Bachelor's project

NTNU
Norwegian University of Science and Technology
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Department of Computer Science

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Experimental comparison of hill climbing variants on CVRP

Bachelor's project in Computer Science
Supervisor: Alexander Holt

May 2021

NTNU
Kunnskap for en bedre verden

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Preface

This paper will describe a project about route optimization for waste collection and research regarding how to use different local search algorithms to solve a variant of the ”capacitated vehicle routing problem.” The purpose of this document is to explain the background theory of the problem at hand, how the product was developed, and discuss the research results from the conducted algorithmic experiment.

Two out of the three people writing this paper worked for the startup that provided the task. The startup worked on an internal project for waste collection and wanted a route optimization tool and an analytic tool for waste collectors. The team was interested in the route optimization part, as the problem is heavily centered around algorithms. All participants in the team are interested in optimization techniques and developing useful algorithms. In addition to working on something of personal interest, the team also had the opportunity to create a valuable product for their startup.

The team thanks the representatives from Favn Software AS and the supervisor from NTNU for the support provided during the project:

- Anders Hallem Iversen, Bjørnar Østtveit and Sveinung Øverland from Favn for providing guidance for the task, helping keep the team on the right track for the product and providing the resources needed to develop the product.
- Alexander Holt from NTNU for guiding the team through the development of the thesis.

[illegible] Kjerand Evje [illegible]

Tobias Meyer Andersen Kjerand Evje Håvard Stavnås Markhus

Task

"The task is to develop a prototype for a service that will be a part of a larger pilot project in cooperation with the waste company Remiks in Tromsø. The goal of the pilot project is to make it possible to order waste pickup via an app, instead of driving oneself to a recycling station (The app is not a part of this task)". This statement from Favn declares the background of the project. Additionally, their suggestion of what precisely should be created was the following: "The task is to create an interface in the form of a website, and a back-end that will handle orders, route optimization, and data analysis."

Including developing the system tasked by Favn, the team will develop and compare the performance of several hill climbing algorithms for route optimization.

Summary

Routing problems in its different forms are ubiquitous in society. From postman deliveries to computer chip manufacturing, finding paths that minimize some criteria play a central role in many different industries. This thesis involves exploring such a concrete optimization problem, namely the vehicle routing problem. Determining an optimal solution to this problem is NP-hard, meaning that computing an optimal route in complex cases requires an extreme amount of computing power. This is not practical for applications that should quickly and reliably deliver solutions to these problems. It is therefore common to use heuristics that approximate the solution. The challenge with this is that the performance of different heuristics are situational and specific to the problems.

This thesis saw the creation of a system that could handle truck fleets and orders to be picked up, in addition to efficient routing for picking up the orders utilizing the available fleet. Within the thesis, experiments were conducted to test how certain known algorithms would perform when adjusted to work on a specific use case of the vehicle routing problem. Very broadly the algorithms tested all work by quickly producing a placeholder solution and gradually improving it in small increments, when done in a certain way it is known as hill climbing.

This document introduces the problem first, then lays the theoretical foundation of the terms used and the problem to be studied. Subsequently, the method of both development and the scientific experiments is described before the results are presented. The conducted experiments will answer the thesis question, which revolves around comparing different route-optimizing algorithms that build on the principle of hill climbing. The experiment tested the run time of different algorithms and the quality of the routes produced.The engineering professional approach and the development process of the product utilizing the algorithms are also documented. After the results, a chapter will discuss how to interpret the results and what the data indicates. The thesis is concluded with some pointers to further work for both the product developed and to route optimization will be left for the reader.

The experiments indicated that the teams implementation of Adaptive Iterated Local Search scaled worse than normal hill climbing and simulated annealing, but reliably provided better routes. The difference in scalability is significant as the sheer size of the route grows large limiting certain applications using this algorithm.

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1 Introduction

1.1 Background

Favn Software is a startup company located in Trondheim, Norway. It was founded in 2020 by students from NTNU (Norwegian University of Science and Technology). Favn is a consulting company specializing in developing software. This includes developing software for customers, including websites, mobile applications, back-end systems, and more.

Favn had for some time been interested in developing a system for waste collectors. The company had been in talks with Remiks (a waste company located in Tromsø), and there seemed to be interest in such a product. This system would include a route optimizing engine for the waste collectors and a front-end system for ordering waste pick-up.

The primary purpose of this system is to make the waste-collecting process more effective by computing optimized routes for the waste trucks. Normal people would not have to drive themselves to a recycling station, generating pollution and traffic in the process. Having a select few trucks collecting the items is, therefore, an opportunity for the waste company and the customers.

The role of the team during development will be to focus on the routing optimizing part of the system. This will include implementing different algorithms and optimizing them to produce the best possible results.

1.2 Research question

Formally the optimization problem the team received from Favn Software AS can be described as following: given a set of trucks with weight and space capacities, and a set of orders with a location, weight, and space assigned, produce a set of routes for the trucks such that each order is picked up within a time limit, minimizing the total travel time. It is also given that all the trucks start at the depot where they should return with all the orders. This is a variant of what is known as the capacitated vehicle routing problem, first formally studied in 1959 by Dantzig and Ramser[1].

Given the time at disposal, the approach of the team will be to implement different versions of hill climbing algorithms that are known to work for very similar problems and compare how they perform on this exact application. Hill climbing was chosen in particular as various sources state that iterative route optimizers using metaheuristics have been very successful[2][3].
In addition, the goal is to find out what works well in practice on realistic data and not to optimize theoretical worst-case scenarios. The thesis question is then:

How do different versions of hill climbing compare when adjusted to solve the capacitated vehicle routing problem with time limits on realistic map data?

1.3 Document structure

Here is a brief overview of the structure of the document.

- **Introduction** - background information and thesis question.
- **Theory** - brief summary of relevant background theory to understand the method, results and discussion.
- **Method** - explanation of how the algorithms were made and tested, development process of the team and technologies used.

- **Results -** a chapter showing the experimental results of the algorithms, team results in terms of development and administrative documentation generated in the project.
- **Discussion -** a discussion trying to put the results into context and reflecting on their meaning.
- **Conclusion -** answer of the thesis question using the results and their interpretation from the discussion, also a brief note on future work to be done around the thesis question.
- **References -** list of sources cited in the document.
- **Attachments -** list of attachments that constitute the rest of the delivery for the thesis.

1.4 Nomenclature

AILS	Adaptive Iterated Local Search
API	Application Programming interface
GCP	Google Cloud Platform
HC	Hill Climb
ILS	Iterated Local Search
LS	Local Search
OSRM	Open Street Routing Machine
SA	Simulated Annealing
UI	User Interface
UX	User Experience
VM	Virtual Machine

2 Theory

2.1 Greedy algorithms

An algorithm is considered greedy if it computes something by making the immediate best move, with no regard for future planning or past computation, usually in an iterative fashion. To illustrate this, consider trying to purchase as many items as possible from a store. An optimal greedy solution would be to pick items one by one, always selecting the cheapest item available. This strategy will guarantee the most items being purchased, and the greedy part is to select the cheapest one available. Although this particular greedy algorithm is optimal, that is not the case in general. Despite that, greedy algorithms are widely used within optimization, including route optimization. This is mostly because results can be computed very fast when the simplification of not considering the future or past is made.

2.2 Relevant problems

2.2.1 Travelling Salesman

The travelling salesman problem or TSP is an NP-hard problem, meaning the problem cannot be exactly solved in polynomial time.

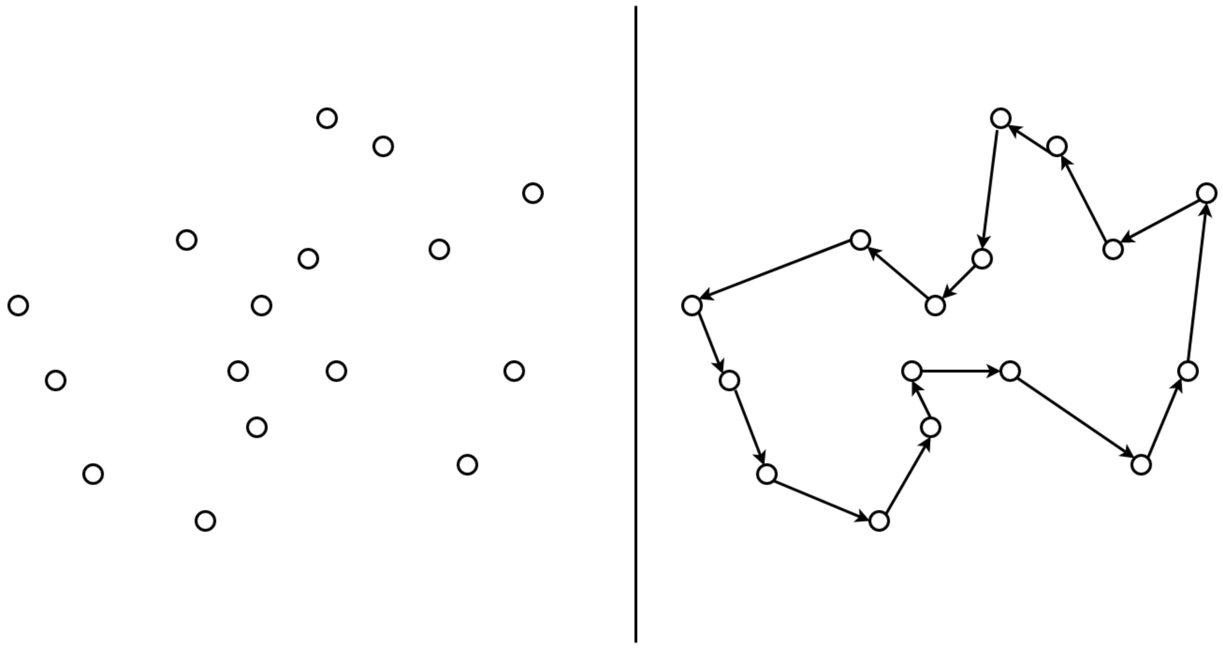


Figure 1: Illustration of TSP solution

The problem is "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?". The complexity of solving this NP-hard problem perfectly using brute force algorithms is O(n!). Using dynamic programming, the problem can be solved in $O(n^2 \cdot 2^n)$ time. The problem is one of the most intensely studied problems in optimization, as it appears in numerous real-life optimization problems. Transport logistics, manufacture of microchips and routing of trucks for parcel pickup all benefit from TSP algorithms. In many of these real-life cases, numerous "cities" need to be visited, and computing the optimal solution is impractical. Most applications therefore utilize approximate algorithms that are often based on meta-heuristics and iterative improvement of a constructed solution.

2.2.2 Capacitated Vehicle Routing Problem (CVRP)

CVRP is a generalization of the traveling salesman problem. The problem statement usually is defined as: "Given a list of cities to visit with a weight assigned, and a list of vehicles with a given capacity, minimize the total distance traveled while having the trucks combined visit all cities exactly once without surpassing their capacity."

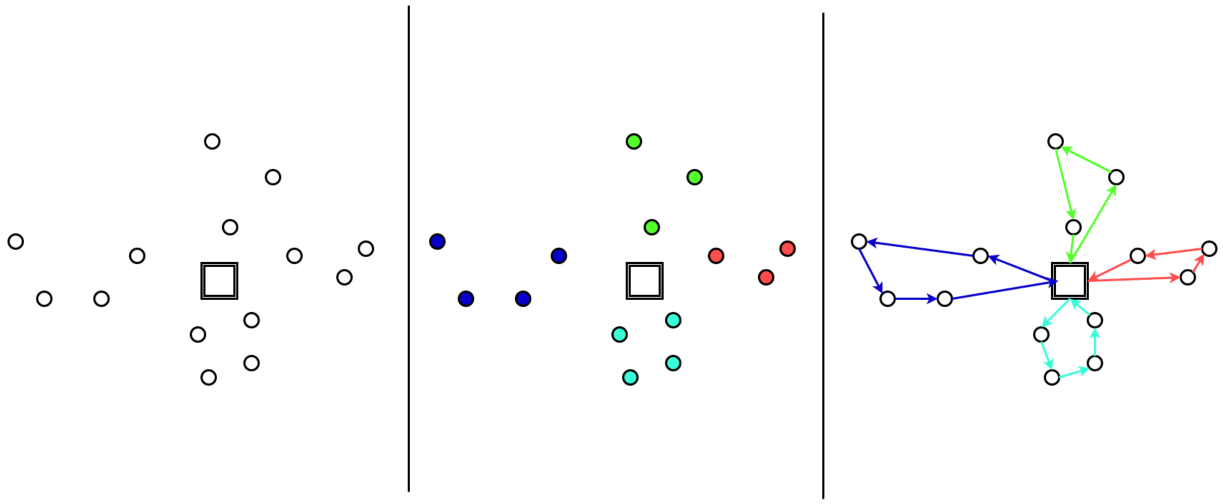


Figure 2: Illustration of CVRP solution based on clustering then solving multiple TSP's

CVRP can therefore explained as a traveling salesman problem with capacity constraints and multiple salesmen. As a generalization of an NP-hard problem, this is also a NP-hard problem. The simplest edge case is when one vehicle can visit all the cities, that special case is the regular traveling salesman. Otherwise, one must also figure out which orders should be picked up by which vehicles and optimize those individual trips. As with most NP-hard problems, finding an exact solution requires a large amount of computation. This suggests that approximation is needed in order to be able to compute good routes within a reasonable amount of time for practical applications. Different approximations will be explained later.

2.3 Clustering

Clustering is trying to single out meaningful groups of data. For instance, one can group books by genre, publisher, or release date. Clustering algorithms receive a dataset as input and tries to categorize them based on some metric of similarity. Clustering algorithms are often divided into two types, supervised and unsupervised. In the CVRP problem, the vehicles are groups, and the goal is to distribute the cities into the group to which they belong. Since an analyst defines the groups before running the clustering algorithm, it is categorized as supervised. The clustering methods described in this document are all supervised.

2.3.1 Nearest neighbor clustering

Nearest neighbor clustering groups data by iteratively finding the most similar data point to the previous, sometimes limited by some sort of group size constraint. This is a typical example of a greedy algorithm. This can be illustrated by using it as a heuristic to quickly produce a valid solution for CVRP. With this algorithm, the solution would be constructed node by node. Start by selecting any truck; the first order to pick up will be the closest one. The next order will be the closest order to the previous one that is not yet visited; the pattern repeats until the truck cannot pick up any of the other orders, at which point one returns to the starting node. This procedure is repeated for all the trucks available until all the orders are picked up. The solutions of the CVRP should minimize total distance. Hence the next order to visit is always the closest unvisited one.

The nearest neighbor clustering method is widely used within data science and optimization algorithms, primarily because it is intuitive, very quick in its greedy nature, and often yielding useful results, for instance, in many TSP algorithms. In that case, ignore the capacity of the trucks, allowing a single one to pick up all the orders. For symmetric TSP, it has been experimentally shown to usually produce solutions 25% worse than the global minima [4]. Symmetric means that the graph used by the traveling salesman algorithm is undirected, the distance between a pair of nodes is the same in both directions. As this thesis uses Open Street Map data from real cities, this problem is an example of an unsymmetric traveling salesman.

2.3.2 Randomized proximity clustering

Another clustering method is Randomized Proximity Clustering. In light of the application, the algorithm will be explained using the terms in the thesis question where orders need to be picked up by trucks that drive a given route instead of vehicles visiting cities. This initial cluster is based on the clustering method described in [5]. The algorithm is defined as:

1. At the start of the algorithm, a number m is calculated, representing the number of routes that should be driven. S is the set of orders, and s_c is the capacity needed to pick up a certain order, $\bar{q}$ average truck capacity. Therefore, m is the number of routes needed to pick up all of the orders with the following formula:
$$m = [\frac{1}{\bar{q}} \sum_{s \in S} s_c] \tag{1}$$
2. Assign a random order to each of the routes.
3. Go through all the unassigned orders and assign them to the route which they are the closest to in terms of some proximity measure. This could be implemented in many different ways; for instance, the team has used the proximity insertion metric described in 2.4.3.4.
4. Although all the orders now should be picked up by precisely one truck; the route may not satisfy the capacity restriction. In that case, the suggested clustering needs to go through some sort of correction to guarantee that a valid solution is produced by the clustering algorithm. The team implemented the feasibility function described in 2.4.6 to assure the creation of valid clusters.

2.4 Optimization

2.4.1 Local search

In many optimization problems, the goal is to find a configuration that maximizes some evaluation function. Often it can be very complicated and even unpractical to produce this solution from the bottom up. If it is easy to generate an arbitrary solution, although it may not be optimal, local search can be a very efficient way to improve that solution into a good one.

Local search is usually implemented as a graph search, where each node represents a possible solution to the problem. In the context of optimization problems, the goal will then be to find the node with the greatest value given some starting node. The adjacent nodes are those solutions one would have by making a certain change to the current one. The most common way to implement a local search is to clearly define what types of changes one can make in a solution to obtain another valid solution. In this way, it is possible to generate all of the neighboring nodes given the current solution, making the memory usage O(1) with respect to the number of nodes in the graph. Local search in the context of optimization problems can be thought of as making small adjustments to a solution repeatedly until some criteria are satisfied.

