Master's thesis

NTNU
Norwegian University of Science and Technology
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NTNU Business School

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Applying GARCH-EVT-Copula Forecasting in Active Portfolio Management

Master's thesis in Economics and Business Administration
Supervisor: Becker, Denis

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Preface

We would like to thank our supervisor, Denis Becker, for his guidance in writing this thesis. His advice and critical insight has been crucial for our research. Lastly, we would thank our fellow students in office 3044 at NTNU Business School for some rewarding discussions. It has been a pleasure.

Disclaimer

We hereby declare that the following research is our own original work. The content in this thesis is at the authors' own expense.

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Abstract

This thesis uses historical out-of-sample backtesting to evaluate the performance of short-term portfolio selection strategies, based on the inputs supplied by 20 different GARCH-EVT-Copula simulation models. Evaluations are made with respect to three allocation objectives: maximum Sharpe, minimum variance and minimum CVaR, and allocations based on historical inputs under each objective are used as benchmarks. The strategies are backtested over the period from Aug. 1st, 2001 to Dec. 31st, 2020, and the portfolio is based on the Dow 30 index composition, as of 2021. Our main finding is that the performance of these models appears to be time-variant, and dependent on which type of allocation problem is being solved. Under minimum CVaR, the exact choice of simulation model seem less important; while under minimum variance and maximum Sharpe, these choices appear more important.

Sammendrag

Denne masteroppgaven benytter out-of-sample backesting for å prestasjonsevaluere ulike kortsiktige porteføljeoptimeringsstrategier som bygger på 20 forskjellige GARCH-EVT-Copula simulerings- modeller. Vi løser tre ulike optimeringsproblemer for hver modell: maksimum Sharpe, minimum varians og minimum CVaR, og bruker tilsvarende optimeringsproblem for historiske data som sammenligningsgrunnlag. Strategiene testes i tidstommet 1. Aug. 2001 til 31. Des. 2020, og den aktuelle portføljen er basert på sammensetningen til Dow 30-indeksen. Hovedfunnet vårt er at prestasjonene til disse strategiene ser ut til å variere for ulike tidsrom, og avhenge av hvilket type optimeringsproblem som løses. For minimum CVaR synes valget av konkret simuleringsmodell å være mindre viktig, mens for maksimum Sharpe og minimum varians ser dette ut til å være av større betydning.

Abbreviations

Acronyms

ARIMA	Autoregressive Integrated Moving Average
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
ARCH	Autoregressive Conditional Heteroskedasticity
EVT	Extreme Value Theory
EVM	Extreme Value Mixture
POT	Peek-over-threshold
GPD	Generalized Pareto Distribution
GEC	GARCH-EVT-Copula
MLE	Maximum Likelihood Estimation
MPLE	Maximum Pseudo Likelihood Estimation
VaR	Value-at-Risk
CVaR	Conditional Value-at-Risk
MV	Mean-Variance

Chapter 1

Introduction

A fundamental problems faced by investors is how to select the optimal portfolio allocation. The introduction of Markowitz (1952)'s mean-variance framework marks the beginning of decades of tireless research on the subject, and a world of different models have been proposed, under new assumptions, constraints and objectives. As their input parameters, these types of problems generally require users to state expectations about the future behavior of asset returns. For the purpose of input prediction, we take a closer look at the GARCH-EVT-Copula models described by Wang et al. (2010). This multi-component simulation methodology takes advantage of several key econometric innovations, and allows for many alternative specifications. A few have already been examined in previous studies, and many have not. We specify a handful of these models, and try to determine which is better in context of the Dow 30 equity portfolio.

1.1 Background and Research Question

For simplicity, Markowitz (1952) used the means and covariances of historical returns as input estimates for his allocation problem. He never claimed this was appropriate, but neither was forecasting the objective of his paper. Using out-of-sample backtesting, DeMiguel et al. (2009) evaluated the historical performance for a range of different MV models under this approach. Somewhat disappointingly, they concluded that none were able to consistently outperform the naive (1/N) portfolio in terms of realized Sharpe ratio. Following the decision of the Basel Committee to impose mandatory VaR evaluations in banking risk management, the research into risk management and forecasting has received more attention over the past couple of decades. In a recent literature review into new approaches, Milhomem and Dantas (2020) concluded that most progress can be made from devoting more attention to the task of input estimation.

The GARCH-EVT-Copula approach represent a different strategy for supplying input estimates for the allocation problem. These simulation models can be seen as multi-asset extensions of the single-asset GARCH-EVT models suggested by McNeil and Frey (2000). The GARCH-EVT approach applies GARCH filtering and EVT-tail modelling to simultaneously account for volatility clustering and heavy tails in financial return distributions. Wang et al. (2010) proposed the implementation of copulas as a measure of association between multiple GARCH-EVT models. They minimize the CVaR of a Chinese currency portfolio, using the Standard-GARCH(1,1) and three different copulas: Normal, Clayton and Student t. In later studies on these models, out-of-sample backtesting is introduced as a means of performance evaluation. It is suggested that allocations based on the historical inputs be used as a benchmark of comparison. Huang and Hsu (2015) minimize CVaR for a global equity portfolio, using similar simulation models as above, less the Clayton copula. They find evidence to suggest that the simulation models outperforms the benchmark in terms of Sharpe ratio for (very) short balancing intervals. A similar study was performed by Sahamkhadam et al. (2018), who found evidence suggesting that the use of these simulation models helped decreased portfolio risk, also in the very shorter term.

The objective of our thesis is to reveal new evidence regarding the optimal choice of GARCH-EVT-Copula model. Taking inspiration from the works above, we specify a handful of models to be evaluated against the historical input-strategy. Our 20 combinations are based on based on five different copulas, and four different GARCH models. We use the Standard-GARCH(1,1) and GJR-GARCH(1,1)[1], and specify them using both a constant and time-variant ARIMA(1,0,1) mean equation. Our five copulas are the Normal, Student t, Clayton, Gumbel and Frank. Our research question is to determine which of these model combinations performs better.

1.2 Outline of Thesis

In this chapter, we have tried to provide some brief context for our project. In the next chapter, we will go deeper into the literature, and thus try to further motivate our choices of model simulation models and allocation objective(s). The methodology chapter is two-fold: sections (3.1), (3.4) and (3.6) concerns the research design, analysis and data applied, whereas sections (3.2) and (3.3) concerns the simulation and portfolio allocation methodology. Then follows our results and subsequent discussion, and also the conclusion of our thesis.

[1]Proposed by Bollerslev (1986) and Glosten et al. (1993), respectively.

Chapter 2

Literature

2.1 Portfolio Selection Objectives

When solving a portfolio selection problem, we derive the optimal asset allocation in relation to some objective. These are typically risk or performance measures, and so it follows that the development of different allocation problems is closely related to the development of these metrics. Markowitz (1952) built his classical problem of optimal allocation on the assumption that investors had *mean-variance preferences*, and sought to allocate such as to maximize the ratio of return-to-volatility. He assumed multivariate normality of asset returns, which implies that their joint behavior can be described by their means and covariances. The weighted sums of these parameters amounts to the overall portfolio expected return and variance, and allocations optimal in relation to the mean-variance criterion are located along the *efficient frontier*. On the same distributional assumptions, Sharpe (1963) introduced his *reward-to-variability* ratio. Using this ratio as the allocation objective, one can reduce the efficient set to a single most optimal portfolio. This is often referred to as the *tangency portfolio*, and it has important theoretical implications, as it connects the MV model to the Capital Asset Pricing Model (CAPM). This relationship is closely described in the concluding chapter of Alexander (2008*a*). Another common objective under the same distributional assumptions is the minimization of variance. This portfolio is located at the lower end-point of the efficient frontier.

In the 1960's, asymmetric risk metrics like *semivariance* were first introduced. In his review of this early literature, Nawrocki (1999) emphasize two main reasons for this development. First, researchers came to realize that investors were often more concerned with avoiding large losses than pursuing large gains, suggesting that the symmetric criteria described above were less representative of investors' actual preferences. Second, the validity of the normality assumption started getting questioned in empirical research. Among the earliest evidence was Mandelbrot (1963) and Fama (1965), and specific problems related to normality is discussed in section (2.3). According to the literature review, semivariance metrics have proved useful in the case of skewed distributions. Other notable developments include the Lower Partial Moment (LPM), and performance metrics based on downside risk metrics have been proposed.

The Value-at-Risk (VaR) have received much attention in later years, following its mandatory implementation to banking risk management by the Basel Committee in 1995. VaR originated at JP Morgan in the 1980's, and is used for *tail loss* evaluation. The term refers to the maximum (expected) loss to occur at given probability, within a given period. Rockafellar and Uryasev (2000) proposed the closely related Conditional Value-at-Risk (CVaR), often referred to as the *expected tail loss*. CVaR is considered more informative, as it analyzes how big a loss is expected to occur by the chance that VaR is exceeded. Furthermore, CVaR is generally preferred to VaR for portfolio selection purposes (see Uryasev (2000) and Pflug (2000)), as it has more desirable mathematical properties in relation to the coherency requirements suggested by Artzner et al. (1999). A good overview of asymmetric risk metrics is found in Alexander (2008*c*).

2.2 Further Developments

It goes without saying that the MV model has been widely influential to the overall development of modern financial theory, and Markowitz received the Nobel Price of 1992 for his contributions. One the other hand, the model has always been treated with a great deal of caution due to its many limitations (see for example Michaud (1989)). Despite the early concerns related to the normality assumption, research addressing more secondary concerns related to the MV model has continued, which has brought a few interesting perspectives. In a recent survey, Zhang et al. (2018) discuss how the MV literature has evolved into multiple distinct branches. One such branch is concerned with how to reshape the problem so that more practical investment-related issues are accounted for, like transaction costs and trading rules. Another branch discusses how to implement the MV framework in a *dynamic* context, meaning how to accommodate investors' desire to reallocate as market conditions changes. The original model is *static*, meaning its allocations are constant and based on information available pre-investment, only. It can be undesirable to hold these allocations over long periods of time, as they are unlikely to remain truly optimal as time passes. The simplest strategy for solving this problem, is to divide the total investment horizon into multiple sub-periods and treat each sub-period as a separate (static) MV problem. This strategy is referred to as *discrete* rebalancing, and there are also more sophisticated methodologies that deal with the problem of *continuous* rebalancing.

A third branch of the literature addresses the *robustness* of the model. This is understood as the sensibility of allocations to changes in input parameters, and multiple authors have raised concerns related to the low robustness of the mean parameters in particular (see Merton (1980), Michaud (1989) and others). Along these lines, Chopra, Vijay K. and Ziemba (1993) even suggest that under most practical circumstances, it would be most feasible to abandon the mean-variance objective in favor of the minimum variance objective, just to avoid the problem of erroneous mean estimates. In conclusion, a variety of different MV models have been proposed, and even greater the paradox is the general lack of empirical evidence of these models actually performing well as real investment strategies (DeMiguel et al. (2009)).

2.3 The Non-Normality of Asset Returns

The rise of GARCH-EVT-Copula and related methodologies, can be seen as an attempt to battle the many undesirable statistical properties observed in financial return distributions. It is very convenient to handle asset returns under assumptions of normality, but loads of empirical evidence to suggest it is not very realistic. citecont2001empirical summarized decades of empirical research on the matter into eleven "stylized statistical properties of asset returns". As a starter, there is the widespread occurrence of *volatility clustering* (or *heteroskedasticity*), meaning that volatility levels do not remain constant over time. The tendency of a negative correlation of volatility and return has also been described, and is often referred to as *asymmetric* volatility. The *leverage effect* (Black (1976)) and *volatility-feedback effect* (French et al. (1987)) are the two leading theoretical explanations of this phenomenon, and more recent evidence for equity markets include the findings of Bekaert and Wu (2000). Volatility clustering imply that the return generating process is not i.i.d. (*independently and identically distributed*), and this is highly problematic to the validity of distribution parameter estimates. Second, there is the common observation of *leptokurtic* return distributions. Returns they to be "sharp peaked and heavy-tailed", and these higher moments represent a problem in relation to the ability of common distribution models to fit the data accurately. Cont (2001) described the heavy tails as being caused by extreme market events, and that they tend to persist even after making corrections for the effects of volatility clustering. A third property is that of gain/loss asymmetries, more specifically that asset return interdependencies tend to be stronger in the case of market downturns than upturns. This phenomenon can be referred to as *asymmetric* interdependence, and imply that covariances and linear correlations can give poor descriptions of the true co-movements of asset returns.

2.4 The GARCH-EVT Approach

In relation to the second problem discussed above, multiple authors have suggested the use of Extreme Value Theory (EVT) for financial modelling, with some early works including those of Embrechts et al. (1997) and Nystrom and Skoglund (2002). EVT is a family of statistical methodologies, used for describing the extreme observations of a sample, with perhaps the most common application being the Generalized Pareto Distribution (GDP). Using EVT, the second problem becomes connected to the first, since these are parametric methods and thereby subject to the i.i.d. requirement. As a solution to the problem, McNeil and Frey (2000) suggested the use of GARCH *filtering* to prepare data for EVT-modelling. The filtering procedure can be understood as the reduction of total variability into white noise only, by the removal of heteroskedasticity. The filtered observations are referred to as the *standardized residuals*, and these should be perfect for parametric methods, provided the GARCH is well fitted. The ability of this approach to improve VaR forecasting accuracy has been confirmed by several later studies (see Byström (2004) and Fernandez (2005) and others), and a similar approach was independently proposed by Barone-Adesi et al. (1999, 2002) for non-parametric VaR forecasting.

The Generalized Autoregressive Conditional Heteroskedasticity model was proposed by Bollerslev (1986), and is a generalization of Engle (1982)'s previously introduced ARCH model. Contrary to the traditional assumption of constant volatility, GARCH assume volatility is time-variant and conditional on past information, more specifically the lagged variances and residuals. This has become a popular tool for volatility modelling, and wide family of similar models have since emerged from the same concept. These new models are designed to capture other characteristics of volatility behavior, and a review of the early literature on ARCH/GARCH models can be found in Bollerslev et al. (1992). Bollerslev et al. (2008) provide a more general overview of the field, with related terminology and credits to the many co-contributors. Notable developments include the the introduction of *non-linear* GARCH models, designed to also deal with asymmetric volatility clustering. This include the three early models proposed by Nelson (1991), Engle and Ng (1993) and Glosten et al. (1993). There is a strong case for the general ability of ARCH/GARCH type models to capture volatility behavior (see Bollerslev et al. (1992) and references), but still appears to be no clear consensus to what is the optimal choice of model. Several comparative studies have found that no single model remains consistently superior over time, indicating that the optimal choice is likely to be period-specific, see Brailsford and Faff (1996) and Loudon et al. (2000). As noted in the discussion of Awartani and Corradi (2005), there are also many conflicting findings in relation to the ability of non-linear GARCH models to outperform the linear ones.

