Specifics of the autonomous energy power unit dynamic processes' modeling

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Abstract. Significant features of modern autonomous power units, along with increasing complexity of structures and modes and increasing requirements for quality of operation, are the heterogeneity of their composition, the presence of links with distributed parameters, changing parameters during operation, the presence of computer components control, monitoring and diagnostics. A characteristic feature of controlled autonomous power units is the diversity of elements that are part of them. This, in turn, usually creates inhomogeneity in the mathematical description of the system. Physical heterogeneity is caused by the presence of elements that are different in physical essence (mechanical, electrical, optical, magnetic and electromagnetic, etc.). Functional heterogeneity is manifested in the presence of different purpose signals (control, measuring, duplicating, auxiliary, etc.). Thus, modern autonomous power units belong to the class of complex inhomogeneous dynamic systems.These features cause the emergence of new requirements for methods and tools for mathematical modeling of dynamic processes in autonomous power units, in particular the need to move from common in practice autonomous study of individual components, to a comprehensive study of the system as a whole using macromodels, model building according to experimental data, increasing the adequacy of mathematical models by rationally taking into account the distribution of parameters of the relevant links, the possibility of rapid changes in the composition of computer models, taking into account physical and functional heterogeneity.The article considers the methods of construction of mathematical models of dynamic processes in modern autonomous power units and in their control systems, diagnostics and testing on the basis of expansion of forms of mathematical descriptions with involvement of integral operators and Volterra equations.

Keywords: autonomous energy power unit (AEPU), mathematical modeling, dynamic model, computer-integrated system, integral models.

1. Introduction

One of the urgent problems of modern energy development is the development of new models and modernization of existing autonomous high efficiency energy power units. The process of development of modern autonomous power units is characterized by a steady trend to expand the scope of their application and increase the level of complexity [1]. Autonomous power units have evolved from systems with simple analog regulators to modern systems with computer control and, as a consequence, with new possibilities, in particular flexibility of control algorithms, providing remote control, etc. Requirements for the functionality and reliability of such installations, improvement of their dynamic characteristics, development of control and diagnostic subsystems, expansion and complexity of control functions with the use of new smart technologies have significantly increased. One of the ways to ensure these requirements is the further development of methods and tools for mathematical and computer modeling of the most autonomous power units and their components, as well as their involvement in the processes of design, control, monitoring and diagnosis. Therefore, the scientific and technical problem of creating and further development based on a structurally-oriented approach to methods and tools of mathematical and computer modeling of dynamic processes in autonomous power units and their control systems, diagnostics to solve problems of analysis, synthesis and construction of controls, is relevant.

Previous experience of research and practical development in the field of mathematical modeling of power units shows that effective ways to solve these problems include the development and expanded application of a structural approach to the formation of algorithms and software, involving dynamic models in the form of integrated operators and integrated equations that have a number of positive properties such as high versatility (the structure of the model remains unchanged, and the properties of the object are set by one function - the core of the integral operator), potentially high adequacy of modeling processes, smoothing experimental data (possibility of use in real systems with significant level of high-frequency noise spectra), high convergence of iterative computational processes, the possibility of efficient construction of macromodels, etc. [2;3].

Research in the field of mathematical and computer modeling in the development of mathematical models and methods, algorithms and software and computer tools for their implementation to ensure the development of modern AEPU is expanding every year in the world. The most famous research centers that carry out developments in this area are divisions of the world's leading companies: "General Electric", "Siemens", "Electricite de France". Development of such systems is performed by power engineering companies such as "Westinghouse ", "Babcock", "Toshiba", "Hitachi". Areas of research of foreign scientists are complex and component modeling of processes in power units and test systems based on the use of multilevel modeling, object-oriented software, distributed architecture, information integration, multidisciplinary approach to building models of real objects. At the same time, an important trend that has been actively developing over the last decade is the creation of complex multi-program modeling complexes.

2.Alternativeness of the dynamic models.

The desire to ensure high quality computer tools and take into account the limitations of resources leads to the need to explore the possibility of using different types of mathematical descriptions as models of dynamic objects. The problem of research and use of nonparametric dynamic models in the form of integral equations and operators, which have significant positive features, as well as features in the description of a wide class of processes and in computer implementation, is not completely solved.