2.4.2 Hill climbing

Hill climbing is a greedy local search algorithm. It is common to generate a random solution as the starting node for the search, which can be far from optimized. As hill climbing is a local search algorithm, all the neighboring nodes of a solution can be generated and evaluated in terms of desirability. The most common greedy variant of hill climbing will move to the first generated neighbor with higher desirability than the current one. This is repeated until it converges to a local (possibly global) maximum in terms of the evaluation function. Another notable implementation of this is the steepest ascent hill climb, where all the neighboring nodes are generated, and one visits the node with the greatest desirability. Hill climbing is a straightforward technique that is quite fast but will not guarantee the optimal solution in general. It is possible to run several subsequent hill climbs starting from random solutions in hopes of converging to a better local optimum. Having no specific strategy when generating the next starting point is known as the "Random Restart Hill Climb." If one instead applies a specific modification to the result of the hill climb and start the next search from there, one has an implementation of what is known as "Iterated Local Search".

Intra route heuristics

In this thesis, the team used the heuristics described below to modify valid or invalid solutions into solutions that are better and more feasible. These are "intra heuristics," meaning that they modify a specific route for a single truck. Since these transformations changes the current solution, these heuristics will generate all neighboring nodes for a particular state. To make it clear, the graphs shown below represent a single trip, or route, that will be driven by a truck. Note that the classic TSP terminology of cities is used in the following figures showing each type of move used by the team to optimize single trips. These intra heuristics are described in [2].

2.4.2.1 Intra-route 2-opt

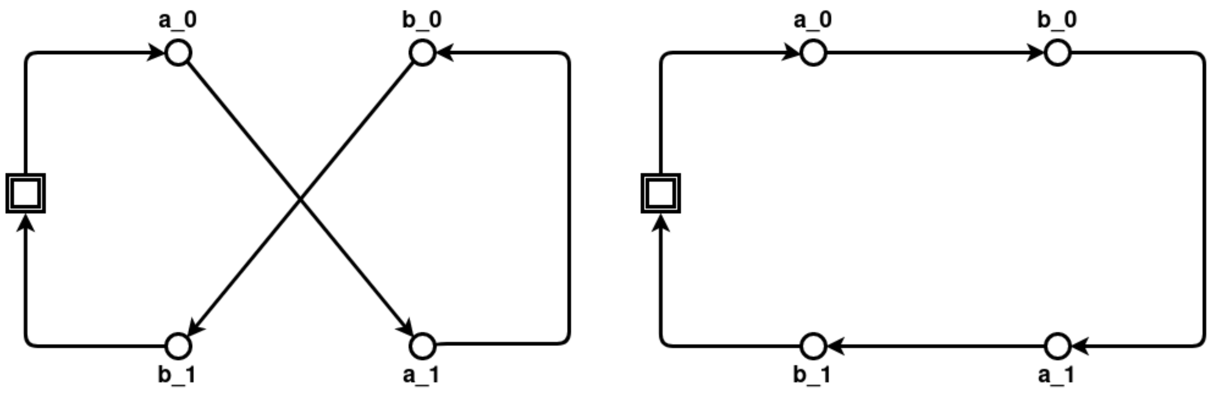


Figure 3: TSP 2-opt heuristic

2-opt tries to minimize the distance by reversing the order of the nodes from b_0 to a_1

2.4.2.2 Intra-route exchange

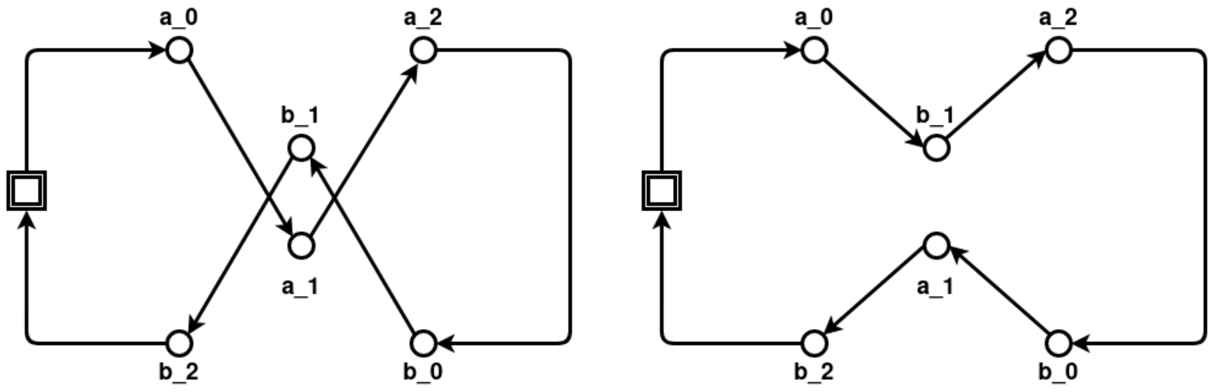


Figure 4: TSP exchange heuristic

The exchange heuristic swaps the position of two cities in the route. a_1 is swapped to be visited between b_0 and b_2, while b_1 is swapped to be visited between a_0 and a_2. See figure 4.

2.4.2.3 Intra-route or-opt

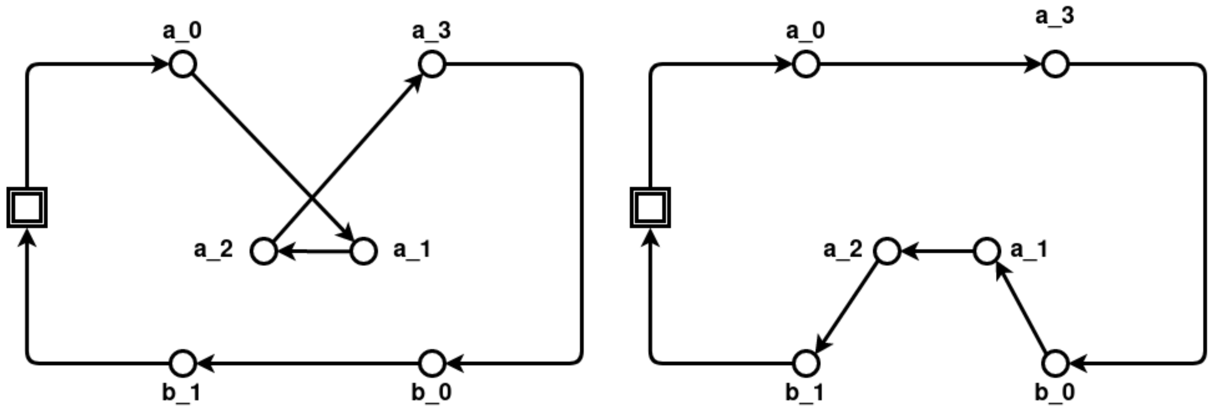


Figure 5: TSP or-opt heuristic

The or-opt heuristic checks for a segment of 1, 2 or three consecutive Cities can be relocated to another place in the route order. In the case of figure 5 two cities are moved.

2.4.2.4 Intra-route relocate

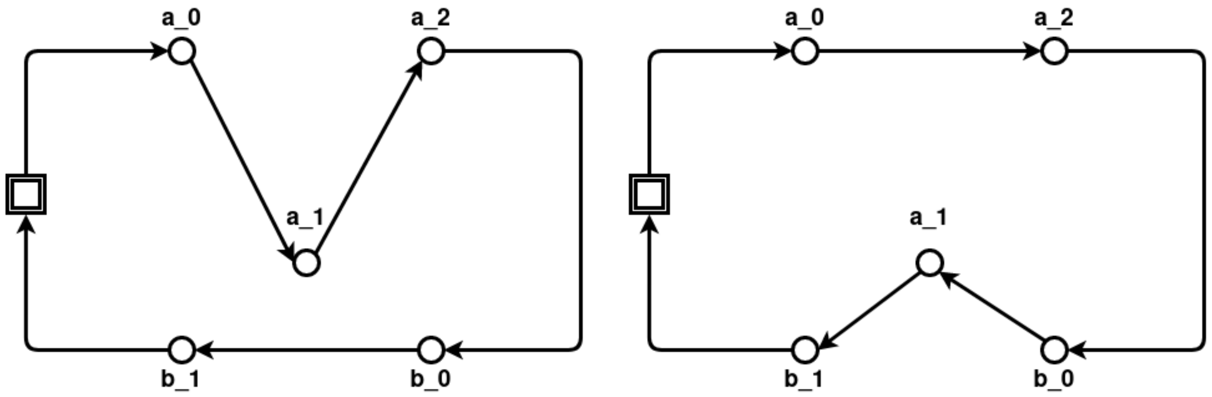


Figure 6: TSP relocate heuristic

The relocate heuristic checks for a single city to be relocated to another place in the route.

Inter route heuristics

Inter route heuristics are move heuristics that moves cities/orders between two routes. These heuristics in the HC implementation also modify valid or invalid solutions to solutions that are better and, if invalid, more feasible. The ones implemented by the team in the HC algorithm described in 3.9 are shown below; note again that the graphs shown represent the actual changes in a solution. These inter heuristics are described in [2].

2.4.2.5 Inter-route relocate

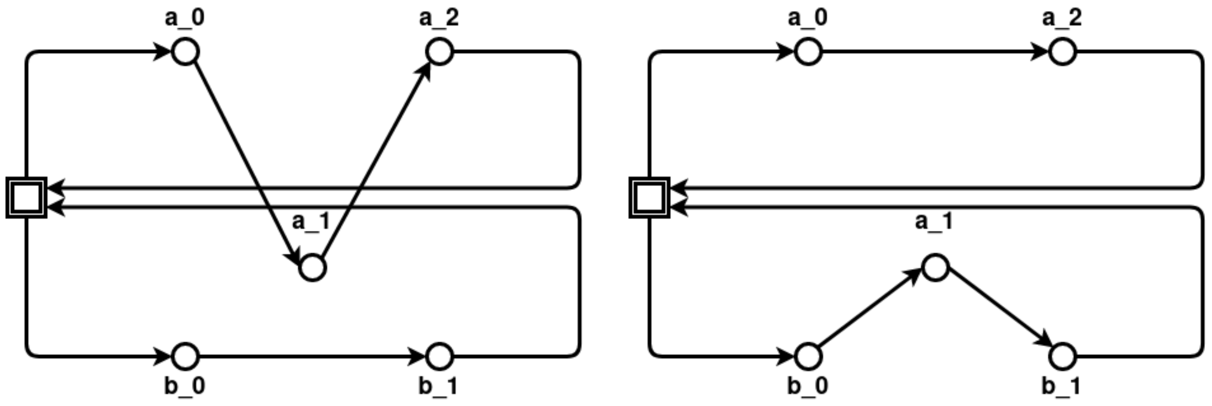


Figure 7: VRP relocate optimization heuristic

This optimization technique checks a candidate node in one route cluster for relocation to another candidate position in another cluster.

2.4.2.6 Inter-route exchange

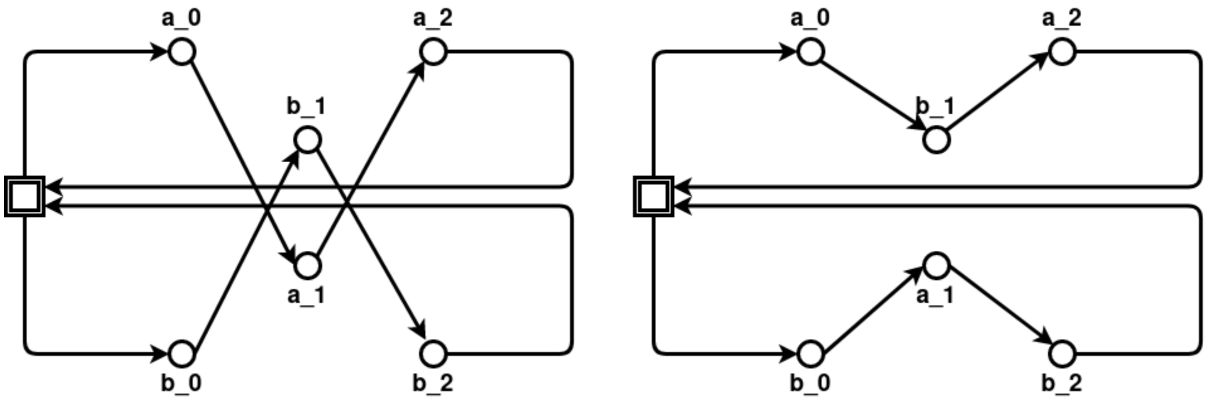


Figure 8: VRP exchange optimization heuristic

This technique checks one node from two different routes and exchanges the position of the nodes in the two routes.

2.4.2.7 Inter-route cross exchange

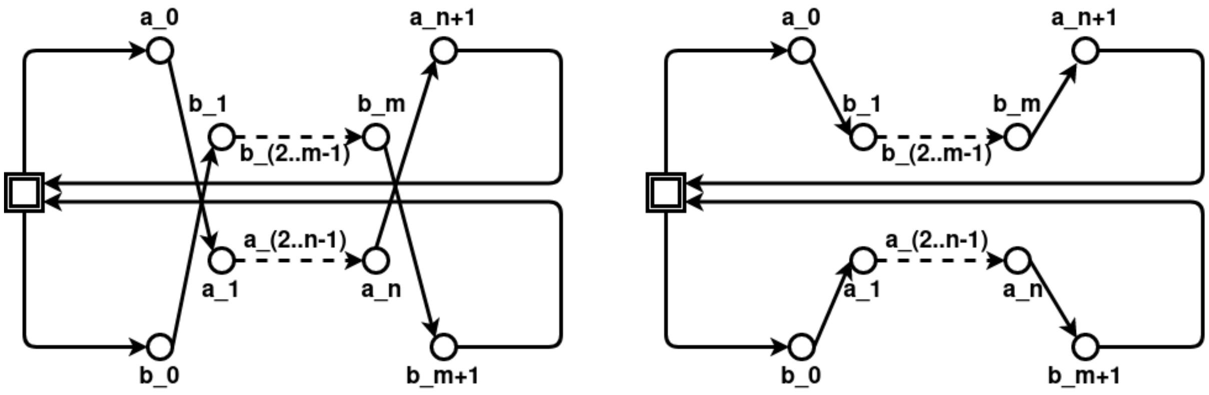


Figure 9: VRP cross exchange

The cross exchange heuristic swaps two intervals of consecutive orders from different clusters with each other.

2.4.2.8 Inter-route icross exchange

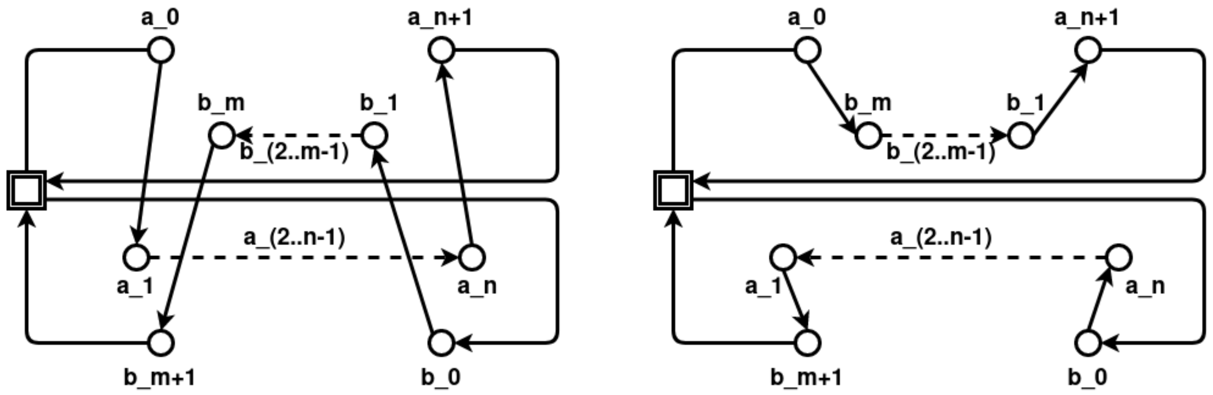


Figure 10: VRP icross exchange

The i-cross exchange heuristic swaps and reverses two intervals of consecutive orders from different clusters with each other.

2.4.2.9 Inter-route 2-opt

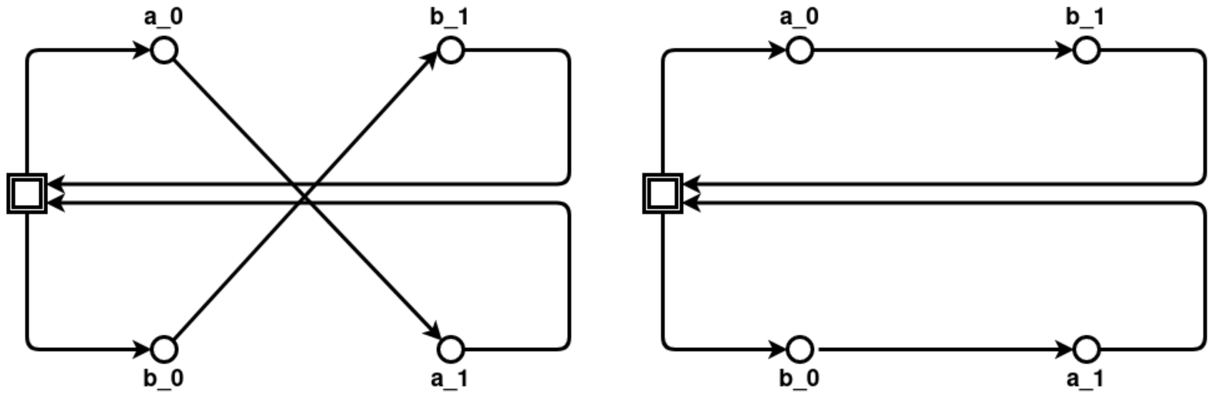


Figure 11: VRP 2-opt

The inter route 2-opt heuristic swaps the destination of two edges from different routes.