In all four studies on GARCH filtering named above, Standard-GARCH(1,1) is used. We find this to be the common choice, including studies more similar to our own (see Wang et al. (2010), Huang and Hsu (2015) and Sahamkhadam et al. (2018)). Based on the wide variety of GARCH models available, we are interested to see whether non-linear models are more suitable for filtering than the linear ones. In addition, Barone-Adesi et al. (2002) suggest the idea of using ARIMA-GARCH models, which has the potential to correct the data of possible *autocorrelation*.

These are also adopted in some of the models tested by Sahamkhadam et al. (2018) with promising results, despite the fact that autocorrelation is not considered a widespread problem in financial data according to Cont (2001)'s stylized properties.

2.5 The Interdependence of Returns

According to Alexander (2008*b*), linear correlations and covariances can only model a certain type of risk. This is relevant with respect to the third problem of financial return distributions mentioned above, and Embrechts et al. (2002) were among the early to argue the versatility of copulas, among alternative metric of interdependence. A copula can be understood as a function describing the dependency structure among a set of distributions, and the fundamentals were provided already by Sklar (1959). Several families of copulas have been described, with the Elliptical and Archimedean copulas being the most commonly applied in finance. As the GARCH-EVT models are only for single-asset simulations, some metric of dependency is necessary to extend the methodology to the multi-asset perspective. From what we have been able to find, Wang et al. (2010) were the first to propose the use of copulas for this purpose. In their application of the GARCH-EVT-Copula models, Huang and Hsu (2015) test both the Normal and Student-t copulas, and optimize their equity index portfolio under the minimum CVaR objective. They find evidence to suggest that allocations resulting from both simulation models are able to outperform the "historical" CVaR allocation in the post-financial crisis period. In addition to these elliptical copulas, Sahamkhadam et al. (2018) apply three Archimedean copulas, under different portfolio selection objectives. In relation to minimum CVaR, they conclude that the elliptical copulas yield better portfolio performance.

They conclude that the Student t and Clayton copulas generally better portray the dependency structures of their portfolio than the Normal copula.

Chapter 3

Methodology

3.1 Research Design

We assume an investment universe limited to 26 of the 30 equities included in the Dow 30 index composition, as of 2021. For this portfolio, we want to study the potential benefits of using GARCH-EVT-Copula models to supply inputs for the allocation problem, as oppose to solving the problem directly upon historical data. Our research methodology is a *backtesting*[1] approach, in which we calculate and evaluate the hypothetical performance of allocations resulting from these different strategies over an historical period. At fixed h-day increments over this period, we solve the portfolio selection problem to derive sequences of historical optimal allocations. The problems are solved using only data that was available at that time, and backtested against historical returns over the subsequent h days to determine the outcome of the strategy. The outputs are series of historical returns for different strategies, which can then be analysed using performance metrics and strategical tools. As means to cross-validate the results, we evaluate the simulation models in light of three separate allocation objectives: maximum Sharpe ratio, minimum variance and minimum CVaR. We also find allocations for five different rebalancing intervals, ranging from 1 day to 5 days. Including the benchmark strategy, there is a total of $(20 + 1) \times 5 \times 3 = 315$ unique allocation and return sequences being generated.

[1]This is one of several understandings of the term "backtesting", see Christoffersen (2010).

3.2 Simulation Methodology

In these following sections, we go more into debt about the GARCH-EVT-Copula simulation methodology, sourcing the approach described by Wang et al. (2010) as our main reference. Related calculations are performed in R, and code is attached in the appendix. We comment on package choices and estimation techniques. In the first three steps of this methodology, we use historical data to respectively fit the GARCH model, the EVT model, and then the copula. In the next few step, we simulate returns by applying the fitted models in the opposite order. In the concluding step, we solve the allocation problem upon the simulated returns.

3.2.1 GARCH Models

Denote by $i = 1...n$ the portfolio assets, and by $t = 1...T$ the historical returns. For each asset i, we use $T = 1500$ observations of returns (r_{it}) to fit the GARCH models. These models requires a *variance equation*, a *mean-equation*, and a distributional assumption for the residuals. We use the linear Standard-GARCH(1,1) model proposed by Bollerslev (1986), and the non-linear GJR-GARCH(1,1) model proposed by Glosten et al. (1993). Their variance equations have the following representations, respectively (see Alexander (2008*b*), p. p. 135 and 150):

$$\sigma_{it}^2 = \omega_i + \alpha\epsilon_{i,t-1}^2 + \beta\sigma_{i,t-1}^2 \tag{3.1}$$

$$\sigma_{it}^2 = \omega_i + \alpha\epsilon_{i,t-1}^2 + \lambda_i 1_{\{\epsilon_{i,t-1}<0\}}\epsilon_{i,t-1}^2 + \beta\sigma_{i,t-1}^2 \tag{3.2}$$

Where σ_{it} and ϵ_{it} are the conditional variances and residuals, ω_i is a constant, and α_i and β_i are the coefficients of the residual and variance terms. λ_i is the coefficient of the GJR term, and the indicator function takes the value of 1 if $e_{i,t-1} < 0$ and 0 otherwise. In standard-GARCH(1,1), the parameters are constrained to $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$ and $\alpha_i + \beta_i < 1$, and to $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$ and $\alpha_i + \beta_i + \frac{1}{2}\lambda_i < 1$ in GJR-GARCH(1,1).

The *mean equation* holds the GARCH model's assumption about the return generating process. We use a constant mean, and a time-variant ARMA(1,1) mean. Similar to GARCH, ARMA assume the current return can be described as a function of lagged returns and residuals, and is a special case of the ARIMA model with the *order of integration* set to zero. This is a common design in the analysis of financial returns time series (see Studenmund (2016)), since they already represent the first (log) difference of the original (price) series. The mean equations have the following representations, respectively (see Alexander (2008*b*), p. 136 and 205):

$$r_{it} = \mu_i + \epsilon_{it} \tag{3.3}$$

$$r_{it} = \mu_i + \phi r_{i,t-1} + \theta\epsilon_{i,t-1} + \epsilon_{i,t} \tag{3.4}$$

In the constant mean equation, μ_i is the average historical return. In the time-variant mean equation, μ_i is a constant, and ϕ_i and θ_i the coefficient of the return and residual terms.

We assume the residuals of all four model combinations are subject to the Student-t distribution, i.e. $\epsilon_i \sim Student\ t(0, \sigma^2)$, and we assume no ARCH-in-mean effects for the ARMA-based models. Provided that the GARCH model is appropriately fitted to the data, the series of standardized residuals (z_{it}) is characterized as a strict white noise process, with the filtering being performed as follows (see McNeil and Frey (2000), p. 6):

$$z_{ti} = \frac{\epsilon_{it}}{\sigma_{it}} \sim (i.i.d.) \tag{3.5}$$

Parameters are estimated using the MLE method, an all GARCH-related calculations are performed using the `rugarch` package. We use the `ugarchfit` function for fitting, and later the *ugarchsim* for simulations. Specifications cross-checked against other theoretical references, using Ghalanos (2020).

3.2.2 Extreme Value Theory

The second step is to use the 1500 standardized residuals to fit the distribution model, for each asset i. The distribution type applied in Wang et al. (2010) is often referred to as an Extreme Value Mixture (EVM) model, as it applies different distributional assumptions to different parts of the sample. The standardized residuals are divided into three subsets: the *lower tail*, *centre* and *upper tail*, based on whether they exceed some threshold values or not. Tail observations are categorized as *extreme*, and described with the Generalized Pareto Distribution (GPD). GPD is a generalization of an EVT methodology known as *peek-over-threshold* (POT). Centre observations are categorized as *non-extreme*, and described with a Gaussian KDE (Kernel Density Estimator). KDEs are non-parametric smoothing techniques, used for inferring the population density from the empirical density of the sample. Closer individual descriptions of both of these concepts can be found in the third chapter of Alexander (2008*a*). The EVM model has the following representation (see Wang et al. (2010), p. 4920):

$$G_i(z_i) = \begin{cases} \frac{k_L}{T}\left(1 + \frac{\xi_L}{\beta_L}(u_L - z_i)\right)^{-\frac{1}{\xi_L}} & , \; z_i < u_L \\ \phi(z) & , \; u_L < z_i < u_R \\ 1 - \frac{k_R}{T}\left(1 + \frac{\xi_R}{\beta_R}(u_R - z_i)\right)^{-\frac{1}{\xi_R}} & , \; z_i > u_R \end{cases} \tag{3.6}$$

Where subscripts L and R denote the lower and upper tail parameters. Denote by u the threshold, and by β and ξ the scale and shape parameters of the GPD estimator. The fraction k/T measure the number of tail observations to total sample size. ϕ denotes the Gaussian KDE.

In order to apply the model, we need to make a decision about what are the appropriate threshold values. In practical use of POT, the choice of threshold values often means striking a balance between *bias* and *variance* in the model. Bias relates to the ability of the GPD estimator to fit the data properly, and arises from misplacement of the threshold. Variance relates to the accuracy of the GPD coefficient estimates, and arises from failure of the *central limit theorem* to hold in small samples. The freedom to select high thresholds is often limited by sample size, and one must often accept some bias to achieve accurate coefficient estimates. Multiple authors have addresses this problem in relation to POT. DuMouchel (1983) argues that GPD is a flexible distribution, and suggest that an indiscriminate threshold of 10 % for each tail will strike a good balance between bias and variance. A similar rule of 5 % for each tail is proposed by Neftci (2000). According to the findings of Nystrom and Skoglund (2002), both of these rules are appropriate. In this study, GPD estimates are compared over a range of thresholds, for different sample sizes and underlying distributional assumptions, and found to be similar for thresholds in the range of 5 % to 13 %. In previous applications of the GARCH-EVT-copula simulation model, a 5 % rule is adopted by Huang and Hsu (2015), while a 10 % rule is adopted by Wang et al. (2010) and Sahamkhadam et al. (2018). In our models, we define the thresholds (u_L and u_R) such that each tail holds 10 % of the sample, i.e. for both tails we fix $k/T = 0,1$ so that $k = 150$ for a sample of $T = 1500$ observations. The model parameters are estimated using the MLE method, and calculations are performed using the `spd` package in `R`. The `spdfit` function is used for fitting, and the `pspd` and `qspd` functions for inversion of samples. Our specifications are cross-checked against other theoretical references using Ghalanos (2013).

3.2.3 Copulas

After having determined the distributions ($u_i = G_i(z_i)$) of the standardized residuals, the third step is to describe the relationship between them. Wang et al. (2010) suggested that copulas could be used for this purpose. According to Sklar (1959)'s theorem, a joint distribution can be expressed as a function C of its marginals, given that the marginals are continuous. C is referred to as the *copula distribution function* (see Alexander (2008*b*), p. 259-260):

$$\mathbf{F}(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)) \tag{3.7}$$

Where $F_1, ..., F_n$ denotes a set of marginal distributions, and $\mathbf{F}$[2] the corresponding joint distribution. It is not possible to determine a copula specific for our portfolio, as the joint distribution of standardized residuals is dependent on the portfolio allocation. However, many copulas have already been derived on basis of the relationships between known univariate and multivariate distribution functions, and these can be applied to our standardized residuals. We can then evaluate how the results are impacted by the different assumptions of association they represent. Five copulas have been chosen for evaluation, and these belong to two different families: the elliptical and Archimedean copulas. These represent different *tail dependency* assumptions, which according to Alexander (2008*b*), can be loosely defined as the conditional probability that one variable takes a value in a tails, given that others do the same. We use two elliptical copulas: the normal copula that assume no tail dependence, and the Student-t copula that assume symmetric tail dependence (both tails). Based on the univariate and multivariate standard normal distribution functions (here noted Φ and $\boldsymbol{\Phi}$), the normal copula is derived as follows (see Alexander (2008*b*), p. 266, also 90 and 115):

$$C(u_1, ..., u_n \ ; \ \boldsymbol{\Sigma}) = \boldsymbol{\Phi}\Big(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_n)\Big) \tag{3.8}$$

And similarly, based on the univariate and multivariate Student-t distributions (t_ν and $\mathbf{t}_\nu$), we see that the Student-t copula is derived as (see Alexander (2008*b*), p. 268 , also 97 and 117):

$$C_\nu(u_1, ..., u_n \ ; \ \Sigma) = \mathbf{t}_\nu\left(t_\nu^{-1}(u_1), ..., t_\nu^{-1}(u_n)\right) \tag{3.9}$$

[2] Be careful to note that font is not to be understood as matrix notation in this specific context.

Archimedean copulas are derived using so-called *generator functions* ($\Psi(u)$), instead of real distribution functions. However, these are based on an otherwise similar concept as the ones above. We use three Archimedean copulas: described by Gumbel (1960), Clayton (1978) and Frank (1979). The Gumbel copula is based on the generator function $\Psi(u) = -(\ln\ u)^{\alpha}$ where $\alpha \geq 1$, and assumes dependence for the upper tail only (see Alexander (2008*b*), p. 272):

$$C(u_1, ..., u_n\ ;\ \alpha) = \exp\left(-\left[(-\ln\ u_1)^{\alpha} + ... + (-\ln\ u_n)^{\alpha}\right]^{1/\alpha}\right) \tag{3.10}$$

The Clayton copula is derived from the generator function $\Psi(u) = \alpha^{-1}(u^{-\alpha} - 1)$ where $\alpha \neq 0$, and assumes dependence for the lower tail only (see Alexander (2008*b*), p. 271):

$$C(u_1, ..., u_n\ ;\ \alpha) = \left(u_1^{-\alpha} + ... + u_n^{-\alpha} - n + 1\right)^{-1/\alpha} \tag{3.11}$$

The Frank copula is based on the generator function $\Psi(u) = -\ln\left[(e^{-\alpha u} - 1)(e^{-\alpha} - 1)^{-1}\right]$ where $\alpha \geq 0$ (see Yan et al. (2007) p. 4) and assumes dependence for both tails. We do not present the multivariate representation of this copula as it becomes fairy complicated.

In these expressions, we see that the copulas are not functions of the standardized residuals directly, but indirectly through the corresponding cumulative probabilities. Using the EVM models described in the previous section, the series of standardized residuals are first inverted into new series of cumulative probabilities, i.e. we calculate $u_{it} = G_i(z_{it})$ for each asset i. We then compose a $T \times n$ matrix of these series, and estimate (or *calibrate*) the relevant copula parameters accordingly. The parameters are estimated using the MLE method, and all copula-related calculations are performed using the `copula` package in `R`. The `copulafit` function is used for fitting, and later the `rCopula` function for simulations. Our specifications are cross-checked against other theoretical references using Yan et al. (2007). After deriving the copula parameters, the model is fitted to the data and simulations may begin.

