tion discussed here. One of these is the avoidance of saddle points from which the descent might take the wrong path. Several others are listed in [3]. A most important investigation is that of using random in-service signals instead of special test inputs.

If the input is a stationary random signal the MSE will be a function similar to ISE [2]. It therefore seems likely that the logical scheme devised for descent using ISE measurements could also be used for the in-service inputs with measurements made on MSE. One main question which arises here is that of the truncation or averaging time over which MSE will be measured. Investigations into this problem have been started at the University of Washington by Prof. R. N. Clark and P. V. V. S. Sastry and preliminary results show that a truncation time very much longer than 3 sec is required for a successful descent. These results seem to be in accord with the work of McGrath and Rideout, as reported in [9].

References

[1] J. J. Levin, "Methods for the realization of self-optimizing systems," *Instrument Society of America Paper FCS-2-58;* 1958.

[2] G. C. Newton, L. A. Gould, and J. F. Kaiser, "Analytic Design of Linear Feedback Controls," John Wiley and Sons, Inc., New York, N. Y.; 1957.

[3] R. N. Clark, and E. E. Gould, "A self-adjusting control system," *Proc. 1960 Conf. on Adaptive Control Systems,* Garden City, N. Y., October, 1960, Pergamon Press, New York, N. Y.; 1962.

[4] R. N. Clark, "Integral of the error squared as a performance index for automatic control systems," *Trans. AIEE, Paper 60-1016,* pp. 467–471; January, 1961.

[5] R. N. Clark, and E. E. Gould, "Final Report on Adaptive Systems," Elec. Engrg. Dept., University of Washington, Seattle, Wash.; 1959.

[6] E. F. Beckenbach, "Modern Mathematics for the Engineer," McGraw-Hill Book Co., Inc., New York, N. Y.; 1956.

[7] P. C. Wheeler, "Descent Methods for a Two-Variable Performance-Measuring Adaptive Control System," University of Washington Engrg. Exper. Station, Seattle, Wash.; 1960.

[8] P. C. Wheeler, "Application of the Truncated Integral-Squared Error in a Performance-Measuring Adaptive Control System," M.S. Thesis, University of Washington, Seattle, Wash.; 1960.

[9] R. J. McGrath and V. C. Rideout, "A simulator study of a two-parameter adaptive system," IRE Trans. on Automatic Control, vol. AC-6, pp. 35–42; February, 1961.

Adaptive Sampling Frequency for Sampled-Data Control Systems*

R. C. DORF†, member, ire, M. C. FARREN‡, and C. A. PHILLIPS||

Summary—Sampled-data control systems generally have fixed sampling frequencies which must be set high enough to give satisfactory performance for all anticipated conditions. A study is made here of an adaptive system which varies the sampling frequency by measuring a system parameter. It is shown that a sampler followed by a zero-order hold whose sampling period is controlled by the absolute value of the first derivative of the error signal will be a more "efficient" sampler than a fixed-frequency sampler. That is, over a given time interval, fewer samples are needed with the variable-frequency system than with a fixed-frequency system while maintaining essentially the same response characteristics.

Analog computer studies of simple Type I and Type II sampled-data servo systems with error sampling and unity feedback verified the method. Standard analog computer components were used to set up a simulated servo system, a rate detector, absolute-value detector, a voltage-controlled oscillator, and a sampler and zero-order hold.

The system described reduced the number of samples required for response to a step input to about three-quarters that required in a fixed-sampling-frequency system. Over a long period of time savings in the number of samples required can be expected to be between 25 and 50 per cent. In many applications the savings produced by reducing the over-all number of samples required may outweigh the added complexity of the adaptive-sampling-frequency system.

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Introduction

THIS PAPER presents the investigation of a sampled-data feedback control system employing an adaptive, variable-frequency sampler. The basic area of investigation was concerned with finding a system signal and the functions of that signal for controlling the variable-frequency sampler so that the sampling would be more efficient. Sampling is defined to be more efficient when similar output response characteristics are obtained with fewer samples. The investigation was limited to the extent that only readily available system signals or signals which could be generated in a simple manner from some system signal were considered as controlling signals for the variable-frequency sampler.

The problem was attacked with the aid of analog computer studies. All of the necessary system component circuits were simulated on the analog computer. The entire system was composed of a simple servo, a variable-frequency sampler and zero-order hold circuit, a differentiating circuit, an absolute-value detection circuit, a voltage-controlled, variable-frequency oscillator and a transistorized relay control circuit. Type I and Type II servo systems were investigated and it was found that the number of samples necessary for specified output response characteristics could be substantially reduced using aperiodic sampling. A method of sampling frequency control using a function of the first derivative of the error signal was developed. It was found that, in the interest of efficiency, the sampling frequency should be increased as necessary toward an upper bound during the transient state and decreased toward a lower bound during steady-state conditions.

A survey of the literature indicated that this sampling problem had not yet been investigated. Many studies have recently been made concerning sampled-data feedback control systems with periodic sampling.[1–4] Direct applications of the periodic-sampling principle are, however, relatively limited. A more difficult and more promising problem is that of aperiodic sampling control.[5] Methods have been developed for analyzing aperiodic sampling systems as an aid toward understanding and synthesizing these nonlinear systems. However, the problem of designing or implementing the aperiodic sampling control has not been investigated. The authors could find no published or unpublished literature treating the problem of aperiodic sampling as applied to total sample reduction without response characteristic degradation.

I. The Basic Sampled-Data Control System

One basic sampled-data feedback control system was chosen for study. The block diagram of the system is shown in Fig. 1. The basic sampled-data control system was investigated using two different transfer functions $G(s)$. This section will consider the basic sampled-data control system with fixed-period sampling only. The desired minimum and maximum fixed-frequency sampling periods will be discussed. The discussion is separated into two parts; the first describes the Type II servo system and the second describes the Type I servo system.

