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Capacity optimization of an innovating firm

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ABSTRACT

This article considers an incumbent's product innovation decision within an uncertain framework, where the firm decides whether to continue selling the established product. The model being dynamic allows to analyze the trade-off between an early innovation where the new product only slightly improves the existing one, or innovating late with a much better new product. We find that the effect of uncertainty is that it raises the value of the strategy where the firm keeps on producing the old product after innovating. This results in earlier investment if the firm stays active on the established product market after adopting the new product, and that it keeps on producing the established product for a longer time after the product innovation. Limited uncertainty could lead to a non-monotonicity: with a better new product it is not optimal to innovate, whereas innovating is optimal with a worse one.

1. Introduction

In order to keep demand for its products at a sustainable level, from time to time a firm needs to carry out product innovations. When innovating the firm needs to think about many aspects. First of all it is the timing. Innovating early has the advantage that the firm increases revenue soon. However, on the other hand it takes time to develop a new product of reasonable quality, so if a firm innovates early the improvement will not be too big compared to the quality of the existing products, and the resulting revenue increase will be limited. Innovating late obviously means that for a long time the firm just sells the established product(s), so that there will be no early boost in its revenue level. The upside of innovating late is that due to technological progress the innovation frontier has moved up considerably compared to what the firm is currently producing. This means that the innovative product's quality level is much higher, which in general will result in a considerable increase of the firm's product demand level and therewith, revenue.

A second aspect that a firm needs to take into account when it carries out a product innovation is what to do with the existing product market. In most of the literature models are developed in which product innovation implies that the firm *replaces* the established product by the new one (see, among many others, (Saha, 2007) and (Letina, 2016)). In other words, the firm stops selling its old product at the moment the innovative product is launched. However, what frequently happens in reality is that the new product is *added* to the firm's existing product portfolio. Hence, the firm simultaneously sells the existing and the new product, the obvious benefit being that it collects revenues from both these products. On the other hand starting to sell the new product will cannibalize demand of the existing product. This is the case when products are strategic substitutes. Moreover, continuing to sell the old product also jeopardizes the revenue of the new product. This is because consumers exist that are not willing to pay extra for some new features associated with the innovation. So they prefer to keep on buying the old product. The result is that the firm is "competing with itself", which will reduce the revenue of both the old and the new product. The focus of our article is on a problem that, although of practical importance, did not receive too much attention in the literature. Namely, we focus on the choice between "*add*" and "*replace*", i.e., whenever it carries out its product innovation the firm has to choose between abolishing the old market, or keeping it. Of course, in the latter case the firm still has the option to stop selling the established product after some time.

GM's investment in robotaxis is a recent example of such a decision problem. GM paid $1bn in 2016 for Cruise, an artificial-intelligence

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startup, therewith, adopting the technological know how to produce autonomous vehicles.[1] And the "marriage of cutting-edge technology and large-scale manufacturing seems to be paying off" according to The Economist (January 25th, 2018). GM is now able to mass-produce self-driving cars. GM expects that the costs of ride-hailing services, will fall from $2.50 a mile now to about $1 as the main expense–the driver–is eliminated. On the long term this would mean that car buyers become car users as *drivers are lured from their cars to robotaxis* (The Economist, January 25th, 2018). Therewith, GM's current main business of supplying vehicles to drive will decline, eventually being completely *replaced* by the new business of self-driving cars.

The third aspect of product innovation is how large the production capacity of the new product should be. Such a decision has obvious trade-offs. In general it is not known how consumers will react to new products. Therefore, firms have to take into account that new product demand could be lower than expected. This will obviously be bad for profitability especially if the firm has installed a large new product capacity. However, in case consumers like this new product a lot, revenue will grow sky-high once the firm is able to respond by putting large quantities on the market. Moreover, the decision to keep on selling the existing product or not, will have its own implications for the optimal capacity level.

Many contributions can be found just analyzing the first aspect, i.e. the timing of the investment. This is carried out in the real options literature, see, e.g., (Dixit and Pindyck, 1994) and (Trigeorgis, 1996), and, more related to innovation, in Reinganum (1981) and Farzin et al. (1998) (see also (Doraszelski, 2001)). We also take a real options approach, which is especially suitable because our demand system is subject to stochastic shocks. A contribution that combines the first and the second aspect, so how to time the innovation and deciding on whether to keep on producing the established product or not, is (Hagspiel et al., 2019). Combining the first and the third aspect, i.e. let the firm not only decide on the timing but also on the size of the investment, has been done for the first time in Dangl (1999) and Bar-Ilan and Strange (1996), and later in Huisman and Kort (2015). These contributions and the original real options models have in common that they treat investment projects as being lumpy. A different approach in which capacity/capital stock develops continuously over time and investment is incremental, is followed in, e.g., (Grenadier, 2002) and (Evans and Guthrie, 2012). However, to the best of our knowledge, no contributions exist that combine all three aspects in one framework. This is what the present article does.

We employ a stochastic dynamic model of the firm to analyze the firm's innovation decision in such a setting. Initially the firm is active on an established product market. Over time new technologies arrive at beforehand unknown points in time. The firm has the option to carry out a product innovation. The way the firm innovates is that it is not active in pursuing R&D itself.[2] Instead, what the firm is doing is that it chooses the optimal time to adopt a new technology to produce a corresponding new product. This new product is horizontally and vertically differentiated from the established product. The longer it waits with adopting, the better the new product is that it can produce, implying that the level of vertical differentiation is higher. Innovating goes along with investing in the capacity level of the new product. The firm needs to choose the optimal capacity level, which determines the size of the innovation investment. In addition the firm needs to decide whether it will keep on producing the established product or not. If it decides to continue this production process it still has the option to discontinue this process at some later point in time, after which it will solely be active on the innovative product market. In other words, after the firm has chosen for the add strategy, it needs to determine the optimal time to switch from "add" to "replace", if at all.

Analyzing a framework that simultaneously takes into account the timing of the innovation, the decision whether to *add* or to *replace*, and the size of the capacity level associated with producing the new product, has led to the following results. First, we consider the effect of uncertainty, which is the model input extending the current literature of versioning (see, e.g., (Agi and Yan, 2020) and (Liu et al., 2015)). Surprisingly enough, we get that when the firm pursues an *add* strategy the firm accelerates investing in a more uncertain economic environment. The intuition for our result is that investing according to the add strategy creates the option to replace and the value of this option increases if the economic environment gets more uncertain. A similar result is obtained by Yatsenko and Hritonenko (2017). They study a machine replacement problem and find that cost uncertainty accelerates investment when they consider technological uncertainty. The reason for this result is that in their model cost-saving technological change has smaller future gains due to exponentially decreasing effectiveness.

Since abolishment of producing the established product is an irreversible decision, the switch from an add to a replace strategy takes place later if there is more uncertainty. This all implies that, when the economic environment is more uncertain, the firm is more inclined to keep on producing the established product after it has introduced the new product.

Applying an add strategy implies that the firm's capacity investment associated with the innovative product will be smaller than in the case of the replace strategy. The reason is that when the firm keeps on producing both products, increasing capacity of the new product, and thus selling more of that product, cannibalizes sales of the established product. If the initial size of the new product market is small, a firm will prefer the add above the replace strategy. Then the firm invests less in new product capacity and it will require that the market of the new product has to grow more before the firm will decide to stop selling the established product.

A third major result is that a non-monotonicity in the innovation decision arises when the size of the new product market is stable. In such a case it could happen that when a new product with a higher quality level is available, still it is not optimal for the firm to innovate, while at the same time it is optimal to do so and apply the add strategy when this quality level is lower. This typically happens if it can be expected that a new innovation will arrive soon, so that the quality of the best available new product increases and a replace strategy will be optimal. Since a replace strategy goes along with a larger capacity investment, optimality of this investment requires existence of high demand for the new product, which should thus be of high quality.

If upon innovation the quality, and thus demand, of the new product is high, the firm will carry out a large capacity investment. This implies that the firm will more often choose a replace strategy. Only in the case that the initial size of the new product market is low, it will pursue the add strategy. However, then still the firm will change to the replace strategy relatively soon. In other words it will stop producing the established product as soon as the new product market has reached a larger but still relatively small size.