3.2.4 Simulating h-day Returns

After fitting the model components, we want to simulate an $(S \times n)$ matrix of h-day returns, as input for the allocation problem. Seeing as we have fitted the models to daily data, we first need to simulate sequences of h daily returns, and then accumulate them in the end to find the h-day return. For each day within the forecasting horizon, we perform the following two steps:

1. For each j trial, we use the fitted copula to simulate[3] n-length semi-random vectors of , i.e. $y_j = [y_{1j}, ..., y_{nj}]$. Compiling these vectors yields an $(S \times n)$ matrix, with numbers similar to the cumulative probabilities of section (3.2.3).

2. For each i column in the semi, random matrix, we use the fitted inverse distribution functions from section (3.2.2) to impose on the numbers the same distribution as the standardized residuals of section (3.2.1), i.e. $x_i = [x_{i1}, ..., x_{iS}] = [G_1^{-1}(y_{i1}), ..., G_1^{-1}(y_{iS})]$.

Based in the corresponding elements of these matrices, we have $S \times n$ paths of simulated standardized residuals over the forecasting horizon, i.e. $\{x_{1ji}, x_{2ji}, ..., x_{hji}\}$. The last step is to use the corresponding GARCH models of section (3.2.1) to perform the filtering in reverse[4]. We use as starting values the last observations of return (r_{iT}), residual (e_{iT}) and volatility (σ_{iT}) in the historical data. This procedure yields $S \times n$ paths of simulated returns over the forecasting horizon, i.e. $\{r^*_{1ji}, r^*_{2ji}, ..., r^*_{hji}\}$. The last step is to accumulate them into h-day returns.

[3]See Wang et al. (2010), p. 4920-4922 for in-debt descriptions of copula simulations.
[4]See Barone-Adesi et al. (1999) for closer descriptions on GARCH simulations.

3.3 Portfolio Selection Objectives

Next, we provide the details for the three portfolio allocation problems. As explained above, the inputs for these problems are the $(S \times n)$ matrices of simulated returns in the case of our GARCH-EVT-Copula models, and the $(T \times n)$ matrices of daily historical returns in the case of out benchmarks. Input matrices are denoted $\mathbf{r}$, and the resulting allocation vector as $\mathbf{w} = [w_1, ..., w_n]'$. The problems are solved assuming no transaction costs or short-selling, and for calculations we use the `fPortfolio` package in `R`.

3.3.1 Mean-Variance Framework

As explained above, the mean-variance framework assume asset behavior can be described by the normal distribution parameters: expectation and variance. Let $E(\mathbf{r}) = [E(r_1), ..., E(r_n)]'$ denote the vector of expected asset returns, $\mathbf{\Sigma}$ the covariance matrix and r_f the risk-free rate. The portfolio expected return and variance are defined as (see Alexander (2008*a*), p. 238-239):

$$E(r_p) = \mathbf{w}'E(\mathbf{r}) \tag{3.12}$$

$$\sigma_p^2 = \mathbf{w}'\mathbf{\Sigma}\mathbf{w} \tag{3.13}$$

Furthermore, the Sharpe (1964) ratio is defined as (see Alexander (2008*b*), p. 250):

$$SR = \frac{E(r_p) - r_f}{\sigma_p} = \frac{\mathbf{w}'E(\mathbf{r}) - r_f}{\sqrt{\mathbf{w}'\mathbf{\Sigma}\mathbf{w}}} \tag{3.14}$$

I.e., the minimum variance problem can be formulated as (see Alexander (2008*a*), p. 243):

$$\begin{aligned} \min_{\mathbf{w}\in\mathbb{R}^n} \quad & \mathbf{w}'\mathbf{\Sigma}\mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T\mathbf{1} = 1 \\ & w_i \geq 0 \; \forall i \in [1, ..., n] \end{aligned} \tag{3.15}$$

And the maximum Sharpe problem as (see Alexander (2008*a*), p. 244):

$$\begin{aligned} \max_{\mathbf{w}\in\mathbb{R}^n} \quad & \frac{\mathbf{w}'E(\mathbf{r}) - r_f}{\sqrt{\mathbf{w}'\mathbf{\Sigma}\mathbf{w}}} \\ \text{s.t.} \quad & \mathbf{w}^T\mathbf{1} = 1 \\ & w_i \geq 0 \; \forall i \in [1, ..., n] \end{aligned} \tag{3.16}$$

3.3.2 Conditional Value-at-Risk

The VaRα is understood as the maximum loss that is likely to occur at a given probability level $\alpha \in [0, 1]$. The corresponding CVaRα is understood as the expectation of losses when VaRα is exceeded. In the context of portfolio selection, VaRα and CVaRα are usually defined in relation to the portfolio *loss distribution*, in which losses are located in the upper tail, and gains in the lower tail (see Salahi et al. (2013), p. 3-4):

$$f(\mathbf{w}, \mathbf{r}) = -\mathbf{w}'\mathbf{r} \tag{3.17}$$

For a given allocation, the portfolio VaRα is expressed:

$$\text{VaR}_\alpha(\mathbf{w}) = \min\{\gamma : \Pr\left(f(\mathbf{w}, \mathbf{r}) \leq \gamma\right) \geq \alpha\} \tag{3.18}$$

In relation to a certain VaRα (or α-level), CVaRα can formulated using the expression proposed by Rockafellar and Uryasev (2000):

$$\text{CVaR}_\alpha = \bar{F}_\alpha(\mathbf{w}, \gamma) = \gamma + ((1 - \alpha)S)^{-1} \sum_{j=1}^{S} [-\mathbf{w}'\mathbf{r} - \gamma]^+ \tag{3.19}$$

I.e. the minimum CVaRα problem can be formulated as:

$$\begin{aligned} \min_{\mathbf{w} \in \mathbb{R}^n, \alpha} \quad & \bar{F}_\alpha(\mathbf{w}, \gamma) \\ \text{s.t.} \quad & \mathbf{w}_t^T \mathbf{1} = 1 \\ & w_i \geq 0 \; \forall i \in [1, ..., n] \end{aligned} \tag{3.20}$$

3.4 Analysis

Next, we explain how we analyze the 315 output sequences of portfolio returns. Sahamkhadam et al. (2018) apply the same allocation objectives as ourselves, and analyse performance using standard deviation, and the 1st and 99th percentiles. Huang and Hsu (2015) apply the minimum CVaR objective, and analyze performance in terms of CVaR for different sub-periods of their series (under and after the 2007/09 financial crisis). We use a similar approach, in which we analyze performance separately for the periods *before*, *under* and *after* the financial crisis[5]. Also, we want performance metrics to be specific to the underlying allocation objective, and run separate comparisons for each objective based on which performance metrics seems appropriate. For each sub-period, the 105 maximum Sharpe portfolios are compared in terms (annual) Sharpe ratio, the 105 minimum CVaR portfolios in terms of CVaR, and the 105 minimum variance portfolios in terms of (annual) standard deviation[6]. Like Sahamkhadam et al. (2018), we run (dummy) regressions to compare performance across different input models and rebalancing intervals, nine such models in total. Regressions are based on two sets of dummy variables: input-strategy (20 dummies) and forecasting horizon (4 dummies). Both sets apply the one-day historical-inputs portfolio as their common reference category.

$$SR_{\text{Pre-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.21}$$

$$SR_{\text{Sub-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.22}$$

$$SR_{\text{Post-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.23}$$

$$CVaR_{\text{Pre-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.24}$$

$$CVaR_{\text{Sub-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.25}$$

$$CVaR_{\text{Post-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.26}$$

[5]See section (3.1) for more information

[6]The daily Sharpe ratios and volatilities are annualized before regression for visual purposes, without further impact to the regression outputs. We assume a year of 252 trading days.

$$SD_{\text{Pre-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.27}$$

$$SD_{\text{Sub-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.28}$$

$$SD_{\text{Post-Crisis}} = \beta_0 + \beta_{1,m} M_{1,m} + \beta_{2,h} H_h + \epsilon_i \tag{3.29}$$

Where $m = [1, 2, ..., M]$ *refers to the dummies representing different input-strategies, and* $h = [1, 2, ..., H]$ *refers to the dummies representing different investment horizons. 105 observations apply to each regression, as explained above.*

Although the regression provide information about the statistical inference for the target functions w.r.t. the applied model composition, it can be difficult to distinguish between the models. To get a better overview and a more intuitive understanding of the underlying relations, we find it appropriate to further present accompanying box-plots to the regression results. This enables us to visualize the dispersion in the target function w.r.t. model composition, where we get information about the median, 1^{st} (Q1) and 3^{rd} (Q3) quantile, the interqantile (IQR) and further the upper and lower whisker as well as potential outliers. The box-plots are presented as a supplement in B.1, B.2 and B.3.

3.5 Quality

In these concluding sections of the chapter, we evaluate our approach with respect to reliability, validity, and the potential to generalize results. The evaluations apply mainly to the research design (choice of dataset, the backtesting approach, the choice of performance metrics and sub-periods, and the comparisons by regression), whereas the design of our trading strategies are subject to evaluation in the results and discussion chapter.

3.5.1 Generalization

This point concerns the choice of data. Calculations related to the trading strategies are computationally demanding, and so it is very time-consuming to backtest these types of models for large quantities of data. Our strategies are backtested for only one portfolio composition, but still over a lengthy period of about 18.5 years. The world economy has experienced several different states over this period, and strategy performance under many circumstances are represented in the resulting sequences of strategy return. With 4800 observations in each sequence, we should be well equipped to perform sound statistical analysis of these results, also when differentiating into shorter sub-periods. Provided our subsequent choice of analysis is reasonable, we are confident in our ability to draw good conclusions for this specific portfolio. The big but is of course the world of equities that are not included in our study, and therefore we conclude that the potential to really generalize results to other portfolio compositions is fairly limited.

3.5.2 Reliability

The question of reliability is highly relevant when examining a framework such as GEC. The outline of such research includes a certain amount of data being digested through a software program in which we do not know for sure what the program does with the data. This problem is referred to as the black box problem within the computer science environment. Thus, it is critical to examine whether the input data is treated the right way and end up as correct output data. To ensure that the filtering process described in Section 3.2.4 is correctly specified in `R`, we have performed the same calculations manually in Excel. The same yields for the optimizations for the portfolio selection strategies, where we have performed a random sample cross-examination using the Excel solver to compare the optimal weights allocated through R. We are confident that the applied software packages in this thesis are reliable as they are widely used in the literature. It is worth noting that some raw data went missing in the correspondence of the asset returns and risk free rate due to date differences. Further, some data also went missing due to estimation errors when running the models (See Section 3.6 below). All in all, the missing data appears random, expect that we observe some larger proportion of estimation errors related to 2008 period.

3.5.3 Validity

This thesis is a relatively broad examination of different model compositions within the GEC framework. To ensure that the analysis in this thesis is sufficient to announce anything about the theoretical validity for the GEC framework, the research design is set up in a way that isolates the portfolio performance for each model composition across the three portfolio selection models. The well-established portfolio selection models optimizes certain performance metrics, in which we use these respective metrics as measurement on how the model compositions are performing with and without the application of the GEC-framework. The analysis of these performance metrics are set up in a dummy regression that compares each distinct model performance against the traditional in-sample historical strategy. Thus, the analysis provides information about the performance as is and equally important the performance relative to the traditional portfolio strategies. A final remark should be that the broad examination naturally put some constrains on the the ability to provide robust statistical tests on the results obtained. Since our research examine 300 distinct portfolios, we suffer from the magnitude it would cause to perform robust pairwise test on the portfolio performance metrics. That said, our research is able to provide reasonably good guidelines for further research within this framework.

3.6 Data and Descriptive Statics

3.6.1 Dataset

Our investment universe is limited to 26 of the 30 equities included in the Dow Jones Industrial Average (DJIA) index composition, as of 2021. The remaining four equities are removed due to insufficient data history, and changes to the index composition during the period of the analysis are not accounted for. The index is chosen due to its position as a key financial indicator, the manageable size of the underlying portfolio, and the lengthy histories of (most) assets included. We acquire daily price data from Eikon, and calculate series of log returns for the period of 22/08/1995 to 31/12/2020. A rolling estimation window of 1500 observations (about 6 years of data) of returns are used for each model, and so we need returns also prior to 31/07/2001. In addition, we acquire daily observations of a risk-free rate proxy for the period of 31/07/2001 to 30/12/2020, needed for the Maximum Sharpe problem. Our rates are based on the 1 Month U.S. Treasure Bill, and downloaded from the US Department Of The Treasury (2021*c*). These are Constant Maturity Treasury (CMT) rates, and represent the annualized bond equivalent yield (BEY) for securities that pay semi-annual interest[7]. The annualized rates are scaled down by a factor of 365[8] to reflect the daily risk-free dates. After lining up the interest rates and returns, making sure that the dates are corresponding after 31/07/2001, they have lengths 4849 and 6349, respectively. Next, we look at some descriptive statistics for the returns.

3.6.2 Descriptive Statistics

In table (3.1), we present descriptive statics over the length of the asset returns series. We note that normality of returns is rejected for all stocks by the Jarque-Bera (JB) statistic[9], which is not surprising since all are leptokurtic, or "sharp-peaked and heavy-tailed". In the case of APPL and PG, kurtosis is extreme, and the majority of stock are also negatively skewed. We note as well that stationary of all series is rejected by the Augmented Dickey-Fuller (ADF) test[10].

[7]See US Department Of The Treasury (2021*c*) and US Department Of The Treasury (2021*a*).

[8]See US Department Of The Treasury (2021*b*)

[9]See Alexander (2008*a*), p. 158.

[10]See Alexander (2008*b*), p. 218.

Table 3.1: Descriptive statistics of raw data

Descriptive statistics from the full sample returns of the 26 individual stocks in the investment universe. The full sample period yields from 22/08/1995 to 31/12/2020 (6449 observations). The average return and the standard deviation is annualized under the assumption of 252 trading days per year.