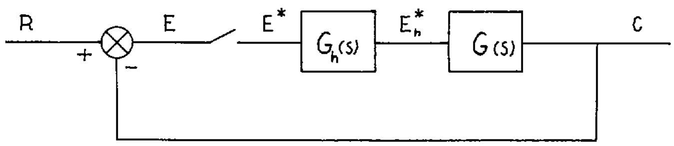


Fig. 1—Basic sampled-data control system.

The transfer function $G(s)=K(s+a)/s^2$ produces a Type II servo system. $G_h(s)$ is a zero-order hold. The z-plane transfer function of this Type II sampled-data system is

$$G(z) = \frac{K_1(z+z_1)}{(z-1)^2}$$

where

$$K_1 = KT\left(1+\frac{aT}{2}\right)$$

$$z_1 = \frac{aT-2}{aT+2}.$$

The maximum period T was determined by the steady-state stability requirements. Two conditions determine the region of stable response of the closed-loop system. These simultaneous conditions are

$$0 < aT \leq 2$$

and

$$0 < KT \leq 2.$$

Therefore with both a and K given as fixed values, the range of T for stable closed-loop response is determined by

$$0 < T \leq \frac{2}{a} \qquad \text{for } a > K$$

and

$$0 < T \leq \frac{2}{K} \qquad \text{for } a < K.$$

The minimum sampling period T was determined by the largest value of T that produced a response similar to that of a continuous system. Table I contains pertinent experimental values attained on an analog computer. System parameters were $a=10$ and $K=10$. The table shows that, with respect to output response characteristics, very little is gained by sampling faster than $T=0.043$. That value of T can then be used as the lower limit to the sampling period.

[1] B. Friedland, "A technique for the analysis of time-varying sampled-data systems," *Trans. AIEE*, vol. 76, pp. 407–413; January, 1957.

[2] R. E. Hufnagel, "Analysis of cyclic rate sampled-data feedback control systems," *Trans. AIEE*, vol. 77, pp. 421–423; 1958.

[3] J. T. Tou, "Digital and Sampled-Data Control Systems," McGraw-Hill Book Co., Inc., New York, N. Y.; 1959.

[4] E. I. Jury and F. J. Mullin, "The analysis of sampled-data control systems with a periodically time-varying sampling rate," IRE Trans. on Automatic Control, vol. AC-4, pp. 15–21; No. 1, May, 1959.

[5] R. E. Hufnagel, "Analysis of Aperiodically-Sampled-Data Feedback Control Systems," unpublished Ph.D. dissertation, Co-nell University, Ithaca, N. Y.; 1959.

TABLE I
COMPARISON OF FAST SAMPLING RESPONSES

T Seconds	Rise Time	Peak Overshoot	Time of Peak Overshoot	5 Per Cent Settling Time
0.016	0.125	40%	0.30	0.813
0.025	0.125	40%	0.30	0.813
0.043	0.115	45%	0.28	1.075
0.080	0.075	60%	0.25	1.150

The Type I servo is produced by the transfer function $G(s) = K/s(s+b)$. The z-plane transfer function is

$$G(z) = \frac{K_1(z + z_1)}{(z - p_1)(z - 1)}$$

where

$$K_1 = \frac{K}{b^2}(bT + e^{-bT} - 1)$$

$$z_1 = \frac{1 - e^{-bT}(1 + bT)}{bT + e^{-bT} - 1}$$

$$p_1 = e^{-bT}$$

If $bT > 3.71$, then the relationship between T and K for the stable response region is

$$0 < K \leq \frac{2b^2(1 + e^{-bT})}{bT(1 + e^{-bT}) - 2(1 - e^{-bT})}.$$

and if $bT < 3.71$, the relationship is

$$0 < K \leq \frac{b^2(1 - e^{-bT})}{1 - e^{-bT}(1 + bT)}.$$

In the same manner as for the Type II system, the minimum sampling period was determined by the largest value of T that produced continuous response characteristics. With system parameters $b = 40$ and $K = 1200$, a reasonable range of T is given by $0.023 \leq T \leq 0.083$. The system parameters b and K were chosen so that the range of the sampling period T would be similar to the range used for the Type II system.

The purpose of this section was to demonstrate how the ranges of the various system parameters were determined and to justify the ranges so determined. Treating T as a dependent variable allows a range of T to be determined based on a stability criterion. This range of T determines an upper bound to the duration of the sample period. The lower bound to the sample period was determined by analog computer tests and could also be determined by bandwidth considerations. For any given set of system parameters, two fixed limits can be found so that 1) it is not desirable to sample with longer periods than the upper bound to T because of stability considerations and, 2) it is not necessary to sample with shorter periods than the lower bound to T since faster sampling provides relatively little additional information in the error channel to improve the system response. The upper and lower bounds to T determine the permissible range of variation of T when it is continuously or discretely varied during the transient periods of the system response, as will be described in Section III.

II. VARIABLE-FREQUENCY SAMPLING

The function of a sampler and hold in a servo system is to approximate a continuous signal as accurately as possible. One way to improve the accuracy of approximation is to increase the sampling frequency. Another way is to use a higher order hold than the zero-order hold which is generally used. Both these methods have distinct limitations. The first method demands more performance of components and more time of communication channels, the second method requires more complicated circuitry. This section presents a method of improving the efficiency of a sampler-zero order hold combination by using variable-frequency sampling. An efficient sampling system is defined to be one which "satisfactorily" approximates its input with a minimum number of samples over any period of time.