The remainder of the article is organized as follows. The model setup is introduced in Section 2. Section 3 presents the solution, first deriving the optimal time to switch from add to replace in Section 3.1, then the optimal capacity level of the new product in Section 3.2, and finally the optimal innovation time in Section 3.3. The economic analysis of the results is presented in Section 4. Section 5 concludes. All proofs can be found in the appendix. The appendix also contains a list of the main symbols used along the paper presented in Table 1.

2. Model

We consider a firm that is currently producing an established product

[1] https://www.economist.com/business/2018/01/25/gm-takes-an-unexpected-lead-in-the-race-to-develop-autonomous-vehicles.

[2] A recent study on R&D investments is carried out by Peters et al. (2017). They employ data of German manufacturing firms to analyze the effect of R&D choice, product and process innovations, on future productivity and profits.

with price $p_0 = \xi_0 - \alpha K_0$,

$$p_0 = \xi_0 - \alpha K_0, \tag{1}$$

where K_0 is the installed capacity and $\xi_0 > \alpha K_0$. ξ_0 is the maximum willingness to pay for the established product and $\alpha > 0$ is a constant parameter reflecting the sensitivity of the quantity with respect to the price. Similar to (Marvel et al., 1997), (Anand and Girotra, 2007), (Goyal and Netessine, 2007) and (Huisman and Kort, 2015), we assume that the firm produces up to capacity.[3] The instantaneous profit on the established product market then equals

$$\pi_0 = (\xi_0 - \alpha K_0) K_0. \tag{2}$$

The development of technologies over time is governed by an uncertain process, which is exogenous to the firm. Similar to (Farzin et al., 1998) and (Huisman, 2001), the state of the technological progress is given by a compound Poisson process, $\boldsymbol{\theta} = \{\theta_t, t \geq 0\}$. We may express

$$\theta_t = \theta_0 + u N_t, \tag{3}$$

where θ_0 denotes the state of technology at the initial point in time, $u > 0$ is the jump size and $\{N_t, t \geq 0\}$ follows a homogeneous Poisson process with rate $\lambda > 0$. This implies that new technologies arrive at rate λ, where each arrival increases the technology level by u. Note that the process $\boldsymbol{\theta}$ is non-decreasing over time, reflecting the non-declining nature of technological progress. The aircraft industry is only one example, where ongoing innovations play a crucial role in the decision process. This industry faces a coming wave of technological change, including "engine electrification, artificial intelligence and advanced connectivity that would change how aircraft are developed, manufactured, flown, powered and serviced. It means increased use of new materials (see article) and 3D printing, and greater efforts to reduce greenhouse-gas emissions" (The Economist, April 11th, 2019).[4]

Adopting a new technology allows the firm to introduce a new and more innovative product to the market. To do so it has to incur a sunk cost δK_1, where δ is the unit investment cost and K_1 is the capacity level of the new product. Also for the new product, we assume that the firm produces up to capacity. We denote the time of adoption of the new technology by τ_1. At the moment of adoption the firm also has to decide in how much capacity K_1 to invest in order to produce the innovative product. When adopting new technology, the firm has two options. It can either add the new product to the product portfolio (for a certain time) or abolish the production of the established product and replacing it by the innovative one. In case the firm decides to replace the first product by the new one, the price of the new product is given by the following inverse demand function

$$p_1^R(X_t, \theta_{\tau_1}) = (\theta_{\tau_1} - \alpha K_1) X_t. \tag{4}$$

Note that the price of the new product is positively influenced by the state of technological progress θ. The larger θ is, the more advanced is the new product, and the higher the price the consumers are willing to pay for it. The sensitivity of quantity with respect to price is influenced by the same parameter α. Furthermore, we assume that demand for the new product develops in an uncertain way. Uncertainty is introduced via the stochastic demand shift parameter $\mathbf{X} = \{X_t, t \geq \tau_1\}$ that is assumed to follow a geometric Brownian motion with drift μ and volatility $\sigma > 0$:

$$dX_t = \mu X_t dt + \sigma X_t dz_t, \quad \text{with } X_{\tau_1} = x, \tag{5}$$

in which dz_t is the increment of a Wiener process. The firm knows the value of X at the innovation time τ_1, so that $X_{\tau_1} = x$, where x is a parameter. We impose that $r - \mu > 0$, where $r > 0$ is the (constant) interest rate. This is a standard assumption, that guarantees that the optimal investment time is finite (therewith, excluding trivial cases), as in this case the expected value of the project in perpetuity is finite. The value of the drift μ depends on the scenario under consideration. Our analysis allows for both positive and negative values of the drift parameter. In the numerical example of Section 4 we let μ admit a neutral value in that we fix it to be equal to zero.

We assume the following utility function when the firm is producing both products (the established and the new one),

$$U = \xi_0 K_0 - \frac{1}{2}\alpha K_0^2 - \gamma K_0 K_1 x + \theta K_1 x - \frac{1}{2}\alpha K_1^2 x - p_0 K_0 - p_1 K_1, \tag{6}$$

where γ represents the horizontal differentiation parameter. We assume γ to be positive to reflect that the two products are strategic substitutes.

From this utility function, the following demand system can be derived for the established and the new product:

$$p_0^A(X_t, \theta_{\tau_1}) = \xi_0 - \alpha K_0 - \gamma K_1 X_t, \tag{7}$$

$$p_1^A(X_t, \theta_{\tau_1}) = (\theta_{\tau_1} - \alpha K_1 - \gamma K_0) X_t. \tag{8}$$

The upper bound of γ is given by α, i.e. $\gamma < \alpha$, meaning that it can never be the case that the quantity of the other product has a larger effect on the product price than the quantity of the product itself. In order to make sure that the price of the old market stays positive everywhere, implying that X should not get too large before the firm decides to abolish the established product, we need to impose the additional assumption $r + \mu > \sigma^2$.

After adoption of the new technology at the moment τ_1, demand on the established market also becomes uncertain. This is due to the fact that the variation of the demand of the new product influences the demand of the established one through the cannibalization effect. This is represented by the term $-\gamma K_1 X_t$ in expression (7). Therewith, the instantaneous profit functions are given by

$$\pi_1^R(X_t, \theta_{\tau_1}) = (\theta_{\tau_1} - \alpha K_1) X_t K_1, \quad t \geq \tau_1, \tag{9}$$

for the case that only the new product is produced, and

$$\pi_1^A(X_t, \theta_{\tau_1}) = (\xi_0 - \alpha K_0 - \gamma K_1 X_t) K_0 + (\theta_{\tau_1} - \alpha K_1 - \gamma K_0) X_t K_1, \quad t \geq \tau_1, \tag{10}$$

in case that both products are produced. After having decided to add the innovative product to the established one, the firm will eventually abandon the established product, which happens at the moment that the price of the established product falls too low. We denote the time of abandonment of the established product by τ_2. The possible timelines of the problem are described in Fig. 1. As can been seen in the left graph of Fig. 1 the price of the new product jumps upwards once the firm decides to abolish the old product. This effect is due to the fact that we consider the firm to be the only actor in the market. If consumers can suddenly not buy an old generation of some product anymore, they have to look for an alternative. Therefore, they end up being one of the potential consumers of the new product. As a consequence, the demand and therefore, the price of this new product jump up.

The optimization problem of the firm is then defined as follows:

$$V(x,\theta) = \sup_{\tau_1} \mathbb{E}\left[\int_0^{\tau_1} \pi_0 e^{-rs} ds + e^{-r\tau_1} \max_{K_1} \left\{ \sup_{\tau_2 : \tau_2 \geq \tau_1} \vartheta(X_{\tau_1}, \theta_{\tau_1}) - \delta K_1 \right\} \middle| X_{\tau_1} = x, \theta_0 = \theta \right] \tag{11}$$

with

[3] (Goyal and Netessine, 2007) state that firms may find it difficult to produce below capacity due to fixed costs associated with, for example, production ramp-up, commitments to suppliers and labor. See (Dangl, 1999) and (Hagspiel et al., 2016) for studies that release this assumption.