2.4.3 Iterated Local Search

As stated in the previous section 2.4.2, iterated local search is the repetitive strategy of performing a hill climb, modifying that solution in a certain way, and then starting another hill climb from the new solution. The core difference from the random restart algorithm is to have a perturbation step that selects the following starting location based on the found optimum. This stage of the algorithm needs to balance between the following factors; firstly, a very "weak" perturbation step would be equivalent to a random restart. Secondly, the opposite will also cause problems, as having a new starting solution that is very similar to the last optimum will usually converge back to the same state. In normal ILS, the perturbation step is always the same. An example of a common perturbation step for a TSP ILS algorithm is the "double-bridge move" [6]. The essential takeaway here is that ILS uses a perturbation step that is not dependant on solution quality or density; the details of the "double-bridge move" are unimportant. If the algorithm considers the history of local optima reached when choosing how to perturb, one would have an example of an adaptive iterated local search.

In the next few subsections, the heuristics used in the perturbation step of the AILS implementation of the team are shown. All of the perturbation heuristics are based on the heuristics described in [5]. These graphs show routes with multiple trucks to pick up the orders. In the descriptions, the orders are referred to as ”vertices”.

2.4.3.1 Concentric removal

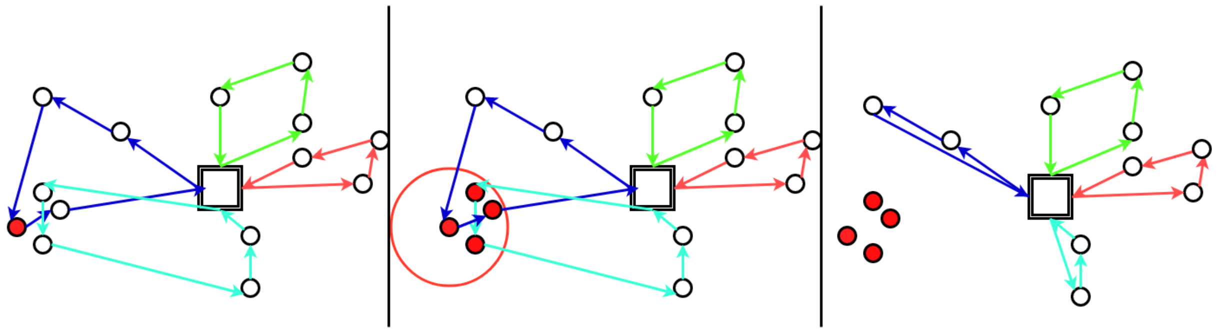


Figure 12: Removing a given amount of vertices closest to a random vertex, including the random vertex

This heuristic selects a random vertex v_r in the graph, removes the vertex and a given amount of vertices closest to v_r. This way the heuristic removes vertices within a coverage area, using the vertex v_r as center.

2.4.3.2 Removal by proximity

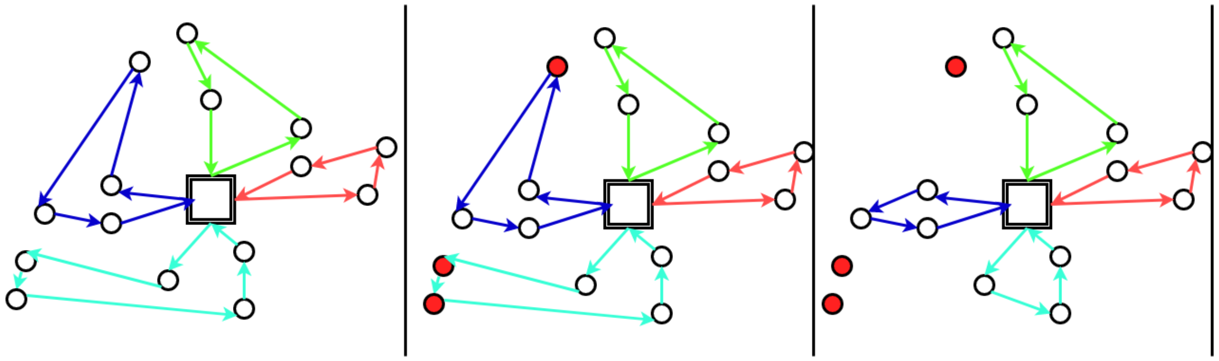


Figure 13: Removing a given amount of vertices that are rated as the furthest away from their routes

In this heuristic a given number of vertices are removed that are rated the furthest away from their routes, based on a proximity index. The proximity index indicates a relative distance from the route, and not a euclidean one. The proximity index for a vertex compared to a route is calculated with this equation:

$$prox(v,p,R_j^S) = \frac{minset(min\{p,|R_j^S|-2\},\Pi_{R_j^S}(v))}{min\{p,|R_j^S|-2\}} \tag{2}$$

The value $\Pi_{R_j^S}(v)$ is a set containing the distance rank of the other vertices in route R_j^S to vertex v. if $R_j^s = \{3,6,7\}$, $v = 6$ in a system of 10 vertices $V = \{1,2,...,9\}$ and vertex 3, 7 are the 3rd and 6th closest

vertices to vertex $v = 6$, then $\Pi_{R_j^S}(6) = \{3, 6\}$.
$minset(a, set)$ is defined as the sum of the a smallest elements in the set "set".
The proximity function (2) gives an index of relatively how close a vertex is to a route. In the proximity removal heuristic one therefore compares all vertices to its route with the proximity function (2) and remove the vertices with the highest proximity indexes.

2.4.3.3 Sequence removal

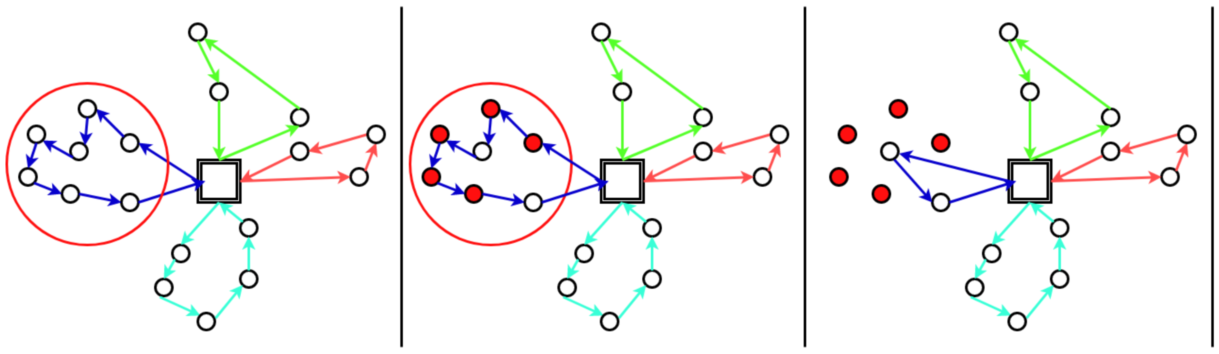


Figure 14: Removing random sequences of a given amount of vertices from a route

In the sequence removal heuristic a given number of vertices are removed by randomly selecting sequences of vertices in a route and removing these from the solution.

2.4.3.4 Insertion by proximity

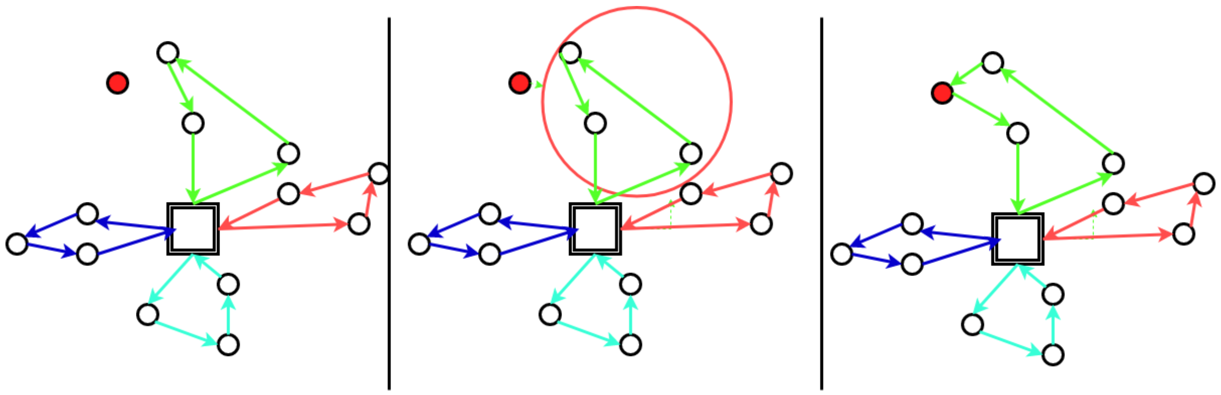


Figure 15: Inserting vertex in the route closest to the vertex, at the position with lowest cost

This is a heuristic in which one inserts a vertex into a route based on proximity. First, one needs to find which route is closest to the vertex. Compare the vertex to every route in the solution using the proximity function (2) and find the route with the smallest proximity index:

$$R_j^S = argmin_{R_j^s \in Routes} prox(v, p, R_j^s) \tag{3}$$

p is a random integer in the interval $[1, [n/m]]$, where n is the amount of vertices, and m is the amount of routes in the current solution. The route R_j^S is the one with the lowest proximity index. Next, find the position within the route to place the vertex. This is done by calculating the insertion cost with following equation for a route $R_j = \{v_0^j, v_1^j, ..., v_{n_j}^j\}$:

$$\hat{i} = arg \min_{i \in (0,1,...n_j)} d(v_i^j, v) + d(v_{i+1}^j, v) - d(v_i^j, v_{i+1}^j) \tag{4}$$

The index $\hat{i}$ to insert the vertex is calculated with this equation, where v is the inserted vertex, v_i is a vertex in the route, and d is the distance between two given vertices. This results in inserting the vertex at the index $\hat{i}$, which is the index with the lowest insertion cost.

2.4.3.5 Insertion by cost

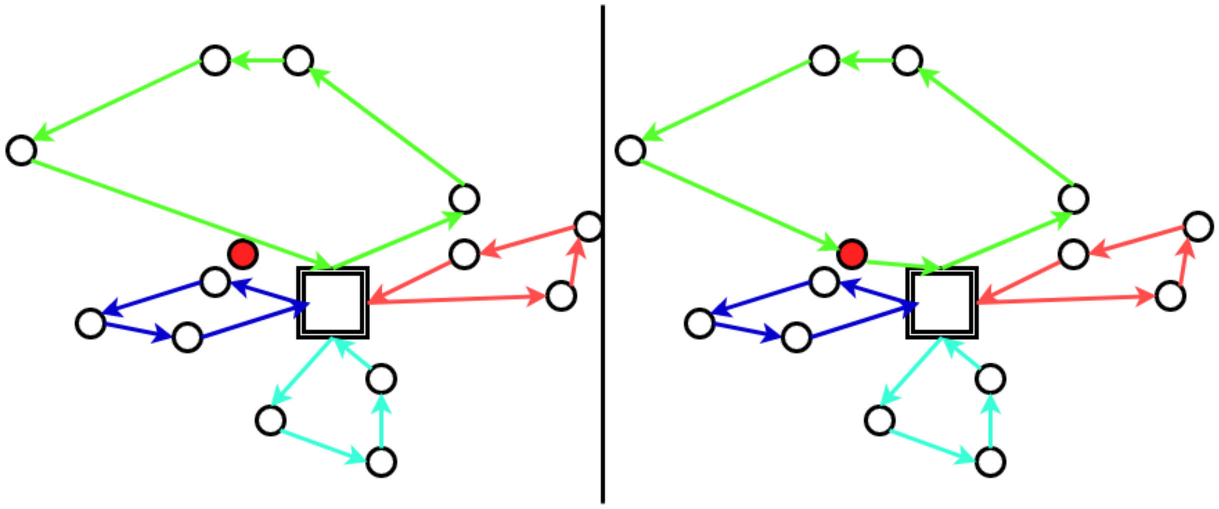


Figure 16: Inserting vertex in the minimum cost position

The cost insertion technique inserts the vertex v in the position with the overall lowest cost. This differs from proximity insertion by deciding which route to insert to solely based on the cost of the insertion. The process is described as following:
Find the route $R_{\hat{j}} = argminR_{\hat{j}} \in Routes\ c(v_i^j, v_{i+1}^j, v)$. Insert the vertex into route $R_{\hat{j}}$ at position i. where $c(v_i^j, v_{i+1}^j, v)$ is the lowest cost in route $R_{\hat{j}}$.

2.4.4 Adaptive Iterated Local Search

As briefly mentioned in 2.4.3, AILS is a form of ILS where the perturbation step depends on the optima found so far. This optimization technique often works well because solutions from the same region share many characteristics. When taking into consideration all of the optima found in the previous iterations, one can determine if the region of the search space is promising or not. One can then choose to perturb more weakly in promising regions to more finely explore solutions expected to be good or take larger steps to avoid spending time in uninteresting regions. The premise that promising regions often contain multiple solutions of similar quality rests on empirical observation [5].

As the core part of AILS is to decide when to explore a new region, an example is provided below in equations 5, 6 and 7. These calculations make up what is known as the acceptation criterion, which decides whether or not to perturb to create a new starting location for the next iteration. This particular acceptation criterion is taken from Vinícius et al. [5]. To start, the user decides the value of k within the range [0, 1]; the greater the value of k, the more likely one is to perturb. If s is a valid state, s has a set of routes such that all the orders are visited, f(s) is the total travel time by all the trucks. γ describes how many iterations one should consider when deciding whether to try a new starting state; this parameter is a user-defined positive integer. $\underline{f}$ is simply the best solution found in the last $\min\{\gamma, it\}$ iterations, where it is the number of the current iteration. $\bar{f}$ is a weighted average of the quality of the last iterations updated as shown in equation 5. Both $\bar{f}$ and $\underline{f}$ both have a starting value of 0. Using equation 7 $\bar{b}$ will be a value in the range $[\bar{f}, \underline{f}]$

$$\bar{f}_{next} = \begin{cases} \bar{f}_{prev} \cdot (1 - \frac{1}{\gamma}) + \frac{f(s)}{\gamma}, & it > \gamma \\ \frac{\bar{f}_{prev} \cdot (it-1) + f(s)}{it}, & it \leq \gamma \end{cases} \tag{5}$$

$$\eta_{next} = max\{\epsilon, \frac{k \cdot \eta_{prev}}{k_r}\} \tag{6}$$

$$\bar{b} = \underline{f} + \eta \cdot (\bar{f} - \underline{f}) \tag{7}$$

All of the equations are calculated each iteration of the AILS algorithm. If the current solution found is better than the calculated $\bar{b}$ then the current solution will be the starting point of the following search; otherwise, continue to search for solutions from the same starting state.

2.4.5 Simulated Annealing

Simulated annealing is a probabilistic version of hill climbing. Analogous to a material cooling down, the algorithm keeps track of a temperature that steadily decreases by a constant factor. The most important difference from the hill climbing algorithm is that there is a nonzero probability of exploring a state of lower desirability. The exact formula for the probability is shown in equation 9, where ΔD is the difference in solution quality of two adjacent states, and T is the temperature at that point in time. If the considered neighboring solution is an improvement, then the search will always explore that state, hence the probability of 1. Note that these equations are written on the form where one seeks to minimize some loss function instead of maximizing some value function.

$$\Delta D = loss(state_{current}) - loss(state_{new}) \tag{8}$$

$$P(\Delta D, T) = \begin{cases} 1, & \Delta D \leq 0 \\ e^{\frac{-\Delta D}{T}}, & \Delta D > 0 \end{cases} \tag{9}$$

In practice, the search will be very exploratory in the beginning and slowly start to converge to a local optimum. This technique will often outperform hill climbing as it gets to explore a greater variety of solutions and converges to a better one. It has even been proved that with infinitely slow cooling, the global optimum can be guaranteed [7]. This is clearly infeasible for complex applications as the computational power needed also goes to infinity. This means that one needs to fine-tune the cooling factor to let the algorithm spend as much time as one can afford in practice. It should be noted that in some problems, only very specific changes to a solution will be improvements. This can result in simulated annealing being worse than hill climbing as it will not have time to converge to any optima at all if it rarely has the chance even to make a single improvement.

2.4.6 Feasibility

In the CVRP, the feasibility of a solution is critical. This, however depends on if the constraints are hard or soft. This paragraph focuses on hard constraints, meaning the solution should not violate the constraints at all. As with many versions of the VRP, some constraints have to be met in the solution. In the CVRP, the constraint is the capacity of the vehicle, where each city to visit has a weight that fills the capacity of a vehicle. In the CVRP, the value of a move is measured by the difference in the total distance of the solution and the gain of feasibility. An infeasible solution might need to visit neighboring solutions that are more feasible to converge toward a potentially feasible solution. In ILS and AILS, the step of perturbing the solution might result in transforming a feasible solution into an infeasible one, but this is often appropriate. This is appropriate because one might find a feasible local optima that otherwise would not have been discovered in the neighborhood search process. By defining a process for evaluating the slack, it can be determined that the feasibility gain for a specific move between neighboring nodes in the search process. In this case slack means spare capacity or the amount a constraint is violated in a route. The feasibility gain can be determined by measuring slack before and after a move has been made.

