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# The Impact of Digital Twins on Local Industry Symbiosis Networks in Light of the Uncertainty Caused by the Public Crisis

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## ABSTRACT

Digital twins provide a solution for information-sharing between enterprises, thereby alleviating uncertainties in the supply chain. In light of the public crisis caused by COVID-19, the authors suggest a signal game model for a two-stage supply chain with two suppliers and two manufacturers. Based on the model, the impact of the digital twin platform on the profits of the local industrial symbiosis network is analyzed. The results show that the uncertainty of supply and demand caused by the public crisis has led to fluctuations in profits and profit volatility. Under this influence, suppliers are willing to participate in information sharing on the digital twin platform, but manufacturers are less willing to participate. Moreover, application of the digital twin platform in information sharing is conducive to maintaining and promoting the smooth operation of the industrial chain under these conditions of uncertainty.

## KEYWORDS

Digital Twin, Game Theory Model, Industrial Symbiosis Network, Information Sharing, Uncertainty

## INTRODUCTION

The industrial symbiosis (IS) network refers to the long-term cooperative symbiosis formed by the transfer and exchange of material, energy, knowledge, and human and technological resources between companies within a region. The network aims to obtain both environmental and competitive benefits (Wang, Mishima, & Adachi, 2021). The enterprise-level IS network and hybrid network, including IS and traditional modes of manufacturing, are newer endeavours in Norway. However, COVID-19 has caused economic turmoil worldwide since the beginning of 2020. Except for some basic industries (i.e., medical, public security, food retailing, etc.), most industries have suffered a severe shock. Thus, Norway is experiencing has its highest unemployment rate since World War II.

COVID-19 has brought uncertainty to the manufacturing industry and production process due to uncertain supplies, transportation disruption, and indeterminate demand (Shrivastava, Ernst, & Krishnamoorthy, 2019). In addition, many companies on the IS network have not established a fixed mode of information communication and transaction. When dealing with shocks like COVID-19, difficulties in information sharing and communication lead to greater challenges than faced by companies in the traditional supply chain. First, the material supply is highly uncertain. It is impossible to order recycled materials or predict their output because recycled materials are not a mainstream

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product of the suppliers. The output of recycled materials depends on the output of mainstream products (Liao & Li, 2016). Greater instability of mainstream product supply chains during COVID-19 makes their supply on the symbiosis network more unstable. Second, the costs and environmental impacts of production plans based on renewable materials must be evaluated. Availability of recycled materials is low and quality is unstable. Compared with traditional methods like landfills and incineration, the renewable remanufacturing processes may lead to unexpectedly high production costs, which cause more environmental pollution (Prosman & Sacchi, 2018). However, during COVID-19, communication between companies was restricted and could not be assessed in due time.

To solve this uncertain challenge, more information sharing between enterprises on the network is necessary (Chan, Liu, & Szeto, 2017; Kiil et al., 2019). Digital Twins (DT), as an important technology for the realization of Industry 4.0, can combine the Internet of things (IoT), artificial intelligence (AI), machine learning, and software analysis with spatial network diagrams to create real-time digital simulation models. These models are updated and changed as the physical copy changes (Zhang et al., 2019). As an emerging solution for data integration and real-time processing to realize intelligent production, the DT platforms have the advantages of real-time data transmission, data analysis, and information visualization (Qi & Tao, 2018). This provides a potential solution for information communication of enterprises on the current IS network. However, there is limited research on the impact of the DT platform on the IS networks.

The authors of this study analyse the impact of the DT-based vertical information sharing between enterprises in the local IS network under a public crisis represented by COVID-19. It aims to illustrate the economic impact of the DT platform's information-sharing function on the IS supply network. First, the authors establish a signal game model framework to describe a mixed IS network composed of two suppliers and two manufacturers. Second, based on the scenario analysis, the authors model three scenarios in which two manufacturing companies agree or disagree to share demand information with suppliers through the DT platform. Based on the solution of the models, the authors compare the consequences of these decisions and discover the influence of the platform on the amount and stability of enterprise profits.

The main contributions of this article are reflected in three aspects. First, regarding the aspect of content, current research on IS and the DT is undergoing rapid development. At present, there is little research on solutions to the information sharing of IS enterprises and application of the DT platform in interenterprise information sharing. This paper demonstrates the role of the DT information-sharing function on the IS network based on the signal game model, enriching the research content of the IS field and the DT in the cross-enterprise application field. Second, regarding the method, this paper constructs an IS network game model of two suppliers and two manufacturers. It enriches not only the research related to such models, but also the research into IS networks. Third, regarding the application, this paper increases the understanding of its application in cross-enterprise information sharing and promotes the digital transformation of IS networks.

This paper is arranged as follows. The second section, the literature review, discusses current research on issues related to information sharing in IS, the application of the DT in information sharing, and the enterprise game model in the supply chain. A supply chain model for IS is constructed in the third section, which includes four companies in the supply chain, their production relationships, and three types of information sharing models among these companies. The fourth section is a review of the results. Based on the reverse induction method, the outputs, prices, and profits of the companies in equilibrium under the information sharing modes are obtained. The fifth section provides a comparative analysis of the equilibrium solutions obtained in the fourth section and discusses the parameters in the model. The last section is the conclusion and implications.

## Background

The literature related to this article includes three topics: (1) information sharing in IS; (2) application of the DT to information sharing; and (3) game models for enterprises in the supply chain. This section reviews the existing bodies of literature on these topics. For each of the research areas, the authors summarize from the aspects of author, subject, and method (see Appendix A).

## Information Sharing in IS

The government promotes information sharing among companies in the symbiosis network (D'Hauwers, van der Bank, & Montakhabi, 2020). Still, information sharing is a main challenge in the development of the current IS system (Florencio de Souza et al., 2020; Shi & Chertow, 2017). Concerns focus on the unwillingness of companies to share their information and technical challenges of building the platform.

Along with the framework of technical solutions, information-sharing technology research focuses on the identification and matching of IS opportunities based on specific cases (Maqbool, Mendez Alva, & Van Eetvelde, 2019; Yazdanpanah, Yazan, & Zijm, 2019; Yeo et al., 2019). Framework technology solutions include a sustainable cooperation paradigm (Xiang & Yuan, 2019). Research on identifying opportunities includes blueprints on energy and material information sharing (Cervo et al., 2020), the opportunity identification model (Cervo et al., 2019), the enterprise-matching model (Ghali & Frayret, 2019), and the demand-matching strategy (Fraccascia & Yazan, 2018).

To improve information sharing in the IS system, it is necessary to solve technical problems and incentivize the willingness to share information between all companies in the symbiosis network (Luciano et al., 2016). Such companies lack confidence in the benefits of information sharing. Therefore, it is necessary to demonstrate the advantages of information sharing to symbiosis enterprises.

## Application of DT to Information Sharing

Multiple enterprises form a dynamic enterprise alliance; therefore, development of the IS system will encourage companies to have a deeper cooperation within the supply chain (Qi & Tao, 2018). Moreover, solving problems related to information sharing is conducive to reducing uncertainties in production for symbiotic enterprises. At present, studies mention that information sharing is a main challenge faced by IS networks within the context of Industry 4.0 (Cervo et al., 2020). However, few studies provide general information-sharing solutions from enterprise pairing on IS networks regarding how to produce cooperation among enterprises (Fraccascia & Yazan, 2018). By analysing the competitive behaviour of upstream and downstream companies in the supply chain, the DT can simulate cooperation by companies at strategic, tactical, and operational levels (Cavalcante et al., 2019; Haag & Simon, 2019; Lutters, 2018). These efforts will enhance decision-making support (Smith, 2020).

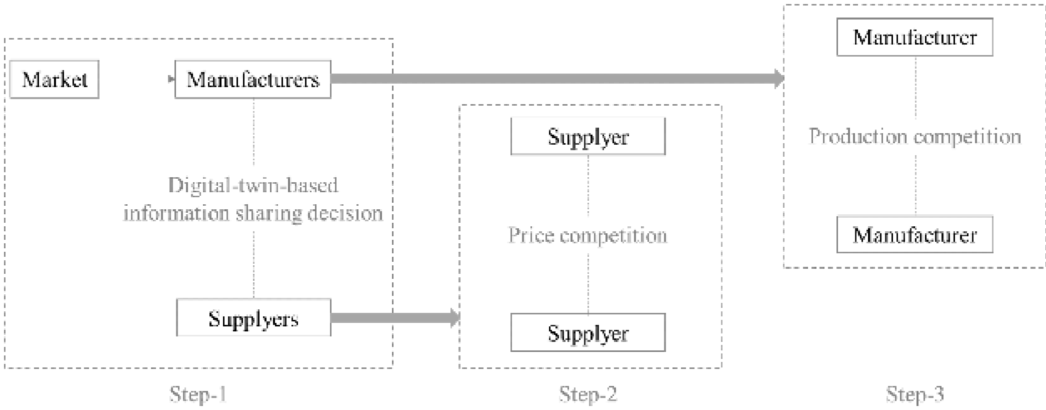
Current DT applications in information sharing can be divided into the DT for supply chains and DT for workshops in factories. Applications at the workshop level are the basis for applications at the industry chain level. Some companies have applied the DT to the production process (D'Angelo & Chong, 2018), strengthening their confidence in the development and application of the DT at the supply chain level. However, few studies analyse the application of the DT in the IS network.