Ticker	Mean(%)	Std.dev(%)	Skewness	Kurtosis	Jarque-Bera	ADF(1)
AXP	9.10	36.20	0.07	10.36	28347^{***}	-58.92^{***}
AMGN	11.18	33.41	0.20	4.94	6494^{***}	-60.77^{***}
AAPL	22.52	45.07	−2.53	75.16	1498750^{***}	-57.37^{***}
BA	7.54	34.67	−0.52	16.19	69511^{***}	-52.88^{***}
CAT	9.13	32.99	−0.25	4.22	4774^{***}	-56.21^{***}
CSCO	10.33	39.38	0.07	6.99	12903^{***}	-59.23^{***}
CVX	4.27	27.41	−0.60	17.96	85572^{***}	-57.63^{***}
HD	12.80	31.59	−0.95	19.79	104395^{***}	-57.33^{***}
HON	9.13	31.22	−0.26	12.82	43458^{***}	-56.51^{***}
IBM	5.77	28.30	−0.09	8.15	17557^{***}	-57.40^{***}
INTC	6.86	38.40	−0.45	8.26	18204^{***}	-58.59^{***}
JNJ	8.15	20.91	−0.41	9.83	25712^{***}	-59.87^{***}
KO	4.36	22.45	−0.24	6.45	11052^{***}	-56.71^{***}
JPM	7.76	38.61	−0.20	12.43	40875^{***}	-58.94^{***}
MCD	9.17	24.58	−0.08	10.19	27439^{***}	-58.76^{***}
MMM	7.09	24.36	−0.24	5.90	9246^{***}	-59.26^{***}
MRK	4.24	27.45	−1.17	22.50	135160^{***}	-57.28^{***}
MSFT	13.60	31.62	−0.18	7.00	12985^{***}	-58.51^{***}
NKE	15.50	32.08	−0.13	9.32	22952^{***}	-57.89^{***}
PG	8.16	23.19	−2.88	75.28	1505422^{***}	-59.86^{***}
TRV	6.15	29.23	−0.26	17.14	77639^{***}	-58.47^{***}
UNH	14.98	34.99	−1.72	27.98	209817^{***}	-54.75^{***}
VZ	3.24	25.19	0.13	5.61	8341^{***}	-59.15^{***}
WBA	7.15	29.68	−0.24	6.80	12271^{***}	-57.48^{***}
WMT	9.19	25.82	0.18	5.78	8866^{***}	-59.63^{***}
DIS	8.57	30.59	−0.11	8.86	20760^{***}	-58.56^{***}

*: p-value<0,1, **: p-value<0,05 and ***: p-value<0,01

JB refers to the Jarque-Bera test for normality. The null hypothesis is that the respective time series is normally distributed.

ADF(1) is short for the Augmented Dickey-Fuller test statistic of order(i.e lag) 1. Since we operate with daily returns, which tend to variate between a mean equal zero, we use the equation with no constant term nor trend variable to test for stationarity.The conclusion of these tests is that all the equity log returns is non-normally distributed, but can be assumed to have eliminated the unit root[11]. Although not reported, the conclusion is consistent for up to 10 lags.

Table 3.2: sGARCH and ARIMA-sGARCH parameters

The table displays the GARCH parameters. The are are estimated by maximizing the log likelihood function w.r.t. to these parameters. The GARCH equations are applied on the full sample returns for the individual stocks in the investment universe.(6449 observations).

	sGARCH(1,1)					ARIMA(1,0,1)-sGARCH(1,1)							
Ticker	ω	α_1	β_1	shape	LogL	μ	ϕ	θ	ω	α_1	β_1	shape	LogL
AXP	0.000	0.087**	0.912***	5.672***	16946, 72	0.000	0.863***	−0.895***	0.000	0.089*	0.910***	5.622***	16969, 48
AMGN	0.000***	0.067***	0.927***	4.982***	16664, 06	0.000**	0.595***	−0.636***	0.000***	0.068***	0.926***	4.972***	16674, 67
AAPL	0.000***	0.053***	0.946***	4.518***	15105, 27	0.001***	0.310	−0.329	0.000***	0.060***	0.939***	4.522***	15124, 97
BA	0.000***	0.078***	0.911***	5.523***	16687, 75	0.000***	0.908***	−0.922***	0.000***	0.080***	0.909***	5.466***	16701, 3
CAT	0.000***	0.056***	0.938***	5.537***	16362, 72	0.000***	−0.600	0.607	0.000***	0.057***	0.936***	5.393***	16369, 09
CSCO	0.000**	0.059***	0939***	4.820***	16039, 71	0.000***	0.605**	−0.632**	0.000**	0.0613***	0.938***	4.785***	16053, 52
CVX	0.000**	0.078***	0.909***	8.306***	17970, 39	0.000***	0.563***	−0.602***	0.000**	0.079***	0.909***	8.136	17983, 19
HD	0.000	0.088***	0.907***	5.992***	17290, 77	0.001***	−0.462*	0.486**	0.000	0.091***	0.904***	6.062***	17304, 38
HON	0.000	0.089***	0.908***	5.490***	17424, 06	0.000***	0.927***	−0.947***	0.000	0.095**	0.903***	5.498***	17446, 49
IBM	0.000***	0.073***	0.923***	4.394***	17931, 67	0.000**	0.650***	−0.676***	0.000***	0.073***	0.922***	4.370***	17937, 8
INTC	0.000	0.046***	0.952***	5.398***	15905, 53	0.001***	−0.070	0.046	0.000	0.047***	0.952***	5.394***	15912, 45
JNJ	0.000	0.110***	−0.883***	5.472***	19735, 4	0.000***	0.945***	−0.957***	0.000	0.110***	0.883***	5.514***	19748, 67
KO	0.000***	0.057***	0.940***	5.277***	19369, 05	0.000***	0.921***	−0.936***	0.000***	0.058***	0.939***	5.230***	19379, 89
JPM	0.000	0.082***	0.917***	5.780***	16638, 81	0.000***	0.858***	−0.879***	0.000	0.083***	0.916***	5.743***	16652, 16
MCD	0.000***	0.0523***	0.946***	5.129***	18666	0.001***	0.776***	−0.806***	0.000***	0.052***	0.944***	5.157***	18682, 95
MMM	0.000***	0.056***	0.942***	4.409***	18544, 07	0.001***	0.844***	−0.871***	0.000***	0.056***	0.942***	4.380***	18563, 29
MRK	0.000	0.071***	0.917***	4.662***	17870, 56	0.000***	0.960***	−0.970***	0.000**	0.073***	0.915***	4.663***	17878, 96
MSFT	0.000	0.088**	0.910***	4.743***	17094, 13	0.001***	0.192	−0.243	0.000	0.089**	0.909***	4.747***	17110, 73
NKE	0.000**	0.039***	0.959***	4.060***	17052, 78	0.001***	0.677***	−0.717***	0.000**	0.039***	0.959***	4.059***	17070, 2
PG	0.000***	0.080***	0.913***	4.826***	19481, 23	0.000***	0.648***	−0.699***	0.000	0.082***	0.913***	4.793***	19503, 99
TRV	0.000*	0.088***	0.906***	4.923***	18107, 86	0.000***	−0.089	0.038	0.000*	0.088***	0.907***	4.851***	18123, 92
UNH	0.000	0.066**	0.925***	4.417***	16738, 21	0.001***	0.898***	−0.923***	0.000*	0.071***	0.920***	4.364***	16764, 15
VZ	0.000***	0.079***	0.912***	6.822***	18399, 04	0.000**	0.479	−0.511	0.000***	0.079***	0.913***	6.789***	18405, 2
WBA	0.000***	0.046***	0.947***	4.680***	17109, 41	0.000**	0.909***	−0.925***	0.000***	0.047***	0.945***	4.659***	17116, 34
WMT	0.000	0.045***	0.952***	5.048***	18446, 88	0.000**	0.677***	−0.714***	0.000	0.046***	0.952***	5.033***	18457, 73
DIS	0.000***	0.057***	0.940***	5.187***	17423, 15	0.001***	0.810***	−0.840***	0.000***	0.056***	0.941***	5.170***	17438, 53
Average LogL					17500.20								17514.39

*: p-value<0,1, **: p-value<0,05 and ***: p-value<0,01

Note that the p-value are related to application of ordinary standard errors (i.e. not robust SE). Thus, we have to bear in mind the risk of Type I-error due to underestimated SE. The coefficient estimates are not expected to be affected by this.

Table 3.3: GJR-GARCH and ARIMA-GJR parameters

The table displays the GARCH parameters. The are are estimated by maximizing the log likelihood function w.r.t. to these parameters. The GARCH equations are applied on the full sample returns for the individual stocks in the investment universe.(6449 observations)..

	GJR(1,1)						ARIMA(1,0,1)-GJR(1,1)							
Ticker	ω	α_1	β_1	γ_1	shape	LogL	μ	ϕ	θ	ω	α_1	β_1	γ_1	shape	LogL
AXP	0.000	0.031	0.918***	0.101***	6.092***	16984, 33	0.001***	0.814***	−0.847***	0.000	0.038***	0.917***	0.087***	5.861***	16998, 61
AMGN	0.000	0.036***	0.917***	0.080***	5.085	16686, 15	0.000	0.554***	−0.596***	0.000	0.038**	0.919***	0.073**	5.052***	16694, 94
AAPL	0.000**	0.052***	0.916***	0.061***	4.660***	15116, 28	0.001***	0.305	−0.321	0.000	0.054***	0.917***	0.055*	4.589***	15134, 85
BA	0.000***	0.037***	0.917***	0.072***	5.670***	16710, 8	0.001***	0.886***	−0.901***	0.000***	0.040***	0.916***	0.063***	5.599***	16719, 77
CAT	0.000	0.019*	0.944***	0.059***	5.516***	16386, 26	0.001***	−0.748***	0.754***	0.000*	0.019***	0.944***	0.057***	5.528***	16390, 3
CSCO	0.000**	0.037***	0.928***	0.069***	4.972***	16056, 01	0.001***	0.515*	−0.543**	0.000**	0.039***	0.929***	0.062***	4.844***	16067, 89
CVX	0.000***	0.035***	0.910***	0.079***	8.930***	17995, 47	0.000***	0.522***	−0.560	0.000***	0.037***	0.911***	0.071***	8.711***	18004, 24
HD	0.000**	0.041***	0.905***	0.102***	6.277***	17322, 65	0.001***	−0.457*	0.480**	0.000**	0.043***	0.904***	0.095***	6.252***	17331, 22
HON	0.000*	0.031***	0.910***	0.114***	5.890***	17468, 04	0.001***	0.860***	−0.883***	0.000	0.037	0.909***	0.098	5.805	17480, 74
IBM	0.000	0.048***	0.908***	0.078**	4.464***	17949, 38	0.000*	0.469*	−0.499*	0.000	0.049*	0.909***	0.072	4.424	17954, 53
INTC	0.000	0.041***	0.951***	0.013*	5.420	15906, 71	0.001***	−0.051	0.026	0.000	0.041***	0.950***	0.012	5.415***	15913, 41
JNJ	0.000**	0.048***	0.885***	0.126***	5.666***	19770, 63	0.000*	0.851	−0.866	0.000	0.052	0.884***	0.113***	5.659***	19777, 1
KO	0.000***	0.037***	0.934***	0.052***	5.453	19381, 19	0.000***	0.906***	−0.920***	0.000***	0.041***	0.933***	0.043***	5.427***	19388, 82
JPM	0.000	0.027**	0.918***	0.108	6.184***	16689, 05	0.000***	0.770***	−0.790***	0.000	0.030**	0.919***	0.010***	6.017***	16695, 32
MCD	0.000***	0.032***	0.944***	0.045***	5.313***	18678, 18	0.001***	0.726***	−0.757***	0.000***	0.034***	0.944***	0.038***	5.250***	18692, 81
MMM	0.000***	0.035***	0.933***	0.055***	4.513***	18556, 24	0.001***	0.795***	−0.827***	0.000***	0.037***	0.934***	0.048***	4.469	18573, 98
MRK	0.000**	0.043***	0.909***	0.066***	4.774***	17883, 02	0.000***	0.959***	−0.967***	0.000**	0.047***	0.907***	0.059***	4.756***	17889, 46
MSFT	0.000**	0.061***	0.904***	0.069***	4.804***	17106, 21	0.001***	0.196	−0.246	0.000	0.064***	0.905***	0.059***	4.769	17120, 39
NKE	0.000**	0.017***	0.954***	0.058***	4.187***	17079, 79	0.001***	0.654***	−0.698***	0.000**	0.017***	0.955***	0.054***	4.156***	17097, 27
PG	0.000**	0.047***	0.907***	0.081***	4.937***	19498, 32	0.000***	0.621***	−0.672***	0.000**	0.049***	0.909***	0.069***	4.920***	19518, 23
TRV	0.000	0.047***	0.907***	0.080***	5.042***	18127, 18	0.000***	−0.103	0.053	0.000**	0.048***	0.911***	0.070***	4.959	18139, 86
UNH	0.000**	0.045***	0.900***	0.087***	4.521***	16755, 62	0.001***	0.893***	−0.918***	0.000***	0.052***	0.902***	0.066***	4.439***	16775, 71
VZ	0.000***	0.043***	0.925***	0.051***	6.942***	18410, 54	0.000	0.488	−0.522	0.000***	0.044***	0.925	0.048***	6.897***	18415, 81
WBA	0.000***	0.041***	0.935***	0.029***	4.725***	17112, 65	0.000**	0.883***	−0.902***	0.000***	0.042***	0.934***	0.027***	4.717***	17119, 52
WMT	0.000***	0.028***	0.948***	0.045***	5.143***	18458, 93	0.000**	0.632***	−0.670***	0.000***	0.029***	0.948***	0.041***	5.103***	18468, 6
DIS	0.000***	0.018***	0.948***	0.062***	5.307***	17449, 57	0.001***	0.793***	−0.823***	0.000**	0.019***	0.950***	0.055***	5.271***	17463, 05
Average LogL						17520.74									17531.79

*: p-value<0,1, **: p-value<0,05 and ***: p-value<0,01

Note that the p-value are related to application of ordinary standard errors (i.e. not robust SE). Thus, we have to bear in mind the risk of Type I-error due to underestimated SE. The coefficient estimates are not expected to be affected by this.

Table 3.4: Estimated long-term volatilities for the individual stocks

The following table presents the annualized unconditional volatilises calculated from applying the respective GARCH equation on the full sample data set. The period yield from 22/08/1995 to 31/12/2020 (6449 observations). Estimates are annualized by using the square-root-of-time rule under the assumption of 252 trading days per year.

Ticker	sGARCH(%)	ARIMA-sGARCH(%)	GJR(%)	ARIMA-GJR(%)
AXP	77.43	78.67	81.29	66.94
AMGN	38.93	38.54	45.1	41.67
AAPL	82.01	88.68	122.66	113.79
BA	35.13	35.58	36.12	33.34
CAT	37.38	37.33	36.96	34.48
CSCO	65.9	67.83	83.07	76.03
CVX	26.53	26.55	25.95	24.67
HD	39.32	39.94	55.01	42.54
HON	51.74	57.09	62.9	40.14
IBM	38.93	38.91	42.25	38.97
INTC	50.11	51.27	48.79	47.02
JNJ	29.68	29.15	35.79	28.5
KO	29.47	29.07	29.71	25.6
JPM	77.35	78.93	87.56	66.08
MCD	37.34	31.59	35.12	28.92
MMM	39.51	40.15	34.88	29.96
MRK	29.31	29.26	29.04	27.88
MSFT	68.85	66.37	99.25	69.74
NKE	42.65	43.07	68.55	50.32
PG	26.87	28.49	31.2	26.29
TRV	35.87	37.77	37.31	33.66
UNH	36.96	38.15	41.75	37.18
VZ	27.69	27.66	28.37	27.24
WBA	32.21	32.52	32.83	32.13
WMT	30.03	30.01	45.49	36.93
DIS	36.53	36.51	39.61	32.14

In tables (3.2) and (3.3), we have run the four different GARCH models described in section (3.2.1) over the full samples of returns (22/08/1995 to 31/12/2020) for each asset. The presence of ARCH and GARCH effects (α and β) is tested in all four models. For all assets, GARCH effects are fully confirmed by all models, while ARCH effects are confirmed by most models. In the asymmetric models, the GJR parameter (γ) is significant for most assets. In the time-variant mean-models, the AR and MA parameters (ϕ and θ) are significant for most assets. Interestingly, there appears to be some negative relationship in which the AR parameter tend to be positive when MA is negative and vise verse. Without further considerations of the differences in parsimony across these models, the ARIMA-GJR appears to provide the best overall fit based calculations of the average log likelihoods across assets.