Parametric dynamic models include differential equations (ordinary and partial derivatives), the solution of which is primarily focused on numerous (serial) software products designed for the study of dynamic objects [5]. Parameters (coefficients) of differential equations are directly or indirectly determined by the given parameters of the object. The class of nonparametric dynamic models consists of different types of integral equations and their systems, the properties of which are determined by the structure of equations and kernels included in integral operators. The kernels of operators in the case of scalar integral dynamic models are functions of two variables formed according to the given dynamic characteristics of the object. In addition to these parametric and nonparametric dynamic models, there are also models of mixed type, namely dynamic models in the form of integro-differential equations, which combine the properties of parametric and nonparametric models.

3. Modeling of processes in computer-integrated systems.

Intensive development of computer and computerized means of information processing in modern technical systems is characterized by the ever-increasing complexity of their structures and modes, increasing requirements for the quality of operation (speed, accuracy, reliability, efficiency, etc.). These factors cause new requirements for methods and means of mathematical modeling of dynamic processes in these systems.

The most obvious example of the effective use of integral equations in the study of dynamics problems are control systems for technical objects. The main tasks to be solved are the analysis of a given system in order to determine the characteristic reactions to actions and obstacles; control, which consists in finding the control signal, as a result of the influence of which on the investigated system its reaction meets the requirements for the quality of the control process [4;5].

Methods for solving these problems significantly depend on what kind of mathematical description of the management process is used. In the classical theory of automatic control, the mathematical model of the system is an ordinary differential equation that depends on the derivatives of the input and output signals of certain orders. If the studied system is linear and stationary, then to develop the most rational methods for solving the main problems of control theory, other equivalent methods of description are also introduced, based on the use of transfer functions, frequency and time characteristics. In addition to stationary objects , linear systems with variable parameters are intensively studied and used, for the mathematical description of which either parametric models in the form of differential equations with variable coefficients or increasingly nonparametric models are used.

In addition, the reason for the increased interest in finding mathematical models that are adequate to the problems solved in the theory of controlled systems is the widespread introduction into practice of object management processes with distributed parameters or with a delayed argument. There is a need to build such a mathematical model of the controlled process, which would allow from a single standpoint to effectively solve the main problems of control theory of all possible classes of continuous systems. As such mathematical descriptions in many cases it is expedient to use integral linear or nonlinear equations of Volterra of the II kind. Significant advantages of integrated models have in the case when the system under study is non-stationary, contains elements of delay or with distributed parameters. Integral Fredholm- type equations arise naturally when modeling oscillatory processes and when solving problems of constructing optimal control. This allows us to build optimization algorithms based on the use of numerical methods for solving integral Fredholm equations. In addition, it is possible to build stable methods for calculating the output signals, simplifying mathematical models, analyzing the main types of connections and checking the stability of the system.

Thus, there are important grounds for the effective use of an integrated approach in solving problems that are essential for the theory and practice of management [5].

In connection with the increasing requirements for modern processes of information processing and management, the problem of implementing mathematical models of dynamics becomes extremely important. Quality control of the system depends on the accuracy of its mathematical model. The experience gained in designing systems shows that it is impossible to build a mathematical model adequate to a real system, only on the basis of theoretical studies of physical processes in the system. Therefore, in the process of designing control systems simultaneously with theoretical research, numerous experiments are conducted, including computational ones, to determine and refine their mathematical models. In the presence of rather complex mathematical models, the efficiency of a computational experiment is determined by the quality of the available algorithmic and software. In particular, the task of developing or selecting software for solving dynamics problems based on the use of integrated dynamic models is relevant.

Depending on the type of facility and the degree of adequacy of the corresponding mathematical model equation, which describes the object may be ordinary differential, one-dimensional or multidimensional integral, integral-differential or operator equations, differential equations with partial differentials and so on. In general, these equations are mixed, as they include functions of various internal and external excitations acting on the system [2].

4. Input-output models.

The "input-output" model is a description of the connection between the input and output signals of a dynamic system. The need for such a description emerges when considering the behavior of individual units and in particular object management, and the entire management system as a whole. Differences in the mathematical description of the blocks and control systems are not fundamental, but need Use cation of different signs.

In the theory of control in the analysis and synthesis of control systems use mathematical models of "input-output", which are given in the form of transfer functions and models given in the state space, which describe the processes occurring in the control system.