[4] https://www.economist.com/business/2019/04/11/airbus-risks-losing-its-competitive-thrust.

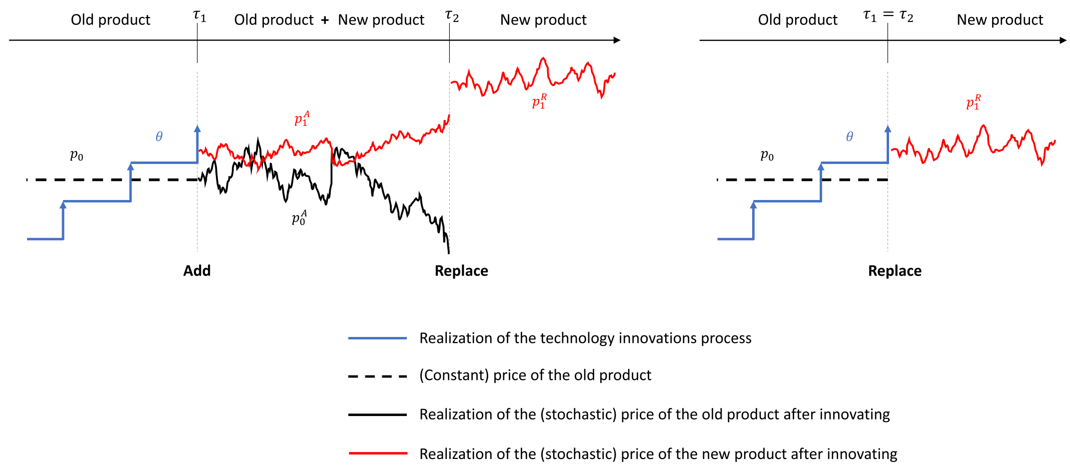

Fig. 1. Possible timelines for the decision problem.

$$\vartheta(X_{\tau_1},\theta_{\tau_1}) = \mathbb{E}\left[\int_{\tau_1}^{\tau_2} \pi_1^A(X_s,\theta_{\tau_1})e^{-r(s-\tau_1)}ds + \int_{\tau_2}^{+\infty} \pi_1^R(X_s,\theta_{\tau_1})e^{-r(s-\tau_1)}ds \middle| X_{\tau_1},\theta_{\tau_1}\right]. \tag{12}$$

Note that the value function depends on the initial state of the technology, $\theta_0 = \theta$. The process $\boldsymbol{\theta}$ is a compound Poisson process (as presented in Equation (3)), and thus increases in its initial level, θ. Since $\boldsymbol{\theta}$ governs both the profitability of innovating and the relative advantage of the new versus the old product, it can be expected that both τ_1 and τ_2 are dependent on θ. Moreover, the value function, besides depending on the current state of the technology, θ, also depends explicitly on $X_{\tau_1} = x$. This assumption is based on the fact that companies in general perform marketing analysis and demand forecasts for new products, which form the basis for deciding the value of the parameter x. After the product is launched the demand will change stochastically based on how it is received in the market (as shown in Equation (5)). Furthermore, note that the second decision, i.e. when the firm abandons the old product and only produces the new product (see Equation (12)), depends on the value of both processes at moment τ_1: X_{τ_1} and θ_{τ_1}. Whereas X_{τ_1} is considered to be constant and equal to x, θ_{τ_1} is a random variable given by $\theta_{\tau_1} = \theta_0 + uN_{\tau_1}$, where N_{τ_1} represents the number of new technologies developed until time τ_1.

Finally, note that τ_1 could be equal to τ_2, which is the case when the firm decides to replace upon innovating. One extreme example of immediate replacement is the fact that luxury companies often destroy unsold inventory in order to avoid cannibalization. In this way they maintain the scarcity of their goods (The Wall Street Journal, September 6, 2018 [5]).

3. Model solution

In order to solve the optimization problem, we first rewrite the optimization problem (11) in a simpler way by applying the strong Markov property and Fubini's theorem,

$$V(x,\theta) = \frac{\pi_0}{r} + \sup_{\tau_1}\mathbb{E}^{\theta_0=\theta}\left[e^{-r\tau_1}\max_{K_1}\rho(x,\theta_{\tau_1},K_1)\right], \tag{13}$$

where we use $E^{\theta_0=\theta}[\ldots]$ to denote the conditional expectation $E[\ldots|\theta_0 = \theta]$ in order to ease notation. The function ρ is defined as follows

$$\rho(x,\theta,K_1) = v(x,\theta,K_1) - \delta K_1 - \frac{\pi_0}{r}, \tag{14}$$

[5] https://www.wsj.com/articles/burning-luxury-goods-goes-out-of-style-at-burberry-1536238351.

where

$$v(x,\theta,K_1) = \sup_{\tau:\tau>0}\mathbb{E}^{X_{\tau_1}=x}\left[\int_0^{\tau} \pi_1^A(X_{\tau_1+s},\theta)e^{-rs}ds + \int_{\tau}^{+\infty} \pi_1^R(X_{\tau_1+s},\theta)e^{-rs}ds\right], \tag{15}$$

with $E^{X_{\tau_1}=x}[\ldots]$ being the conditional expectation $E[\ldots|X_{\tau_1} = x]$.

This representation highlights that we solve the problem in a sequential manner starting backwards. First we solve the optimization problem in (15) for the optimal time to eventually replace the old product after having produced both products for a certain time upon technology adoption. Then we derive for each (x,θ) whether it is optimal to add or replace upon investment and we determine the corresponding capacity level. Finally, we derive the optimal investment thresholds.

In the following subsections we address the three steps in order to solve the problem presented in (13). To shorten notation we will refrain from denoting all functional arguments there where it does not create confusion.

3.1. When to switch from add to replace

We first determine, given the firm has decided to apply the add strategy upon innovation, when to eventually abolish the established product. This means we solve the optimal stopping problem presented in (15). To do so we start out with stating the payoff resulting from the add strategy, which equals

$$v^A(x) = Ax^{\beta_1} + \frac{(\xi_0 - \alpha K_0)K_0}{r} - \frac{\gamma K_1 x K_0}{r-\mu} + \frac{(\theta - \alpha K_1 - \gamma K_0)K_1 x}{r-\mu}, \tag{16}$$

in which

$$A = \frac{1}{\beta_1 - 1}\left(\frac{\beta_1 - 1}{\beta_1}\right)^{\beta_1}\left(\frac{\pi_0}{r}\right)^{1-\beta_1}\left(\frac{2\gamma K_0 K_1}{r-\mu}\right)^{\beta_1}, \tag{17}$$

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \frac{2r}{\sigma^2}}. \tag{18}$$

The payoff of the add strategy can be explained as follows. The first term stands for the value of the option to abandon the established product. The second and the third term are equal to the expected discounted revenue stream of selling the established product. The final term represents the expected discounted revenue stream of the sales of the new product. After abolishing the old product the firm's value is equal to the payoff of the replace strategy given by

$$v^R(x) = \frac{(\theta - \alpha K_1)K_1 x}{r - \mu}. \quad (19)$$

The firm abolishes the established product as soon as the process X hits the value:

$$X^\star = \frac{\beta_1}{(\beta_1 - 1)} \frac{\pi_0}{r} \frac{(r-\mu)}{2\gamma K_0 K_1}. \quad (20)$$

To explain this threshold we rewrite this expression as follows

$$\frac{2\gamma K_0 K_1 X^\star}{(r-\mu)} = \frac{\beta_1}{(\beta_1 - 1)} \frac{\pi_0}{r}. \quad (21)$$

This shows that the decision to abolish producing the established product is in fact equal to taking an irreversible investment with sunk costs $\frac{\pi_0}{r}$ in order to get rid of the cannibalization effect represented by the left side of equation (21). $\frac{\beta_1}{\beta_1 - 1}$ is the familiar term known from real options theory representing the value of waiting. This value of waiting increases with uncertainty indicating that the firm will abolish the established product later when the economic environment is more uncertain. Note that the threshold $X^\star$ is decreasing in K_1, reflecting that the decision to go from the add to the replace strategy is more profitable when the cannibalization effect is large.