## Game Models for Enterprises in the IS Network

At present, game theory is mainly used in researching IS networks to solve problems of enterprise income distribution, resource optimization allocation, and enterprise cooperation mode in IS industrial parks. The income distribution among IS enterprises includes reducing waste discharge costs and total purchasing costs and sharing profits to compensate for technology conversion costs (Tan et al., 2016; Yazan, Yazdanpanah, & Fraccascia, 2020). For example, Parlar, Sharafali, and Goh (2019) introduced a framework for analysing allocations of the conversion costs of an IS system.

The optimal allocation of resources is often solved through a combination method of linear programming (LP) and game theory. For example, Tan et al. (2016) applied a model based on a LP cooperative game to distribute the benefits generated by the integration of factories. Leong et al.

**Figure 1. Players and their connections**




(2016) built a cross-enterprise water cooling network based on multiobjective LP and an analytic hierarchy process to promote cost savings.

The strategic cooperation between IS enterprises predicts the evolution mechanism of the symbiosis system in the industrial park (Luo, Wang, & Shi, 2019). For example, Zare Mehrjerdi and Lotfi (2019), who designed a flexible and sustainable supply chain network based on a two-stage mixed integer LP, took an automobile assembly company as a case for the model. Similarly, Lotfi et al. (2019) designed a closed-loop supply chain to achieve environmental, economic, and social optimization through the launching and operating of material flows. Ramos et al. (2018) devised a multileader follower game (MLFG) model for designing a utility network for an ecological industrial park (EIP), whichwas verified in a case study of an ecoindustrial park in Norway.

## Model

### Model Framework and Decision-Making Process

Using the two-stage supply chain structure of Wu, Wang, and Shang (2019), the authors constructed a local industry symbiosis oligopoly market competition model consisting of two suppliers and two manufacturers. In this game model, four players with asymmetric information participate in the signal game (see Figure 1).

In Figure 1, the game is divided into three steps.

- **Step 1:** Manufacturers predict future market demand through independent observation. They share information based on the use of the DT platform on the supply network. Market demand is uncertain.
- **Step 2:** The supplier also sets the price of raw material. The two suppliers are heterogeneous. One is a supplier of recycled materials; the other supplies primary materials. Manufacturers do not consider the environmental costs; therefore, they only focus on the price. The supply is uncertain, so the competition between the two suppliers is an example of Bertrand competition with supply uncertainty.
- **Step 3:** The manufacturer sets the purchase quantities at the same time. The two manufacturers use different raw materials. The products are the same. Therefore, this paper uses the Cournot model with demand uncertainty to describe the competition among manufacturers.

## Manufacturer

### *Demand Uncertainty of Manufacturers*

The two manufacturers in this article are manufacturer $l$ and manufacturer $f$. Each manufacturer determines its own output based on the observed market demand and condition of its competitors. Then, the manufacturer orders raw materials from suppliers. Due to COVID-19, market demand is uncertain. This affects the manufacturers' output and amount of raw materials ordered from the supplier.

According to Wu et al. (2019), the demand uncertainty in this paper is described by the random variable $\theta$, $\theta \sim \left[0, \sigma_\theta^2\right]$.

In addition, the private demand signal observed by the manufacturer from the market is $X$. The observation error is $\varepsilon$, $\varepsilon \in (0, +\infty)$. For $X$:

1. $X$ is the unbiased estimate of $\theta$, which means that $E\left(X \middle| \theta\right) = \theta$. This illustrates that the market demand estimated by the two manufacturers is not systematically biased.
2. $X_l$ and $X_f$ are independent of each other. Namely, $E\left[\theta \middle| X_l, X_f\right] = \alpha_0 + \alpha_l X_l + \alpha_f X_f$. $\alpha_0$, $\alpha_l$, and $\alpha_f$ are constants. This means that the two manufacturers observe market conditions independently without sharing information.
3. $X_l$ and $X_f$ have the same distribution. This means that there is no difference in the technical level of obtaining market information between the two manufacturers. Neither manufacturer has a stronger ability to obtain more information or more accurate information.

### *Production Competition Between Manufacturers: Cournot Model*

According to the model framework section, production competition between the two manufacturers is described using the Cournot model. The market inverse demand function is:

$$P = a + \theta - \left(Q_l + Q_f\right). \tag{1}$$

where $Q$ is the manufacturers' output. $l$ and $f$ represent the two manufacturers in the same market position.

The manufacturers' production function is: $Q = A \cdot q$. (2)

In Equation (2), $q$ is the quantity of raw materials purchased by the manufacturers from the suppliers. $A$ is a constant. $A = 1$.

The manufacturers' only variable unit cost, $w$, $w \in \left\{w_l, w_f\right\}$, is determined by the quantity and price of the raw materials purchased by the manufacturer from the two suppliers. Therefore, the manufacturers' profit functions are: $\begin{cases} \Pi_l = q_l(P \cdot A - w_l) \\ \Pi_f = q_f(P \cdot A - w_f) \end{cases}$. (3)

## Supplier

### *Supply Uncertainty from Suppliers*

The two suppliers in the model are supplier $t$ and supplier $r$. Supplier $t$ is a supplier based on traditional production (a supplier of raw materials based on first-time production). Supplier $r$ is a raw material supplier based on IS.

The supply uncertainty from the supplier in this article is determined by the external economic environment, including labour market, macroeconomic environment, and policy factors. For example, during COVID-19, traffic blockades and transportation interruptions caused by workers' sick leave led to short-term supply shortages. In the model, supply uncertainty is described by the quantity fluctuation of the supply, $Y$, $Y \in \left(0,1\right] \sim \left[\mu, \sigma_Y^2\right]$. The fluctuation coefficient refers to the ratio of the actual quantity of raw materials provided by the supplier to the quantity of raw materials ordered by the manufacturer. The fluctuation coefficients of the two suppliers $Y_t$ and $Y_r$ are independent of each other.

The actual quantity of raw materials obtained by a manufacturer is the product of the order quantity $q$ and the fluctuation coefficient $Y$. When manufacturer $l$ orders raw materials with the quantity of $q_{tl}$ from supplier $t$, the amount of raw materials actually obtained by the manufacturer is $Y_t q_{tl}$.

The supply uncertainty is $\delta_Y$, $\delta_Y = \frac{\sigma_Y}{\mu}$. When $\delta_Y$ increases, the uncertainty in the supply of raw materials from suppliers becomes larger.

### *Price Competition Between Suppliers: Bertrand Model*

According to the model framework section, the Bertrand model is introduced to describe the pricing competition between two suppliers.

The profit function of the two suppliers is:

$$\begin{cases} \pi_t = \left(w_t - c_t\right) Y_t \left(q_{lt} + q_{ft}\right) \\ \pi_r = \left(w_r - c_r\right) Y_t \left(q_{lr} + q_{fr}\right) \end{cases} . \tag{4}$$

In Equation (4), $w_t$ and $w_r$ are the unit price of raw materials from two suppliers. $c_t$ and $c_r$ are the costs of the two suppliers. The total unit costs of the two suppliers are the same. The cost structures, including transportation and production costs, are different.

## Structure of Information Sharing Between Manufacturers and Suppliers

According to the model structure in the model framework section, the current market has four players, two manufacturers and two suppliers. Each manufacturer makes an independent decision whether to share the market signals it has obtained from its suppliers through the DT platform. Four information-sharing scenarios between suppliers and manufacturers are obtained from this information. In the following figures, the direction of the arrow is the direction of the information flow.

### *Scenario 1: No DT for Either Manufacturer*

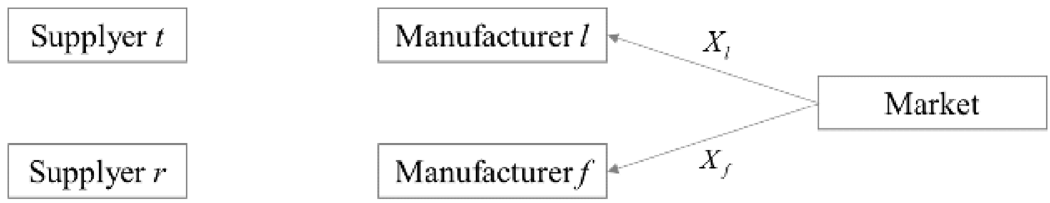
In scenario 1, the two manufacturers do not use the DT platform for information sharing. Therefore, the suppliers cannot obtain market demand signals. The relationship between the manufacturers and suppliers is shown in Figure 2.

In Figure 2, each enterprise is an isolated island regarding its information.

### *Scenario 2: DT for One Manufacturer*

In scenario 2, one manufacturer in the market shares the observed market demand signals with two suppliers through the DT platform. The other manufacturer does not share information. There are two manufacturers in the market; therefore, the situations in the information-sharing structure between manufacturers and suppliers are similar. See Figure 3.

Figure 2. Supply chain information exchange structure without DT




In Figure 3, manufacturer *l* has an information exchange with two suppliers. Manufacturer *f* does not share information with the two suppliers. According to Figure 4, manufacturer *f* has an information exchange with the two suppliers; manufacturer *l* shares no information with the two suppliers.

### *Scenario 3: DT for Both Manufacturers*

In scenario 3, two manufacturers in the market use the DT platform. Both manufacturers also share their observed market demand signals with their suppliers. Figure 5 shows the relationship between the manufacturers and suppliers.

## Results of the Model

Based on the uncertainty mentioned in this research, this section conducts a scenario analysis of the different information-sharing decisions. It also discusses the impact of the DT platform on solving the uncertainty impact of the local supply end-of-life products or coproduction chain.