The control system and any of its elements perform the conversion of the input signal $u(t)$ into an output signal $y(t)$. From a mathematical point of view they perform display

$$y(t) = Au(t), \qquad (1)$$

according to which each element $u(t)$ of the plurality of input signals is associated with a certain element $y(t)$ of the plurality of output signals; A is the system operator that determines the mapping between the input and output signals of the control system (element). A model that has the form (8) is called an operator model of the object.

To describe linear control systems in operator form are often used transfer functions $W(p) = B(p)/A(p)$, the form of which stems from the dependences that look like $Y(p)A(p) = X(p)B(p)$, where $y(p)$, $X(p)$ - Laplace mapping corresponding output $y(t)$ and input $x(t)$ signals. $A(p) = a_0 p^n + a_1 p^{n-1} + ... + a_n$, $B(p) = b_0 p^m + b_1 p^{m-1} + ... + b_m$, $A(p)$ - characteristic polynomial or own operator, and complex numbers p_i, $i = \overline{1, n}$ are the roots of the characteristic equation $A(p) = 0$ that are called the poles of the system; $B(p)$ - characteristic polynomial of the right part or the operator of influence. The roots of the equation $B(p) = 0$ are called the zeros of the system.

Models set in the state space. Of great importance for the study of control systems are models set in the state space. Common form of these models:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases}$$

where $x(t)$, $u(t)$ and $y(t)$ are the state, input and output of the system, respectively. A, B, C and D - constants, predefined, or experimentally obtained matrices.

Discrete models. A dynamic system is called discrete in time if at least one of its signal spaces is a space of sequences, i.e. discrete in time input signals x_n or output y_n, where n - discrete time. The description of the discrete process can be represented as a solution of the difference equation. Discrete models either reflect the dynamics of real time- quantized processes, or are a form of approximate description of continuous-time systems. In the latter case, there is a need to consider the issues of quantization and methods of transforming dynamical systems into discrete forms, ie their sampling.

5. Integral dynamic models.

Integral equations include such functional equations that contain an integral transformation over the desired function. [5] In a fairly general case, linear integral equations can be represented as

$$g(x)y(x) - \lambda\int_{\Omega} K(x,s)y(s)\,ds = f(x), x \in Q, \quad (2)$$

where $K(x,s)$ – kernel of the integral equation, $f(x)$ – right part of the equation with the domain Q, λ – parameter of the equation (often considered equal to 1 or -1), $y(s)$ – the desired function with the domain Ω, variable in the case of equations:

$$y(x) - \int_{a}^{x} K(x,s)y(s)\,ds = f(x), x \in [a,b],$$

$$y(x) - \int_{a(x)}^{b(x)} K(x,s)y(s)\,ds = f(x), x \in Q;$$

or constant – in this case (1) is the Fredholm equation, one-dimensional or multidimensional. The functions $g(x)$, $K(x,s)$, $f(x)$ and domains Q and Ω are considered specified. Meanwhile functions $K(x,s)$, $f(x)$, $g(x)$ and $y(s)$ can be both complex and real, and the variables *x* and *s* – only real.

When $g(x) \equiv 0$ equation (2) is an equation of the first kind, written in the form

$$\int_{\Omega} K(x,s)y(s)\,ds = f(x), x \in Q; \quad (3)$$

in this case, the Volterra equation $\Omega = Q$ and for the Fredholm equation, generally speaking $\Omega \neq Q$. If $g(x) \neq 0$, then equation (2) allows division by $g(x)$, i.e. in this case we can consider the equation of the form

$$y(x) - \lambda\int_{\Omega} K(x,s)y(s)\,ds = f(x), x \in Q, \quad (4)$$

which is an equation of Fredholm or Volterra of the second kind. If $g(x) = 0$ for some, but not all, $x \in Q$, then (2) there is an equation of the third kind.

Note that there are a number of integral transformations (Laplace, Mellin, Hankel, Hartley, Bessel, etc.), which (along with the Fourier transform) can be considered as important examples of the Fredholm integral equation of the first kind and which, in principle, can be called Fourier integral equations, Laplace, Mellin, Hartley, Bessel and so on.