In the AILS algorithm proposed by Vinicıcius et al. [5], The neighborhood search process is split into two parts: a feasibility procedure and a local search procedure. The feasibility procedure receives a potentially infeasible solution, then performs optimizations valuing feasibility gain to converge to a feasible solution. If a feasible solution is not found with the current set of routes, another route is added. This increases the capacity of the routes in the current solution. The process is then repeated with the newly added empty route. This process repeats until a feasible solution is found. The local search procedure then searches the neighborhood of the feasible solution, valuing moves that make the total distance shorter, as well as requiring any move to either gain feasibility or keep feasibility the same.

3 Method

3.1 Process

In this project, the task was to develop a functional system for Favn. This system includes a route optimization algorithm and a user interface with connected functionalities. Due to the project not having all the specific system requirements set in stone at the beginning of the project, and because the thesis question was not yet decided, it made sense to develop iteratively to focus on adaptability. The team consisted only of three developers and decided that the overhead and complications of following a strict version of a comprehensive development style like Scrum would not be worth it. The team also had plans to focus on the algorithmic part of the system, meaning the development process might not need to be very user-centered. However, the team focused on keeping an iterative and lean development strategy.

Before the project began Favn Software AS produced a list of different properties they wished a final product would have and what might be interesting for the team to work on. This list was extensive as it included both details about the algorithms and specific properties of front-end components. This list was the foundation for the vision document. For this reason, the vision document is also quite ambitious. There was an understanding between the team and Favn that the team would have some freedom during the project to pick out what parts they wished to work on as completing all of it seemed unlikely from the start. Throughout the project, the team and Favn agreed that the project would focus on the algorithmic part of the product. Therefore, some front-end goals relating to user functionality were deprioritized to let the team delve deeper into route optimization techniques.

The team discussed creating a Gantt diagram for long-term planning. However, since the project time frame was long and the specifics of the task were uncertain, the team and the supervisor deemed it unnecessary to produce such an extensive diagram that might quickly grow obsolete during the development. The team instead created a road map for long-term planning at the start of the development. The road map contained clear overarching goals for each two-week sprint. The document was created to have some oversight of the overall progress compared to the time remaining for the project, without having to use a lot of time planning, making it easy to change this during development. The goals created for the road map are based on the design document and its user stories and should line up with these stories and requirements when initially creating it.

During the development phase, the team had sprints that lasted two weeks. As the team consists of only three people, it was preferable to have more than one week for each sprint. This was done to set a realistic short-term goal for each sprint more easily and still successfully meet the goal within one sprint. Since the time frame for the project is set to several months, it also seems feasible to have some length to the sprints, but not so much that it is hard to determine how much can be accomplished in one sprint. For these reasons, a two-week-long sprint is the most reasonable. At the end of each sprint, there was a sprint review/sprint planning meeting with Favn, where the team presented the progress achieved during the sprint and planned on what to do for the next sprint. The team received feedback on what was implemented to ensure that the project was moving in the right direction. During the meeting, the goal for the next sprint from the road map would also be discussed. A clear goal for the sprint would be set, and potential changes to the road map would also be discussed in these meetings. This way, the process stayed iterative and agile.

In addition to the sprint review and planning meeting, the team also held a couple of stand-up meetings each week. The purpose of these meetings was to inform the team of the status of the current task each person was working on. This means informing what the person has done since last meeting, what task the person is doing currently, and what might block them from doing the task. These meetings also force people to be assigned a task if they are not doing something productive. Usually, with bigger teams, these sorts of meetings should be held every day. The team normally kept in contact, which kept the team sufficiently up to date on what was being worked on. This made performing daily stand-up meetings superfluous.

As part of the agile development process, the team used a Github task board to keep track of the tasks during a sprint. The task board is populated by using the clear goal produced in the sprint planning. The product to produce and the vision for the project are susceptible to change under development. Therefore, the team decided not to use much time to produce an overall product backlog but instead decided on a clear goal and produced tasks for each sprint based on the goal set for the sprint. The task-board set up for this project in Github consists of four columns: "Tasks, Doing, Testing, Review." Meaning each task has to be tested and reviewed by the rest of the team before it is considered done. This ensures quality assurance on completed tasks as long as the process is followed.

3.2 Choice of technologies

3.2.1 Rust

The route optimization server is written in Rust [8]. As this server is made to run extensive calculations, a very efficient language was a prerequisite for real-time results. Rust also has a strict compiler that ensures memory safety, increasing the confidence of the team that specific bugs are not present in the code after compilation. Rust is also a very modern language with libraries (called crates in Rust), so one can easily create web APIs as an interface to communicate with the server. This is where Rust differentiated itself from C++, which is another fast programming language typically used for rapid computations. C++ has also been used to write route optimizing APIs, such as the open-source project VROOM. C++ is an older language where working with web APIs and JSON structures would have taken us quite a bit more time to implement, which would leave the team less time to focus on the actual problem at hand. The web framework used to set up the endpoints and the rest API was Actix, one of the fastest web frameworks in Rust [9].

In the vision document one of the functional properties of the system is a simulation tool, where there is a critical need for the possibility of rapidly generating solutions. The tool should be able to simulate route generation long-term. This means that it should be possible to generate hundreds of solutions that show how the results would be over a long time period. The simulation tool therefore indicates it is critical that good solutions can be generated as quickly as possible. This means the drawbacks in Rust of needing more time to implement features is worth the payoff of getting optimal execution times. Another functional property in the vision document is the ability to generate a route for a user and to visualize it. For this use case, a quick route optimizer server is vital, as responsiveness is very important in user systems. This is another reason why Rust is a logical choice for the system.

3.2.2 Firebase

For this project, the team used Firebase as the back-end platform. This was a forced decision by Favn as a non-functional requirement of the project described in the vision document. Firebase is a platform made by Google which offers a range of back-end functionality like cloud functions, NoSQL database, authentication, hosting, machine learning, and more. One of the main reasons the team chose Firebase is that it is the main platform used by the client, Favn Software. As the project is part of a larger system, it makes sense for the team to use the same platforms as implementations used in the other parts of the larger system. When it is time to merge the different parts of th system, it will be easier to manage than using different platforms.

Firebase also offers all the back-end functionalities that the team needed for the project, including cloud functions and a NoSQL database. Another reason the team chose Firebase is that it integrates very well with the React framework described in 3.2.4.

3.2.2.1 Firebase Firestore (NoSQL)

Firebase Firestore is a NoSQL database integrated into the Firebase platform [10]. For this project, it is preferable with a NoSQL database as it is very fast and scalable. This is important since the team needed to store large amount of route data into the database. It also allows for both structured and unstructured data, which is very useful as the data is quite dynamic. NoSQL makes it easy to change the structure of the data while developing, which is very likely in the project. The set structure and metadata of the routes constantly change depending on requirements figured out while developing the system. Firestore database is also accessible directly from front-end applications for a lot of languages, including javascript. This means that data can be fetched directly from the database, using firebase authentication and security rules in the database.

3.2.2.2 Cloud Functions for Firebase

Firebase also allows for Cloud Functions, which is an essential feature for the project. Cloud functions are serverless endpoints managed by google, which scale automatically based on demand. They also have good support for testing and running these functions locally, so the development of the back-end solution stays simple while also ensuring scalability. The drawback of this serverless solution is that the functions have a slow start-up time, meaning that data gathered from a cloud function might take a lot longer time than with a server-based distributed system. However, by using cloud functions for features that are bound to be somewhat slow anyway, this drawback is negligible. For just reading and fetching data from the database, this can be done directly in the UI. This is feasible by having good security rules in the database. Cloud Functions can also be triggers. A trigger is a cloud function that activates when something is added, changed, or removed in the database. As the system is planned to automatically update routes for different time interval instances as soon as a new order is added, the trigger function feature is very useful.

3.2.2.3 Cloud pub/sub functions

pub/subs are similar to Cloud functions, as they are computing endpoints that scale automatically. However, pub/subs read messages with data put on a queue by other cloud services in the GCP project. The pub/sub reads incoming messages on the queue and handle the data passed in the message to perform computations with these. This computation flow is a good fit for the simulation tool planned in the vision document, as huge workloads and long computation times are needed to generate all the results needed from a simulation.

3.2.3 Google Cloud Platform

Google Cloud Platform is a platform for cloud computing services [11], including Firebase(3.2.2). Favn has most of its applications built with GCP technologies, and the same was planned for the task. While already using other technologies within Google Cloud, integrating new technologies under the same group of computing services is simple. As well as using firebase and cloud pub/subs, The team utilized the virtual machines service in Google cloud platform to host the route optimization API and OSRM API. This was because the integration of this service was simple and could be connected to the same Google Cloud project as the rest of the back-end system.

3.2.4 React

The product was initially planned to have a web application for garbage collection participants, so a user interface that utilizes the system is needed. For developing this front-end, web application the technology used is React [12]. One of the reasons for choosing this technology was that it was recommended by Favn, as they have expertise with this technology and can help out if needed. React is also easy to learn and has a lot of community support. Also, as the intention of the task was to focus on the optimization part of the system and developing this, it is critical that the team can quickly create components in the web application to test the core of the task. React makes it easy to create high-quality components quickly, and it is easy to create and reuse other react components.

Another alternative that was considered during planning was vue.js. This framework works well for smaller projects and has an easy learning curve. This seems very appropriate for the scale of UI development that was planned for the task. However, this project has a bigger vision for the web application than what was planned in the task. React is then the better option. React works well for larger scale applications, which is the plan for future work on this project. Since the application needs maps to show resulting routes for participants to use, React is also a good choice, as it has good libraries for map solutions like Google Maps.

3.2.5 TypeScript

TypeScript was chosen as the development language in the front-end components and cloud functions [13]. TypeScript is a superset of Javascript made by Microsoft. Our choice was between TypeScript and JavaScript, as these are languages that are well suited for web development and integrate well with React, Firebase, and Google Maps. The reason TypeScript was chosen over JavaScript is because of the ability to add static types in TypeScript. This makes handling orders and complex routes more manageable, while it also makes for better error handling. It also makes the code more maintainable for future work, since types make the code more readable.

3.2.6 Google Maps

The user interface of the system requires a map in order to show the generated routes and order information. The route data on the server-side uses standard WGS84/EPSG 4326 coordinate standard, the same as Google Maps. This makes it easy to integrate the route data to a library utilizing the google maps API. Google Maps has the most extensive API for web applications, making it easy to configure and integrate features to the map, like order information and markers for generated routes. Since React already has some well-implemented libraries for Google Maps, this was the simplest and best choice. Another reason for this is that the project mostly already utilizes google technology via Google Cloud Platform. This makes a Google Maps API key easily accessible and simple to set up.

3.2.7 Docker

The system developed in this task uses several services in the backend. The user interface interacts with Firebase, which is serverless, and firebase ensures scalability by default. Both cloud functions and Firestore database scale automatically based on demand. However, firebase cloud functions utilize the route-optimizer service, a REST API, and not serverless. The route-optimizer is also built on top of the OSRM API, which is also a server-based API. To ensure simple scalability in these services, Docker [14] has been utilized to containerize these APIs. This allows the APIs to be more portable. The OSRM service is dockerized by having a large container that fetches the latest Norwegian map data and sets up the newest version of OSRM from git. The route-optimizer API is also dockerized and automated for the same reasons as OSRM; simpler scaling. Both of the APIs are stateless, except for OSRM needing to specify which map to build the service on. However, since the project is planned for Norwegian roads, the state is the same in every case, rendering the service technically stateless. By dockerizing these APIs, the process of setting up virtual machines or future clusters for these services is simplified.

3.2.8 Open Source Routing Machine (OSRM)

OSRM is an open-source routing API written in C++ [15]. It is among the quickest of its sort and allows all the key features required for the project. Most importantly, it allows for sending a list of coordinates and receiving the shortest path matrix back. This matrix contains the shortest path between all the pairs of the points sent to it. In this context, the length of the shortest path is measured in seconds, as the time it takes to drive from one place to another is the most interesting. This happens extremely quickly and lets the team greatly reduce the complexity of the problem by compressing the map. OSRM gets its map data from Open Street Map, which is another open-source project. Open Street Map is regularly updated and allows for getting the travel time between points. All of this considered, OSRM is a well-performing, completely free way to get necessary geographic data and help reduce the complexity of the problem. As this solution is satisfactory, paid options like Google Maps were looked away from for fetching shortest path data quickly. Additionally, OSRM is free to use in commercial settings, which is another requirement of Favn Software AS.

3.3 System architecture

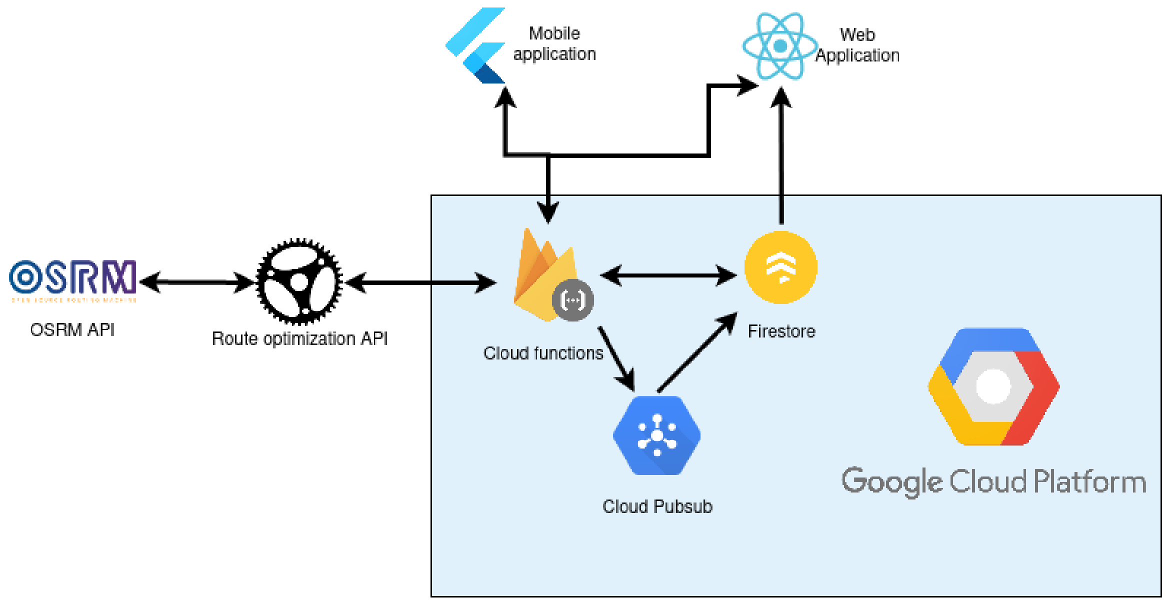


Figure 17: illustration of the overall system technology and how the system communicates

The overall system is planned to have a mobile application that is also connected to the back-end system the team was planned to develop. Since other applications are being developed on top of the same technologies with GCP, the team also needed to use these technologies.

The implementation of the team focuses mostly on the route optimization API and the system within the GCP while developing some features on the web application. To ensure that what the team is working on can easily be integrated with the overall system, the route optimization was developed as a separate service any application can build on top of. The service was also built on top of an OSRM API. This means that any application that can generate a distance table and routes between orders can easily replace the OSRM API.

Because of this, mocking the OSRM API and testing the route optimization API by itself becomes a simple process. The web application communicates with the Firebase Cloud Functions, but also receives data directly from the Firestore database, with the intention of using security rules to ensure security when authentication is implemented. The cloud functions communicate with the route optimization API to create route data and store these in Firestore. The cloud functions also send jobs to Google Cloud pubsub, which in turn write data to Firestore. These jobs are mostly created for the simulation tool described in 3.8.