In table (3.4), we have estimated the long-term volatilities[12] for the four different GARCH models described in section (3.2.1) over the full samples of returns (22/08/1995 to 31/12/2020) for each asset. This provides some intuition of what level of volatility is assumed by the respective GARCH model as oppose to the standard deviations of table (3.1). We see that the long-term volatilise supplied by the GARCH models are generally much higher than the standard deviations, suggesting that volatility would generally be underestimated under the assumption of normality. We also observe some differences between the different GARCH models, but these are generally small in comparisons. The GJR-GARCH models seem to generate the highest volatility estimates, e.g. APPL has an estimated annualized volatility of over 100 % for both GJR-GARCHs, compared to only 45 % in table (3.1).

3.6.3 Calculation Outputs and Sub-Periods

Regarding the simulation and portfolio selection procedure described in sections (3.2) and (3.3), 49 dates were removed from the sequences of allocations and portfolio returns due to failure of the `R` solver to fit one or more GARCH model at these dates. As there were 4849 observations within the test period to begin with, the output allocation and return sequences have an exact length 4800 (!). For 1-day strategies, the rolling calculations did produce 4800 unique set of weights, and then respectively 2400, 1600, 1200 and 960 for the longer balancing intervals. In these series, we define the first 1348 observations (01/08/2001 to 31/12/2006) as pre-financial crisis, the next 1000 (03/01/2007 to 10/02/2011) as under-financial crisis, and the last 2453 (11/02/2011 to 31/12/2020) as post-financial crisis. The pre-financial crisis period contains historical events such as the 9/11 terror attack, resulting in closed markets for a certain period of time and high uncertainty. In addition, this period contains the ending of the Dot-Com bubble, which further developed to become an expansion period. The sub-financial is defined extra long, as to include both the early events and the recession that followed. The post-crisis period is defined by generally high growth in the financial markets, but also events like collapse of the oil-price and the COVID-19 crisis.

[12]See (Alexander (2008*b*), p.136)

Chapter 4

Results

4.1 Regression Outputs

Table (4.1) display the regression outputs, based on the nine models explained in section (3.4). The (dummy) coefficients measures the response of different simulation models to the respective performance metrics. Returns of maximum Sharpe portfolios are analyzed in terms of Sharpe ratio, and positive coefficients are to be interpreted as improvements in risk-adjusted performance relative to the benchmark. Returns of minimum CVaR portfolios are analyzed in terms of CVaR, and positive coefficients are to be interpreted as risk reductions relative to the benchmark. Minimum Variance portfolios are analyzed in terms of (annualized) Standard Deviation, and negative coefficients are to be interpreted as risk reductions relative to the benchmark. We report significance at levels 0.1, 0.05 and 0.01, and interpret coefficients with significance greater than 0.05 as zero-difference.

To evaluate the reliability for the analysis presented in this thesis, the classical Ramsey´s Regression Specification Error test (RESET) are presented, which aim to determine whether a regression model is correctly specified. For each cross-sectional (dummy) regression in Table 4.1, it emerges that 6 out of 9 regressions rejects the null hypothesis of correct specification on 5 percent level, which in isolation imply a high uncertainty related to the regression output. Though, further examination of central specification criteria offer some leverage to the analysis.

Table 4.1: Cross-sectional dummy regression

The table displays the results of the dummy regression, for three allocation objectives, and three sub-periods for each objective. The coefficients measures the response of different simulation models and forecasting horizons to the respective performance metrics. Performance metrics are the Sharpe ratio for maximum Sharpe allocations, CVaR for minimum CVaR allocations, and Standard Deviation for minimum variance allocations. Benchmark are the one-day historical allocation for the respective strategies. We note that Sharpe ratios and Standard Deviations are annualized based on the assumption of 252 trading days per year.

	Maximum Sharpe (MSR)			Minimum CVaR (CVaR)			Minimum Variance (GMV)		
Simulation Model	Pre-Crisis	Sub-Crisis	Post-Crisis	Pre-Crisis	Sub-Crisis	Post-Crisis	Pre-Crisis	Sub-Crisis	Post-Crisis
sGARCH-EVT-Normal	0.2998*** (0.0576)	−0.1016 (0.1658)	0.0378 (0.093)	0.0035*** (0.0005)	−0.0012* (0.0006)	−0.0012*** (0.0004)	−0.0073*** (0.0015)	0.0003 (0.0021)	0.0028 (0.0018)
sGARCH-EVT-Student	0.3328*** (0.081)	−0.1704 (0.1731)	0.0882 (0.1371)	0.0055*** (0.0004)	−0.0004 (0.0006)	−0.0007 (0.0005)	−0.0097*** (0.0016)	0.0026 (0.0017)	0.0017 (0.0015)
sGARCH-EVT-Clayton	−0.112 (0.0718)	−0.5228*** (0.1719)	−0.1873** (0.0798)	0.003*** (0.0004)	−0.0019*** (0.0006)	0.0004 (0.0003)	−0.0038** (0.0016)	0.0069*** (0.0018)	0.0056** (0.0023)
sGARCH-EVT-Gumbel	0.0189 (0.1541)	0.0964 (0.2337)	−0.2063** (0.0966)	0.0034*** (0.0004)	−0.0066*** (0.0007)	−0.0045*** (0.0002)	−0.0106*** (0.0016)	0.0032* (0.0019)	0.0005 (0.0016)
sGARCH-EVT-Frank	0.2092*** (0.0534)	0.0013 (0.1886)	0.2074* (0.1121)	0.0021*** (0.0005)	−0.0051*** (0.0005)	−0.0044*** (0.0005)	−0.0089*** (0.0015)	0.0031* (0.0018)	0.0017 (0.0015)
GJR-EVT-Normal	0.7672*** (0.1162)	−0.0919 (0.1813)	−0.2013** (0.0829)	0.0035*** (0.0004)	−0.0027*** (0.0007)	−0.0011*** (0.0004)	−0.0071*** (0.0015)	−0.0067*** (0.0023)	−0.0001 (0.0014)
GJR-EVT-Student	0.6499*** (0.1226)	−0.222 (0.1928)	−0.133 (0.1066)	0.0043*** (0.0006)	−0.0017*** (0.0006)	0.0009 (0.0004)	−0.0079*** (0.0015)	−0.0046 (0.0031)	0.0002 (0.0015)
GJR-EVT-Clayton	0.1727 (0.1279)	−0.4118** (0.1637)	−0.3051*** (0.0841)	0.0003 (0.0007)	−0.0038*** (0.0006)	−0.0017* (0.0009)	−0.0031* (0.0016)	0.0001 (0.0028)	0.0022 (0.0017)
GJR-EVT-Gumbel	0.0747 (0.0657)	−0.0269 (0.1718)	−0.3187*** (0.0917)	0.0033*** (0.0003)	−0.006*** (0.0005)	−0.003*** (0.0002)	−0.0092*** (0.0015)	−0.0045* (0.0024)	−0.0008 (0.0017)
GJR-EVT-Frank	0.5992*** (0.0954)	0.1141 (0.1702)	−0.1559* (0.0876)	0.0021*** (0.0003)	−0.0053*** (0.0006)	−0.0041*** (0.0004)	−0.0078*** (0.0015)	−0.0025 (0.0034)	0.0006 (0.0015)
ARIMA-sGARCH-EVT-Normal	0.261*** (0.0834)	−0.0069 (0.2135)	0.3535*** (0.0971)	0.0036*** (0.0004)	−0.0001 (0.0006)	−0.0017*** (0.0003)	−0.0069*** (0.0015)	−0.0002 (0.0023)	0.0035* (0.0018)
ARIMA-sGARCH-EVT-Student	0.2699*** (0.0933)	−0.0981 (0.22)	0.3546*** (0.0749)	0.0045*** (0.0007)	−0.0005 (0.0007)	−0.0014*** (0.0004)	−0.0093*** (0.0016)	0.0034* (0.0018)	0.0031* (0.0016)
ARIMA-sGARCH-EVT-Clayton	0.1024 (0.1075)	−0.3000* (0.1728)	0.2213*** (0.0843)	0.0031*** (0.0004)	−0.0018*** (0.0005)	−0.001* (0.0005)	−0.0039** (0.0016)	0.005*** (0.0019)	0.0058*** (0.002)
ARIMA-sGARCH-EVT-Gumbel	0.2249*** (0.0857)	0.1432 (0.1792)	0.278*** (0.0821)	0.0032*** (0.0005)	−0.0058*** (0.0006)	−0.0047*** (0.0002)	−0.0113*** (0.0017)	0.0032 (0.0019)	0.0012 (0.0015)
ARIMA-sGARCH-EVT-Frank	0.3026*** (0.0678)	0.2273 (0.1807)	0.4044*** (0.0834)	0.0022*** (0.0005)	−0.0044*** (0.0005)	−0.0045*** (0.0005)	−0.009*** (0.0015)	0.0027 (0.0019)	0.0022 (0.0015)
ARIMA-GJR-EVT-Normal	0.6804*** (0.0592)	−0.0142 (0.1672)	0.1149 (0.1516)	0.003*** (0.0004)	−0.0024*** (0.0006)	−0.001*** (0.0004)	−0.0065*** (0.0015)	−0.0003 (0.0023)	−0.0003 (0.0015)
ARIMA-GJR-EVT-Student	0.6161*** (0.1158)	−0.2009 (0.182)	0.1363 (0.113)	0.0038*** (0.0006)	−0.0014** (0.0006)	−0.0003 (0.0003)	−0.0072*** (0.0016)	0.0036 (0.0017)	−0.0004 (0.0017)
ARIMA-GJR-EVT-Clayton	0.1753* (0.096)	−0.2465 (0.1681)	0.0466 (0.1297)	0.0014*** (0.0004)	−0.0033*** (0.0005)	−0.0016** (0.0007)	−0.0025 (0.0016)	0.0076*** (0.0017)	0.0022 (0.0017)
ARIMA-GJR-EVT-Gumbel	0.3039*** (0.0355)	0.0111 (0.1757)	0.0728 (0.0906)	0.0031*** (0.0004)	−0.0057*** (0.0006)	−0.0035*** (0.0003)	−0.0094*** (0.0017)	0.0031 (0.0021)	−0.0001 (0.0016)
ARIMA-GJR-EVT-Frank	0.4228*** (0.0819)	0.2305 (0.1995)	0.1374 (0.1383)	0.0023*** (0.0004)	−0.0052*** (0.0006)	−0.0043*** (0.0004)	−0.008*** (0.0015)	0.0042** (0.0018)	0.0004 (0.0014)
2-day simulations	0.0672 (0.0555)	0.1993** (0.0774)	−0.0524 (0.0402)	0.0001 (0.0002)	−0.0001 (0.0002)	0.0003 (0.0003)	0.0004 (0.0004)	0.0015 (0.0012)	−0.0003 (0.0005)
3-day simulations	0.0407 (0.0547)	−0.0413 (0.0688)	−0.0563 (0.0453)	0.0001 (0.0002)	−0.0005*** (0.0002)	−0.0009*** (0.0003)	0.0019*** (0.0004)	0.0029** (0.0011)	0.0018*** (0.0005)
4-day simulations	0.0678 (0.0704)	−0.5024*** (0.0757)	−0.2782*** (0.0451)	−0.0002 (0.0002)	−0.0006** (0.0003)	−0.0002 (0.0003)	0.0047*** (0.0005)	0.0039*** (0.0012)	0.0051*** (0.0005)
5-day simulations	−0.0858 (0.062)	−0.5698*** (0.0695)	−0.3223*** (0.0605)	−0.001*** (0.0003)	−0.0019*** (0.0003)	0.0003 (0.0003)	0.0075*** (0.0005)	0.0097*** (0.0014)	0.0068*** (0.0008)
Constant	0.0749 (0.0508)	0.5539*** (0.1562)	0.7354*** (0.0787)	−0.0359*** (0.0003)	−0.0476*** (0.0006)	−0.038*** (0.0003)	0.1385*** (0.0015)	0.1747*** (0.0019)	0.1366*** (0.0014)
Observation	105	105	105	105	105	105	105	105	105
R^2	0.6803	0.7522	0.7563	0.7789	0.9223	0.8368	0.9192	0.7541	0.7942
Ramsey RESET	4.03**	2.13	2.60*	18.93***	3.74**	1.24	28.45***	10.98***	30.96***
Jarque-Bera	1.095	2.198	0.4923	0.7022	0.2321	1.861	9.839***	17.63***	8.018**

*: p-value<0,1, **: p-value<0,05 and ***: p-value<0,01

All of the regressions report a R^2 above 75 %, except from one reporting 68 %. This, in relation with the overall significant coefficients across the regressions imply that the they do not suffer from severe multicollinearity. We have also accounted for heteroskedasticity by applying robust standard errors for the hypothesis tests. It further provides confidence to the analysis that the accompanying box plots for each regression, presented in Appendix B, overall imply the same effects as the regression. Thus, the analysis is informative and appear reliable, although the regression output in isolation needs to be interpreted with some caution. We note also note that the assumption of normally distributed residuals seem to be satisfied in the first two periods, but not for the latter period.

Our research design is based on the idea of cross-validating results across different allocation objectives (and performance metrics), and we mainly look for coefficients being significant across multiple objectives within the same sub-period. The general impression is that the simulation models perform considerably better in the pre-financial crisis period. As many as 14/20 simulation models outperform the benchmark under all three objectives in this period, 17/20 under at least two objectives, and 19/20 models under at least one objective. Only one model perform the same as the benchmark, and no model performs significantly worse. Also, we see that the least favorable results are all related to the asymmetric copulas, the Clayton in particular. We also note that fewer models perform well under the maximum Sharpe objective. For the second and third periods however, the story is very different. Here, the best-performing models do so under only one objective each: and this goes for just 1/20 models during the financial crisis, and 5/20 models post-financial crisis. We also observe that the most brutal shift in performance is under the minimim CVaR objective: from 19/20 models outperforming the benchmark in the first period, to 16/20 and 14/20 models performing significantly worse than the benchmark in the second and third period.