Figure 3. Supply chain information exchange structure of enterprise l using DT

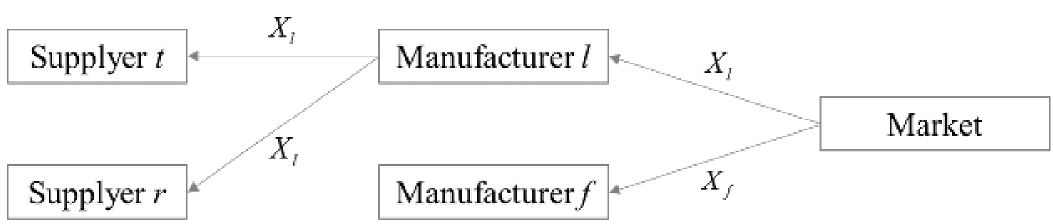



Figure 4. Supply chain information exchange structure of one enterprise f using DT

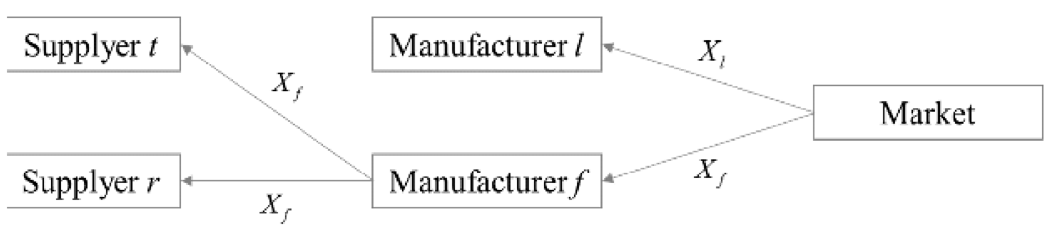
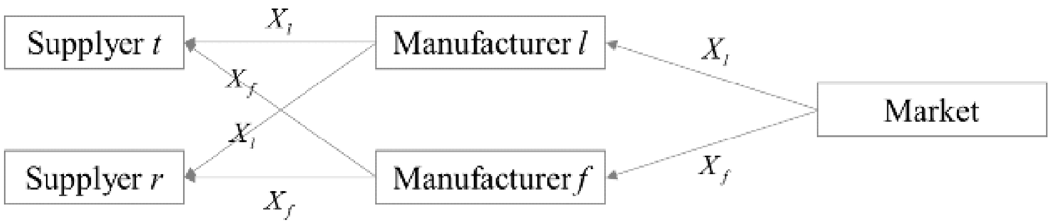


Figure 5. Supply chain information exchange structure with DT




## Scenario 1: No DT for Either Manufacturer

In Figure 2, when the two manufacturers do not share information, the profit functions of the two suppliers are:

$$E\left[\pi_t\right] = E\left[\left(w_t - c_t\right) Y_t \left(q_{lt} + q_{ft}\right)\right]$$
$$E\left[\pi_r\right] = E\left[\left(w_r - c_r\right) Y_r \left(q_{lr} + q_{fr}\right)\right]. \tag{5}$$

The profit functions of the two manufacturers are:

$$E\left[\Pi_l \middle| X_l, X_f\right] = E\left[(a + \theta - Y_t q_{lt} - Y_t q_{ft} - Y_r q_{lr} - Y_r q_{fr})(Y_t q_{lt} + Y_r q_{lr}) - \left(Y_t w_t q_{lt} + Y_r w_r q_{lr}\right) \middle| X_l, X_f\right]$$
$$E\left[\Pi_f \middle| X_l, X_f\right] = E\left[(a + \theta - Y_t q_{lt} - Y_t q_{ft} - Y_r q_{lr} - Y_r q_{fr})(Y_t q_{ft} + Y_r q_{fr}) - \left(Y_t w_t q_{ft} + Y_r w_r q_{fr}\right) \middle| X_l, X_f\right]. \tag{6}$$

According to the two game stages in the model framework section, the suppliers first conduct a price game to determine the price of raw materials. Subsequently, the manufacturers conduct a production game to determine the quantity of raw materials to be ordered from the suppliers. According to the backward induction, after solving the optimal order quantity of the two manufacturers, the optimal supply price of the two suppliers is solved (see Appendix B). Therefore, the authors can obtain the raw material prices without the DT platform.

$$w_t^{NN*} = w_r^{NN*} = \frac{a\delta_Y^2 + c\left(\delta_Y^2 + 1\right)}{2\delta_Y^2 + 1}. \tag{7}$$

In Equation (7), $\delta_Y$ is the supply uncertainty caused by supply shock. $a$ is the market size and $c$ is the unit cost of the suppliers.

The order quantities of the two manufacturers from the two suppliers are:

$$\begin{cases} q_{lt}^{NN*} = q_{lr}^{NN*} = \dfrac{\left(a - c\right)\left(\delta_Y^2 + 1\right)}{3\mu(\delta_Y^2 + 2)(2\delta_Y^2 + 1)} + \dfrac{1}{\mu(\delta_Y^2 + 2)(2\varepsilon + 3)} X_l \\ q_{ft}^{NN*} = q_{fr}^{NN*} = \dfrac{\left(a - c\right)\left(\delta_Y^2 + 1\right)}{3\mu(\delta_Y^2 + 2)(2\delta_Y^2 + 1)} + \dfrac{1}{\mu(\delta_Y^2 + 2)(2\varepsilon + 3)} X_f \end{cases}. \tag{8}$$

In Equation (8), $\mu$ is the mean value of the external impact of the macro-environment on the production quantity of the suppliers. $X_l$ is the unbiased estimate of the market demand signal observed by supplier *l*. $X_f$ is the unbiased estimate of the market demand signal observed by supplier *f*. $\varepsilon$ is the error of the signal observed by the suppliers.

The suppliers' profits in equilibrium are:

$$\pi_t^{NN*} = \pi_r^{NN*} = \frac{2\delta_Y^2(\delta_Y^2+1)(a-c)^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2}. \tag{9}$$

The manufacturers' profits in equilibrium are:

$$\Pi_l^{NN*} = \Pi_f^{NN*} = \frac{2(\delta_Y^2+1)^2(a-c)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{2\sigma_\theta^2(1+\varepsilon)}{(\delta_Y^2+2)(3+2\varepsilon)^2}. \tag{10}$$

## Scenario 2: DT for One Manufacturer

In Figure 3, when one manufacturer shares information on the DT platform, the profit functions of the two suppliers are:

$$\begin{aligned} E\left[\pi_t \middle| X_l\right] &= E\left[\left(w_t - c_t\right)Y_t\left(q_{lt}+q_{ft}\right)\middle| X_l\right] \\ E\left[\pi_r \middle| X_l\right] &= E\left[\left(w_r - c_r\right)Y_r\left(q_{lr}+q_{fr}\right)\middle| X_l\right] \end{aligned}. \tag{11}$$

The profit functions of the two manufacturers are:

$$\begin{aligned} E\left[\Pi_l \middle| X_l, X_f\right] &= E\left[(a+\theta - Y_t q_{lt} - Y_t q_{ft} - Y_r q_{lr} - Y_r q_{fr})(Y_t q_{lt} + Y_r q_{lr}) - \left(Y_t w_t q_{lt} + Y_r w_r q_{lr}\right)\middle| X_l, X_f\right] \\ E\left[\Pi_f \middle| X_l, X_f\right] &= E\left[(a+\theta - Y_t q_{lt} - Y_t q_{ft} - Y_r q_{lr} - Y_r q_{fr})(Y_t q_{ft} + Y_r q_{fr}) - \left(Y_t w_t q_{ft} + Y_r w_r q_{fr}\right)\middle| X_l, X_f\right] \end{aligned}. \tag{12}$$

According to the two game stages in the model framework section, the suppliers first conduct a price game to determine the price of raw materials. Subsequently, the manufacturers conduct a production game to determine the quantity of raw materials to be ordered from the suppliers. According to the backward induction, after solving the optimal order quantity of the two manufacturers, the optimal supply price of the two suppliers is solved (see Appendix B). Therefore, the authors can obtain the raw material prices with just one manufacturer applying the DT platform.

$$w_t^{DN*} = w_r^{DN*} = \frac{a\delta_Y^2 + c\left(\delta_Y^2+1\right)}{2\delta_Y^2+1} + \frac{\delta_Y^2}{(2\delta_Y^2+1)(1+\varepsilon)}X_l. \tag{13}$$

In Equation (13), $\delta_Y$ is the supply uncertainty caused by the supply shock. *a* is the market size; *c* is the unit cost of the suppliers. $\varepsilon$ is the error of the signal observed by the suppliers.

The order quantities of the two manufacturers from the two suppliers are:

$$\begin{cases} q_{lt}^{DN*} = q_{lr}^{DN*} = \dfrac{(a-c)(\delta_Y^2+1)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)} + \dfrac{\delta_Y^2+1}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+1)} X_l \\ q_{ft}^{DN*} = q_{fr}^{DN*} = \dfrac{(a-c)(\delta_Y^2+1)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)} + \dfrac{2(\delta_Y^2+1)(\varepsilon+2)-3(2\delta_Y^2+1)}{6\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+1)(\varepsilon+2)} X_l + \dfrac{1}{2\mu(\delta_Y^2+2)(\varepsilon+2)} X_f \end{cases}. \tag{14}$$

In Equation (14), $\mu$ is the mean value of the external impact of the macro-environment on the production quantity of the suppliers. $X_l$ is the unbiased estimate of the market demand signal observed by supplier *l*. $X_f$ is the unbiased estimate of the market demand signal observed by supplier *f*.