3.2. Capacity level of the new product K_1

Here we determine the optimal capacity level for both the add and replace strategy.

It follows from (20) that the cannibalization effect introduces an implicit upper bound for the capacity choice in the add region. Defining

$$K_b = \frac{\beta_1}{(\beta_1 - 1)} \frac{\pi_0}{r} \frac{(r-\mu)}{2\gamma K_0}, \quad (22)$$

we see that capacity larger than $\frac{K_b}{x}$ makes cannibalization too expensive for the add strategy. Therefore, replace is the optimal choice.

So when the firm wants to install a capacity level such that $K_1 > \frac{K_b}{x}$, this can only be optimal when it applies the replace strategy. Note that analogously it can be derived that for capacity levels lower than $\frac{K_b}{x}$ only the add strategy needs to be considered. We conclude that in order to maximize the value of the firm under the two strategies we have to solve

$$\max_{K_1} \rho(K_1) = \max \left\{ \max_{K_1 : 0 < K_1 < \frac{K_b}{x}} \left[v^A(K_1) - \delta K_1 \right], \max_{K_1 : K_1 \geq \frac{K_b}{x}} \left[v^R(K_1) - \delta K_1 \right] \right\} - \frac{\pi_0}{r} \quad (23)$$

This leads to the results presented in Proposition 1.

Proposition 1.
- *Given that it is optimal to add, the capacity level of the new product depends on the value of β_1.*
  - *For parameter values such that $\beta_1 = 2$, the capacity level of the new product is equal to*

$$K_1^A = \frac{\left[\frac{\theta - 2\gamma K_0}{r-\mu} x - \delta\right] \pi_0}{\frac{2x}{r-\mu}\left[\alpha \pi_0 - \frac{r(\gamma K_0)^2 x}{r-\mu}\right]}. \quad (24)$$

  - *For parameter values such that $\beta_1 \neq 2$, K_1^A is implicitly determined by*

$$\frac{2\gamma K_0 x}{r-\mu}\left[\frac{\beta_1 - 1}{\beta_1} \frac{r}{\pi_0} \frac{2\gamma K_0 x}{r-\mu} K_1^A\right]^{\beta_1 - 1} + \frac{\theta - 2\gamma K_0 - 2\alpha K_1^A}{r-\mu} x = \delta. \quad (25)$$

For $1 < \beta_1 < 2$, K_1^A is equal to the largest root of this equation and for $\beta_1 > 2$, it is equal to the smallest one.

- *Given that it is optimal to replace, the capacity level of the new product is equal to*

$$K_1^R = \frac{\theta x - \delta(r-\mu)}{2\alpha x}. \quad (26)$$

As a special case we study the scenario where the demand for the new product is constant, i.e. $\mu = \sigma = 0$. Proposition 2 presents the resulting capacity size of the new product for this case.

Proposition 2. *In the following assume that $\mu = \sigma = 0$.*

- *Given that it is optimal to add, the capacity level of the new product is equal to*

$$K_1^A = \frac{(\theta - 2\gamma K_0)x - \delta r}{2\alpha x}. \quad (27)$$

- *Given that it is optimal to replace, the capacity level of the new product is given by*

$$K_1^R = \frac{\theta x - \delta r}{2\alpha x}. \quad (28)$$

It can be easily verified that these capacity sizes are the result of taking the first order conditions of the value functions given in equations (16) and (19). *Note that for this case the trend μ is equal to zero and the option to replace after having added is worthless. Since the demand of the new product is constant over time, it holds that if the add strategy is optimal upon innovation it is always optimal.*

By now it is clear that the choice of the capacity level for the new product depends on whether the firm will keep on producing the established product. Therefore, the last part of this section is devoted to the choice between the add and the replace strategy. The following proposition gives an expression for the boundary between the add and the replace region in the (θ, x) plane.

Proposition 3. *Given that it is optimal for the firm to innovate, the indifference curve, θ_b, between the regions where it is optimal to add or to replace is obtained when the two terms of the maximization in* (23) *are equal. The indifference curve is given by*

$$\theta_b(x) = \frac{\delta(r-\mu) + 2\alpha K_b}{x}. \quad (29)$$

For obvious reasons, we call this curve the add/replace boundary. Note that this boundary (29) *is a decreasing function of x. This makes sense, because a larger market for the new product makes it more profitable to just produce the new product and abolish the established one. In the following proposition we study how θ_b depends on several parameters.*

Proposition 4. *The add/replace boundary, θ_b, defined in (29), decreases with γ, K_0 and μ; increases with δ, ξ_0 and σ; and does not depend on λ and u. Regarding α and r it holds that.*

- *for $\alpha_1 < \alpha_2$, if $\alpha_1 + \alpha_2 < \frac{\xi_0}{K_0}$ then $\theta_b(x; \alpha_1) < \theta_b(x; \alpha_2)$, otherwise $\theta_b(x; \alpha_1) > \theta_b(x; \alpha_2)$;*
- *for $r_1 < r_2$, if $\delta(r_2 - r_1) + \frac{\alpha \pi_0}{\gamma K_0}\left[\frac{(r_2-\mu)\beta_1(r_2)}{r_2(\beta_1(r_2)-1)} - \frac{(r_1-\mu)\beta_1(r_1)}{r_1(\beta_1(r_1)-1)}\right] > 0$ then $\theta_b(x; r_1) < \theta_b(x; r_2)$, otherwise $\theta_b(x; r_1) > \theta_b(x; r_2)$.*

The results of this proposition can be nicely economically interpreted. The degree of horizontal differentiation is large when the parameter γ is small. Therefore, the established and the innovative product are especially competing if γ is large, which results in a large cannibalization effect. This makes it less profitable to produce both products at the same time. Therefore, the replace strategy will be more attractive for a higher value of γ, which explains that the add/replace boundary decreases with γ. If the firm produces a lot of the established product, i.e. K_0 is large, this reduces the output price of the new product a lot. This explains that the add strategy is less profitable in such a situation. If demand for the new product is expected to grow faster over time, i.e. μ increases, the new product market is more profitable making it more attractive for the firm to be solely active on this market and stop with producing the established product. Hence, the add/replace boundary decreases with. μ.

If the unit investment cost δ is large, the firm invests less in capacity of the new product. This reduces the cannibalization effect, making it more attractive to also produce the established product. This explains that the add/replace boundary is increasing with δ. If ξ_0 increases, the established product becomes more profitable. Therefore, the firm prefers producing it and thus the add/replace boundary is increasing with this parameter. An increase of the uncertainty parameter σ causes an increase of the value of the option to abandon the established product. Therefore, the firm wants to acquire this option, which makes the add strategy more attractive, and the add/replace boundary increases with σ.

The parameters λ and u govern the speed of technological progress. This is not relevant anymore in the stopping region where the firm has already innovated, which explains why these parameters do not influence the add/replace boundary.

The effect of α is more involved because this parameter covers both the negative effect of the quantity of the established and the new product on their respective output prices. So, on the one hand an increased level of α reduces the profitability of the established product. On the other hand is also the new product less profitable, and, moreover, α also has an effect on the chosen capacity size K_1.

Similarly, increasing the discount rate r has multiple effects. First, since it reduces the net present value of an investment, the firm will invest less in capacity for the new product. This in turn reduces the cannibalization, and therefore it is more profitable to simultaneously produce the established and the new product.On the other hand, an increased discount rate decreases the value of the option to abandon the established product, which diminishes the attractiveness of the add strategy.

3.3. When to innovate

The decision of when to innovate depends on the level of θ. The larger θ the more profitable the current innovation is. It follows that it is optimal to innovate once θ is large enough, i.e. when it passes some threshold level θ^*. The next proposition presents this threshold level given that the firm applies the replace strategy upon innovating. For the add strategy only a numerical approximation is available, for which we present the algorithm in Appendix A.5.