3.3.1 Web application and Firebase entity system

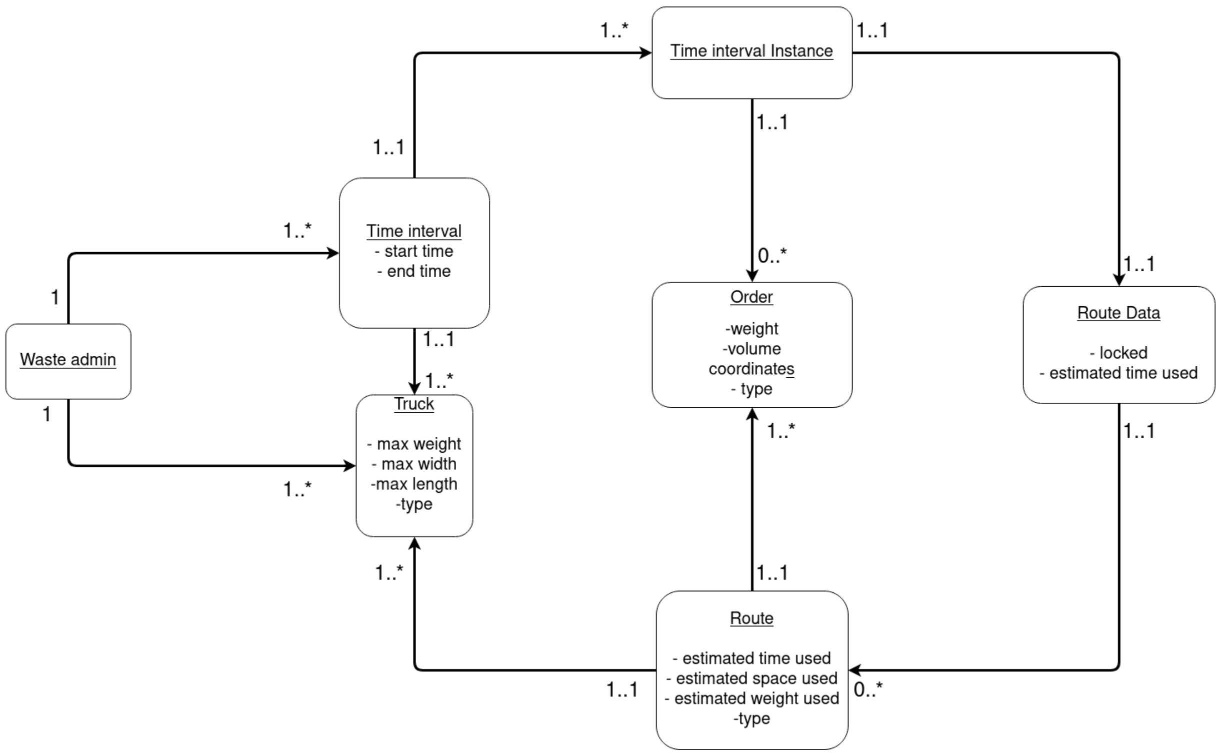


Figure 18: model of the domains in the system

In this section, order and truck will described as in section 3.5. In the system, each waste administrator or organization can have several trucks registered to their organization. They can also have several time intervals set up. These time intervals contain information about which trucks are driving during the time interval, what time the interval starts and what time it ends. The intervals also contain data about how the optimizer should prioritize utilizing the trucks at disposal. This means that the optimizer can be configured to initialize the solution by either spreading out the orders between trucks or filling trucks one by one. The intervals also contain data about whether the time at disposal should be considered or not. The decision to not consider the time should generate better solutions but might violate the time limit on individual trucks. The reason for this is to give the admins some room to explore different ways of generating routes. The configuration of intervals with trucks and time should give administrators room to manage work times and adapt to orders while still automatically getting optimized routes for each truck. Based on information from waste companies, each truck can pick up only one type of garbage. This means each order and truck needs to specify the type of waste.

Each time interval can have instances of the time intervals. An instance has the same data as the time interval parent but refers to a specific time window. These instances are the windows orders can theoretically be placed in. Each instance can, because of this, have zero to many orders. When an order is placed to a time interval instance, Route data is automatically updated. The route optimizer API does not consider types, so the system separates the order and truck on each type and sends these to the route optimizer to update each time interval instance. The route data on each interval instance contains a set of routes, each listing the entire route as WGS84 Coordinates. The route data also lists the designated truck for the route, metadata about weight, time, and space used, and an ordered list of all the orders to be picked up in the route.

3.4 Testing, integration and deployment

The team decided to implement unit tests and use both continuous integration and continuous deployment. The unit tests were made in Rust and tested that all route optimization heuristics worked as intended. Passing the tests would mean that only legal actions were performed on the solutions; in this way, the team could trust the result. There is also an integration test that checks if the route API is stable. The test sends a standard http request with a data set of trucks and orders to the main endpoint for route generation on the server, and checks if the response is valid. The test also calculates the total distance of the solution it receives. These tests were also connected to Github Actions, which can perform certain actions if a success criterion is met. The continuous integration part of the workflow is then that all the tests are run on Github on the latest commit to see if the latest version is stable or not. In addition, the team also used a Github action that listened for commits on the Dev branch. If there was a commit and it passed all the tests, the results were deployed on a development server that Favn has access to. This was agreed upon with Favn to have an overview of our progress and give feedback on the completed features. Only completed features were pulled into the dev branch to ensure that Favn could inspect a stable version. This continuous deployment would be healthy for the cooperation between the team and Favn.

3.5 Trucks and orders implementation

Orders have been mentioned many times so far, but this shall be clearly defined before the implementation of the algorithms are discussed. An order is associated with a geographical coordinate, a weight, and some volume. The order represents something someone wants to be picked up by a truck; that is, a truck needs to travel to where the order is placed and pick it up. The coordinate is a basic pair of latitude and longitude, which is where it should be picked up. The weight and volume describe the thing to be picked up itself, and this is taken into consideration because they are the limiting factor for the trucks to be used.

What exactly is meant by truck shall be fleshed out now. All the algorithms take in a parameter which is the set of trucks that can be used. These trucks can be different. Each truck has the following attributes clearly defined; a max weight, width, and length of the truck's cargo space. In addition to this, the trucks are assumed to have the same starting point; this is where all the orders should be returned

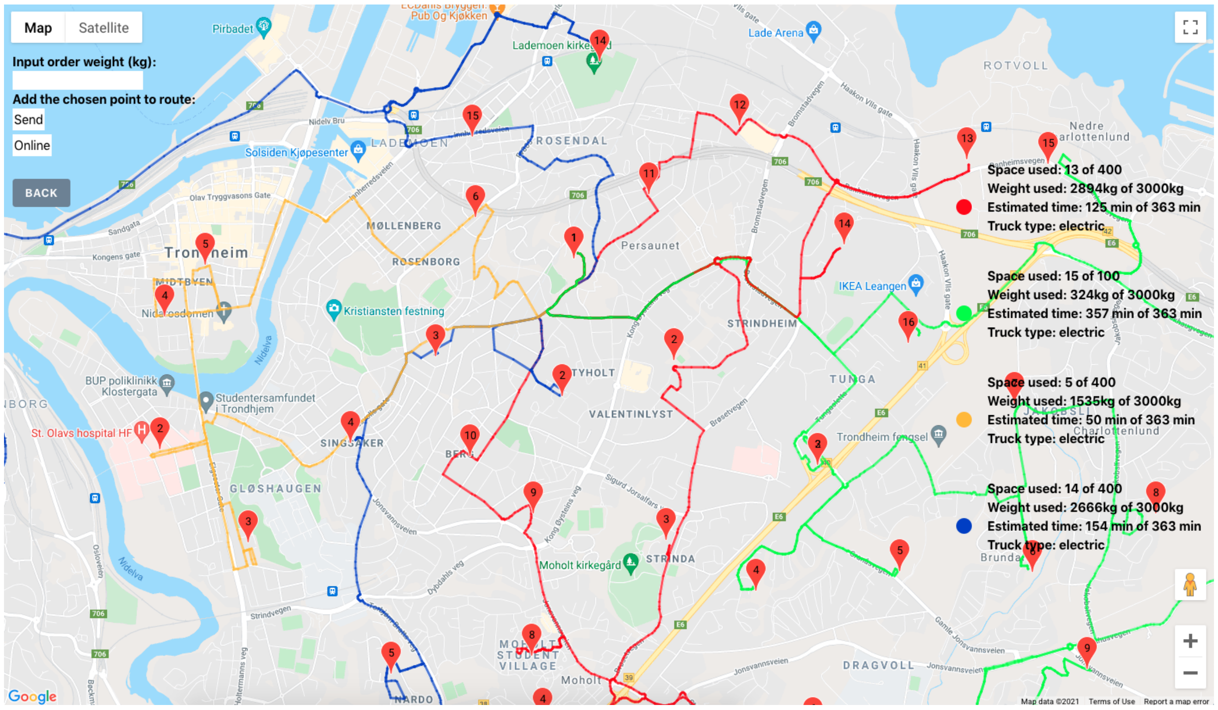
3.6 Graph compression and representation

The input to all the optimization algorithms is, as stated earlier, the set of orders and trucks. To run any variant hill climbing, the state must be clearly defined. A state is a current solution; that is, all the orders are distributed in some way among the trucks without breaking the weight or space constraints. Before the actual optimization algorithm is run, all the orders are sent to the OSRM service to retrieve the shortest path matrix. This matrix stores the shortest path among all pairs of orders. This matrix represents a complete graph (all nodes are related to all other nodes). From now on, the program will work with this graph. The graph has thus been simplified from the entire road network of the city into an n by n matrix where n is the number of orders plus one for the starting point of the trucks. This will make all future path computations very fast as one can find the shortest path between two orders in O(1) time. This compression is lossless in the sense that one always wants to drive between these points in the final route, so no critical data is lost.

When working with routes, one has to decide how to represent a path in the graph. All the paths that the trucks will drive start and end in the same place; in graph theory terminology, this type of path is known as a circuit. These circuits only visit each node/order once, so the team decided to store the paths simply as a list associated with a truck. This list contains all the orders to be picked up by the associated truck. This was chosen because it is then easy to see which orders a truck should pick up and because a list is ordered. Due to the inherent order of a list, information can be extrapolated about the path. Since one always drives to and from an order only once, one can assume that there is an edge between consecutive orders in the list. With this assumption, a truck always starts at the first order in its list and then visits the second, then the third, and so on. Note that there is an extra edge from the last element of the list back to the first one to complete the circuit.

3.7 Visualization tool

As part of the system, it is necessary to have tools to visualize and communicate the route that is generated. Without such tools, the route optimizer would not be usable for practical purposes. The generated route is a list of latitudes and longitudes and would therefore be incomprehensible without a visualization. Such a tool is also useful when testing out different algorithms. When visualizing a route, it is often possible to see whether the route makes logical sense or not. If the route does not make sense, it is likely something wrong with the implementation of the algorithm.



🟥 Space used: 13 of 400
Weight used: 2894kg of 3000kg
Estimated time: 125 min of 363 min
Truck type: electric

🟩 Space used: 15 of 100
Weight used: 324kg of 3000kg
Estimated time: 357 min of 363 min
Truck type: electric

🟧 Space used: 5 of 400
Weight used: 1535kg of 3000kg
Estimated time: 50 min of 363 min
Truck type: electric

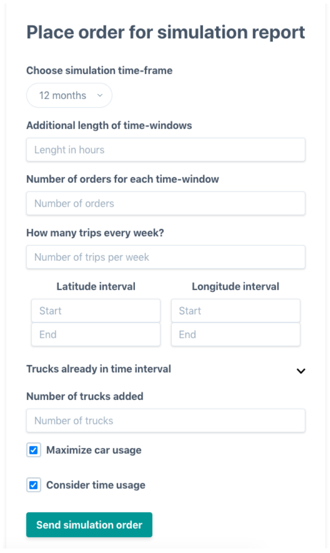
🟦 Space used: 14 of 400
Weight used: 2666kg of 3000kg
Estimated time: 154 min of 363 min
Truck type: electric

Figure 19: Example of visualization of a set of routes

The visualization tool is part of the front-end part of the system and is built on top of Google Maps. It provides a series of different functionalities for the user. It displays all the different routes for the chosen time interval by color-coded lines. Each route is coded by color, and the relevant information for that route is displayed on the right. This includes information such as space used by each truck, weight used, how much time is used and what type of waste that truck collects. Each order in each route is also enumerated based on the order the truck will drive. Information about a given order is accessible by clicking on it, displaying a popup with information about the given order, such as an address, postcode, weight, volume, latitude, and longitude. Part of the tool is also the functionality for adding new orders to the route. It is possible to place a marker anywhere on the map, then input the weight in the top left corner and then send it to the route. This will trigger a cloud function that will generate a new set of routes that includes the new order.

3.8 Simulation tool

Another functionality implemented in the system is the ability to run simulations on the routing algorithm. This is a useful function for companies to simulate what impact a potential change might have. For example, simulating the impact of acquiring a new truck will have on fuel and time use over a year. Such a tool could guide a company on what changes might be beneficial economically and environmentally.



Place order for simulation report

Choose simulation time-frame: 12 months

Additional length of time-windows: Lenght in hours

Number of orders for each time-window: Number of orders

How many trips every week?: Number of trips per week

Latitude interval	Longitude interval
Start	Start
End	End

Trucks already in time interval

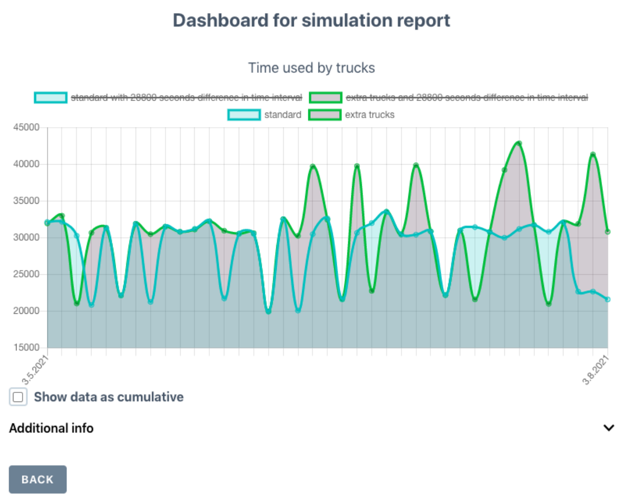
Number of trucks added: Number of trucks

[✓] Maximize car usage

[✓] Consider time usage

Send simulation order

(a) A form where the user can specify the attributes of the report



(b) Dashboard with a graph displaying the total time used for the different simulations per day

Figure 20: The simulation report tool

In the UI, one is presented with a form where it is possible to specify all of the following parameters:

- Length of simulated time-span
- Time limit of a route
- Daily order frequency
- Number of trips per week
- Area the generated orders are in
- Number of extra trucks
- whether to prioritize distributing the orders evenly when clustering

When the attributes have been chosen and sent, the routing server will run the algorithm for all the days in the given time frame. All the different attributes specified by the user are taken into consideration. After the simulation is done, the user is presented with a dashboard displaying the results. Among other things, it is then possible to see what effect the different choices have on the time used. In figure 20, the standard route is marked by the blue line, and the route with the additional truck is marked with the green line. In this case, the standard route uses less time than the route with more trucks, with some days there being quite a big difference. Therefore for this scenario, the additional truck might not be that beneficial.

3.9 Hill climbing implementation

The hill climbing algorithm implementation needs a starting solution to begin the local search. It uses a nearest neighbor clustering method to very quickly create a route satisfying the weight and space limitations. Note that the time limit constraint can be broken in this starting solution. The reason for using a clustering algorithm and not just a completely random solution is that a better starting solution will require fewer iterations to converge, making it faster.

The algorithm takes a parameter that dictates the max amount of iterations that will be run. An iteration consists of generating all conceivable improvement heuristics, performing them if they reduce the total travel time while respecting the weight and volume constraints. Generating all conceivable improvement heuristics means trying all pairs of indices in the same route of a truck and using those as the starting points for all the intra-route heuristics, or all pairs of indices of different routes as the starting points for the inter-heuristics. The algorithm also keeps track of whether an improvement has been made in this iteration. This way, it can terminate when it has completed the maximum number of iterations or run a whole iteration without improving. This can safely be done as one would be guaranteed not to find an improvement in any of the subsequent iterations anyway. All the heuristics are generated in the same order each time, and the clustering phase depends on the order that the trucks are received. Therefore the algorithm is completely deterministic.

Note that there exists a problematic edge case without allowing the trucks to drive multiple trips within the time limit. Imagine that there are many orders right next to the depot, but they are so heavy that a truck can only drive one order at a time. In this case, the trucks can drive back and forth within the time limit, but multiple trips are usually not allowed in default implementations. This will make the algorithm unable to find a valid solution. The team came up with a trick to solve this without increasing the complexity. By copying the trucks, each of the copies can have its own circuit. To validate whether the set of trucks constitute a valid solution, one has to summarize the distances traveled by the different copies of the same car to make sure it can make all the trips within the time limit; everything else will work as normal. This trick is implemented in the hill climbing algorithm and in the simulated annealing one, but not for AILS as it has its own mechanism for adjusting the number of trucks to use.

3.10 Simulated Annealing implementation

Equivalent to the hill climbing implementation, the starting solution of SA is based on the deterministic nearest neighbor clustering algorithm. The simulated annealing implementation takes in two parameters, starting temperature and cooling factor. The starting temperature will dictate just how exploratory the algorithm is, especially in the beginning. The cooling factor reduces the temperature over time; the algorithm terminates when the temperature has reached 1 degree. Each time the temperature is decreased, a random heuristic move is generated, which is a suggestion for a change in the current solution. The change to be analyzed can be any conceivable move as described in the theory section of TSP and CVRP. The total distance of the current and candidate solution is calculated and feed to equation 9 to calculate the chance of making a move. The equation states that all improvements should be performed. If the temperature is high, or the move is only somewhat bad, then there is a chance to perform the move to explore more of the search space. Additional restraints still hold; changes in the solution must respect the weight and space limits of the truck.

3.11 Adaptive Iterated Local Search implementation

The Adaptive Iterative Local Search implementation is based on the AILS algorithm explained in [5]. The implementation of the team is written in rust on the route-optimizer service in the system of the team. The paper describes an AILS algorithm for the CVRP explained in 2.4.4. The implementation is different in that the problem of the system is not directly the CVRP described in 2.2.2, but a problem where the vehicles can have different weight and space constraints. The vehicles also have a potential time constraint that is the same for all cars, but each order can be picked up by any truck. For these reasons, the team tweaked the AILS implementation to fit their application.

