Master's thesis

NTNU
Norwegian University of Science and Technology
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Department of Economics

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The Fama-French five-factor asset pricing model: A replication across the globe

Master's thesis in Financial Economics
Supervisor: Costanza Biavaschi
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Abstract

Despite the powerful empirical performance of the five-factor model of Fama and French (2015), the literature on modern asset pricing is heavily influenced by findings in the US stock market. This thesis extends the current literature and seeks to evaluate whether the five-factor model directed at capturing the size, value, profitability, and investment patterns in average stock returns is replicable across the globe. In particular, this paper empirically investigates the performance of the five-factor model and subset of its factors in the US, North America, Asia Pacific (excluding Japan), Japan, Europe, and Emerging Markets. The five-factor model consistently outperforms the three-factor model in the US, North America, Asia Pacific (excluding Japan), Europe, and Emerging Markets. In Japan, the five-factor model does not offer an improvement over the three-factor model. Results indicates that the five-factor model performs well, but does not replicate across the world, and different markets need to account for different sets of factors. Practical applications of the five-factor model, such as cost of equity capital calculations and performance evaluations, may be best performed on a country-specific basis.

Sammendrag

Til tross for den kraftige empiriske ytelsen til fem-faktor-modellen til Fama og French (2015), er litteraturen om moderne kapitalverdimodeller (verdsetting av finansielle aktiva) sterkt påvirket av funn i det amerikanske aksjemarkedet. Denne oppgaven utvider dagens litteratur og søker å evaluere om fem-faktor-modellen rettet mot å fange størrelse, verdi, lønnsomhet og investeringsmønster i gjennomsnittlig aksjeavkastning er replikerbar over hele kloden. Denne oppgaven undersøker spesielt den empiriske ytelsen til fem-faktor-modellen og ulike varianter av den i USA, Nord-Amerika, Asia-Stillehavet (utenom Japan), Japan, Europa og fremvoksende markeder. Fem-faktor-modellen overgår konsekvent tre-faktor-modellen i USA, Nord-Amerika, Asia-Stillehavet (utenom Japan), Europa og fremvoksende markeder. I Japan tilføyer ikke fem-faktor-modellen en forbedring over tre-faktor-modellen. Resultatene indikerer at fem-faktor-modellen presterer bra, men at den ikke kan bli replikert over hele verden, og forskjellige markeder må ta hensyn til forskjellige sett med faktorer. Praktiske anvendelser av fem-faktor-modellen, som beregning av avkastningskrav til egenkapitalen og porteføljens ytelse kan være best ufført på landsspesifikk basis.

Preface

This thesis concludes my master's degree in Financial economics at the Norwegian University of Science and Technology. Throughout my master's degree I have assigned a comprehensive knowledge within financial economic theory, capital markets, and econometric methods used for empirical financial market analysts. I would like to give a sincere thank you to my supervisor Costanza Biavaschi for guiding me through the process of writing this thesis.

Trondheim, June 2021.

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1 Introduction

The Capital Asset Pricing Model (CAPM) of William Sharpe (1964) and John Lintner (1965) was a pioneering breakthrough for modern financial economics. This is a single-factor model which describes the relationship between systematic risk and expected return for assets (Fama and French, 2004). Over the years, additional theory-based asset pricing models have been developed, such as the Intertemporal CAPM (Merton, 1973), and the consumption based CAPM of Lucas (1978) and Breeden (1979). “A survey of 392 CFOs conducted by Professors John Graham and Campbell Harvey found that 73.5% of the firms that they questioned use the CAPM to calculate the cost of capital” (Berk and DeMarzo, 2017, p. 506). Though often used in applications, the CAPM has been exposed to much criticism for its unrealistic simplification and assumptions, as well as for its acknowledged empirical failure.

In 1992, Fama and French published “The cross section of expected stock returns” and found empirical evidence that other explanatory variables could help explain average stock returns. Banz (1981) documented a strong negative relation between average return and firm size, and Rosenberg, Reid, and Lanstein (1985) documented a positive relation between average return and book-to-market equity for U.S. stocks. These findings contradict with the CAPM which states that expected return of a firm only depends on its relation towards the market premium. From these findings, the three-factor model was developed by Fama and French in 1993. The three-factor model is an extension to the CAPM with two additional factors, such as, small minus big (SMB) and high minus low (HML) to capture the size and value premiums. Since then, researchers have found several new “anomalies” that the three-factor model fails to capture, which has resulted in the development of new models. In 1997, Carhart presented a four-factor model which captures the momentum “anomaly” from Jegadeesh and Titman (1993). He extended the three-factor model of Fama and French and added the up minus down (UMD) factor to capture the momentum risk premium. Furthermore, from the work of Novy-Marx (2013) and Titman, Wei, and Xie (2004), Fama and French (2015) extended their three-factor model with two additional factors, robust minus weak (RMW) and conservative minus aggressive (CMA) to capture the profitability and investment premiums. Empirical tests of the five-factor model shows that the additional factors have explanatory power and hence, was able to outperform the three-factor model in explaining average stock returns in the US.

Today, the five-factor model of Fama and French (2015) is one of the most famous empirical asset pricing models in the world. In their paper "A five-factor asset pricing model", they empirically investigated the cross section of expected stock returns with the five-factor model and were able to explain between 71% and 94% of the variance in returns of the examined portfolios in the US.

However, despite the powerful empirical performance of the five-factor model, the literature on modern asset pricing is heavily influenced by findings in the US stock market. Also, while the model is often used by academics and practitioners to evaluate portfolio performance, little is known about its generalizability in context outside the US. This thesis seeks to replicate the work of Fama and French with the five-factor model in the US with and without additional observations and is also the first to provide a comprehensive overview of its replicability across several markets around the world, both in developed and emerging markets. There are a number of studies that have tested the three- and five-factor model in a wide range of developed and Emerging Markets (Fama and French, 1993; Griffin, 2002; Fama and French, 2015; Fama and French, 2017; Lin, 2017; Foye, 2018). Yet, to the best of my knowledge, this thesis is the first to assemble a dataset comprising of information on all continents across the globe, including a broad sample of developed and emerging markets. This offers fresh evidence in a single source on the usefulness of adding profitability and investment factors to the original three-factor model, as well as allows the reader to easily compare the performance of the five-factor model across space using a single source. In doing this, I address the following research question:

"Does the five-factor model of Fama and French replicate across the world?"

In the first part of the thesis, I replicate the findings of Fama and French and find consistent results with some minor discrepancies in the data collected, but which achieves the same conclusion as the replicated paper, "A five-factor asset pricing model". As Fama and French (2015), the value factor for the US seems to be redundant in explaining average return, which also goes for North America. Further, I expand and find that the five-factor model offers an improved explanation of average stock returns over the three-factor model in the US, North America, Asia Pacific (excluding Japan), Europe, and Emerging Markets. However, for Japan the five-factor model does not offer an improvement over the three-factor model. Interestingly, all markets examined in this thesis, outside the US, markets seem to have no size effect nor investment effect when accounting for all the five factors. Japan has neither market, size, profitability nor investment patterns. This thesis concludes that the five-factor model of Fama and French (2015) does *not* replicate across the world, and different markets need to account for different sets of factors. Combining my findings with Fama and French (2017) indicates that national rather than regional factor models may be a better choice in the markets I have examined in this thesis.

The format of this paper is as follows. In section 2, I build the theoretical foundation of the thesis by presenting the CAPM and the Arbitrage Pricing Theory (APT) which the factor models of Fama and French are built on. I also give a brief summary of the literature on modern asset pricing tests. In section 3, I establish central definitions and tools needed to empirically test and evaluate the Fama and French five-factor model. In section 4, I replicate the paper "A five-factor asset pricing model" of Fama and French (2015) in the first part, and in the second part I replicate the model to the other markets examined in the thesis. In section 5 I discuss and compare my results, and in section 6 I present my conclusions in light of my research question.

2 Theory and literature review

2.1 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) of William Sharpe (1964) and John Lintner (1965) is a single-factor model which describes the relationship between systematic risk and expected return for assets. The CAPM is widely used throughout finance for pricing risky securities and generating expected returns for assets given the risk of those assets and cost of capital. The goal of the CAPM formula is to evaluate whether a stock is fairly valued when its risk and time value of money are compared to its expected return.

The CAPM assumes that investors are risk-averse and maximize expected utility of wealth. They are also price takers and have homogeneous expectations about asset returns that follow a normal distribution. In a frictionless market, all investors will hold the market portfolio (Fama and French, 2004).

The formula for calculating the expected return of an asset given its risk is as follows:

$$E(R_i) = R_f + \beta_i\left[E(R_m) - R_f\right] \tag{1}$$

Were;

$E(R_i) = Expected\ return\ for\ security\ i$
$R_f = Risk - free\ rate$
$\beta_i = Beta\ of\ security\ i\ (Systematic\ risk)$
$E(R_m) = Expected\ return\ of\ the\ market\ portfolio$
$E(R_m) - R_f = Expected\ risk\ premium\ for\ the\ market$

The CAPM formula consists of two parts for calculating the expected return of asset i. Investors expect to be compensated for both risk and the time value of money. The risk-free rate in the formulae accounts for the time value of money, whilst the second part of equation (1) on the right-hand side accounts for the additional risk the investor is taking. The beta of an asset i is a measure of its systematic risk and is measured by how much an individual security co-varies with the market portfolio and is shown by equation (2):

$$\beta_i = \frac{Cov(R_{i,}R_m)}{Var(R_m)} \tag{2}$$

When investing in an asset there is always two parts of risk to consider, and the total risk is shown for equation (3):

$$Total\ risk = Systematic\ risk + Unsystematic\ risk \tag{3}$$

As equation (3) above shows, the total risk of an investment is a combination of both systematic and unsystematic risk. Systematic risk can be interpreted as "market risk" (economy as a whole) and is not possible to eliminate by diversification. Unsystematic risk on the other hand can be eliminated if one diversifies the portfolio. So, in other words, a well-diversified portfolio will only contain systematic risk, and achieve a beta which equals to one. If $\beta_i = 1$, then the expected return of asset i, is equal to the expected return of the market portfolio. If $\beta_i > 1$, the asset is measured to be riskier than investing in the market portfolio and hence, the expected return of the asset should be greater than the market portfolio. Equivalently, if $\beta_i < 1$, then the asset is less risky than the market portfolio and the expected return of asset i is less than the expected return of the market portfolio. In other words, if $\beta_i = 0$, then security i is considered to be risk free.

