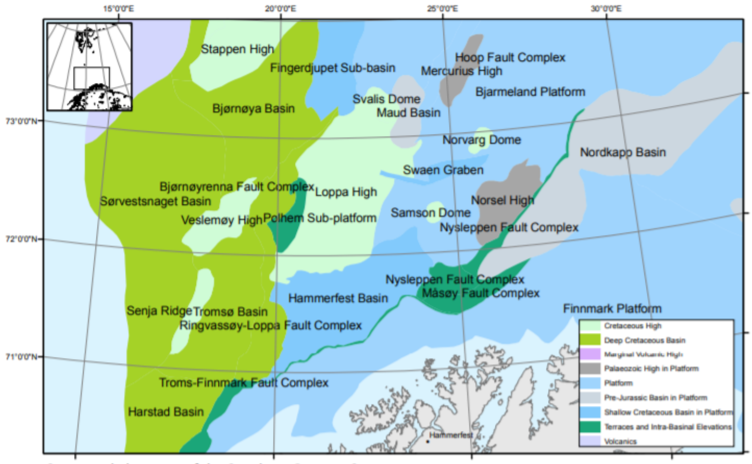
4. Evaluate the effect of uncertainties in Initial Oil in Place (IOIP), water and gas production profiles, cost figures, production potential, and oil price. The analysis is carried out using the probability tree technique.

1.3 Field Overview

Due to confidentiality clauses, the field in this study was called 'Field X'. The field was an offshore field located at the Loppa High area. Figure 1.2 points out the location of Loppa high in the southern part of the Barents sea. Field X consisted of two non-communicating reservoirs: reservoir X1 and reservoir X2. The distance between the two reservoirs was approximately 20 km away.

The reservoirs were saturated oil reservoirs with the presence of a gas cap. Reservoir X1 was considered the more significant reservoir with higher recoverable reserves of approximately 15.6 M Sm^3, and heavier oil with a lower solution gas-oil ratio. It will use a water-gas injection system to maintain reservoir pressure. All the wells are going to be drilled horizontally and installed with the gas-lift system. On the other hand, Reservoir X2 had smaller recoverable reserves of 6.5 M Sm^3, and higher solution gas-oil ratio. The reservoir pressure will be supported using the gas reinjection system. The wells are also drilled horizontally, but without any artificial lift installed.

The production from reservoir X1 and X2 will be commingled at the separator located at the Floating Production Storage and Offloading (FPSO). The water depth for this field was 390 m above the seabed. The location of the FPSO is set to be closer to reservoir X1 with a distance of about 500 m away. In the initial design, there are pipelines to transport production from reservoir X2 and riser to flow the production from reservoir X1. The



Structural elements of the Southern Barents Sea.

Figure 1.2: Location of Loppa high at the Barents Sea (retrieved from NPD (2019))

production for this field is expected to start in 2025 and was scheduled for abandonment in 2045.

1.4 Structure of The Report

Chapter 1 introduced the background and objective of the thesis. A simple overview of the field, including the reservoirs, was provided. The chapter finished with a description of the report structure.

Chapter 2 described the basic theory for this project. It included the concepts of production potential and mathematical optimization.

Chapter 3 encompassed the methodology to achieve the objective. The methodology consisted of building the cost proxy model, the formulation for NPV optimization, generating several water and gas production profiles as a function of cumulative oil production, and performing the uncertainty analysis.

Chapter 4 mainly discussed the derived cost proxy model from the data figures, the result comparison of NPV optimization with different settings, the generated water and gas production profile, the result of NPV optimization using these new profiles, and, finally, the result of uncertainty study.

Chapter 5 concluded this report.

Chapter 2

Literature Review

2.1 Production Potential

This section has been reproduced from the specialization project of Alkindira (2019) and serves as a brief introduction to the following literature used in the thesis.

2.1.1 Concept of production potential

Production potential may be interpreted as the maximum unique rate of a field at a particular time (Stanko, 2020). This rate is reached when the field is operated in its optimum condition within a given constraint (e.g., at a fully open choke, maximum gas-lift injection rate, and maximum water injection rate). A reasonable limitation is that the field is not able to deliver a rate higher than the production potential. However, the rate can be lowered by controlling the adjustable components such as choke (Stanko, 2020).

The production potential is determined from the intersection between Inflow Performance Relationship (IPR) and Tubing Performance Relationship (TPR). Therefore, the deliverability of the reservoir and production system affect production potential behavior. IPR and TPR behaviors differ during production due to the changes in production instruments or reservoir properties. The reservoir deliverability declines with time as the consequence of fluid being produced from the reservoir. Accordingly, the reservoir deliverability also depends on cumulative oil production. It can be said that the production potential at a given time, $q_{pp}(t)$, is also dependent on the total oil production, $N_p(t)$. (Stanko, 2020).

Equation 2.1 represents the relationship between the production potential and cumulative production when the field is produced at its potential.

$$N_p(t) = \int_0^t q_{pp}(t)dt \tag{2.1}$$

Coupled reservoir-production network simulation is run to yield the maximum oil production rate over time. Then, the production potential curve is generated by plotting the cumulative production, from equation (2.1) against the production potential, as illustrated in Figure 2.1.

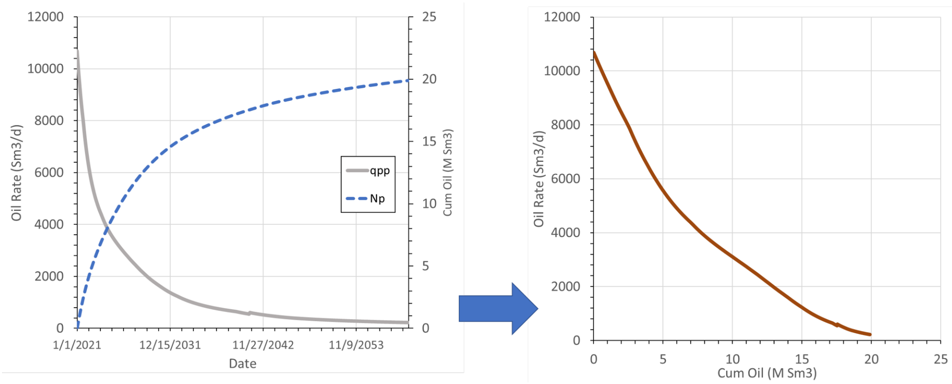


Figure 2.1: Production potential curve is constructed by plotting the q_{pp} at particular time t with its cumulative production $N_p(t)$

2.1.2 Production planning using production potential curve

The objective of production scheduling is to predict how much the field will produce with time. There are two types of production schemes used in the field: plateau mode and decline mode.

During plateau mode, the field production rate is maintained as constant at the desired rate until it is equal to the potential production rate. As discussed in section 2.1.1, production potential is a function of cumulative production and declines as more oil is withdrawn. Afterward, the field enters the decline mode because the production cannot keep the plateau rate. In decline mode, the field produces as much as possible. The production rate usually reaches production potential. Thus, it follows the same trend as the potential.

The production potential curve is used to estimate the plateau and decline mode of the field. The cumulative production $N_p(t_i)$ at time t_i from equation (2.1) is approximated by discretizing the time using rectangular integration as the following:

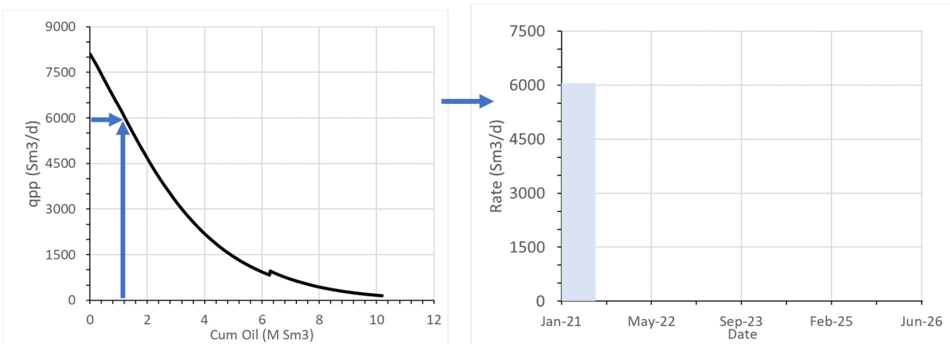
$$N_p(t_i) = q(t_{i-1}) \cdot (t_i - t_{i-1}) + N_p(t_{i-1}) \tag{2.2}$$

where $N_p(t_{i-1})$ is cumulative production at previous time step t_{i-1}, $q(t_i)$ is production rate at current time step t_i.

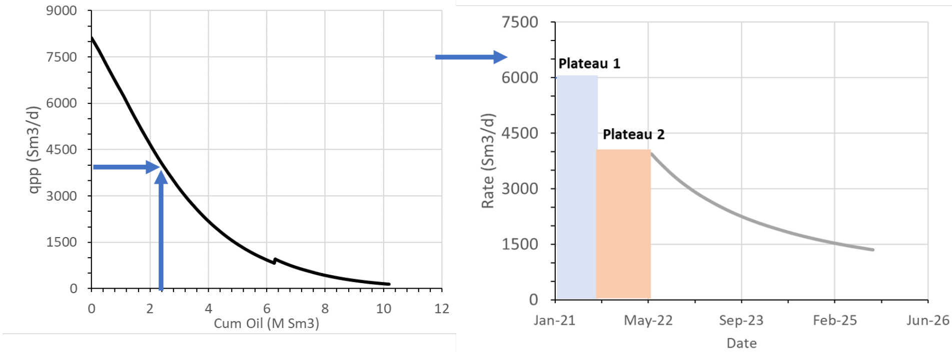
During the plateau duration, the oil production rate has to satisfy the following condition:

$$q_{pp}(N_p(t_i)) \geq q_{plateau}(t_i) \tag{2.3}$$

where $q_{pp}(N_p(t_i))$ is the production potential after a certain cumulative production at time t and $q_{plateau}(t_i)$ is the plateau rate at time t.



(a) Planning the $q_1(t_1)$



(b) Planning the $q_2(t_2)$

Figure 2.2: Production planning with production potential approach

Graphically, the plateau length can be determined by dividing the production potential curve's cumulative production by the production rate. Suppose the field is produced with several plateau periods, the time when k period ends, t_k, can be calculated mathematically by reformulating equation (2.2) into:

$$t_k = t_{k-1} + \frac{N_p(t_k) - N_p(t_{k-1})}{q_{plateau}(t_k)} \tag{2.4}$$

During the post-plateau period or in decline mode, the condition in equation (2.3) is violated. Therefore, the field production rate, $q(t_i)$, should be equal to the production potential. It can be expressed as:

$$q(t_i) = f(N_p(t_i)) \tag{2.5}$$

With the production potential curve, $q(t_i)$ can be obtained by interpolating $N_p(t_i)$ to q_{pp} in the curve. Therefore, $N_p(t_i)$ and $q(t_i)$ need to be determined concurrently for every time step.

An illustration of the application of the production potential is provided in Figure 2.2. In the beginning (Figure 2.2a), $t = 0$, there is still no oil production. The production rate can be chosen between 0 - 8000 Sm^3/d. Suppose that the first plateau rate, $q_1(t_1)$ is 6000 Sm^3/d, the first plateau duration, t_1 can be estimated using equation (2.4) and by referring to production potential on the left.

Once t_1 is surpassed (Figure 2.2b), $q_1(t_1)$ cannot be maintained anymore. Afterward, by referring to production potential, it is known that the $q_1(t_1)$ decline after reaching N_p =1.2 M Sm^3. Now, the production rate can only be ranged between 0 - 6000 Sm^3/d. Thus, $q_2(t_2)$ is selected to be 4000 Sm^3/d, and then t_2 is calculated. After t_2 is reached, a new plateau rate can be chosen by the same procedure. Otherwise, the production rate will produce at its potential and deplete, starting from the last plateau rate.

2.1.3 Remarks about the production potential

The characteristics of production potential concept discussed by other works from Stanko (2020) and Angga (2019) are as follow:

1. The production potential curve shows discontinuity if there are abrupt modifications to a production system or reservoir. Such modifications on production system can involve drilling a new well and changing separator pressure, while the adjustments applied to reservoir cover pressure maintenance, stimulation activities or Enhanced Oil Recovery (EOR)

2. Reservoir pressure, Water Cut (WC), production GOR, and injection-production ratio can be defined as a function of cumulative production. However, this concept is only applicable to the reservoir with cumulative oil production as an input (e.g., material balance model). The curve representing the relationship between these properties and cumulative production needs to be generated from the same simulation scenario as the corresponding production potential curve.

3. The field production potential consisting of one reservoir is unique for given field cumulative production. If the field has several wells, the wells' production potentials are dependent on the accumulation of all well productions.

4. If all wells in the field are identical and produced from the same reservoir, the field production potential can be estimated by multiplying the number of wells with the production potential of a single well.

5. If the field has two or more reservoirs tied at a fixed pressure, the field production potential is not unique and depends on each reservoir's production schedule. However, the production potential of a reservoir is independent of other reservoirs and only dependent on its cumulative production.

6. If the field has two or more reservoirs tied at a non-fixed pressure node (e.g., junction), each reservoir's production potential depends on the production of other reservoirs.

2.2 Mathematical programming

2.2.1 Linear Programming

Linear programming model is an optimization technique to solve a linear objective function subject to a system of linear equality and inequality constraints. Mathematically, a linear programming model is expressed as (Van Roy and Mason, 2005):

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{2.6}$$

Where the vector $x \in \mathbb{R}^N$ is a set of decision variables, $c \in \mathbb{R}^N$ is the vector values for each x_i, and the matrix $x \in \mathbb{R}^{MxN}$ and the vector $b \in \mathbb{R}^M$ is the inequality constraints. A set of decision variables that fulfills all of the constraints is called a feasible solution. An optimal solution refers to a feasible solution reaching the desired maximum value. If no solution satisfies all the constraints, the problem becomes infeasible. Also, there is an unbounded problem which refers to a feasible problem with infinite objective value (Vanderbei, 2020). This condition happens if the problem does not have an upper bound; thus, it is unsolvable.

Feasible region and optimal solution

In linear programming, the set of all feasible solutions is a feasible polytope. The term refers to the approach to find the solution based on the intersections of the linear constraints. The boundaries of the two-dimensional problems are line segments, while three-dimension problems have the boundaries of flat planes. The feasible polytope is convex, such that any convex combination of two-element within the polytope stays inside. One of the linear programming theorems states that a feasible convex region has the optimal value at one of its extreme points (Luenberger et al., 1984). The extreme points are the points that do not connect the other two points in the feasible set. For instance, a triangle has the extreme points located at its vertices.

To have clearer illustration, let consider the two dimensional problem below:

$$\begin{aligned} \text{minimize} \quad & -2x_1 - x_2 = z \\ \text{subject to} \quad & x_1 + \frac{8}{3}x_2 \leq 4 \\ & x_1 + x_2 \leq 2 \\ & 2x_1 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned} \tag{2.7}$$

An illustration of the example above is depicted in Figure 2.3. The feasible region of this problem is the area inside the bold lines. The objective function lines are parallel to each other, and it shifts to search for the optimal solution within the region. The red dot denotes the location of the optimal solution attained at the extreme point of feasible region.

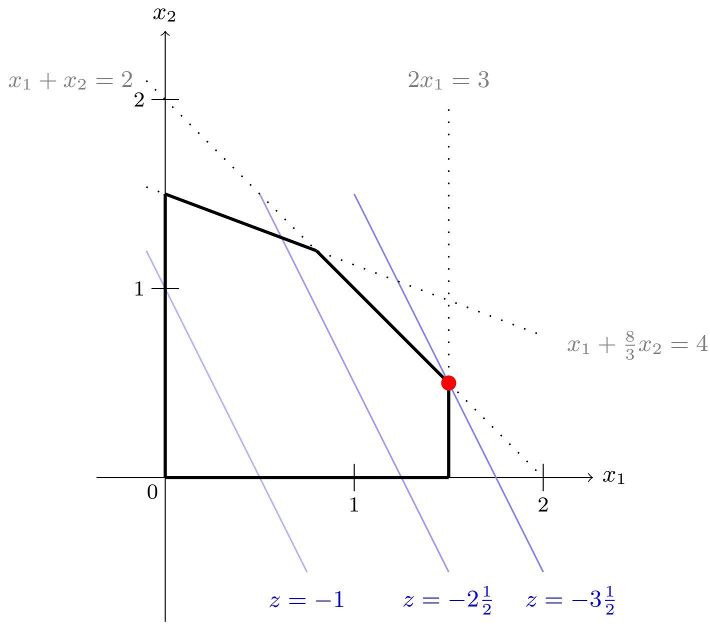


Figure 2.3: Illustration of the two dimension example solution. The extreme points of $(\frac{3}{2}, \frac{1}{2})$ yields the optimal objective value of $z = -3\frac{1}{2}$ (adapted from Luenberger et al. (1984))

2.2.2 Mixed Integer Linear Programming

Mixed Integer Linear Programming (MILP) is a branch of Linear Programming that restricts some of the decision variables to be integers. If all the decision variables are integer, it is called Integer Programming (IP). Such programming can be stated as:

$$\begin{aligned} \text{maximize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \\ & x = (x_i, x_j) \geq 0 \\ & x_i \in \mathbb{Z}^m \\ & x_j \in \mathbb{R}^{N-m} \end{aligned} \tag{2.8}$$

Linear Programming (LP) Relaxation

Having integer variables increases the complexity of the optimization model. For any MILP or IP problem, linear programming (LP) relaxation can be performed by switching the integer restriction with a continuous constraint. This changes the IP problem into an LP problem.

Let consider an example below:

$$
\begin{aligned}
\text{maximize} \quad & 5x_1 + 8x_2 = z \\
\text{subject to} \quad & x_1 + x_2 \leq 6 \\
& 5x_1 + 9x_2 \leq 45 \\
& x_1, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
\tag{2.9}
$$

Figure 2.4 shows the graphical solution to the problem. The optimal solution of LP relaxation gives greater or equal value to the optimal integer solution since its feasible region is bigger than the IP problem. In other words, the solution from LP relaxation acts as the upper bound. In this example, the optimal solution of LP relaxation is attained at a blue point $(\frac{9}{4}, \frac{15}{4})$. Rounding off this point does not necessarily give the optimal integer solution, and in this case, the point is infeasible. The nearest integer point also has a lower solution than the LP solution and the optimum integer solution. Therefore, the optimum solution is not determined by simply rounding the points or picking the nearest point.

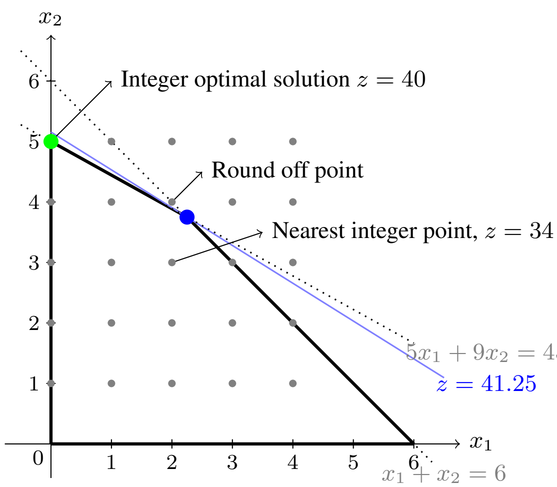


Figure 2.4: Illustration of the Integer Programming example. The LP relaxation obtains the optimal solution of $z = 41.25$ at point $(\frac{9}{4}, \frac{15}{4})$. The integer optimal solution is lower, which is $z = 40$ at point $(0, 5)$ (adapted from Bradley et al. (1977))

2.2.3 Methods to Solve (Mixed) Integer Programming

Numbers algorithms have been developed for solving the integer problem. Two of them are introduced in this section.

Cutting Plane

The cutting plane method was first proposed by Gomory (1958). The idea of the cutting plane method is to generate new inequality constraints into the IP problem. These new constraints narrow the feasible region until the integer solution is obtained. The inequality constraint has to be valid that it does not violate any feasible integer solution (Mitchell, 2009).

The same example from the previous section is used to discuss this method. Suppose two new constraints are introduced:

$$\begin{aligned} C_1: &\quad 4x_1 + 7x_2 \leq 35 \\ C_2: &\quad 2x_1 + 3x_2 \leq 15 \end{aligned}$$

As shown in Figure 2.5, adding the C_1 and C_2 into the problem reduced the area of feasible region. There is no integer solution being removed; both constraints are valid. The IP problem can be solved by linear relaxation, and it shall have the solution attained at the integer point. In practice, this method cuts a small section of the region in each iteration. First, add C_1. This still does not lead to an optimal solution. Only after C_2 is added that the optimal solution can be found.

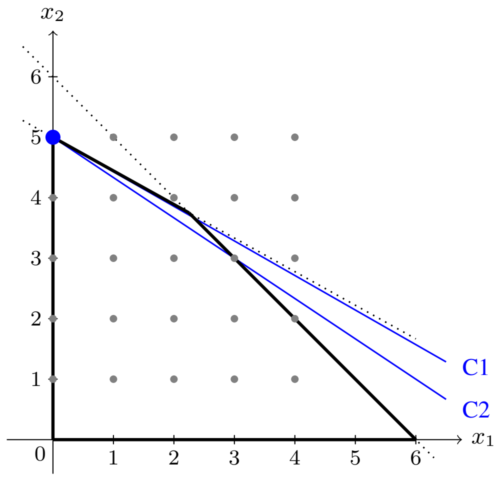


Figure 2.5: Illustration of the cutting plane method (adapted from Bradley et al. (1977))

Branch and Bound

The branch-and-bound method partitions the problem into subdivisions and solves them to find a better integer solution. In the maximization problem, the optimal value of LP relaxation is the upper bound of the IP problem. The objective value of a feasible integer solution is the lower bound. The branch and bound continues to divide the problem until it finds the feasible integer solution with a value close to the upper bond. Considering the same example as the previous section, initially, the problem P_0 is not decomposed and solved directly using the LP relaxation. The objective value from the relaxation, z_0, is considered the upper bound of the maximization problem. The z_0 has both variables as non-integer. Subsequently, the problem is divided into two sub-problems (SP_k) to change the variables to be an integer. The value of x_2 lies between $x_2 = 3$ and $x_2 = 4$, therefore the subproblems are assigned to be $x_2 \leq 3$ and $x_2 \geq 4$.

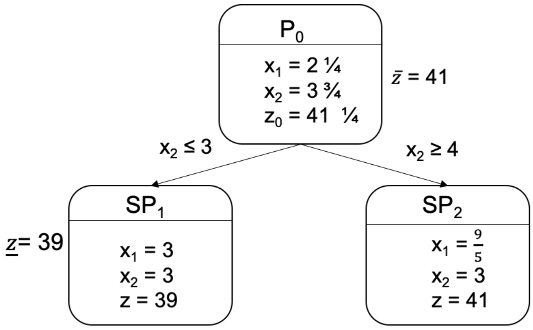


Figure 2.6: The first subdividing of the problem. One of the subproblems obtains an integer solution, while the other still has a continuous variable. However, the branching can be continued because the z is larger than the lower bound (adapted from Bradley et al. (1977))

If one of the subproblems obtains integer solution, its objective value would be the lower bound, $\underline{z}$. The $\underline{z}$ keeps updated every time new integer solution is found to be greater than current $\underline{z}$. See enumeration stored in Figure 2.6. First, let consider the branch from SP_1. The value of $\underline{z} = 39$. For the next steps, the only considered solution is when $39 \leq z \leq 41.25$.

The next step is to consider the branch SP_2. As shown in Figure 2.7, the solution in this branch is neither integer nor feasible until it reaches SP_5 and SP_6. If the SP_5 is computed first, the solution is feasible but not optimal with lower $z = 39$. Accordingly, the solution from this subproblem is discarded. New optimal solution is disclosed in SP_6 with higher $z = 40$, thus the new bound is set to be $40 \leq z \leq 41.25$. The SP_k is not efficiently solvable, if either of the following conditions is met:

1. The (SP_k) is infeasible
2. The (SP_k) has integer feasible solution
3. The solution of (SP_k) is worse than the current $\underline{z}$.

To improve problem-solving efficiency, the algorithm discards SP_k with any of these conditions. Note that the SP_4 is no longer decomposed because it is infeasible. The same termination is found in SP_1, SP_5, and SP_6 because those subproblems have found an integer solution. Therefore, the optimal integer solution is determined by SP_6.

2.3 Piecewise Linear Approximation

Piecewise Linear (PWL) approximation is one of the approaches to reformulate a non-linear function into a linear function. PWL reshapes the non-linear function as a sequence of linear segments translated into linear functions.

Let $f(x)$ be a 1D non linear function. PWL approximation generates new linear functions $g(x)$ to estimate $f(x)$ by selecting some breakpoints in its domain. The general form

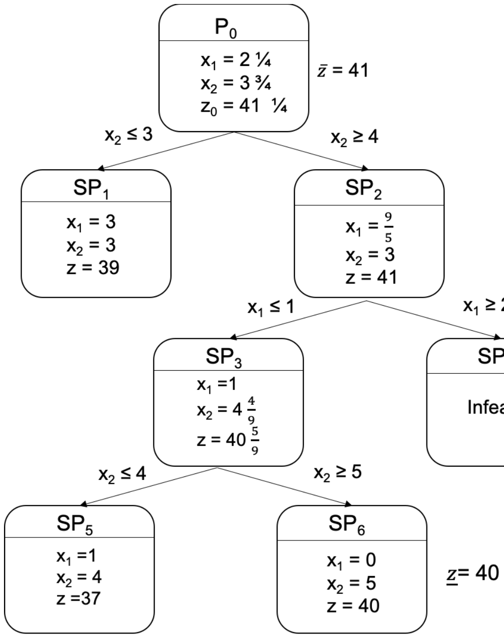


Figure 2.7: The final enumeration tree of the branch-and-bound method (adapted from Bradley et al. (1977))

of 1D PWL function are defined as:

$$
\begin{aligned}
g(x) &= \sum_{i=1}^{n} \lambda_i \cdot f(x_i) \\
x &= \sum_{i=1}^{n} \lambda_i \cdot x_i \\
1 &= \sum_{i=1}^{n} \lambda_i
\end{aligned} \tag{2.10}
$$

with n as the number of breakpoints and λ_i is the non-negative weighting factor of breakpoint-i.

Special Order Set (SOS) Formulation

There are two types of SOS models:

1. SOS1: a set of variables with only one element be a non-zero value
2. SOS2: a set of variables with at most two consecutive elements be a non-zero value

In PWL, the λ_i is set to be SOS1/SOS2 in order to limit the number of non-zero values in the breakpoints set. If SOS1 is used, the approximation must only select one point from the set. If SOS2 is used, the algorithm includes two points in the approximation.

Illustration

The illustration of the $f(x)$ and PWL approximation $g(x)$ is depicted in Figure 2.8. Suppose a set of breakpoints $\{x_1, x_2, x_3, x_4\}$. The PWL approximation of this non-linear function is found to be:

$$g(x) = \begin{cases} g_1(x) & x \in \{x_1, x_2\}. \\ g_2(x) & x \in \{x_2, x_3\}. \\ g_3(x) & x \in \{x_3, x_4\}. \end{cases} \tag{2.11}$$

By looking at the expression above and applying λ as SOS2, one only need to use $g_2(x)$, x_2 and x_3 to approximate $f(a)$. Then, they are defined as:

$$\begin{aligned} g_2(a) &= \lambda_2 \cdot f(x_2) + \lambda_3 \cdot f(x_3) \\ a &= \lambda_2 \cdot x_2 + \lambda_3 \cdot x_3 \end{aligned} \tag{2.12}$$

The sum of λ_i has to be equal to one, with the value of λ_1 and λ_2 equal to zero.

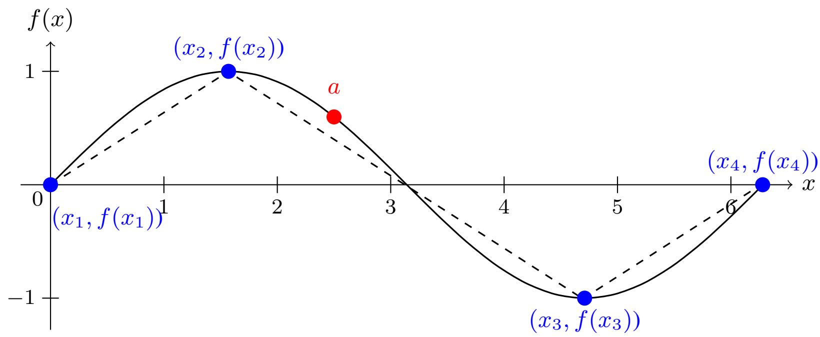


Figure 2.8: 1D PWL approximation of $f(x)$. The dashed lines are the linear functions $g(x)$ estimating the non-linear function in solid line

2.3.1 Piecewise Linear Approximation in Multidimension

The PWL has been applied in research works for multivariate problems. Kosmidis et al. (2005) formulated an MILP of the gas lift well with two variables; manifold pressure and the gas injection rate; into a MILP using the PWL. Silva and Camponogara (2014) performed a comparison study on several multidimensional PWL formulation of a gas-lifted oil field. Similarly, as the previous section, only the PWL using the SOS formulation is discussed.