3.11.1 Summary of the implementation

The algorithm creates an initial solution using proximity clustering described in 2.3.2.
After getting an initial solution, which might not be feasible based on the time, space, and weight constraints, the algorithm uses a feasibility heuristic to find the closest feasible solution. After this pre-process, the iterations start. In each iteration, the algorithm perturbs the current reference solution. This means that it swaps up the orders in the solution and creates a different and potentially infeasible solution. With this perturbed solution, it runs it through the feasibility heuristic again to find a feasible solution. After getting a feasible solution, it performs a neighborhood search to find a local optimum, a temporary best solution. The algorithm performs acceptance criteria to see if this new solution should be set as the reference solution for the next iteration. The algorithm also updates some parameters for the perturbation step to make this step adapt to the results of previous iterations. If the solution is better than any other solution found in a previous iteration, it is saved as the current best solution. The iteration repeats until the max amount of iterations is run and returns the best solution found. The overall algorithm is described in Algorithm 5

3.11.2 Initial clustering

The initial clustering in the implementation calculates the minimum amount of needed trucks based on weight, initializes empty routes using the calculated amount of trucks needed, and randomly assigns each route in one order. The rest of the orders are then inserted based on proximity. The implementation is just like the one described in 2.3.2. It does not consider the additional constraints in the problem at hand, as the initial solution does not need to be feasible. The initial solution needs only be quickly generated and somewhat reasonable.

3.11.3 Perturbation

A perturbation step is needed in an iterative local search. The algorithm should explore different parts of the global scope to find in an attempt to find the global optima and not just find the first local optima as a solution. A perturbation step perturbs the current reference solution in an iteration of the algorithm by removing orders from routes and inserting them differently. This potentially creates a solution far enough away from the reference solution that the algorithm can potentially find another local optimum than the previous iteration. The AILS implementation uses the removal and insertion heuristics described in 2.4.3 to perturb a solution. This step in the algorithm also does not require a feasible solution but actually potentially transforms a feasible solution into an infeasible one. Because of this, the implementation of this step is the same as with the CVRP. The implementation is described like this:

Algorithm 1 Perurbation Procedure

```
Data: Solution s and removal heuristic $H^r$
Result: Solution $s'$
$s' \leftarrow s$
  Remove random route with probability $1/\gamma$ as long as $m \geq \underline{m}$
  Remove $\omega_{H^r}$ orders from solution $s'$ based on heuristic $H^r$
  Randomly select an insertion heuristic $\{H_1^a, H_2^a\}$ and add all removed orders with the heuristic
```

In the perturbation algorithm, $\{H_1^a, H_2^a\}$ are the insertion heuristics described in 2.4.3.4 and 2.4.3.5. H^r is an assigned removal heuristic described in 2.4.3.2, 2.4.3.1 or 2.4.3.3. ω_{H^r} is an integer related to the removal heuristic that decides how many orders will be removed from the solution. γ is a constant calibrated by the team based on testing results. $\underline{m}$ is the estimated least amount of routes needed to get a feasible solution.

3.11.4 Neighborhood search

The neighborhood search explores the neighborhood and tries to find the local optima closest to the solution it optimizes on. The neighborhood search also includes a feasibility heuristic that looks for moves with high feasibility gain and requires having a feasible solution before it finishes optimizing. The implementation of the neighborhood search is different from the one described for the CVRP, as the feasibility heuristic has to account for several constraints. The feasibility search algorithm is described like this:

Algorithm 2 feasibility heuristic

```
$LM \leftarrow \emptyset$
repeat
    foreach $R_i^S \in Routes$ do
        Update $LM(R_i^S, LM, s)$
    end
    while $|LM| > 0$ do
        Find the inter-route move $mov$ between route $R_i^S$ and $R_j^S$ resulting in $s'$ in LM with highest feasibility gain
        $s \leftarrow s'$
        remove all moves from LM involving the routes $R_i^S$ $R_i^S$
        Do intra-route local search to routes $R_i^S$ and $R_j^S$
        Update $LM(R_i^S, LM, s)$ and Update $LM(R_j^S, LM, s)$
    end
    if $s$ is infeasible then
        Add a new route $Routes \leftarrow Routes \cup R_{m+1}^S$ with the truck $T_i$ that has used the least amount of time.
    end
until $s$ is feasible OR $it > it_{max}$
```

The algorithm for updating the list of moves LM is described like this:

Algorithm 3 Update LM

```
Data: $R_i^s, LM, s$
Result: Updated LM
foreach $v_k^i \in R_i^S$ do
    foreach $v_l^j \in \delta(v_k^i) | i \neq j$ and $R_i^S$ and $R_j^S$ meet feasibility condition described in equation 10 do
        foreach inter-route move heuristic $N_k \in$movement heuristics do
            if $N_k$ gives a solution $s''$ that respects condition 12 then
                if $\exists$ (move between $v_k^i$ and $v_l^j$) $\in LM$ but s" has higher feasibility gain then
                    replace move between $v_k^i$ and $v_l^j$ in LM with new move
                end
                if $\nexists$(move between $v_k^i$ and $v_l^j$) $\in LM$ then
                    $LM \leftarrow LM \cup$ (move between $v_k^i$ and $v_l^j$ using move heuristic $N_k$)
                end
            end
        end
    end
end
```

$\delta(v_k^i)$ is the set of orders closest (by proximity) to R_i^S

the movement heuristics are the ones described in 2.4.2.6, and 2.4.2.5, 2.4.2.9

$$slack(R_i^S) \geq 0 \ XOR \ slack(R_j^S) \geq 0 \tag{10}$$

In condition 10 slack describes a number indicating the capacity of a given route.
In the implementation, this slack function returns negative if any constraints are violated: used time, weight, or space. Any violations of these constraints are summed up using the real value of the constraint violation. e.g: if a route uses a truck T_i with 50 max weight, and the orders in the route add up to 60 weight, and no other constraints are violated, the slack function gives a value of -10. If time is also violated, meaning the total time of all the routes R_x^S the truck $T_i^S = T_x^S$ in R_i^S is driving, this violation is also added to the slack value. So if truck T_i^S has an estimated time use of 3000, and the limit is 2500, the slack adds up to -510. These values are also weighted in an attempt to balance these numbers out, valuing time, space, and weight equally.

The feasibility condition for a given move candidate can be described using the teams implementation of feasibility gain:

$$\Omega(s,s') = min\{0, slack(R_i^{s}{'})\} + min\{0, slack(R_j^{s}{'})\} - min\{0, slack(R_i^S)\} - min\{0, slack(R_j^S)\} \tag{11}$$

$$\Omega(s,s'') > 0 \tag{12}$$

Equation 11 calculates a number indicating the feasibility gain by getting the difference in slack between the solutions s and s' when committing to a move between routes R_i and R_j

3.11.4.1 Local search heuristic

The neighborhood search also has a similar process to the local search process that optimizes a feasible solution. The process is very similar to the feasibility heuristic, except some of the conditions are different.
In the update LM algorithm for the local search heuristic, every other route is considered when considering move candidates as no route should have a $slack()$ value under 0. Candidate moves must also result in a shorter total distance, as well as having positive feasibility gain (condition 12). This is to explore the neighborhood for better solutions without finding infeasible solutions.

3.11.5 Acceptance criterion and adaptivity

For the algorithm to be adaptive, there is an acceptance criterion that manages the reference solution used in each iteration of the algorithm. There is also a system for updating the ω values for the removal heuristics described in 2.4.3. The implementation of these systems is similar to those explained in [5] as they as well do not account for feasibility in any way, making it the same for the thesis problem. Because of this, the implementation of the acceptance criterion is described in 2.4.4.
The update of the removal heuristic ω values is implemented like this:

Algorithm 4 Update $\omega_{H_k^r}$

```
Data: Solutions $s$ and $s^r$, $it_{H_k^r}$, $d_{H_k^r}$, $d_\beta$ and $n$
Result: Updated $\omega_{H_k^r}$
$it_{H_k^r} \leftarrow it_{H_k^r} + 1$
$d_{H_k^r} \leftarrow \frac{d_{H_k^r}(it_{H_k^r}-1)+d(s,s^r)}{it_{H_k^r}}$
if $it_{H_k^r} = \gamma$ then
    $\omega_{H_k^r} \leftarrow \frac{\omega_{H_k^r} d_\beta}{d_{H_k^r}}$
    $\omega_{H_k^r} \leftarrow min\{n, max\{1, \omega_{H_k^r}\}\}$
    $it_{H_k^r}, d_{H_k^r} \leftarrow 0$
end
```

$it_{H_k^r}$ describes the number of times the removal heuristic has been used since the last time $\omega_{H_l^r}$ was updated. $d_{H_k^r}$ is the average distance between local optima found by the removal heuristic and the reference solution s^r since the last update of ω. d_β and γ are, in this case, constant integers with a value calibrated by the team based on testing. n is the number of orders in the solution.

Algorithm 4 updates the omega value for one specific removal heuristic H_l^r. The goal of this algorithm is to compare the current solution with the reference solution from before the iteration and give ω a new value based on this.

3.11.6 The algorithm

The team combines all the parts described in 3.11.2, 3.11.3, 3.11.4 and 3.11.5 to create an AILS algorithm tailored for the teams task and system. The overall algorithm is quite similar to the one described in [5], but tweaked to fit the custom CVRP the team is tasked with.

Algorithm 5 AILS

```
Data: orders, truck data, time limit per truck
Result: Best solution $s^*$
$s^r \leftarrow$ Initial solution from initial clustering described in 3.11.2
$s^r, s* \leftarrow$ feasibility heuristic($s^r$) - as described in Algorithm 2
$it \leftarrow 1$ repeat
    $H^r \leftarrow$ Choose a random removal heuristic $H^r \in \{H_1^r.H_2^r, H_3^r\}$ - described in 2.4.3.1, 2.4.3.2 and 2.4.3.3
    $s \leftarrow$ Perturbation procedure($s^r, H^r$) - as described in Algorithm 1
    $s \leftarrow$ feasibility heuristic($s$) - as described in Algorithm 2
    $s \leftarrow$ Local search heuristic($s$) - as described in Algorithm 2 using tweaks explained in 3.11.4.1.
    Update $\omega_{H_k^r}$ - as described in Algorithm 4
    $s^r \leftarrow$ Apply from $s$ based on acceptation criterion as described in 2.4.4
    if $s$ is a better solution than $s^*$ then
        $s^* \leftarrow s$
    end
    $it \leftarrow it + 1$
until $it \geq it_{max}$;
```

3.12 Run time experiments

To experimentally test the solution quality and run time of the different algorithms, the team has run a series of tests. All the tests were run on a Google Cloud virtual machine. The VM is running on a 2.00 GHz Intel Xeon CPU. The results shown are an average from running 50 tests in three different cities

(Oslo, Trondheim, Tromsø). Each test is to optimize a route that picks up all the orders. The set of trucks are the same throughout all the tests, although the generated trucks themselves do not need to be identical. All trucks need to return to the starting point within a 6-hour time limit for the routes generated. The 50 requests sent are unique, but the same set is sent to the same city during the same iteration for all the algorithms. This is done so that the results are comparable as each algorithm tries to find a route based on the same orders and trucks. The orders are generated on the spot, the number of sections and weights used in the order are random within a limit. The weights for each order are between 1 and 100, but have a ten percent chance of being ten times larger. The reason for this randomness is based on information from waste companies, saying that about 1 in 10 waste pickups are much larger containers of waste. The sections used for each order is a random amount between 1 and 5. The most important thing is that the weight and space usage was generated so that both can break the constraints of a car, meaning that the improvements must carefully balance both constraints.

The requests are sequentially sent to the route optimization server; note that the run time is calculated on the optimization server to avoid tracking the network lag. The timer used is from the 'time' module of the standard Rust library. What is referred to as distance in the result section is the total driving time of all the trucks; this is the "distance" metric that the algorithm minimizes. The purpose of the experiment is to experimentally show how the different proposed algorithms scale with regard to the number of orders, the number of iterations, and how that affects the quality of the solution. For this reason, there is more than one test for each algorithm; HC and AILS are tested with 1, 5, and 20 iterations to see how run time and solution quality scales with the number of iterations. The SA algorithm is always run with a starting temperature of 100 degrees as it was tested to give good results, and the cooling factors are 0.9999, 0.99998, and 0.999996. The AILS takes two parameters more than HC, namely k, which is set to 0.5, and γ, which is set to 10. All of the parameters were selected as they seemingly yielded the best results.

3.13 Distribution of roles

The role distribution for the development of the product was not very established. This means that every team member worked on most aspects of the product, but some worked more on specific areas than others. However, the role distribution for the process was very specific. The roles were distributed like this:

- Håvard Stavnås Markhus: For system development focused on system architecture, automation, system structure, testing, and backend development. Responsible for setting up meetings and being chairman in these meetings.
- Tobias Meyer Andersen: For system development focused on backend development, algorithm research, and algorithm development. Responsible for writing meeting reports and quality assurance for documents.
- Kjerand Evje: For system development focused on front-end development, Responsible for structuring documentation and making sure documents are available to whomever the document concerns.

4 Results

4.1 Scientific results

4.1.1 Hill climbing results

HC	1 iteration		5 iterations		20 iterations	
orders/trucks	run time(ms)	distance(s)	run time(ms)	distance(s)	run time(ms)	distance(s)
10/2	35.0	7604.0	40.0	7480.3	40.3	7480.3
25/3	56.0	17316.3	90.3	16946.3	91.3	16946.3
50/4	103.7	32977.0	213.3	32511.0	223.7	32503.7
99/6	218.3	62010.0	487.3	61476.3	500.0	61470.3

4.1.2 Simulated Annealing results

The starting temperature for all SA runs was 100 degrees. CF is an abreviation for cooling factor.

SA	cf=0.9999		cf=0.99998		cf-0.999996	
orders/trucks	run time(ms)	distance(s)	run time(ms)	distance(s)	run time(ms)	distance(s)
10/2	56.7	7501.0	223.3	7441.0	1056.0	7427.0
25/3	64.0	17200.0	235.3	16901.7	1092.7	16754.7
50/4	76.3	32971.7	251.7	32457.3	1260.0	31973.0
99/6	107.0	62320.3	289.3	61646.0	1199.3	60805.7

4.1.3 Adaptive Iterated Local Search results

AILS parameters: k = 0.5, γ=10

AILS	1 iteration		5 iterations		20 iterations	
orders/trucks	run time(ms)	distance(s)	run time(ms)	distance(s)	run time(ms)	distance(s)
10/2	18.3	7422.3	27.7	7370.7	56.0	7349.3
25/3	64.0	16700.0	115.3	16524.0	283.3	16384.7
50/4	248.0	32048.0	481.0	31646.0	1381.0	31373.3
99/6	1375.7	60899.0	2648.0	60115.7	7129.0	59353.3

4.2 Engineering professional results

4.2.1 Final product overview

The overall goal of the project was to develop a route optimization engine with an administrative web application for waste collection companies. The administration tool would have access to route generation for waste collection orders coming from a mobile app for waste collection customers. Orders would come in from paying users, and waste administrators would manage these orders and have routes generated for picking these up optimally. The task initially had a vast scope, and the plan was for the team to choose which parts to focus on. The team focused on developing the routing engine and prioritizing this but developing some key features to the administration application. The final product has a competitively fast routing engine with the route optimization API built with Rust and Actix web on top of the OSRM API. The engine is also open for utilizing more sophisticated algorithms for close to optimal solutions with the drawback of longer run times and worse scalability in regards to the number of orders. The routing performance of the routing engine is extensively experimented with and tested, giving the team a good indication of how the engine performs in different situations using different algorithms. The team also built some key features in the web application and firebase that utilize the routing engine, including truck management, a simulation tool for the route generator, and visualization of the routes.

The system was built in Google Cloud Platform, which merges well with the mobile application built on top of firebase and the rest of the project the team did not focus on. The system is also horizontally scalable as the routing engine, and OSRM API is stateless and dockerized. The OSRM API is configurable for different countries by initializing the API using different map files. This makes the system configurable for other countries as well as the intended country, Norway.

4.2.2 Functional requirements

In this chapter, how and in what way different functional requirements defined in the vision document were met will be described. The order is the same as in the vision document. Recall that the vision document was intentionally somewhat over-ambitious so the team could have some freedom in deciding what part of the system to develop based on personal interest.

4.2.2.1 Authentication system

As the focus of the project was mostly centered around algorithmic optimization, it was both unnecessary and unnatural to spend time developing an authentication system that would restrict the usage of the optimizer to only those with access. This was agreed upon by all parties.

4.2.2.2 Interactive front-end showing the orders

The team implemented functionality to view data of the orders when accessing a time interval instance, as this was the most important feature while developing the optimization engine. The orders are stored and used in the system but do not have relevant, interactive components, as this was deprioritized after discussing with Favn and the team. The feature was originally planned as a feature the team would implement in the roadmap. However, later in the development of the system, the team and Favn decided that the team would focus on key parts of the front-end system that helps reinforce testing the optimization engine. This resulted in the deprioritization of the feature.

4.2.2.3 Communication from waste company to users app

This was also looked away from as the user part of the system was not central to the project; this was agreed upon by all parties. The feature is not at all related to the route-optimization, and therefore not prioritized in the same way as other features.

4.2.2.4 Generic route optimization

The goal stated in the vision document was to create a generic route optimizer that takes into consideration space, weight, and time. This criterion was met by creating a route optimizer that takes into consideration the weight and space limits of the trucks. It is general in that it can be applied for varying amounts of trucks. It is not limited to some specific geographical location; it also works for a varying number of orders and any starting point for the trucks. The algorithm that was finally chosen for the optimizer was HC.