The suppliers' profits in equilibrium are:

$$\pi_t^{DN*} = \pi_r^{DN*} = \frac{2\delta_Y^2(\delta_Y^2+1)(a-c)^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{2\delta_Y^2(\delta_Y^2+1)\sigma_\theta^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)}. \tag{15}$$

The manufacturers' profits in equilibrium are:

$$\begin{cases} \Pi_l^{DN*} = \dfrac{2(\delta_Y^2+1)^2(a-c)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \dfrac{2(\delta_Y^2+1)^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)} \\ \Pi_f^{DN*} = \dfrac{2(\delta_Y^2+1)^2(a-c)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \dfrac{2(\delta_Y^2+1)^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)} + \dfrac{2\sigma_\theta^2\varepsilon}{4(\delta_Y^2+2)(\varepsilon+1)(\varepsilon+2)} \end{cases}, \tag{16}$$

where $\sigma_\theta^2$ is the variance of demand fluctuations, illustrating the demand uncertainty.

The market positions of the two suppliers are symmetrical. Therefore, the equilibrium solution based on Figure 4 is:

$$w_t^{ND*} = w_r^{ND*} = \frac{a\delta_Y^2 + c(\delta_Y^2+1)}{2\delta_Y^2+1} + \frac{\delta_Y^2}{(2\delta_Y^2+1)(1+\varepsilon)} X_f. \tag{17}$$

$$\begin{cases} q_{lt}^{ND*} = q_{lr}^{ND*} = \dfrac{(a-c)(\delta_Y^2+1)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)} + \dfrac{2(\delta_Y^2+1)(\varepsilon+2)-3(2\delta_Y^2+1)}{6\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+1)(\varepsilon+2)} X_f + \dfrac{1}{2\mu(\delta_Y^2+2)(\varepsilon+2)} X_l \\ q_{ft}^{ND*} = q_{fr}^{ND*} = \dfrac{(a-c)(\delta_Y^2+1)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)} + \dfrac{\delta_Y^2+1}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+1)} X_f \end{cases}. \tag{18}$$

$$\pi_t^{ND*} = \pi_r^{ND*} = \frac{2\delta_Y^2(\delta_Y^2+1)(a-c)^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{2\delta_Y^2(\delta_Y^2+1)\sigma_\theta^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)}. \tag{19}$$

$$\begin{cases} \Pi_l^{ND*} = \dfrac{2(\delta_Y^2+1)^2(a-c)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \dfrac{2(\delta_Y^2+1)^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)} + \dfrac{2\sigma_\theta^2\varepsilon}{4(\delta_Y^2+2)(\varepsilon+1)(\varepsilon+2)} \\ \Pi_f^{ND*} = \dfrac{2(\delta_Y^2+1)^2(a-c)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \dfrac{2(\delta_Y^2+1)^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)} \end{cases}. \tag{20}$$

### Scenario 3: DT for Both Manufacturers

In Figure 5, when both manufacturers share information on the DT platform, the profit functions of the two suppliers are:

$$\begin{aligned} E\left[\pi_t \middle| X_l, X_f\right] &= E\left[\left(w_t - c_t\right)Y_t\left(q_{lt} + q_{ft}\right)\middle| X_l, X_f\right] \\ E\left[\pi_r \middle| X_l, X_f\right] &= E\left[\left(w_r - c_r\right)Y_r\left(q_{lr} + q_{fr}\right)\middle| X_l, X_f\right] \end{aligned}. \tag{21}$$

The profit functions of the two manufacturers are:

$$\begin{aligned} E\left[\Pi_l \middle| X_l, X_f\right] &= E\left[(a+\theta - Y_t q_{lt} - Y_t q_{ft} - Y_r q_{lr} - Y_r q_{fr})(Y_t q_{lt} + Y_r q_{lr}) - \left(Y_t w_t q_{lt} + Y_r w_r q_{lr}\right)\middle| X_l, X_f\right] \\ E\left[\Pi_f \middle| X_l, X_f\right] &= E\left[(a+\theta - Y_t q_{lt} - Y_t q_{ft} - Y_r q_{lr} - Y_r q_{fr})(Y_t q_{ft} + Y_r q_{fr}) - \left(Y_t w_t q_{ft} + Y_r w_r q_{fr}\right)\middle| X_l, X_f\right] \end{aligned}. \tag{22}$$

According to the two game stages in the model framework section, the suppliers first conduct a price game to determine the price of raw materials. Subsequently, the manufacturers conduct a production game to determine the quantity of raw materials to be ordered from the suppliers. According to the backward induction, after solving the optimal order quantity of the two manufacturers, the optimal supply price of the two suppliers is solved (see Appendix B). Therefore, the authors can obtain the raw material prices from the DT platform.

$$w_t^{DD*} = w_r^{DD*} = \frac{a\delta_Y^2 + c\left(\delta_Y^2+1\right)}{2\delta_Y^2+1} + \frac{\delta_Y^2}{(2\delta_Y^2+1)(2+\varepsilon)}\left(X_l + X_f\right). \tag{23}$$

In Equation (23), $\delta_Y$ is the supply uncertainty caused by the supply shock. $a$ is the market size; $c$ is the unit cost of the suppliers. $\varepsilon$ is the error of the signal observed by the suppliers.

The order quantities of the two manufacturers from the two suppliers are:

$$q_{lt}^{DD*} = q_{lr}^{DD*} = q_{ft}^{DD*} = q_{fr}^{DD*} = \frac{\left(a-c\right)\left(\delta_Y^2+1\right)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)} + \frac{\delta_Y^2+1}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+2)}\left(X_l + X_f\right). \tag{24}$$

In Equation (24), $\mu$ is the mean value of the external impact of the macro-environment on the production quantity of the suppliers. $X_l$ is the unbiased estimate of the market demand signal observed by supplier *l*. $X_f$ is the unbiased estimate of the market demand signal observed by supplier *f*.

The suppliers' profit in equilibrium are:

$$\pi_t^{DD*} = \pi_r^{DD*} = \frac{2\delta_Y^2(\delta_Y^2+1)(a-c)^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{4\delta_Y^2(\delta_Y^2+1)\sigma_\theta^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+2)}. \tag{25}$$

where $\sigma_\theta^2$ is the variance of demand fluctuations, illustrating the demand uncertainty.

The manufacturers' profit in equilibrium are:

$$\Pi_l^{DD*} = \Pi_f^{DD*} = \frac{2(\delta_Y^2+1)^2(a-c)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{4(\delta_Y^2+1)^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+2)}. \tag{26}$$

## Discussion

In the previous section, the authors obtained the equilibrium of the purchase unit price, purchase quantity, and profits of two suppliers and two manufacturers under different information-sharing conditions. The sensitivity and rationality of the model are also discussed (see Appendix F). Profit is the main concern of enterprises. Therefore, in this section, the authors explain how the profits of each enterprise and the total profit of the supply chain are affected by the information-sharing mechanism.

### Profits of Suppliers Under the Three Modes of Information Sharing

The authors will first compare the profits of the two suppliers in the three scenarios. Next, they will compare the profit variation rates under the shocks. The profits are compared by calculating the differences between Equation (9), Equation (15), Equation (19), and Equation (25), as is shown in Equation (27).

$$\pi_t^{NN*} = \pi_r^{NN*} < \pi_t^{DN*} = \pi_r^{DN*} = \pi_t^{ND*} = \pi_r^{ND*} < \pi_t^{DD*} = \pi_r^{DD*}. \tag{27}$$

| Topic | Author | Subject | Method |
|---|---|---|---|
| Information Sharing in IS | Cervo et al. (2019) | Identify, evaluate, and promote symbiotic cooperation | Engineering and project management-oriented support (EPOS) methodology |
| | Cervo et al. (2020)* | Blueprint for industrial symbiosis | Case study |
| | D'Hauwers et al. (2020) | Government's role in sharing economy information | Case study |
| | Florencio de Souza et al. (2020) | Diagnose the presence of industrial symbiosis practices in five domains | Circular economy modelling |
| | Fraccascia & Yazan (2018)* | Impact of an online information-sharing platform on an industrial symbiosis network | Agent-based model and case study |
| | Ghali & Frayret (2019) | Framework for the initiation of industrial synergies | Social semantic Web |
| | Luciano et al. (2016) | Potential method improvement of regional industrial symbiosis | Interview, meeting, and case study |
| | Maqbool et al. (2019) | Evaluation of information technology development in European IS | Content análisis |
| | Shi & Chertow (2017) | Organizational boundary change of an industrial symbiosis company | Case study |
| | Xiang & Yuan (2019) | Demand and collaboration-driven model of smart industrial parks | SWOT analysis and case study |
| | Yazdanpanah et al. (2019) | Decision support for industrial symbiosis opportunities | Industrial symbiosis opportunity filtering method |
| | Yeo et al. (2019) | Tools to promote industrial symbiosis | Review |
| Application of DT to Information Sharing | Cavalcante et al. (2019) | Supplier performance risk under uncertainty | Machine learning and simulation |
| | Cervo et al. (2020)* | Blueprint for industrial symbiosis | Case study |
| | D'Angelo & Chong (2018) | Logistics for companies | Discrete event simulation model |
| | Fraccascia & Yazan (2018)* | Impact of an online information-sharing platform on the environment and economic benefits of an industrial symbiosis network | Agent-based model and case study |
| | Haag & Simon (2019) | Material and information flow for horizontal and vertical integration | Web-based models |
| | Lutters (2018) | Production environment platform for stakeholders | Resources and processes model |
| | Qi & Tao (2018) | Application comparison of big data and digital twins in manufacturing | Review |
| | Smith (2020) | Value of AI to future agriculture | Review |
| Game Models for Enterprises in the IS Network | Leong et al. (2016) | Modelling of cooling and cooling water networks between factories | Multi-objective linear programming |
| | Lotfi et al. (2019) | Decision optimization in a flexible closed-loop supply chain | Two-stage, mixed integer linear programming |
| | Luo et al. (2019) | Industrial symbiosis strategy of an e-commerce industrial park | Dynamic evolution game model |
| | Parlar et al. (2019) | Resource optimization on the industrial symbiosis network | Optimal control and cooperative game model |
| | Ramos et al. (2018) | Optimization of public utility resources in the ecoindustrial park | Multileader-follower game model |
| | Tan et al. (2016) | Process integration (PI) tools in industrial ecology (IE) applications | Cooperative game model |
| | Yazan et al. (2020) | Strategies to share additional costs of operating industrial symbiosis | Noncooperative game-theoretical model |
| | Zare Mehrjerdi & Lotfi (2019) | Design of a resilient and sustainable supply chain network | Two-stage, mixed integer linear programming |