Proposition 5. *When the optimal decision is to replace immediately, the threshold, $\theta_R^\star$, is given by*

$$\theta_R^*(x) = \frac{\delta(r-\mu)}{x} + \frac{u\lambda}{r} + \sqrt{\left(\frac{u}{r}\right)^2 \lambda(\lambda+r) + 4\alpha(r-\mu)\frac{\pi_0}{rx}}. \tag{30}$$

4. Economic analysis

This section is organized as follows. First, we consider the case where demand of the innovative product is constant. Here we compare our results to the findings of Hagspiel et al. (2019) who analyze the problem without capacity optimization. We then proceed by analyzing the general case where demand of the innovative product is stochastic.

4.1. Constant demand innovative product

A typical solution of the problem with constant demand of the innovative product is presented in Fig. 3. Note that in this case, before the innovation is undertaken the movement in the (θ, x)-plane takes place vertically with upward jumps. After the innovation the movement stops and constant demand means that there is also no horizontal movement. Fig. 3 shows that it is optimal to innovate when the technology level θ is sufficiently large. We also see that next to the innovative product the firm keeps on producing the established product if the innovative product market is small, i.e. x is small. If the technology level is larger, the firm wants to invest in a larger capacity level for the innovative product. Then the replace strategy becomes more profitable compared to the add strategy. This is because it is optimal for the firm to invest in a larger capacity level for the innovative product for the replace strategy. For the add strategy the drawback of installing a larger capacity is that the innovative product cannibalizes a larger part of the revenue of the established product. This makes the add strategy less profitable compared to replace. Fig. 3 confirms this by illustrating that the add/replace boundary is decreasing in the (x, θ)-plane. Note that if the firm does not optimize capacity this effect would not appear. Therefore, as already shown by Hagspiel et al. (2019), the boundary between the add and replace region would be vertical as illustrated in Fig. 2.

An interesting feature that occurs in Fig. 3 is the inaction region that arises between the add and replace region. This inaction region implies a non-monotonicity in the innovation decision in the sense that when a new product with a higher quality level is available it is not optimal to innovate whereas it is optimal to do so applying the add strategy when this level is lower. The intuition is that innovating in the replace region goes along with a larger capacity investment. So, in this inaction region the potential quality of the new product is still not good enough to justify a replace-innovation decision. Of course, it is always optimal to replace once the θ is sufficiently large. Such an inaction region never occurs when capacity investment size is assumed to be fixed as can be seen in

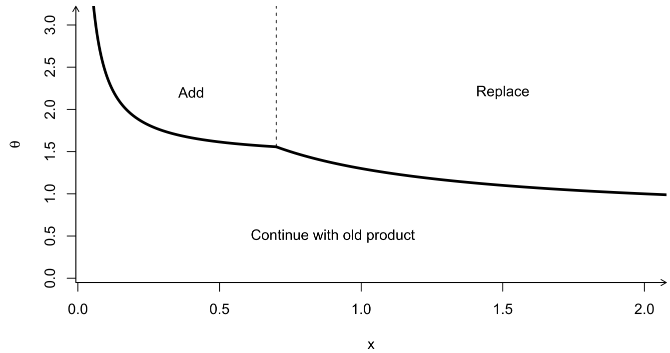

Fig. 2. Strategy state space diagram where the demand of the innovative product is constant without capacity optimization. [Parameter values: $\xi_0 = 1$, $\alpha = 0.07, \gamma = 0.05, \delta = 1, \mu = \sigma = 0, r = 0.1, \lambda = 0.1, u = 0.2$ and $K_0 = K_1 = \xi_0/2\alpha \approx 7.14$.].

Fig. 3. Strategy state space diagram where the demand of the innovative product is constant. [Parameter values: $\xi_0 = 1, \alpha = 0.07, \gamma = 0.05, \delta = 1, \mu = \sigma = 0, r = 0.1, \lambda = 0.1, u = 0.2$ and $K_0 = \xi_0/2\alpha \approx 7.14$.].

Fig. 2 (see (Hagspiel et al., 2019)).

4.2. Stochastic demand innovative product

We now consider the general case that the demand of the innovative product is stochastic. In the (θ, x)-plane this implies that after the innovation there are continuous horizontal movements. Introducing uncertainty related to the demand level of the new product implies that at some point it can be optimal for the firm to switch to the replace strategy after having originally decided to produce both products at the same time. The firm in fact receives the option to replace upon investing in the add region. It will exercise this option once the demand level for the innovative product is sufficiently large. The reason is that also the cannibalization effect grows with x, making the add strategy less profitable. This replace option value increases with uncertainty. Therefore, the incentive to invest in the add strategy goes up. In Fig. 4 this is reflected by a lower boundary for the add region for larger σ.

The add region is also expanding on the right side of the (x,θ)-plane as illustrated in Fig. 4. Since the value of adding increases in σ (due to the added option value), and the value of replacing does not change, the relative attractiveness of adding versus replacing improves.

As can be seen in Fig. 4, this means that the boundary separating the two stopping regions moves to the right. So, essentially what we can conclude from Fig. 4 is that the add region expands with uncertainty. In fact, the following proposition formally shows that the value of applying the add strategy increases with the uncertainty parameter σ.

Proposition 6. *The value of the firm when the optimal strategy is to add increases with uncertainty, i.e.*

$$\max_{K_1:0<K_1<\frac{K_b}{x}} \left[v^A(K_1) - \delta K_1 \right] - \frac{\pi_0}{r}$$

increases with σ. The explanation rests on the fact that option values increase with uncertainty. This implies that in a more uncertain economic environment the firm keeps an option alive for a longer time, or is more eager to acquire an option. The first explains why after innovating the firm prefers to keep on producing the established product when there is a lot of uncertainty. In this way it still holds the valuable option to abandon this product at some time in the future. The second explains that when applying the add strategy uncertainty accelerates investment: by investing the firm acquires the option to switch from add to replace at some point in the future.

These two effects of changing the uncertainty (σ) in the demand shift parameter x are consistent with the real options literature (see (Dixit and Pindyck, 1994)). The underlying reasons may be interpreted most readily in the context of two different scenarios the firm may find itself in:

1. The decision maker has chosen to add (i.e. producing both the old and new product simultaneously) after having acquired the capacity K_1 for the new product. At every point in time, the decision maker needs to choose between continuing like this or to scrap/abandon the old product (producing only the new product). In Fig. 4 the firm enters the add-region at point a for either the case when $\sigma = 0.2$ or $\sigma = 0.5$. As expected, and illustrated by points A_1 and A_2, the decision maker will abandon the old product later (i.e. for a higher x-value) when the uncertainty is larger.
2. Only the old product is being produced and at every point in time the firm needs to choose between continuing or innovating; if innovating is chosen, then it should be either the add or the replace strategy. Increasing the uncertainty (σ) in the demand shift parameter x does not change the value of the replace strategy or the dynamics in the continuation region (since only the development of the technology is uncertain at that point), but it does unambiguously increase the value of choosing the add strategy as explained above and more formally in Proposition 6. As illustrated in Fig. 4, increasing σ therefore results in a replace boundary that is unchanged (where this is still the optimal strategy) and boundaries for the add region that expands in all directions, also at the expense of the replace region.

It is important to note that the boundary separating the add and replace strategies inside of the stopping region is not an exercise boundary. This boundary only indicates which is the better strategy to choose when entering the stopping region. The optimal exercise threshold for replacing after one has first chosen the add strategy depends on which initial point (x,θ), and at what capacity K_1, the add-decision was first made, as can be seen from expressions (20) and (25). Both for higher levels of x and θ a larger capacity will be chosen for the add strategy. In both cases this leads to a lower replacement threshold X^*. This effect is illustrated in Fig. 5 for three different starting points a, b and c, in the add region.