Fig. 2(a) shows the continuous input and sampled-held output of a sampler and zero-order hold using ordinary fixed-frequency sampling. Since, in the servo system, the error signal is the input to the sampler, the

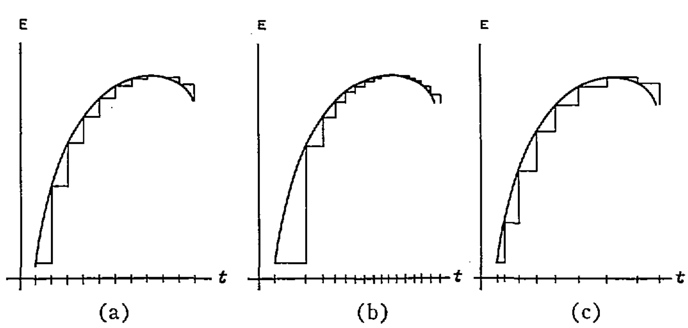


Fig. 2—Input and output of sampler and zero-order hold.
(a) Fixed T.
(b) T controlled by E.
(c) T controlled by $\dot{E}$.

symbols E and E_h^* will be used for the input and output of the sampler and hold. In this section the terms "input" and "output" will refer only to the signals in and out of the sampler and hold, not to the reference variable R and command variable C of the servo system. In Fig. 2(a) it can be seen that fixed-frequency sampling results in a better approximation of the input near the maximum of the curve where the first derivative $\dot{E}$ approaches zero, than it does in the portion of rapid rise where $\dot{E}$ is large.

Since in a servo system it is always the purpose to minimize E, the authors at first tried $|E|$ as a frequency-controlling variable. Fig. 2(b) demonstrates that attempting to control f_s as a function of E can actually decrease the efficiency of the system. When f_s is proportional to $|E|$ it can be seen that, in the rapidly rising portion of the curve, sampling is too slow, and in the region of the maximum error the system is sampling unnecessarily fast. In Fig. 2(c) f_s is a function of $|\dot{E}|$ and the accuracy of approximation appears to be nearly constant over the entire curve. This is the type of sampling frequency control found suitable for improving the efficiency.

Fig. 3 is a magnification of the input and output of the sampler over one sample period. The total area between the input and output in the sample period is the integral difference for one period, ID:

$$ID = \int_{t_i}^{t_{i+1}} (E - E_h^*)dt$$
$$= \int_0^{T_i} (E - E_h^*)dt.$$

In any successful sampled-data servo system, the sampling frequency must be several times the highest system signal frequency of importance. Hence it is asumed that $\dot{E}$ does not vary radically during a sample period. Then a reasonable approximation of E over the period is a straight line approximation,

$$E = E_i + \dot{E}_a t,$$

where $\dot{E}_a$ is the first derivative of E at some instant a in the period. Fig. 3 shows approximations based on $\dot{E}_i$ and a value of $\dot{E}$ which has the same slope as the chord line.

Then an approximation of the integral difference is

$$ID = \int_0^{T_i} (E_i + \dot{E}_i t - E_h^*)dt,$$

but $E_h^* = E_i$ over the entire period, hence:

$$ID = \int_0^{T_i} \dot{E}_i t dt = \dot{E}_i \frac{T_i^2}{2} \cdot$$

Now if T is made a function of $\dot{E}$ such that

$$T_i = \frac{C}{\sqrt{|\dot{E}_i|}}$$

where C is a constant, the integral difference per sample period will be a constant

$$ID = \pm \frac{C^2}{2},$$

the algebraic sign being the same as $\dot{E}_i$. Such a function for T_i is difficult to generate but can be approximated over a given range by simpler functions.

In the preceding section it was shown that in general a sampled-data servo system has a usable range of T between upper and lower limits determined by the stability and the bandwidth of the system. The simplest way to control T was found to make it a linear function of $|\dot{E}|$ between those limits,

$$T = T_{\max} - A|\dot{E}| \qquad 0 \le |\dot{E}| \le \frac{T_{\max} - T_{\min}}{A},$$

$$T = T_{\min} \qquad |\dot{E}| \ge \frac{T_{\max} - T_{\min}}{A},$$

which can be readily generated on an analog computer.

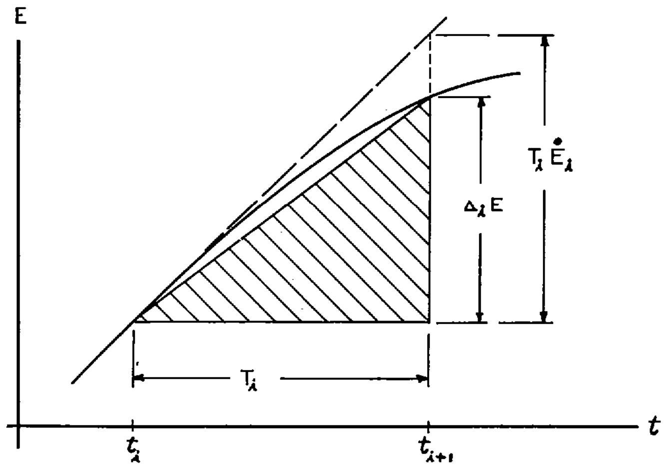


Fig. 3—Input and output of sampler and zero-order hold; expansion of one sample period showing integral difference (ID).