Despite the CAPM´s popular usage by practitioners and academics, the model rests on some unrealistic assumptions that have received much criticism. This went so far as the point made by Fama and French (1992), who in their paper "The cross-section of expected stock returns" eventually declared the beta "dead".

Empirical tests of the CAPM can be done by testing if the market portfolio is mean-variance efficient, i.e., all investors holding the tangency portfolio[1]. However, since the market portfolio cannot be observed, we need to find a proxy which would represent the market portfolio. Fama and French (1992) use a combination of NYSE, AMEX, and NASDAQ stocks as proxy for the market portfolio. Since the CAPM includes a risk-free rate component we also need to find a proxy for R_f. For R_f, Fama and French (1992) use one-month Treasury bills. Once we have the proxies for the model, we need to determine a sample period, holding period, and a sample size. Moreover, we need to estimate the security characteristic line [2].

First stage regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{M,t} - r_{f,t}\right) + \varepsilon_{i,t} \qquad (4)$$

[1] Tangent portfolio is the one intersect with the tangent line, so it has the highest Sharpe ratio than other portfolios sitting on the efficient frontier. The portfolios with the best trade-off between expected returns and risk (Std. dev) lie on this line. The tangency point is the optimal portfolio of risky assets, known as the market portfolio.

[2] A security characteristic line (SCL) is a straight line formed using regression analysis that summarizes a particular security's systematic risk and rate of return.

We use the regression above (4) to solve for each stock i, obtain average excess return for security i ($\overline{r_{i,t} - r_{f,t}}$), average excess return for the market ($\overline{r_{M,t} - r_{f,t}}$), estimate for β_i (b_i), and estimate for the unsystematic risk $\sigma^2(\varepsilon_i)$.

Furthermore, we calculate the second stage regression:

$$\overline{r_{i,t} - r_{f,t}} = \gamma_0 + \gamma_1 b_i + \gamma_2 \sigma^2(\varepsilon_i) \tag{5}$$

From the second pass regression (5) we can conclude that the CAPM is a realistic representation of the relationship between systematic risk and expected return for assets if $\gamma_0 = 0, \gamma_1 = r_M - r_f$, and $\gamma_2 = 0$ (Jensen, Black, Scholes, 1972). However, a large amount of literature has evaluated empirically CAPM testing these hypotheses, often rejecting the idea that only the market portfolio could explain returns. Further, I will investigate earlier empirical research of the CAPM.

There are several studies on the empirical performance of the CAPM. In 1977, research conducted by Sanjay Basu challenged the CAPM when they sorted stocks by earnings price characteristics. The findings were that stocks with higher earnings yields tended to have better returns than the CAPM would have predicted. Further, Banz (1981) found evidence that other variables seemed to help explain stock returns. He found that the size of the firm explains variation of return, and that small firms substantially outperforms large firms in average returns after adjusting for risk. Fama and French (1992) support Banz (1981) and found that other explanatory variables, such as firm size (market equity) and value (book-to-market equity ratio), seem to explain the cross-section of expected stock returns better than the beta. Fama and French (1992) also found several other variables such as price earnings ratio and leverage to explain stock returns. In their paper, Fama and French (1992) use data from NYSE, AMEX, and NASDAQ. From merged CRSP COMPUSTAT they collected security prices and annual industrial files of income-statement and balance-sheet data from 1962-1989. Fama and MacBeth (1973) found a positive simple relation between average return and market β during the early years (1926–1968) of the CRSP NYSE returns file. However, Reinganum (1981) and Lakonishok and Shapiro (1986), found that the simple relation between β and average return disappears during the more recent 1963–1990 period. Fama and French (1992), found that the relation between β and average return is also weak in the last half century (1941–1990) of returns on NYSE stocks. In brief, Fama and French (1992) do not support that average stock returns are positively related to market β. Furthermore, this evidence became the origin of the three-factor model of Fama and French (1993) as will be presented later in the thesis.

2.2 Arbitrage Pricing Theory (APT)

The APT was developed by Stephen Ross (1976) and is a multi-factor asset pricing model. The idea of the model is that an asset´s return can be predicted using the linear relationship between the asset´s expected return and several variables which identifies systematic risk. The model is useful to analyze portfolios and identify mispricing, and hence, the opportunity for a risk-free profit.

In the APT, the return of asset i is assumed to be generated by a linear factor model:

$$\tilde{r}_i = \alpha_i + \beta_{i,1}\widetilde{F_1} + \beta_{i,2}\widetilde{F_2} + \cdots + \beta_{i,K}\widetilde{F_K} + \tilde{\varepsilon}_i \tag{6}$$

In the equation above (6) the α_i is a constant, $\beta_{i,K}$ is the risk associated with the K factor, F_K is the macroeconomic factor to capture the systematic risk of factor K, and $\tilde{\varepsilon}_i$ is the unsystematic risk component for asset i. When subtracting expectations from the equation, and assuming $E(\tilde{\varepsilon}_i) = 0$, we get:

$$\tilde{r}_i = E(\widetilde{r_i}) + \beta_{i,1}\widetilde{X_1} + \beta_{i,2}\widetilde{X_2} + \cdots + \beta_{i,K}\widetilde{X_K} + \tilde{\varepsilon}_i \tag{7}$$

Were;

$\widetilde{X_K} = \widetilde{F_K} - E(\widetilde{F_K})$, i.e., the deviation of factor K from its expected value.

Assumptions on the error term:

$$E(\widetilde{\varepsilon_i}) = 0 \tag{8}$$

$$E(\widetilde{\varepsilon_i}\widetilde{X_K}) = 0 \qquad \forall i, K \tag{9}$$

$$E\left(\tilde{\varepsilon}_i \tilde{\varepsilon}_j\right) = 0 \qquad \forall i \neq j \tag{10}$$

The APT further assumes perfectly competitive and frictionless capital markets, homogenous expectations on the K-factor model, and that the number of assets is much larger than the number of factors. No-arbitrage states that there is no opportunity for a risk-free profit, and if two assets are the same in terms of risk, they cannot sell at different prices (Law of One Price). Unlike the CAPM, which assumes that markets are perfectly efficient, APT assumes markets sometimes misprice securities before the market eventually corrects and securities move back to fair value (Ross, 1967). Whilst the CAPM relies on only one-factor (market premium), APT allows the return to be determined by several factors. In other words, the CAPM could be considered a special case of the APT.

Compared to the CAPM, APT has several advantages because it relies on less assumptions and allows returns to be determined by many factors and hence, is considered to be more robust. Nonetheless, although APT is elegant and theoretically powerful, one essential drawback applies. The theory neither tells us what the number of K factors should be, nor does it tell us what these risk factors are. Therefore, the model cannot be truly tested.

Furthermore, in modern finance, several practitioners and academics search for proxies which could help explain the variance in average stock returns. “The currently dominant approach to specifying factors as candidates for relevant sources of systematic risk uses firm characteristics that seem on empirical grounds to proxy for exposure to systematic risk. The factors chosen are variables that on past evidence seem to predict average returns well and therefore may be capturing risk premiums” (Bodie et al., 2014, p. 340). As we will see, Fama and French three-factor and five-factor model use APT in terms of firm characteristics to explain average stock returns.

2.2.1 Three-factor model

In 1993, Fama and French published the three-factor model, "Common risk factors in the returns on stocks and bonds" to explain stocks and bond returns. The three-factor model is an extension of the CAPM by adding two additional factors. The factors added from Fama and French are: (1) small minus big (SMB), which is the difference in returns on a portfolio of small stocks and a portfolio of big stocks, and (2) high minus low (HML), which is the difference in return on a portfolio of high book-to-market equity stocks (value) and a portfolio of low book-to-market equity stocks (growth). Fama and French (1993) empirically tested the model using regressions of the excess stock returns on the excess market returns and the mimicking returns for the size (SMB) and book-to-market equity (HML) factors. Their data is from July 1963 to December 1991 (342 months), and is collected from NYSE, AMEX, and NASDAQ stock files.

In practice they estimate the following model:

$$R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it} \tag{11}$$

Their findings showed that SMB and HML factors have explanatory power, and the intercept is close to zero, as it should be when the regressions use excess returns. They found that, on average, returns typically increased from low book-to-market equity to high book-to-market equity. They also found that small stocks have persistently higher average returns than big stocks. The three-factor model of Fama and French has come to dominate empirical research and is considered to be a benchmark when other models are tested.

Despite the evidence of Fama and French (1993), the three-factor model does not capture all the variation in average stock returns. Jegadeesh and Titman (1993) found evidence that trading strategies that buy past winners and sell past losers realize significant abnormal returns. They used data over the 1965 to 1989 period from the NYSE and AMEX files. From this evidence, Carhart (1997) added a momentum factor to the three-factor model to capture more of the variation in stock returns.

2.2.2 Five-factor model

In 2015, Fama and French published “A five-factor asset pricing model”, which is an extension of the three-factor model by two additional factors inspired by the dividend discount model[3]. First, Fama and French added a profitability factor (robust minus weak). The idea is that companies reporting higher future earnings have higher returns in the stock market. Second, the investment (conservative minus aggressive) factor suggesting that companies directing profit towards major growth projects are likely to experience losses in the stock market. Data collection is similar as for Fama and French (1993), in which they use NYSE, AMEX, and NASDAQ stocks on both CRSP and COMPUSTAT files, but in the latest paper extended their timeframe with 21 years of new data from July 1963 to December 2013.