Let $f(x, y)$ be a 2D non linear function and $g(x, y)$ be its PWL approximation. A set of breakpoints in both x and y direction need to be introduced to construct $g(x)$. The general form of 2D PWL function are (Silva and Camponogara, 2014; Hoffmann, 2014):

$$
\begin{aligned}
g(x,y) &= \sum_{i=1}^{N_x}\sum_{i=1}^{N_y} \lambda_{i,j} \cdot f(x_i, y_j) \\
x &= \sum_{i=1}^{N_x}\sum_{i=1}^{N_y} \lambda_{i,j} \cdot x_i \\
y &= \sum_{i=1}^{N_x}\sum_{i=1}^{N_y} \lambda_{i,j} \cdot y_i \\
1 &= \sum_{i=1}^{N_x}\sum_{i=1}^{N_y} \lambda_{i,j} \\
\eta_{x,i} &= \sum_{i=1}^{N_y} \lambda_{i,j} \qquad \forall i \in 1, 2..N_x \\
\eta_{y,j} &= \sum_{i=1}^{N_x} \lambda_{i,j} \qquad \forall\forall y \in 1, 2..N_y
\end{aligned}
\tag{2.13}
$$

where N_x and N_y are the number of breakpoints in each direction. η_x and η_y are additional variables to be defined as either SOS1 or SOS2. These variables restrict the number of non-zero breakpoints in x and y directions.

Illustration

Figure x visualizes variable grids of 2D PWL. The $f(x,y)$ domain are discretized into a set of breakpoints in x-direction $\{x_1, x_2, x_3, x_4\}$ and y-direction $\{y_1, y_2, y_3\}$. The PWL approximation constructed from this set is:

$$
g(x,y) = \begin{cases}
g_1(x,y) & x \in \{x_1, x_2\}, y \in \{y_1, y_2\}. \\
g_2(x,y) & x \in \{x_2, x_3\}, y \in \{y_1, y_2\}. \\
g_3(x,y) & x \in \{x_3, x_4\}, y \in \{y_1, y_2\}. \\
g_4(x,y) & x \in \{x_1, x_2\}, y \in \{y_2, y_3\}. \\
g_5(x,y) & x \in \{x_2, x_3\}, y \in \{y_2, y_3\}. \\
g_6(x,y) & x \in \{x_3, x_4\}, y \in \{y_2, y_3\}.
\end{cases}
\tag{2.14}
$$

To approximate the $f(a,b)$, $\eta_{x,i}$ and $\eta_{y,j}$ are assigned to be SOS2. At most four neighboring breakpoints are included into the computation. Based on the expression in Eq. 2.14, it can be solved by applying linear equation $g_2(x,y)$ thus only $\lambda_{2,1}, \lambda_{2,2}, \lambda_{3,1}$, and $\lambda_{3,2}$ are set to be non-zero (Eq. 2.15). Accordingly, $\eta_{x,2}$, $\eta_{x,3}$, $\eta_{y,1}$, and $\eta_{y,2}$ are considered.

$$
\begin{aligned}
g_2(a,b) &= \lambda_{2,1} \cdot f(x_2,y_1) + \lambda_{2,2} \cdot f(x_2,y_2) + \lambda_{3,1} \cdot f(x_3,y_1) + \lambda_{3,2} \cdot f(x_3,y_2) \\
a &= \lambda_{2,1} \cdot x_2 + \lambda_{2,2} x_2 + \lambda_{3,1} \cdot x_3 + \lambda_{3,2} \cdot x_3 \\
b &= \lambda_{2,1} \cdot y_1 + \lambda_{2,2} y_2 + \lambda_{3,1} \cdot y_1 + \lambda_{3,2} \cdot y_2
\end{aligned} \tag{2.15}
$$

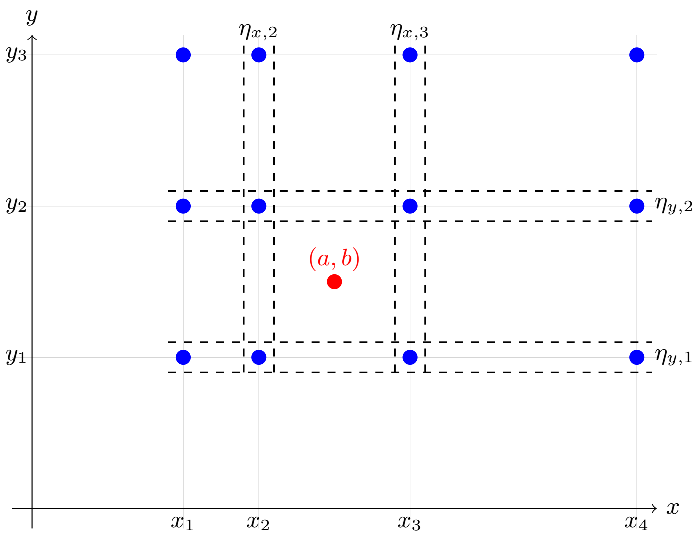


Figure 2.9: 2D PWL Approximation to estimate $f(a,b)$ (adapted from Hoffmann (2014); Angga (2019))

Angga (2019) shows that 3D PWL has a similar formulation with 2D PWL. One only needs to have additional variable of η_z for non-linear function $f(x,y,z)$. Consequently, η_z also imposes SOS1/SOS2. Therefore, this work implements this formulation for a higher degree of a non-linear function.

Chapter 3

Methodology

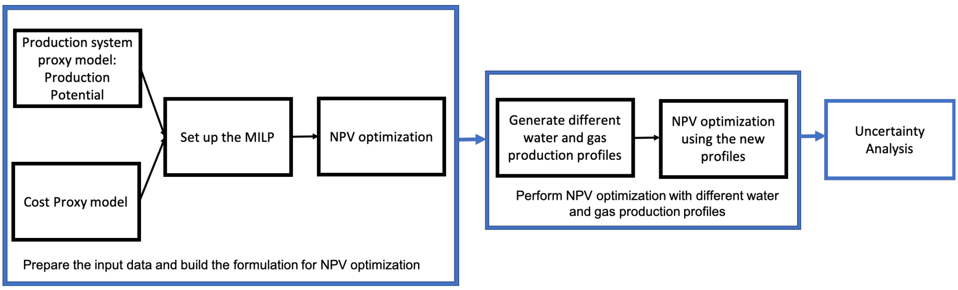


Figure 3.1: The main workflow of this thesis

In general, this thesis focuses on performing an early development study on Field X. Figure 3.1 shows the three main stages of the study. The first stage is to build a MILP for NPV optimization. The goal is to maximize the Net Present Value (NPV) of the field by adjusting the production splitting of the reservoirs and the drilling schedule. The decision variables are fluid production rates and the number of wells drilled throughout the field lifetime.

The optimization uses the proxy models of reservoir model and cost estimation as the input. The proxy model for the reservoirs implements the production potential from Alkindira (2019). Cost estimation of the field includes the Capital Expenditure (CapEx), Drilling Expenditure (DrillEx), and Operational Expenditure (OpEx). The CapEx and OpEx proxy models are generated from a multivariate linear regression. Once the problem formulation is completed, the NPV optimization result is compared with another optimization routine as a reference case, which applied fixed-well scheduling. This well schedule is obtained from the original production profile. Therefore, the reference case only adjusts fluid production profiles.

The next step is to perform another NPV optimization for several water and gas production schemes. Different cumulative water and gas productions of the field are generated.

They are calculated from the available water and gas profile from the original production plot. Lastly, the effects of uncertainty in oil price, initial oil in place, production potential, and cost estimation are analyzed. The uncertainty analysis is carried out using the probability tree approach.

3.1 Production System Proxy Models

The derivation of the production system proxy model is further explained in the specialization project of Alkindira (Alkindira, 2019).

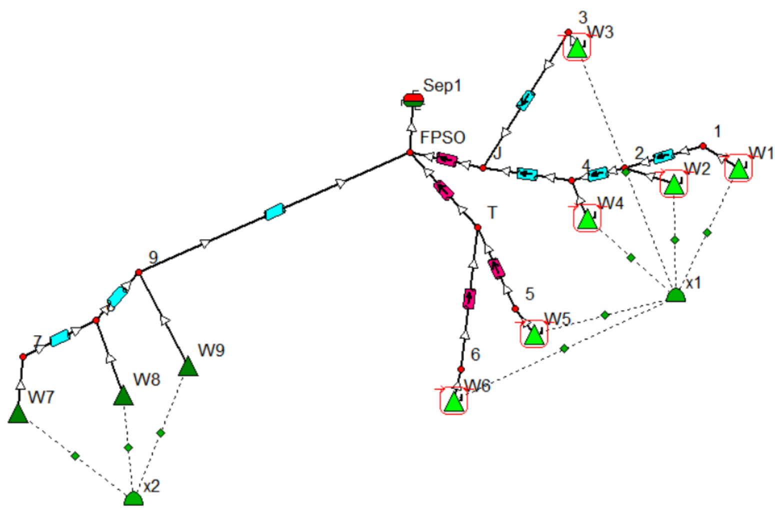


Figure 3.2: Production System Layout of Field X (retrieved from Alkindira (2019))

Field X has two reservoirs with nine producer wells; six wells in Reservoir X1 and three in Reservoir X2. The production of each reservoir is commingled at the separator (Figure 3.2). Therefore, each reservoir has its production potential and independent of each other.

The production potential of the production system was derived from the original production plot in Figure 3.3. The plot is generated from the reservoir simulator model. The derivation only employed the data points in the plot where all the producers are in decline mode (blue region). This region is selected because it represents the trend of production potential. A curve-fit to production potential from coupled MBAL-GAP model is performed to construct the production potential from the data points. The MBAL-GAP model is another production system model of Field X employing a material balance approach to create the reservoir.

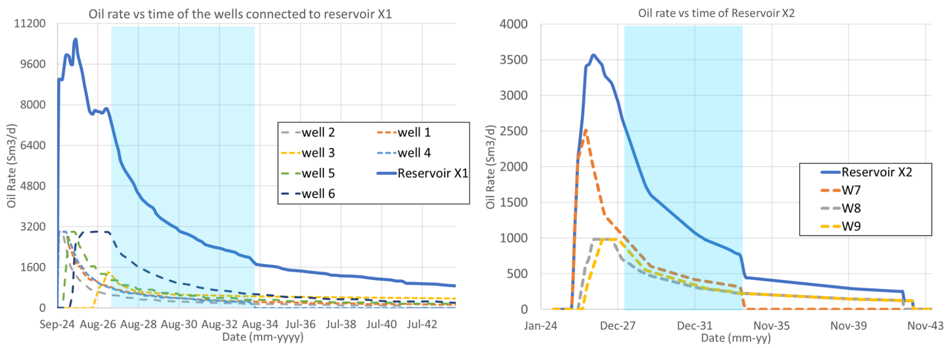
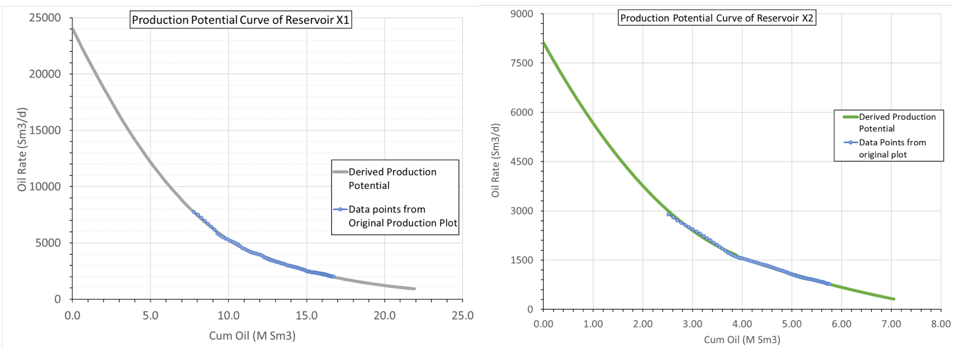


Figure 3.3: Original production plot.



(a) Production potential of Reservoir X1

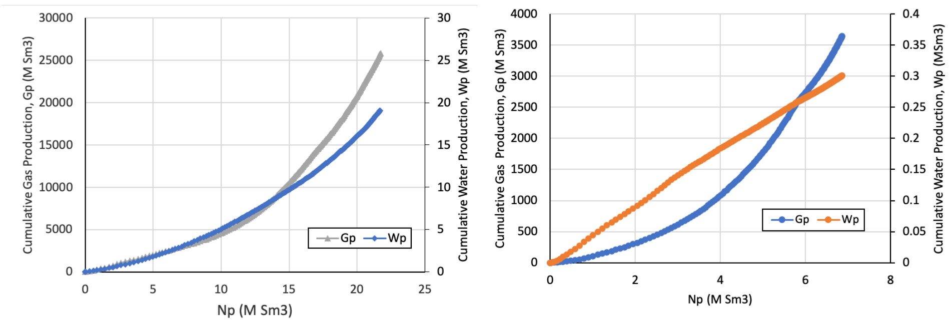
(b) Production potential of Reservoir X2

Figure 3.4: Constructed production potential derived from the data points

Figure 3.4 shows the production potential for Reservoir X1 and Reservoir X2. The derived production potential was only suitable when all producer wells in the corresponding reservoir are active or producing. Furthermore, the production potential curve with fewer producer wells was obtained by multiplying the derived curves with the contributing factors. Mathematically, it can be defined as :

$$q_{opp,n}(N_p) = f_n \cdot q_{opp,f}(N_p) \tag{3.1}$$

With $q_{opp,n}$ as the production potential of a particular producer well status scenario n, $q_{opp,f}(N_p)$ as production potential at a particular N_p with all wells in the reservoir active and f_n as the contribution factor of scenario n. In other words, the contribution factor is also a multiplier factor to the $q_{opp,f}(N_p)$ for a certain producer well status scenario. Note that each scenario has a unique, well status combination. The example of the contributing factor in Reservoir X1 and its translation are shown in Table 3.1. Similar with q_{opp}, the water and gas production are modelled as a function of N_p only (Figure 3.5). There is no modification applied for these relationship curves, so they are retrieved directly from the original production profiles.



(a) Reservoir X1 **(b)** Reservoir X2

Figure 3.5: The cumulative water and gas production profiles

Table 3.1: Well status scenario and contribution factors of Reservoir X1. The value ”1” means the well active and ”0” means the well inactive

n	fn	W1	W2	W3	W4	W5	W6
1	0	0	0	0	0	0	0
2	0.166	1	0	0	0	0	0
3	0.249	0	1	0	0	0	0
4	0.194	0	0	1	0	0	0
5	0.293	0	0	0	1	0	0
:	:	:	:	:	:	:	:
64	1.000	1	1	1	1	1	1

Pipelines connect the wells to manifolds and manifolds to the separator. For each well status, the field requires a certain pipeline installment. Hence, the assembly of the pipeline is different between one well status combination and others. Table 3.2 stores the example of the required pipeline for each scenario and its total length. For example, scenario 2 has only well 1 producing thus only P_{12}, P_{12}, P_{24},P_{4J} and P_{12} should be installed. Therefore, the total pipeline length of $Pl_{sum,2}$ is 8.5 km.

Table 3.2: Example of the required pipeline for each scenario. P_{ij} is the pipeline connecting joint i to joint j

P_{12} (km)	P_{24} (km)	P_{4J} (km)	P_{3J} (km)	P_{JFPSO} (km)	P_{6T} (km)	P_{5T} (km)	P_{TFPSO} (km)	$\bar{P}l_{sum,n}$ (km)	$\bar{n}$
0	0	0	0	0	0	0	0	0	1
3.1	2.3	2.6	6.5	0.5	0	0	0	8.5	2
0	2.3	2.6	0	0.5	0	0	0	5.5	3
:	:	:	:	:	:	:	:	:	:

3.2 Cost Proxy Models

In general, the cost estimation development requires many detailed aspects (e.g., specific tool price and cost of injector well) to be considered. It is updated every time a decision is changed. Nunes et al. (2018) gave an example of a simplified cost estimation based on historical data for screening field development alternatives. In this work, the cost proxy model is formulated as a linear function. Therefore, the model can be included in the optimization problem. The expense for the field development is categorized into four groups:

1. DrillEx: the cost of drilling well(s).

2. CapEx - Subsea: the cost of subsea equipment such as pipeline, flowline, Xmas tree, and template.

3. CapEx - Topside: the cost of offshore structure and handling facilities. Both of these two parameters are considered to be dependent on the fluid capacity. It is modeled as a function of maximum fluid rates.

4. OpEx: the expense considering all ongoing costs for running the field, such as operator, utilities, supply, and well maintenance cost. It is modeled as a function of maximum fluid rates and the number of well.

The cost proxy model is built based on the provided data (Table 3.3, Table 3.4, and Table 3.5). Both CapEx and OpEx are estimated using different fluid rates. These rates are adjusted to be $\pm 25\,\%$ of the maximum fluid rates from the original production plot. This ensures that the proxy model is compatible for a wider range of fluid capacity. For the cost that depends on multiple variables, a linear regression is performed to build the function using Add-in Excel (Cameron, 2009). In general, the multivariate linear regression is expressed as:

$$y = \alpha + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \beta_3 \cdot x_3 \tag{3.2}$$

Where y is the dependent variable, x_i is a set of independent variables, α is the y-intercept, and $beta_i$ is a set of the coefficient for each x_i. In this work, the regression uses the oil, gas, and water rates as the independent variables.

Table 3.3: The cost of the production system instruments

Well cost (Million NOK/well)	500
Xmas Tree (Million NOK/Xmas Tree)	50
Subsea Template (Million NOK/template)	500
Prodution Line (Million NOK/km)	25

Table 3.4: The input for CapEx proxy model

Required capacity			FPSO	Circular FPSO (Sevan)
qo_max (Sm3/d)	qw_max (Sm3/d)	qg_max (1000 Sm3/d)	CapEx Topside (Million NOK)	CapEx Topside (Million NOK)
13750	13750	6750	16150	17650
13750	13750	4050	15500	17000
13750	8250	6750	15955	17455
13750	8250	4050	15305	16805
11000	13750	6750	13625	15125
11000	13750	4050	12975	14475
11000	8250	6750	13495	14995
11000	8250	4050	12845	14345
8250	13750	6750	11545	13045
8250	13750	4050	10895	12395
8250	8250	6750	11100	12600

Table 3.5: The input for OpEx proxy model

Required capacity			FPSO	Circular FPSO (Sevan)
qo_max (Sm3/d)	qw_max (Sm3/d)	qg_max (1000 Sm3/d)	OpEx Topside (Million NOK)	OpEx Topside (Million NOK)
13750	13750	6750	1050	1050
13750	13750	4050	1050	1050
13750	8250	6750	1050	1050
13750	8250	4050	1050	1050
11000	13750	6750	980	980
11000	13750	4050	980	980
11000	8250	6750	980	980
11000	8250	4050	980	980
8250	13750	6750	920	920
8250	13750	4050	920	920
8250	8250	6750	920	920
8250	8250	4050	920	920

3.3 NPV Optimization Formulation

This optimization mainly uses PWL approximation to reduce the complexity of an oil field problem by transforming it into a simpler linear problem. As mentioned in section 2.1, The production potential $q_{opp,n}$ acts as a bound to oil rates, and it is a non-linear function dependent to N_p and well status (Alkindira, 2019). A particular well combination status zw_i is represented by unique scenario number sn for distinction. Accordingly, a lookup table is provided to link the well status with a scenario number. This table can be converted to a linear function by applying a SOS1 constraint. Scenario number also defines the pipeline length, Pl, if few wells are drilled. Therefore, the PWL Approximation is implemented to reformulate the non linear function $q_{opp} = f(N_p, sn)$, $sn = f(zw_1, zw_2 ... zw_N)$ and $Pl = f(sn)$ with N as the number of well into an MILP.

The objective and related equations

The main purpose of this optimization problem is to maximize the net present value of the project:

$$NPV = NPV_{preprod} + \sum_{n \in T} DCF_n \tag{3.3}$$

where n indicates the year and T is all the production years.

$NPV_{preprod}$ is the development cost before production starts. It includes the expenses on CapEx. Mathematically, it is expressed as:

$$NPV_{preprod} = CapEx_{topside} + CapEx_{subseapipe} \tag{3.4}$$

CapEx consists of two separate components: Topside and subsea. As mentioned in section 3.2, the CapEx topside depends on the fluid processing capacity of oil, water and gas, while the CapEx subsea depends on the pipeline length Pl and the number of xmas tree. However, the CapEx subsea xmas tree is not included $NPV_{preprod}$. Thus, the components for $NPV_{preprod}$ is:

$$\begin{aligned} CapEx_{topside} &= f(q_{o,max}, q_{w,max}, q_{g,max}) \\ CapEx_{subseapipe} &= f(Pl) \end{aligned} \tag{3.5}$$

Meanwhile, Discounted Cashflow (DCF_n) at particular time n is modelled as the discounted revenue subtracted with the discounted cost:

$$DCF_n = \frac{Revenue_n - Cost_n}{(1 + disc)^n} \tag{3.6}$$

The revenue at year n is the return obtained from the field oil production at that particular year only, $\Delta N_{p,n}$, while assuming that the oil price P_o remains constant throughout the production times. The revenue can be written as:

$$Revenue_n = P_o \cdot XR \cdot VC \cdot \Delta N_{p,n} \tag{3.7}$$

where XR is the exchange rate from USD to NOK and VC is the volume conversion. Generally, $\Delta N_{p,n}$ are determined from the oil production rates of both X1 and X2. The equations related to the field oil production computation are:

$$\begin{aligned} \Delta N_{p,n} &= \frac{360 \cdot q_{o,X1,n-1} + 360 \cdot q_{o,X2,n-1}}{1E06} \\ N_{p,X1,n} &= N_{p,X1,n-1} + 360 \cdot q_{o,X1,n-1} \\ N_{p,X2,n} &= N_{p,X2,n-1} + 360 \cdot q_{o,X2,n-1} \end{aligned} \tag{3.8}$$

where $q_{o,X1,n}$ is oil rate at year n and $N_{p,X1,n}$ is the cumulative oil production after producing for n years from reservoir X1. The same definition is applied to reservoir X2. Similarly, the cumulative gas and water production are determined from the corresponding fluid production rates:

$$\begin{aligned} G_{p,X1,n} &= G_{p,X1,n-1} + 360 \cdot q_{g,X1,n-1} \\ W_{p,X1,n} &= W_{p,X1,n-1} + 360 \cdot q_{w,X1,n-1} \end{aligned} \tag{3.9}$$

where $q_{g,X1,n-1}$ and $q_{w,X1,n-1}$ is gas and water rate at year n of reservoir X1, while $G_{p,X1,n}$ and $W_{p,X1,n}$ is the cumulative gas and water production respectively after producing for n years from reservoir X1 also. The similar expression is applied to reservoir X2.

The expenses after the production starts are originated from the cost of drilling new wells (DrillEx), OpEx and the installation of new xmas tree (or the CapEx subsea xmas tree):

$$\begin{aligned} Cost_n &= DrillEx_n + OpEx_n + CapEx_{subseaxt,n} \\ DrillEx_n &= f(N_{w,d,n}) \\ CapEx_{subseaxt,n} &= f(N_{w,d,n}) \\ OpEx_n &= f(N_{w,F,n}, q_{o,F,n}, q_{w,F,n}, q_{g,F,n}) \end{aligned} \tag{3.10}$$

where $q_{o,F,n}$, $q_{w,F,n}$, $q_{g,F,n}$ are the field oil, water and gas production, respectively, $N_{w,d,n}$ is the the number of well drilled. $N_{w,F,n}$ is the number of wells in the field expressed as:

$$N_{w,F,n} = N_{w,X1,n} + N_{w,X2,n} \tag{3.11}$$

where $N_{w,X1,n}$ and $N_{w,X2,n}$ is the number of well in reservoir X1 and X2 respectively.

Optimization Variables

The optimization is carried out by changing:

- the oil production rates per year per reservoir, $q_{o,X1,n}$ and $q_{o,X2,n}$.
- the status of each well in each time step, $zw_{i,n}$ where $i \in \{1, 2, ..., 6\}$ for reservoir X1 and $zw_{j,n}$ where $j \in \{7, ..., 9\}$ for reservoir X2 . All the well statuses are binary (0-1).

Optimization Constraints

The optimization problem is subject to the following constraints:

$$\forall n \in T :$$

- The oil production rates should not be greater than the production potential of each reservoir:

$$\begin{aligned} q_{o,X1,n} &\leq q_{opp,X1,n} \\ q_{o,X2,n} &\leq q_{opp,X2,n} \end{aligned} \tag{3.12}$$

$q_{opp,X1,n}$ and $q_{opp,X2,n}$ are the potential oil rates at time step n for reservoir X1 and X2.

- Maximum number of wells (for borth reservoirs) allowed to drill each year is three, and it is not allowed to shut down wells:

$$0 \leq N_{w,F,n} - N_{w,F,n-1} \leq 3 \tag{3.13}$$

- Relationship between the number of wells and the well status:

$$\begin{aligned} N_{w,X1,n} &= \sum_{i=\{1,2,\ldots,6\}} zw_{i,n} \\ N_{w,X2,n} &= \sum_{j=\{7,..9\}} zw_{j,n} \end{aligned} \tag{3.14}$$

- Once a well is activated (drilled), it should not be shut down:

$$\begin{aligned} zw_{i,n} &\geq zw_{i,n-1} \\ zw_{j,n} &\geq zw_{j,n-1} \\ \forall i &\in \{1,2,..6\} \\ \forall j &\in \{7,...,9\} \end{aligned} \tag{3.15}$$

- The production oil, gas and water capacities of the procession facilities are obtained as follow:

$$\begin{aligned} q_{o,max} &\geq q_{o,F,n} \\ q_{g,max} &\geq q_{g,F,n} \\ q_{w,max} &\geq q_{w,F,n} \end{aligned} \tag{3.16}$$

  where $q_{o,F,n}$, $q_{g,F,n}$ and $q_{w,F,n}$ are the field oil, gas and water production rates respectively.

- The number of well in reservoir X1 has to be greater than 3.

The potential oil rates of each reservoir is a non-linear function of the cumulative oil produced from each reservoir and status of wells in that reservoir:

$$\begin{gathered} \forall i \in \{1,2,..6\}: \\ q_{opp,X1,n} = f(N_{p,X1,n}, zw_{i,n}) \\ \forall j \in \{7,...,9\}: \\ q_{opp,X2,n} = f(N_{p,X2,n}, zw_{j,n}) \end{gathered} \tag{3.17}$$

Meanwhile, the G_p and W_p are non-linear functions of the cumulative oil produced only from each reservoir. Table 3.6 shows the example of reservoir X1.

Table 3.6: The production potentials illustration of Reservoir X1

zw1	zw2	zw3	zw4	zw5	zw6	Np	qopp,X1	Gp,X1	WpX1
0	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0	0	0	0	0	0	21.9	0	25839.5	19.07
1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	4.2	2301.31	1673.6	1.26
1	0	0	0	0	0	7.1	1445.49	2973.43	2.86
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	1	1	0	24010.6	0	0
1	1	1	1	1	1	4.2	13832.7	1673.6	1.26
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

3.3.1 Formulation

The formulation of the problem is further explained in the following.

Objective function

The main purpose of this optimization problem is to yield the highest NPV. Therefore, the objective function can be expressed as:

$$\text{Maximize} \qquad NPV \tag{3.18}$$

Sets

- T = $\{0, 1...nt\}$ is set of timestep where nt is number of years
- R: Set of reservoirs $\{1, 2\}$.
- S: set of well status breakpoints $\{1, 2\}$.
- i_{Np} = $\{1, ..., bp_{np}\}$: Set of N_p breakpoint, with bp_{np} as number of N_p breakpoint used for PWL
- $i_{sn[ir]}$= $\{1, ..., bp_{sn[ir]}\}$: Set of the sn breakpoints with $bp_{sn[ir]}$ as the number of scenario breakpoints for reservoir-ir. Reservoir X1 reduces the number of scenarios used for $q_{opp} = f(N_p, sn)$. It is decided that scenario with N_w less than 3 in reservoir X1 is discarded to improve the computational efficiency. Reservoir X1 have 43 scenarios instead of 64, while Reservoir X2 uses 8 scenarios.

Indices

The followings are the indices used in the sets:

- ir : reservoir in R

- n : time in T.
- k : cumulative oil production in i_{Np}.
- l : scenario number in $i_{sn[ir}$ of reservoir-ir.
- m : scenario number in $i_{sn[ir}$ defining the pipeline length.
- a : well 1 status in S.
- b : well 2 status in S.
- c : well 3 status in S.
- d : well 4 status in S.
- e : well 5 status in S.
- f : well 6 status in S.
- g : well 7 status in S.
- h : well 8 status in S.
- i : well 9 status in S.