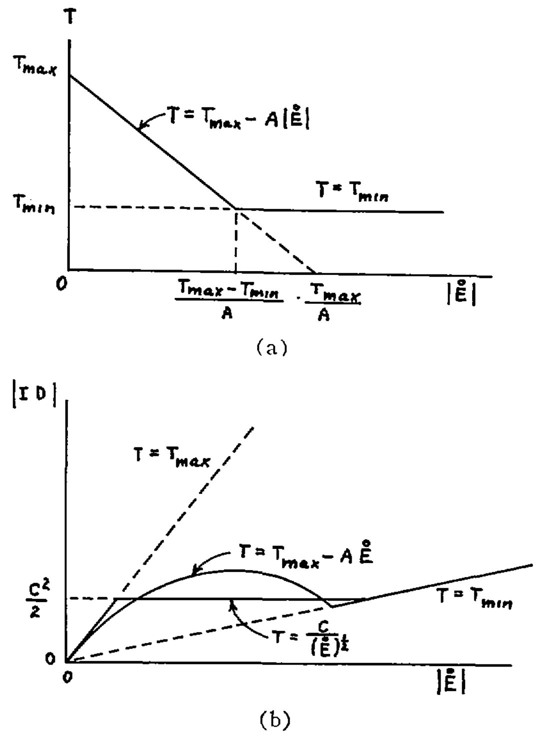


Fig. 4—Variable-sampling-frequency control.
(a) Sample period T vs error derivative $\dot{E}$.
(b) Integral difference per sample period ID vs error derivative $\dot{E}$.

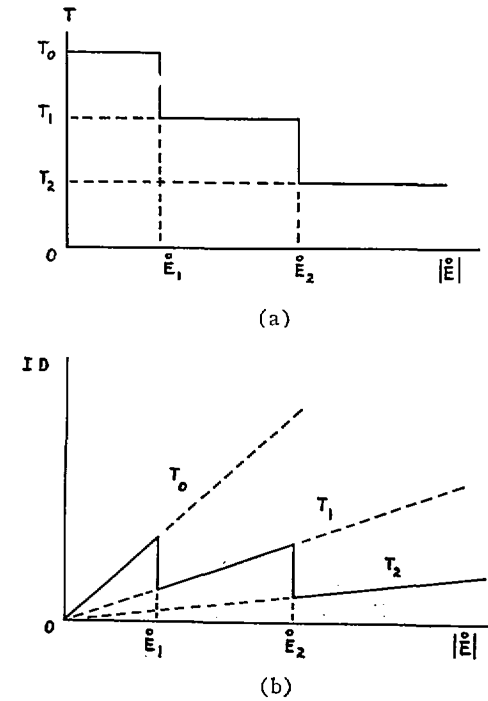


Fig. 5—Discrete-sampling-frequency control.
(a) Sample period T vs error derivative $\dot{E}$.
(b) Integral difference per sample period ID vs error derivative $\dot{E}$.

The function is shown in Fig. 4(a). The ID per sample as a function of $|\dot{E}|$ is sketched in Fig. 4(b). It can be seen that the ID can be held essentially constant over a limited range by using the linear function. In use with the simulated servo system the linear function was found to improve the sampling efficiency as will be described in the next section.

Another method of controlling the sampling frequency is by a number of fixed frequencies which are successively used as $|\dot{E}|$ increases. A system using two discrete frequencies was found to improve efficiency and is also discussed in the next section. Curves of T and ID as functions of $|\dot{E}|$ are shown in Fig. 5. Therefore, two methods of sampling-frequency control have been investigated to improve efficiency. That these methods can be used to decrease the amount of sampling needed to control a servomechanism will be demonstrated next.

III. Servo System with Adaptive Sampling Frequency

The characteristics of two sampled-data systems with fixed-frequency sampling were discussed in Section I. This section presents the characteristics of systems incorporating the variable-frequency sampling discussed in Section II. Fig. 6 is a block diagram of the system with the sampling-frequency controller. The principal method of analysis was an analog computer simulation.

In Section II it was demonstrated that the integral difference between input and output of a sampler and zero-order hold could be controlled by varying the sampling period T. The method used was to make T a function of the first derivative of the input signal. Since the input to the sampler in this servo system is the error signal E,

$$T = T(|\dot{E}|). \tag{1}$$

Since the z-plane transfer function $G(z)$ is a function of T, it becomes a function of $|\dot{E}|$ when a variable-frequency sampler is used. The servo system is then nonlinear. Furthermore, the nonlinearity is an unusually complicated one since the poles and zeros of the transfer function as well as the gain are all functions of $|\dot{E}|$.

Because of the nonlinearity of the system an analog computer simulation was undertaken as being the most direct approach to the problem. With the simulation, all the nonlinearities are accounted for by actually using a variable-frequency sampling system. Thus the record of the computer output is a direct measure of system performance.

Since it was determined that the sampling frequency should increase as a function of the absolute value of the first time-derivative of the error signal, the controller consisted of a differentiator, an absolute-value detector, shaping circuits, and a variable-frequency oscillator. The function generated for control was:

$$\begin{aligned} T &= T_{\max} - A|\dot{E}| \qquad 0 \le |\dot{E}| \le \frac{T_{\max} - T_{\min}}{A} \\ T &= T_{\min} \qquad\qquad\quad |E| \geqq \frac{T_{\max} - T_{\min}}{A}. \end{aligned} \tag{2}$$

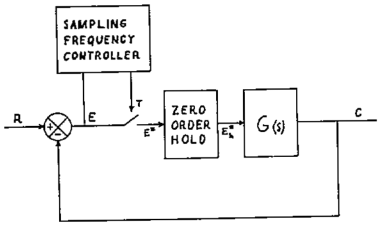


Fig. 6—Block diagram of adaptive-sampling-frequency sampled-data control system.

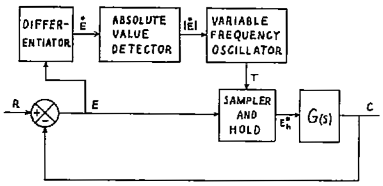


Fig. 7—Block diagram of sampled-data system with sample period controlled by $|\dot{E}|$.