The five-factor model takes the form:

$$R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it} \tag{12}$$

Empirical tests of the five-factor model shows that the model can explain between 71% and 94% of the cross-section variance of expected returns for the Size, Book-to-market equity, Operating profitability, and Investment portfolios examined. Even though the five-factor model fails the GRS-test[4], Fama and French (2015) conclude that the five-factor model is the superior model when comparing the three-factor and the five-factor model to explain average stock returns in the US.

2.3 Similar studies

The paper “A comprehensive test of the Fama-French five-factor model in emerging markets” by James Foye (2018), uses the five- and three-factor model of Fama and French to evaluate whether the five-factor model is able to offer a better description of emerging market equity returns than the three-factor model. In the paper, Foye (2018) applies the five- and three-

[3] The dividend discount model is a method for valuing a company's stock based on the theory that its stock is valued for the sum of all its future dividend payments discounted back to their present value. In other words, it is used to value stocks based on the net present value of the expected future dividends. (Fama and French, 2015).
[4] The GRS test is what finance calls a statistical F-test for the hypothesis that all the alphas (from a set of time-series regressions) are zero.

factor models in 18 different countries, and across three different regions (Eastern Europe, Latin America, and Asia). Data are retrieved from Datastream and consists of firms that have a stock market listing at some point during the period between December 1996 to June 2016. Empirical tests show that all three regions exhibit a strong value premium and strong profitability premiums in Eastern Europe and Latin America, but not Asia. Profitability and investment premium cannot be distinguished in the Asian sample and hence, the five-factor model is superior to explain stock returns in Eastern Europe and Latin America but does not offer an improvement over the three-factor model in Asia.

Fama and French published the paper, “International tests of a five-factor asset pricing model” in 2017. They empirically investigated the performance of the five-factor model in four different regions, such as North America, Europe, Japan, and Asia Pacific. Data are collected primarily from Bloomberg, supplemented by Datastream and Worldscope, and the sample period is from July 1990 to December 2015. Evidence shows that for North America, Europe, and Asia Pacific, average stock returns increase with the book-to-market ratio and profitability and are negatively related to investment. For Japan, the relationship between average returns and book-to-market ratio is strong, but average returns show little relation to profitability and investment. Thus, they concluded that the five-factor model is superior to explain stock returns in North America, Europe, and Asia Pacific, but does not offer an improvement over the three-factor model in Japan.

Other applications have also applied the factor models of Fama and French in their studies, such as Hou, Xue and Zhang (2014) Hou, Karolyi, and Kho (2011), Griffin (2002), and others. We cannot go into all the papers that have mimicked Fama and French´s models because there are so many. Despite a large amount of asset pricing studies using Fama and French´s models conducted all over the world, there are to my knowledge zero studies that applies Fama and French five-factor asset pricing model to several markets across the globe into one paper and seek if the model can replicate across the world. Replication studies are considered a hallmark of good scientific practice and I seek to contribute to a greater understanding of capital asset pricing in modern finance.

3 Methodology

Throughout this study, I follow the same procedure and methods as Fama and French (2015) in their paper "A five-factor asset pricing model". In the replication-part of the thesis I collected the same data as Fama and French (2015), in which I applied the same model and methods. Further, I extended their work by adding more observations to get an up-to-date dataset for the US. I also applied the five-factor model to several markets across the world, such as North America, Asia Pacific (excluding Japan), Japan, Europe, and Emerging Markets, to answer my research question.

My thesis and research question are of such a nature that a quantitative methodological approach is the best procedure to provide in-depth knowledge on the topic. It is also implied for achieving a replication of "A five-factor asset pricing model". Even though Fama and French (2015) described the procedure of their model thoroughly, a great understanding of the theory behind the model and the construction of the portfolios behind the factors is challenging.

As mentioned earlier in the thesis, data is collected from both French´s website[5] and from Wharton Research Data Services for the US market. For the replication part of my thesis, I collected observations from July 1963 to December 2013, including all NYSE, AMEX, and NASDAQ stocks, same as Fama and French (2015). In the extension part of my thesis, I collected additional data, giving a dataset with observations from July 1963 to February 2021 (692 months), for NYSE, AMEX, and NASDAQ stocks for the US market. Data for all other markets (North America, Asia Pacific (excluding Japan), Japan, Europe, and Emerging Markets) were collected from French´s website and Bloomberg database. For North America, Asia Pacific (excluding Japan), Japan and Europe, the data is from July 1990 to February 2021 (368 months), whilst for the Emerging Markets data is from July 1992 to February 2021 (344 months).

[5] See https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html , last retrieved on February 2021.

3.1 Factor constructions

To replicate and test the five-factor model, I had to create the five factors from equation (12). Fama and French five-factor model is constructed using the 6 value-weight portfolios formed on size and book-to-market equity, the 6 value-weight portfolios formed on size and operating profitability, and the 6 value-weight portfolios formed on size and investment.

Book-to-market equity for June of year t is the book equity for the last fiscal year end in $t-1$ divided by market equity for December of $t-1$. *Operating profitability* for June of year t is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in $t-1$. *Investment* is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets. The Book-to-market, OP, and Inv breakpoints are the 30th and 70th NYSE percentiles, and the size breakpoint for year t is the median NYSE market equity at the end of June of year t.

$R_M - R_F$ is the excess return on the market which is constructed by using the value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares, and price data at the beginning of t, and good return data for t minus the one-month Treasury bill rate (from French's website). For the other markets $R_M - R_F$ is constructed for July of year t to June of $t+1$ that include all stocks for which they have market equity data for June of t.

SMB, HML, RMW, and CMA for July of year t to June of $t+1$ include all stocks for which we have market equity data for December of $t-1$ and June of t, (positive) book equity data for $t-1$ (for SMB, HML, and RMW), non-missing revenues and at least one of the following: cost of goods sold, selling, general and administrative expenses, or interest expense for $t-1$ (for SMB and RMW), and total assets data for $t-2$ and $t-1$ (for SMB and CMA) (French, 2021).

Table 1. Construction of Size, B/M, profitability, and investment factors. SMB, HML, RMW, and CMA factors are constructed using the 6 value-weight portfolios formed on size and book-to-market, the 6 value-weight portfolios formed on size and operating profitability, and the 6 value-weight portfolios formed on size and investment.

SMB(B/M)	=	1/3 (Small Value + Small Neutral + Small Growth) – 1/3 (Big Value + Big Neutral + Big Growth)
SMB(OP)	=	1/3 (Small Robust + Small Neutral + Small Weak) – 1/3 (Big Robust + Big Neutral + Big Weak)
SMB(Inv)	=	1/3 (Small Conservative + Small Neutral + Small Aggressive) – 1/3 (Big Conservative + Big Neutral + Big Aggressive)
SMB	=	1/3 (SMB(B/M) + SMB(OP) + SMB(Inv))
HML	=	1/2 (Small Value + Big Value) - 1/2 (Small Growth + Big Growth)
RMW	=	1/2 (Small Robust + Big Robust) - 1/2 (Small Weak + Big Weak)
CMA	=	1/2 (Small Conservative + Big Conservative) - 1/2 (Small Aggressive + Big Aggressive)

Table 1 shows the factors used in the five-factor asset pricing model (Equation 12) of Fama and French (2015), and how they are constructed. SMB (Small Minus Big) is the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios. HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios. RMW (Robust Minus Weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios. CMA (Conservative Minus Aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios.

3.2 Market Portfolio

When applying asset pricing models that contain the market portfolio, such as CAPM, three-factor model, five-factor model, and similar models, we first need to identify the market portfolio. The market portfolio is the total supply of all securities, so the proportion of each security should correspond to the proportion of the total market which each security represents (Berk and DeMarzo, 2017, p. 441). So, because of how the market portfolio is constructed, the portfolio contains more of the largest stocks and less of the smallest stocks. Equation (13) shows how the market value of security i is calculated:

$$MV_i = (Number\ of\ Shares\ of\ i\ Outstanding) * (Price\ of\ i\ per\ Share) \tag{13}$$

From this we can calculate the portfolio weights (x_i) for each security as:

$$x_i = \frac{Market\ Value\ of\ Security\ i}{Total\ Market\ Value\ of\ All\ Securities\ in\ the\ Portfolio} = \frac{MV_i}{\Sigma_j MV_j} \tag{14}$$

A portfolio like the market portfolio, where each security i is held in proportion to its market capitalization, is called a value-weighted (VW) portfolio. The return on a VW portfolio is the sum of the weighted return of all portfolio securities. The alternative to the VW portfolio is the equal-weighted (EW) portfolio. The EW portfolio gives every stock the same weight regardless of their market capitalization and hence, gives smaller stocks a greater influence in the portfolio than the VW portfolio would have. Since the market portfolio cannot truly be observed (since it contains all securities i, at time t for the whole market), we seek to form portfolios that are the best proxy for the market as a whole (Berk and DeMarzo, 2017). In this thesis, the VW portfolio is preferred to the EW portfolio because of how the market portfolio is constructed, and thus, I will concentrate on using VW portfolios when constructing factor mimicking portfolios for the asset pricing models.

3.3 Model performance

When evaluating an asset pricing model, I seek to identify the model that is the best (but imperfect) at explaining average returns (Fama and French, 2015). Tests of asset pricing models commonly use either the cross-section regression approach of Fama and MacBeth (1973) or the time-series regression approach that centers on the GRS test of Gibbons, Ross, and Shanken (1989) (Fama, 2015). In my thesis, I test the models using the GRS test statistics approach as Fama and French (2015)[6]. If an asset pricing model completely captures all expected returns, the intercept is zero in a regression of an asset´s excess returns on the model´s factor returns.