Parameters

A fixed value in the optimization problem is called parameter. It consists of breakpoints data used during PWL approximation and general data that is not change during the computation. The parameters are classified into four segments:

1. General parameters:
   - $N_{w,peryear}$: max number of new wells in a year (3 wells).
   - $N_{w,pd}$: number of predrilled wells (3 wells).
   - Po: Oil Price (60 USD/bbl).
   - xr: Currency exchange rate from USD to NOK (8.5).
   - VC: Volume conversion ratio from Sm^3 to bbl (6.29).
   - P_{well}: the drilling cost per well.
   - P_X: the installment cost per one X-mas tree
   - P_{pipe}: the cost of pipeline installment.
   - $P_{Capex,i}$: coeffiecient of linear function CapEx with $i \in \{1, 2, 3, 4\}$.
   - $P_{Opex,i}$: coeffiecient of linear function OpEx $i \in \{1, 2, 3, 4, 5\}$.
2. Parameters for the scenario lookup tables:
   - $\bar{zw}_{1,a,b,c,d,e,f}$: Binary variable for well 1 status at breakpoint (a,b,c,d,e,f). The well status lookup tables (Table 3.7) is modelled with 6D PWL approximation.

- $\bar{zw}_{2,a,b,c,d,e,f}$: The status of well 2 at breakpoint (a,b,c,d,e,f).
- $\bar{zw}_{3,a,b,c,d,e,f}$: The status of well 3 at breakpoint (a,b,c,d,e,f).
- $\bar{zw}_{4,a,b,c,d,e,f}$: The status of well 4 at breakpoint (a,b,c,d,e,f).
- $\bar{zw}_{5,a,b,c,d,e,f}$: The status of well 5 at breakpoint (a,b,c,d,e,f).
- $\bar{zw}_{6,a,b,c,d,e,f}$: The status of well 6 at breakpoint (a,b,c,d,e,f).
- $\bar{sn}_{x1,a,b,c,d,e,f}$: the scenario number of reservoir X1 at breakpoint (a,b,c,d,e,f). It is important to note that the scenario number is a function of unique well status zw_i combination.

Table 3.7: The look up table illustration for Reservoir X1, $sn_{x1}(zw_1, zw_2, zw_3, zw_4, zw_5, zw_6)$. This table stores all combinations of the well status and its scenario number

a	b	c	d	e	f	$\bar{zw}_1$	$\bar{zw}_2$	$\bar{zw}_3$	$\bar{zw}_4$	$\bar{zw}_5$	$\bar{zw}_6$	$\bar{sn}_{x1}$
1	1	1	1	1	1	0	0	0	0	0	0	1
2	1	1	1	1	1	1	0	0	0	0	0	2
:	:	:	:	:	:	:	:	:	:	:	:	:
2	2	2	2	2	2	1	1	1	1	1	1	64

Table 3.8: The look up table illustration for Reservoir X2, $sn_{x2}(zw_7, zw_8, zw_9)$

g	h	i	$\bar{zw}_7$	$\bar{zw}_8$	$\bar{zw}_9$	$\bar{sn}_{x2}$
1	1	1	0	0	0	1
2	1	1	1	0	0	2
:	:	:	:	:	:	:
2	2	2	1	1	1	8

- $\bar{zw}_{7,g,h,i}$: Binary variable representing the status of well 7 at breakpoint (g,h,i) in the lookup tables for reservoir X2 (Table 3.8). The table reformulated with 3D PWL approximation.
- $\bar{zw}_{8,g,h,i}$: The status of well 8 at breakpoint (g,h,i).
- $\bar{zw}_{9,g,h,i}$: The status of well 9 at breakpoint (g,h,i).
- $\bar{sn}_{x2,g,h,i}$: the scenario number of reservoir X2 attained at breakpoint (g,h,i) defined by unique well status combination.

3. Parameters related to 2D PWL approximation of $q_{opp} = f(N_p, sn)$ and other properties that depends on the N_p and sn (Table 3.9):

Table 3.9: The illustration for 2D PWL approximation data

ir	k	l	$\bar{N_p}$ (M Sm3)	$\bar{sn}$	$\bar{q_{opp}}$ (Sm3/d)	$\bar{G_p}$	$\bar{W_p}$
1	1	43	0	1	0	0	0
1	2	43	4.16	1	0	1673.6	1.26
1	3	43	7.08	1	0	2972.4	2.85
1	4	43	10.12	1	0	4650.4	5.035448
1	5	43	15.00	1	0	10523.2	9.62
1	6	43	18.51	1	0	17247.8	13.79
1	7	43	21.73	1	0	25839.5	19.07
1	1	1	0	23	12386.6	0	0
1	2	1	4.16	23	7136.0	1673.6	1.26
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	6	42	18.51	64	1515.9	17247.87	13.79
1	7	42	21.73	64	943.8	25839.5	19.07
2	1	1	0	50	0	0	0
2	2	1	1.01	50	0	106.7	0.044
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

- $\bar{N}_{p,ir,k,l}$: The cumulative oil production of reservoir-ir at breakpoint (k,l). Index-k represents the breakpoint for N_p.
- $\bar{sn}_{ir,k,l}$: The scenario number for production potential curve of reservoir-ir at breakpoint (k,l).
- $\bar{q}_{opp,ir,k,l}$: The production potential of reservoir-ir. The value of q_{opp} is different as a subject to the N_p and sn.
- $\bar{G}_{p,ir,k,l}$: The cumulative gas production of reservoir-ir. Here, it is defined to be a subject to N_p and sn.
- $\bar{W}_{p,ir,k,l}$: The cumulative water production of reservoir-ir as a subject to the N_p and sn.

4. Parameters related to 1D PWL approximation of pipeline length $\bar{Pl} = f(sn)$ (Table 3.10):

Table 3.10: The illustration for 1D PWL approximation $\bar{Pl} = f(sn)$

ir	m	$\bar{sn}$	$\bar{Pl}_{sum}$
1	1	1	0
1	2	23	15
1	3	24	8.5
⋮	⋮	⋮	⋮
2	1	1	0
2	2	2	17
⋮	⋮	⋮	⋮

- $\bar{P}l_{sum,ir,m}$: The total length of pipeline connecting wells in reservoir-ir to separator at breakpoint-m.
- $\bar{sn}_{pipe,ir,m}$: The scenario number for $\bar{P}l = f(sn)$ at breakpoint-m.

Variables

Variables are adjustable element in the optimization. It is changed to yield the optimum objective value. Note that most of the variables in this optimization are non negative, except DCF and NPV. Other than the variables declared as integer or binary, all the variables are continuous number. Furthermore, the variables are listed as:

- $N_{w,F,n} \in \mathbb{Z}$: total number of well of the field at time-n.
- $N_{w,ir,n} \in \mathbb{Z}$, : total number of well of reservoir-ir at time-n.
- $q_{o,F,n}$: field oil production rate at time-n.
- $q_{o,ir,n}$: oil production rate of reservoir-ir at time-n.
- $q_{g,ir,n}$: gas production rate of reservoir-ir at time-n.
- $q_{w,ir,n}$: water production rate of reservoir-ir at time-n.
- $q_{opp,ir,n}$: production potential rate of reservoir-ir at time-n.
- $N_{p,ir,n}$: cumulative oil production of reservoir-ir at time-n.
- $sn_{ir,n} \in \mathbb{Z}$: scenario number of reservoir-ir at time-n.
- $G_{p,ir,n}$: cumulative gas production of reservoir-ir at time-n.
- Wp, ir, n: cumulative water production of reservoir-ir at time-n.
- Np, F, n: total cumulative oil production of the field at time-n.
- $zw_{1,n} \in \{0, 1\}$: status of well 1 at time-n.
- $zw_{2,n} \in \{0, 1\}$: status of well 2 at time-n.
- $zw_{3,n} \in \{0, 1\}$: status of well 3 at time-n.
- $zw_{4,n} \in \{0, 1\}$: status of well 4 at time-n.
- $zw_{5,n} \in \{0, 1\}$: status of well 5 at time-n.
- $zw_{6,n} \in \{0, 1\}$: status of well 6 at time-n.
- $zw_{7,n} \in \{0, 1\}$: status of well 7 at time-n.
- $zw_{8,n} \in \{0, 1\}$: status of well 8 at time-n.
- $zw_{9,n} \in \{0, 1\}$: status of well 9 at time-n.

- $pipe_{ir,n}$: total length of the pipeline connected to reservoir-ir at time-n.
- $pipe_{max}$: maximum pipeline length in the field.
- $q_{o,max}$: maximum oil production rate.
- $q_{w,max}$: maximum water production rate.
- $q_{g,max}$: maximum gas production rate.
- PV_{top}: present value representing CapEx Topside in Million NOK.
- $PV_{op,n}$, $\forall n \in \{1, 2, ..., nt\}$: present value representing OpEx at time-n in Million NOK.
- $PV_{d,n}$: present value representing DrillEx at time-n in Million NOK.
- $PV_{sub,n}$: present value representing CapEx Topside at time-n in Million NOK.
- $PV_{r,n}$, $\forall n \in \{1, 2, ..., nt\}$: present value representing revenue at time-n in Million NOK.
- DCF_n: discounted cash flow at time-n in Million NOK.
- NPV: net present value in Million NOK.

Variables induced by PWL approximation constraints:

- Auxiliary variables for a particular well status in Reservoir X1. They enforce the SOS1 constraint for 6D PWL Approximation of the lookup table at time-n:
  $\eta_{zw1,n,a}$, $\eta_{zw2,n,b}$, $\eta_{zw3,n,c}$, $\eta_{zw4,n,d}$, $\eta_{zw5,n,e}$, and $\eta_{zw6,n,f}$
- $\lambda_{x1,n,a,b,c,d,e,f}$: The weighting coefficient of breakpoints (a,b,c,d,e,f) in the 6D PWL Approximation at time-n.
- Auxiliary variables for a particular well status in reservoir X2. They impose the SOS1 constraint for 3D PWL Approximation of the lookup table at time-n:
  $\eta_{zw7,n,g}$,$\eta_{zw8,n,h}$, and $\eta_{zw9,n,i}$
- $\lambda_{x2,n,g,h,i}$: The weighting coefficient of breakpoints (g,h,i) in the 3D PWL Approximation at time-n.
- Auxiliary variables for a particular sn and N_p of reservoir-ir at time-n. They impose the SOS1/SOS2 constraint for the 2D PWL Approximation:
  $\eta_{sn,ir,n,l}$ and $\eta_{N_p,ir,n,k}$.
- $\lambda_{qopp,ir,n,k,l}$: The weighting coefficient of breakpoints (k,l) in the 2D PWL Approximation correspond to reservoir-ir at time-n.
- $\lambda_{pipe,ir,n,m}$: The weighting coefficient of breakpoint-m in the 1D PWL Approximation correspond to reservoir-ir at time-n.

Inequality Constraints

The inequality constraints enclose the condition that may be binding or not binding as long as the criteria are fulfilled. In particular, binding means enforcing the condition to be the same. The inequality constraints are typically used to bound certain variables.

The oil rates are bounded to be less than or equal to the production potential:

$$q_{o,ir,n} \leq q_{opp,ir,n} \qquad \forall n \in T, \qquad \forall ir \in R \tag{3.19}$$

It is assumed that $q_{opp,n}$ and $q_{o,n}$ are remained constant from n to $n-1$.

The maximum fluid production rates are required to determine the CapEx Topside. It is defined that any rate in the entire years does not exceed the maximum production rates:

$$\begin{aligned} q_{o,F,n} &\leq q_{o,max} && \forall n \in T \\ \sum_{i \in R} q_{g,ir,n} &\leq q_{g,max} && \forall n \in T \\ \sum_{i \in R} q_{w,ir,n} &\leq q_{w,max} && \forall n \in T \end{aligned} \tag{3.20}$$

The optimization restricts the value of maximum rates to be the highest rates of the field such that Equation 3.20 is satisfied.

The $zw_{i,n}$ are the binary variables representing the status of well-i. The $zw_{i,n}$ equal to 1 means that the well-i is drilled or active, while $zw_{i,n}$ equals to 0 indicates that the well-i not drilled or inactive. Once a well is drilled, it could not be undrilled. Then, the well should be present in the following years:

$$\begin{aligned} zw_{1,n} - zw_{1,n-1} &\geq 0 \\ zw_{2,n} - zw_{2,n-1} &\geq 0 \\ zw_{3,n} - zw_{3,n-1} &\geq 0 \\ \vdots \quad \vdots \quad \vdots \quad & \vdots \quad \vdots \\ zw_{9,n} - zw_{9,n-1} &\geq 0 \\ \forall n \in \{1, .., nt\} & \end{aligned} \tag{3.21}$$

Note that Equation 3.21 is applied to all well status $zw_{i,n}$.

Consequently, the number of wells in each reservoir cannot be reduced:

$$N_{w,ir,n} - N_{w,ir,n-1} \geq 0 \qquad \forall ir \in R \qquad \forall n \in \{1, .., nt\} \tag{3.22}$$

The number of well allowed to be drilled at the early time is not more than the number of pre-drilled well:

$$N_{w,F,1} \leq N_{w,predrilled} \tag{3.23}$$

In the following years, the field should add at most three wells per year:

$$N_{w,F,n} - N_{w,F,n-1} \leq N_{w,peryear} \qquad \forall ir \in R \qquad \forall n \in \{2, .., nt\} \tag{3.24}$$

When new wells are drilled, there are two possibilities; installing another pipeline installed or just making use of the existing pipelines. Either way, the length of the pipeline used in the field does not decrease. Therefore, the same constraint as Eq. 3.22 is applied to the pipeline length:

$$Pipe_{ir,n} - Pipe_{ir,n-1} \geq 0 \qquad \forall ir \in R \qquad \forall n \in \{2,..,nt\} \tag{3.25}$$

The maximum pipeline length are required to determine the CapEx subsea. Similar to Eq- 3.20, the maximum pipeline length imposes the following:

$$\sum_{i \in R} pipe_{ir,n} \leq pipe_{max} \qquad \forall n \in T \tag{3.26}$$

Equality Constraints

Equality constraints strictly enforce the variable to be a specific condition or value (binding). The equality constraints are mostly used to determine the equations that calculate certain variables, e.g., field cumulative oil production and the number of wells.

In the beginning, the field does not have any oil production. Thus, the cumulative oil production is initiated as:

$$N_{p,ir,0} = 0 \qquad \forall ir \in R \tag{3.27}$$

The common way to compute gas rate q_g is by multiplying the oil rate and Gas Oil Ratio (GOR). However, by multiplying two variables, the problem becomes non-linear. One approach is to use the PWL approximation for GOR as a function of N_p and thus determined the q_g from 2D PWL approximation $q_g = f(q_o, GOR)$ (González et al., 2020). Another approach is to compute q_g from cumulative production potential, $G_p = f(N_p)$. The optimization finds the gas rate that would give the corresponding value of G_p. The second approach with G_p is preferred due to faster computation and comparable results as the first approach (Angga, 2019). Therefore, it is decided to use cumulative production to calculate q_g. This approach is also applied to determine the q_w.

All cumulative production terms are approximated using the forward integration as:

$$\begin{aligned} N_{p,ir,n} &= N_{p,ir,n-1} + \frac{q_{o,ir,n-1} \cdot 365}{10^6} \\ G_{p,ir,n} &= G_{p,ir,n-1} + \frac{q_{g,ir,n-1} \cdot 365}{10^3} \\ W_{p,ir,n} &= W_{p,ir,n-1} + \frac{q_{w,ir,n-1} \cdot 365}{10^6} \\ &\forall ir \in R, \qquad \forall n \in \{1,..,nt\} \end{aligned} \tag{3.28}$$

Since the well status is a binary, the total number of well of a particular reservoir can be directly expressed as the summation of the well status in the corresponding reservoir:

$$\begin{aligned} N_{w,1,n} &= zw_{1,n} + zw_{2,n} + zw_{3,n} + zw_{4,n} + zw_{5,n} + zw_{6,n} \\ N_{w,2,t} &= zw_{7,n} + zw_{8,n} + zw_{9,n} \\ &\forall t \in T \end{aligned} \tag{3.29}$$

For all variables related to a field, it is the summation product of the variable of each reservoir:

$$q_{o,F,n} = \sum_{i \in R} q_{o,ir,n} \qquad \forall n \in T \qquad ir \in R \tag{3.30}$$

$$N_{w,F,n} = \sum_{i \in R} N_{w,ir,n} \qquad \forall n \in T \qquad ir \in R \tag{3.31}$$

$$N_{p,F,n} = \sum_{i \in R} N_{p,ir,n} \qquad \forall n \in T \qquad ir \in R \tag{3.32}$$

PWL Approximation Constraints

Four groups of PWL approximation are listed as below:

1. A lookup table is used to find the scenario number's translation to the well-status combination. Well status only have 2 possible values; 1 $\rightarrow$ positive and 0 $\rightarrow$ inactive. The scenario number sn is non-linearly dependent on the unique combination of well status. It is important to define which well are producing in order to impose Eq. 3.21 and maintain well scheduling consistency. 6D PWL Approximation for Reservoir X1 is used to model the non-linear function $sn_{x1}(zw_1, zw_2, \ldots, zw_6)$.

   The formulation of this multidimensional PWL adapts the standard form of 2D PWL introduced in section 2.3.1. Now, the formulation adds more variables of η to impose SOS1 constraint in each dimension. The following represents the 6D PWL constraints for three of six wells:

$$\forall n \in T :$$

$$sn_{1,n} = \sum_{a \in S} \sum_{b \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \cdot \bar{sn}_{x1,a,b,c,d,e,f} \tag{3.33}$$

$$z_{w1,n} = \sum_{a \in S} \sum_{b \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \cdot \bar{z}_{w1,a,b,c,d,e,f} \tag{3.34}$$

$$z_{w2,n} = \sum_{a \in S} \sum_{b \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \cdot \bar{z}_{w2,a,b,c,d,e,f} \tag{3.35}$$

$$z_{w3,n} = \sum_{a \in S} \sum_{b \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \cdot \bar{z}_{w2,a,b,c,d,e,f} \tag{3.36}$$

$$\eta_{zw1,n,a} = \sum_{b \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \qquad \forall a \in S \tag{3.37}$$

$$\eta_{zw2,n,b} = \sum_{a \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,nt,a,b,c,d,e,f} \qquad \forall b \in S \tag{3.38}$$

$$\eta_{zw3,n,c} = \sum_{a \in S} \sum_{b \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \qquad \forall c \in S \tag{3.39}$$

$$1 = \sum_{a \in S} \sum_{b \in S} \sum_{c \in S} \sum_{d \in S} \sum_{e \in S} \sum_{f \in S} \lambda_{x1,n,a,b,c,d,e,f} \tag{3.40}$$

$$\{\eta_{zw1,n,a}, \eta_{zw2,n,b}, \text{ and } \eta_{zw3,n,c}, \qquad \forall a, b, c \in S\} \text{is SOS1} \tag{3.41}$$

By setting all the set of η_{zwi} to SOS1, it ensures that values of variable $zw_{i,n}$ are chosen among the breakpoints. Therefore, the scenario number would only be translated based on the defined well status combination.

2. 3D PWL Approximation is implemented to model the lookup table for the three wells statuses of reservoir X2 . Again, the formulation of this multidimensional PWL modifies the standard form in section 2.3.1. The following shows the 3D PWL constraints:

$$\forall n \in T :$$
$$sn_{2,n} = \sum_{g \in S} \sum_{h \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \cdot s\bar{n}_{x2,g,h,i} \tag{3.42}$$

$$z_{w7,n} = \sum_{g \in S} \sum_{h \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \cdot \bar{z}_{w7,g,h,i} \tag{3.43}$$

$$z_{w8,n} = \sum_{g \in S} \sum_{h \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \cdot \bar{z}_{w8,g,h,i} \tag{3.44}$$

$$z_{w9,n} = \sum_{g \in S} \sum_{h \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \cdot \bar{z}_{w9,g,h,i} \tag{3.45}$$

$$\eta_{zw7,n,g} = \sum_{h \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \qquad g \in S \tag{3.46}$$

$$\eta_{zw8,n,h} = \sum_{g \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \qquad h \in S \tag{3.47}$$

$$\eta_{zw9,n,i} = \sum_{i \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \qquad i \in S \tag{3.48}$$

$$1 = \sum_{g \in S} \sum_{h \in S} \sum_{i \in S} \lambda_{x2,n,g,h,i} \tag{3.49}$$

$$\{\eta_{zw7,n,g}, \eta_{zw8,n,h}, \text{ and } \eta_{zw9,n,h}, \qquad \forall a, b, c \in S\} \text{is SOS1} \tag{3.50}$$

3. Production potential is non-linearly dependent on N_p and sn. Alkindira (2019) determined the $q_{opp,t}$ at certain N_p for any combination of s_n by multiplying the $f(s_n)$ with the $q_{opp,64}$ (see section 3.1). To cope with this, the $q_{opp}(N_p, S_n)$ is pre-calculated for every N_p and sn before included into the formulation. Subsequently,

the non-linear function $q_{opp}(N_p, S_n)$ can reformulated directly using 2D PWL Approximation. The constraints for the $q_{opp}(N_p, S_n)$ formulation are:

$$
\begin{aligned}
&\forall n \in T \text{ and } \forall ir \in R: \\
q_{opp,ir,n} &= \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{q}_{opp,ir,k,l} \\
sn_{ir,t} &= \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{sn}_{ir,k,l} \\
N_{p,ir,t} &= \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{N}_{p,ir,k,l} \\
\eta_{Np,ir,n,k} &= \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \qquad \forall k \in i_{N_p} \\
\eta_{sn,ir,n,l} &= \sum_{k \in i_{N_p}} \lambda_{qopp,ir,n,k,l} \qquad \forall l \in i_{sn[ir]} \\
1 &= \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l}
\end{aligned}
\tag{3.51}
$$

$$
\begin{aligned}
\{\eta_{sn,ir,n,l}, \qquad \forall l \in i_{sn[ir]}\} \text{is SOS1} \\
\{\eta_{Np,ir,n,k}, \qquad \forall l \in i_{Np}\} \text{is SOS2}
\end{aligned}
\tag{3.52}
$$

The scenario number is iteratively picked among the defined selection and the interpolation is executed between the N_p of the particular sn. Only $\eta_{sn,ir,n,l}$ imposes SOS1 constraint.

The W_p and G_p are solely dependent on the N_p, but stay the same for any scenario number. Instead of creating another PWL approximation for these two variables, the W_p and G_p are included to the 3D Approximation and determined using the same $\lambda_{qopp,ir,n,k,l}$. This is to reduce the number of variables in the optimization. The G_p and W_p are expressed as:

$$
G_{p,ir,t} = \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{G}_{p,ir,k,l} \qquad \forall n \in T \qquad \forall ir \in R \tag{3.53}
$$

$$
W_{p,ir,t} = \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{W}_{p,ir,k,l} \qquad \forall n \in T \qquad \forall ir \in R \tag{3.54}
$$

4. Lastly, this optimization employs 1D PWL Approximation to find the pipe length for a particular sn. The total length of pipeline is pre-calculated based on the configuration of the required pipeline connecting the separator and the wells for each

scenario (Table 3.2). Again, this PWL approximation has SOS1 constraint:

$\forall n \in T$ and $\forall ir \in R$:

$$\begin{aligned} Pipe_{ir,t} &= \sum_{m \in i_{sn[ir]}} \lambda_{pipe,ir,n,m} \cdot \bar{Pl}_{sum,ir,m} \\ sn_{ir,t} &= \sum_{m \in i_{sn[ir]}} \lambda_{pipe,ir,n,m} \cdot \bar{sn}_{pipe,ir,m} \\ 1 &= \sum_{m \in i_{sn[ir]}} \lambda_{pipe,ir,n,m} \end{aligned} \tag{3.55}$$

$$\{\lambda_{pipe,ir,n,m}, \qquad \forall m \in i_{sn[ir]}\} \text{is SOS1} \tag{3.56}$$

Economic Constraints

These constraints are composed based on the cost proxy model in section 3.2. the price (i.e. P_{well}, P_X and P_{pipe} included in the cost constraints are obtained from Table 3.3. The coefficient of CapEx and OpEx in the cost constraints are obtained through linear regression. Several assumptions made to the constraints are:

- Drillex is computed when the well starts producing on that year
- CapEx Topside and Pipeline is made at year “0” or the beginning of production.
- CapEx X-mas tree is computed when the well starts producing
- OpEx and Revenue start at year “1” when the field has operated for one year

The following is the list of economic constraints:

1. DrillEx

   At the beginning ($t = 0$), the cost of drilling the predrilled wells is expressed as:

   $$PV_{dp,0} = P_{well} \cdot N_{w,F,0} \tag{3.57}$$

   Starting from the $t = 1$ year, the cost of drilling additional wells of each year is formulated as:

   $$PV_{d,n} = P_{well} \cdot (N_{w,F,n} - N_{w,F,n-1}) \qquad \forall n \in \{1..nt\} \tag{3.58}$$

   The additional wells are calculated from the difference in the number of well between the two consecutive years:

2. CapEx Subsea

   The payment of the pipeline installment takes place at the beginning of production, thus it directly calculate cost of the maximum pipeline length. On the other hand, the schedule of the X-mas tree payment follows DrillEx’s schedule. The CapEx Subsea is expressed as:

   $$\begin{aligned} PV_{sub,0} &= P_{pipe} \cdot pipe_{max} + P_X \cdot N_{w,F,0} \\ PV_{sub,n} &= P_X \cdot (N_{w,F,n} - N_{w,F,n-1}) \qquad \forall n \in \{1..nt\} \end{aligned} \tag{3.59}$$

3. CapEx Topside The CapEx Topside depends on the maximum oil, water and gas rates:

$$\begin{aligned} PV_{top} = &P_{Capex,1} + P_{Capex,2} \cdot q_{o,max} \\ &+ P_{Capex,3} \cdot q_{w,max} + P_{Capex,4} \cdot q_{g,max} \end{aligned} \tag{3.60}$$

   CapEx Topside only has one value since it is fully paid at the beginning of the production.

4. OpEx

   The operating expense is paid after one year of production or at the end of the year. Thus, it is computed using the variables from the previous year , $n-1$. The operating cost per year can be determined as:

$$\begin{aligned} PV_{op,n} = &P_{Opex,1} + P_{Opex,2} \cdot N_{w,F,n-1} + P_{Opex,3} \cdot q_{o,ir,n-1} + ... \\ &P_{Opex,4} \cdot q_{w,ir,n-1} + P_{Opex,5} \cdot q_{g,ir,n-1} \qquad \forall n \in \{1..nt\} \end{aligned} \tag{3.61}$$

5. Revenue

   By considering the exchange rate xr, volume conversion VC and oil price Po, the revenue per year is computed as:

$$PV_{r,n} = P_o \cdot XR \cdot VC \cdot (N_{p,F,n} - N_{p,F,n-1}) \qquad \forall n \in \{1..nt\} \tag{3.62}$$

   The return of this field only comes from the sales of cumulative oil production. The revenue is inputted from yearly production which is the difference between $N_{p,n}$ and $N_{p,n-1}$.

6. Discounted Cash Flow (DCF) and NPV

   The DCF are segmented into two parts:

$$\begin{aligned} DCF_0 &= -(PV_{top} + PV_{d,0} + PV_{sub,0}) \\ DCF_n &= \frac{PV_{r,t} - PV_{op,t} - PV_{d,t} - PV_{sub,n}}{(1 + disc)^n} \qquad n \in \{1..nt\} \end{aligned} \tag{3.63}$$

   Lastly, the NPV is the summation of the DCF set:

$$NPV = \sum_{n \in T} DCF_n \tag{3.64}$$

3.4 Setting up the Optimization

The optimization is executed using a tool called AMPL. AMPL is a modeling language widely used for setting up optimization problems, e.g., production distribution and scheduling. The way the problem created is practical because it uses similar algebraic notation used in mathematical programming (AMPL, 2020). Many powerful solvers are available in this tool. This optimization chooses GUROBI as the solver for the MILP. In general, the AMPL user divides the problem formulation into several files with different information. In this work, the formulation presented in the previous section is segmented into five file categories:

- $case.mod$: .mod file stores the main formulation. All variables, constraints, parameters, and objective functions are declared in this file.

- $combinationfactor.dat$: a file with .dat extension saves the parameter data implemented in the problem. This data file contains the lookup table data, nt, nr, $N_{w,py}$ and $N_{w,pd}$.

- $PWL\_Table\_x.dat$: this data file consists of data related to the 2D PWL approximation, including the number of N_p and sn breakpoints.

- $Econ.dat$: it stores data related to economic constraints.