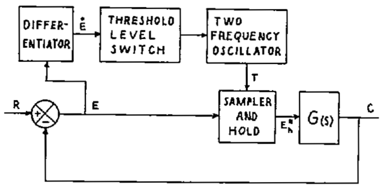


Fig. 8—Block diagram of sampled-data system with discrete frequency control.

Fig. 7 is a block diagram showing these components in the system. Another method of control found to be satisfactory was to use two discrete frequencies instead of continuously variable frequencies. Fig. 8 is a block diagram of the system with a discrete frequency controller which switches frequencies at $|\dot{E}| = |\dot{E}_1|$.

The two systems analyzed for fixed-frequency sampling in Section I had acceptable response characteristics as continuous systems. For each system a pair of sampling frequency limits were found such that, above the upper limit no improvement in response occurred and below the lower limit the system was unstable. In the analog computer study a fixed frequency between these limits was chosen such that system performance was satisfactory and only slightly more oscillatory than the comparable continuous system. Its performance was then used as a standard against which the variable-frequency and multiple-frequency systems were compared. If the variable f_s system could produce similar response with fewer samples per unit time then it was said to be more efficient.

Fig. 9 (pages 44, 45) is a series of recordings of the response of the Type II system to a unit step input. Table II (page 46) is a summary of the response characteristics determined from the recordings. For each variable period run $T_{\max} = 0.1$ sec, $T_{\min} = 0.05$ sec. Examples include runs at constant $T_{\min}$ and $T_{\max}$, bi-frequency runs switching from $T_{\max}$ to $T_{\min}$ at $|\dot{E}_1|$, a run using continuous control between $T_{\min}$ and $T_{\max}$, and three high-frequency runs (i, j, k) for comparison.

Run (g) having $T = T_{\min}$ was used as the standard of performance. In Section I it was shown that the response at $T = 0.05$ sec closely approximates the response of the continuous system. The variable and bi-frequency systems all sample at half the frequency of the standard system when $\dot{E} = 0$, thus sampling is reduced by 50 per cent under quiescent steady-state conditions.

The settling time for response to a step input is approximately 0.5 sec for each type of control. Over the settling time, sampling has been reduced 10 per cent using bi-frequency sampling in run (f) and 20 per cent using variable frequency in run (h). Both these runs demonstrate a reduction in sampling while maintaining essentially the same response characteristics as the standard. By setting $|\dot{E}_1|$ at higher levels, the number of samples over the settling period is reduced at the expense of higher peak overshoots. Specifications and system requirements would dictate which type of control and what sensitivity to use in a particular case. Results with the Type I system were similar over the same type of frequency range.

It can be seen that the variable-frequency control system definitely accomplishes its purpose of more efficient sampling. In general, the continuously-variable control system was more efficient than the two-frequency control but even the latter was a distinct improvement over constant-frequency sampling.

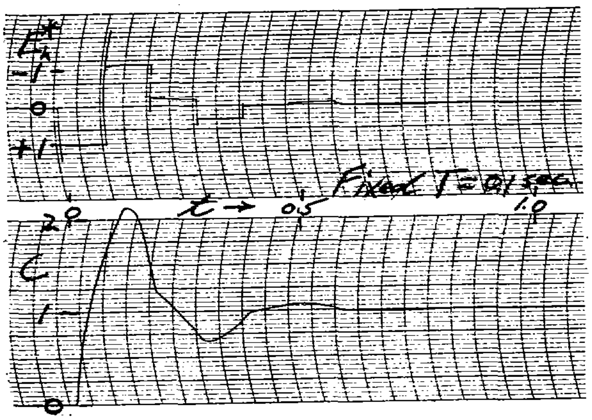
IV. Adaptability

This section will discuss the adaptive qualities of the variable-sampling-frequency system to variations of plant parameters, *i.e.*, open-loop system pole or zero variations. A second consideration will be whether the variable-sampling-frequency system can be programmed to be adaptive to plant parameter variations.

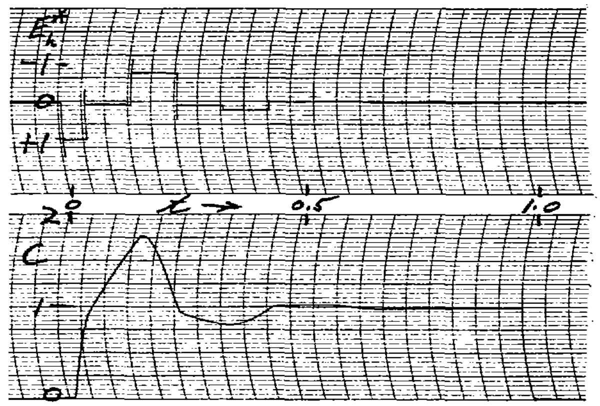
Consider first the adaptivity of the variable-sampling-frequency system to plant parameter variations. It is seen that this system is inherently adaptive to parametric variations in the sense that the variable-sampling frequency will compensate for the parametric variations which increase or decrease the error rate.

The sampling frequency is controlled by a function of the first derivative of the error. In the sense that the varying plant parameter will cause a variation of the error derivative then, the sampling frequency will be adaptive to this parameter variation. That is, if the parameter is varying or varied in such a manner as to reduce the error derivative, then the sampler will sample more slowly, and if the parameter variation increases the error derivative, then the sampler will sample more rapidly.

Consider the situation where it is possible to measure the changes of a varying system parameter, and as a result of this information correspondingly change the value of the open-loop gain or the sampling frequency.