The GRS test is what finance calls a statistical F-test for the hypothesis that all the alphas (from a set of time-series regressions) are zero. The null hypothesis is stated as: H_0: $\alpha_i = 0,\ \forall i$. If the null hypothesis is rejected ($a_i \neq 0$), the intercept is statistically significantly different from

[6] Program used was the STATA package "grstest2" (Ibert, 2014) to calculate the Gibbons, Ross, Shanken (1989) F-test for the joint null hypothesis that, N estimated intercepts from N time-series regressions are equal to zero.

zero, and the model has cross-sectional pricing errors. I also show the chi-square test (χ^2) in equation (16) which is also commonly used, for comparison.

The GRS test statistics is shown by the following equation:

$$GRS = \left(\frac{T - N - K}{N}\right) \frac{\left(\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\right)}{(1 + \mu'_f \hat{\Sigma}_f^{\ -1} \mu_f)} \sim F_{N,T-N-K} \qquad (15)$$

And the χ^2 test statistics as "J":

$$J = T \frac{\left(\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\right)}{(1 + \mu'_f{}' \hat{\Sigma}_f^{\ -1} \mu_f)} \sim \chi^2(n) \qquad (16)$$

Where T is the number of observations in the time-series, N is the number of test assets, K is the number of factors. $\hat{\alpha}$ is an N × 1 vector with the estimated intercepts from the individual time-series regressions, $\hat{\Sigma}$ is an N × N unbiased estimate of the residual covariance matrix, μ'_f is a K × 1 vector of the factor portfolios means, and $\hat{\Sigma}_f$ is an K x K unbiased estimate of the factor portfolios covariance matrix. To derive the GRS test statistics, errors are assumed normally distributed, uncorrelated over time, and are homoscedastic. One could also look at the chi-square test for equation (16) as a goodness-of-fit test. Whereas the GRS test statistics are assumed normally distributed in errors, the chi-square test ($\chi^2(n)$) do not need this assumption but is only asymptotically valid (Fama, 2015). Further, I will only concentrate on the GRS test statistics for this thesis.

3.4 Goodness of Fit Statistic – "R^2"

The R^2 can be interpreted as a measure of how well a model fits the data. Given the dependent variable on the left-hand side, and the independent (explanatory) variables on the right-hand side, the R^2 measures the explained variance of the regression model. R^2 is a value between 0 and 1 and is interpreted in percentage. A regression model that captures most of the variance in the data, has a R^2 close to 1, and a model that fits the data poorly has a low R^2.

The goodness of fit statistic is given by:

$$R^2 = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS} \tag{17}$$

Were;

$TSS = ESS + RSS$

$TSS = Total\ sum\ of\ squares$

$ESS = Explained\ sum\ of\ squares$

$RSS = Residual\ sum\ of\ squares$

Equation (15) above shows how the R^2 is calculated from the regression model (Brooks, 2019). If the model fits the data well, the explained sum of squares would be close to the total sum of squares, which would give a high R^2 for the regression model. When looking at model performance for asset pricing models, R^2 has been treated mainly as a descriptive statistic, and not the realistic explanatory power of the model. It is however a great tool for performance comparison for different models against each other.

4 Analysis and discussion

4.1 Replication

In this part of the thesis, I am going to use the same procedure as Fama and French to replicate their paper "A five-factor asset pricing model". I have collected the same data as Fama and French (2015), returns from both CRSP and Compustat through Wharton Research Data Service (WRDS) and from Kenneth R. French`s website from July 1963 to December 2013. By using the same data and models as in "A five-factor asset pricing model", the goal of this replication is to achieve the same results as Fama and French (2015). I will not replicate all the models and portfolios as Fama and French (2015) because of time and space limits. Instead, I will only focus on the most relevant models and portfolios from their paper. First, I describe the portfolios used in the model and the data (Table A1 and A2 in appendix). Following, I discuss and show model performance by the GRS test (Table 2), and regression slopes and intercepts (Table A5, A6, and A7 in appendix). Finally, I discuss and conclude my results.

4.1.1 Asset pricing tests

From this point on, I look at model performance for the original three- and five-factor model. I also include a four-factor model that combine RM-RF, SMB, and pairs of HML, RMW, and CMA. I consider in total 5 asset pricing models: (1) one three-factor model that combine RM−RF, SMB and HML; (2) three four-factor models that combine RM−RF, SMB, and pairs of HML, RMW, and CMA; and (3) the five-factor model that include all the five factors. Furthermore, I test how well the sets of factors explain average excess returns on the portfolios from Table A1 (Appendix). The factor statistics and correlation are shown in Table A3 and A4 (Appendix) To test the model performance, I use the GRS test statistics. If an asset pricing model completely captures all expected returns, the intercept is zero in a regression of an asset´s excess returns on the model´s factor returns. As Fama and French (2015), the GRS test easily rejects all the models. However, we are less interested in whether the models are rejected than in their relative performance, which we judge using GRS test statistics. The lower the mean absolute alpha, the better is the model.

Table 2. The table tests the ability of three-, four-, and five-factor models to explain monthly excess returns on 25 Size-B/M portfolios (Panel A), 25 Size-OP portfolios (Panel B), 25 Size-Inv portfolios (Panel C). For each set of 25 regressions, the table shows the GRS test statistics, mean absolute alpha ($\boldsymbol{A|\alpha_i|}$), P-value, and Mean adjusted $\boldsymbol{R^2}$. On the top of the table, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the bottom we can see data for the replication part.

	Fama and French	
	2x3 Factors	
	GRS	$A\|\alpha_i\|$
Panel A: 25 Size-B/M Portfolios		
MKTRF SMB HML	3,62	0,10
MKTRF SMB HML RMW	3,13	0,10
MKTRF SMB HML CMA	3,52	0,10
MKTRF SMB RMW CMA	2,84	0,10
MKTRF SMB HML RMW CMA	2,84	0,09
Panel B: 25 Size-OP Portfolios		
MKTRF SMB HML	2,31	0,11
MKTRF SMB HML RMW	1,64	0,06
MKTRF SMB HML CMA	3,02	0,14

MKTRF SMB RMW CMA	1,87	0,08
MKTRF SMB HML RMW CMA	1,87	0,07
Panel C: 25 Size-Inv Portfolios		
MKTRF SMB HML	4,56	0,11
MKTRF SMB HML RMW	4,40	0,11
MKTRF SMB HML CMA	4,00	0,10
MKTRF SMB RMW CMA	3,33	0,09
MKTRF SMB HML RMW CMA	3,32	0,09

	Replication 2x3 Factors			
	GRS	$A\|\alpha_i\|$	P-value	Mean adj $\boldsymbol{R^2}$
Panel A: 25 Size-B/M Portfolios				
MKTRF SMB HML	3,75	0,10	0,00	0,91
MKTRF SMB HML RMW	3,37	0,10	0,00	0,92
MKTRF SMB HML CMA	3,64	0,10	0,00	0,91
MKTRF SMB RMW CMA	3,15	0,10	0,00	0,89
MKTRF SMB HML RMW CMA	3,20	0,10	0,00	0,92
Panel B: 25 Size-OP Portfolios				
MKTRF SMB HML	2,37	0,11	0,00	0,91
MKTRF SMB HML RMW	1,68	0,06	0,02	0,93
MKTRF SMB HML CMA	2,96	0,13	0,00	0,91
MKTRF SMB RMW CMA	1,93	0,07	0,00	0,93
MKTRF SMB HML RMW CMA	1,94	0,07	0,00	0,93
Panel C: 25 Size-Inv Portfolios				
MKTRF SMB HML	4,73	0,12	0,00	0,92
MKTRF SMB HML RMW	4,50	0,11	0,00	0,92
MKTRF SMB HML CMA	4,16	0,10	0,00	0,93
MKTRF SMB RMW CMA	3,60	0,09	0,00	0,93
MKTRF SMB HML RMW CMA	3,60	0,09	0,00	0,93

From Table 2, Panel A, B and C, we can see that the five-factor model provides a lower GRS statistic than the three-factor model for both the Fama and French and the Replication, which is what I expected. This implies that the five-factor model is better at explaining average stock returns in the US. Also, as Fama and French (2015), the HML factor seems to be redundant for the five-factor model when explaining average stock returns, since the GRS statistics is not very affected when leaving the factor out from the test. In some cases, the GRS is even lower when dropping the HML factor. From Table 2, there are very similar results as when I tested the five-factor model and the four-factor model (without HML). The Replication provides similar results as Fama and French but gives a higher GRS test statistic on average for all the models tested. The only exception is for the four-factor model (RM-RF, SMB, HML and CMA) in Panel B that shows a lower GRS statistic.

4.1.2 Conclusion

After replicated "A five-factor asset pricing model" of Fama and French (2015), I obtained mostly the same results in all the tests. At first, in the summary statistics (A1 in appendix), I found some discrepancies between the data I collected and from the tables in Fama and French (2015). This may be because CRSP has recently completed the Pre62 Daily Data Series Project (French, 2021). The project involved backfilling of shares outstanding data for some stocks prior to 1947, and these changes affect the early history of the return series. Because of this, I also found some significant differences in portfolio returns for the RMW slopes and the CMA slopes in the regression portfolios of 25 Size-B/M, 25 Size-OP, and 25 Size-Inv (Table A5, A6, and A7 in appendix). My results are consistent with Fama and French (2015), and I conclude that the five-factor model performs well and is able to explain between 71% and 94% of the cross-section variance of expected returns for the Size, B/M, OP, and Inv portfolios I examined. Furthermore, from the GRS test statistics, I conclude that the five-factor model is superior to the three-factor model, even though all the models were rejected. Nonetheless, the five-factor model does not offer an improvement over the four-factor model (without the HML factor) at least for the US data for 1963–2013. This is in line with Fama and French (2015).

4.2 Extension

In this part of my thesis, I will extend the five-factor model with additional data from the US and compare the replicated results in the last section of the paper. To answer my research question: *"Does the five-factor model of Fama and French replicate across the world?"*, I will also apply the five-factor model in markets across the world, such as, North America, Asia Pacific (excluding Japan), Japan, Europe, and Emerging Markets. Furthermore, I will test and conclude whether the five-factor model is superior to the three-factor and sorts of the four-factor models across all the markets. First, I present summary statistics for all the markets. Second, I discuss and show model performance by the GRS test, regression slopes, and intercepts. Finally, I discuss and conclude my results.