- $Case\_x.run$: the .run file put all .mod and .dat files together. It states the selected solver along with its solver option for customization. It also declares the SOS1/SOS2 constraints and states which variables and parameters supposed to be displayed as the output.

- $log.tmp$: This file prints the result and the log messages during the computation.

3.4.1 Selecting the Breakpoints

The selection of the breakpoints plays a vital role in the computational efficiency of PWL Approximation. A large number of breakpoints have presentable non-linear behavior but much slower computation. In contrast, reducing the breakpoints shortens the running time, but the result is less accurate.

Table 3.11: List of cases for evaluating number of breakpoints

Number of Breakpoints	Case ID
5	$Case_{5bp}$
7	$Case_{7bp}$
10	$Case_{10bp}$
15	$Case_{15Bp}$

This section determines the optimal breakpoints selection with a shorter run time and an average error of less than 10% compared to the reference result. The error is calculated for the NPV and phase production rates. The optimization has a fixed drilling schedule (i.e., scenario number). Thus, only NPV and oil rates are assessed. The cases for this study are listed in Table 3.11. The author wisely selected the breakpoints to fit the non-linear curve of q_{opp} vs. N_p. The reconstructed curves with the corresponding number of breakpoints are shown in Figure 3.6.

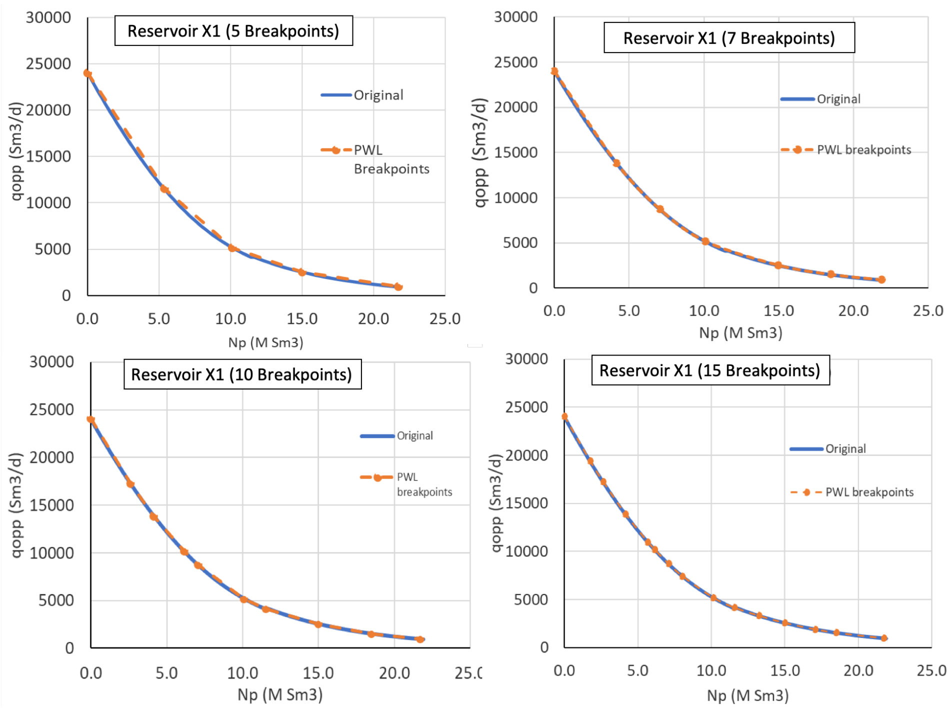


Figure 3.6: The reconstructed production potential with 5, 7, 10 and 15 breakpoints

3.4.2 NPV Optimization Cases

In this section, two optimization cases being compared are:

- $Case_1$: maximizing NPV by changing the production schedule and drilling schedule. The decision variables are Oil production rates and the number of well
- $Case_2$: maximizing NPV by changing only the production schedule. The decision variable is oil production rates.

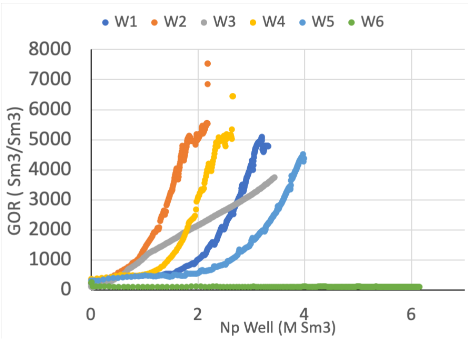
$Case_2$ employs the drilling schedule from the original plot. The input for this case can be seen in Table 3.12. The objective is to compare the NPV of the two results. $Case_1$ is expected to have higher NPV since it has more decision variables to adjust.

Table 3.12: The drilling schedule for $Case_2$

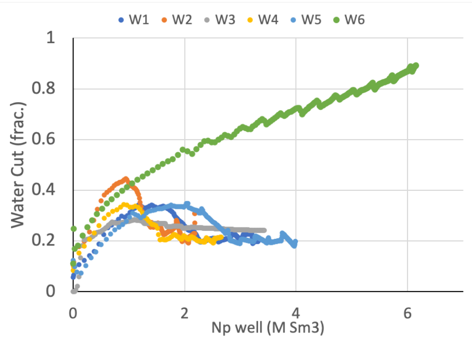
Year	Active wells	Timestep	Scenario X1	Scenario X2
2025	1, 2, 4	0	24	1
2026	1, 2, 4, 5, 6, 7	1	61	2
2027	1, 2, 3, 4, 5, 6, 7, 8, 9	2	64	8

3.5 NPV Optimization for Different Cumulative Gas and Water Production Profiles

The cumulative water and gas production is computed as a function of cumulative oil production only. However, the W_p and G_p profiles of the field are not fixed but change depending on the well scheduling. Each well has a separate WC and GOR profiles (See Figure 3.7), which might be caused by the well's location in the reservoir model.



(a) GOR



(b) Water Cut

Figure 3.7: GOR and Water Cut of the wells in Reservoir X1

In this section, several W_p and G_p curves of each reservoir are generated using W_p and G_p of the well. In this regard, the upper and lower limit of W_p and G_p can be computed. Another NPV optimization routine is performed by employing these curves to determine which curve has the highest and lowest NPV. Moreover, the GOR and WC from the NPV result are computed to assess the generated W_p and G_p curves. The workflow of this section is illustrated in Figure 3.8.

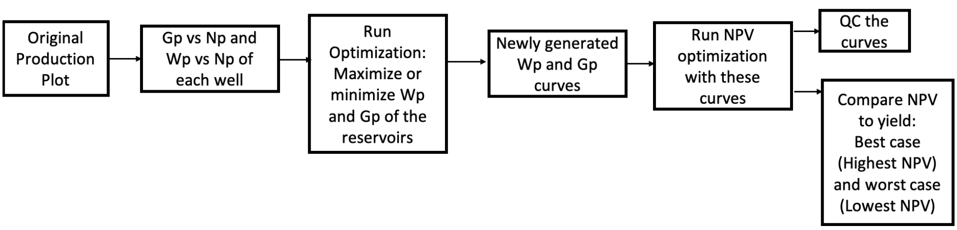


Figure 3.8: The workflow of running NPV optimization with different W_p and G_p curves

3.5.1 Formulation for Generating New Gas and Water Cumulative Production

The curves are computed through an optimization routine. Again, the PWL Approximation is employed to transform the optimization problem into MILP. The problem includes some non-linear functions, e.g. $W_{p,wellj} = f(N_{p,wellj})$ and $G_{p,wellj} = f(N_{p,wellj})$. with $j \in \{1, 2, ..., 9\}$.

Objective

In this optimization, the highest and lowest bound of W_p and G_p are determined. Both W_p and G_p are dependent on N_p, thus it might not possible to find the profile that maximize or minimize water and gas production at the same time. To capture all the possible bounds, six extreme cases are formulated along with its objective function:

- Maximizing Water and Gas Production case (MaxWpGp)

$$\text{Maximize} \qquad G_{p,total} + W_{p,total} \tag{3.65}$$

- Minimizing Water and Gas Production case (MinWpGp)

$$\text{Minimize} \qquad G_{p,total} + W_{p,total} \tag{3.66}$$

- Maximizing Gas Production case (MaxGp)

$$\text{Maximize} \qquad G_{p,total} \tag{3.67}$$

- Maximizing Water Production case (MaxWp)

$$\text{Maximize} \qquad W_{p,total} \tag{3.68}$$

- Minimizing Gas Production case (MinGp)

$$\text{Minimize} \qquad G_{p,total} \tag{3.69}$$

- Minimizing Water Production case (MinWp)

$$\text{Minimize} \qquad W_{p,total} \tag{3.70}$$

Sets

- iNp : the set of N_p reservoir breakpoints $\{1, 2, ..., 15\}$. Both reservoir use the same number of breakpoints.
- I: is the set of N_p well breakpoints $\{1, 2, ..., 15\}$. All wells have the same number of breakpoints.

Parameters

- $N_{p,x1,k}$, $\forall k \in iNp$: the predefined oil cumulative production of Reservoir X1 at breakpoint-k. The optimization determines the G_p and W_p at this particular $N_{p,k}$. The values of $N_{p,x1,k}$ get higher for increasing k.

- $N_{p,x2,k}$, $\forall k \in iNp$: the predefined oil cumulative production of Reservoir X2 at breakpoint-k. The values of $N_{p,x2,k}$ get higher for increasing k.

- Some of the parameters are the breakpoints of all linear function. The following are the examples for some of the wells:
  - $\bar{G}_{p,w1,i}$, $\forall i \in I$: the gas cumulative production of well-1 defined at breakpoint-i as a subject to $N_{p,well1}$.
  - $\bar{W}_{p,w1,i}$, $\forall i \in I$: the water cumulative production of well-1 at breakpoint-i as a subject to $N_{p,well1}$.
  - $\bar{N}_{p,w1,i}$, $\forall i \in I$: the oil cumulative production of well-1 at breakpoint-i. The index is marked by i.
  - $\bar{G}_{p,w2,i}$, $\forall i \in I$: the gas cumulative production of well-1 at breakpoint-i as a subject to $N_{p,well2}$.
  - $\bar{W}_{p,w2,i}$, $\forall i \in I$: the water cumulative production of well-2 at breakpoint-i as a subject to $N_{p,well2}$.
  - $\bar{N}_{p,w2,i}$, $\forall i \in I$: the oil cumulative production of well-2 at breakpoint-i. The index is marked by i.

  Note that parameter $G_{p,wj,i}$, $W_{p,wj,i}$, and $N_{p,wj,i}$ are declared also for the remaining wells $\{j = 3, 4, .., 9\}$.

Variables

- $W_{p,x1,k}, \forall k \in iNp$: the water cumulative production of Reservoir X1 at breakpoint-k. It also can be stated as the W_p at $N_{p,k}$ of Reservoir X1.

- $G_{p,x1,k}, \forall k \in iNp$: the gas cumulative production of Reservoir X1 at breakpoint-k. It also can be stated as the G_p at $N_{p,k}$ of Reservoir X1.

- $W_{p,x2,k}, \forall k \in iNp$: the water cumulative production of Reservoir X2 at breakpoint-k. It also can be stated as the W_p at $N_{p,k}$ of Reservoir X2.

- $G_{p,x2,k}$, $\forall k \in iNp$: the gas cumulative production of Reservoir X2 at breakpoint-k. It also can be stated as the G_p at $N_{p,k}$ of Reservoir X2.

- $G_{p,total}$: total gas cumulative production.

- $W_{p,total}$: total water cumulative production.

- The oil cumulative production of each well at breakpoint-k:

  $N_{p,w1,k}$, $N_{p,w2,k}$, $N_{p,w3,k}$, $N_{p,w4,k}$, $N_{p,w5,k}$, $N_{p,w6,k}$, $N_{p,w7,k}$
  $N_{p,w8,k}$, and $N_{p,w9,k}$

- The gas cumulative production of each well at breakpoint-k:

  $G_{p,w1,k}$, $G_{p,w2,k}$, $G_{p,w3,k}$, $G_{p,w4,k}$, $G_{p,w5,k}$, $G_{p,w6,k}$, $G_{p,w7,k}$,
  $G_{p,w8,k}$, and $G_{p,w9,k}$

- The water cumulative production of each well at breakpoint-k:
  $W_{p,w1,k}$, $W_{p,w2,k}$, $W_{p,w3,k}$, $W_{p,w4,k}$, $W_{p,w5,k}$, $W_{p,w6,k}$, $W_{p,w7,k}$,
  $W_{p,well8,k}$, and , $W_{p,well9,k}$
- Weighting coefficient of each well at breakpoint-(i,k) in 1D PWL Approximation:
  $\lambda_{w1,i,k}$, $\lambda_{w2,i,k}$, $\lambda_{w3,i,k}$, $\lambda_{w4,i,k}$, $\lambda_{w5,i,k}$, $\lambda_{w6,i,k}$, $\lambda_{w7,i,k}$, $\lambda_{w8,i,k}$, and $\lambda_{w9,k}$

Constraints

The constraints imposed in this problem are the following:

1. In every reservoir, the N_p at breakpoint-k is the summation of $N_{p,wi}$ of all wells, existing in that reservoir, at point-k :

   $$\begin{aligned} &\forall k \in iNp : \\ &\quad N_{p,x1,k} = N_{p,w1,k} + N_{p,w2,k} + N_{p,w3,k} + N_{p,w4,k} + N_{p,w5,k} + N_{p,w6,k} \\ &\quad N_{p,x2,k} = N_{p,w7,k} + N_{p,w8,k} + N_{p,w9,k} \end{aligned} \tag{3.71}$$

   This constraint enforces at least one well to have more oil production for every increasing N_p reservoir.

2. The cumulative oil production of every well is non-decreasing:

   $$\begin{aligned} &\forall k \in \{1, 2, ..., 14\} : \\ &\qquad N_{p,w1,k+1} - N_{p,w1,k} \geq 0 \\ &\qquad N_{p,w2,k+1} - N_{p,w1,k} \geq 0 \\ &\qquad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ &\qquad N_{p,w9,k} - N_{p,w9,k-1} \geq 0 \end{aligned} \tag{3.72}$$

   Note that this condition is applied to all wells.

   The constraint keeps the well production consistent. The N_p of reservoir is ascending order in k. Therefore, Once a well has contributed to a particular N_p , it could not be discarded or reduced.

3. When a certain well produces a certain $N_{p,wi}$ to fulfil Equation 3.71, it also have produced certain amount of water and gas production as the consequence. Therefore, the G_p and W_p of a particular reservoir is also the summation of G_p and W_p of all wells in that reservoir:

   $$\begin{aligned} &\forall k \in iNp : \\ &\quad G_{p,x1,k} = G_{p,w1,k} + G_{p,w2,k} + G_{p,w3,k} + G_{p,w4,k} + G_{p,w5,k} + G_{p,w6,k} \\ &\quad G_{p,x2,k} = G_{p,w7,k} + G_{p,w8,k} + G_{p,w9,k} \\ &\quad W_{p,x1,k} = W_{p,w1,k} + W_{p,w2,k} + W_{p,w3,k} + W_{p,w4,k} + W_{p,w5,k} + W_{p,w6,k} \\ &\quad W_{p,x2,k} = W_{p,w7,k} + W_{p,w8,k} + W_{p,w9,k} \end{aligned} \tag{3.73}$$

   This constraint would determine G_p vs N_p and W_p vs N_p relationships for each reservoir.

4. The total production of a fluid is the summation of the related fluid production for all breakpoints and all reservoirs:

$$G_{p,total} = \sum_{k \in iNp} \frac{(G_{p,x1,k} + G_{p,x2,k})}{1000} \tag{3.74}$$

$$W_{p,total} = \sum_{k \in iNp} W_{p,x1,k} + W_{p,x2,k} \tag{3.75}$$

To normalize the G_p, the total gas production is divided by 1000.

5. A 1D PWL Approximation is implemented to convert the $W_{p,wellj} = f(N_p, wellj)$ and $G_{p,wellj} = f(N_p, wellj)$ for every well j into linear functions. Therefore, there are 9 1D PWL Approximations in this problem. Similar to the formulation in Equation 3.53 and 3.54, G_p and W_p of each well are compounded into one PWL formulation. The following are the PWL induced constraints for two out of nine 1D PWL Approximations:

$$\forall k \in iNp : N_{p,w1,k} \quad = \sum_{i \in I} \lambda_{w1,i,k} \cdot N_{p,w1,i} \tag{3.76}$$

$$G_{p,w1,k} = \sum_{i \in I} \lambda_{w1,i,k} \cdot G_{p,w1,i} \tag{3.77}$$

$$W_{p,w1,k} = \sum_{i \in I} \lambda_{w1,i,k} \cdot W_{p,w1,i} \tag{3.78}$$

$$1 = \sum_{i \in I} \lambda_{w1,i,k} \tag{3.79}$$

$$N_{p,w2,k} = \sum_{i \in I} \lambda_{w2,i,k} \cdot N_{p,w2,i} \tag{3.80}$$

$$G_{p,w2,k} = \sum_{i \in I} \lambda_{w2,i,k} \cdot G_{p,w2,i} \tag{3.81}$$

$$W_{p,w2,k} = \sum_{i \in I} \lambda_{w2,i,k} \cdot W_{p,w2,i} \tag{3.82}$$

$$1 = \sum_{i \in I} \lambda_{w2,i,k} \tag{3.83}$$

$$\{\lambda_{w1,i,k} \text{ and } \lambda_{w2,i,k}, \qquad \forall i \in I\} \text{is SOS2} \tag{3.84}$$

Note that the constrains imposed in Equation 3.76- 3.84 are implemented also for the other seven wells $\{j = 3, 4, .., 9\}$.

3.5.2 NPV Optimization

Once the new curves are generated, NPV optimizations are carried out using these curves. The best case is the case with the highest NPV, while the case with the lowest NPV is defined as the worst case. These cases are used further in the uncertainty analysis.

The optimization changes both production and drilling schedule. Although the generated W_p and G_p curves already have a particular well schedule, it is assumed that the curves are fixed and only dependent on the N_p during the optimization. The idea of this optimization is to find the best strategy if such W_p and G_p occur.

3.5.3 Quality Control (QC) of the Cumulative Gas and Water Production Curves

The generated curves from section 3.5.1 are cross-checked to ensure that the computer GOR and Water cuts do not exceed the maximum producing well GOR and Water cuts. One way to do it is by comparing the GOR and WC computed from each NPV optimization result and from the original production plot. First, the GOR and WC are determined from NPV optimization result as:

$$\begin{aligned}
GOR_{ir,n} &= \frac{G_{p,ir,n+1} - G_{p,ir,n}}{N_{p,ir,n+1} - N_{p,ir,n}} \\
WC_{ir,n} &= \frac{W_{p,ir,n+1} - W_{p,ir,n}}{W_{p,ir,n+1} + N_{p,ir,n+1} - W_{p,ir,n} - N_{p,ir,n}} \\
\forall ir \in R \qquad & \forall n \in T
\end{aligned} \tag{3.85}$$

From these result, the maximum and minimum of GOR and WC of every case are selected. Afterwards, GOR and WC of each well and reservoir from the original plot are calculated based on monthly production. This is to obtain precise value of these parameters. The maximum and minimum values are determined as well during this phase.

Since the GOR and WC from the optimization results are calculated every year, it combines the production for longer time duration and larger N_p. This might cause a significant difference with the values from the original plot. Accordingly, the production for longer timesteps is compounded to yield another GOR and WC from the original plot. By doing so, the values of GOR and WC from both approaches can be compared. The workflow of this QC is illustrated in Figure 3.9. If the GOR and WC are between the max and min values of monthly GOR and WC from the original plot and have the similar values with the compound ones, the curves of W_p and G_p are valid.

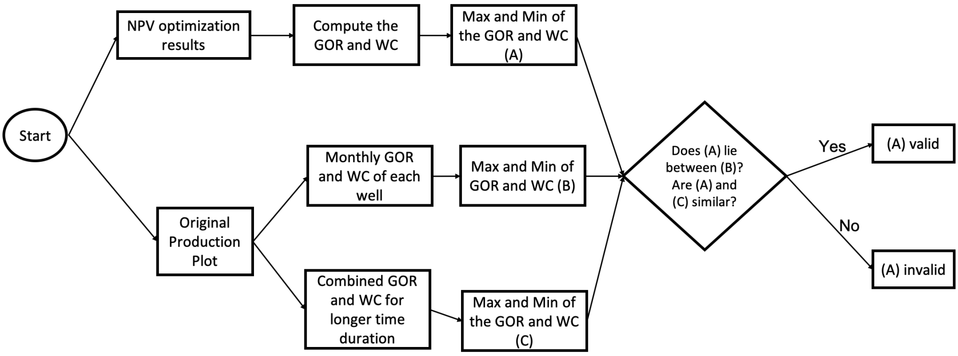


Figure 3.9: The Quality Control (QC) workflow

3.6 Uncertainty Analysis

In general, early field development has high uncertainty due to the variability of inputs used for data processing (with many assumptions). The uncertainty concerns the reservoir modeling, cost estimation, and the oil price. It could build up affecting the production forecast and further into revenue and NPV calculation. If the uncertainty is not considered, it may lead to false design due to over or underestimated NPV and selection or inadequate design parameters. Therefore, it is essential to include the impacts of uncertainty during the development study.

This section discusses the uncertainties in Initial Oil in Place (IOIP), production potential (q_{opp}), cost estimation, oil price, and cumulative water and gas production. The uncertainty analysis is executed for different G_p and W_p curves that obtain the best NPV, worst NPV, and the base case (original W_p and G_p). The gas and water production are included because of their dynamic behavior (as mentioned in section 3.5). The uncertainty analysis is used to quantify the possible change in NPV, optimal oil rates, and the number of wells to make a better judgment. The probability tree approach is selected for the uncertainty analysis.

3.6.1 The Effect of Uncertainty to the Problem Formulation

Some modifications are made to simplify the computation, as well as to include the uncertain parameters. Only three parameters affect the mathematical formulation in section 3.3:

- Initial Oil in Place (IOIP)

  In this optimization, the IOIP is not directly applied in the calculation. Despite that, the change in IOIP could change the production performance i.e. prolong or shorten the field lifetime, thus the production potential. Angga (2019) studied the effect of changing the IOIP to the production potential curves. It was found that that changing IOIP by x% from the base value would move the N_p of the production potential curves by the same x% (Figure 3.10). This finding is also valid in G_p vs

N_p and W_p vs N_p curves. By including the uncertain parameter, the cumulative oil production are determined as:

$$N_{p,ir,n} = N \cdot \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{N}_{p,ir,k,l} \tag{3.86}$$

with N as the uncertainty multiplier of IOIP.

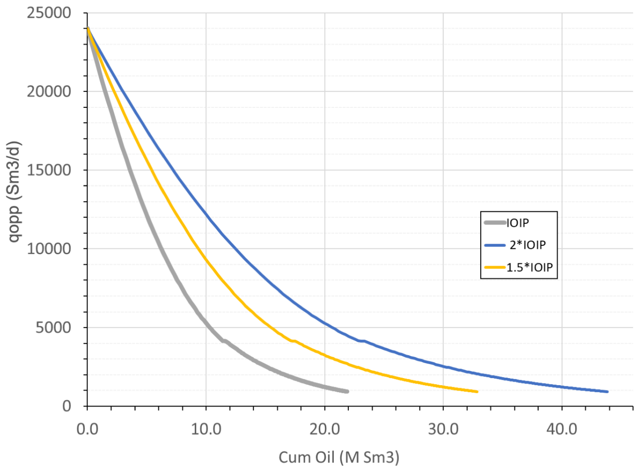


Figure 3.10: The effect of varying IOIP to the production potential. When the IOIP is changed into double the size, the N_p shifts into same ratio (adapted from Angga (2019))

- Production potential

  Referring to the section 3.1, the production potential of a particular scenario number is the multiplication product of the production potential and combination factor. Implementing the same principal, it is assumed that the q_{opp} could be shifted for a certain extend by adding another multiplier (the uncertain parameter) into the Equation 3.51. In other words, it takes account the uncertainty of the factor. Therefore, the q_{opp} is calculated as:

$$q_{opp,ir,n} = Q \cdot \sum_{k \in i_{N_p}} \sum_{l \in i_{sn[ir]}} \lambda_{qopp,ir,n,k,l} \cdot \bar{q}_{opp,ir,k,l} \tag{3.87}$$

  with Q as the uncertainty multiplier of q_{opp}.

- Cost proxy model

  The cost value is varied with desired range. Similar to the production potential, the uncertain parameter is set as a multiplier to the actual equations. Some economical constraints for cost proxy model are modified as:

- DrillEx equations:

$$\begin{aligned} PV_{dp,0} &= C \cdot P_{well} \cdot N_{w,F,0} \\ PV_{d,n} &= C \cdot P_{well} \cdot (N_{w,F,n} - N_{w,F,n-1}) \end{aligned} \tag{3.88}$$

- CapEx Subsea equations:

$$\begin{aligned} PV_{sub,0} &= C \cdot (P_{pipe} \cdot pipe_{max} + P_X \cdot N_{w,F,0}) \\ PV_{sub,n} &= C \cdot P_X \cdot (N_{w,F,n} - N_{w,F,n-1}) \end{aligned} \tag{3.89}$$

- CapEx Topside equation

$$\begin{aligned} PV_{top} = C \cdot (&P_{Capex,1} + P_{Capex,2} \cdot q_{o,max} \\ &+ P_{Capex,3} \cdot q_{w,max} + P_{Capex,4} \cdot q_{g,max}) \end{aligned} \tag{3.90}$$

- OpEx equation

$$\begin{aligned} PV_{op,n} =&C \cdot (P_{Opex,1} + P_{Opex,2} \cdot N_{w,F,n-1} + P_{Opex,3} \cdot q_{o,ir,n-1} + ... \\ &P_{Opex,4} \cdot q_{w,ir,n-1} + P_{Opex,5} \cdot q_{g,ir,n-1}) \end{aligned} \tag{3.91}$$

With C as the standard multiplier representing the uncertain cost estimation.

3.6.2 The Uncertain Distribution

There is a variation of five elements to be studied based on the uncertainty analysis:

- The uncertainty of the initial oil in place is quantified as a multiplier N representing the ratio between the new and base values of the corresponding parameter. The distribution for N is assumed, similar to that of the Visund Field (Visund PDO report (1995)). It is normally distributed with a mean and standard deviation of 1.0 and 0.2, respectively.

- A multiplier C is also used as a measure of the uncertainty of cost estimation. It is modeled with the normal distribution that has a mean of 1.0 and standard deviation of 0.2

- The multiplier Q represents the uncertainty of production potential and the combination factor. It is assumed that the value is normally distributed with a mean and standard deviation of 1.0 and 0.2.

- The oil price is uniformly distributed between 20 and 80. The large range of this distribution is to take account of the high variation of oil prices.

- Unlike the other parameters, the uncertainty in water and gas production is already categorized into three types: best case, base case, and worst case. The curves of G_p and W_p used in the uncertainty analysis are derived from the NPV optimization in Section 3.5.1.

3.6.3 The Probability Tree

The probability tree captures all possible combinations of events enclosing the uncertain parameters and their probability of occurrence. Starting with the first parameter, the branches of the possible outcomes and the probability, are drawn. Then, from each branch, the same method is done for the next parameter.

A method to encode probability from a cumulative distribution function (CDF) is introduced by McNamee and Celona (2008). The principle is to discretize the CDF into several outcomes and probability. The example for the discretization can be seen in Figure 3.11. The first step is to select the desired probabilities that the summation of them is equal to 1. In this example, the probabilities are 0.25, 0.75, and 0.25. Horizontal lines are drawn at these points. Starting from the lower legion, choose a point A and draw a vertical line at A, such that the left shaded area and right shaded area of the vertical line are equal. Then, it can be said that point A has a probability of 0.25. The same method is performed in the middle and upper regions.

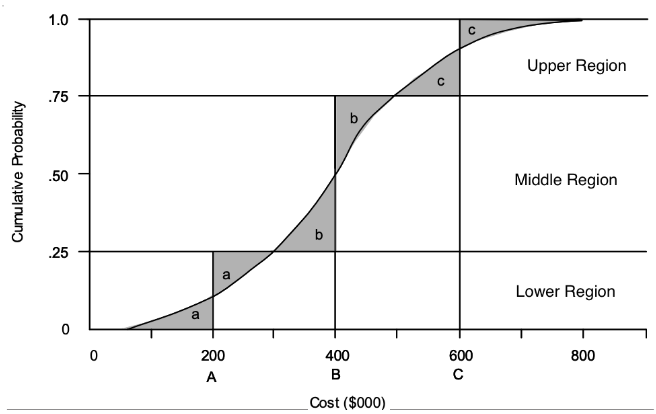


Figure 3.11: The example of discrete probability distribution cost (McNamee and Celona, 2008)

In this work, three outcomes for each uncertain parameter are used. Production potential, cost proxy model, and IOIP apply the same probabilities: 0.25, 0.75, and 0,25. The probabilities for the oil price outcomes are 0.2, 0.6, and 0.2. Afterward, the procedure from McNamee and Celona (2008) is used to determine the outcome with associated probability from the CDF. Lastly, each G_p-W_p curve has an equal probability of $1/3$. The probability tree for this uncertainty analysis can be seen in Figure 3.12. With five uncertain parameters used in the computation, there are 243 optimization cases.