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## APPENDIX B. DATA BASED ON MANUFACTURERS SHARING INFORMATION WITH SUPPLIERS

This is based on the condition that both manufacturers share information with suppliers. Taking the first-order partial differential of $E[\Pi_l | X_l, X_f]$ with respect to $q_{lt}$, let $\frac{\partial E[\Pi_l | X_l, X_f]}{\partial q_{lt}} = 0$. The authors obtain:

$$q_{lt} = \frac{(a + E[\theta | X_l, X_f] - w_t)}{2\mu(1+\delta_Y^2)} - \frac{q_{lr}}{1+\delta_Y^2} - \frac{1}{2}E[q_{ft} | X_l, X_f] - \frac{1}{2(1+\delta_Y^2)}E[q_{fr} | X_l, X_f]. \quad \text{(A.1)}$$

Taking the second-order partial differential of $E[\Pi_l | X_l, X_f]$, the authors obtain:

$$\begin{pmatrix} \frac{\partial^2 E[\Pi_l | X_l, X_f]}{\partial q_{lt}^2} & \frac{\partial^2 E[\Pi_l | X_l, X_f]}{\partial q_{lt}\partial q_{lr}} \\ \frac{\partial^2 E[\Pi_l | X_l, X_f]}{\partial q_{lr}\partial q_{lt}} & \frac{\partial^2 E[\Pi_l | X_l, X_f]}{\partial q_{lr}^2} \end{pmatrix} = \begin{pmatrix} -2\mu^2(1+\delta_Y^2) & -2\mu^2 \\ -2\mu^2 & -2\mu^2(1+\delta_Y^2) \end{pmatrix}. \quad \text{(A.2)}$$

Equation (A.2) is negative; $E[\Pi_l | X_l, X_f]$ is joint concave in $q_{lt}$ and $q_{lr}$. The best response function is:

$$q_{lt} = \frac{a - w_t}{2\mu\delta_Y^2} - \frac{2a - w_t - w_r}{2\mu\delta_Y^2\left(\delta_Y^2 + 2\right)} + \frac{E[\theta | X_l, X_f]}{2\mu\left(\delta_Y^2 + 2\right)} - \frac{1}{2}E[q_{ft} | X_l, X_f], \quad \text{(A.3)}$$

$$q_{lt} = q_{lr} = q_{ft} = q_{fr} = \frac{a - w_t}{3\mu\delta_Y^2} - \frac{2a - w_t - w_r}{3\mu\delta_Y^2\left(\delta_Y^2 + 2\right)} + \frac{X_l + X_f}{3\mu\left(\delta_Y^2 + 2\right)\left(\varepsilon + 2\right)}. \quad \text{(A.4)}$$

Let $q_t = q_{lt} + q_{ft}, q_r = q_{lr} + q_{fr}$. Substitute $q_t$ into Equation (21). Take the first-order partial differential with respect to $w_t$. The authors obtain:

$$\frac{\partial E[\pi_t | X_l, X_f]}{\partial w_t} = 0. \quad \text{(A.5)}$$

Therefore, the authors obtain the best response function as:

$$w_t = \frac{a + c}{2} - \frac{a - w_r}{2(\delta_Y^2 + 1)} + \frac{\delta_Y^2(X_l + X_f)}{2(\delta_Y^2 + 1)(\varepsilon + 2)}. \quad \text{(A.6)}$$

According to Wu et al. (2019):

$$q_{lt}^{DD*} = q_{lr}^{DD*} = q_{ft}^{DD*} = q_{fr}^{DD*} = \frac{(a-c)(\delta_Y^2+1)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+2)} + \frac{(\delta_Y^2+1)(X_l+X_f)}{3\mu(\delta_Y^2+2)(2\delta_Y^2+1)(\varepsilon+2)}. \tag{A.7}$$

Substitute Equation (A.6) and Equation (A.7) into Equation (22):

$$\Pi_l^{DD*} = \Pi_f^{DD*} = \frac{2\left(a-c\right)^2\left(\delta_Y^2+1\right)^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{4\left(\delta_Y^2+1\right)^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2\left(\varepsilon+2\right)}. \tag{A.8}$$

Substitute Equation (A.6) and Equation (A.7) into Equation (21):

$$\pi_t^{DD*} = \pi_r^{DD*} = \frac{2\left(a-c\right)^2\delta_Y^2\left(\delta_Y^2+1\right)}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2} + \frac{4\delta_Y^2\left(\delta_Y^2+1\right)\sigma_\theta^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2\left(\varepsilon+2\right)}. \tag{A.9}$$

Like the solution procedure of this solution, the authors can obtain solutions for the other conditions.

## APPENDIX C. FIRST-ORDER PARTIAL DIFFERENTIAL OF EQUATIONS 9, 15, 19 AND 25

Taking the first-order partial differential of Equation (9), Equation (15), Equation (19), and Equation (25) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\frac{\partial \pi_t^{NN*}}{\partial \sigma_\theta^2} = \frac{\partial \pi_r^{NN*}}{\partial \sigma_\theta^2} = 0. \tag{B.1}$$

$$\frac{\partial \pi_t^{DN*}}{\partial \sigma_\theta^2} = \frac{\partial \pi_r^{DN*}}{\partial \sigma_\theta^2} = \frac{\partial \pi_t^{ND*}}{\partial \sigma_\theta^2} = \frac{\partial \pi_r^{ND*}}{\partial \sigma_\theta^2} = \frac{2\delta_Y^2\left(\delta_Y^2+1\right)}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)} > 0. \tag{B.2}$$

$$\frac{\partial \pi_t^{DD*}}{\partial \sigma_\theta^2} = \frac{\partial \pi_r^{DD*}}{\partial \sigma_\theta^2} = \frac{4\delta_Y^2\left(\delta_Y^2+1\right)}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+2\right)} > 0. \tag{B.3}$$

$$\frac{\partial \pi_t^{NN*}}{\partial \delta_Y^2} = \frac{\partial \pi_r^{NN*}}{\partial \delta_Y^2} = \frac{-2\left(a-c\right)^2\left[2\left(\delta_Y^2\right)^3 + 3\left(\delta_Y^2\right)^2 - 2\right]}{3\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3}. \tag{B.4}$$

$$\frac{\partial \pi_t^{DN*}}{\partial \delta_Y^2} = \frac{\partial \pi_r^{DN*}}{\partial \delta_Y^2} = \frac{\partial \pi_t^{ND*}}{\partial \delta_Y^2} = \frac{\partial \pi_r^{ND*}}{\partial \delta_Y^2} = \left[\frac{2\left(a-c\right)^2}{3} + \frac{2\sigma_\theta^2}{3\left(\varepsilon+1\right)}\right]\frac{-\left[2\left(\delta_Y^2\right)^3 + 3\left(\delta_Y^2\right)^2 - 2\right]}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3}. \tag{B.5}$$

$$\frac{\partial \pi_t^{DD*}}{\partial \delta_Y^2} = \frac{\partial \pi_r^{DD*}}{\partial \delta_Y^2} = \left[\frac{2\left(a-c\right)^2}{3} + \frac{4\sigma_\theta^2}{3\left(\varepsilon+2\right)}\right]\frac{-\left[2\left(\delta_Y^2\right)^3 + 3\left(\delta_Y^2\right)^2 - 2\right]}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3}. \tag{B.6}$$

$$\frac{\partial \pi_t^{NN*}}{\partial \varepsilon} = \frac{\partial \pi_r^{NN*}}{\partial \varepsilon} = 0. \tag{B.7}$$

$$\frac{\partial \pi_t^{DN*}}{\partial \varepsilon} = \frac{\partial \pi_r^{DN*}}{\partial \varepsilon} = \frac{\partial \pi_t^{ND*}}{\partial \varepsilon} = \frac{\partial \pi_r^{ND*}}{\partial \varepsilon} = \frac{-2\delta_Y^2\left(\delta_Y^2+1\right)\sigma_\theta^2}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)^2} < 0. \tag{B.8}$$

$$\frac{\partial \pi_t^{DD*}}{\partial \varepsilon} = \frac{\partial \pi_r^{DD*}}{\partial \varepsilon} = \frac{-4\delta_Y^2\left(\delta_Y^2+1\right)\sigma_\theta^2}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+2\right)^2} < 0. \tag{B.9}$$

Therefore, Table 1 is proved.
Calculating the differences of Equation (9), Equation (15), Equation (19), and Equation (25) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\pi_t^{NN*} - \pi_t^{DN*} = \frac{-2\delta_Y^2(\delta_Y^2+1)\sigma_\theta^2}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)}. \tag{B.10}$$

$$\pi_t^{DD*} - \pi_t^{DN*} = \frac{2\delta_Y^2(\delta_Y^2+1)\sigma_\theta^2\varepsilon}{3(\delta_Y^2+2)(2\delta_Y^2+1)^2(\varepsilon+1)(\varepsilon+2)} > 0. \tag{B.11}$$

Therefore, $\pi_t^{NN*} = \pi_r^{NN*} < \pi_t^{DN*} = \pi_r^{DN*} = \pi_t^{ND*} = \pi_r^{ND*} < \pi_t^{DD*} = \pi_r^{DD*}$ is proved.