In a nonlinear integral equation, the integral containing the desired function $y(s)$ is written as the Urison operator $A_Y[y] \overset{\Delta}{=} \int_{\Omega} K[x,s,y(s)]\,ds$ or as the Hammerstein operator $A_\Gamma[y] \overset{\Delta}{=} \int_{\Omega} K(x,s)F[y(s)]\,ds.$

The most common types of nonlinear equations:

Urison's equation of the second kind - $y(x) - \int_{\Omega} K[x,s,y(s)]\,ds = f(x), x \in \Omega;$ (5)

Urison's equation of the first kind - $\int_{\Omega} K[x,s,y(s)]\,ds = f(x), x \in \Omega;$ (6)

Hammerstein's equation of the second kind –

$$y(x) - \int_{\Omega} K(x,s)F[y(s)]\,ds = f(x), x \in \Omega; \quad (7)$$

Hammerstein equation of the first kind $\int_{\Omega} K(x,s)F[y(s)]\,ds = f(x), x \in \Omega.$ (8)

For a wide class of nonlinear dynamic objects, the relationship between influence and reaction in explicit form can be represented by a functional power (integro-power) series. The "input-output" ratio for a continuous dynamic object with one input and one output can be represented by a series of Volterra in the form

$$y(t)=w_0(t)+\int_0^t w_1(\tau)x(t-\tau)d\tau+\int_0^t\int_0^t w_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2+ \\ +\int_0^t\int_0^t\int_0^t w_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3+... \qquad (9)$$

where $x(t)$ and $y(t)$ – respectively, the input and output signals of the system; $w_n(\tau_1,\tau_2,...,\tau_n)$ – weight function or Volterra nucleus of the n^{th} order (n = 1,2,3,…), function, symmetric with respect to real variables $\tau_1,\tau_2,...,\tau_n$; $w_0(t)$ is a free member of the Volterra series under zero initial conditions $w_0(t)\equiv 0$; t is the current time.

The Volterra series can be written in abbreviated form under zero initial conditions

$$y(t)=\sum_{n=1}^{\infty}y_n[x(t)]=\sum_{n=1}^{\infty}\int_0^t...\int w_n(\tau_1,\tau_2,...,\tau_n)\prod_{r=1}^{n}x(t-\tau_r)d\tau_r, \qquad (10)$$

where $y_n[x(t)]=y_n(t)$ is the n^{th} partial component of the system response.

The system of linear integral equations of type (1) is written in the form

$$g_i(x)y_i(x)-\lambda\sum_{j=1}^{N}\int_{\Omega_j}K_{ij}(x,s)y_j(s)ds=f_i(x), \quad x\in Q_i,\ i=\overline{1,N}, \qquad (11)$$

either as (2), but $y(s)$, $f(x)$, $g(x)$ should be vector functions, and $K(x,s)$ – matrix function.

The growing complexity and expansion of modern tasks of analysis and design of dynamic systems have significantly changed the situation. Along with the classic problem of analyzing the practice of investigation of dynamic operator means includes signal processing tasks describe objects without prior knowledge of the laws of their functioning, problem inverse problem analysis - the identification, control synthesis, diagnostics systems and so on . In these circumstances the possibility of differential equations in many cases proved inadequate, which primarily relates to the problems of unsteady simulation systems, control systems with distributed parameters, recovery signals and so on . Interpretation of the properties of dynamic objects based on the concept of aftereffect, the development of structural research method led to the practical use of integral operators as mathematical models for elements of systems and systems in general. As a result, nowadays integral equations have become widely used to solve many problems of modeling dynamic objects and systems.

Formation of integrated dynamic models.

Integral dynamic models can be obtained by the following three main groups of methods: 1) based on the use of physical laws, which is characteristic of the problems of modeling dynamic processes in environments and systems that obey the provisions of existing theories (for example, processes in electrical devices , or processes in the mechanisms described by the rules of the theory of machines and mechanisms, etc.); 2) using mathematical methods of equivalent transformations of known models in the form of differential equations into integral models, 3) based on the structural method, which allows to form integral models for a given structure and their dynamic characteristics or characteristics of elements included in this structure. This method is closest to the established practice of research and design of technical facilities based on a systems approach, in connection with which it is considered below [2;3].

Structural method. A linear model of a dynamic control object with one input and output is formed on the basis of dynamic characteristics, i.e. dimensional physical dependencies that reflect the properties of the object. The study of these characteristics is important for determining the class of the object.