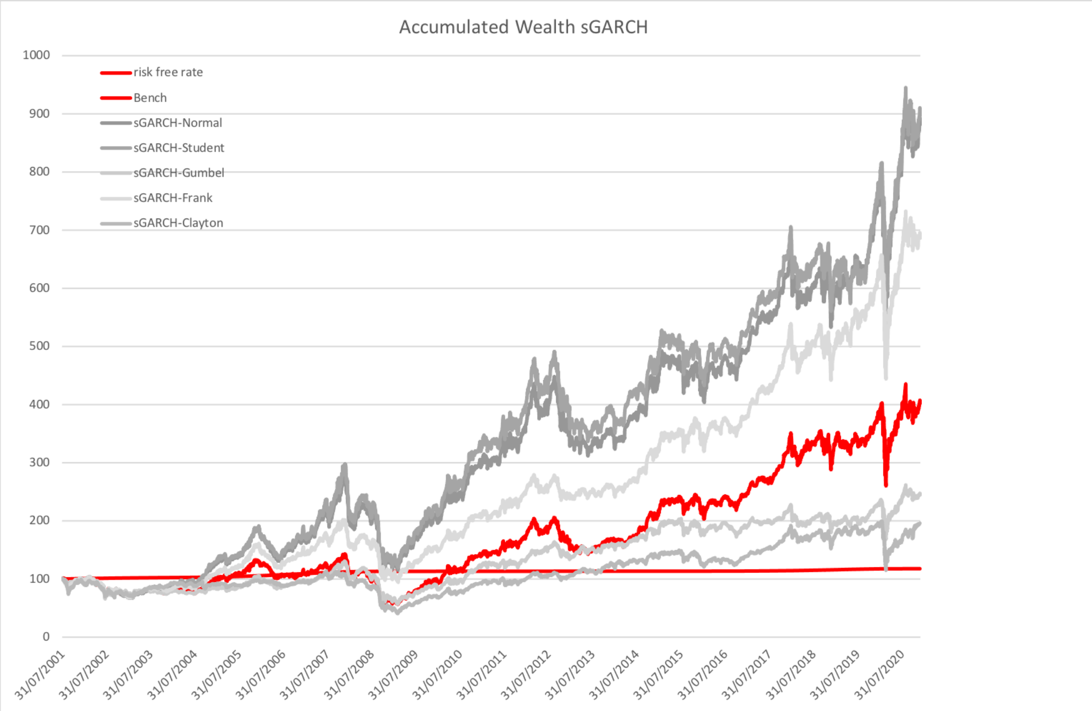
4.2 Accumulated Wealth

For the 1-day maximum Sharpe strategies, we have also calculated the accumulated wealth on basis of the series of return. These are reported in figure (4.1), and appear consistent with the

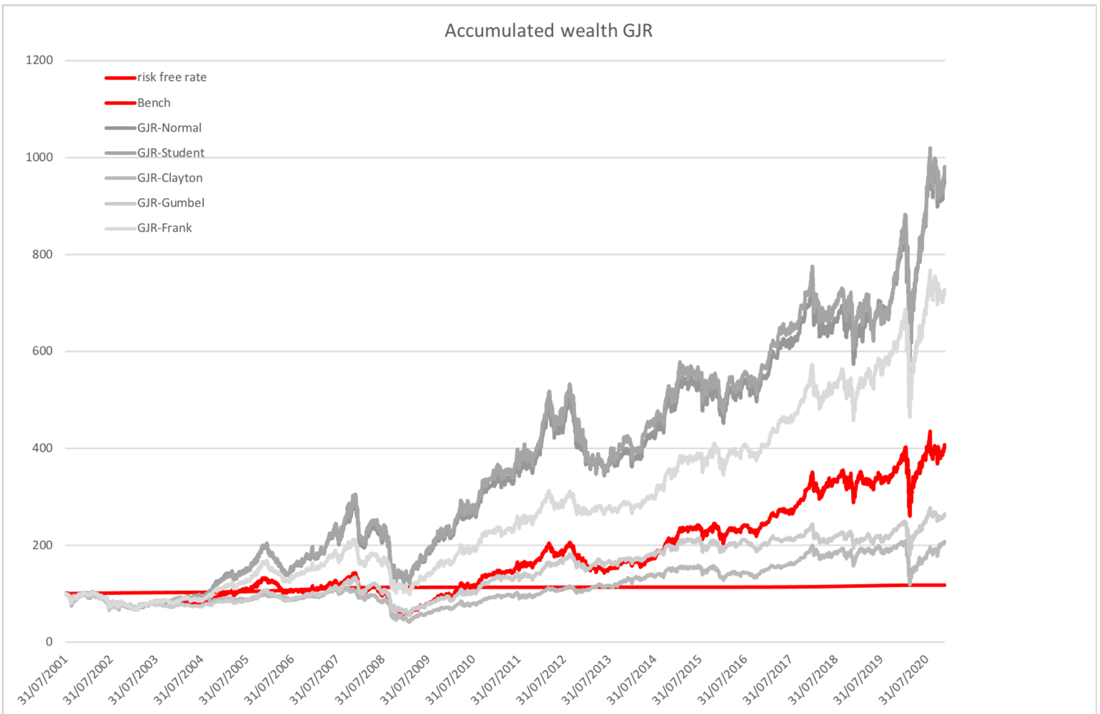
Figure 4.1: Accumulated wealth for the maximum Sharpe strategy

Accumulated wealth from an initial investment of 100 for the 20 model compositions within the MSR optimization that use 1 day a head out-of-sample return simulation. The in-sample historical strategy and the risk free rate are marked in red.

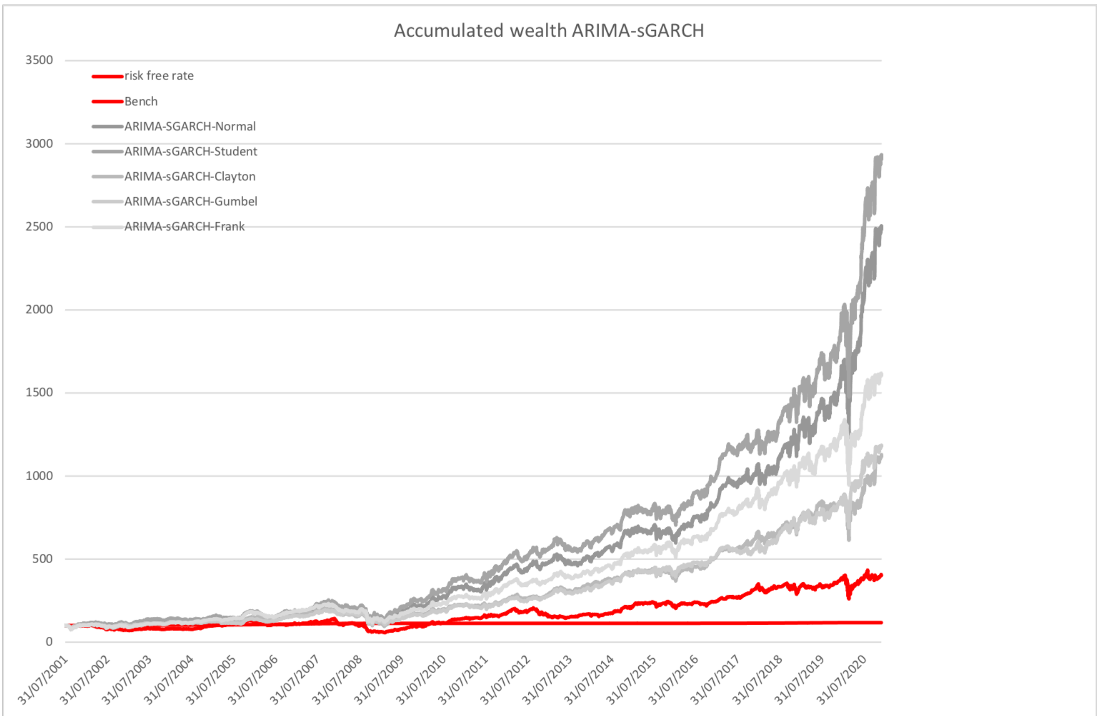
(a) Accumulated wealth for standard GARCH models:



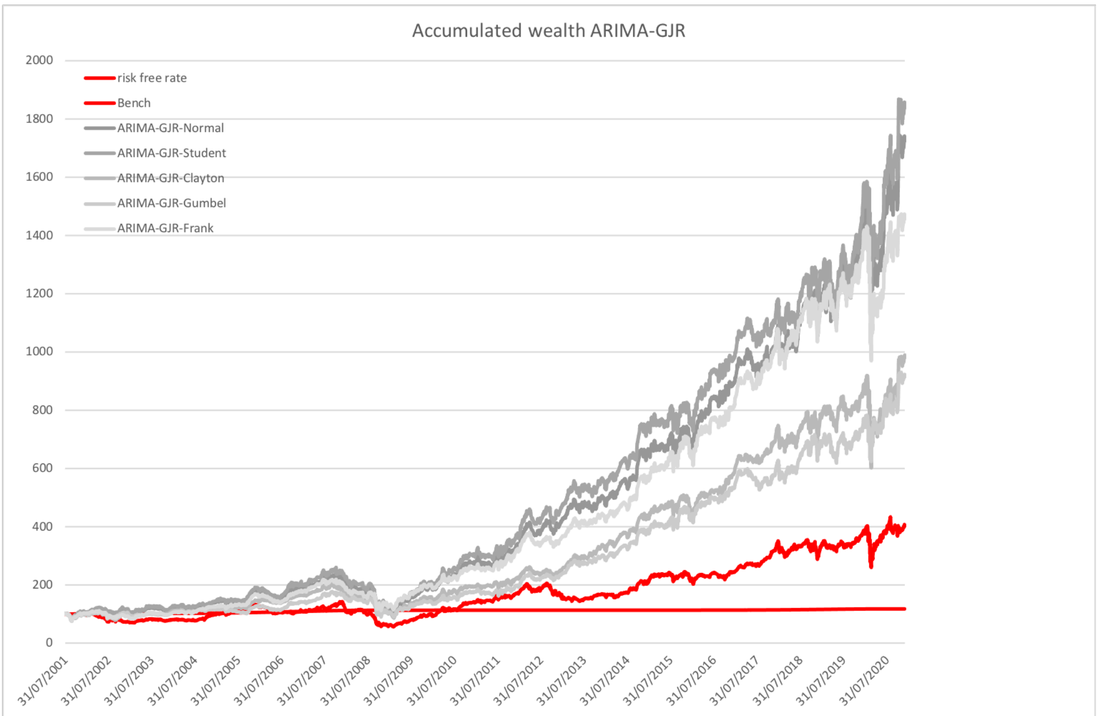
(b) Accumulated wealth for GJR GARCH models:



(c) Accumulated wealth for ARIMA-sGARCH models:



(d) Accumulated wealth for ARIMA-GJR models:



regression outputs. In all plots, we see that the Gumbel and Student t copulas yields the highest performance, and then the Frank copula. In the case of sGARCH and GJR-GARCH under constant means, the Clayton and Gumbel are both outperformed by the benchmark strategy. We do not see fit to use the same type of analysis for the minimum variance and minimum CVaR objectives, as these are risk-based strategies, and not necessarily expected to outperform their benchmarks in terms of accumulated wealth.

Chapter 5

Discussion

5.1 Overall Remarks

As explained in the introduction, the aim of our analysis is to evaluate whether we can find GARCH-EVT-Copula simulation models with favorable applications in the field of portfolio selection. Evaluations are based on cross-comparisons over the table (4.1), in which we look for patterns and consistencies with respect to which type of strategy is being used. We look at overperformance, underperformance and equal performance, relative to the benchmark. The three chosen allocation objectives have differences and similarities with respect to two dimensions: investor preferences and underlying assumptions about the behavior of asset returns. The minimum CVaR impose no particular distributional assumptions on the assets, whereas the two others are bound within the assumption of multivariate normality. The maximum Sharpe objective aims to improve risk-adjusted performance, whereas the two others seek to reduce portfolio risk. With respect to the second dimension, we feel as if we have represented a broader group of investors: the preferences of personal investors and mutual funds on one hand, and those of commercial banks bound by the Basel requirements on the other hand. In other words, if the benefits of GARCH-EVT-Copula models can be cross-validated for all objectives, they would have proven wide applications within the risk management field. Based on the comments in the results section, we know that this is a high bar in the second and third period. Before drawing further conclusions, we want to extend our comments in the results chapter with some more in-dept interpretations in light of the theory previously discussed.

In our interpretation, the most prevalent finding is the general development from the first period to the last. The majority of simulation models outperforms the benchmark under all objectives in the pre-financial crisis period, and then performance appears to deteriorate into the next two periods. This is especially clear under the CVaR objective, which has the highest number of simulation models outperforming the benchmark in the first period, and the highest number of models underperforming the benchmark in the second and third periods. For the maximum Sharpe and minimum variance objectives, these results are less consistent across different models. Also, these are mostly zero-difference results. In relation to CVaR minimization, Huang and Hsu (2015) found evidence to suggest that GARCH-EVT-Copula simulation models perform better under periods of economic expansion. These results are consistent with our findings with respect to the first and second periods, but not for the third however, which has been characterized by mostly strong economic growth. We have not been able to find previous literature on GARCH-EVT-Copula that can support or disprove these pattern for the maximum Sharpe and minimum variance objectives.

5.2 Choices of GARCH and Copula

Under the maximum Sharpe and minimum variance objectives, it clearly emerges from the table that lack of performance under maximum Sharpe and minimum variance is related to the Clayton and Gumbel, both of which asymmetric and Archimedean. These findings are not nearly as prevalent under minimum CVaR. This is interesting, because Wang et al. (2010) reports that the Clayton copula (and Student-t) provide more accurate measure of interdependency than the Normal Copula. This in mind, an interpretation could be that the Clayton copula provides better information into the simulation models and thus the optimization problem, which would yield in better portfolio allocation. Our findings suggests the opposite, where the overall impression is that the elliptical Normal and Student-t copula, following each other relative closely, outperform the other copulas in regards of generating higher proportion of positive coefficients (relative to negative) as well as generally more appealing coefficient values. In his stylized facts, Cont (2001) emphasized the prevalence of gain/loss asymmetries in return distributions. This is not necessarily inconsistent with our findings, since our copulas are applied to the error distributions instead of the return distributions directly.