3.6.4 Running Time Limitation

Due to the time constraint of the project, each optimization case's running time needs to be limited. Some cases need computation time for 4-6 hours in order to find the optimal solution. Therefore, it could take at most 1458 hours (61 days) to finish all 243 runs.

The MILP implements the branch and bound method to find the best solution. Optimization determines the best-bound while searching for the best integer solution, usually

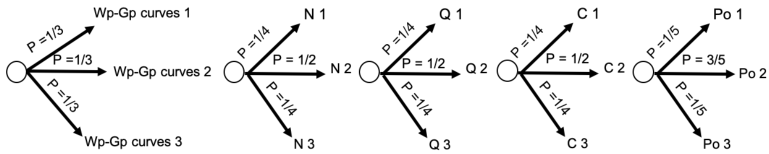


Figure 3.12: The probability tree for the uncertainty analysis. Each parameter have three branches of outcomes along with its probability P

called incumbent (GUROBI, 2020). As time passes, the best-bound decreases, and the incumbent increases. The optimization keeps running until the best-bound equals to the incumbent. However, the optimization tends to stop finding a better integer solution after extending the running time. Then, the optimization stops because the best bound evolves and reaches the incumbent, without any increment in the incumbent.

In this section, several cases are run to determine the shorter runtime that could yield similar NPV compared to the one obtained from longer runtime. The cases that being evaluated are the base, worst, and best cases determined in section 3.5.2 and also randomly picked from 243 cases of the uncertainty analysis cases. The evaluated runtime is 2, 4, and 6 hours. 3.13.

Table 3.13: The tested case with evaluated runtime of 2, 3 and 6 hours

Tested Case
BaseCase
MaxWpGp
MinWpGp
BaseCase_20
BaseCase_40
Best_Case_20
Best_Case_40
Worst_Case_20
Worst_Case_40

Chapter 4

Result and Discussion

4.1 Cost Proxy Model

DrillEx

The cost of drilling well usually depends on the daily rig rates, water depth, and distance from the shore. A single cost for DrillEx is used as a function of the number of well: 500 Million NOK/well. The DrillEx (in Million NOK) is expressed as:

$$\text{DrillEX}_t = 500 \cdot N_{w,t} \tag{4.1}$$

Where $N_{w,t}$ is the number of well at time-t.

CapEx

CapEx is the investment cost for the company to build a field. There are two categories of CapEx introduced:

1. CapEx - Subsea. Referring to Figure 3.2, the wells are not installed in a template. Therefore, it is assumed to be linearly dependent on maximum pipeline length and subsea X-mas tree. The cost of Pipeline is 25 Million Nok/km and the cost of X-mas tree is 50 Million Nok/km. The proxy model for CapEx Subsea in Million NOK is expressed as follow:

$$\begin{aligned} \text{CapEx (Subsea Pipe)} &= 25 \cdot Pl_{max} \\ \text{CapEx (Subsea xmas tree)}_t &= 50 \cdot N_{w,t} \end{aligned} \tag{4.2}$$

   where Pl_{max} is the maximum length of the pipeline in km.

2. CapEx - Topside. CapEx Topside is modelled as a dependent variable with multiple independent variables: maximum oil rates, maximum gas rates and maximum water rates. The cost proxy model for CapEx Topside in Million NOK is presented below:

$$
\begin{aligned}
\text{CapEx}_{top,FPSO} &= 2150 + 0.8562 \cdot q_{o,max} \\
&+ 0.0442 \cdot q_{w,max} + 0.2.35 \cdot 10^{-1} \cdot q_{g,max} \\
\text{CapEx}_{top,Sevan} &= 3650 + 0.8562 \cdot q_{o,max} \\
&+ 0.0442 \cdot q_{w,max} + 2.35 \cdot 10^{-1} \cdot q_{g,max}
\end{aligned}
\tag{4.3}
$$

where $q_{o,max}$ and $q_{w,max}$ are maximum oil and water rates in sm$^3/d$ and $q_{g,max}$ is maximum gas rate in 1000 sm$^3/d$.

OpEx

OpEx is expressed as a function of fluid rates and number of well. The well maintenance cost is 6.5 Million nok/well/year. The data stored in table 3.5 is the yearly expense for managing the fluid production. The OpEx for both FPSO and Sevan are assumed to be the same, thus only one cost proxy model for OpEx per year are defined in Million NOK as:

$$
\begin{aligned}
\text{OpEx}_{op,t} &= 723.3 + 6.5 \cdot N_{w,t} + 0.0236 \cdot q_{o,t-1} \\
&+ 1.855E - 18 \cdot q_{w,t} + 3.04E - 18 \cdot q_{g,t}
\end{aligned}
\tag{4.4}
$$

where $q_{o,t}$ and $q_{w,t}$ are in oil and water rates at time t (in sm$^3/d$) and $q_{g,t}$ is gas rate at time t (in 1000sm$^3/d$).

CapEx Topside and OpEx linear equations are retrieved from multilinear regression. The standard error for this regression is shown in Table 4.1. It can be shown that the errors are considerably low for their actual value.

Table 4.1: Linear regression of CapEc and OpEx

	Standard Error	R Square
CapEx FPSO	116.125	0.997
CapEx Sevan	116.125	0.997
OpEx	2.886	0.998

Furthermore, the cost proxy model is employed into the MILP as the economic constraints. The optimization only executed for the FPSO structure. The economic parameters previously introduced in section 3.3 have the values stored in Table 4.2.

Parameters	Value	Parameters	Value	Parameters	Value
P_{well}	500	$P_{Capex,1}$	2150	$P_{Opex,1}$	723.3
P_{pipe}	25	$P_{Capex,2}$	0.8562	$P_{Opex,2}$	6.5
P_X	50	$P_{Capex,3}$	0.0442	$P_{Opex,3}$	0.0236
		$P_{Capex,4}$	$2.35 \cdot 10^{-1}$	$P_{Opex,4}$	1.855E-18
				$P_{Opex,5}$	3.04E-18

Table 4.2: The economical parameter data value

4.2 NPV Optimization

4.2.1 The Number of Np Breakpoints

To select the number of breakpoints, the NPV optimization results with several numbers of breakpoints were compared. Table 4.3 shows the NPV result from the breakpoint evaluation.

Case	NPV (Million NOK)
$case_{5bp}$	28037.4
$case_{7bp}$	27753.9
$case_{10bp}$	27542.9
$case_{15bp}$	27590

Table 4.3: The NPV obtained from cases with different number of N_p

It can be seen that the NPV with 5 N_p breakpoints has the most significant difference with $case_{15bp}$ by 500 Million NOK. To summarize, more breakpoints give a more accurate representation of the non-linear behavior of the function. Hence, $case_{15bp}$ was considered as the reference to compute the average error of the other cases.

Table 4.4: Average error of the $case_{5bp}$, $case_{7bp}$, $case_{10bp}$ and $case_{15bp}$

Breakpoint	5		7		10		16	
Runtime (s)	2		6		50		190	
			Average Error				Reference Value	
	X1	X2	X1	X2	X1	X2	X1	X2
q_o	4.3%	5.1%	2.2%	3.2%	2.5%	0.9%	-	-
q_g	13.9%	8.4%	4.7%	7.3%	1.9%	1.1%	-	-
q_w	12.3%	4.0%	4.1%	5.3%	4.2%	1.3%	-	-
$q_{o,F}$	6.8%		2.5%		1.4%		-	
NPV	1.6%		0.6%		0.2%		-	

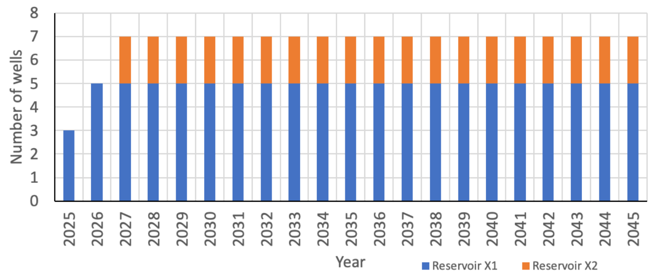
The average errors of the NPV and the production rates are shown in Table 4.4. The average error mainly went up with an increasing number of breakpoints, while the runtime got longer. If only oil rates and NPV were considered, all cases had met the criteria. However, $case_{5bp}$ was proven to obtain a relatively high error in gas and water rate. It was because the N_p reconstruction aimed to have a good fit in the q_{opp} curves only. Despite that, the errors of gas and water production rate of $case_{7bp}$ were still below the allowable error while being the second-fastest to reach the optimum solution. Therefore, considering the error and the runtime, $case_{7bp}$ was selected for this optimization formulation.

4.2.2 The NPV optimization result

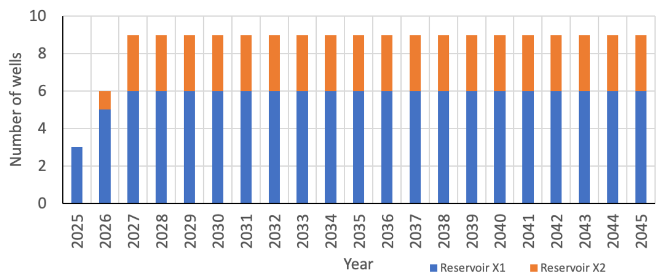
Table 4.5: The NPV comparison of $Case_1$ and $Case_2$

	NPV (Million NOK)	Runtime (s)
$Case_1$	28968.5	18327
$Case_2$	28332.3	8

The comparison of optimization result between $Case_1$ and $Case_2$ is presented in Table 4.5. Corresponding with the statement in section 3.4.2, $Case_1$ yields higher NPV by 2.2% compared to $Case_2$. An optimization with more decision variables tends to be more flexible in finding the optimum solution. Consequently, it also required longer runtime to obtain the best solution because the optimization had more possible solutions to assess. However, considering the small improvement from $Case_1$ implied that the drilling schedule used for $Case_2$ was nearly optimal.



(a) $Case_1$



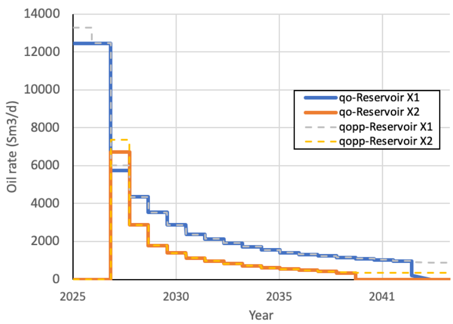
(b) $Case_2$

Figure 4.1: The drilling schedule of both cases

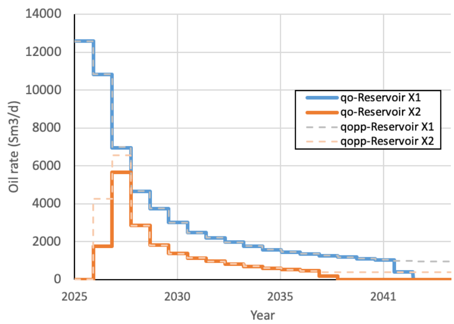
Table 4.6 tabulated the well schedule of both cases and Figure 4.1 shows the comparison between the drilling schedule of $Case_1$ and $Case_2$. Based on the result, $Case_1$ yields higher NPV by applying the following measures: (1) drill one less well in both reservoirs Same as $Case_2$, and (2) delay the drilling of well in reservoir X2. Adding new wells increased not only the production potential but also the development expense. In this particular problem, the DrillEx for having two more wells was higher than the added revenue.

Table 4.6: The well schedule of $Case_1$ and $Case_2$

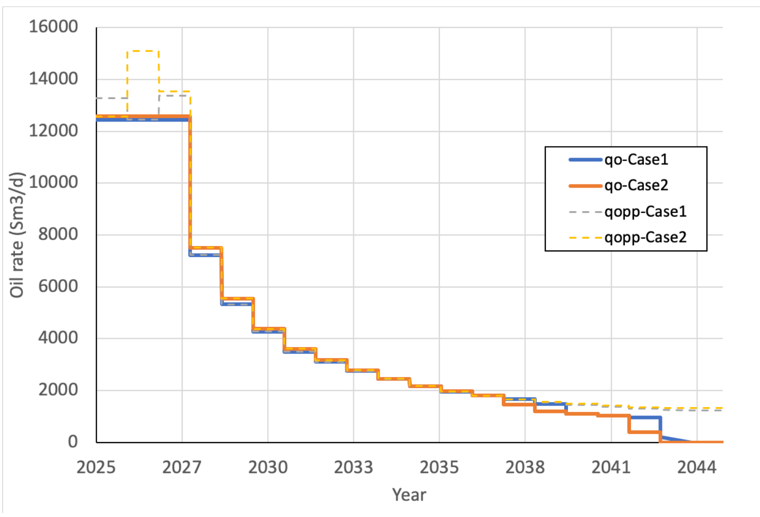
Year	Active wells $Case_1$	Active wells $Case_2$
2025	2, 5, 6	1, 2, 4
2026	2,3,4,5,6	1, 2, 4,5, 6, 7
2027	2, 3, 4, 5, 6, 8, 9	1, 2,3, 4,5, 6, 7, 8,9
:	:	:
2029	2,3,4, 5, 6, 8, 9	1, 2, 3, 4, 5, 6, 7, 8, 9



(a) $Case_1$

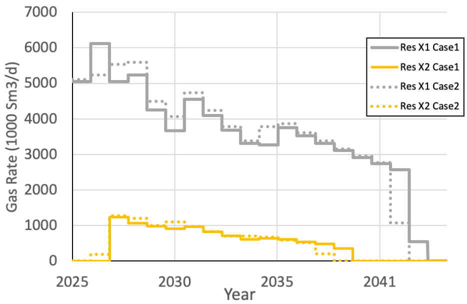


(b) $Case_2$

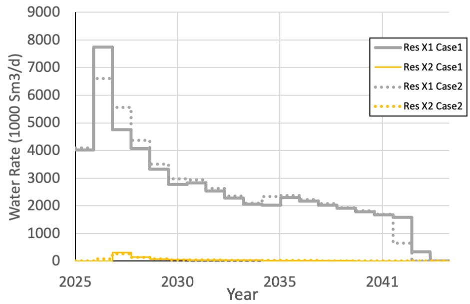


(c) Field oil rates of both cases

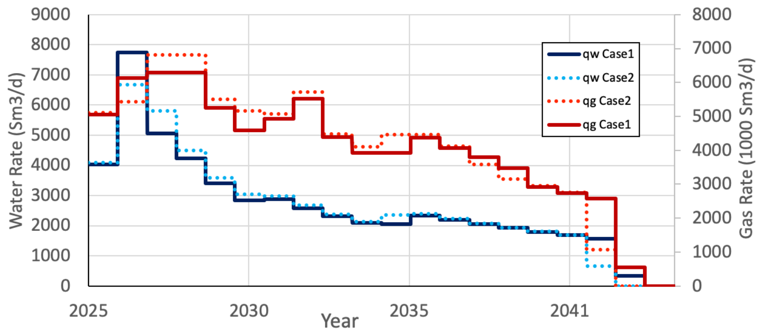
Figure 4.2: The comparison of oil production rates and production potential between case1 and case2



(a) Gas rates from the reservoirs in both cases



(b) Water rates from the reservoirs in both cases



(c) Field water and gas production rates of both cases

Figure 4.3: The comparison of gas and water production rates between $Case_1$ and $Case_2$

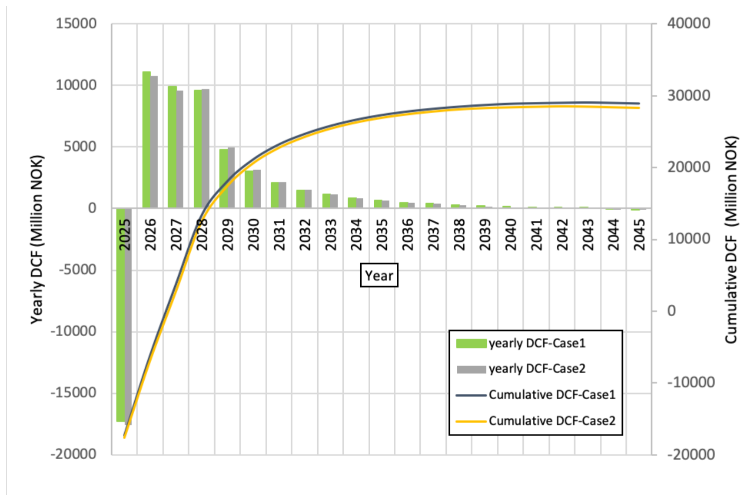


Figure 4.4: Comparison of DCF between $Case_1$ and $Case_2$

Thus the optimization suggested canceling the drilling of additional wells. $Case_1$ choose to discard the well 1 and well 7 for the development. The reason could be that the oil productions from these wells did not give significant contributions to the field production. Nonetheless, both optimization agreed to drill three wells initially, which is the maximum number of predrilled wells.

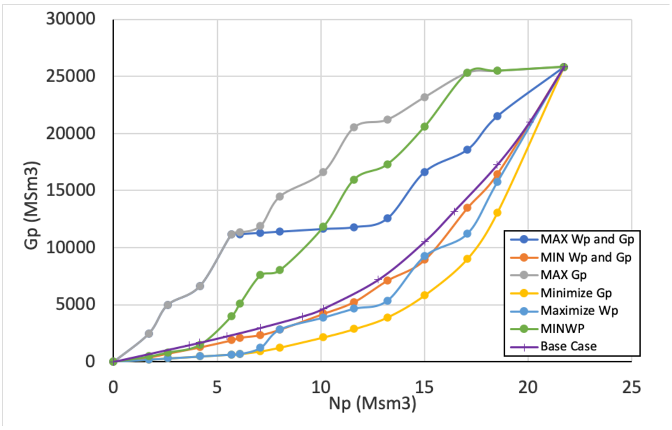
Although the number of wells was distinct, the field oil production rates of $Case_1$ and $Case_2$ were quite similar (Figure 4.2). It could be said that the plateau rate of 12000 Sm^3/d is the optimal rate for this field in most cases. On closer look, $Case_2$ did have slightly higher oil production to increase the revenue at the beginning. $Case_2$ chose not to increase the production rates even though it had much higher production potential. Increasing production rates were not always beneficial because it also required higher fluid capacity, thus increasing the CapEx expenses. Corresponding to this, the $Case_1$ optimization considered that it was unnecessary to enhance the oil production rates, not to mention adding more wells.

Both cases decided to prioritize reservoir X1, as the result of having higher production potential. $Case_1$ proposed to delay the production from Reservoir X2 until the time when Reservoir X1 could not afford to deliver the plateau rate. Besides, $Case_2$ always set the oil production from reservoir X1 at its potential, even though reservoir X2 was able to contribute with more production starting from the second year. It is surmised that the delay is to reduce the maximum field gas production rate (Figure 4.3). Referring to the gas rate coefficient in Equation 4.3, the gas rate was second to the most sensitive parameter to CapEx. Therefore, having a high production of gas was not favorable.

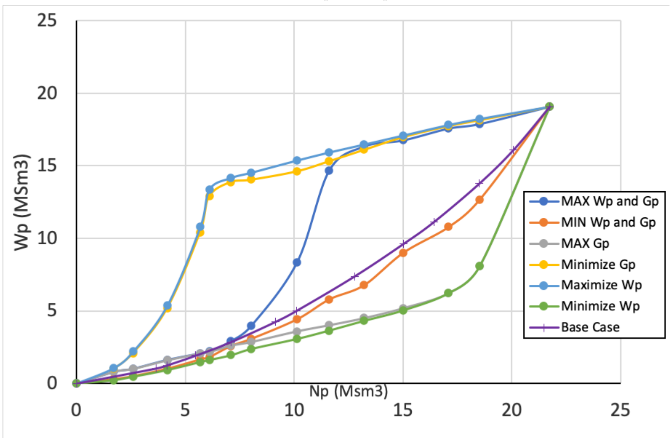
From the observation in Figure 4.4, both cases have relatively identical DCF. Moreover, the cashflow of $Case_1$ excelled slightly early. The decrease of $Case_2$ compared to $Case_1$ in 2026 and 2027 occurred due to the cost of additional wells. $Case_2$ had higher field plateau rates but did not significantly boost the DCF due to the high expense on DrillEx. This supports the fact that drilling more wells gave inadequate returns. In conclusion, the $Case_1$ has successfully increased the NPV with fewer wells drilled in both reservoirs while having a similar production plot.

4.3 The Extreme Curves of Cumulative Water and Gas Production

Figure 4.5 depicted the cumulative water production (W_p) and cumulative gas production (G_p) curves of reservoir X1 from the optimization cases. Here, the gas and water production curves retrieved from the original plot are referred to as ”base case”. It showed that there was no exact upper or lower limit that applied for both W_p and G_p simultaneously. Both W_p and G_p are dependent on the N_p, such that adjusting the N_p to achieve maximum production of one fluid production affects the other fluid's production. For instance, the MaxGp case constructed the curve with the highest gas production; in turn, the optimization has considerably low production of water. This observation typically occurred in cases that maximized or minimized only one of the fluid production. MinWpGp lowered both water and production such that the line in both curves falls between the maximum and minimum cases. On the other hand, MaxWpGp determined to optimize gas and water

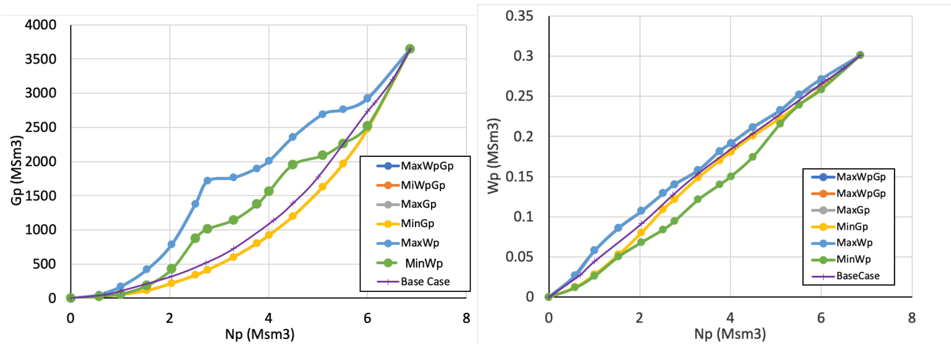


(a) G_p vs N_p



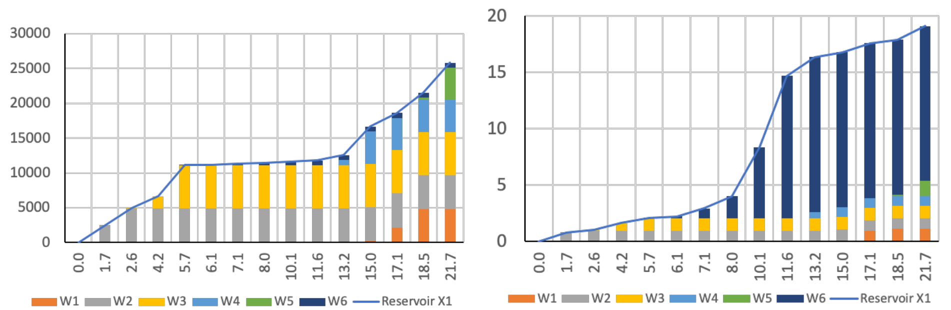
(b) W_p vs N_p

Figure 4.5: The extreme curves of Reservoir X1



(a) G_p vs N_p **(b)** W_p vs N_p

Figure 4.6: The extreme curves of Reservoir X2



(a) G_p vs N_p **(b)** W_p vs N_p

Figure 4.7: The production contribution of each well in MaxWpGp case for Reservoir X2

production separately. At first, It maximized the G_p until N_p = 7 MSm3, then increased the W_p in range of N_p = 7-13 MSm3. This phenomenon was indicated by a sharp change of slope at that range in both MaxWpGp curves.

In contrast with Reservoir X1, the optimization for Reservoir X2 shows more consistent results among the cases (Figure 4.6). All the maximization cases have the same W_p and G_p profiles. That is to say, the upper bound of both fluid production for this reservoir was found. For minimization cases, only MinGp shows a distinct profile while the others were the same.

These extreme curves were built by adjusting the well production scheduling in each reservoir. For example, Figure 4.7 shows how much the well contributed to the gas and water production at each N_p breakpoint in MaxWpGp case. The columns under the G_p and W_p curves were the gas and water production share respectively from each well.

Initially, the reservoir produced from well with high GOR, i.e., well 2, and well 3 (see Figure 3.7). Since the non-decreasing constraint was imposed, the production from each well at the next N_p was either constant or increasing. Once the N_p of a particular well has reached its maximum, the production stayed constant for the following N_p. After well 2 and 3 reached the maximum N_p, the optimization chose to continue the production with the highest WC well, i.e., well 6. Although the WC of this well was the highest, the GOR was the lowest compared to the other wells. Considering Equation 3.85, the GOR of the reservoir can be defined as the slope of G_p vs. N_p curve. Therefore, the insignificant increment of G_p in range of N_p = 7-13 MSm3 represented the low GOR production from Well 6.

The other production share plot is presented in the Appendix.

4.4 Npv Optimization using The Extreme Curves

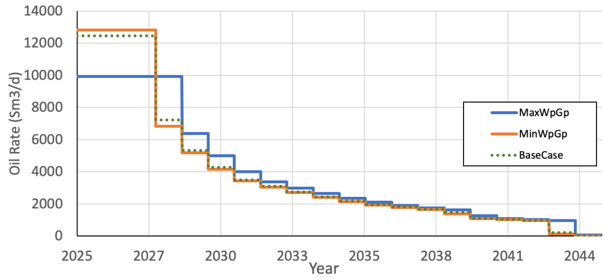
The NPV obtained from optimizations with the extreme curves are summarized in Table 4.7. The base case refers to the $Case_1$ defined in section 4.2.2. It used the water and gas production profiles from the original plot. Based on the result, case MinWpGp was named the best case of water and gas production with the highest NPV, while MaxWpGp was the worst case. The NPV of the base case and MinWpGp were not very distinctive, possibly due to W_p and G_p curves of both being almost the same. However, considering the MaxWpGp curves into the optimization had lowered the NPV by 3 billion NOK approximately, compared to the base case.

Table 4.7: NPV optimization result of the extreme cases

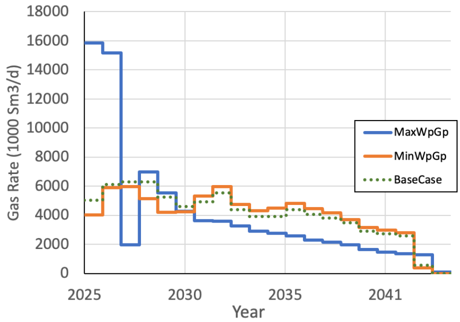
	NPV (Million NOK)	Runtime (sec.)
Base case	28968.5	18327
MaxWpGp	26110.6	40679
MaxGp	26347.3	19781
MaxWp	28002.9	38315
MinWpGp	29141.1	20870
MinGp	28555.5	16056
MinWp	27017	12388

Figure 4.8 exhibits the production profile comparisons between the three cases. Again, the results between the MinWpGp and base case were quite similar. MinWpGp slightly raised the oil plateau rate, because it was still able to lower the rates of the other fluid production. In contrast, MaxWpGp reduced the oil production by 2000 Sm^3/d as the consequence of having high productions of water and gas. Figure 4.8b shows that there was sudden drop of gas rate after high gas production during the early years in MaxWpGp plot. This occurrence happened due to the well schedule forming the W_p and G_p curves as discussed in section 4.3. For MaxWpGp in particular, there would be a time when the field have high gas - low water production rates then continued with low gas - high water production rates. This justified why the field had high gas and high water rates consecutively.