(a)



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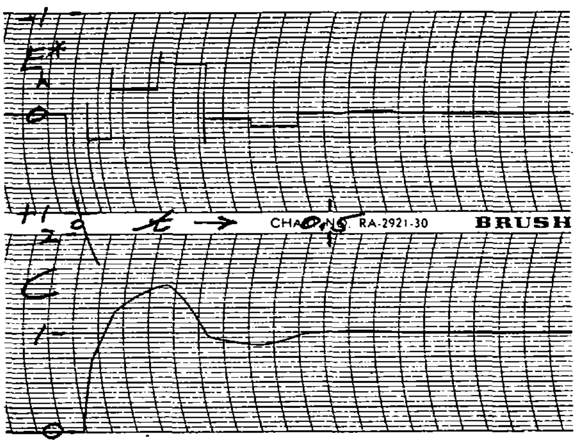
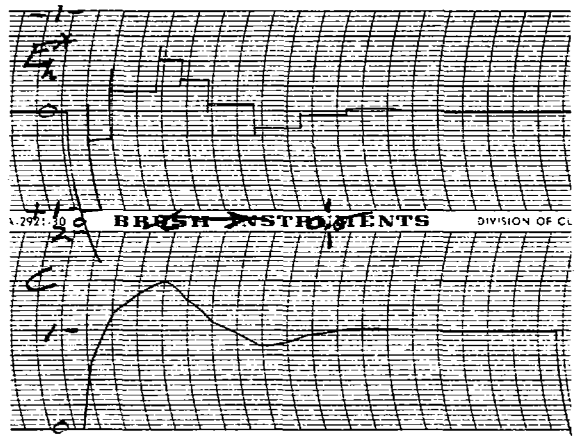
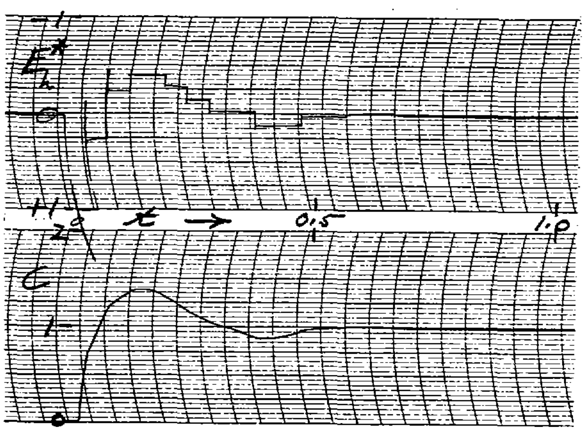


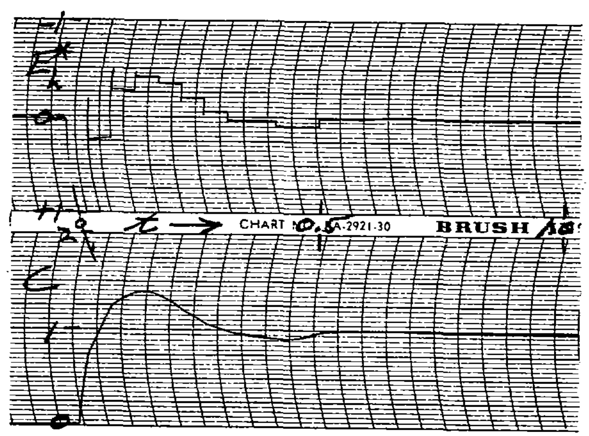
(c)



(d)



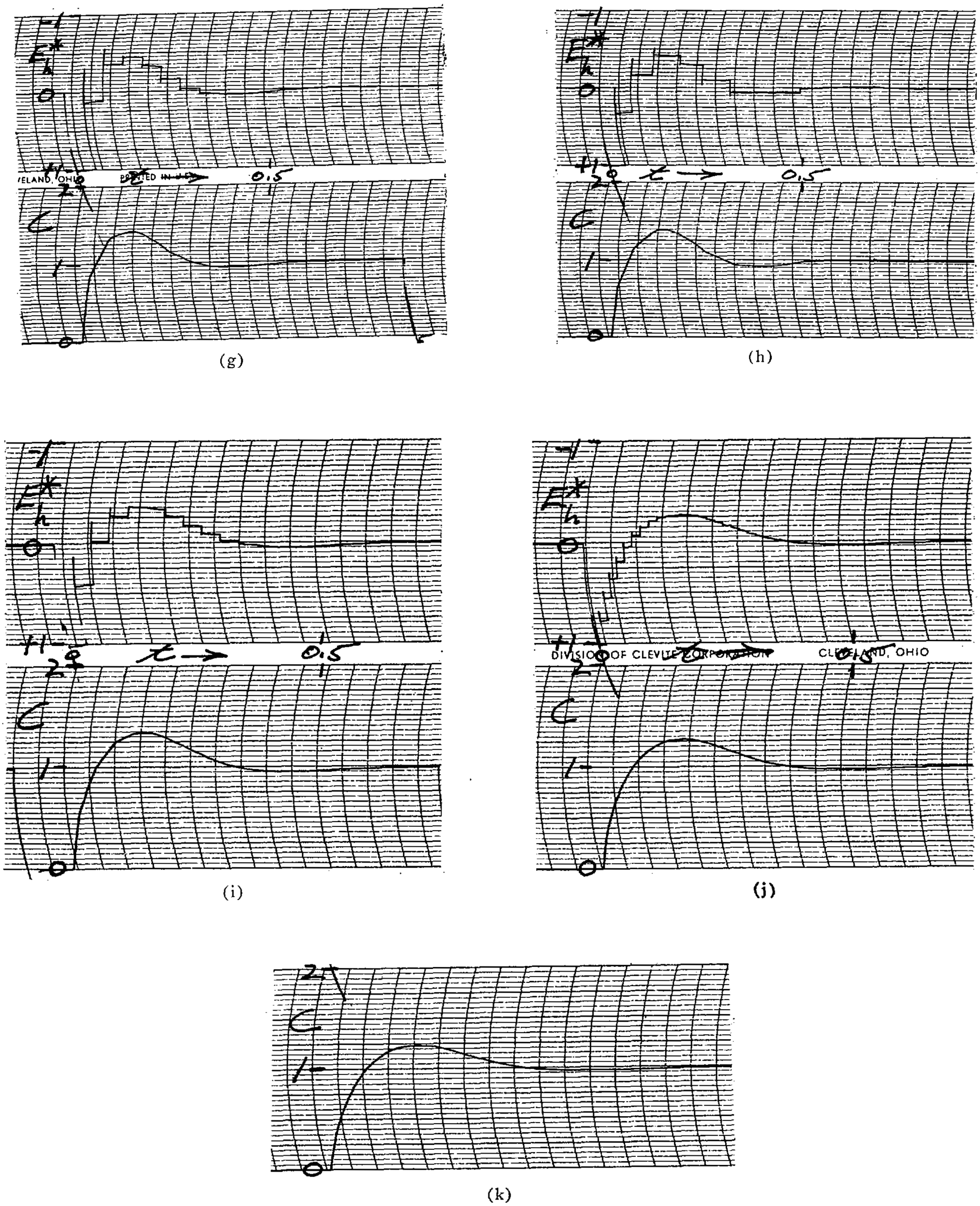
(e)