Calculating the differences of Equation (B.2) and Equation (B.3) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\left|\frac{\partial \pi_t^{DN*}}{\partial \sigma_\theta^2}\right| - \left|\frac{\partial \pi_t^{DD*}}{\partial \sigma_\theta^2}\right| = \frac{-2\delta_Y^2\left(\delta_Y^2+1\right)\varepsilon}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)\left(\varepsilon+2\right)} < 0\,. \tag{B.12}$$

Because of $\left|\frac{\partial \pi_t^{NN*}}{\partial \sigma_\theta^2}\right| = 0$, $\left|\frac{\partial \pi_t^{DN*}}{\partial \sigma_\theta^2}\right| > 0$ and $\left|\frac{\partial \pi_t^{DD*}}{\partial \sigma_\theta^2}\right| > 0$,

$\left|\frac{\partial \pi_t^{NN*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \pi_r^{NN*}}{\partial \sigma_\theta^2}\right| < \left|\frac{\partial \pi_t^{DN*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \pi_r^{DN*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \pi_t^{ND*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \pi_r^{ND*}}{\partial \sigma_\theta^2}\right| < \left|\frac{\partial \pi_t^{DD*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \pi_r^{DD*}}{\partial \sigma_\theta^2}\right|$ is proved.

Calculating the differences of Equation (B.4), Equation (B.5). and Equation (B.6) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\left|\frac{\partial \pi_t^{NN*}}{\partial \delta_Y^2}\right| - \left|\frac{\partial \pi_t^{DN*}}{\partial \delta_Y^2}\right| = -\frac{2\sigma_\theta^2}{3\left(\varepsilon+1\right)}\left|\frac{2\left(\delta_Y^2\right)^3+3\left(\delta_Y^2\right)^2-2}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3}\right| < 0\,, \tag{B.13}$$

$$\left|\frac{\partial \pi_t^{DN*}}{\partial \delta_Y^2}\right| - \left|\frac{\partial \pi_t^{DD*}}{\partial \delta_Y^2}\right| = \left(\frac{1}{\varepsilon+1} - \frac{2}{\varepsilon+2}\right)\frac{2\sigma_\theta^2}{3}\left|\frac{2\left(\delta_Y^2\right)^3+3\left(\delta_Y^2\right)^2-2}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3}\right| < 0\,. \tag{B.14}$$

$\left|\frac{\partial \pi_t^{NN*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \pi_r^{NN*}}{\partial \delta_Y^2}\right| < \left|\frac{\partial \pi_t^{DN*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \pi_r^{DN*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \pi_t^{ND*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \pi_r^{ND*}}{\partial \delta_Y^2}\right| < \left|\frac{\partial \pi_t^{DD*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \pi_r^{DD*}}{\partial \delta_Y^2}\right|$ is proved.

## APPENDIX D. FIRST-ORDER PARTIAL DIFFERENTIALS ON EQUATIONS 10, 16, 20 AND 26

Taking the first-order partial differential of Equation (10), Equation (16), Equation (20), and Equation (26) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\frac{\partial \Pi_l^{NN*}}{\partial \sigma_\theta^2} = \frac{\partial \Pi_f^{NN*}}{\partial \sigma_\theta^2} = \frac{2\left(1+\varepsilon\right)}{\left(\delta_Y^2+2\right)\left(2\varepsilon+3\right)^2} > 0\,. \tag{C.1}$$

$$\frac{\partial \Pi_l^{DN*}}{\partial \sigma_\theta^2} = \frac{\partial \Pi_f^{ND*}}{\partial \sigma_\theta^2} = \frac{2\left(\delta_Y^2 + 1\right)^2}{9\left(\delta_Y^2 + 2\right)\left(2\delta_Y^2 + 1\right)^2\left(\varepsilon + 1\right)} > 0\,. \tag{C.2}$$

$$\frac{\partial \Pi_f^{DN*}}{\partial \sigma_\theta^2} = \frac{\partial \Pi_l^{ND*}}{\partial \sigma_\theta^2} = \frac{2\left(\delta_Y^2 + 1\right)^2}{9\left(\delta_Y^2 + 2\right)\left(2\delta_Y^2 + 1\right)^2\left(\varepsilon + 1\right)} + \frac{2\varepsilon}{4\left(\delta_Y^2 + 2\right)\left(\varepsilon + 1\right)\left(\varepsilon + 2\right)} > 0\,. \tag{C.3}$$

$$\frac{\partial \Pi_l^{DD*}}{\partial \sigma_\theta^2} = \frac{\partial \Pi_f^{DD*}}{\partial \sigma_\theta^2} = \frac{4\left(\delta_Y^2 + 1\right)^2}{9\left(\delta_Y^2 + 2\right)\left(2\delta_Y^2 + 1\right)^2\left(\varepsilon + 2\right)} > 0\,. \tag{C.4}$$

$$\frac{\partial \Pi_l^{NN*}}{\partial \delta_Y^2} = \frac{\partial \Pi_f^{NN*}}{\partial \delta_Y^2} = \frac{-2\left(a - c\right)^2\left(\delta_Y^2 + 1\right)\left[2\left(\delta_Y^2\right)^2 + 5\delta_Y^2 + 5\right]}{9\left(\delta_Y^2 + 2\right)^2\left(2\delta_Y^2 + 1\right)^3} - \frac{2\left(1 + \varepsilon\right)\sigma_\theta^2}{\left(\delta_Y^2 + 2\right)^2\left(2\varepsilon + 3\right)^2} < 0\,. \tag{C.5}$$

$$\frac{\partial \Pi_l^{DN*}}{\partial \delta_Y^2} = \frac{\partial \Pi_f^{ND*}}{\partial \delta_Y^2} = \left[\frac{2\left(a - c\right)^2}{9} + \frac{2\sigma_\theta^2}{9\left(\varepsilon + 1\right)}\right]\frac{-\left(\delta_Y^2 + 1\right)\left[2\left(\delta_Y^2\right)^2 + 5\delta_Y^2 + 5\right]}{\left(\delta_Y^2 + 2\right)^2\left(2\delta_Y^2 + 1\right)^3} < 0\,. \tag{C.6}$$

$$\begin{aligned}\frac{\partial \Pi_l^{ND*}}{\partial \delta_Y^2} = \frac{\partial \Pi_f^{DN*}}{\partial \delta_Y^2} &= \left[\frac{2\left(a - c\right)^2}{9} + \frac{2\sigma_\theta^2}{9\left(\varepsilon + 1\right)}\right]\frac{-\left(\delta_Y^2 + 1\right)\left[2\left(\delta_Y^2\right)^2 + 5\delta_Y^2 + 5\right]}{\left(\delta_Y^2 + 2\right)^2\left(2\delta_Y^2 + 1\right)^3} \\ &\quad - \frac{\varepsilon\sigma_\theta^2}{2\left(\varepsilon + 1\right)\left(\varepsilon + 2\right)\left(\delta_Y^2 + 2\right)^2} < 0\end{aligned}\,. \tag{C.7}$$

$$\frac{\partial \Pi_l^{DD*}}{\partial \delta_Y^2} = \frac{\partial \Pi_f^{DD*}}{\partial \delta_Y^2} = \left[\frac{2\left(a - c\right)^2}{9} + \frac{4\sigma_\theta^2}{9\left(\varepsilon + 2\right)}\right]\frac{-\left(\delta_Y^2 + 1\right)\left[2\left(\delta_Y^2\right)^2 + 5\delta_Y^2 + 5\right]}{\left(\delta_Y^2 + 2\right)^2\left(2\delta_Y^2 + 1\right)^3} < 0\,. \tag{C.8}$$

$$\frac{\partial \Pi_l^{NN*}}{\partial \varepsilon} = \frac{\partial \Pi_f^{NN*}}{\partial \varepsilon} = \frac{-2\left(4\varepsilon^2 + 8\varepsilon + 3\right)\sigma_\theta^2}{\left(\delta_Y^2 + 2\right)\left(3 + 2\varepsilon\right)^4} < 0\,. \tag{C.9}$$

$$\frac{\partial \Pi_l^{DN*}}{\partial \varepsilon} = \frac{\partial \Pi_f^{ND*}}{\partial \varepsilon} = \frac{-2\left(\delta_Y^2 + 1\right)^2 \sigma_\theta^2}{9\left(\delta_Y^2 + 2\right)\left(2\delta_Y^2 + 1\right)^2 \left(\varepsilon + 1\right)^2} < 0\,. \tag{C.10}$$

$$\frac{\partial \Pi_l^{ND*}}{\partial \varepsilon} = \frac{\partial \Pi_f^{DN*}}{\partial \varepsilon} = \frac{-2\left(\delta_Y^2 + 1\right)^2 \sigma_\theta^2}{9\left(\delta_Y^2 + 2\right)\left(2\delta_Y^2 + 1\right)^2 \left(\varepsilon + 1\right)^2} - \frac{2\left(\varepsilon^2 - 2\right)\sigma_\theta^2}{4\left(\delta_Y^2 + 2\right)\left(\varepsilon + 1\right)^2 \left(\varepsilon + 2\right)^2}\,. \tag{C.11}$$

$$\frac{\partial \Pi_l^{DD*}}{\partial \varepsilon} = \frac{\partial \Pi_f^{DD*}}{\partial \varepsilon} = \frac{-4\left(\delta_Y^2 + 1\right)^2 \sigma_\theta^2}{9\left(\delta_Y^2 + 2\right)\left(2\delta_Y^2 + 1\right)^2 \left(\varepsilon + 2\right)^2} < 0\,. \tag{C.12}$$