We also take a closer look at the results for the four different GARCH models. Based on the stylized facts proposed by Cont (2001), we expected the stand-alone GJR-GARCH would prove the better alternative (prevalent asymmetric volatility, and less prevalent autocorrelations). In the pre-financial crisis period, we see that most GJR-based models prove better than the sGARCH-based models, at least under the maximum Sharpe and minimum CVaR objectives, and the mean equation appears to be of less importance. In the post-financial crisis period however, we see all 5/60 coefficients implying satisfactory performance are related to the ARIMA-sGARCH model, all under the maximum Sharpe objective. The post-crisis findings are interesting, because they are highly consistent with the findings of Sahamkhadam et al. (2018). There may appear to have been a shift from the third period from the last, which is not unlikely with respect to the findings discussed in section (2.4), but we do not want to draw any solid conclusions since these results are not cross-validated across multiple objectives in the third period. All-in-all, the ARIMA-sGARCH is archives the highest numbers of satisfactory results across the table.

5.3 Rebalancing Intervals

The overall impression of applied investment horizon across the different portfolio selection strategies is rather consistent. We recall that the return paths are simulated h-days ahead, and that the dispersion of these paths naturally should increase as an effect of increasing the forecasting horizon. Longer rebalancing intervals increases the probability of estimation error, consistent with what one would expect. Table (4.1) report only one single coefficient that suggest a positive impact of increasing the length of forecasting horizons. The remaining coefficients are either insignificant, or significant but indicate worse performance. Although, it is fairly important to (once again) highlight that our research do not account for transaction cost, meaning that the one-day strategies might appear more favorable than what they would do under normal circumstances.

Chapter 6

Conclusion

In our thesis, we have backtested the performance of portfolio selection strategies based on 20 different GARCH-EVT-Copula simulation models, over a period of 18,5 years. The relevant strategies apply three different allocation objectives, and intends for rebalancing in the very short-term (1-5 days). We have discussed whether any of the simulation models are related to superior performance across multiple objectives and sub-periods simultaneously, and 1) the conclusion to this question is negative. Instead, we have found rather strong indications that 2) the performance of these models generally varies across different time frames, and 3) the optimal choice of simulation model appears related to which type of portfolio selection problem is being solved. In relation to Sharpe optimization and variance minimization (both within the mean-variance framework), there are weaker indications that 3) performance may be positively related the ARIMA-sGARCH-EVT-Copual model, and 5) negatively related to the use of asymmetric copulas (Clayton and Gumbel). Also, 6) we have not found similar model-specific indications for CVaR minimization. As the literature on these simulation models is still fairly limited, it has been generally hard to cross-validate findings in relation to similar findings in other studies, *all else equal* (even though we have identified a few consistencies). In addition, we have experienced some problems in our regression analysis. Our all-in-all conclusion is that we can be fairy sure of conclusions (1)-(3), while (4)-(6) must be carefully interpreted as indications.

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Appendix A

R-code

The `rugarch` package is used for specifying and fitting our ARIMA and GARCH models.

```
model1 <- ugarchspec( variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
mean.model = list(armaOrder = c(1, 1), include.mean=TRUE),
distribution.model = "std")
```

$$model1 < -ugarchspec(variance.model = list(model = "sGARCH", garchOrder = c(1,1)), \tag{A.1}$$

$$mean.model = list(armaOrder = c(1,1), include.mean = TRUE), \tag{A.2}$$

$$distribution.model = "std") \tag{A.3}$$

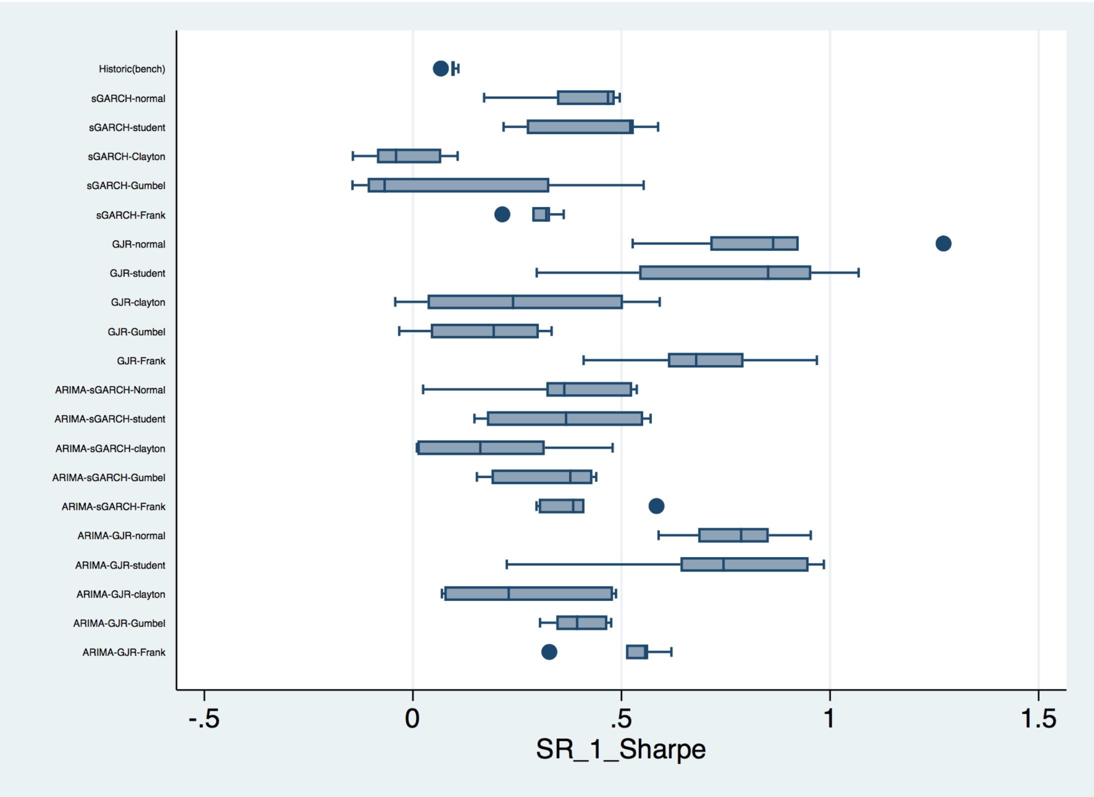
Appendix B

Box plots

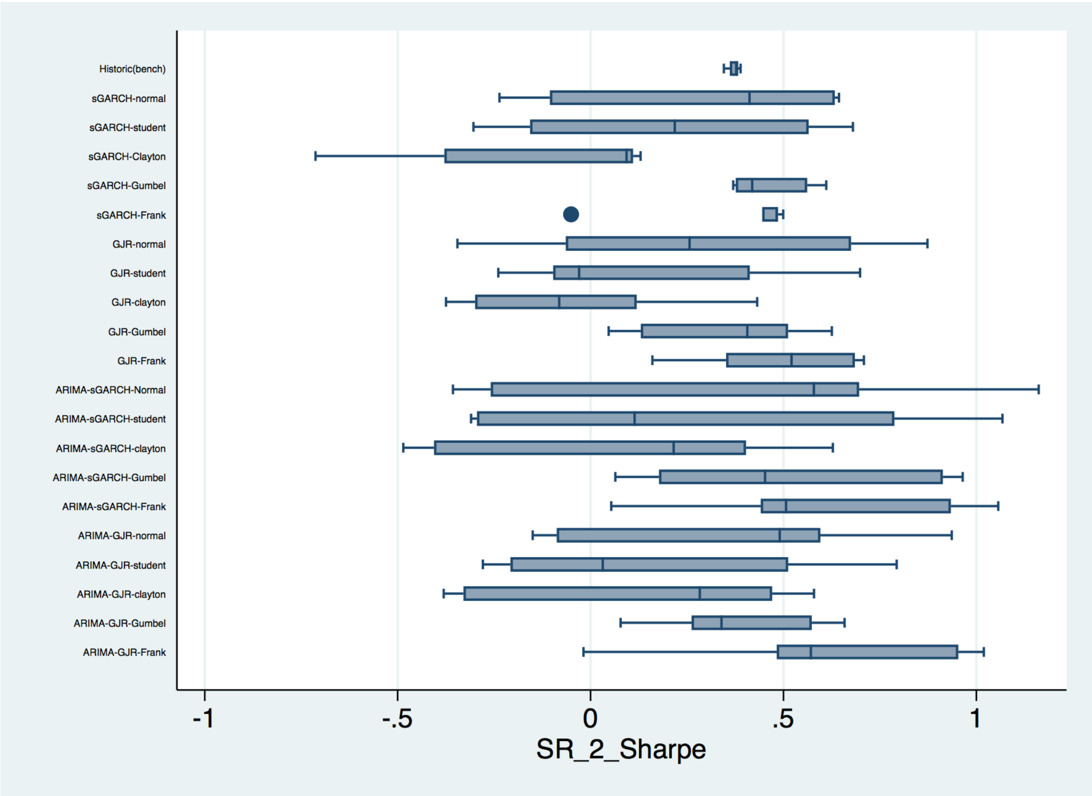
Figure B.1: Box Plots MSR

The figures below displays the calculated Sharpe Ratios from the 20 model compositions as well as the benchmark strategy for the 3 sub-periods (Before, during and after the financial crisis). They are presented in box plots that include observations from applying the respective model composition across the 1-5 day investment horizon. The attractive models are located at the right hand side with low dispersion.

(a) Box plot MSR pre-financial crisis:



(b) Box plot MSR sub-financial crisis:



(c) Box plot MSR post-financial crisis:

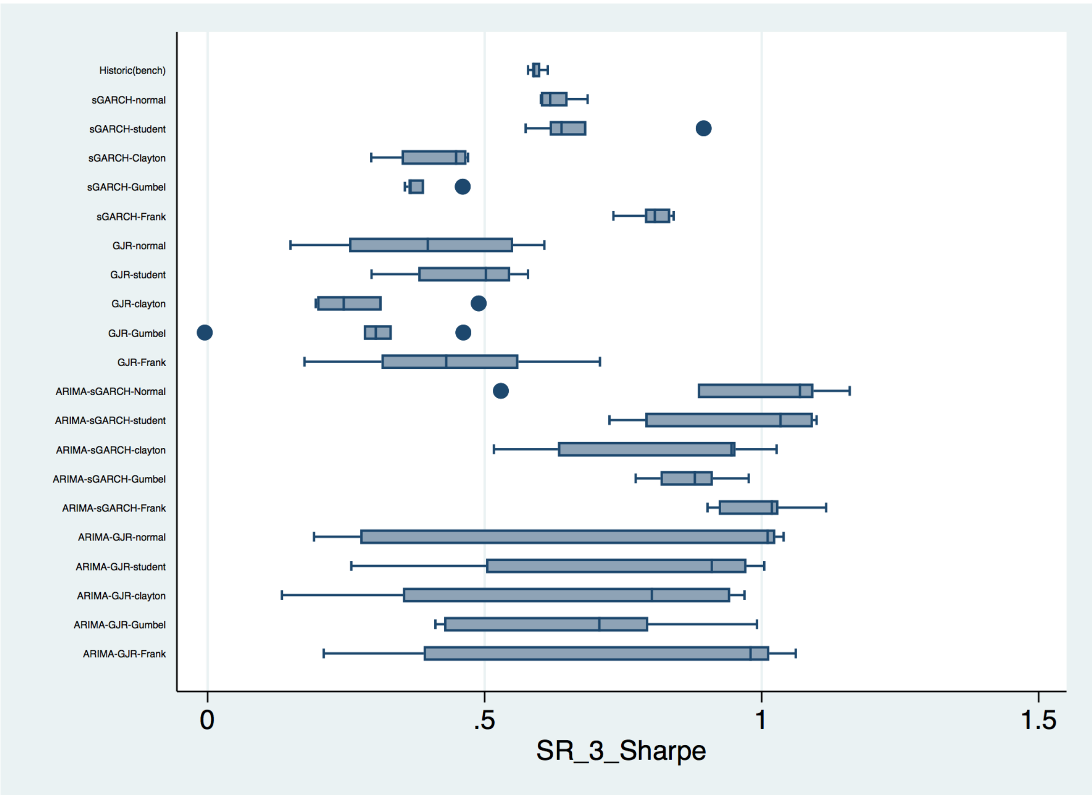
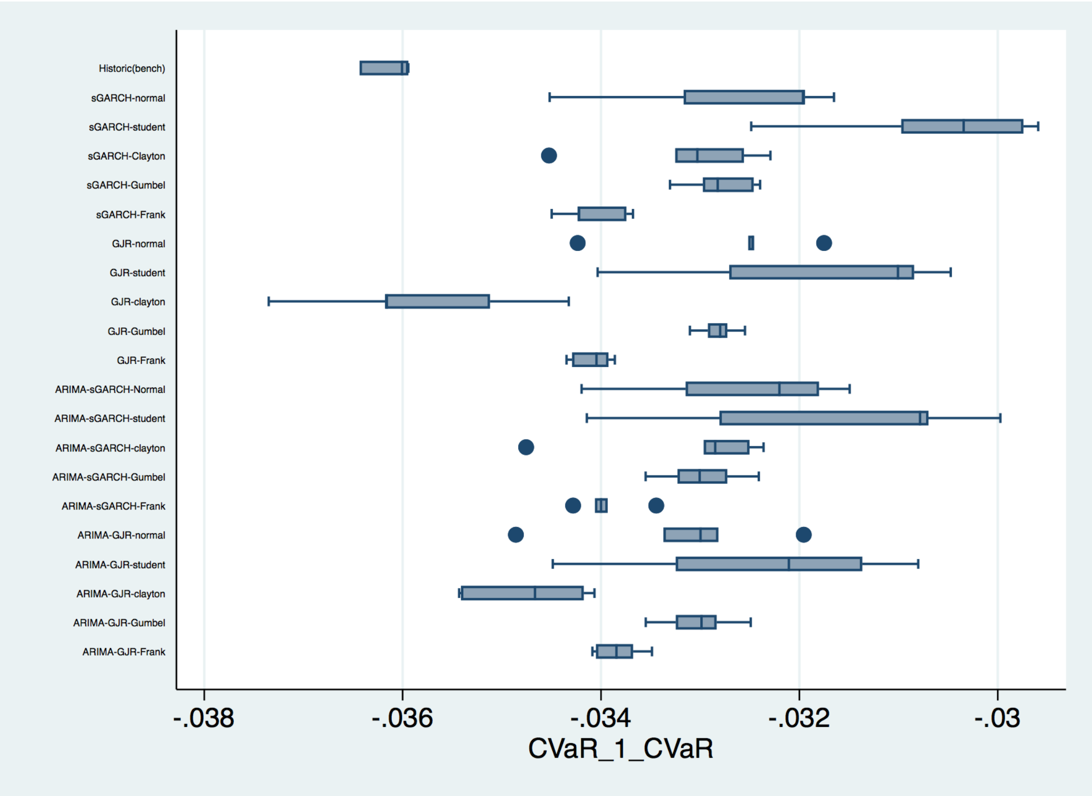