(f)

(a) $T=0.1$ sec.
(b) $T_0=0.1$ sec; $T_1=0.05$ sec; $|E_1|=20$ rad/sec.
(c) $T_0=0.1$ sec: $T_1=0.05$ sec: $|\dot{E}_1|=10$ rad/sec.
(d) $T_0=0.1$ sec: $T_1=0.05$ sec; $|\dot{E}_1|=5.0$ rad/sec.
(e) $T_0=0.1$ sec; $T_1=0.05$ sec; $|\dot{E}_1|=2.50$ rad,/sed.
(f) $T_0=0.1$ sec; $T_1=0.05$ sec; $|\dot{E}_1|=1.25$ rad/sec.

Fig. 9—Type II system responses to a unit step.



(g) $T = 0.05$ sec.
(h) $T_{\max} = 0.1$ sec; $T_{\min} = 0.05$ sec; $A = 0.005$ [Eq. (2)].
(i) $T = 0.036$ sec.
(j) $T = 0.016$ sec.
(k) $T < 0.005$ sec.

Fig. 9.—(continued)

TABLE II

COMPARATIVE RESPONSES TO ONE RADIAN STEP INPUT (FROM FIG. 9)

Sample Period T	Samples over		Overshoot	Undershoot
	0.5 sec	1.0 sec		
a) $T_0 = T_{\max} = 0.1$ sec	5	10	110 per cent	33 per cent
b) $\lvert\dot{E}_1\rvert = 20$ rad/sec	6	11	75	20
c) $\lvert\dot{E}_1\rvert = 10$ rad/sec	7	12	50	12
d) $\lvert\dot{E}_1\rvert = 5.0$ rad/sec	8	13	50	20
e) $\lvert\dot{E}_1\rvert = 2.5$ rad/sec	8.5	13.5	40	10
f) $\lvert\dot{E}_1\rvert = 1.25$ rad/sec	9	14	40	8
g) $T_1 = T_{\min} = 0.05$ sec	10	20	40	6
h) $T = T_0 - 0.005\lvert\dot{E}\rvert$ sec [Eq. (2)]	8	13	40	8

Specifically, consider the Type II system with a fixed sampling period T, where

$$G(z) = KT\left(1 + \frac{aT}{2}\right)\left[\frac{z + \dfrac{aT - 2}{aT + 2}}{(z - 1)^2}\right].$$

Let the s-plane open-loop zero be changed from a to $c_1 a$ where c_1 is a positive real constant. As a compensation investigation let T be changed from T to T/c_1 and K be changed from K to c_1K. Therefore the products aT and KT are unchanged and the z transform $G(z)$ is exactly the same expression as before the disturbance c_1 was introduced. However the real time output response has been changed. Since the z transform is unchanged, it is seen that the amplitude of the ith sample (where $i = 0, 1, 2, \cdots, n$) is unchanged, however the ith sample now occurs at real time $t = (T)(i)/c_1$ instead of at $t = (T)(i)$ as before. Therefore this method of compensation preserves all relative response amplitudes but changes the time of the response, increasing the speed of response as the zero (or pole of the Type I system) becomes larger in value and decreases the speed of response as the zero (or pole of the Type I system) becomes smaller in value.

If it is desired to maintain exactly the same response characteristics while the open-loop zero value changes, then T and K may be varied in some programmed manner to achieve the desired response, but such an investigation is beyond the scope of this paper.

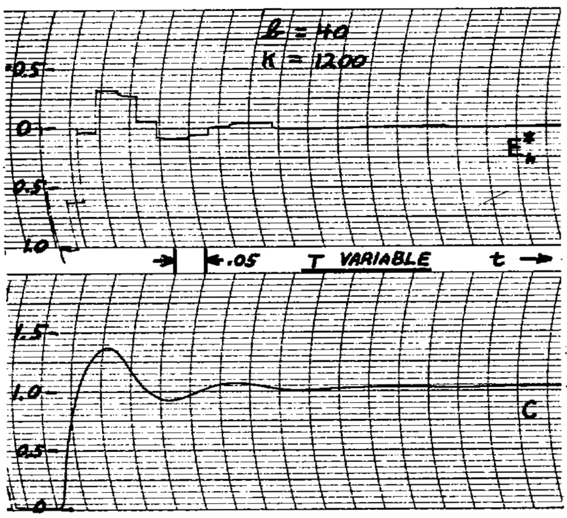
Fig. 10 (opposite) illustrates the effect of introducing a change in the value of the Type I system open-loop pole. Fig. 10(a) shows a desired system response where variable-frequency sampling $0.03 \leq T \leq 0.07$ is employed and system parameters are fixed constants. Fig. 10(b) shows system response degradation as a result of permanently changing the open-loop pole value from 40 to 20, even though a relatively fast $T = 0.024$ fixed sampling frequency is employed. Figs. 10(c) and 10(d) show the system response when the open-loop pole value changes from $b = 20$ to $b = 40$ at time $t = 0.9$ during the step input response. Fig. 10(c) shows the response of the fixed-frequency-sampling system and Fig. 10(d) shows the response of the variable-frequency-sampling system. Comparison of Figs. 10(c) and 10(d) shows that the variable-frequency-sampling output response requires less samples and is more desirable than the fixed-frequency-sampling output response and thus, with respect to fixed-frequency-sampling control, the variable-frequency-sampling control is adaptive to plant parameter changes. Table III lists pertinent characteristics from Figs. 10(c) and 10(d) for comparison.