4.2.1 Statistics

Table 3. Average monthly percent excess returns for 25 portfolios formed on Size and B/M (Panel A), Size and OP (Panel B), and Size and Inv (Panel C). The US shows data from July 1963–February 2021, 692 months. For North America, Asia Pacific (excluding Japan), Japan and Europe the data is from July 1990-February 2021, 368 months. Emerging Markets are not included in this table.

Panel A: 25 Size-B/M Portfolios											
The US						**North America**					
	Lo	**2**	**3**	**4**	**High**		**Lo**	**2**	**3**	**4**	**High**
Small	0,38	0,87	0,82	1,00	1,10	**Small**	0,53	0,72	1,00	0,86	1,10
2	0,59	0,83	0,90	0,90	1,00	**2**	0,61	0,71	0,85	0,80	0,84
3	0,60	0,83	0,78	0,89	1,01	**3**	0,89	0,75	0,80	0,75	0,89
4	0,69	0,66	0,71	0,85	0,86	**4**	0,99	0,75	0,88	0,75	0,78
Big	0,58	0,54	0,59	0,50	0,65	**Big**	0,80	0,70	0,65	0,64	0,58
Asia Pacific						**Japan**					
	Lo	**2**	**3**	**4**	**High**		**Lo**	**2**	**3**	**4**	**High**
Small	0,46	0,42	0,68	0,94	1,28	**Small**	0,27	0,35	0,41	0,37	0,49
2	-0,03	0,28	0,25	0,58	0,80	**2**	0,21	0,00	0,15	0,30	0,22
3	0,15	0,35	0,64	0,61	0,68	**3**	-0,03	0,01	0,08	0,11	0,26
4	0,69	0,75	0,56	0,87	0,87	**4**	-0,07	0,10	0,13	0,19	0,20
Big	0,62	0,74	0,77	0,72	0,86	**Big**	0,06	0,14	0,14	0,24	0,40
Europe											
	Lo	**2**	**3**	**4**	**High**						
Small	0,09	0,43	0,50	0,60	0,74						
2	0,36	0,55	0,55	0,67	0,74						
3	0,47	0,64	0,56	0,56	0,69						
4	0,57	0,58	0,54	0,54	0,62						
Big	0,42	0,55	0,55	0,61	0,49						

Panel B: 25 Size-OP Portfolios											
The US						**North America**					
	Lo	**2**	**3**	**4**	**High**		**Lo**	**2**	**3**	**4**	**High**
Small	0,63	0,95	0,88	1,01	0,85	**Small**	0,85	1,06	0,98	1,03	1,06
2	0,68	0,79	0,84	0,83	0,98	**2**	0,53	0,83	0,92	1,10	1,11
3	0,64	0,75	0,78	0,80	0,95	**3**	0,67	0,79	0,83	0,91	1,00
4	0,64	0,70	0,73	0,75	0,84	**4**	0,64	0,83	0,89	0,83	1,00

Big	0,38	0,46	0,54	0,56	0,63	**Big**	0,35	0,59	0,63	0,81	0,79
Asia Pacific						**Japan**					
	Lo	**2**	**3**	**4**	**High**		**Lo**	**2**	**3**	**4**	**High**
Small	0,72	1,20	1,07	1,07	1,10	**Small**	0,29	0,44	0,39	0,37	0,66
2	0,19	0,45	0,65	0,72	0,76	**2**	0,07	0,19	0,25	0,24	0,30
3	0,10	0,66	0,54	0,82	0,82	**3**	0,06	0,10	0,14	0,16	0,19
4	0,41	0,82	0,72	0,84	0,91	**4**	-0,06	0,10	0,24	0,25	0,11
Big	0,55	0,75	0,80	0,80	0,69	**Big**	0,01	0,10	0,23	0,13	0,20
Europe											
	Lo	**2**	**3**	**4**	**High**						
Small	0,21	0,68	0,78	0,91	0,78						
2	0,27	0,60	0,63	0,77	0,99						
3	0,29	0,63	0,72	0,64	0,83						
4	0,28	0,54	0,63	0,74	0,71						
Big	0,17	0,52	0,54	0,49	0,61						

Panel C: 25 Size-Inv Portfolios											
The US						**North America**					
	Lo	**2**	**3**	**4**	**High**		**Lo**	**2**	**3**	**4**	**High**
Small	1,03	0,98	0,97	0,90	0,44	**Small**	1,21	1,08	1,00	0,96	0,52
2	0,92	0,91	0,93	0,92	0,56	**2**	0,90	0,96	0,86	0,88	0,47
3	0,92	0,93	0,80	0,83	0,60	**3**	0,88	0,86	0,90	0,81	0,65
4	0,79	0,76	0,77	0,78	0,66	**4**	0,90	0,93	0,88	0,88	0,68
Big	0,72	0,58	0,54	0,56	0,55	**Big**	0,78	0,66	0,67	0,72	0,64
Asia Pacific						**Japan**					
	Lo	**2**	**3**	**4**	**High**		**Lo**	**2**	**3**	**4**	**High**
Small	1,04	1,10	1,18	1,00	0,52	**Small**	0,36	0,37	0,47	0,37	0,49
2	0,48	0,80	0,51	0,59	0,08	**2**	0,21	0,20	0,22	0,28	0,10
3	0,49	0,87	0,58	0,70	0,17	**3**	0,15	0,18	0,11	0,07	0,10
4	0,55	0,76	0,91	0,85	0,57	**4**	0,14	0,12	0,22	0,04	0,14
Big	0,84	0,67	0,71	0,75	0,62	**Big**	0,14	0,08	0,02	0,17	0,11
Europe											
	Lo	**2**	**3**	**4**	**High**						
Small	0,58	0,69	0,72	0,67	0,27						
2	0,57	0,74	0,74	0,66	0,42						
3	0,63	0,63	0,69	0,51	0,43						

4	0,59	0,57	0,65	0,63	0,45
Big	0,49	0,59	0,48	0,46	0,49

Panel A of Table 3 shows average monthly excess returns (returns in excess of the one-month US Treasury bill rate) for 25 VW portfolios from independent sorts of stocks into Size and B/M quantiles. Panel B of Table 3 shows average excess returns for 25 VW portfolios from independent sorts of stocks into Size and profitability quintiles, and Panel C shows average excess returns for 25 VW portfolios into Size and Investment quantiles. In this section I introduce and apply the five-factor model to five new markets (North America, Asia Pacific (excluding Japan), Japan, Europe, and Emerging Markets), and an extension of the US stock market. Further, I investigate patterns in the 25 Size-B/M, 25 Size-OP, and 25 Size-Inv portfolios. Note that I will typically refer to the smallest and biggest size quintiles as microcap and megacap.

The US

Panel A, B and C of Table 3 shows average monthly excess returns for the US. In the replication part of my thesis, I have thoroughly investigated the performance of the five-factor model and models that include subsets of its factors for this market. For this section I use most of the same data as for the replication, but with an updated dataset with additional 86 observations (months). The US shows data from July 1963 to February 2021 (692 months) which is collected from WRDS and from Kenneth R. French`s website. As expected, the average excess returns for the US are very similar to the statistics observed in the replication part, whereas the size effect (returns tends to decrease from small stocks to big stocks) on average returns are strong in almost all B/M, OP, and Inv portfolios. The exception is for the extreme growth stocks in B/M portfolios (Panel A) and extreme investment stocks for Inv portfolios (Panel B) where there is no clear relationship between size and average returns. The value effect is also prominent for most portfolios in Panel A of Table 3, whereas average excess returns for stocks with low B/M increases towards stocks with high B/M. Big stocks in B/M portfolios show little to none patterns between average excess returns and value. Panel B of Table 3 reports that the profitability effect is evident for the extreme low OP and extreme high OP quantiles, whereas the middle three quantiles show less patterns, but typically increases with higher OP. At last, we look at Panel C of Table 3 for portfolios on Size and Investment. The investment effect is also prominent for these portfolios, where stocks with low investment has higher average excess returns than stocks with high investment. The patterns are clearly shown for the low and high investment quantiles, but a little less for the portfolios between high and low investment.

North America

Panel A, B and C of Table 3 shows average monthly excess returns for North America. North America includes data from the US and Canada. For North America, the data is from July 1990 to February 2021 (368 months) which is collected from Bloomberg database and from Kenneth R. French`s website. Panel A shows portfolios for 25 Size-B/M, where the size effect is strong for the three biggest B/M quantiles but indicates less patterns for the two lowest B/M quantiles. In fact, for the lowest B/M quantile average excess returns increase for the lowest size quantile towards the next highest size quantile. For Panel B, the size effect is prominent for the microcap and megacap portfolios, but for the middle size quantiles there are no clear patterns. Panel C shows similar size patterns as for Panel B, but for the highest investment quantiles there are no patterns at all. The value effect for Panel A in North America seems to be absent for the portfolios examined. It is only the microcap quantile that shows some value patterns, whereas the rest of the portfolios indicates no patterns at all. I find these results abnormal since the US returns are also included for this market. Panel B shows 25 Size-OP portfolios, and the profitability effect seems to be present in most of the portfolios, particularly for low and high OP quantiles. The investment effect is also present in North America, whereas average excess returns for all portfolios in the lowest investment quantile are higher than in the highest investment quantile.

Asia Pacific (excluding Japan)

Panel A, B and C of Table 3 shows average monthly excess returns for Asia Pacific. Asia Pacific includes Australia, Hong Kong, New Zealand, and Singapore. For Asia Pacific (excluding Japan) the data is from July 1990 to February 2021, (368 months) and is collected from Bloomberg database and from Kenneth R. French`s website. For Asia Pacific, Panel A, B and C displays little to none patterns between average excess returns and size. Panel A has no patterns, whereas for Panel B the size effect is only present for the microcap and megacap portfolios. The same goes for Panel C, but only for the four lowest investment quantiles. Furthermore, there seems to be a value effect for almost all Size-B/M portfolios in Panel A. The relationship between average excess returns and profitability in Panel B is particularly present for the microcap and megacap, and somewhat weak but present for the rest of the size quantiles. Finally, for Panel C, the investment effect seems to be present when looking at the lowest investment quantile compared to the highest investment quantile. However, the returns in the middle investment quantiles seem to be higher, so the patterns between average excess returns and investment are unclear and weak for Asia Pacific.