Therefore, Table 2 is proved.
Calculating the differences of Equation (10), Equation (16), Equation (20), and Equation (26) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\Pi_l^{DN*} - \Pi_l^{DD*} = \frac{-2(\delta_Y^2 + 1)^2 \sigma_\theta^2 \varepsilon}{9(\delta_Y^2 + 2)(2\delta_Y^2 + 1)^2(\varepsilon + 1)(\varepsilon + 2)} < 0\,. \tag{C.13}$$

$$\Pi_l^{ND*} - \Pi_l^{DD*} = \frac{\sigma_\theta^2 \varepsilon\left[34\left(\delta_Y^2\right)^2 + 32\delta_Y^2 + 7\right]}{18(\delta_Y^2 + 2)(2\delta_Y^2 + 1)^2(\varepsilon + 1)(\varepsilon + 2)} > 0\,. \tag{C.14}$$

Therefore, $\Pi_l^{DN*} = \Pi_f^{ND*} < \Pi_l^{DD*} = \Pi_f^{DD*} < \Pi_f^{DN*} = \Pi_l^{ND*}$ is proved.
Calculating the differences of Equation (C.1), Equation (C.2), Equation (C.3), and Equation (C.4) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\left|\frac{\partial \Pi_l^{DN*}}{\partial \sigma_\theta^2}\right| - \left|\frac{\partial \Pi_l^{DD*}}{\partial \sigma_\theta^2}\right| = \frac{-2(\delta_Y^2 + 1)^2 \varepsilon}{9(\delta_Y^2 + 2)(2\delta_Y^2 + 1)^2(\varepsilon + 1)(\varepsilon + 2)} < 0\,, \tag{C.15}$$

$$\left|\frac{\partial \Pi_l^{ND*}}{\partial \sigma_\theta^2}\right| - \left|\frac{\partial \Pi_l^{DD*}}{\partial \sigma_\theta^2}\right| = \frac{\varepsilon\left[34\left(\delta_Y^2\right)^2 + 32\delta_Y^2 + 7\right]}{18\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)\left(\varepsilon+2\right)} > 0\,. \tag{C.16}$$

$\left|\frac{\partial \Pi_l^{DN*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \Pi_f^{ND*}}{\partial \sigma_\theta^2}\right| < \left|\frac{\partial \Pi_l^{DD*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \Pi_f^{DD*}}{\partial \sigma_\theta^2}\right| < \left|\frac{\partial \Pi_l^{ND*}}{\partial \sigma_\theta^2}\right| = \left|\frac{\partial \Pi_f^{DN*}}{\partial \sigma_\theta^2}\right|$ is proved.

Calculating the differences of Equation (C.5), Equation (C.6), Equation (C.7), and Equation (C.8) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\left|\frac{\partial \Pi_l^{DN*}}{\partial \delta_Y^2}\right| - \left|\frac{\partial \Pi_l^{DD*}}{\partial \delta_Y^2}\right| = \frac{-2\left(\delta_Y^2+1\right)\left[2\left(\delta_Y^2\right)^2 + 5\delta_Y^2 + 5\right]\sigma_\theta^2\varepsilon}{9\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3\left(\varepsilon+2\right)\left(\varepsilon+1\right)} < 0\,, \tag{C.17}$$

$$\left|\frac{\partial \Pi_l^{DN*}}{\partial \delta_Y^2}\right| - \left|\frac{\partial \Pi_l^{ND*}}{\partial \delta_Y^2}\right| = -\frac{\sigma_\theta^2\varepsilon}{2\left(\varepsilon+1\right)\left(\varepsilon+2\right)\left(\delta_Y^2+2\right)^2} < 0\,. \tag{C.18}$$

Therefore, $\begin{cases} \left|\frac{\partial \Pi_l^{DN*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \Pi_f^{ND*}}{\partial \delta_Y^2}\right| < \left|\frac{\partial \Pi_l^{DD*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \Pi_f^{DD*}}{\partial \delta_Y^2}\right| \\ \left|\frac{\partial \Pi_l^{DN*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \Pi_f^{ND*}}{\partial \delta_Y^2}\right| < \left|\frac{\partial \Pi_l^{ND*}}{\partial \delta_Y^2}\right| = \left|\frac{\partial \Pi_f^{DN*}}{\partial \delta_Y^2}\right| \end{cases}$ is proved.

## APPENDIX E. FIRST-ORDER PARTIAL DIFFERENTIAL ON EQUATIONS 33-35

Taking the first-order partial differential of Equation (33), Equation (34), and Equation (35) with $\sigma_\theta^2 > 0$, $\delta_Y^2 > 0$ and $\varepsilon > 0$, the authors obtain:

$$\frac{\partial J^{NN*}}{\partial \sigma_\theta^2} = \frac{4\left(1+\varepsilon\right)}{\left(\delta_Y^2+2\right)\left(3+2\varepsilon\right)^2} > 0\,. \tag{D.1}$$

$$\begin{aligned}\frac{\partial J^{DN*}}{\partial \sigma_\theta^2} = \frac{\partial J^{ND*}}{\partial \sigma_\theta^2} = \frac{4\delta_Y^2\left(\delta_Y^2+1\right)}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)} + \frac{4\left(\delta_Y^2+1\right)^2}{9\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)} \\ + \frac{\varepsilon}{2\left(\delta_Y^2+2\right)\left(\varepsilon+1\right)\left(\varepsilon+2\right)} > 0\end{aligned}\,. \tag{D.2}$$

$$\frac{\partial J^{DD*}}{\partial \sigma_\theta^2} = \frac{8\delta_Y^2\left(\delta_Y^2+1\right)}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+2\right)} + \frac{8\left(\delta_Y^2+1\right)^2}{9\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+2\right)} > 0\,. \tag{D.3}$$

$$\frac{\partial J^{NN*}}{\partial \delta_Y^2} = \frac{-4\left(a-c\right)^2\left[2\left(\delta_Y^2\right)^3+3\left(\delta_Y^2\right)^2-2\right]}{3\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3} - \frac{4\left(a-c\right)^2\left(\delta_Y^2+1\right)\left[2\left(\delta_Y^2\right)^2+5\delta_Y^2+5\right]}{9\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3} - \frac{4\left(1+\varepsilon\right)\sigma_\theta^2}{\left(\delta_Y^2+2\right)^2\left(2\varepsilon+3\right)^2}\,. \tag{D.4}$$

$$\frac{\partial J^{DN*}}{\partial \delta_Y^2} = \frac{\partial J^{ND*}}{\partial \delta_Y^2} = \left[\frac{4\left(a-c\right)^2}{3} + \frac{4\sigma_\theta^2}{3\left(\varepsilon+1\right)}\right]\frac{-\left[2\left(\delta_Y^2\right)^3+3\left(\delta_Y^2\right)^2-2\right]}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3} - \left[\frac{4\left(a-c\right)^2}{9} + \frac{4\sigma_\theta^2}{9\left(\varepsilon+1\right)}\right]\frac{\left(\delta_Y^2+1\right)\left[2\left(\delta_Y^2\right)^2+5\delta_Y^2+5\right]}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3} - \frac{\varepsilon\sigma_\theta^2}{2\left(\varepsilon+1\right)\left(\varepsilon+2\right)\left(\delta_Y^2+2\right)^2}\,. \tag{D.5}$$

$$\frac{\partial J^{DD*}}{\partial \delta_Y^2} = \left[\frac{4\left(a-c\right)^2}{3} + \frac{8\sigma_\theta^2}{3\left(\varepsilon+2\right)}\right]\frac{-\left[2\left(\delta_Y^2\right)^3+3\left(\delta_Y^2\right)^2-2\right]}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3} - \left[\frac{4\left(a-c\right)^2}{9} + \frac{8\sigma_\theta^2}{9\left(\varepsilon+2\right)}\right]\frac{\left(\delta_Y^2+1\right)\left[2\left(\delta_Y^2\right)^2+5\delta_Y^2+5\right]}{\left(\delta_Y^2+2\right)^2\left(2\delta_Y^2+1\right)^3}\,. \tag{D.6}$$

$$\frac{\partial J^{NN*}}{\partial \varepsilon} = \frac{-4\left(4\varepsilon^2+8\varepsilon+3\right)\sigma_\theta^2}{\left(\delta_Y^2+2\right)\left(3+2\varepsilon\right)^4} < 0\,. \tag{D.7}$$

$$\frac{\partial J^{DN*}}{\partial \varepsilon}=\frac{\partial J^{ND*}}{\partial \varepsilon}=\frac{-4\delta_Y^2\left(\delta_Y^2+1\right)\sigma_\theta^2}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)^2}-\frac{4\left(\delta_Y^2+1\right)^2\sigma_\theta^2}{9\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+1\right)^2}-\frac{2\left(\varepsilon^2-2\right)\sigma_\theta^2}{4\left(\delta_Y^2+2\right)\left(\varepsilon+1\right)^2\left(\varepsilon+2\right)^2}. \tag{D.8}$$

$$\frac{\partial J^{DD*}}{\partial \varepsilon}=\frac{-8\delta_Y^2\left(\delta_Y^2+1\right)\sigma_\theta^2}{3\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+2\right)^2}-\frac{8\left(\delta_Y^2+1\right)^2\sigma_\theta^2}{9\left(\delta_Y^2+2\right)\left(2\delta_Y^2+1\right)^2\left(\varepsilon+2\right)^2}<0. \tag{D.9}$$

Therefore, Table 3 is proved.