Comparing points a and c shows an example of entering the stopping region at two different values of θ for the same initial demand level of the new market. Indeed for the larger θ the capacity investment is larger. In the case of point a, the optimal capacity is equal to $K_1 = 4.1$ and for point c, equal to $K_1 = 6.5$. Therefore, the firm replaces earlier due to the larger cannibalization effect. This can be seen comparing points A and C, where C is located more to the left so that x has a lower value there. The same can be observed comparing points a and b. A larger initial demand level of the new market (i.e. larger x) for the same new product quality leads to a larger capacity investment. Specifically, for point b, $K_1 = 5.8$. Therefore, for this case earlier exercise of the replace strategy is optimal. The latter can be inferred from the fact that point B is associated with a smaller x than point A.

A major difference between the constant demand solution illustrated

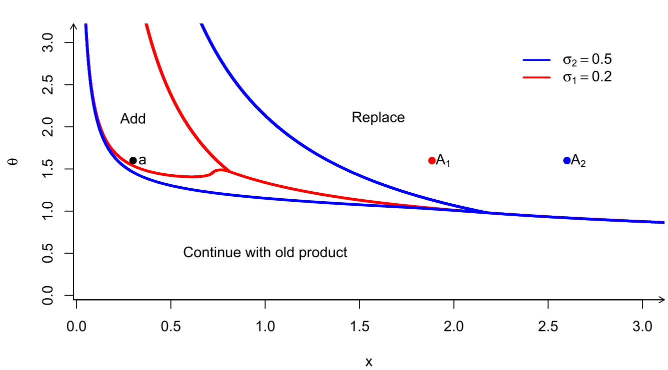

Fig. 4. Strategy state space diagram for different levels of uncertainty σ. [Parameter values: $\xi_0 = 1, \alpha = 0.07, \gamma = 0.05, \delta = 1, \mu = 0, r = 0.1, \lambda = 0.1, u = 0.2$ and $K_0 = \xi_0/2\alpha \approx 7.14$.].

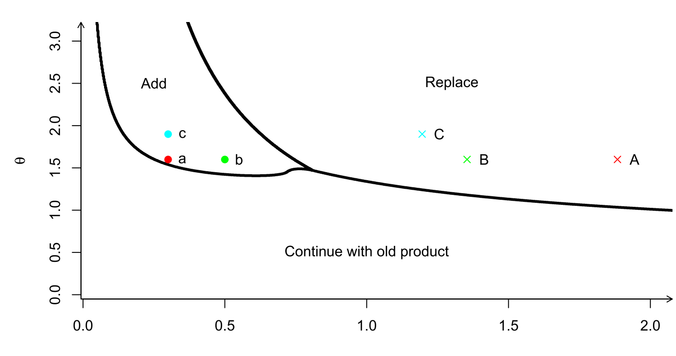

Fig. 5. Strategy state space diagram indicating optimal add and replace decisions for different levels of x and θ. [Parameter values: $\xi_0 = 1, \alpha = 0.07, \gamma = 0.05, \delta = 1, \mu = 0, \sigma = 0.2, r = 0.1, \lambda = 0.1, u = 0.2$ and $K_0 = \xi_0/2\alpha \approx 7.14$.].

in Fig. 3 and the stochastic demand solution, is that in the latter case no inaction region arises in the sense that for a relatively low θ and a relatively high θ the firm innovates, while it waits for intermediate values of θ. The reason is that due to the uncertainty the value of *X* is fluctuating. Therefore, it can be expected that if a firm adopts an add strategy while being close to replace in fact the replace region will be entered soon. The firm will anticipate this change to a replace strategy by choosing a larger capacity size. If σ is sufficiently small then an inaction region can again occur as in the constant demand case.

5. Conclusion

This article considers a firm being active on an established product market, which has an option to carry out a product innovation. In exercising this option there is a value of waiting, because innovating at a later point in time makes that the new product is of better quality due to technological progress. The obvious trade-off is that at the same time there is an opportunity cost of waiting in that, as long as the firm does not innovate, it misses the profit generated by being active on an innovative product market. This article solves this timing problem by determining a threshold value for the state of the technological progress indicating that it is optimal for the firm to innovate once this threshold is reached.

The article also focuses on the problem of what the firm should do with its established product after it has innovated. Essentially there are two possibilities in that, *first*, the firm stops activities on the established product market at the moment that it launches the new product, implying that the firm *replaces* the established product by the new one. *Second,* after it has innovated, the firm keeps on producing its established product so that it *adds* the new product to its existing product portfolio. In the latter case it still has the option to go for the *replace* strategy, thus abolishing the established product, once the innovative product market has grown sufficiently. We find that, compared to the *replace* option, the *add* strategy goes along with a lower capacity investment, because sales of the new product take away demand from the established product and vice versa. One of our main results is that for higher levels of uncertainty in the innovative product market, the firm invests sooner given it adopts the *add* strategy.

One simplifying assumption in our current model is that the established product capacity is given. An interesting extension could be to look at the problem of how the option of a product innovation could influence capacity investments in the current product. Related to this, one could investigate whether the model may be extended to consider a (potentially perpetual) sequence of capacity and add-or-replace decisions for an arbitrary number of new products. Another feature we did not take into account is competition. So, another interesting extension would be to analyze our innovation model in an incumbent-entrant framework or a duopoly. An idea here is to combine the current model with the duopoly analyzed in Huisman and Kort (2015).

Acknowledgements

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A. Appendix

A.1. List of symbols

Table 1
List of relevant notation used in the article.

Processes	$\mathbf{X} = \{X_t, t \geq \tau_1\}$	demand at time t
	$X_t \in \mathbb{R}^+$	
	$\boldsymbol{\theta} = \{\theta_t, t \geq 0\}$	state of the technology at time t
	$\theta_t = \theta_0 + uN_t,\ \ \theta_0, u > 0$	
	$\{N_t, t \geq 0\}$	number of new technologies arrived until time t
	$N_t \in \mathbb{N}$	
Parameters	$x,\ \ x > 0$	initial demand for the new product
	$K_0,\ \ K_0 > 0$	installed capacity of the established product
	$\alpha,\ \ \alpha > 0$	sensitivity of the quantity w.r.t. price
	$\xi_0,\ \ \xi_0 > \alpha K_0$	maximum willingness to pay for the established product
	$\gamma,\ \ 0 < \gamma < \alpha$	horizontal differentiation
	$\delta,\ \ \delta > 0$	unitary investment cost
	$\lambda,\ \ \lambda > 0$	intensity of arrival of new technologies
	$u,\ \ u > 0$	jump size of arrival of new technologies
	$\mu,\ \ \mu \in \mathbb{R}$	drift of the demand
	$\sigma,\ \ \sigma > 0$	volatility of the demand
	$r,\ \ r > \mu$	interest rate
Products	p_0	price of the established product
	p_1^R	price of the new product, in case of replace strategy
	$p_0^A \cdot (p_1^A)$	price of the established (new) product, in case of add strategy
Optimal quantities	X^*	price at which the established product stops being produced
	K_b	upper bound for the capacity, in case the add strategy is chosen
	$K_1^A \cdot (K_1^R)$	optimal capacity of the innovative product in add (replace) strategy
	$\theta^\star(x)$	level of technology where the firm innovates
	$\theta_b(x)$	indifference curve between add or replace

A.2. Proof of Proposition 1

The proof of (26) - the replace case - is trivial, and therefore we just show the proofs for the add case. For this case, we need to maximize the following function w.r.t. K_1:

$$f(K_1) = A(K_1)x^{\beta_1} + \frac{(\theta - \alpha K_1 - 2\gamma K_0)K_1 x}{r - \mu} - \delta K_1, \tag{31}$$

with $K_1 > 0$, where A is given by (17). Simple calculations lead to the following expression for the first order derivative of f,

$$f^{'}(K_1) = \frac{2\gamma K_0}{r-\mu}\left(\frac{K_1}{K_b}\right)^{\beta_1 - 1} x^{\beta_1} + \frac{(\theta - 2\alpha K_1 - 2\gamma K_0)x}{r-\mu} - \delta,$$

where K_b is defined in (22). Therefore the zeros of its first order derivative are solution of equation (25) .[6] In case $\beta_1 = 2$ then $f^{'}$ is an increasing linear function, and thus the minimum of f may be computed explicitly, being K_1^A given by (24). In case $\beta_1 \neq 2$, then in general we are not able to find the zeros of $f^{'}$ explicitly. Nevertheless, the second order derivative of f is given by

$$f''(K_1) = \frac{2\gamma K_0(\beta_1 - 1)}{(r-\mu)K_b} x^{\beta_1}\left(\frac{K_1}{K_b}\right)^{\beta_1 - 2} - \frac{2\alpha x}{r-\mu}.$$

As f'' has a unique zero, it follows that this is the unique inflection point of f. Then, given that $f(0) = 0$,

- in case $1 < \beta_1 < 2$, the function is convex until that inflection point, and afterwards is concave. Thus, the maximizer of f w.r.t. K_1 must be the larger solution of (25).
- in case $\beta > 2$, then the function is concave until the inflection point, and afterwards is convex. Thus, the maximizer of f w.r.t. K_1 must be the smallest solution of (25).