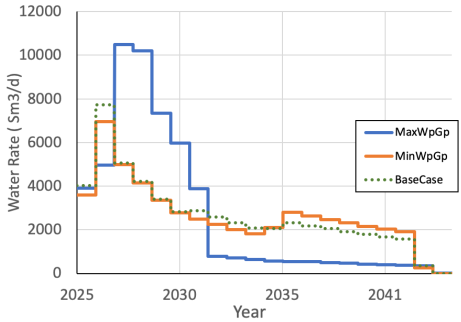
The drilling schedule for MaxWpGp and MinWpGp is depicted in Figure 4.9. MaxWpGp delayed the drilling of additional well by one year. The field could maintain plateau rates until 2027. Thus it was considered unnecessary to add more wells before that year. MinWpGp drill more wells in 2026 compared to the base case (Figure 4.1a). Presumably,



(a) Field oil rates

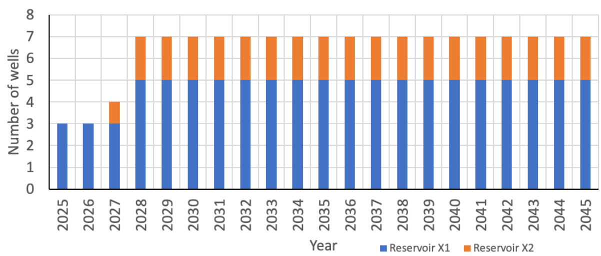


(b) Gas rates

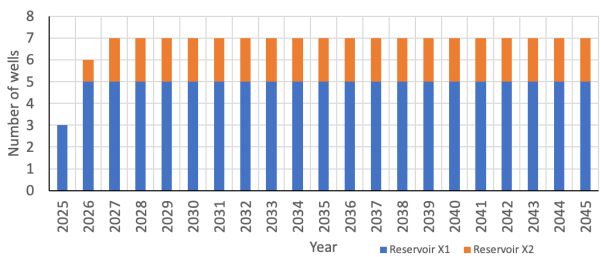


(c) Water rates

Figure 4.8: The comparison of fluid production rates between MaxWpGp, MinWpGp and base case $(Case_1)$



(a) MaxWpGp



(b) MinWpGp

Figure 4.9: The drilling schedule of MaxWpGp and MinWpGp

the reason is that it has a higher plateau rate; thus, the field could not deliver the plateau rates with the existing number of well.

It is important to note that the actual well schedule of the W_p and G_p curves was not employed. In reality, the W_p and G_p would change if the different well schedules were assigned. In this regard, one could use the fixed well scheduling to have a more realistic result.

4.5 The Quality Check of the Wp and Gp Curves

The comparison of the GOR and WC values are tabulated in Table 4.8 and 4.9. Base case(1) retrieved the GOR and WC from monthly production, while base case(2) calculated the values from compounded production.

Table 4.8: The GOR and WC of the extreme cases (Reservoir X1)

Case	GOR_min	GOR_max	WC_min	WC_max
MaxWp	115.2	1855.8	0.24	0.75
MaxGp	654.7	1769	0.25	0.45
MaxWpGp	112.1	1594.7	0.24	0.64
MinWp	358.3	1829.6	0.18	0.46
MinGp	117.9	2064	0.25	0.62
MinWpGp	312	2146	0.22	0.51
Base case (1)	95.4	7536.5	0.01	0.89
Base case (2)	112.3	2241.15	0.24	0.69

Table 4.9: The GOR and WC of the extreme cases (Reservoir X2)

Case	GOR_min	GOR_max	WC_min	WC_max
MaxWp	373.2	843.1	0.03	0.05
MaxGp	168.2	680.3	0.03	0.05
MaxWpGp	205.2	686.4	0.03	0.05
MinWp	150.4	777.9	0.03	0.05
MinGp	144.5	851.5	0.04	0.05
MinWpGp	54	935	0.03	0.05
Base case (1)	32.3	1796.9	0.01	0.07
Base case (2)	163.2	1019.3	0.03	0.05

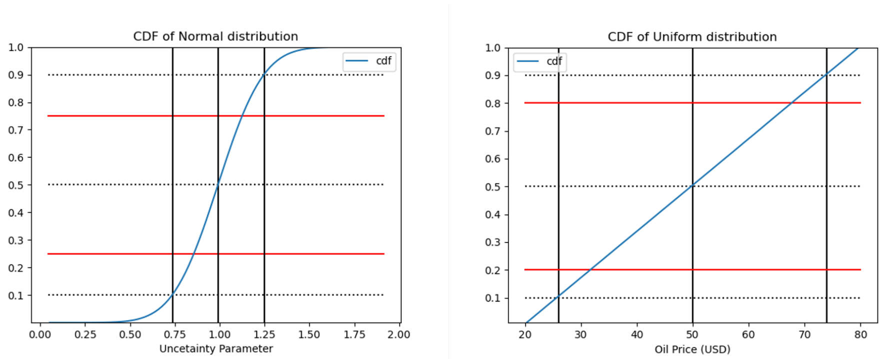
The values retrieved from the base case (1) were considered the lower and upper range of the GOR and WC in the corresponding reservoir. The GOR and WC from base case (1) have higher precision of the gas and water behaviors. It is indicated from the significant difference between its maximum and minimum values of both parameters. The NPV optimization used timestep of one year; hence it averaged the GOR and WC for one year (more considerable value of ΔNp). This caused a massive difference in the values between the NPV optimizations and base case (2). The base case (2) averages the water and gas production to find the GOR and WC. For Reservoir X1, the compounded GOR and WC were calculated from the production at $t = 0$ until the end of the production time. Reservoir X2 compounded the production by dividing the G_p and W_p vs. N_p into two segments. By doing so, the GOR and WC of NPV optimization results and base cases were similar. Hence, it can be said that GOR and WC's value would depend on the timestep or the Np range used.

The GOR and WC of the optimization cases vary within the base case's upper and lower bound (1). Moreover, all optimization cases have similar GOR and WC as the base case (2). Based on these observations, the curves of W_p and G_p are considered credible.

It is worth mentioning that one could use more breakpoints in G_p and W_p curves to improve GOR and WC's quality during the optimization. Since the GOR is the slope of G_p vs. N_p, more breakpoints would enable the computation to capture more GOR and WC changes. The more the breakpoints and the smaller the time step, the more precise the result would be. Nevertheless, it is decided to keep using the same number of breakpoints in this work, considering the result in section 4.2.1.

4.6 The Uncertainty Analysis

IOIP, production potential, and cost proxy model have a normal distribution with the same mean value and standard deviation. Accordingly, thus these parameters would have identical CDF, whereas the oil price had separate CDF.



(a) CDF of the normal distribution mode **(b)** CDF of the uniform distribution model

Figure 4.10: The discretization of Cumulative Distribution Function (CDF) plot

The generation for the possible outcomes with the associated probabilities is depicted in Figure 4.10. The distribution for IOIP, production potential and cost proxy are discretize into: 0.74 with probability of 0.25, 0.99 with probability of 0.5 and 1.26 with probability of 0.25. Concurrently, the oil price CDF are segmented into 26, 50 and 74 with probabilities of 0.25, 0.5 and 0.25 respectively. Using these values, the probability tree diagram for this uncertainty analysis are presented in Figure 4.11. The W_p and G_p used in the study are the curves that produced the best and worst Npv as well as the original curves. Thus, only MaxWpGp, MinWpGp and base case curves are included.

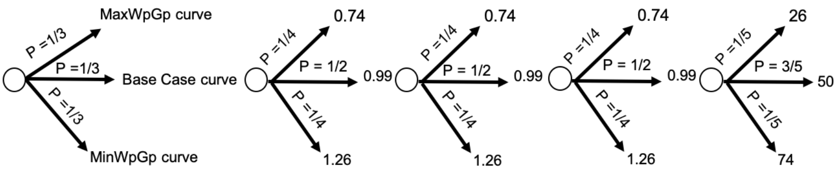


Figure 4.11: The complete tree diagram with the possible outcomes

4.6.1 The Runtime Limit

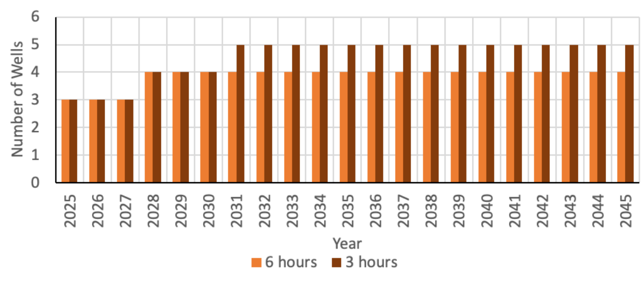
Table 4.10: The result of runtime evaluation

Runtime	Base case		Base case20		Base case40	
	NPV	Relative Gap	NPV	Relative Gap	NPV	Relative Gap
7200	28968.5	6.2%	6516.7	14%	449.3	195 %
10800	28968.5	2.8%	6516.7	10%	449.3	135 %
21600	28968.5	0.0%	6516.7	0.0%	485.2	0.0 %
	MaxWpGp		MaxWpGp20		MaxWpGp40	
	NPV	Relative Gap	NPV	Relative Gap	NPV	Relative Gap
7200	26110.6	3.5%	4023.1	32%	-949.1	77 %
10800	26110.6	2.8%	4023.1	25%	-922.7	58 %
21600	26110.6	1.8%	4023.1	18%	-922.7	35 %
	MinWpGp		MinWpGp20		MinWpGp40	
	NPV	Relative Gap	NPV	Relative Gap	NPV	Relative Gap
7200	29141.1	5.4%	6649.3	19%	615.8	50 %
10800	29141.1	3.6%	6649.3	17%	615.8	0 %
21600	29141.1	0.0%	6649.3	13%	-	

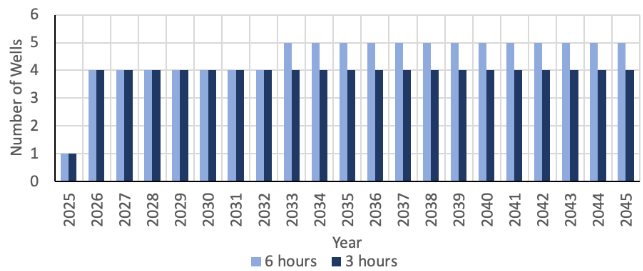
Table 4.10 summarizes the result of the tested cases, presenting the NPV obtained with the particular runtime. Most of the cases have the same result after running 2, 3, and 6 hours. In each of the runtime, it is written the relative gap obtained between the running. The relative gap is the relative difference between the best bound and the incumbent solution. Therefore, there is a possibility to increase the NPV by a certain % of the relative gap. However, there was no assurance that it always happens. The table shows that the optimization typically decreased the relative gap instead of improving the NPV. Only Base case 40 cases showed that 3-hour runtime produced lower NPV than the 6-hour runtime. Meanwhile, there were two cases pointed out that setting 2-hour runtime had resulted in lower NPV. From this finding, it was preferable to limit the time to 3 hours. To have a better evaluation of how different the result would be if a shorter time is used, the oil rate comparison is tabulated in Table 4.11. Base case 40 with 3-hour runtime and MaxW-

Table 4.11: The oil rate errors of Base case 40 and MaxWpGp40. The result from the 6-hour runtime is used as the reference

	Base case 40 (3 hour)	Max40 (2 hour)
Max qo Error	9 %	17 %
Average Error	5 %	5 %



(a) Base case 40



(b) MaxWpGp40

Figure 4.12: The number of well comparison

pGp40 with 2-hour runtime were compared with the result with a 6-hour runtime of the corresponding case. Both cases obtained the same average error of 5 %, although the MaxWpGp40 had a bigger maximum error. Besides, Figure 4.12 also shows that there were differences by one well in the optimal number of wells. Therefore, it should be anticipated that the uncertainty analysis might have a 5 % error in the optimal oil rate and a small deviation in the number of well results due to time limitation. Indeed, one could increase the running time to get a more reliable result. However, it was not convenient to be done in this thesis. Therefore, based on the result in Table 4.10, it was assumed that 3-hour runtime was sufficient to be implemented in the uncertainty study.

4.6.2 The Result

Figure 4.13 shows the cumulative distribution function of the NPV obtained from the uncertainty analysis. It can be seen most of the NPV are positive. However, there were

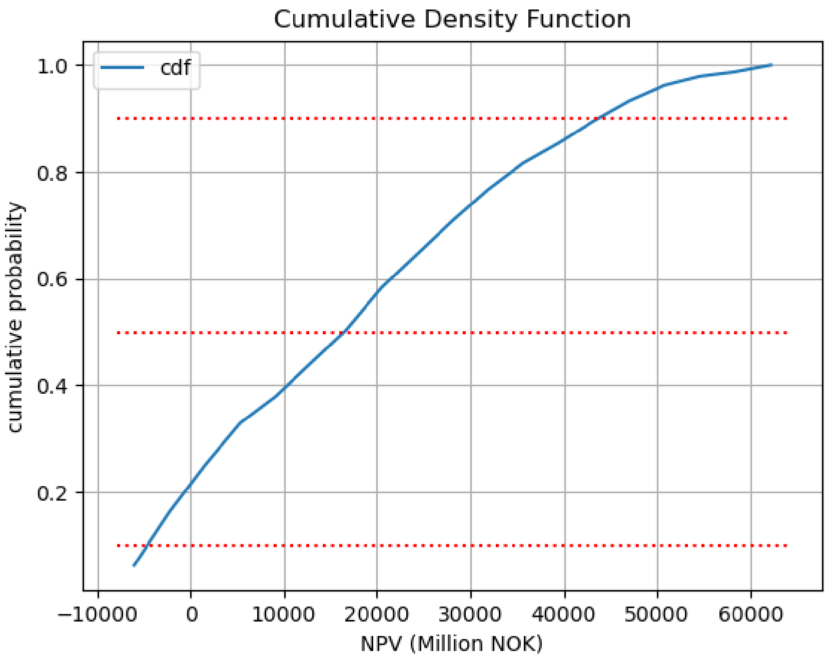


Figure 4.13: The cumulative distribution function of NPV

20% of the data showing that the field may be unprofitable. From the cdf plot, the P10, P50 and P90 are tabulated in Table 4.12.

Table 4.12: The NPV of P10, P50, and P90

	P10	P50	P90
NPV (Million NOK)	-4331	17549	45180

The distribution of an optimal number of well throughout the field lifetime is represented in Figure 4.14. The result suggested that the field should be drilled at its maximum number of predrilled wells (three wells), which corresponds to the result from the base case. It is also shown that 50% of the cases would have the optimal number of the well of only six wells.

Referring to Figure 4.15, 50% of the optimal oil rates suggested to keep a plateau rate of 9800 Sm^3/d, which is lower than the result provided in the base case. It is also important to note that the field had no oil production at the later year, especially in the year 2043 and 2044. It shows that during that year, 25 % of the cases proposed the field to stop producing. One of the causes was that the reservoir had reached its maximum N_p during the computation, thus no more oil could be produced.

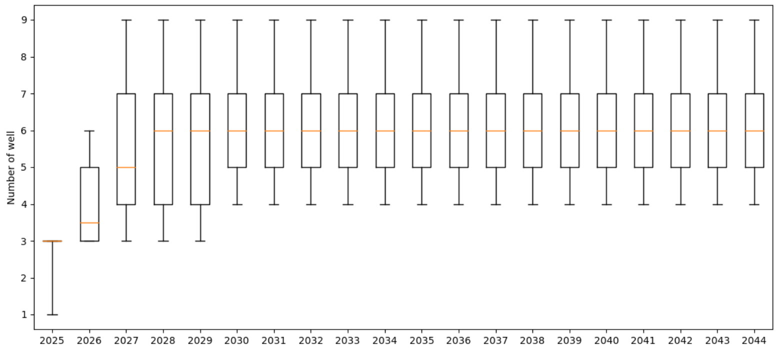


Figure 4.14: The optimal number of well boxplot

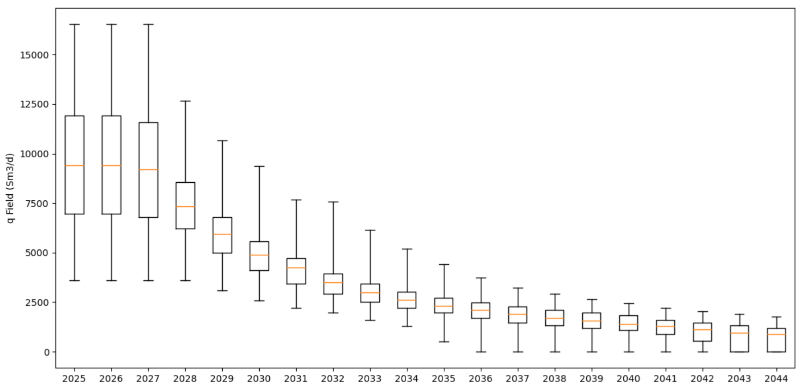


Figure 4.15: The optimal oil rates boxplot

Chapter 5

Conclusion

- The cost proxy model was successfully built based on the cost figures for CapEx, OpEx, and DrillEx. CapEx and OpEx were derived from multilinear regression and obtained low standard error.
- The NPV optimization problem was formulated by implementing the production potential curves that are distinct for a particular active well combination. Multidimensional PWL approximation was used to transform the non-linear behaviour of q_{opp} at particular N_p and well combination scenario s_n. Two cases were run to test the formulation: fixed drilling schedule case and varied drilling schedule case. The optimization included the drilling schedule as decision variables had increased the NPV by 2.2% while having fewer wells than the fixed drilling schedule case.
- In order to quantify the uncertainty in GOR and WC, the following actions are performed:
  - Six extreme curves of W_p and G_p curves were successfully generated to capture all the possible changes of GOR and WC profiles in the reservoir using an optimization formulation. The main idea of the formulation was (1) to use individual well GOR and WC profiles as the input and (2) 1D PWL approximation to reformulate the non-linear functions of $W_p(N_p)$ and $G_p(N_p)$ to linear functions
  - NPV optimizations using these extreme curves were run. It showed that the NPV could be slightly increased by having low production of gas and water simultaneously. On the other hand, the NPV was significantly decreased by three billion NOK by taking into account the high production of both gas and water.
  - From the quality control of W_p and G_p, it verified that the GOR and WC produced from the extreme curves are similar to the base case and does not exceed the maximum and minimum values of the base case.

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- Uncertainty analysis was performed for five parameters; thus, 243 optimizations were run. In this particular problem, it was proved that the limitation of 3-hour runtime could produce representative results of the optimization without time limitation. The uncertainty analysis shows that the field were apparent to be profitable. However, 20% of the cases was indicated for having negative NPV. Futhermore, the study also suggested to have plateau rates of 9800 Sm^3/d and the maximum number of well of 6 wells.

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Appendix A

Well combination factor

Table A.1: Full translation of scenario number and combination factor of reservoir X1

n	fn	Status					
		W1	W2	W3	W4	W5	W6
1	0	0	0	0	0	0	0
2	0.166	1	0	0	0	0	0
3	0.249	0	1	0	0	0	0
4	0.195	0	0	1	0	0	0
5	0.293	0	0	0	1	0	0
6	0.198	0	0	0	0	1	0
7	0.113	0	0	0	0	0	1
8	0.332	1	1	0	0	0	0
9	0.357	1	0	1	0	0	0
10	0.413	1	0	0	1	0	0
11	0.365	1	0	0	0	1	0
12	0.279	1	0	0	0	0	1
13	0.436	0	1	1	0	0	0
14	0.467	0	1	0	1	0	0
15	0.448	0	1	0	0	1	0
16	0.362	0	1	0	0	0	1
17	0.477	0	0	1	1	0	0
18	0.393	0	0	1	0	1	0
19	0.308	0	0	1	0	0	1
20	0.492	0	0	0	1	1	0

Continue to next page..

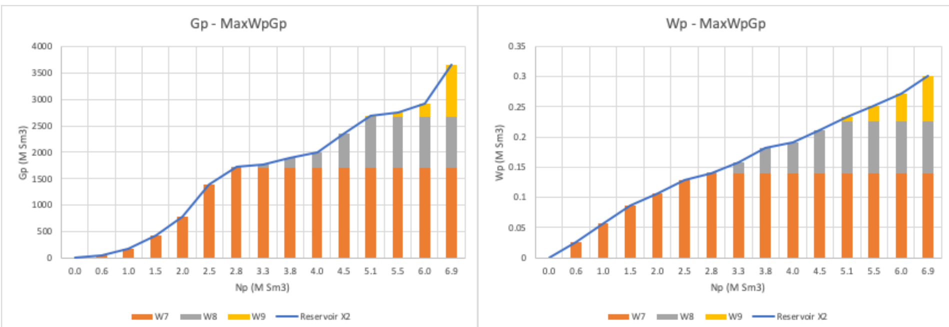
n	fn	Status					
		W1	W2	W3	W4	W5	W6
21	0.406	0	0	0	1	0	1
22	0.304	0	0	0	0	1	1
23	0.516	1	1	1	0	0	0
24	0.524	1	1	0	1	0	0
25	0.530	1	1	0	0	1	0
26	0.445	1	1	0	0	0	1
27	0.591	1	0	1	1	0	0
28	0.556	1	0	1	0	1	0
29	0.470	1	0	1	0	0	1
30	0.611	1	0	0	1	1	0
31	0.526	1	0	0	1	0	1
32	0.470	1	0	0	0	1	1
33	0.642	0	1	1	1	0	0
34	0.635	0	1	1	0	1	0
35	0.549	0	1	1	0	0	1
36	0.665	0	1	0	1	1	0
37	0.580	0	1	0	1	0	1
38	0.553	0	1	0	0	1	1
39	0.676	0	0	1	1	1	0
40	0.590	0	0	1	1	0	1
41	0.499	0	0	1	0	1	1
42	0.597	0	0	0	1	1	1
43	0.781	0	0	1	1	1	1
44	0.771	0	1	0	1	1	1
45	0.740	0	1	1	0	1	1
46	0.755	0	1	1	1	0	1
47	0.841	0	1	1	1	1	0
48	0.717	1	0	0	1	1	1
49	0.661	1	0	1	0	1	1
50	0.704	1	0	1	1	0	1
51	0.790	1	0	1	1	1	0
52	0.636	1	1	0	0	1	1
53	0.637	1	1	0	1	0	1
54	0.722	1	1	0	1	1	0
55	0.629	1	1	1	0	0	1
56	0.714	1	1	1	0	1	0
57	0.696	1	1	1	1	0	0
58	0.895	1	1	1	1	1	0
59	0.809	1	1	1	1	0	1
60	0.820	1	1	1	0	1	1
61	0.828	1	1	0	1	1	1
62	0.895	1	0	1	1	1	1
63	0.946	0	1	1	1	1	1
64	1.000	1	1	1	1	1	1

Table A.2: Full translation of scenario number and combination factor of reservoir X2

n	fn	Status		
		W7	W8	W9
1	0.000	0	0	0
2	0.527	1	0	0
3	0.606	0	1	0
4	0.644	0	0	1
5	0.807	1	1	0
6	0.878	1	0	1
7	0.907	0	1	1
8	1.000	1	1	1

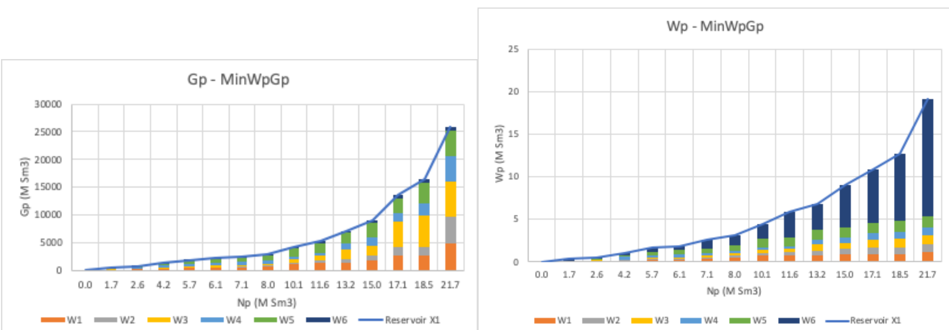
Appendix B

Well Production Contribution



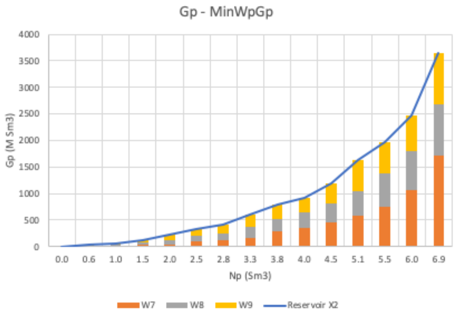
(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

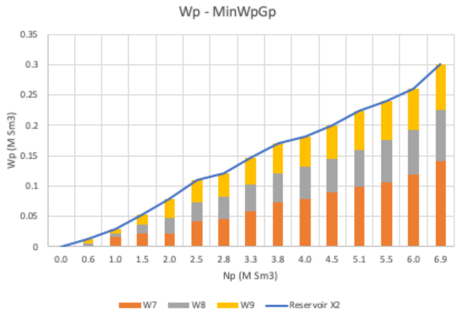
Figure B.1: The production contribution of MaxWpGp case in reservoir X2



(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

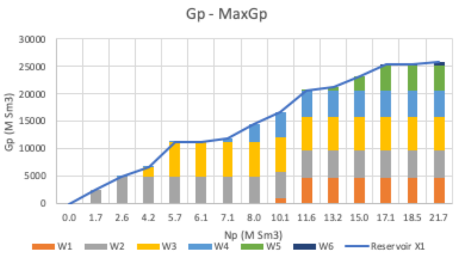
Figure B.2: The production contribution of MinWpGp case in reservoir X1

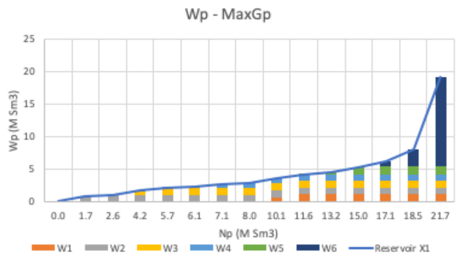




(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

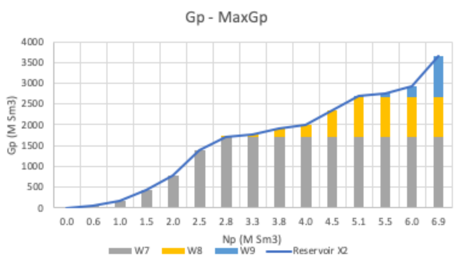
Figure B.3: The production contribution of MinWpGp case in reservoir X2

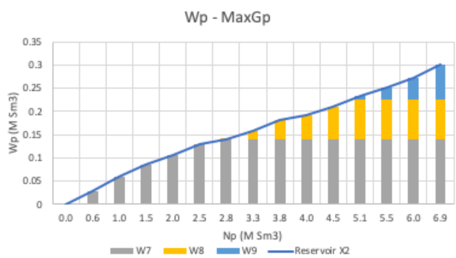




(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

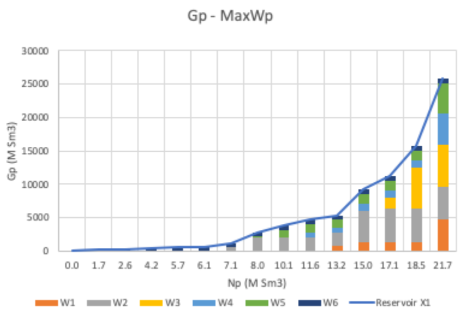
Figure B.4: The production contribution of MaxGp case in reservoir X1