Japan

Panel A of Table 3 shows average monthly excess returns for Japan. Data for Japan is from July 1990 to February 2021, (368 months) and is collected from Bloomberg database and from Kenneth R. French`s website. Panel A and B show size effect for the microcap and megacap portfolios but is weak between the other size quantiles. There seems to be highest size effect for 25 Size-Inv portfolios in Panel C, but the pattern between average excess returns and size are not very strong. The value patterns in Panel A seem to be prominent for the three biggest size quantiles, whereas for the two lowest size quantiles the patterns are weak. Panel B shows that the profitability effect is present for the lowest OP quantile and the highest OP quantile. Panel C shows no patterns between average excess returns and investment for Japan.

Europe

Panel A of Table 3 shows average monthly excess returns for Europe. Europe include Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Ireland, Italy, Netherlands, Norway, Portugal, and Sweden. Data for Europe is from July 1990 to February 2021, (368 months) and is collected from Bloomberg database and from Kenneth R. French`s website. Panel A and C indicate no patterns between average excess returns and size, whereas for Panel B there seems to be some size patterns for the three highest OP portfolios. Value patterns are present for the three lowest size quantiles, whereas for the two biggest size quantiles there are only value effect for the lowest and highest B/M quantile. Panel B shows strong profitability patterns when comparing the lowest and highest profitability quantiles. I also observe that the returns tend to increase from low to high investment quantiles, but not persistently. Investment patterns seem to be present for the lowest and highest investment quantiles, but for the middle investment quantiles there are none.

Emerging Markets

Table A8 (Appendix) shows average monthly excess returns for Emerging Markets. The Emerging Markets include Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Qatar, Russia, Saudi Arabia, South Africa, South Korea, Taiwan, Thailand, Turkey, and United Arab Emirates. Data for Emerging Markets is from July 1992 to February 2021 (344 months) and is collected from Bloomberg database and from Kenneth R. French`s website. Panel A of Table A8 shows average excess return on 6 portfolios on Size-B/M, Size-OP, and Size-Inv. The first is small (S) or big (B), the second is the B/M group, high (H), neutral (N), or low (L), the OP group, robust (R), neutral (N), or weak (W), or the Inv group, conservative (C), neutral (N), or aggressive (A). There seems to be some size effect for Size-OP and Size-Inv portfolios, but not very strong. Further, there seems to be a strong value effect where all returns increase from low to high B/M portfolios. The profitability effect also seems to be prominent for the Emerging Markets as returns typically increases with higher OP. Finally, it also seems to be an investment effect for Emerging Markets. Despite these results, I am careful to conclude significant patters and effects on only six portfolios. Nonetheless, it gives an indication for how the Emerging Market returns behave.

Table 4. Summary statistics for monthly factor percent returns, Standard deviations (STD), and t-statistics. The US shows data from July 1963–February 2021, 692 months. For North America, Asia Pacific (excluding Japan), Japan and Europe the data is from July 1990-February 2021, 368 months. For the Emerging Markets the data is from July 1992-February 2021, 344 months.

Panel A: Averages, standard deviations, and t-statistics for monthly returns						**Panel B: Correlation between factors**					
2x3 Factors											
The US											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,57	0,25	0,26	0,24	0,26	Rm-Rf	1,00				
STD	4,46	3,04	2,88	2,17	1,99	SMB	0,29	1,00			
t-statistic	3,36	2,13	2,42	2,95	3,42	HML	-0,21	-0,02	1,00		
						RMW	-0,20	-0,34	0,07	1,00	

						CMA	-0,37	-0,10	0,67	-0,03	1,00
North America											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,72	0,13	0,07	0,31	0,21	Rm-Rf	1,00				
STD	4,36	2,82	3,33	2,37	2,58	SMB	0,26	1,00			
t-statistic	3,17	0,90	0,43	2,50	1,59	HML	-0,19	-0,05	1,00		
						RMW	-0,34	-0,42	0,37	1,00	
						CMA	-0,39	-0,12	0,77	0,36	1,00
Asia Pacific											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,69	-0,13	0,51	0,31	0,29	Rm-Rf	1,00				
STD	5,85	2,87	3,00	2,67	2,39	SMB	-0,02	1,00			
t-statistic	2,27	-0,86	3,24	2,22	2,31	HML	0,10	0,06	1,00		
						RMW	-0,34	-0,19	-0,62	1,00	
						CMA	-0,47	-0,06	0,21	0,14	1,00
Japan											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,13	0,10	0,21	0,16	0,03	Rm-Rf	1,00				
STD	5,64	3,17	2,93	2,11	2,36	SMB	0,11	1,00			
t-statistic	0,44	0,58	1,40	1,46	0,22	HML	-0,19	0,06	1,00		
						RMW	-0,23	-0,16	-0,42	1,00	
						CMA	-0,01	0,20	0,55	-0,67	1,00
Europe											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,51	0,09	0,22	0,38	0,11	Rm-Rf	1,00				
STD	4,96	2,14	2,53	1,58	1,80	SMB	-0,11	1,00			
t-statistic	1,97	0,80	1,67	4,55	1,12	HML	0,23	0,01	1,00		
						RMW	-0,30	-0,01	-0,57	1,00	
						CMA	-0,26	-0,01	0,57	-0,21	1,00
Emerging Markets											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,68	0,08	0,57	0,18	0,22	Rm-Rf	1,00				
STD	6,14	2,11	2,29	1,62	1,94	SMB	-0,21	1,00			
t-statistic	2,06	0,72	4,64	2,05	2,14	HML	0,12	0,00	1,00		
						RMW	-0,29	-0,17	-0,53	1,00	
						CMA	-0,29	0,03	0,35	-0,15	1,00

Panel A in Table 4 shows averages, standard deviations, and t-statistics for monthly returns for all factors in the five-factor model. The factors are constructed using the portfolios of Table A8, which shows relatively large spreads between different markets across the world. The market premium is positive for all markets and is on average 0,55% per month, but only seems to be statistically significant for the US and North America. SMB has a wider spread than the market premium between the different markets. With an average on 0,086% per month and an average standard deviation of 2,7. Surprisingly, the size premium for Asia Pacific shows a negative return of -0,13% per month, which is quite different from the other markets. The t-statistics is also quite low for Asia Pacific (-0,86), which tells us that the size premium is not significantly different from zero. Only the US has a t-statistic above two, which indicates that the SMB is not significant for the most markets examined in Table 4. Furthermore, the HML factor provides an average of 0,30% per month and seems to be more stable than SMB across the different markets. The RMW factor looks to be the factor with the lowest spread between the markets and is on average 0,26% per month. Also, the CMA provides a low spread and is positive for all markets in Table 4 with an average of 0,186% per month.

Panel B of Table 4 represents the correlation between factors. In finance it is well known that small stocks tend to have higher market betas than big stocks, as they are considered to be riskier. So, not surprisingly, like Fama and French (2015) the size factor in the US is positively correlated with the excess market return. However, for Asia Pacific, Europe, and Emerging Markets, Panel B presents a negative correlation for these markets which I find very surprising, especially for Asia Pacific (excluding Japan) and Europe where there are strong and developed capital markets. Given the positive correlation between profitability and investment, it is perhaps surprising that the correlation between the profitability and investment factors is low (-0,67 to 0,14) for the US, Asia Pacific, Japan, Europe, and Emerging Markets. In North America the correlation between profitability and investment is 0,36. It is also known that high B/M value firms tend to be low investment firms, so HML should be positively correlated with CMA. There seems to be high correlation between HML and CMA factors for most markets, except for Asia Pacific (0,21) and Emerging Markets (0,35) where they are quite low but still positive.

4.2.2 Asset pricing tests

I turn now to the primary task of the thesis, testing how well the three sets of factors explain average excess returns on the portfolios of Table 3 (25 Size-B/M, OP, and Inv portfolios) and Table A8 (6 Size-B/M, OP, and Inv portfolios). To evaluate model performance, I present in Table 5 GRS test statistics, average absolute value of intercepts $A|\alpha_i|$ and adjusted R^2 values, which is the same approach as for the replication part of my thesis. I look at model performance for the original three- and five-factor model, and I include a four-factor model that combine RM-RF, SMB, and pairs of HML, RMW, and CMA. I consider in total 5 asset pricing models: (1) one three-factor model that combine RM−RF, SMB and HML; (2) three four-factor models that combine RM−RF, SMB, and pairs of HML, RMW, and CMA; and (3) the five-factor model that include all the five factors. Furthermore, I test how well the sets of factors explain average excess returns on the portfolios from Table 3 and A8. In each panel, every row represents a model and its factors. To test the model performance, I use the GRS test statistics. I repeat the key point in testing model performance: if an asset pricing model completely captures all expected returns, the intercept is zero in a regression of an asset´s excess returns on the model´s factor returns.

Table 5. The table tests the ability of three-, four-, and five-factor models to explain monthly excess returns on 25 Size-B/M portfolios (Panel A), 25 Size-OP portfolios (Panel B), 25 Size-Inv portfolios (Panel C). The US shows data from July 1963–February 2021, 692 months. For North America, Asia Pacific (excluding Japan), Japan and Europe the data is from July 1990-February 2021, 368 months. For the Emerging Markets the data is from July 1992-February 2021, 344 months. For each set of 25 or 6 regressions, the table shows the GRS test statistics, mean absolute alpha ($\boldsymbol{A|\alpha_i|}$), P-value, and Mean adjusted $\boldsymbol{R^2}$.