Calculating the differences of Equation (33), Equation (34), and Equation (35) with $\sigma_\theta^2>0$, $\delta_Y^2>0$ and $\varepsilon\to 0$, the authors obtain:

$$J^{DD*}-J^{DN*}=0. \tag{D.10}$$

$$J^{DD*}-J^{NN*}=\frac{-4\delta_Y^2\sigma_\theta^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2}<0. \tag{D.11}$$

Therefore, when $\varepsilon\to 0$, $J^{DD*}=J^{DN*}=J^{ND*}<J^{NN*}$ is proved.

Calculating the differences of Equation (D.1), Equation (D.2), and Equation (D.3) with $\sigma_\theta^2>0$, $\delta_Y^2>0$ and $\varepsilon\to 0$, the authors obtain:

$$\left|\frac{\partial J^{DN*}}{\partial \sigma_\theta^2}\right|-\left|\frac{\partial J^{DD*}}{\partial \sigma_\theta^2}\right|=0, \tag{D.12}$$

$$\left|\frac{\partial J^{DD*}}{\partial \sigma_\theta^2}\right|-\left|\frac{\partial J^{NN*}}{\partial \sigma_\theta^2}\right|=\frac{-4\delta_Y^2}{9(\delta_Y^2+2)(2\delta_Y^2+1)^2}<0. \tag{D.13}$$

When $\varepsilon\to 0$, $\left|\frac{\partial J^{DD*}}{\partial \sigma_\theta^2}\right|=\left|\frac{\partial J^{DN*}}{\partial \sigma_\theta^2}\right|=\left|\frac{\partial J^{ND*}}{\partial \sigma_\theta^2}\right|<\left|\frac{\partial J^{NN*}}{\partial \sigma_\theta^2}\right|$ is proved.

**Table 2. Suppliers' profit changes under the shocks**

| | $\sigma_\theta^2 \uparrow$ | $\delta_Y^2 \uparrow$ | $\varepsilon \uparrow$ |
|---|---|---|---|
| $\pi_t^{NN*} = \pi_r^{NN*}$ | - | | - |
| $\pi_t^{DN*} = \pi_r^{DN*} = \pi_t^{ND*} = \pi_r^{ND*}$ | ↑ | | ↓ |
| $\pi_t^{DD*} = \pi_r^{DD*}$ | ↑ | | ↓ |

## APPENDIX F. UNCETAINTIES OF SUPPLIERS' AND MANUFACTURERS' AND SUPPLY CHAIN PROFITS

### UNCERTAINTY OF SUPPLIERS' PROFITS

The profit changes of suppliers in different scenarios are shown in Table A2 (see Appendix 3 for the calculation process).

**Variance of Demand Uncertainty** $\sigma_\theta^2$ **:** In scenario 1 (neither manufacturer uses the digital twin platform), the increase in demand uncertainty, $\sigma_\theta^2$ , will not affect the suppliers' profits. The authors believe this is caused by the manufacturers not sharing information with the suppliers. In scenario 2 and scenario 3, the increase in demand uncertainty, $\sigma_\theta^2$ , is conducive to the increase in suppliers' profits. Manufacturers can observe demand changes in the market; therefore, at least one manufacturer is willing to share the demand signals with suppliers through the DT platform in scenario 2 and scenario 3. The suppliers can make a production plan based on the signals. This promotes the increase in supplier profits.

**Variance of Supply Uncertainty** $\delta_Y^2$ **:** In all three scenarios, the increase in supply uncertainty, $\delta_Y^2$ , leads to uncertain changes in suppliers' profits, which is related to the value of the supply shock. When the supply shock is weak, the suppliers' profits rise slightly. When the supply uncertainty is greater, the suppliers' profits decline.

**Variance of Prediction Error** $\varepsilon$ **:** It should be noted that in scenario 1, the increase in prediction error $\varepsilon$ has no effect on the suppliers' profit. In scenario 1, there is no information sharing between the supplier and manufacturer. Therefore, the manufacturers' prediction error is not passed to the suppliers. In scenario 2 and scenario 3, when the manufacturers' prediction error $\varepsilon$ increases, the suppliers' profit decreases. If the manufacturers' forecast of market demand is not accurate enough, information sharing will bring additional loss to the suppliers. In addition, in each scenario, the profits of the two suppliers are the same. However, the nature of the two suppliers differs. One is a traditional supplier; the other is a supplier in an industrial symbiosis network. Environmental factors of their products are not considered in the manufacturers' procurement process. Therefore, the two suppliers conduct undifferentiated price competition. The profits, prices, and output of the two suppliers are the same.

### UNCERTAINTY OF MANUFACTURERS' PROFITS

The profit changes of manufacturers in different scenarios are shown in Table A3 (see Appendix 4 for the calculation process).

**Table 3. Manufacturers' profit changes under the shock**

| | $\sigma_\theta^2 \uparrow$ | $\delta_Y^2 \uparrow$ | $\varepsilon \uparrow$ |
|---|---|---|---|
| $\Pi_l^{NN*} = \Pi_f^{NN*}$ | ↑ | ↓ | ↓ |
| $\Pi_l^{DN*} = \Pi_f^{ND*}$ | ↑ | ↓ | ↓ |
| $\Pi_f^{DN*} = \Pi_l^{ND*}$ | ↑ | ↓ | ↓ |
| $\Pi_l^{DD*} = \Pi_f^{DD*}$ | ↑ | ↓ | |

**Variance of Demand Uncertainty $\sigma_\theta^2$ :** In the three scenarios, the increase in demand uncertainty, $\sigma_\theta^2$ , leads to higher profits for manufacturers. The authors believe that this is achieved when manufacturers observe market demand signals in time, adjusting production plans according to changes in market demand.

**Variance of Supply Uncertainty $\delta_Y^2$ :** In the three scenarios, the increase of supply uncertainty $\delta_Y^2$ leads to the decline of the manufacturers' profits. The authors believe that suppliers are unable to pass on the supply situation to manufacturers, resulting in manufacturers' weak control over raw materials. From the perspective of information sharing, traditional suppliers can reduce supply fluctuations by enhancing information sharing with the manufacturers. The suppliers on the industrial symbiosis network have difficulties forecasting the production of these products. The products provided by the suppliers on the symbiosis network to the manufacturers are by-products or end-of-life (EoL) products rather than mainstream products.

**Variance of Prediction Error $\varepsilon$ :** When the manufacturers' prediction error of market demand, $\varepsilon$ , increases, in most cases, the manufacturers' profit declines. If the manufacturers' prediction of market demand is not accurate enough, it will bring additional loss to their own profits under the condition of information sharing. This decline is not affected by the information- sharing structure because the manufacturers' prediction of market demand directly affects the manufacturers' order quantity. However, it has nothing to do with information sharing. It also illustrates that, regarding the production process of this supply chain, manufacturers are in a dominant position.

## UNCERTAINTY OF SUPPLY CHAIN PROFITS

The profit changes of the supply chain in different scenarios are shown in Table A4 (see Appendix 5 for the calculation process).

**Variance of Demand Uncertainty $\sigma_\theta^2$ :** In the three scenarios, the increase in demand uncertainty, $\sigma_\theta^2$ , will lead to increased profits of the supply chain. Regardless of whether the suppliers obtain the information shared by the manufacturer, the manufacturer realizes the increase in profit of the entire supply chain by observing the market demand signal in time and adjusting the production plan.

**Variance of Supply Uncertainty $\delta_Y^2$ :** In the three scenarios, the increase in supply uncertainty, $\delta_Y^2$ , leads to uncertain changes in the profits of the supply chain. This is related to the degree of the supply uncertainty shock. When the supply shock is weak, the profit of the supply chain rises slightly. When the supply uncertainty is greater, the profit of the supply chain decreases. The authors believe

**Table 4. Supply chain's profit variations under the shocks**

| | $\sigma_\theta^2 \uparrow$ | $\delta_Y^2 \uparrow$ | $\varepsilon \uparrow$ |
|---|---|---|---|
| $J^{NN*}$ | $\uparrow$ | | $\downarrow$ |
| $J^{DN*} = J^{ND*}$ | $\uparrow$ | | |
| $J^{DD*}$ | $\uparrow$ | | $\downarrow$ |

that it is because suppliers cannot pass supply information to manufacturers before ordering. The communication of information within the industry chain is not smooth.

**Variance of Prediction Error** $\varepsilon$ **:** When the manufacturers' prediction error of market demand, $\varepsilon$ , increases, in most cases, the supply chain's profit declines. If the manufacturers' prediction of market demand is not accurate enough, it will bring additional loss to the entire supply chain under the condition of information sharing.

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