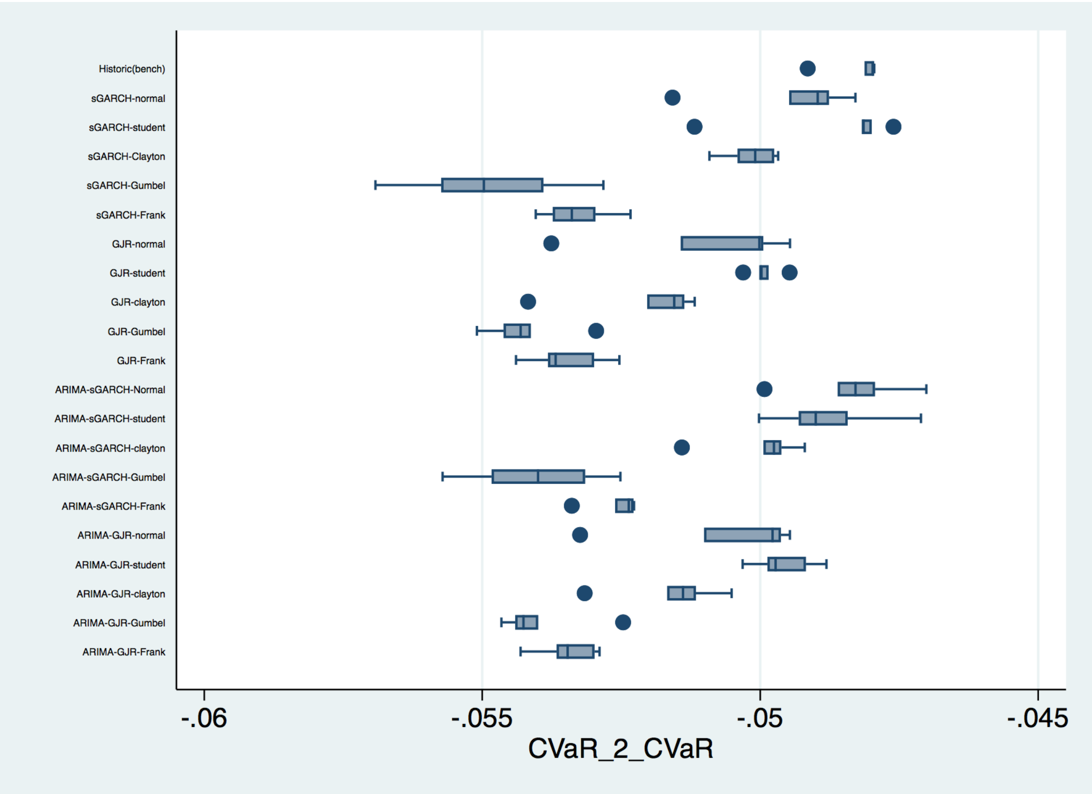
Figure B.2: Box Plots CVaR

The figures below displays the calculated CVaR from the 20 model compositions as well as the benchmark strategy for the 3 sub-periods (Before, during and after the financial crisis). They are presented in box plots that include observations from applying the respective model composition across the 1-5 day investment horizon. The CVaR is expressed in negative values (i.e. losses). The attractive models are located at the right hand side with low dispersion

(a) Box plot CVaR pre-financial crisis



(b) Box plot CVaR sub-financial crisis



(c) Box plot CVaR post-financial crisis

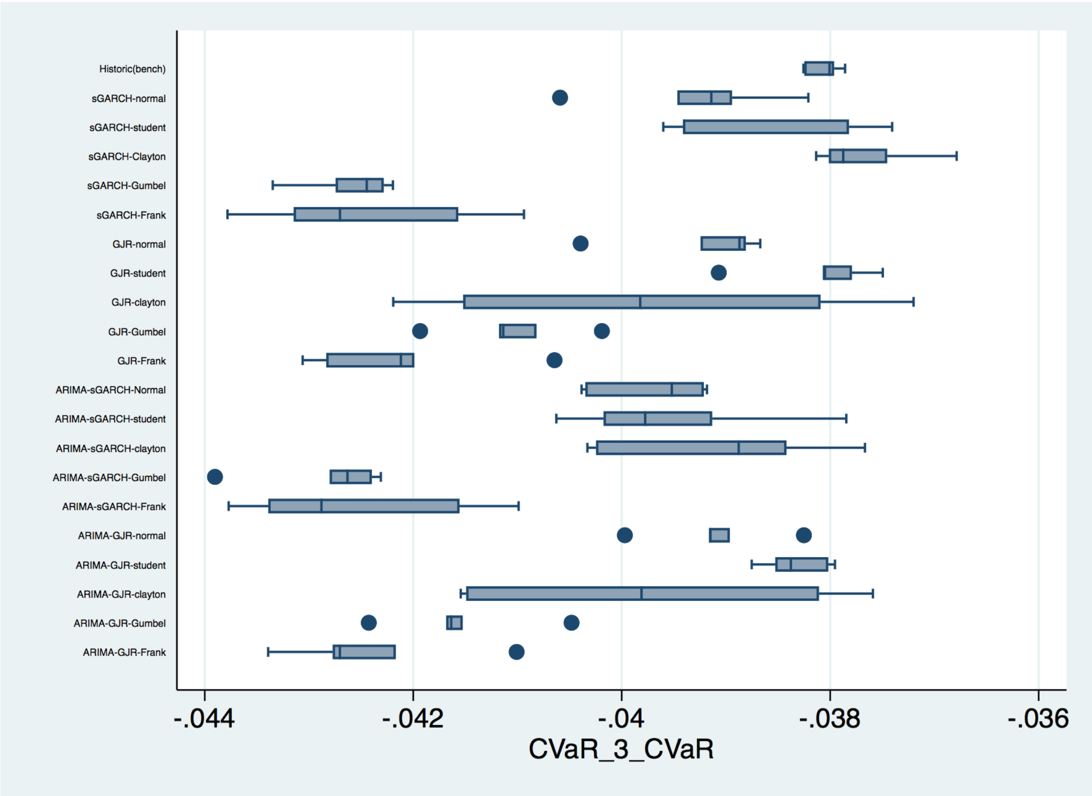
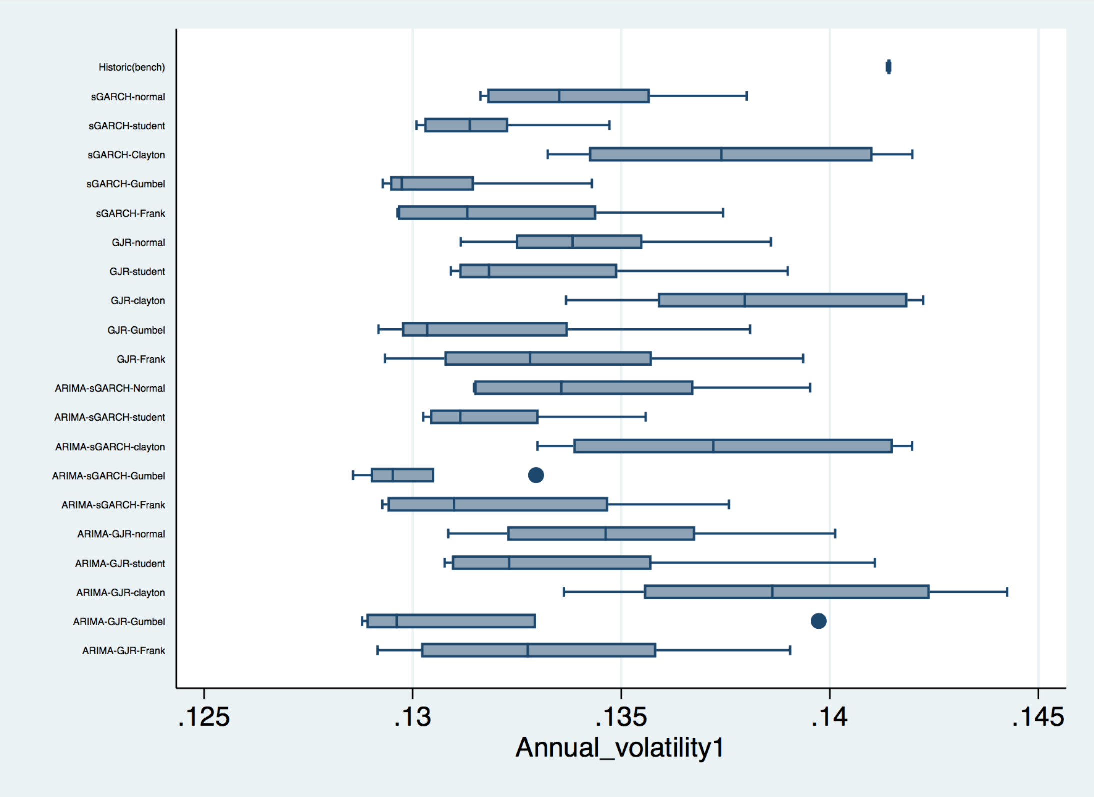


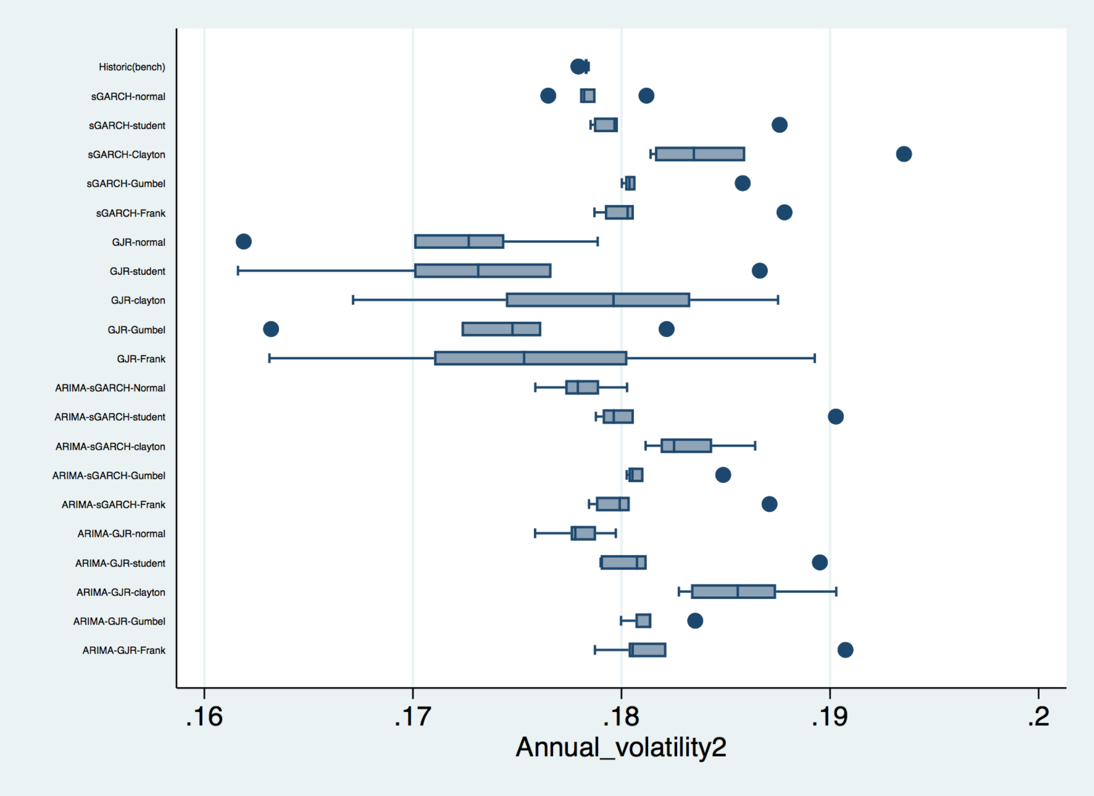
Figure B.3: Box-plot GMV

The figures below displays the calculated annualized unconditional volatility from the 20 model compositions as well as the benchmark strategy for the 3 sub-periods (Before, during and after the financial crisis). They are presented in box plots that include observations from applying the respective model composition across the 1-5 day investment horizon. The higher volatility, the higher risk is estimated. The attractive models are located on the left hand side with low dispersion:

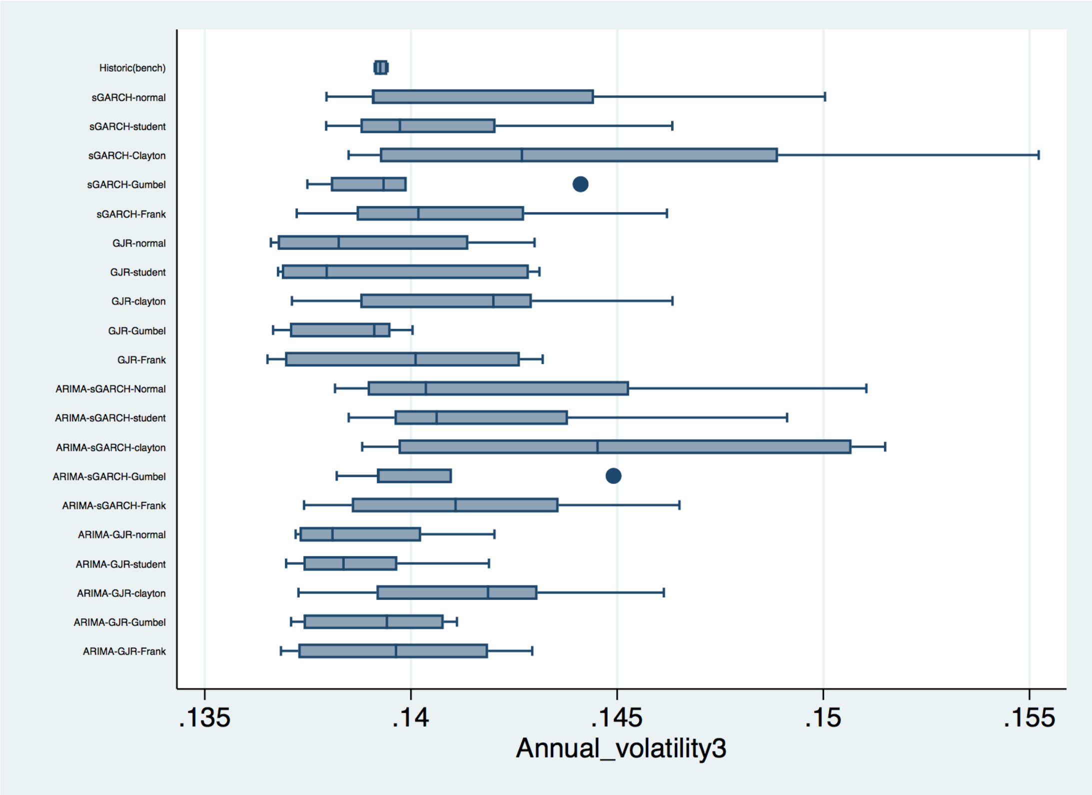
(a) Box plot GMV pre-financial crisis



(b) Box plot GMV sub-financial crisis



(c) Box plot GMV post-financial crisis



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