The US	GRS	$A\|\alpha_i\|$	P-value	R^2
Panel A: 25 Size-B/M Portfolios				
MKTRF SMB HML	3.784	.098	0.00	.914
MKTRF SMB HML RMW	3.341	.089	0.00	.917
MKTRF SMB HML CMA	3.619	.091	0.00	.915
MKTRF SMB RMW CMA	3.119	.097	0.00	.888
MKTRF SMB HML RMW CMA	3.081	.084	0.00	.918
Panel B: 25 Size-OP Portfolios				
MKTRF SMB HML	2.437	.097	0.00	.899
MKTRF SMB HML RMW	1.891	.060	.006	.919
MKTRF SMB HML CMA	2.883	.114	0.00	.900
MKTRF SMB RMW CMA	2.062	.068	.002	.914
MKTRF SMB HML RMW CMA	2.025	.060	.002	.919
Panel C: 25 Size-Inv Portfolios				

North America	GRS	$A\|\alpha_i\|$	P-value	R^2
Panel A: 25 Size-B/M Portfolios				
MKTRF SMB HML	3.466	.114	0.00	.931
MKTRF SMB HML RMW	3.063	.113	0.00	.934
MKTRF SMB HML CMA	3.306	.107	0.00	.931
MKTRF SMB RMW CMA	3.347	.165	0.00	.914
MKTRF SMB HML RMW CMA	3.028	.106	0.00	.935
Panel B: 25 Size-OP Portfolios				
MKTRF SMB HML	2.865	.167	0.00	.912
MKTRF SMB HML RMW	2.220	.087	.001	.933
MKTRF SMB HML CMA	2.777	.173	0.00	.913
MKTRF SMB RMW CMA	2.021	.097	.003	.927
MKTRF SMB HML RMW CMA	2.159	.083	.001	.934
Panel C: 25 Size-Inv Portfolios				

Asia Pacific	GRS	$A\|\alpha_i\|$	P-value	R^2
Panel A: 25 Size-B/M Portfolios				
MKTRF SMB HML	3.057	.202	0.00	.885
MKTRF SMB HML RMW	2.551	.189	0.00	.888
MKTRF SMB HML CMA	2.738	.178	0.00	.887
MKTRF SMB RMW CMA	3.612	.224	0.00	.876
MKTRF SMB HML RMW CMA	2.434	.178	0.00	.890
Panel B: 25 Size-OP Portfolios				
MKTRF SMB HML	4.724	.313	0.00	.882
MKTRF SMB HML RMW	2.945	.193	0.00	.893
MKTRF SMB HML CMA	4.309	.281	0.00	.884
MKTRF SMB RMW CMA	3.446	.189	0.00	.889
MKTRF SMB HML RMW CMA	2.835	.189	0.00	.894
Panel C: 25 Size-Inv Portfolios				

MKTRF SMB HML	4.572	.107	0.00	.919	MKTRF SMB HML	3.151	.150	0.00	.926	MKTRF SMB HML	4.041	.209	0.00	.875
MKTRF SMB HML RMW	4.189	.098	0.00	.922	MKTRF SMB HML RMW	3.130	.131	0.00	.929	MKTRF SMB HML RMW	4.306	.219	0.00	.878
MKTRF SMB HML CMA	4.053	.097	0.00	.927	MKTRF SMB HML CMA	2.627	.115	0.00	.935	MKTRF SMB HML CMA	3.768	.201	0.00	.887
MKTRF SMB RMW CMA	3.342	.080	0.00	.926	MKTRF SMB RMW CMA	2.296	.107	.001	.935	MKTRF SMB RMW CMA	4.809	.223	0.00	.886
MKTRF SMB HML RMW CMA	3.370	.083	0.00	.930	MKTRF SMB HML RMW CMA	2.599	.104	0.00	.938	MKTRF SMB HML RMW CMA	4.208	.227	0.00	.889

Japan					**Europe**					**Emerging Markets**				
	GRS	$A\|\alpha_i\|$	P-value	R^2		GRS	$A\|\alpha_i\|$	P-value	R^2		GRS	$A\|\alpha_i\|$	P-value	R^2
Panel A: 25 Size-B/M Portfolios					**Panel A: 25 Size-B/M Portfolios**					**Panel A: 6 Size-B/M Portfolios**				
MKTRF SMB HML	1.229	.100	.210	.927	MKTRF SMB HML	2.538	.092	0.00	.943	MKTRF SMB HML	27.482	.187	0.00	.970
MKTRF SMB HML RMW	1.239	.092	.201	.927	MKTRF SMB HML RMW	2.061	.082	.002	.945	MKTRF SMB HML RMW	22.833	.171	0.00	.970
MKTRF SMB HML CMA	1.219	.096	.219	.928	MKTRF SMB HML CMA	2.479	.084	0.00	.945	MKTRF SMB HML CMA	27.287	.193	0.00	.971
MKTRF SMB RMW CMA	1.405	.120	.097	.912	MKTRF SMB RMW CMA	2.361	.093	0.00	.934	MKTRF SMB RMW CMA	30.600	.309	0.00	.956
MKTRF SMB HML RMW CMA	1.230	.095	.209	.928	MKTRF SMB HML RMW CMA	2.044	.081	.003	.947	MKTRF SMB HML RMW CMA	22.443	.178	0.00	.971
Panel B: 25 Size-OP Portfolios					**Panel B: 25 Size-OP Portfolios**					**Panel B: 6 Size-OP Portfolios**				
MKTRF SMB HML	1.559	.117	.045	.925	MKTRF SMB HML	6.660	.181	0.00	.942	MKTRF SMB HML	31.861	.228	0.00	.974
MKTRF SMB HML RMW	1.254	.091	.190	.934	MKTRF SMB HML RMW	4.346	.103	0.00	.949	MKTRF SMB HML RMW	23.952	.184	0.00	.981
MKTRF SMB HML CMA	1.587	.107	.039	.929	MKTRF SMB HML CMA	6.577	.179	0.00	.943	MKTRF SMB HML CMA	32.344	.232	0.00	.974
MKTRF SMB RMW CMA	1.329	.090	.137	.933	MKTRF SMB RMW CMA	4.739	.114	0.00	.948	MKTRF SMB RMW CMA	29.905	.205	0.00	.981
MKTRF SMB HML RMW CMA	1.282	.094	.169	.934	MKTRF SMB HML RMW CMA	4.324	.103	0.00	.949	MKTRF SMB HML RMW CMA	24.212	.181	0.00	.981

Panel C: 25 Size-Inv Portfolios					**Panel C: 25 Size-Inv Portfolios**					**Panel C: 6 Size-Inv Portfolios**				
MKTRF SMB HML	.985	.097	.487	.926	MKTRF SMB HML	2.029	.090	.003	.943	MKTRF SMB HML	19.034	.197	0.00	.963
MKTRF SMB HML RMW	1.107	.084	.331	.930	MKTRF SMB HML RMW	1.072	.063	.373	.944	MKTRF SMB HML RMW	18.164	.206	0.00	.963
MKTRF SMB HML CMA	.977	.088	.497	.938	MKTRF SMB HML CMA	1.966	.089	.004	.950	MKTRF SMB HML CMA	18.698	.199	0.00	.975
MKTRF SMB RMW CMA	1.259	.088	.186	.936	MKTRF SMB RMW CMA	1.377	.077	.111	.949	MKTRF SMB RMW CMA	24.393	.233	0.00	.974
MKTRF SMB HML RMW CMA	1.103	.087	.336	.938	MKTRF SMB HML RMW CMA	1.050	.060	.401	.951	MKTRF SMB HML RMW CMA	17.599	.209	0.00	.975

Asset pricing models are simplified propositions about expected returns that are rejected in tests with power (Fama and French, 2015). In my tests, GRS typically reject the models with high confidence. I am, however, more interested in the relative performance of competing models, which one judge using GRS and other statistics. The lower the mean absolute alpha, the better the model.

Panel A, B, and C in Table 5 shows that Japanese average returns pose no problems for the models I consider. For the portfolios used, almost all models pass the GRS test, often with F-statistics close to 1,0. However, when comparing the different models, the five-factor model does not seem to be superior to the three-factor model, nor the four-factor models that exclude CMA or RMW. For Size-B/M portfolios, the three- and five-factor model report the same GRS statistics, whereas for Size-OP the five-factor model has a lower GRS statistic, and for Size-Inv the three-factor model has the lowest GRS statistics of the two models. My result for Japan is similar to Fama and French (2017), where they report a strong positive relation between B/M and average returns (Table 3 in Panel A), and average returns that are at best weakly related to profitability or investment. They concluded that, aside from a strong value effect, there is not enough variation in 1990–2015 average returns for Japan to challenge the three-factor model or any other model they consider.

Turning over to the US, I observe similar result as Fama and French (2015) and results from the replication part of my thesis. The five-factor model seems to outperform the three-factor model for all Panels in the US. Interestingly, the four-factor model that exclude the CMA factor in Panel B seems to give a lower GRS statistic than the rest of the models compared. All the GRS tests are easily rejected, but the model looks to capture most of the average expected returns. Also, as Fama and French (2015), the HML factor seems to be redundant for explaining average stock returns in the US. For the US in Panel A, B, and C, dropping the HML factor does not seem to affect the results. Furthermore, when comparing the US with North America, I report similar results, whereas the five-factor model report a lower GRS statistic than the three-factor model in all Panels in Table 5 for North America. The HML factor continues to look redundant, also in North America. In Panel B and C, the GRS statistics is even lower for the four-factor model which exclude the HML factor.

Moreover, looking towards Asia Pacific in Table 5, all the models get easily rejected by the GRS test statistics. For the Size-B/M (Panel A) and Size-OP (Panel B) portfolios, the five-factor model adds value to explaining average stock returns, whereas for Size-Inv portfolios in Panel C, the five-factor model reports a higher GRS statistic than the three-factor model. Despite the low value HML gives to the five-factor model in the US and North America, HML seems to be quite powerful in explaining average stock returns for Asia Pacific. This is in line with statistics from Table 3, that shows a strong pattern between average excess returns and B/M.