A.3. Proof of Proposition 2

In this case, the price after investment in the new product is also known, and equal to x. Moreover, upon investment in the new price, the decision to replace or to add does not change with time, as x is fixed. Indeed, regarding equation (11), either $\tau_1 = \tau_2$, and therefore the firm replaces the old product by the new one as soon as it invests, or $\tau_2 = \infty$, and therefore it produces both forever (see (Hagspiel et al., 2019) for more details). Then the maximization problem boils down into the following maximization problems:

- in case the firm produces both products forever, its return is

$$\int_0^{+\infty} \pi_1^A(x,\theta)e^{-rs}ds - \delta K_1 = \frac{\pi_1^A(x,\theta)}{r} - \delta K_1$$

- Maximizing it w.r.t. K_1 gives the expression provided in equation (27).
- in case the firm produces only the new product forever, its return is:

$$\int_0^{+\infty} \pi_1^R(x,\theta)e^{-rs}ds - \delta K_1 = \frac{\pi_1^R(x,\theta)}{r} - \delta K_1$$

- Therefore, maximizing it w.r.t. K_1 gives precisely (28).

A.4. Proof of Proposition 3

For (x,θ) in the indifference curve, K_1^A and K_1^R are equal. Thus, K_1^R should verify Equation (25) for these pairs. Then, plugging (26) into (25), one obtains the expression for θ_b presented in (29).

A.5. Proof of Proposition 4

We want to investigate how θ_b, defined in (29), changes with the different parameters. It is easy to see that $\theta_b(x) = \frac{\delta(r-\mu)+2\alpha K_b}{x}$ does not depend on λ and u, increases with δ, and has the same monotony as K_b (defined in (22)) for σ, γ, ξ_0 and K_0. It remains to study the μ, α and r cases.

Note that θ_b, as a function of μ, can be written as $\frac{1}{x}\left[\delta(r-\mu) + \frac{\alpha\pi_0}{r\gamma K_0}(r-\mu)\frac{\beta_1(\mu)}{\beta_1(\mu)-1}\right]$. Then, we need to study $\varrho_1(\mu) = (r-\mu)\frac{\beta_1(\mu)}{\beta_1(\mu)-1}$. Note that $\varrho_1{}^{'}(\mu) = -\frac{\beta_1(\mu)[\beta_1(\mu)-1]+(r-\mu)\frac{\partial\beta_1(\mu)}{\partial\mu}}{[\beta_1(\mu)-1]^2}$, which can be simplified as $\varrho_1{}^{'}(\mu) = -\frac{\beta_1(\mu)}{\sigma^2\nabla}[\sigma^2\nabla(\beta_1(\mu)-1)-(r-\mu)]$, where $\nabla = \sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$. After some calculations we end up with $\varrho_1{}^{'}(\mu) = -\frac{\beta_1(\mu)}{\nabla}\left[\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2 + \frac{r+\mu}{\sigma^2} - \left(\frac{1}{2}+\frac{\mu}{\sigma^2}\right)\nabla\right]$, to which we can apply the conjugate and after some comprehensive calculations, we get $\varrho_1{}^{'}(\mu) = -\frac{\beta_1(\mu)\left(\frac{r-\mu}{\sigma^2}\right)^2}{\nabla\left[\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2 + \frac{r+\mu}{\sigma^2} + \left(\frac{1}{2}+\frac{\mu}{\sigma^2}\right)\nabla\right]} < 0$. Given that $(r-\mu)\frac{\beta_1(\mu)}{\beta_1(\mu)-1}$ is a decreasing function of μ, we conclude that θ_b decreases with μ too.

Regarding r case, note that θ_b can be written as $\frac{1}{x}(r-\mu)\left[\delta + \frac{\alpha\pi_0}{\gamma K_0}\frac{\beta_1(r)}{r[\beta_1(r)-1]}\right]$. Firstly, we need to study $\varrho_2(r) = \frac{r-\mu}{r}\frac{\beta_1(r)}{\beta_1(r)-1}$. Note that $\varrho_2{}^{'}(r) =$

[6] In this proof we are considering the case where $f^{'}$ has two zeros. The cases where $f^{'}$ has no zeros or only one zero can be considered, leading to much more complex derivations. The complete analysis can be found in Appendix 5.C of Pimentel (2018).

$\frac{\mu\beta_1(r)(\beta_1(r)-1)-r(r-\mu)\frac{\partial\beta_1(r)}{\partial r}}{[r(\beta_1(r)-1)]^2}$, which is equivalent to $\varrho_2{}'(r) = \frac{2\mu\nabla\left[r-\mu\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)-\mu\nabla\right]-r(r-\mu)}{\nabla[r\sigma(\beta_1(r)-1)]^2}$. Applying again the conjugate, we obtain

$$\varrho_2{}'(r) = -\frac{(r-\mu)^2}{\nabla[\sigma(\beta_1(r)-1)]^2\left[2\mu\nabla\left[r-\mu\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)\right]+\left[2\mu^2\left[\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}\right]+r(r-\mu)\right]\right]} < 0.$$

Considering the function $\varrho_3(r) = (r-\mu)\left[\delta + \frac{\alpha\pi_0}{\gamma K_0}\frac{\beta_1(r)}{r[\beta_1(r))}\right]$, we have[7]

$$\varrho_3(r_2) - \varrho_3(r_1) = \delta(r_2 - r_1) + \frac{\alpha\pi_0}{\gamma K_0}\left[\frac{(r_2-\mu)\beta_1(r_2)}{r_2(\beta_1(r_2)-1)} - \frac{(r_1-\mu)\beta_1(r_1)}{r_1(\beta_1(r_1)-1)}\right] \tag{32}$$

$$= \delta(r_2 - r_1) + \frac{\alpha\pi_0}{\gamma K_0}[\varrho_2(r_2) - \varrho_2(r_1)]. \tag{33}$$

Then, if $\delta(r_2 - r_1) + \frac{\alpha\pi_0}{\gamma K_0}\left[\frac{(r_2-\mu)\beta_1(r_2)}{r_2(\beta_1(r_2)-1)} - \frac{(r_1-\mu)\beta_1(r_1)}{r_1(\beta_1(r_1)-1)}\right] > 0$ the θ_b increases and decreases otherwise.

To investigate the α case, we need to study the function $\varrho_4(\alpha) = \alpha(\xi_0 - \alpha K_0)$ for $0 < \alpha < \frac{\xi_0}{K_0}$. Given that ϱ_4 is a concave function with zeros on 0 and $\frac{\xi_0}{K_0}$, then it is not monotonic. Let us consider $\alpha_1 < \alpha_2$. Then $\varrho_4(\alpha_2) - \varrho_4(\alpha_1) > 0 \Leftrightarrow (\alpha_2 - \alpha_1)[\xi_0 - (\alpha_1 + \alpha_2)K_0]\rangle 0 \Leftrightarrow \alpha_1 + \alpha_2 < \frac{\xi_0}{K_0}$. Consequently, if $\alpha_1 + \alpha_2 < \frac{\xi_0}{K_0}$ the θ_b increases with α and decreases otherwise.

A.6. Proof of Proposition 5

Given that here we are interested on studying the functions regarding to θ, we write the dependency on x as a subscript.