An arbitrary linear dynamic model of an object has the form

$$y(t)=\int_{-\infty}^{t}k(t,\tau)x(\tau)d\tau, \qquad (12)$$

where $k(t,\tau)$ is the impulse transition function, which is a dynamic characteristic. It follows from (12) that the momentum transition function is a reaction of a linear model of an object to perturbations in the form of a δ-function at the moment $t=\tau$ (under zero initial conditions). In this case, the values $y(t)$ at the time t are affected by the values $x(t)$ in all previous moments of time. The pulse transition function plays the role of a weight function and characterizes the degree of influence of the input signal $x(t)$ applied at the moment τ on the formation $y(t)$

at the moment *t*. Examples of objects for which the impact of previous input signals decreases but does not disappear, there are objects with properties: $k(t, \tau) = e^{-\alpha(t-\tau)}$, $k(t, \tau) = e^{\alpha(t-\tau)} \cos \omega\tau$, $\alpha > 0$ and others. An object with finite memory is an object whose pulse transition function vanishes after a finite period of time T after the signal is given in the form of a δ-function at the moment τ, ie $k(t, \tau) = 0$ at $t - \tau > T$. In this case

$$y(t) = \int_{t-T}^{t} k(t, \tau) x(\tau) d\tau. \quad (13)$$

In the general case, the type of reaction to the signal in the form of δ-function depends on the moment of its submission τ and the moment of its observation *t*, that is, $k(t, \tau)$ is a function of two arguments. For objects with constant parameters (stationary), the type of reaction depends only on the time from the moment of application to the perturbation of the signal, ie on the difference $t - \tau$. Therefore, for stationary objects $k(t, \tau) = k(t - \tau)$, the relation (12) will take the form

$$y(t) = \int_{-\infty}^{t} x(\tau) k(t-\tau) d\tau = \int_{0}^{\infty} k(u) x(t-u) du , \quad (14)$$

For linear models of objects with *m* inputs and *n* outputs, the dynamics model has the form

$$y_i(t) = \sum_{s=1}^{m} \int_{-\infty}^{t} k_{js}(t, \tau) x_s(\tau) d\tau , \ j = \overline{1, n} , \quad (15)$$

where $k_{js}(t, \tau)$ is the pulse transition function on the js^{th} channel, which is defined as the reaction on the j^{th} output to the perturbation $x_s(u) = \delta(u - \tau)$ at $x_p \equiv 0$ (for all $p \neq s$).

Implicit integral models of dynamic objects. In describing the dynamic objects with complex layouts, including the presence of inverse relationships formed implicit object model in the form of equations for the unknown functions. Integral equations of the Volterra type are used as dynamic models in this case . In the case of oscillating modes, the model takes the form of the Fredholm equation . Consider a dynamic object with feedback, which contains one nonlinear link, the static characteristic of which has the form

$$y = F(x), \quad (16)$$

linear part with transfer function $W(p) = \dfrac{K(p)}{D(p)}$. Then the equation of motion of the system has the form

$$\bar{x} - \left[W(p) \bar{y} + \bar{f} \right] = 0 , \quad (17)$$

where *f* is the external influence.

Taking into account (16), equation (17) in the real domain is written as

$$x(t) = \int_{0}^{t} g(t-\tau) y(\tau) d\tau + f(t), \quad (18)$$

where $g(t-\tau)$ is the weight function corresponding to the transfer function *W*(*p*). Substituting (16) in (18), we obtain the integral equation that determines the transients *x*(*t*) in the system, in the form

$$x(t) = \int_{0}^{t} g(t-\tau) F\left[x(\tau) \right] d\tau + f(t) . \quad (19)$$

Also promising is the application of the method of integral equations to studies of periodic solutions of nonlinear systems. In this general case, the weight function of the linear part will be an arbitrary function of two variables $g(t, \tau)$, and $g \equiv 0$ for $\tau > t$, and instead of (18) will be valid

$$x(t) = \int_{-\infty}^{t} g(t, \tau) y(\tau) d\tau + f(t) , \quad (20)$$

Conclusion

Thus, different approaches to models' creation for dynamic objects and systems have been considered and the ones based on the Volterra integral equations were shown to take into account the most of the heterogeneous dynamic models' specifics.

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