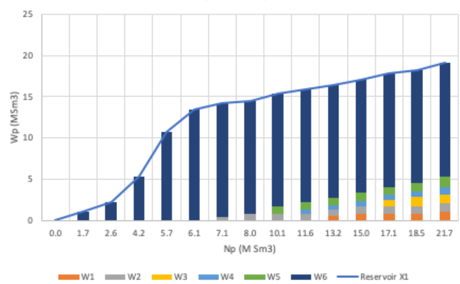


(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

Figure B.5: The production contribution of MaxGp case in reservoir X2

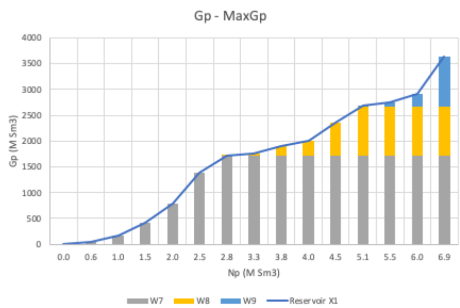


(a) Cumulative Gas Production Potential

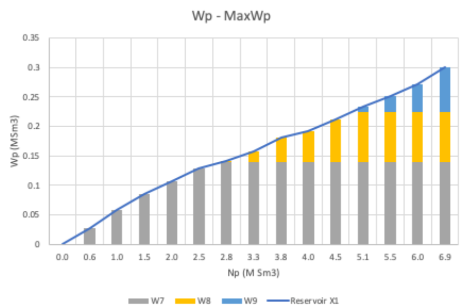


(b) Cumulative Water Production Potential

Figure B.6: The production contribution of Maxwp case in reservoir X1

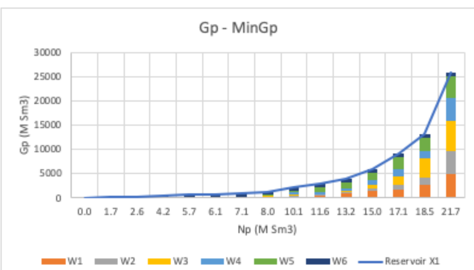


(a) Cumulative Gas Production Potential

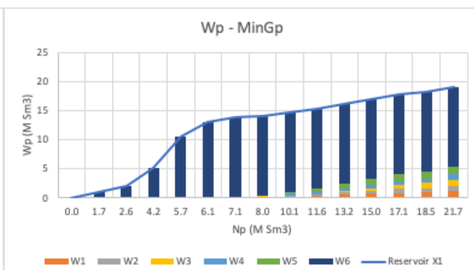


(b) Cumulative Water Production Potential

Figure B.7: The production contribution of MaxWp case in reservoir X2

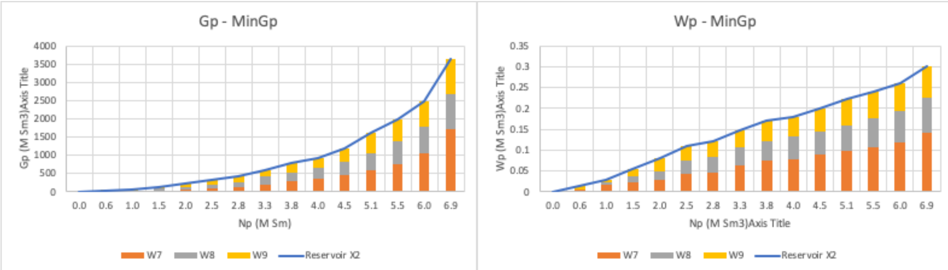


(a) Cumulative Gas Production Potential



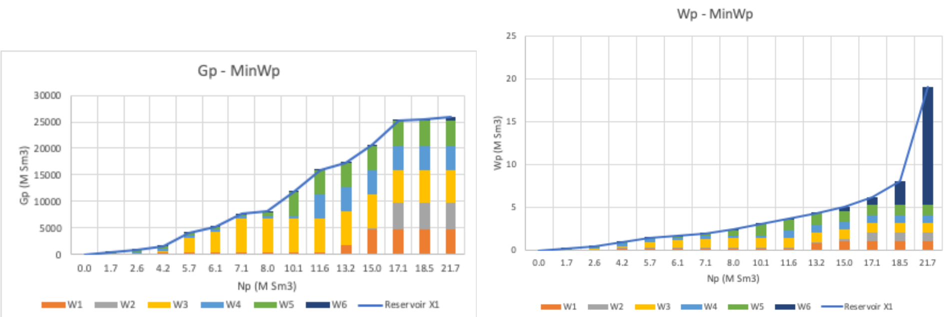
(b) Cumulative Water Production Potential

Figure B.8: The production contribution of MinGp case in reservoir X1



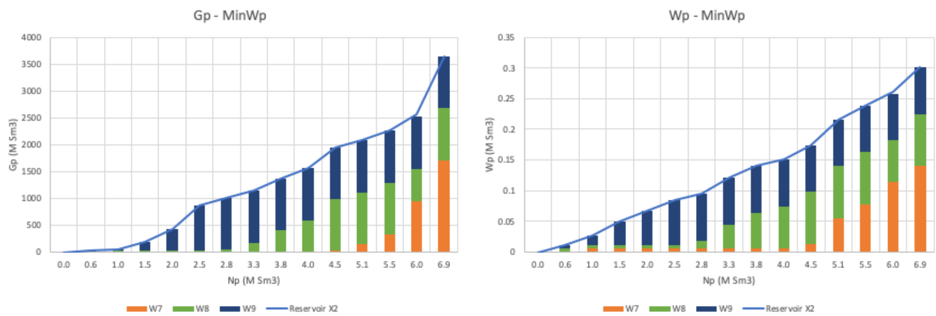
(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

Figure B.9: The production contribution of MinGp case in reservoir X2



(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

Figure B.10: The production contribution of MinWp case in reservoir X1



(a) Cumulative Gas Production Potential **(b)** Cumulative Water Production Potential

Figure B.11: The production contribution of MinGp case in reservoir X2

Appendix C

NPV Optimization Source Code

Case.mod

```
param nr;
set R:= 1..nr; #set of reservoir
param nt;
set T:=0..nt; #set of time
param qo_F_des; #desired plateau rate
param Nw_pd; #predrilled well
param Nw_py; #new well per year

#PWL for scenario
set S := 1..2;
param zw1_bar {a in S,b in S, c in S, d in S, e in S,f in S};
param zw2_bar {a in S,b in S, c in S, d in S, e in S,f in S};
param zw3_bar {a in S,b in S, c in S, d in S, e in S,f in S};
param zw4_bar {a in S,b in S, c in S, d in S, e in S,f in S};
param zw5_bar {a in S,b in S, c in S, d in S, e in S,f in S};
param zw6_bar {a in S,b in S, c in S, d in S, e in S,f in S};
param sn_x1 {a in S,b in S, c in S, d in S, e in S,f in S};
param zw7_bar {g in S, h in S, i in S};
param zw8_bar {g in S, h in S, i in S};
param zw9_bar {g in S, h in S, i in S};
param sn_x2 {g in S, h in S, i in S};


#PWL for qopp,gp,wp, pr
param bp_Np; #number of Np breakpoints
set i_Np := 1..bp_Np;
param bp_sn {ir in R}; #number of scenario break point for each
    reservoir
```

```
set i_sn {ir in R} := 1..bp_sn[ir];
param Np_bar {ir in R, k in i_Np, l in i_sn[ir]};
param sn_bar {ir in R, k in i_Np, l in i_sn[ir]};
param qopp_bar {ir in R, k in i_Np, l in i_sn[ir]};
param gp_bar {ir in R, k in i_Np, l in i_sn[ir]};
param wp_bar {ir in R, k in i_Np, l in i_sn[ir]};


#PWL for pipe length
param pl_sum {ir in R, m in i_sn[ir]};
param sn_pipe {ir in R, m in i_sn[ir]};

#economic value
param Po;
param XR;
param VC;
param disc;
param P_well;
param Px;
param P_pipe;
param P_top {o in {1..4}};
param P_opex {o in {1..5}};


var Nw {ir in R, n in T} integer>=0; #number of well in reservoir
    i for each year
var Nw_F {n in T} integer>=0; #number of well in Field for each
    year
var qo_F {n in T}>=0;
var qo {ir in R, n in T}>=0;
var qg {ir in R, n in T}>=0;
var qw {ir in R, n in T}>=0;


var qopp {ir in R, n in T } >=0;
var Np {ir in R, n in T } >= 0 ;
var sn {ir in R, n in T} integer >= 0 ;
var gp {ir in R, n in T}>=0;
var wp {ir in R, n in T}>=0;
var Np_F {n in T}>=0;


var zw1 {n in T} binary ; #well status per year
var zw2 {n in T} binary ;
var zw3 {n in T} binary ;
var zw4 {n in T} binary ;
var zw5 {n in T} binary ;
var zw6 {n in T} binary ;

var zw7 {n in T} binary ;
```

```
var zw8 {n in T} binary ;
var zw9 {n in T} binary ;

var pipe {ir in R, n in T}>=0; #pipeline length per year
var pipemax>=0;

#2D PWL Variables to estimate qopp, Gp and Wp
var eta_Np {ir in R, n in T, k in i_Np}>=0;
var eta_sn {ir in R, n in T, l in i_sn[ir]}>=0;
var lambda_qopp {ir in R, n in T, k in i_Np, l in i_sn[ir]}>=0;

#MULTI-D PWL Variables to find the well status
var eta_zw1 {n in T, a in S} >=0;
var eta_zw2 {n in T, b in S} >=0;
var eta_zw3 {n in T, c in S} >=0;
var eta_zw4 {n in T, d in S} >=0;
var eta_zw5 {n in T, e in S} >=0;
var eta_zw6 {n in T, f in S} >=0;
var lambda_x1 {n in T, a in S,b in S, c in S, d in S, e in S,f in
    S} >= 0;


var eta_zw7 {n in T, g in S} >=0;
var eta_zw8 {n in T, h in S} >=0;
var eta_zw9 {n in T, i in S} >=0;
var lambda_x2 {n in T, g in S, h in S, i in S} >=0;

#1D PWL Variables to find the pipelength
var lambda_pipe {ir in R, n in T, m in i_sn[ir]} >=0;

var qomax>=0;
var qwmax>=0;
var qgmax>=0;

var PVtop;
var PVop {n in {1..nt}}>=0;
var PVd {p in {0..nt}}>=0;
var PVsub {p in {0..nt}}>=0;
var PVr {n in {1..nt}} >=0;
var DCF {n in T};
var NPV;


maximize obj: NPV;
#Inequality constraint
#operation constraint
subject to c1 {ir in R, n in T } : qo [ir,n]<= qopp[ir, n];
subject to c63 {n in T}: qo_F[n]<=qomax;
subject to c64 {n in T}: qwmax>= sum{ir in R} qw [ir,n];
subject to c65 {n in T}: qgmax>= sum{ir in R} qg [ir,n];
```

```
#number of well constraint
subject to c42 {n in {1..nt} }:zw1[n]-zw1[n-1]>=0;
subject to c43 {n in {1..nt} }:zw2[n]-zw2[n-1]>=0;
subject to c44 {n in {1..nt} }:zw3[n]-zw3[n-1]>=0;
subject to c45 {n in {1..nt} }:zw4[n]-zw4[n-1]>=0;
subject to c46 {n in {1..nt} }:zw5[n]-zw5[n-1]>=0;
subject to c47 {n in {1..nt} }:zw6[n]-zw6[n-1]>=0;
subject to c48 {n in {1..nt} }:zw7[n]-zw7[n-1]>=0;
subject to c49 {n in {1..nt} }:zw8[n]-zw8[n-1]>=0;
subject to c50 {n in {1..nt} }:zw9[n]-zw9[n-1]>=0;
subject to c5 {ir in R, n in {1..nt} } : Nw [ir,n]- Nw[ir,n-1]
    >=0;
subject to c6 {ir in R, n in {1..nt} } : Nw_F [n]- Nw_F[n-1]
    <=Nw_py;
subject to c7 : Nw_F [0]<=Nw_pd;

#pipeline constraint
subject to c59 {ir in R, n in {1..nt}}: pipe[ir,n]-pipe[ir,n-1]
    >=0;
subject to c76 {n in T}: sum {ir in R } pipe [ir,n] <= pipemax;


#equality constraint
#Np, qo_F and Nw definition
subject to c8 {ir in R} : Np[ir,0] =0 ;
subject to c9 {ir in R, n in {1..nt}}: Np[ir,n] =
    Np[ir,n-1]+qo[ir,n-1]*365/10^6;
subject to c54 {ir in R, n in {1..nt}}: gp[ir,n] =
    gp[ir,n-1]+qg[ir,n-1]*365/10^3;
subject to c55 {ir in R, n in {1..nt}}: wp[ir,n] =
    wp[ir,n-1]+qw[ir,n-1]*365/10^6;
subject to c10 {n in T} : Nw[1,n] = zw1[n]+
    zw2[n]+zw3[n]+zw4[n]+zw5[n]+zw6[n];
subject to c11 {n in T} : Nw[2,n] = zw7[n]+ zw8[n]+zw9[n];
subject to c12 {n in T} : Nw_F[n] = sum {ir in R} Nw [ir,n];
subject to c13 {n in T} : qo_F [n] = sum {ir in R} qo [ir,n];
subject to c71 {n in T} : Np_F[n] = sum {ir in R} Np [ir,n];



#PWL for well x1
subject to c14 {n in T, a in S}: eta_zw1 [n,a] = sum {b in S, c
    in S, d in S, e in S, f in S} lambda_x1 [n,a,b,c,d,e,f];
subject to c15 {n in T, b in S}: eta_zw2 [n,b] = sum {a in S, c
    in S, d in S, e in S, f in S} lambda_x1 [n,a,b,c,d,e,f];
subject to c16 {n in T, c in S}: eta_zw3 [n,c] = sum {a in S, b
    in S, d in S, e in S, f in S} lambda_x1 [n,a,b,c,d,e,f];
subject to c17 {n in T, d in S}: eta_zw4 [n,d] = sum {a in S, b
    in S, c in S, e in S, f in S} lambda_x1 [n,a,b,c,d,e,f];
```

```
subject to c18 {n in T, e in S}: eta_zw5 [n,e] = sum {a in S, b
    in S, c in S, d in S, f in S} lambda_x1 [n,a,b,c,d,e,f];
subject to c19 {n in T, f in S}: eta_zw6 [n,f] = sum {a in S, b
    in S, c in S, d in S, e in S} lambda_x1 [n,a,b,c,d,e,f];
subject to c20 {n in T}: 1 = sum {a in S,b in S, c in S, d in S,
    e in S,f in S} lambda_x1 [n,a,b,c,d,e,f];
subject to c21 {n in T}: zw1 [n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * zw1_bar
    [a,b,c,d,e,f];
subject to c22 {n in T}: zw2 [n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * zw2_bar
    [a,b,c,d,e,f];
subject to c23 {n in T}: zw3 [n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * zw3_bar
    [a,b,c,d,e,f];
subject to c24 {n in T}: zw4 [n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * zw4_bar
    [a,b,c,d,e,f];
subject to c25 {n in T}: zw5 [n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * zw5_bar
    [a,b,c,d,e,f];
subject to c26 {n in T}: zw6 [n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * zw6_bar
    [a,b,c,d,e,f];
subject to c27 {n in T}: sn [1,n] = sum {a in S,b in S, c in S, d
    in S, e in S,f in S} lambda_x1 [n,a,b,c,d,e,f] * sn_x1
    [a,b,c,d,e,f];

#PWL for well x2
subject to c28 {n in T, g in S}: eta_zw7 [n,g] = sum {h in S, i
    in S} lambda_x2 [n,g,h,i];
subject to c29 {n in T, h in S}: eta_zw8 [n,h] = sum {g in S, i
    in S} lambda_x2 [n,g,h,i];
subject to c30 {n in T, i in S}: eta_zw9 [n,i] = sum {g in S, h
    in S} lambda_x2 [n,g,h,i];
subject to c31 {n in T}: 1 = sum {g in S, h in S, i in S}
    lambda_x2 [n,g,h,i];
subject to c32 {n in T}: zw7 [n] = sum {g in S, h in S, i in S}
    lambda_x2 [n,g,h,i]* zw7_bar [g,h,i];
subject to c33 {n in T}: zw8 [n] = sum {g in S, h in S, i in S}
    lambda_x2 [n,g,h,i]* zw8_bar [g,h,i];
subject to c34 {n in T}: zw9 [n] = sum {g in S, h in S, i in S}
    lambda_x2 [n,g,h,i]* zw9_bar [g,h,i];
subject to c35 {n in T}: sn [2,n] = sum {g in S, h in S, i in S}
    lambda_x2 [n,g,h,i] * sn_x2 [g,h,i];

#PWL for qopp
subject to c36 {ir in R, n in T,k in i_Np}: eta_Np[ir,n,k] = sum
    {l in i_sn[ir]} lambda_qopp[ir,n,k,l];
subject to c37 {ir in R, n in T,l in i_sn[ir]}: eta_sn[ir,n,l] =
```

```
    sum {k in i_Np} lambda_qopp[ir,n,k,l];
subject to c38 {ir in R, n in T}: 1 = sum {k in i_Np, l in
    i_sn[ir]} lambda_qopp[ir,n,k,l];
subject to c39 {ir in R, n in T}: Np[ir,n] = sum {k in i_Np, l in
    i_sn[ir]} lambda_qopp[ir,n,k,l]*Np_bar[ir,k,l];
subject to c40 {ir in R, n in T}: sn[ir,n] = sum {k in i_Np, l in
    i_sn[ir]} lambda_qopp[ir,n,k,l]*sn_bar[ir,k,l];
subject to c41 {ir in R, n in T}: qopp[ir,n] = sum {k in i_Np, l
    in i_sn[ir]} lambda_qopp[ir,n,k,l]*qopp_bar[ir,k,l];
subject to c51 {ir in R, n in T}: gp[ir,n] = sum {k in i_Np, l in
    i_sn[ir]} lambda_qopp[ir,n,k,l]*gp_bar[ir,k,l];
subject to c52 {ir in R, n in T}: wp[ir,n] = sum {k in i_Np, l in
    i_sn[ir]} lambda_qopp[ir,n,k,l]*wp_bar[ir,k,l];

#PWL for pipe
subject to c56 {ir in R, n in T}: 1 = sum {m in i_sn[ir]}
    lambda_pipe[ir,n,m];
subject to c57 {ir in R, n in T}: pipe[ir,n]= sum {m in i_sn[ir]}
    lambda_pipe[ir,n,m]*pl_sum[ir,m];
subject to c58 {ir in R, n in T}: sn[ir,n]= sum {m in i_sn[ir]}
    lambda_pipe[ir,n,m]*sn_pipe[ir,m];

#Cost constraint
#cost proxy at first oil date
#drillex for predrilled well, paid at the year it starts producing
subject to c60: PVd [0] = P_well*Nw_F[0];
#drillex for producing well in the next timestep, paid when it
    started producing
subject to c67 {n in {1..nt}}: PVd[n]=
    (P_well*(Nw_F[n]-Nw_F[n-1]));

#capexsubsea for predrilled well xmas and pipe
subject to c61: PVsub[0] = P_pipe*pipemax+ Px*Nw_F[0];
subject to c68 {n in {1..nt}}: PVsub[n]= (Px*(Nw_F[n]-Nw_F[n-1]));
#capextopside
subject to c62: PVtop=
    P_top[1]+P_top[2]*qomax+P_top[3]*qwmax+P_top[4]*qgmax*1000;

#opex and revenue calculated with rate at the previous timestep
subject to c69 {n in {1..nt}}: PVop[n] = P_opex[1]+
    P_opex[3]*qo_F[n-1]+ P_opex[4]*(sum{ir in R} qw[ir,n-1]) +
    P_opex[5]*1000*(sum{ir in R} qg[ir,n-1])+ P_opex[2]*Nw_F[n-1];
subject to c70 {n in {1..nt}}: PVr [n] =
    Po*XR*VC*(Np_F[n]-Np_F[n-1]);

subject to c73 :DCF [0] = - (PVtop+PVd[0] + PVsub[0]);
subject to c74 {n in {1..nt-1}}: DCF [n] = (-PVop[n]+ PVr [n]-PVd
    [n] - PVsub [n])/(1+disc)^(n);
subject to c75 : DCF [nt] = (-PVop[nt]+ PVr [nt])/(1+disc)^(nt);
subject to c72: NPV = sum {n in T} DCF [n];
```

Case-basecase.run

```
option log_file "logbase.tmp";
option eexit;
model case.mod;
data combinationfactor.dat;
data PWL_table_basecase.dat;
data Econ.dat;

option show_stats 1;
option solver gurobi;
option gurobi_options "threads=8 timelim=43200 outlev=1
    mipgap=0.0001 nodefilestart=8 bestbound=1";

suffix sosno;
suffix ref;
param iter1;
let iter1 :=0;
for {n in T} {
  let iter1:=iter1+1;
  let {a in S} eta_zw1[n,a].sosno := iter1;
  let {a in S} eta_zw1[n,a].ref := zw1_bar[a,1,1,1,1,1];
  let iter1:=iter1+1;
  let {b in S} eta_zw2[n,b].sosno := iter1;
  let {b in S} eta_zw2[n,b].ref := zw2_bar[1,b,1,1,1,1];
  let iter1:=iter1+1;
  let {c in S} eta_zw3[n,c].sosno := iter1;
  let {c in S} eta_zw3[n,c].ref := zw3_bar[1,1,c,1,1,1];
  let iter1:=iter1+1;
  let {d in S} eta_zw4[n,d].sosno := iter1;
  let {d in S} eta_zw4[n,d].ref := zw4_bar[1,1,1,d,1,1];
  let iter1:=iter1+1;
  let {e in S} eta_zw5[n,e].sosno := iter1;
  let {e in S} eta_zw5[n,e].ref := zw5_bar[1,1,1,1,e,1];
  let iter1:=iter1+1;
  let {f in S} eta_zw6[n,f].sosno := iter1;
  let {f in S} eta_zw6[n,f].ref := zw6_bar[1,1,1,1,1,f];
  let iter1:=iter1+1;
  let {g in S} eta_zw7[n,g].sosno := iter1;
  let {g in S} eta_zw7[n,g].ref := zw7_bar[g,1,1];
  let iter1:=iter1+1;
  let {h in S} eta_zw8[n,h].sosno := iter1;
  let {h in S} eta_zw8[n,h].ref := zw8_bar[1,h,1];
  let iter1:=iter1+1;
  let {i in S} eta_zw9[n,i].sosno := iter1;
  let {i in S} eta_zw9[n,i].ref := zw9_bar[1,1,i];
  }
```

```
for {ir in R, n in T} {
  let iter1:=iter1+1;
  let {l in i_sn[ir]} eta_sn[ir,n,l].sosno:=iter1;
  let {l in i_sn[ir]} eta_sn[ir,n,l].ref:= sn_bar[ir,1,l];

  let iter1:=iter1+1;
  let {m in i_sn[ir]} lambda_pipe[ir,n,m].sosno:=iter1;
  let {m in i_sn[ir]} lambda_pipe[ir,n,m].ref:=sn_pipe[ir,m];
  }

param iter2;
let iter2:= 0;
for {ir in R, n in T} {
  let iter2:=iter2-1;
  let {k in i_Np} eta_Np[ir,n,k].sosno:=iter2;
  let {k in i_Np} eta_Np[ir,n,k].ref:= Np_bar[ir,k,1];
}

solve;
display qopp ;
display qo ;
display qg ;
display qw ;
display qo_F ;
display zw1 ;
display zw2 ;
display zw3 ;
display zw4 ;
display zw5 ;
display zw6 ;
display zw7 ;
display zw8 ;
display zw9 ;
display sn ;
display Np ;
display gp ;
display wp ;
display Nw ;
display Nw_F ;
display pipe;
display PVtop;
display PVd;
display PVsub;
display PVr;
display PVop;
display DCF;
display NPV ;
```

Combinationfactor.dat

,

```
param nr:= 2;
param nt:= 20;
param Nw_pd := 3; #predrilled well
param Nw_py := 3;

param: sn_pipe pl_sum:=
#ir ip
#ir ip
1 1 1 0
1 2 23 15
1 3 24 8.5
1 4 25 11.2
1 5 26 19
1 6 27 15
1 7 28 17.7
1 8 29 25.5
1 9 30 11.2
1 10 31 19
1 11 32 21.2
1 12 33 11.9
1 13 34 14.6
1 14 35 22.4
1 15 36 8.1
1 16 37 15.9
1 17 38 18.1
1 18 39 12.3
1 19 40 20.1
1 20 41 19.7
1 21 42 15.8
1 22 43 22.3
1 23 44 18.1
1 24 45 24.6
1 25 46 22.4
1 26 47 14.6
1 27 48 21.2
1 28 49 27.7
1 29 50 25.5
1 30 51 17.7
1 31 52 21.2
1 32 53 19
1 33 54 11.2
1 34 55 25.5
1 35 56 17.7
1 36 57 15
1 37 58 17.7
1 38 59 25.5
1 39 60 27.7
```

```
1  40  61  21.2
1  41  62  27.7
1  42  63  24.6
1  43  64  27.7
2  1  1  0
2  2  2  17
2  3  3  15.7
2  4  4  14
2  5  5  17
2  6  6  17
2  7  7  15.7
2  8  8  17

;


param: zw1_bar zw2_bar zw3_bar zw4_bar zw5_bar zw6_bar sn_x1 :=
#a b c d e f
1  1  1  1  1  1  0  0  0  0  0  0  1
2  1  1  1  1  1  1  0  0  0  0  0  2
1  2  1  1  1  1  0  1  0  0  0  0  3
1  1  2  1  1  1  0  0  1  0  0  0  4
1  1  1  2  1  1  0  0  0  1  0  0  5
1  1  1  1  2  1  0  0  0  0  1  0  6
1  1  1  1  1  2  0  0  0  0  0  1  7
2  2  1  1  1  1  1  1  0  0  0  0  8
2  1  2  1  1  1  1  0  1  0  0  0  9
2  1  1  2  1  1  1  0  0  1  0  0  10
2  1  1  1  2  1  1  0  0  0  1  0  11
2  1  1  1  1  2  1  0  0  0  0  1  12
1  2  2  1  1  1  0  1  1  0  0  0  13
1  2  1  2  1  1  0  1  0  1  0  0  14
1  2  1  1  2  1  0  1  0  0  1  0  15
1  2  1  1  1  2  0  1  0  0  0  1  16
1  1  2  2  1  1  0  0  1  1  0  0  17
1  1  2  1  2  1  0  0  1  0  1  0  18
1  1  2  1  1  2  0  0  1  0  0  1  19
1  1  1  2  2  1  0  0  0  1  1  0  20
1  1  1  2  1  2  0  0  0  1  0  1  21
1  1  1  1  2  2  0  0  0  0  1  1  22
2  2  2  1  1  1  1  1  1  0  0  0  23
2  2  1  2  1  1  1  1  0  1  0  0  24
2  2  1  1  2  1  1  1  0  0  1  0  25
2  2  1  1  1  2  1  1  0  0  0  1  26
2  1  2  2  1  1  1  0  1  1  0  0  27
2  1  2  1  2  1  1  0  1  0  1  0  28
2  1  2  1  1  2  1  0  1  0  0  1  29
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2  1  1  2  1  2  1  0  0  1  0  1  31
2  1  1  1  2  2  1  0  0  0  1  1  32
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```
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2  2  2  1  1  2  1  1  1  0  0  1  55
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2  2  1  2  2  2  1  1  0  1  1  1  61
2  1  2  2  2  2  1  0  1  1  1  1  62
1  2  2  2  2  2  0  1  1  1  1  1  63
2  2  2  2  2  2  1  1  1  1  1  1  64

;

param: zw7_bar zw8_bar zw9_bar sn_x2:=
1  1  1  0  0  0  1
2  1  1  1  0  0  2
1  2  1  0  1  0  3
1  1  2  0  0  1  4
2  2  1  1  1  0  5
2  1  2  1  0  1  6
1  2  2  0  1  1  7
2  2  2  1  1  1  8
;
```

Econ.dat

,

```
param Po := 60;
param XR := 8.5;
param VC := 6.29;
param disc := 0.12;
param Px := 50;

param P_well := 500;

param: P_top :=
1 2150
2 0.856242424
3 0.044161616
4 0.000235638
;
param: P_opex :=
1 723.3333333
2 6.5
3 0.023636364
4 1.8552E-18
5 3.03876E-21
;

param P_pipe := 25;
```