In order to prove the result, we use Theorem 1, Section 3.3. of Hagspiel et al. (2019). In view of such result, we need to consider the function

$$h_x(\theta) = (r+\lambda)\psi^R(x,\theta) - \lambda\psi^R(x,\theta+u), \tag{34}$$

with $\psi^R_x(\theta) = \rho_x(\theta, K^R_1) = v^R_x(\theta, K^R_1) - \delta K^R_1 - \frac{\pi_0}{r}$, where ρ and v^R are defined in (14) and (19), respectively, and K^R_1 is given by (26). We need to prove that the function h, as a function of θ, has only one zero, and it is positive after this zero. At the same time we prove that such a point is precisely $\theta^*_R(x)$ given by (30).

After some calculations, we get

$$h_x(\theta) = \frac{r(\theta x - \delta(r-\mu))^2 - 2u\lambda x(\theta x - \delta(r-\mu)) - \left[(2\gamma K_0)^2 rx + \lambda(ux)^2\right]}{4\alpha(r-\mu)x}, \tag{35}$$

which is a convex quadratic function and its minimum is attained at $\frac{\delta(r-\mu)}{x} + \frac{u\lambda}{r}$. Doing some more calculations we can prove that $\theta^\star_R$ is a zero of h. Moreover, $\theta^\star_R(x) > \frac{\delta(r-\mu)}{x} + \frac{u\lambda}{r}$, which implies that $h_x(\theta) > 0, \forall\, \theta > \theta^\star_R(x)$, which proves the proposition.

■

A.7. Proof of Proposition 6

For each pair (x,θ), we want to prove that, if $\sigma_1 < \sigma_2$ then

$$\max_{K_1(\sigma_1):0<K_1(\sigma_1)<\frac{K_b(\sigma_1)}{x}}\left[v^A(x,\theta,K_1(\sigma_1);\sigma_1) - \delta K_1(\sigma_1)\right] \le \max_{K_1(\sigma_2):0<K_1(\sigma_2)<\frac{K_b(\sigma_2)}{x}}\left[v^A(x,\theta,K_1(\sigma_2);\sigma_2) - \delta K_1(\sigma_2)\right].$$

As a first step, we will prove that, for a fixed pair (x,θ), and a fixed K_1, such that $K_1 < \frac{K_b(\sigma)}{x}$, v^A increases with σ. We can observe from Expression (16) that only the first term of v^A depends on σ. Let us define the function $g(\sigma) = Ax^{\beta_1(\sigma)}$. Taking into account the definition of A given by (17), we can rewrite g as

$$g(\sigma) = \frac{\pi_0}{r}\frac{1}{\beta_1(\sigma)-1}\left[\frac{K_1 x}{K_b(\sigma)}\right]^{\beta_1}.$$

The first derivative of g, after some calculations, can be written as

$$g'(\sigma) = \frac{\pi_0}{r}\frac{1}{\beta_1(\sigma)-1}\left[\frac{K_1 x}{K_b(\sigma)}\right]^{\beta_1}\left[-\frac{\beta_1{}'(\sigma)}{\beta_1(\sigma)-1} - \frac{\beta_1(\sigma)K_b{}'(\sigma)}{K_b} + \ln\left[\frac{K_1 x}{K_b(\sigma)}\right]\beta_1{}'(\sigma)\right].$$

Considering the definition of K_b in (22), it is straightforward to show that $K_b{}'(\sigma) = -\frac{\beta_1(\sigma)}{\beta_1(\sigma)-1}$. Then,

$$g'(\sigma) = \frac{\pi_0}{r}\frac{1}{\beta_1(\sigma)-1}\left[\frac{K_1 x}{K_b(\sigma)}\right]^{\beta_1}\ln\left[\frac{K_1 x}{K_b(\sigma)}\right]\beta_1{}'(\sigma).$$

[7] Note that $\varrho_2(r_2) - \varrho_2(r_1) < 0$ because ϱ_2 decreases with r.

Given that $K_1 < \frac{K_b(\sigma)}{x}$, then $\ln\left[\frac{K_1 x}{K_b(\sigma)}\right] < 0$. Moreover, it is simple to show that $\beta_1{}'(\sigma) < 0$. These two results together, and also the fact that $\beta_1(\sigma) > 1$, imply that $g'(\sigma) > 0$. Thus, v^A increases with σ.

Let us denote $K_1^\star(\sigma)$ the optimal capacity when the volatility is σ. Let us consider a pair (x, θ) and $\sigma_1 < \sigma_2$. By definition

$$\max_{K_1(\sigma_1):0<K_1(\sigma_1)<\frac{K_b(\sigma_1)}{x}} \left[v^A(x,\theta,K_1(\sigma_1);\sigma_1) - \delta K_1(\sigma_1)\right] = v^A\left(x,\theta,K_1^\star(\sigma_1);\sigma_1\right) - \delta K_1^\star(\sigma_1).$$

Given that v^A increases with σ, we can say that[8]

$$v^A\left(x,\theta,K_1^\star(\sigma_1);\sigma_1\right) - \delta K_1^\star(\sigma_1) \le v^A\left(x,\theta,K_1^\star(\sigma_1);\sigma_2\right) - \delta K_1^\star(\sigma_1).$$

However, the optimal capacity when the volatility is σ_2 is $K_1^\star(\sigma_2)$, thus any other capacity is sub-optimal, i.e.

$$\begin{aligned} v^A\left(x,\theta,K_1^\star(\sigma_1);\sigma_2\right) - \delta K_1^\star(\sigma_1) &\le v^A\left(x,\theta,K_1^\star(\sigma_2);\sigma_2\right) - \delta K_1^\star(\sigma_2) \\ &= \max_{K_1(\sigma_2):0<K_1(\sigma_2)<\frac{K_b(\sigma_2)}{x}} \left[v^A(x,\theta,K_1(\sigma_2);\sigma_2) - \delta K_1(\sigma_2)\right], \end{aligned}$$

which concludes the proof.

■

A.8. Algorithm for the numerical approximation

In the following we state the pseudo-code for the numerical approximation.

Algorithm 1 Numerical approximation

```
% Create equispaced vectors for x and theta, with spacing "gridSpacing" (jump-size needs to be divisible by the spacing-
  size for theta) and vector-length of M
    xVector = vector(start = xStart, end = xEnd, spacing = gridSpacing, length = M)
    thetaVector = vector(start = thetaStart, end = thetaEnd, spacing = gridSpacing, length = M)
  % Create matrix/grid for the total asset value based on the possible x- and theta-values
    assetValue = matrix(rows = M, columns = M)
  % Determine how many indices a jump (of size u) equates to in the theta-vector
    indexJumpSize = u/gridSpacing
  % Outer-loop; loop through each possible x-value
    For (i = 1 to M){
      % Inner-loop 1; loop through the theta-values which are less than one jump away from the largest theta-value,
  starting with the largest value: thetaVector[M]. Assume that one must either invest (adding or replacing) or the value is
  set to 0
      For (j = M to (M-indexJumpSize+1)){
        % Solve equation (19) numerically to find optimal capacity if adding and plug into equation (20) to get the
  optimal capacity if replacing.
        stoppingValue = max(addValue, replaceValue) -π0/r % As in equation (17)
        assetValue[i,j] = max(stoppingValue,0)
      }
      % Inner-loop 2; loop through the theta-values which are more than (or equal to) one jump away from the largest
  theta-value, starting with the largest value: thetaVector[M-indexJumpSize]. The maximum value is chosen between
  either investing (adding or replacing) or waiting (the discounted value one jump away)
      For (j = (M - indexJumpSize) to 1){
        % Solve equation (19) numerically to find optimal capacity if adding and plug into equation (20) to get the
  optimal capacity if replacing.
        stoppingValue = max(addValue, replaceValue) -π0/r % As in equation (17)
        waitingValue = λ/(λ + r) assetValue[i, j + indexJumpSize]
        assetValue[i,j] = max(stoppingValue, waitingValue)
      }
    }
```

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[8] Note that $K_1^\star(\sigma_1) < \frac{K_b(\sigma_2)}{x}$ because K_b increases with σ.

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