4.2.2.5 Estimating time and fuel use, number of trips required

The route optimizer creates metadata for estimated time for each route and estimated total time used for all routes. Estimated fuel use is not implemented, as the team and Favn agreed to focus on the time metric provided by OSRM and optimize based on this. The route optimizer API supports giving the same truck several trips if necessary. All implemented optimization algorithms take the possibility of one truck driving several trips into consideration.

4.2.2.6 Displaying information about space and weight used in trucks

The web application implements this feature by showing the estimated time, space, and weight used for each route in the same component that shows the generated routes on a map. The page indicates which route on the map refers to which route information using color-coding.

4.2.2.7 Possibility to change how many trucks a company has available

This criterion was also met. Users now have front-end access to both add and remove trucks. These trucks are given to the route optimizer when creating the route, as also specified in the vision document.

4.2.2.8 Simulation tool to optimize use

The original criterion was for the simulation tool to determine what sort of parameters or adjustments would give the best route generation over time. However, the team did, during meetings throughout the project, agree that it would be satisfactory to have a simulation tool that would allow the user to test different parameters and simulate how they would affect the routes over time. This was solved with the simulation report function, where users can analyze route trends over long periods of time. The user can test things such as the impact of adding more trucks, area of service, prioritizing filling one truck up before using another, how much load would be expected in the time period, and how long of a period to simulate. The user can also control the time limit, which is analogous to the length of a working shift but can also choose to look away from a time limit altogether. All of this combined lets the user explore different ways to run their pick-up service in order to understand what the data suggests would be beneficial in terms of route length and truck usage. They would also receive feedback if some routes were not possible to be generated due to no route being found that satisfies the time/weight/space constraints.

4.2.3 Non-functinoal requirements

Favn Software AS had two non-functional requirements of the final product, both described in the vision document. The first one was to use Firebase as a back-end solution. This requirement was made as the company primarily uses this and has experience with it, and thus they could, as the further work, merge it with the front end part of the user application. This requirement was met, as made clear earlier. The other non-functional requirement was to use either Vue or React as a front-end framework. This constraint was created for a similar reason, as they would more easily allow Favn to develop the product further later. As also made clear earlier, the team chose to use React to develop the front-end.

4.3 Administrative results

During the completion of this project, different development tools were utilized to ensure that the process was kept systematic. This involved using a sprint-focused development method based on the roadmap made at the start of the project. The project was also documented by status reports, time-sheets, and status meeting logs throughout the project.

4.3.1 Work schedule plan

The work schedule on the project was based on 2-week long sprints. This schedule was kept by the team throughout the project. In the roadmap, it was set up to be nine sprints in total, all of which were completed. There was some variation in the amount of work done and time used on each sprint. As people on the team were busy with other subjects at university and other work, the team used less time on each sprint at the start of the project than at the end. The workload steadily increased from the start in January until the completion in May.

4.3.2 The development process

As mentioned, the team focused on a lean and iterative, sprint-focused development process. All the team members were familiar with and had experience in using Scrum beforehand, so the team quickly became efficient in using this development process. The sprints lasted two weeks each, with some exceptions at the end of the project. Each sprint was concluded with a sprint planning meeting where the team met with Favn to discuss the previous sprint and plan the next sprint. During these meetings, the team continually got feedback on the product, and as the product evolved, the demands for the product also evolved. The team planned the first seven sprints in the road-map for developing the product and the last two sprints for writing the paper and finishing up on documentation.

After the product was finished, the team and Favn came to an agreement it would not be necessary to continue the sprint planning meetings, as Favn was only involved in the development of the product. The team was also successful in using a task board for managing what tasks needed to be done, what tasks were ongoing, and what was finished. At the start of each sprint, the team reviewed the task-board and added the new tasks that would be necessary to achieve the goals for the coming sprint.

Covid-19 also had some impact on the development process. This resulted in that quite a lot of the work on the project was done remotely and over voice chat instead of meeting in person and working. All the sprint planning meetings where held over video chat as well as all the meetings with the supervisor. Favn opened up the possibility of using their offices for the project once a week, which the team often utilized. In the last few weeks of the project the team could meet more often and Favn had more free space in their offices, which led to working more in person when completing the project.

5 Discussion

5.1 Scientific results

What the thesis is trying to explore is "How do different versions of hill climbing compare when adjusted to solve the capacitated vehicle routing problem with time limits on realistic map data." To answer this question, three different forms of hill climbing were implemented. The simplest one is the standard form of hill climbing, another one taking the probabilistic approach of simulated annealing, and a third one building on a more sophisticated series of hill climbs. The algorithms were tested using 150 orders from different cities, using real coordinates and addresses from Open Street Map. It was then tested both how the algorithms scale with regard to the number of orders and compared to the number of iterations/how long the algorithms get to run. The grounds on which the algorithms shall be compared are explained as the metrics used are discussed in the next paragraph.

5.1.1 Experiment metrics

Ass seen in tables of 4.1.1, 4.1.2, and 4.1.3 the metrics measured were run time and distance.

Run time is, as stated earlier, calculated on the rust server; it represents the time it takes for the optimization algorithm itself to run. The time is measured in ms and is, as states before, an average from 50 requests for the same amount of orders in three different cities to increase the empirical foundation of the results. The time measured is a central metric because it uncovers how well an algorithm scales in practice. An algorithm can also have a different application based on how fast or slow it is.

Some trouble was encountered when measuring the time of the execution. Under the heavy tests, which lasted more than one or two minutes, the algorithm seemed to perform very poorly. On closer inspection, the Google Cloud interface showed that the CPU usage went down, even though the program was not finished. This could be due to OS deprioritizing a long-lasting process or due to hardware limitations that, for instance, slowed down the process to avoid generating too much heat. In any case, the team handled this by having the program that sent the requests to sleep between them; this seemingly fixed the program and decreased the run time drastically. This was, however, not done on all the tests, as the less computationally heavy ones did not seem to suffer in the first place; this issue could have produced some inconsistencies by increasing the run time.

Distance in time or the total amount of driving time to complete the route was the other recorded metric. This data is based on OSRM data of how long it would take to drive the sequence of orders of each car, adding 300 seconds per order to give time to pick up the items in the order. This metric decides the quality of a solution, as the total driving time should be minimized.

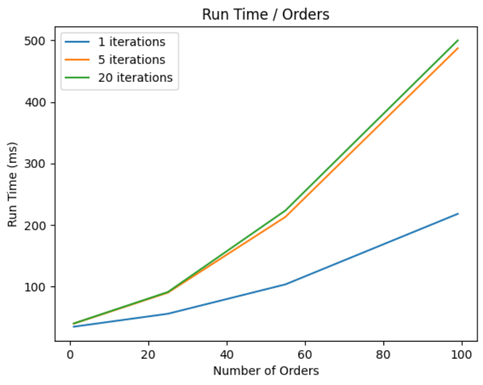
5.1.2 Other notes on the experiment

This paragraph shall highlight some questions that might be raised in relation the run time experiments conducted.

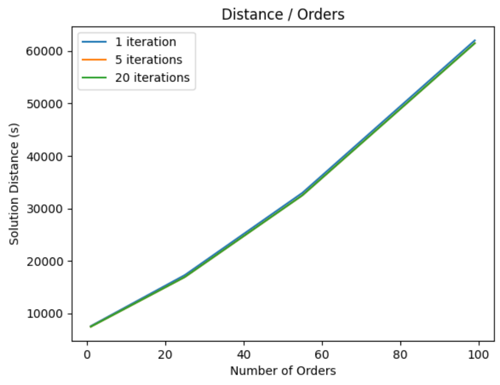
Firstly, lets address why there is no comparison to exact solutions and other benchmark solutions. These questions are both answered by recalling that the exact problem to be solved for Favn is not the standard form of CVRP. Most importantly, the problem at hand features two different capacity constraints (weight, and space) in addition to the time limit. This is not the case for the more common problem which only has one capacity constraint. Due to this the experiments are not truly comparable with other benchmark solutions that solve that problem. In addition this prevents us from using common data-sets used for benchmark purposes as they do not feature the necessary data for this application. This limits the thesis to only using its own implementations as a ground for comparison. For the simpler cases one could feasibly brute force the exact solution, but as that would at best only be done for the simplest cases. The team therefore chose to be consistent and only compare the different solutions found by their own implementations to each other. Had the problem at hand been the standard the team could also have used common benchmark data-sets with known exact solutions. This does limit the conclusion to only answering which of these techniques are better than the others that were implemented, one can not deduce how good these solutions are in absolute terms.

Secondly, lets address why the data-sets stop at 99 orders, and why the amount of orders per car stays about the same through all the test cases. The limitation of 99 orders is unfortunately limited by the configuration of the OSRM server that the team has running to answer requests. This was not fixable within the deadlines used to guide the project. Therefore the team had to accept that the experiment could not feature a test case using 250 orders that was originally planned. Having 250 orders would be significant to verify how the algorithms scale on bigger problem sets. This would particularly have been valuable to distinguish HC implementation of 5 and 20 iterations as will be commented in 5.1.3. Lastly lets address why the number of orders per truck is more or less the same throughout the tests. The purpose of this thesis comes from real life applications of collecting items and returning them to a single place within a workday. The time limit used in the experiment was 6 hours. Due to this the team placed some limitations on how many orders there were compared to the number of trucks. The team ran some tests of their own and noticed the routes could not always be generated within this limit when one got close to 20 orders per truck. Therefore the team stayed clear of this limit in the experiment to ensure that valid routes always were found. Other than that the team did not aim for some exact ratio besides always making sure there were more than one truck to make it a true vehicle problem.

5.1.3 Hill climbing



(a) Graph showing the run time of HC compared to the number of orders

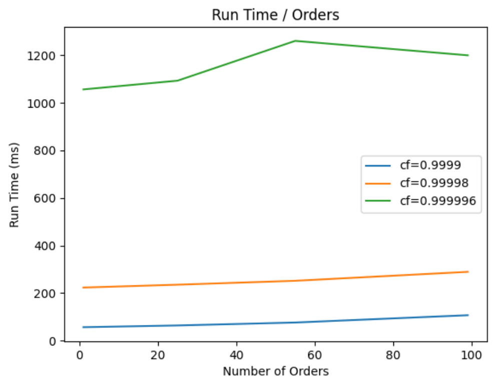


(b) Graph showing the distance of HC solutions compared to the number of orders

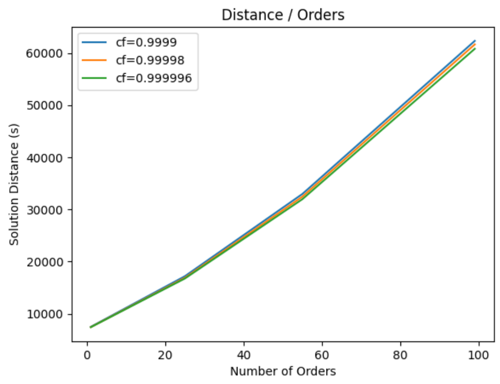
Figure 21: Results of HC graphed and compared to number of orders.

Lets first consider the run time of the HC algorithm using a different number of iterations. It can be seen that the run time is virtually the same when using 5 and 20 iterations, this is caused by the algorithm terminating as soon as a local optima is found. This means that even though the max limit was set to 20 iterations it rarely needed to run more than 5, so computationally they are almost identical for datasets of this size, with similar constraints on the truck. 1 Iteration clearly distinguishes itself as much more efficient, having significantly better running time on the heavier test cases. Interestingly its total distance in the largest test case is still less than a percent worse than the 20 iterations version although it uses more than half the time. So altough a 250 order test set might have separated the 5 and 20 iteration version a bit better it probably would not have a big relative impact on the distance results.

5.1.4 Simulated Annealing



(a) Graph showing the run time of SA compared to the number of orders



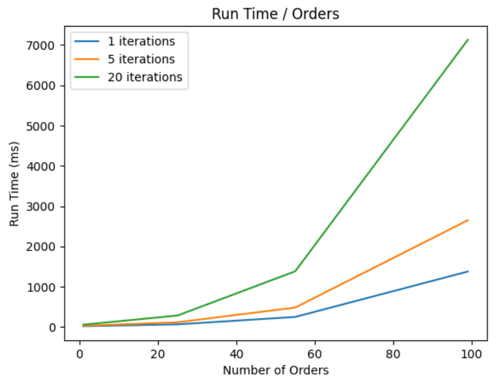
(b) Graph showing the distance of SA solutions compared to the number of orders

Figure 22: Results of SA graphed and compared to number of orders.

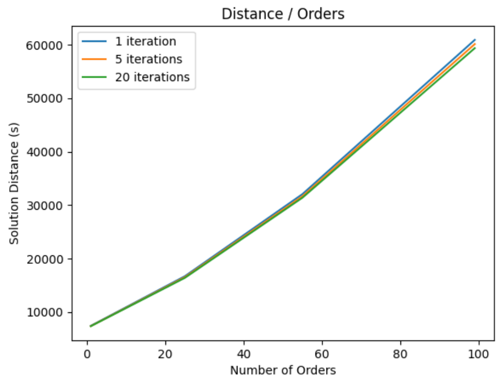
The SA algorithm looks for the same amount of possible moves to improve the solution regardless of the amount of orders. This is because the amount of moves to consider solely depends on starting temperature and cooling factor. This is supported by the graph above showing that the run time does not seem to depend significantly on the amount of orders. The different cooling factors still have very different run times, and provide somewhat different results in regard to the quality of the solutions. At most the worst solution is 3.1% worse than the best (50 orders, 4 trucks). SA is very scalable in terms of run time. Despite this it seems reasonable that the solution quality will eventually drop off in large enough cases. This is expected as some very large cases will require the use of so many heuristics to produce a good solution that it its unlikely for the SA to generate them all without having a slower cooling factor. As will be discussed later this ceiling for the SA is not yet reached in our test cases.

One thing that is worth mentioning is the anomaly in the version of SA with the highest cooling factor The run time is not the highest for the highest amount of orders, he most reasonable explanation for this is the run time issue on the VM described in 5.1.1.

5.1.5 Adaptive Iterated Local Search



(a) Graph showing the run time of AILS compared to the number of orders

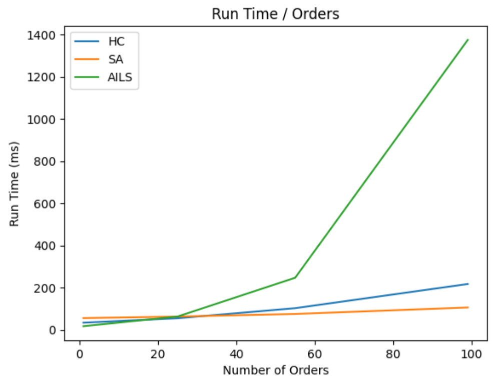


(b) Graph showing the distance of AILS solutions compared to the number of orders

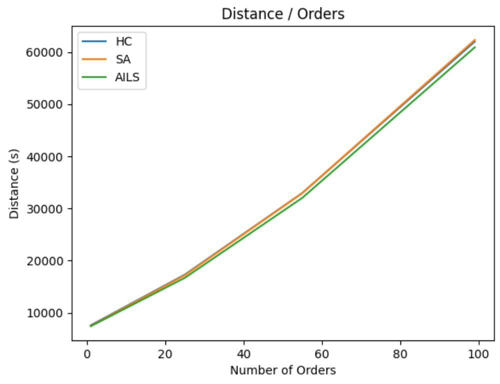
Figure 23: Results of AILS graphed and compared to number of orders.

As clearly seen in the figure the AILS scales unfavourably. From the second last text case (50 orders, 4 trucks), to the last (99 orders, 6 trucks), the run time is a bit more than quadrupled. This suggests a run time just over $\Theta(n^2)$ where n is the number of orders. For this algorithm the different iterations makes the biggest difference at 99 orders and 6 trucks where the worst performing solution using 1 iteration is 2.6% worse than the solution found using 20 iterations. This is an interesting result as the adaptive part of the algorithm probably does not really get into play with just 1 iteration. With 1 iterations the algorithm runs the clustering phase and optimizes it with a neighborhood search. After that it performs a single perturbation phase and tries to find a better solution after making one inter route change.

5.1.6 Comparison



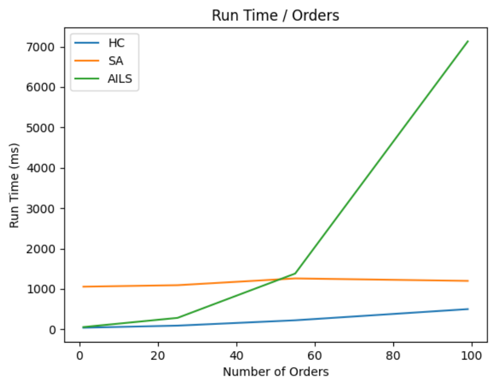
(a) Graph showing the run time of the quickest versions of the algorithms compared to the number of orders



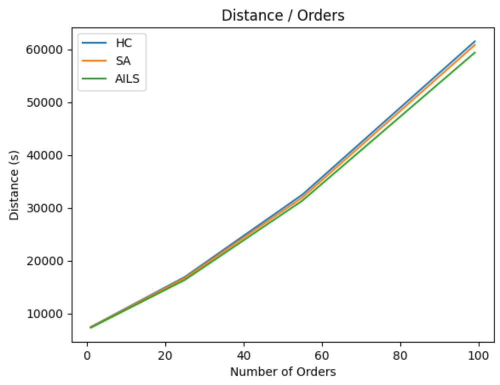
(b) Graph showing the distance of the quickest versions of the algorithms compared to the number of orders

Figure 24: Quickest version of each algorithm compared.