PWL_table_basecase.dat

```
param bp_Np:= 7;

param bp_sn:=
1 43
2 8
;



param: Np_bar sn_bar qopp_bar gp_bar wp_bar:=
#ir k l
1 1 1 0 1 0 0 0
1 2 1 4.163479729 1 0 1673.606623 1.261307784
1 3 1 7.088078361 1 0 2972.430213 2.859564676
1 4 1 10.12279601 1 0 4650.489193 5.035448441
1 5 1 15.00680301 1 0 10523.23814 9.621709902
1 6 1 18.5066877 1 0 17247.86926 13.79690505
1 7 1 21.72981803 1 0 25839.50 19.07
1 1 2 0 23 12386.60318 0 0
1 2 2 4.163479729 23 7136.023275 1673.606623 1.261307784
1 3 2 7.088078361 23 4482.245865 2972.430213 2.859564676
1 4 2 10.12279601 23 2666.786653 4650.489193 5.035448441
```

```
1 5 2 15.00680301 23 1308.206621 10523.23814 9.621709902
1 6 2 18.5066877 23 782.0584154 17247.86926 13.79690505
1 7 2 21.72981803 23 486.9343354 25839.50 19.07
1 1 3 0 24 12581.3048 0 0
1 2 3 4.163479729 24 7248.19247 1673.606623 1.261307784
1 3 3 7.088078361 24 4552.70106 2972.430213 2.859564676
1 4 3 10.12279601 24 2708.705142 4650.489193 5.035448441
1 5 3 15.00680301 24 1328.769963 10523.23814 9.621709902
1 6 3 18.5066877 24 794.3513773 17247.86926 13.79690505
1 7 3 21.72981803 24 494.5883228 25839.50 19.07
1 1 4 0 25 12737.3061 0 0
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1 3 4 7.088078361 25 4609.152063 2972.430213 2.859564676
1 4 4 10.12279601 25 2742.291605 4650.489193 5.035448441
1 5 4 15.00680301 25 1345.245984 10523.23814 9.621709902
1 6 4 18.5066877 25 804.2009 17247.86926 13.79690505
1 7 4 21.72981803 25 500.7209476 25839.50 19.07
1 1 5 0 26 10683.08899 0 0
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1 3 5 7.088078361 26 3865.808144 2972.430213 2.859564676
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1 5 5 15.00680301 26 1128.290585 10523.23814 9.621709902
1 6 5 18.5066877 26 674.5028903 17247.86926 13.79690505
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1 4 6 10.12279601 27 3057.293869 4650.489193 5.035448441
1 5 6 15.00680301 27 1499.772049 10523.23814 9.621709902
1 6 6 18.5066877 27 896.578058 17247.86926 13.79690505
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1 1 7 0 28 13341.51113 0 0
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1 3 7 7.088078361 28 4827.791142 2972.430213 2.859564676
1 4 7 10.12279601 28 2872.374558 4650.489193 5.035448441
1 5 7 15.00680301 28 1409.05888 10523.23814 9.621709902
1 6 7 18.5066877 28 842.3488591 17247.86926 13.79690505
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1 5 8 15.00680301 29 1192.103482 10523.23814 9.621709902
1 6 8 18.5066877 29 712.6508494 17247.86926 13.79690505
1 7 8 21.72981803 29 443.7189868 25839.50 19.07
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1 3 9 7.088078361 30 5312.111803 2972.430213 2.859564676
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1 5 9 15.00680301 30 1550.41469 10523.23814 9.621709902
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1 5 10 15.00680301 31 1333.459292 10523.23814 9.621709902
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1 5 11 15.00680301 32 1192.441451 10523.23814 9.621709902
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1 5 12 15.00680301 33 1628.316696 10523.23814 9.621709902
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1 5 13 15.00680301 34 1609.612188 10523.23814 9.621709902
1 6 13 18.5066877 34 962.2415426 17247.86926 13.79690505
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```

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1 1 26 0 47 20182.86812 0 0
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1 1 38 0 59 19425.76181 0 0
1 2 38 4.163479729 59 11191.34007 1673.606623 1.261307784
```

```
1 3 38 7.088078361 59 7029.452652 2972.430213 2.859564676
1 4 38 10.12279601 59 4182.289655 4650.489193 5.035448441
1 5 38 15.00680301 59 2051.644818 10523.23814 9.621709902
1 6 38 18.5066877 59 1226.492872 17247.86926 13.79690505
1 7 38 21.72981803 59 763.653302 25839.50 19.07
1 1 39 0 60 19682.76395 0 0
1 2 39 4.163479729 60 11339.4011 1673.606623 1.261307784
1 3 39 7.088078361 60 7122.452062 2972.430213 2.859564676
1 4 39 10.12279601 60 4237.621199 4650.489193 5.035448441
1 5 39 15.00680301 60 2078.788006 10523.23814 9.621709902
1 6 39 18.5066877 60 1242.719329 17247.86926 13.79690505
1 7 39 21.72981803 60 773.7564082 25839.50 19.07
1 1 40 0 61 19877.36557 0 0
1 2 40 4.163479729 61 11451.51268 1673.606623 1.261307784
1 3 40 7.088078361 61 7192.87107 2972.430213 2.859564676
1 4 40 10.12279601 61 4279.518158 4650.489193 5.035448441
1 5 40 15.00680301 61 2099.340786 10523.23814 9.621709902
1 6 40 18.5066877 61 1255.005977 17247.86926 13.79690505
1 7 40 21.72981803 61 781.4064644 25839.50 19.07
1 1 41 0 62 21496.47906 0 0
1 2 41 4.163479729 62 12384.29719 1673.606623 1.261307784
1 3 41 7.088078361 62 7778.767351 2972.430213 2.859564676
1 4 41 10.12279601 62 4628.106886 4650.489193 5.035448441
1 5 41 15.00680301 62 2270.342872 10523.23814 9.621709902
1 6 41 18.5066877 62 1357.232658 17247.86926 13.79690505
1 7 41 21.72981803 62 845.0560331 25839.50 19.07
1 1 42 0 63 22713.5892 0 0
1 2 42 4.163479729 63 13085.48428 1673.606623 1.261307784
1 3 42 7.088078361 63 8219.193738 2972.430213 2.859564676
1 4 42 10.12279601 63 4890.145883 4650.489193 5.035448441
1 5 42 15.00680301 63 2398.88752 10523.23814 9.621709902
1 6 42 18.5066877 63 1434.077876 17247.86926 13.79690505
1 7 42 21.72981803 63 892.9022996 25839.50 19.07
1 1 43 0 64 24010.6 0 0
1 2 43 4.163479729 64 13832.7028 1673.606623 1.261307784
1 3 43 7.088078361 64 8688.533171 2972.430213 2.859564676
1 4 43 10.12279601 64 5169.387177 4650.489193 5.035448441
1 5 43 15.00680301 64 2535.870848 10523.23814 9.621709902
1 6 43 18.5066877 64 1515.967818 17247.86926 13.79690505
1 7 43 21.72981803 64 943.8895706 25839.50 19.07
2 1 1 0 1 0 0 0
2 2 1 1.007873034 1 0 106.7302846 0.04453459
2 3 1 2.042479879 1 0 320.597987 0.091392024
2 4 1 3.762329721 1 0 726.1950764 0.153749827
2 5 1 5.008121245 1 0 1389.013126 0.203443716
2 6 1 6.05754954 1 0 2736.948528 0.265414935
2 7 1 6.8607 1 0 3643.818039 0.300889055
2 1 2 0 2 4273.936632 0 0
2 2 2 1.007873034 2 2974.418597 106.7302846 0.04453459
2 3 2 2.042479879 2 1944.09011 320.597987 0.091392024
```

```
2 4 2 3.29 2 1109.970236 726.1950764 0.153749827
2 5 2 4.49 2 692.4721684 1389.013126 0.203443716
2 6 2 6.06 2 342.5436253 2736.948528 0.265414935
2 7 2 6.8607 2 196.8042707 3643.818039 0.300889055
2 1 3 0 3 4915.397104 0 0
2 2 3 1.007873034 3 3420.83887 106.7302846 0.04453459
2 3 3 2.042479879 3 2235.871918 320.597987 0.091392024
2 4 3 3.29 3 1276.561858 726.1950764 0.153749827
2 5 3 4.49 3 796.4029381 1389.013126 0.203443716
2 6 3 6.06 3 393.9548217 2736.948528 0.265414935
2 7 3 6.8607 3 226.3419479 3643.818039 0.300889055
2 1 4 0 4 5222.198198 0 0
2 2 4 1.007873034 4 3634.355111 106.7302846 0.04453459
2 3 4 2.042479879 4 2375.426859 320.597987 0.091392024
2 4 4 3.29 4 1356.240176 726.1950764 0.153749827
2 5 4 4.49 4 846.1114943 1389.013126 0.203443716
2 6 4 6.06 4 418.5440396 2736.948528 0.265414935
2 7 4 6.8607 4 240.4693837 3643.818039 0.300889055
2 1 5 0 5 6549.266422 0 0
2 2 5 1.007873034 5 4557.91967 106.7302846 0.04453459
2 3 5 2.042479879 5 2979.071796 320.597987 0.091392024
2 4 5 3.29 5 1700.888765 726.1950764 0.153749827
2 5 5 4.49 5 1061.125868 1389.013126 0.203443716
2 6 5 6.06 5 524.9047088 2736.948528 0.265414935
2 7 5 6.8607 5 301.5776117 3643.818039 0.300889055
2 1 6 0 6 7120.101246 0 0
2 2 6 1.007873034 6 4955.18848 106.7302846 0.04453459
2 3 6 2.042479879 6 3238.72804 320.597987 0.091392024
2 4 6 3.29 6 1849.138428 726.1950764 0.153749827
2 5 6 4.49 6 1153.613723 1389.013126 0.203443716
2 6 6 6.06 6 570.6554643 2736.948528 0.265414935
2 7 6 6.8607 6 327.8631514 3643.818039 0.300889055
2 1 7 0 7 7359.496494 0 0
2 2 7 1.007873034 7 5121.794056 106.7302846 0.04453459
2 3 7 2.042479879 7 3347.622012 320.597987 0.091392024
2 4 7 3.29 7 1911.310992 726.1950764 0.153749827
2 5 7 4.49 7 1192.401042 1389.013126 0.203443716
2 6 7 6.06 7 589.8422991 2736.948528 0.265414935
2 7 7 6.8607 7 338.8867138 3643.818039 0.300889055
2 1 8 0 8 8114 0 0
2 2 8 1.007873034 8 5646.885899 106.7302846 0.04453459
2 3 8 2.042479879 8 3690.823826 320.597987 0.091392024
2 4 8 3.29 8 2107.260653 726.1950764 0.153749827
2 5 8 4.49 8 1314.647281 1389.013126 0.203443716
2 6 8 6.06 8 650.3135668 2736.948528 0.265414935
2 7 8 6.8607 8 373.6297447 3643.818039 0.300889055
;
```

Appendix D

Extreme Curves Optimization Source Code

casemaxwpgp.mod

```
set S:= 1..15;
param Np {ir in {1,2}, inp in S };

param Np_w1_bar {i in S};
param Wp_w1_bar {i in S};
param Gp_w1_bar {i in S};

param Np_w2_bar {i in S};
param Wp_w2_bar {i in S};
param Gp_w2_bar {i in S};

param Np_w3_bar {i in S};
param Wp_w3_bar {i in S};
param Gp_w3_bar {i in S};

param Np_w4_bar {i in S};
param Wp_w4_bar {i in S};
param Gp_w4_bar {i in S};

param Np_w5_bar {i in S};
param Wp_w5_bar {i in S};
param Gp_w5_bar {i in S};

param Np_w6_bar {i in S};
param Wp_w6_bar {i in S};
param Gp_w6_bar {i in S};
```

```
param Np_w7_bar {i in S};
param Wp_w7_bar {i in S};
param Gp_w7_bar {i in S};

param Np_w8_bar {i in S};
param Wp_w8_bar {i in S};
param Gp_w8_bar {i in S};

param Np_w9_bar {i in S};
param Wp_w9_bar {i in S};
param Gp_w9_bar {i in S};

var Np_w1 {inp in S} >=0;
var Np_w2 {inp in S} >=0;
var Np_w3 {inp in S} >=0;
var Np_w4 {inp in S} >=0;
var Np_w5 {inp in S}>=0;
var Np_w6 {inp in S}>=0;
var Np_w7 {inp in S}>=0;
var Np_w8 {inp in S}>=0;
var Np_w9 {inp in S}>=0;

var Gp_w1 {inp in S}>=0;
var Gp_w2 {inp in S}>=0;
var Gp_w3 {inp in S}>=0;
var Gp_w4 {inp in S}>=0;
var Gp_w5 {inp in S}>=0;
var Gp_w6 {inp in S}>=0;
var Gp_w7 {inp in S}>=0;
var Gp_w8 {inp in S}>=0;
var Gp_w9 {inp in S}>=0;

var Wp_w1 {inp in S}>=0;
var Wp_w2 {inp in S}>=0;
var Wp_w3 {inp in S}>=0;
var Wp_w4 {inp in S}>=0;
var Wp_w5 {inp in S}>=0;
var Wp_w6 {inp in S}>=0;
var Wp_w7 {inp in S}>=0;
var Wp_w8 {inp in S}>=0;
var Wp_w9 {inp in S}>=0;

var lambda_zw1 {inp in S, i in S} >=0;
var lambda_zw2 {inp in S, i in S} >=0;
var lambda_zw3 {inp in S, i in S} >=0;
var lambda_zw4 {inp in S, i in S} >=0;
var lambda_zw5 {inp in S, i in S} >=0;
var lambda_zw6 {inp in S, i in S} >=0;
var lambda_zw7 {inp in S, i in S} >=0;
```

```
var lambda_zw8 {inp in S, i in S} >=0;
var lambda_zw9 {inp in S, i in S} >=0;



var Gp{ir in {1,2},inp in S }>=0;
var Wp{ir in {1,2},inp in S }>=0;

var Gptotal>=0;
var Wptotal>=0;

maximize obj: Wptotal+Gptotal;

#PWL Constraints
subject to c1 {inp in S}: 1 = sum {i in S} lambda_zw1[inp,i];
subject to c2 {inp in S}: Np_w1 [inp]= sum {i in S}
    lambda_zw1[inp,i]*Np_w1_bar[i];
subject to c3 {inp in S}: Gp_w1 [inp]= sum {i in S}
    lambda_zw1[inp,i]*Gp_w1_bar[i];
subject to c4 {inp in S}: Wp_w1 [inp]= sum {i in S}
    lambda_zw1[inp,i]*Wp_w1_bar[i];

subject to c5 {inp in S}: 1 = sum {i in S} lambda_zw2[inp,i];
subject to c6 {inp in S}: Np_w2 [inp]= sum {i in S}
    lambda_zw2[inp,i]*Np_w2_bar[i];
subject to c7 {inp in S}: Gp_w2 [inp]= sum {i in S}
    lambda_zw2[inp,i]*Gp_w2_bar[i];
subject to c8 {inp in S}: Wp_w2 [inp]= sum {i in S}
    lambda_zw2[inp,i]*Wp_w2_bar[i];


subject to c9 {inp in S}: 1 = sum {i in S} lambda_zw3[inp,i];
subject to c10 {inp in S}: Np_w3 [inp]= sum {i in S}
    lambda_zw3[inp,i]*Np_w3_bar[i];
subject to c11 {inp in S}: Gp_w3 [inp]= sum {i in S}
    lambda_zw3[inp,i]*Gp_w3_bar[i];
subject to c12 {inp in S}: Wp_w3 [inp]= sum {i in S}
    lambda_zw3[inp,i]*Wp_w3_bar[i];


subject to c13 {inp in S}: 1 = sum {i in S} lambda_zw4[inp,i];
subject to c14 {inp in S}: Np_w4 [inp]= sum {i in S}
    lambda_zw4[inp,i]*Np_w4_bar[i];
subject to c15 {inp in S}: Gp_w4 [inp]= sum {i in S}
    lambda_zw4[inp,i]*Gp_w4_bar[i];
subject to c16 {inp in S}: Wp_w4 [inp]= sum {i in S}
    lambda_zw4[inp,i]*Wp_w4_bar[i];

subject to c17 {inp in S}: 1 = sum {i in S} lambda_zw5[inp,i];
subject to c18 {inp in S}: Np_w5 [inp]= sum {i in S}
```

```
    lambda_zw5[inp,i]*Np_w5_bar[i];
subject to c19 {inp in S}: Gp_w5 [inp]= sum {i in S}
    lambda_zw5[inp,i]*Gp_w5_bar[i];
subject to c20 {inp in S}: Wp_w5 [inp]= sum {i in S}
    lambda_zw5[inp,i]*Wp_w5_bar[i];

subject to c21 {inp in S}: 1 = sum {i in S} lambda_zw6[inp,i];
subject to c22 {inp in S}: Np_w6 [inp]= sum {i in S}
    lambda_zw6[inp,i]*Np_w6_bar[i];
subject to c23 {inp in S}: Gp_w6 [inp]= sum {i in S}
    lambda_zw6[inp,i]*Gp_w6_bar[i];
subject to c24 {inp in S}: Wp_w6 [inp]= sum {i in S}
    lambda_zw6[inp,i]*Wp_w6_bar[i];

subject to c25 {inp in S}: 1 = sum {i in S} lambda_zw7[inp,i];
subject to c26 {inp in S}: Np_w7 [inp]= sum {i in S}
    lambda_zw7[inp,i]*Np_w7_bar[i];
subject to c27 {inp in S}: Gp_w7 [inp]= sum {i in S}
    lambda_zw7[inp,i]*Gp_w7_bar[i];
subject to c28 {inp in S}: Wp_w7 [inp]= sum {i in S}
    lambda_zw7[inp,i]*Wp_w7_bar[i];


subject to c29 {inp in S}: 1 = sum {i in S} lambda_zw8[inp,i];
subject to c30 {inp in S}: Np_w8 [inp]= sum {i in S}
    lambda_zw8[inp,i]*Np_w8_bar[i];
subject to c31 {inp in S}: Gp_w8 [inp]= sum {i in S}
    lambda_zw8[inp,i]*Gp_w8_bar[i];
subject to c32 {inp in S}: Wp_w8 [inp]= sum {i in S}
    lambda_zw8[inp,i]*Wp_w8_bar[i];

subject to c33 {inp in S}: 1 = sum {i in S} lambda_zw9[inp,i];
subject to c34 {inp in S}: Np_w9 [inp]= sum {i in S}
    lambda_zw9[inp,i]*Np_w9_bar[i];
subject to c35 {inp in S}: Gp_w9 [inp]= sum {i in S}
    lambda_zw9[inp,i]*Gp_w9_bar[i];
subject to c36 {inp in S}: Wp_w9 [inp]= sum {i in S}
    lambda_zw9[inp,i]*Wp_w9_bar[i];


#Consistensy constraints
subject to c37 {inp in S}: Np_w1 [inp]+ Np_w2 [inp]+ Np_w3 [inp]+
    Np_w4 [inp]+ Np_w5 [inp]+ Np_w6 [inp] = Np[1,inp];
subject to c38 {inp in S}: Np_w7 [inp]+ Np_w8 [inp]+ Np_w9 [inp]=
    Np[2,inp];

subject to c39 {inp in {1..14}}: Np_w1 [inp+1]-Np_w1[inp] >=0;
subject to c40 {inp in {1..14}}: Np_w2 [inp+1]-Np_w2[inp] >=0;
subject to c41 {inp in {1..14}}: Np_w3 [inp+1]-Np_w3[inp]>=0;
subject to c42 {inp in {1..14}}: Np_w4 [inp+1]-Np_w4[inp] >=0;
```

```
subject to c43 {inp in {1..14}}: Np_w5 [inp+1]-Np_w5[inp] >=0;
subject to c44 {inp in {1..14}}: Np_w6 [inp+1]-Np_w6[inp] >=0;
subject to c45 {inp in {1..14}}: Np_w7 [inp+1]-Np_w7[inp]>=0;
subject to c46 {inp in {1..14}}: Np_w8 [inp+1]-Np_w8[inp] >=0;
subject to c47 {inp in {1..14}}: Np_w9 [inp+1]-Np_w9[inp] >=0;


subject to c48 {inp in S}: Wp_w1 [inp]+ Wp_w2 [inp]+ Wp_w3 [inp]+
    Wp_w4 [inp]+ Wp_w5 [inp]+ Wp_w6 [inp] = Wp[1,inp];
subject to c49 {inp in S}: Gp_w1 [inp]+ Gp_w2 [inp]+ Gp_w3 [inp]+
    Gp_w4 [inp]+ Gp_w5 [inp]+ Gp_w6 [inp] = Gp[1,inp];
subject to c50 {inp in S}: Wp_w7 [inp]+ Wp_w8 [inp]+ Wp_w9 [inp]=
    Wp[2,inp];
subject to c51 {inp in S}: Gp_w7 [inp]+ Gp_w8 [inp]+ Gp_w9 [inp]=
    Gp[2,inp];

subject to c52: Gptotal= sum { inp in S} (Gp[1,inp] +
    Gp[2,inp])/1000;
subject to c53: Wptotal= sum { inp in S} (Wp[1,inp] + Wp[2,inp]);
```

case_maxwpgp.run

```
reset;

option log_file "logmaxwpgp1.tmp";
option eexit;
model casemaxwpgp.mod;
data PWL_Wp_Gp.dat;


option solver gurobi;
option gurobi_options "threads=8 timelim=1000 outlev=1
    nodefilestart=0.5 bestbound=1";

suffix sosno;
suffix ref;

param iter2;
let iter2:= 0;
for {inp in S} {
  let iter2:=iter2-1;
  let {i in S} lambda_zw1[inp,i].sosno:=iter2;
  let {i in S} lambda_zw1[inp,i].ref:= Np_w1_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw2[inp,i].sosno:=iter2;
  let {i in S} lambda_zw2[inp,i].ref:= Np_w2_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw3[inp,i].sosno:=iter2;
```

```
  let {i in S} lambda_zw3[inp,i].ref:= Np_w3_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw4[inp,i].sosno:=iter2;
  let {i in S} lambda_zw4[inp,i].ref:= Np_w4_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw5[inp,i].sosno:=iter2;
  let {i in S} lambda_zw5[inp,i].ref:= Np_w5_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw6[inp,i].sosno:=iter2;
  let {i in S} lambda_zw6[inp,i].ref:= Np_w6_bar[i];
}
for {inp in S} {
  let iter2:=iter2-1;
  let {i in S} lambda_zw7[inp,i].sosno:=iter2;
  let {i in S} lambda_zw7[inp,i].ref:= Np_w7_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw8[inp,i].sosno:=iter2;
  let {i in S} lambda_zw8[inp,i].ref:= Np_w8_bar[i];
  let iter2:=iter2-1;
  let {i in S} lambda_zw9[inp,i].sosno:=iter2;
  let {i in S} lambda_zw9[inp,i].ref:= Np_w9_bar[i];
}

solve;
display Np;
display Gptotal ;
display Wptotal ;
display Gp ;
display Wp ;
display Np_w1 ;
display Np_w2 ;
display Np_w3 ;
display Np_w4 ;
display Np_w5;
display Np_w6 ;
display Np_w7 ;
display Np_w8 ;
display Np_w9 ;
display Wp_w1 ;
display Wp_w2 ;
display Wp_w3 ;
display Wp_w4 ;
display Wp_w5;
display Wp_w6 ;
display Wp_w7 ;
display Wp_w8 ;
display Wp_w9 ;
display Gp_w1 ;
display Gp_w2 ;
display Gp_w3 ;
```

```
display Gp_w4 ;
display Gp_w5;
display Gp_w6 ;
display Gp_w7 ;
display Gp_w8 ;
display Gp_w9 ;
```

PWL_Wp_Gp.dat

```
param: Np:=
1 1 0
1 2 1.7
1 3 2.6
1 4 4.16348
1 5 5.676587545
1 6 6.1
1 7 7.088078361
1 8 8.028194821
1 9 10.1228
1 10 11.6
1 11 13.2
1 12 15.00680301
1 13 17.07670403
1 14 18.5066877
1 15 21.729
2 1 0
2 2 0.58
2 3 1.007873034
2 4 1.537608201
2 5 2.042479879
2 6 2.52
2 7 2.770615944
2 8 3.29
2 9 3.76
2 10 4.01
2 11 4.49
2 12 5.10
2 13 5.51
2 14 6
2 15 6.8607
;


param: Np_w1_bar Wp_w1_bar Gp_w1_bar:=
1 0 0 0
2 0.288256885 0.031214168 80.96094092
3 0.465165512 0.062620116 149.3432683
4 0.755565581 0.149658701 282.2823623
```

```
5 1.018171586 0.256612197 405.7442709
6 1.275790298 0.383963305 533.3223981
7 1.611020081 0.552166459 711.4866122
8 1.853990239 0.667453077 887.1088925
9 2.002897228 0.724484872 1020.09269
10 2.300950157 0.814772535 1400.84812
11 2.507727078 0.867657345 1784.2556
12 2.802052363 0.943760732 2560.180006
13 3.000692529 0.99943402 3287.473932
14 3.203417269 1.057975708 4236.649151
15 3.318520641 1.086969467 4784.269763
;

param: Np_w2_bar Wp_w2_bar Gp_w2_bar:=
1 0 0 0
2 0.194618395 0.033129412 53.74595096
3 0.438220549 0.108014118 147.2346097
4 0.607234002 0.200259311 241.6290497
5 0.803706271 0.335666542 392.4766218
6 1.008513493 0.493889854 618.4256273
7 1.105420838 0.563168742 765.5239094
8 1.217216612 0.634027506 968.9994549
9 1.506167516 0.743479588 1710.655677
10 1.603461157 0.777370112 2063.847795
11 1.810451887 0.841584861 2969.16151
12 1.912736644 0.877464545 3482.633075
13 2.004526978 0.902543168 3939.246145
14 2.102198152 0.927813748 4447.533676
15 2.183515009 0.95161963 4893.595381
;

param: Np_w3_bar Wp_w3_bar Gp_w3_bar:=
1 0 0 0
2 0.071494043 0.000194454 7.174063148
3 0.20149221 0.016436752 24.14122992
4 0.513034179 0.103675836 108.1096008
5 0.799016576 0.204394834 280.0166745
6 1.015901027 0.28611572 483.8354985
7 1.211436595 0.362736858 726.8962878
8 1.50479555 0.471615205 1163.968016
9 1.808718727 0.580359622 1711.951081
10 2.008677206 0.699670841 1751.700204
11 2.208497283 0.716910515 2572.929493
12 2.510651237 0.818277514 3329.082309
13 2.999209396 0.978162971 4745.260141
14 3.272576498 1.065944302 5654.451177
15 3.43287295 1.117243428 6234.425232
;

param: Np_w4_bar Wp_w4_bar Gp_w4_bar:=
```

```
1 0 0 0
2 0.105299289 0.011119574 38.90824585
3 0.553250747 0.128057126 229.0931524
4 1.018771445 0.347113662 457.4118092
5 1.212016242 0.305728764 494.5150885
6 1.317186696 0.490882003 639.2349644
7 1.418867924 0.530301444 722.2294541
8 1.609897994 0.591317636 934.9727964
9 1.745359457 0.6266902 1142.30094
10 1.858233041 0.658631475 1370.775911
11 1.910539881 0.673111882 1492.404917
12 2.123046091 0.730300269 2124.746961
13 2.222841985 0.756980303 2515.805435
14 2.402084386 0.801779808 3321.952452
15 2.656273116 0.868055268 4621.395457
;

param: Np_w5_bar Wp_w5_bar Gp_w5_bar:=
1 0 0 0
2 0.320002985 0.031710069 103.9534541
3 0.502883611 0.062826003 175.831897
4 0.748775175 0.126606607 278.3811106
5 1.006509563 0.216782732 396.4086008
6 1.212016242 0.305728764 494.5150885
7 1.619356135 0.493138969 686.6429994
8 2.006212458 0.688764339 886.6055975
9 2.393911404 0.874904638 1147.972062
10 2.800485432 1.01921247 1550.153943
11 3.006077466 1.06980651 1837.263824
12 3.204953073 1.120475744 2193.02703
13 3.404515158 1.172962496 2624.451402
14 3.804717552 1.275010044 3849.637916
15 3.986698598 1.320375133 4615.146502
;


param: Np_w6_bar Wp_w6_bar Gp_w6_bar:=
1 0 0 0
2 0.249828031 0.060997479 33.50084177
3 0.515494345 0.165532752 64.02824668
4 1.056181239 0.487374791 124.8790346
5 1.784857442 1.141677289 206.5917366
6 2.130967169 1.554329342 246.2771297
7 2.480973119 2.026983018 285.3806211
8 3.037624168 2.904978879 347.2634136
9 3.508093555 3.81106593 399.5168188
10 4.023491435 4.99541488 457.4193817
11 4.505660493 6.291703986 511.4922378
12 5.004998036 7.91530368 566.6615897
13 5.506067904 9.936142934 621.5240551
```

```
14 5.804637494 11.43104538 653.2841959
15 6.151937713 13.73014415 690.6723081
;


param: Np_w7_bar Wp_w7_bar Gp_w7_bar:=
1  0  0  0
2  0.226601178 0.00620313 11.29507562
3  0.513508216 0.021938782 35.68539389
4  0.784630744 0.042367177 98.24415811
5  0.845077439 0.046775658 116.0731699
6  1.006471458 0.057721318 168.9878566
7  1.21593822 0.06968719 255.2707228
8  1.32934706 0.075783168 308.1963233
9  1.59646365 0.088954105 448.5843238
10 1.815516695 0.097825464 589.5770419
11 2.002032207 0.105221515 746.7178919
12 2.203584618 0.113775718 956.8717017
13 2.306910833 0.118425254 1078.963628
14 2.567613506 0.131396195 1447.100385
15 2.727931115 0.140007997 1714.219314
;

param: Np_w8_bar Wp_w8_bar Gp_w8_bar:=
1  0  0  0
2  0.004052967 0 0.149713481
3  0.100743446 0.000889842 3.931714526
4  0.303373596 0.005630772 14.38402872
5  0.513343797 0.01510258 40.08717395
6  0.59587294 0.019781497 55.42220238
7  0.804137407 0.031480534 106.6066359
8  0.91978532 0.036688467 142.1928487
9  1.098922252 0.044443085 208.5166679
10 1.160184046 0.046830122 234.2134078
11 1.307547259 0.052321471 305.949107
12 1.503712179 0.059961873 428.8248113
13 1.754935886 0.070674956 633.416234
14 1.849553069 0.075025791 725.694873
15 2.047162145 0.085059862 961.6629976
;

param: Np_w9_bar Wp_w9_bar Gp_w9_bar:=
1  0  0  0
2  0.000360561 0 0.011637433
3  0.051203772 0.000102976 1.793729551
4  0.303515453 0.006695388 13.91020544
5  0.514468395 0.016080187 38.14405743
6  0.651391463 0.023189464 63.01987527
7  0.831001825 0.030894843 103.8438753
8  0.935925666 0.035577223 134.7558753
```

```
9 1.014930372 0.038777331 160.4598753
10 1.115724098 0.042502455 196.0758753
11 1.309130372 0.049275505 277.8918753
12 1.500914825 0.055934761 390.4518753
13 1.74622844 0.063504783 584.8278753
14 1.853733499 0.067102702 686.9718753
15 2.085678275 0.075824001 968.0358753
;
```