The variable-frequency sampler is controlled by the error derivative with the sampling period T varying over a given range. The sampler is not aware of the source of the disturbance causing the error derivative value, be it input signal, noise, load variation or system parameter variation; the sampling frequency is determined within its range by the error derivative value and in this sense the variable-frequency-sampling system is adaptive to all error producing disturbances.

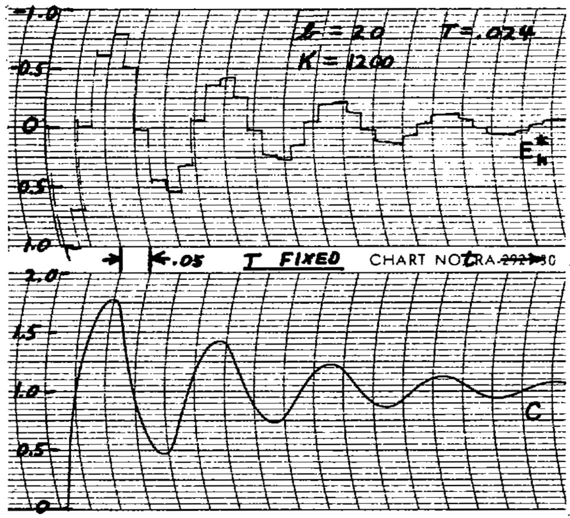
CONCLUSIONS

This paper has described a method of adaptive-sampling-frequency control for sampled-data servomechanisms. In the system described the sampling frequency could be reduced about 20 per cent during the settling time and reduced to about 50 per cent during steady-state conditions from the fixed frequency necessary for the desired transient response. A conservative estimate for the over-all reduction in number of samples required is 25 per cent achieving the desired transient response and steady-state stability for a system which is subject to disturbances often but not continuously.

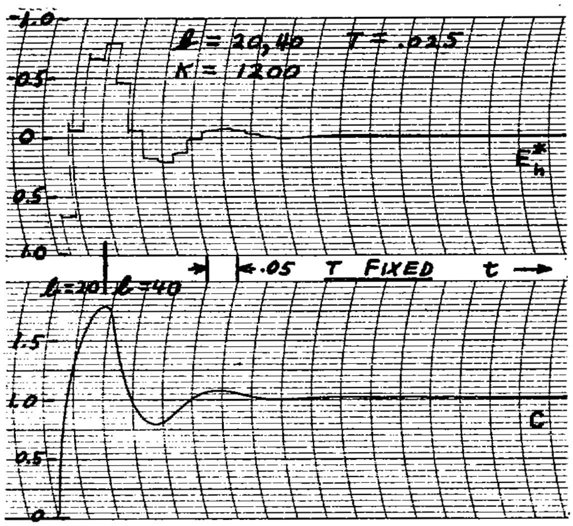
The variable-frequency-sampling control system, with the sampling period varied as a linear function of the absolute value of the first derivative of the error, allows the desired system output response of a fixed-frequency-sampling system while sampling fewer total times, and is adaptive to plant parameter variations or other system disturbances in that response to such disturbances will be lessened because of the nature of the variable-frequency control.



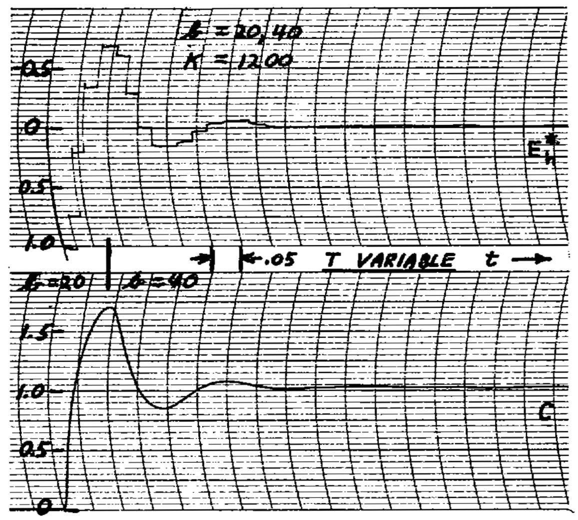
(a)



(b)



(c)



(d)

(a) $b=40$.
(b) $b=20$.
(c) $b=20$, $b=40$. T fixed.
(d) $b=20$, $b=40$. T variable.

Fig. 10—Effect of varying open-loop pole in Type I system. $K=1200$.

TABLE III
COMPARISON OF OUTPUT RESPONSE CHARACTERISTICS

Frequency-Sampling Control	Sample Period T	Samples Over		Overshoot	Undershoot
		0.09 sec	0.40 sec		
Variable	$0.02 \leq T \leq 0.05$	5	15	70 per cent	15 per cent
Fixed	$T=0.025$	4	17	80 per cent	20 per cent