Lets interpret the results shown figure 24. The most clear observation to make is that AILS scales significantly worse than both SA and HC, and HC worse than SA. The latter is to be expected as run time is near constant, independent of the number of orders. The slow run times of AILS seems to be a high price to pay for small improvements. The HC solution is 3.7% worse than AILS at most (25 orders, 3 trucks). Initially this might seem insignificant but always driving 3.7% faster routes over long periods of time could certainly make a noticeable difference in a business. What is still worth noticing is not only that AILS produces the best solutions, but for the two smallest cases is actually the fastest as well. If the cases are small enough then AILS is clearly superior. For the grander cases time must be taken into consideration, although AILS produces the best solutions it might not be useful for all types of applications. An examples of this would be real time applications or those where many requests have to be processed before a final result is reached. An example of the latter would be the simulation tool that the team developed for Favn Software AS, the great amount of requests could mean that analysis is too slow for practical use.



(a) Graph showing the run time of the slowest versions of the algorithms compared to the number of orders



(b) Graph showing the distance of the slowest versions of the algorithms compared to the number of orders

Figure 25: Slowest versions of each algorithm compared.

Studying figure 25 the pattern from figure 24 still holds. That is, the same trends are clearly visible when comparing the slowest iteration of all the algorithms as well. AILS still produces the best solutions, is very fast in the start as well, but scales significantly worse than its alternatives. Additionally the alternatives always stay within a couple per cent in solution quality suggesting that the additional time usage of the large cases, that would be amplified even more with larger test cases would limit the possible range of applications.

In these graphs HC and SA seem very similar. As HC is not close to a constant run time it will eventually run slow on extremely big cases. At the same time the SA solutions will likely be weaker on extremely big cases as the algorithm needs to make many good moves, while it does not get more chances to do so because of the cooling factor. This was unfortunately not experimentally indicated in the experiment as the order limit was to low. Hence one cannot conclude that it would be the case either.

One thing that would be very interesting to know would be the hit-rate of the SA, that is how often was the randomly generated move an improvement and how often does it produce a worse state. The team had somewhat higher hopes for the SA, especially on the smaller test-cases, but it generally placed right in between AILS and HC. This could be because most moves generated are not improving the state, that seems reasonable as there are many more ways to ruin a good route than find the specific move that makes an improvement. If the team had more time this would have been investigated a bit more as writing a heuristic for what sort of improvement could make the hit-rate significantly better so that either the result improves, or satisfactory solutions can be reached with a lower cooling factor.

It seems that if many large routes had to be calculated to analyse long lasting trends as fast as possible SA or HC would be way to go as they scale very well. Keep in mind that solving many routing problems quickly is a non-issue for AILS as long as the individual routes themselves do not consist of a large amount of orders. It seems only certain specific applications would require us the sacrifice the somewhat better routes of AILS for practical reasons. In the context of Favn Software AS one could for instance consider calculating all actual routes with AILS, however having a simulation tool that chooses a routing algorithm that will run in a more practical amount of time, which depends on the parameters given by the user. So if it saw many large cases incoming it could switch to HC or SA as they would only be a little bit off, even in the long run.

5.2 Engineering professional results

5.2.1 Final result

As described in section 4, the team did not develop every feature in the planned project, but focused on the routing engine while building a shell or MVP for the administrative web application and the needed back-end functionality. In early development, the team used a lot of time researching ways to optimize on different Vehicle routing problems and how to get the applicable data needed to develop such an algorithm. The first sprints were used to plan the problem that needed solving and developing a first iteration of the route optimization API. The team used quite a lot of time on developing a first iteration that was not built on top of a routing machine, but instead manually tried to manage an entire road network data set in order to optimize routes. Because of this, the team wrote a lot of code that ended up not being used, as this was later replaced with a separate routing machine the team would build on top of. The team could have saved a lot of time by starting out with this approach. However, the act of actually willing to move over to a better approach is far better than continuing development on the potentially more time consuming approach the team was initially going for. By simplifying the problem and abstracting the map data away from the routing engine, it also follows the loosely coupled pattern and makes it easy to replace any of the services.

When the team initially had a first version of the routing engine, the focus shifted over to developing the platform for utilizing the engine. As there was not a lot of time for focusing on the web application, the team ended up focusing on core features like route visualization and the simulation tool. While developing these tools, the team developed and improved on the routing engine and adapting on it based on information from Favn. This way of developing worked well for the development of the routing engine, but at the cost of not being able to implement all planned features for the web application. After getting the simulation tool down as planned with the road map and iteration over the road map during development, the team decided to continue development on the routing engine. The team and Favn agreed that the progress so far on the administrative tool was far enough, and iterating over the routing engine would be preferable both to the thesis and the product.

The team planned adding alternatives to the algorithm, implementing algorithms that does not only hill climb, but also try to explore several local optima in order to find a better global optima. This made the final work on the product to implement Simulated annealing and AILS in the routing engine as well as a tool for researching the implementations and their performance. These implementations gave a lot of insight on how the normal hill climb compared to more sophisticated hill climbing heuristics with the teams specific routing problem, as well as improving on the routing engine itself, giving better solutions faster.

5.2.2 Strengths

As the team has focused a lot on the algorithmic foundation of the product this is one of the strengths as well. The routes are created quickly and produce similar results to more complex routing algorithms. The algorithm is also robust and general in the sense that the users can send in different forms of output and can expect reasonable results. This is best shown in the report generator that takes in many different parameters and simulates how that set of strategies and limitations would pan out over time.

Another good quality the systems has is that the different components are clearly separated, making it easier to maintain, change components and scale horizontally. This is an example of a loosely coupled architecture. Another key point here it that the route optimization server is stateless, making it particularly easy to horizontally scale this central component.

5.2.3 Weaknesses

As it has not been the primary focus of the project the front-end is not as fleshed out and functional as the team had wished for. Some of it was entirely out of the scope of the project, such as user friendly front-end to create orders, which the team has created the back-end for. An example of something the team would have added if the project had had more time would be more UI for managing the organization. More interfaces to manage time intervals and their trucks etc. This would have made the product more valuable faster as it would be easier to use and more functional, the current method of managing time intervals is directly in Firebase. There are also some known critical bugs, including the simulation tool sometimes not generating a result because of timeouts in the cloud function, the results of the simulations not showing trucks used correctly, and firestore documents becoming too large when the routes contain too many coordinates.

5.2.4 Testing

As mentioned earlier, the team implemented unit tests for the algorithm, combined it with Github Actions to practice continuous integration, and had a server be continuously updated to allow for Favn to inspect the latest stable version. The unit tests worked just as intended; the team was able to quickly identify if their commits had unintentionally harmed the route optimizer. This sometimes happened as there were many changes throughout the project regarding what parameters and metadata were given to the route optimizer. Continuous integration was turned out to be a helpful tool for the developers. Continuous deployment was also implemented for Favn to give feedback on the system more easily and stay updated on the latest stable versions. This was used in, for instance, sprint meetings as the team could in real-time evaluate and discuss the results of the previous sprint.

The team is also aware that there are many other types of tests that could have been implemented; lets go through the most relevant ones, what they do, and how they could have benefited the project.

Integration testing revolves around confirming that different system components manage to cooperate as expected with each other. This is relevant to this project as the system depends on multiple different servers communicating through various APIs. Automating the process of checking that these are functional would quickly give the team confidence that the system works as intended. It would probably have made the development process more robust and somewhat more effective if bugs regarding inter-component communication were automatically detected.

End-to-end tests can be a way to make sure that the system in its entirety is behaving as expected on a macro level. It is usually implemented as simulating a user event and checking to see if the activity propagated through the entire as expected; this is like a broader version of an integration test. These tests can be a bit complicated to maintain if the architecture of the system is varying, and the run time can be a bit slower than other types as well.

Something that would have been particularly relevant to this project, in particular, would be performance tests. Those types of tests measure the run time of different parts of the system, usually to ensure that some upper limit that is specified in the vision document is not broken. For the teams use case, it could have been used to track how different updates changed the run time of the algorithm to automatically detect changes that probably were not implemented in an efficient way to look over. This would be useful in order to have a history of how fast the calculations have been over time and figure out what impact certain commits had on the performance. This was not formally implemented as a test. However, as the development consisted of many requests being sent to check that the resulting solutions came out as expected, the team got a feel of how each change impacted the run time. Having formalized this process would definitely serve as useful documentation and aid the development process of the algorithms.

The reason these types of tests were not implemented in spite of the good reasons listed above came down to multiple reasons. In part, it was because it was not really necessary for a project of this size, though this argument does not hold when the project is expanded. Another reason is that some of them were in part manually tested informally, such as when the run time was observed by the developers frequently. A third reason was that there was a limited amount of time and the team prioritized pure development over always taking the most cautious path to the goal by having formal tests to make sure everything was working perfectly at every step of the way. All in all had the team had more time, these types of tests would have been a great addition to the code base as it could have made the project more maintainable and safer to develop.

5.2.5 Ethical, social and environmental results

In 2018, Norway created 49% more waste than average among the European OECD countries [16]. More waste means a bigger need for environmentally friendly handling of the extra waste. The waste flow project the team has helped with developing aims to streamline the waste collection process in bigger cities and creating a platform for easy disposal of waste. Based on conversations with waste collection companies, the traffic for dropping special waste in recycling facilities is very high on weekends. The collective traffic of people wanting to drop off their waste at the same time creates potential traffic jams. This could be streamlined with a system for ordering waste collection. The project aims to lower the threshold for disposing of waste and potentially prevent unnecessary use of gas, which in turn helps the environment. The area of this project the team has focused on tries to effectively generate well-optimized routes for waste collection companies, which also aims to save gas usage when collecting orders for waste collection.

5.3 Administrative results

5.3.1 Work schedule discussion

The team managed to keep the planned work schedule pretty well throughout the project. Each sprint was marked by a goal, such as finishing a certain part of the system, having finished an MVP, for instance. The team managed to more or less hit these goals throughout the development and therefore managed to keep on track with the work schedule. If some small part of the goals for the last sprint were not fully completed, the team would start the next sprint by finishing up these goals first. As the product evolved, these goals also needed to change. Some changes were made to the road-map during the development, but not any significant changes. As mentioned in the result section, the workload was done by the team steadily increased during the project timeline. This was planned by the team from the start, as the members had more available time towards the end of the project phase.

5.3.2 Development process discussion

As mentioned, the team utilized a lean and iterative Scrum variant. Two of the team members already had experience working together at Favn and utilizing Scrum on projects there. In addition to this, all three team members have worked together on a project before, also using Scrum. For this reason, the development process was adopted easily by all the team members. As the team members had worked together multiple times before, they knew each other's strengths and weaknesses. This meant the team could fit the development process to what had worked in the previous development projects. This led the team to opt-out of some aspects of Scrum, such as using a Scrum master. It was also decided upon between the supervisor and the team members not to use a Gantt diagram in the development, as it was agreed that such a diagram in our case most likely turn out not to be very accurate. Instead, it was agreed to use a roadmap for setting up the overreaching goals of the project.

In addition to the sprint planning meetings with Favn at the end of each sprint, the team had intermediate status meetings where the members updated each other on what was being worked on, what needed to be done, and whether someone needed help with anything. These status meetings were not held on a regular basis but rather when the team deemed it necessary. In retrospect, this worked quite well, but it would not hurt with more structure. The team spent quite a lot of time working over voice chat during the project, where the team discussed and worked together on tasks. Because of this, the team members were updated on what the other members were working on most of the time. For this reason, the team deemed it not to be necessary to have status meetings all that often, but rather when the need arose.

6 Conclusion and further work

6.1 Conclusion

he thesis question was; "How do different versions of hill climbing compare when adjusted to solve the capacitated vehicle routing problem with time limits on realistic map data." The team presented three different approaches to the problem. The first, HC, tries to quickly approach a local optima and uses this as the result. The second, SA, randomizes the hill climb somewhat and aims to use more resources to not get stuck in the local optima, and find better alternative solutions. AILS aims to converge toward an optimal solution. The results conducted were discussed in the previous chapter and now it is time to conclude what the data indicates. AILS is the superior algorithm as it consistently finds the best routes. This is true although for very large cases its run time could be unpractical for some applications. For these very computationally heavy cases the experiments indicate that both SA and HC will find solutions that are worse, but only within the margin of a few per cent. For some very large applications a near-perfect solution all the time might also be a valid sacrifice in order to produce the results quickly. The conclusion is then that for most cases the experiments indicate that AILS and sacrificing some compute time to find a better local optima is superior, but for very large applications one might have to resort to HC and SA, which yield one somewhat worse results, but at a much faster run time.

The vision document of the product described the vision for the entire project. The end product of the team met some of the requirements found in the vision document, but finished most of the requirements the team aimed to complete. The product is currently an MVP application that has the bare minimum features of the waste administrative tool described in the document. As well as a simulation tool for waste administrators. The main focus of the team, the route optimization part, put out acceptable results for routing solutions in short enough execution times, even with large data sets. Since the route optimizer can output good solutions at a fast enough rate, it can be concluded that the core functionality of the project performs within practical limitations and is therefore a good starting point for the project. The administrative platform that utilizes the optimizer, is however currently lacking compared to the project vision, but is reasonable as this was not the primary focus of the project.

6.2 Further work

Before continuing on additional features, it is trivial to mention that bugs, such as specified in 5.2.3. As for the product developed for Favn Software AS, a natural next step would be to advance the front-end and merge the result of the other bachelor thesis, which worked on a mobile application that would be used to, for instance, generate orders to pick up. The front-end and firebase parts of the system need an authentication system and thorough security rules. The security rules are rules specified in Firestore, which is needed for the application to prohibit users from accessing data they are not authenticated for.

The web application still needs an order overview for the waste administrators, a notification feature, the possibility to create new organizations and users for these, the possibility to add time intervals and custom configurations for them, and a way to communicate with users.

Authentication for using the route optimization API is also needed. As of now, the API is not limited in any way, and the deployed optimization API can be accessed and utilized by anyone. This should be limited only to the cloud functions for the project when in production. The same goes for the OSRM API. OSRM should only allow requests from the route optimizer. The routing API should be able to deliver infeasible solutions when the request specifies it as a simulation and report these solutions as violations instead of returning errors.

In terms of route optimization, many different paths could lead to better results that are possibly worth exploring; a brief comment shall now be offered to a couple of these paths.

Improving the optimizations of HC. The current implementation of HC exhaustively goes through all the possible heuristics. It seems natural to analyze how often the different heuristics are used and how good the solution is with and without different heuristics. This could expose that some of them are superfluous or negligible. This seems like a clear way to speed up the algorithm even more at a small cost.

One could create a sophisticated heuristic chooser for SA. The SA generates a random pair of nodes and selects a random improvement heuristic to use on those points. It seems feasible to create some heuristics that either predicts what sort of heuristic should be used or where those improvements should be made so that the algorithm optimizes with a bit more purpose. It is likely, although not confirmed as stated in the discussion, that the SA probably has a very low hit rate right now, and guiding where improvements should be made could make a big difference in how fast decent solutions can be produced. This sort of heuristics could be based on proximity metrics to find outliers that would have a higher probability of being changed, or if one cluster is breaking the time constraint, it could be more likely to be picked for a heuristic.

The AILS implementation is already heavily based on the paper by Viníci us et al. [5], adding the path relinking stage described by them yielded good results for them but was too complex to be done within the limits of this projects. This would probably make the solutions even better, and if the solutions were good enough, one could use fewer iterations to find a good solution making the run time in practice reduce, although this should be verified thoroughly. Fully implementing their algorithm and adjusting it for our constraints, which once again are unusual due to both space and weight limit, could be interesting to test if their results effectively carry over as well. Significantly improved results seem reasonable as relinking has yielded good results in other papers as well, for example, Ribeiro et al. [17]. That paper goes as far as to say that "Path-relinking is a major enhancement to heuristic search methods for solving combinatorial optimization problems." Simply put, path relinking starts by finding a couple of decent solutions. Using the defined heuristics, one of these solutions can gradually morph into another. On this path from one good solution to another, there possibly lies a solution that combines the two qualities that make the individual solutions good. This sort of structure seems to be common in many optimization problems. Path relinking has been successfully applied many times, for instance, on the p-median problem, the Steiner tree problem, the capacitated minimum spanning tree problem according to Resende et al. [18].

Another interesting approach to the CRVP takes an entirely different approach to anything implemented in this project, namely evolutionary algorithms. This group of algorithms keeps track of a group of populations called the population and uses genetic operations to merge different solutions. The goal of the algorithm is to simulate an evolutionary process where the population adapts to the environment, which in this case is analogous to the loss function, which is solution distance. Note that these sorts of solutions could be combined with some of the iterative solutions implemented in this project as "pure genetic algorithms are not competitive with the best published results" [19].

7 Attachments

- **Attachment A** - Vision document
- **Attachment B** - Project handbook
- **Attachment C** - Design document
- **Attachment D** - System document
- **Attachment E** - Hill climbing experiments document

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