The five-factor model in Europe outperforms the three-factor model in all the Panels in Table 5. Interestingly, the three-factor model and the four-factor model which include the CMA factor reports similar statistics. In other words, the CMA factor seems to be redundant in explaining average stock returns in Europe in all the portfolios I examine. These results are in line with findings of Fama and French (2017).

Not surprisingly, all models in the Emerging Markets gets undoubtedly rejected by the GRS test. The mean absolute alpha is quite large in all the models reported in Panel A, B, and C of Table 5. Despite this, my findings show that the five-factor model outperforms the three-factor model for Emerging Markets in all Panels of Table 5. Also, for Emerging Markets as for Europe, the CMA factor seems to be redundant in explaining average stock returns. The three-factor model and the four-factor model which include CMA is approximately the same in all Panels. The same goes for the four-factor model that includes RMW and the five-factor model, which reports similar statistics. One should be careful in interpreting these results as true factor coefficients for this market. Not only are the data for Emerging Markets unreliable, but also hold few observations in their datasets to create enough variation. The GRS test is also run on 6 portfolios on Size-B/M, Size-OP, and Size-Inv compared to 25 for the rest of the markets, which is reflected in the high GRS test statistics.

4.2.3 Regression: 25 Size-B/M portfolios

Panel A in Table 6 (The US, and North America), 7 (Asia Pacific, and Japan), and 8 (Europe, and Emerging Markets) presents intercepts for the 25 Size-B/M portfolios for the three-factor model and Panel B presents intercepts and pertinent slopes (α, h, r, c) for the five-factor model. As Fama and French (2015), I do not include slopes for the market premium nor the size premium. The market premium is always close to 1,0 with a high t-statistic and the size premium are always strongly positive for small stocks and slightly negative for big stocks. This goes for all the markets I have examined. The market (RM-RF) and size (SMB) slopes are similar for different models, so they cannot account for changes in the intercepts observed when factors are added. From Fama and French (2015) and from the replication part of the thesis, I observed that the extreme growth stocks (left column of the intercept matrix) imposed a problem for the three-factor model. For the three-factor model in Panel A in Table 6, I observed that the extreme growth stocks for the US are still a problem when additional observations are added. Most of the portfolio intercepts are all close to zero except for the extreme growth stocks. Not surprisingly, as the US are included in North America, I observe similar problems for extreme growth stocks for this market. Where the US has on average a low intercept for the three-factor model, North America seems to have a slightly higher average across the board but is still able to capture most of the expected returns. Moreover, when moving over to the five-factor model, I can report that the five-factor model reduces these problems for both the US and North America, although the problem is still persistent in the model. This implies that the five-factor model has less cross-sectional pricing errors for the US and North America. Looking towards the five-factor slopes we can see that HML and RMW provides the highest negative slopes (-0,41% and -0,50%) in the troublesome area for the five-factor model. The CMA slope for the microcap stocks in the lowest B/M portfolio reports a coefficient of -0,12%. This value is far lower than what is reported in the replication part of the thesis and for Fama and French (2015). This is interesting because, even though the CMA slope is improved by 45 basispoints the pricing error is improved by 2 basispoints in the same portfolio.

Looking towards Japan and Asia Pacific, Table 7 shows that the three-factor model gives high intercepts for Asia Pacific, and a bit lower for Japan. For the three-factor model, Asia Pacific experience high intercepts for portfolios with extreme low and extreme high B/M. For Japan the biggest problem for the three-factor model is the microcap portfolios. Moreover, the five-factor model does not seem to reduce the intercept problem for Asia Pacific, which is in line with my results from the mean absolute alpha from the GRS test in Table 5. For Japan the additional factors seem to add value in explaining expected stock returns in the B/M portfolios, but not by much.

Europe in Table 8 shows 25 Size-B/M portfolios for Europe and 6 Size-B/M portfolios for Emerging Markets. There are similar results as for the US and North America, whereas for the three-factor model the intercept in the lowest B/M portfolios and the smallest size quantile reports a huge problem. The intercept has a negative value of -0,43% (t=-4,80) which means that the model has large cross-sectional pricing errors. Also, for the Emerging Markets the intercepts show huge pricing errors which means that the model performs poorly in explaining expected stock returns. The five-factor model reduces the problem for Europe in the low B/M and microcap portfolio. However, the five-factor model does not seem to improve average intercepts across the board. For the Emerging Markets almost all intercepts improve when applying the five-factor model.

Table 6. Regressions for 25 value-weight Size-B/M portfolios. The US shows data from July 1963–February 2021, 692 months. For North America the data is from July 1990-February 2021, 368 months. Panel A of the table shows three-factor intercepts (**α**) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (**α**), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

The US										
B/M ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,45	0,05	-0,02	0,16	0,16	-4,89	0,64	-0,31	3,13	2,20
2	-0,17	0,04	0,09	0,06	0,00	-2,83	0,79	1,72	1,20	-0,07
3	-0,08	0,08	0,03	0,08	0,06	-1,30	1,46	0,43	1,42	0,91
4	0,11	-0,04	-0,03	0,06	-0,07	1,84	-0,66	-0,38	0,94	-0,92
Big	0,16	0,00	0,02	-0,22	-0,19	3,77	0,08	0,32	-3,64	-1,91
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,27	0,17	0,00	0,16	0,11	-3,09	2,48	-0,07	3,11	1,51
2	-0,09	0,01	0,01	0,01	-0,02	-1,50	0,24	0,26	0,27	-0,44
3	0,02	0,04	-0,05	0,02	-0,01	0,43	0,62	-0,89	0,39	-0,20
4	0,17	-0,15	-0,12	0,03	-0,09	2,90	-2,34	-1,89	0,52	-1,06
Big	0,11	-0,09	-0,03	-0,23	-0,05	2,75	-1,60	-0,45	-3,74	-0,50
			h					**t(h)**		
Small	-0,41	-0,17	0,13	0,29	0,41	-10,48	-5,51	5,42	12,07	11,89
2	-0,45	-0,03	0,26	0,44	0,64	-16,05	-1,08	11,03	19,59	25,91

North America										
B/M ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,40	-0,16	0,16	0,06	0,26	-2,85	-1,53	1,80	0,90	3,82
2	-0,29	-0,15	0,02	-0,02	-0,05	-2,95	-1,99	0,29	-0,31	-1,02
3	0,03	-0,08	-0,02	-0,05	0,02	0,35	-0,96	-0,31	-0,68	0,27
4	0,19	-0,02	0,12	-0,03	-0,08	1,80	-0,24	1,51	-0,41	-1,14
Big	0,16	0,06	-0,05	-0,08	-0,31	2,76	0,92	-0,71	-1,21	-3,70
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,22	0,00	0,29	0,11	0,30	-1,63	0,00	3,38	1,67	4,25
2	-0,13	-0,07	0,00	-0,04	-0,10	-1,35	-0,96	0,03	-0,73	-1,89
3	0,16	-0,03	-0,07	-0,12	-0,04	1,64	-0,41	-1,04	-1,69	-0,58
4	0,34	0,03	0,05	-0,12	-0,11	3,36	0,37	0,64	-1,49	-1,56
Big	0,07	0,01	-0,03	-0,02	-0,18	1,13	0,14	-0,41	-0,29	-2,24
			h					**t(h)**		
Small	-0,69	-0,32	-0,16	0,10	0,39	-11,11	-6,55	-3,94	3,28	12,11
2	-0,62	-0,40	-0,04	0,26	0,49	-13,75	-11,35	-1,26	9,10	19,96

3	-0,40	0,04	0,31	0,51	0,69	-15,14	1,43	11,67	19,52	21,42	3	-0,52	-0,10	0,11	0,35	0,52	-11,28	-2,49	3,28	10,52	16,27
4	-0,41	0,08	0,34	0,53	0,78	-15,06	2,66	11,45	17,49	20,98	4	-0,62	-0,03	0,13	0,36	0,57	-13,00	-0,81	3,60	9,41	17,11
Big	-0,30	0,02	0,26	0,70	1,06	-16,28	0,97	8,49	24,70	23,56	**Big**	-0,47	-0,13	0,20	0,42	0,84	-17,48	-4,53	6,51	13,87	21,68
			r					**t(r)**						**r**					**t(r)**		
Small	-0,50	-0,40	-0,06	-0,04	0,00	-11,91	-12,04	-2,48	-1,78	0,08	**Small**	-0,54	-0,35	-0,34	-0,15	-0,11	-8,31	-6,77	-8,03	-4,44	-3,22
2	-0,17	0,07	0,22	0,10	0,05	-5,68	2,81	9,02	4,17	1,84	**2**	-0,24	-0,20	0,00	0,07	0,06	-4,99	-5,34	-0,03	2,33	2,20
3	-0,14	0,14	0,21	0,14	0,20	-4,89	4,80	7,29	5,20	5,83	**3**	-0,17	0,07	0,17	0,17	0,11	-3,44	1,73	4,90	4,96	3,21
4	-0,14	0,22	0,23	0,06	0,05	-4,73	6,98	7,24	2,00	1,29	**4**	-0,32	0,01	0,17	0,22	0,07	-6,35	0,24	4,38	5,50	1,96
Big	0,19	0,18	0,07	0,06	-0,16	9,47	6,84	2,14	1,85	-3,37	**Big**	0,21	0,05	-0,04	-0,08	-0,15	7,26	1,62	-1,19	-2,51	-3,69
			c					**t(c)**						**c**					**t(c)**		
Small	-0,12	0,02	0,04	0,06	0,24	-2,02	0,44	1,05	1,55	4,47	**Small**	0,14	-0,08	0,03	0,03	0,03	1,64	-1,22	0,55	0,61	0,69
2	-0,13	0,03	0,03	0,06	0,02	-3,07	0,71	0,82	1,85	0,61	**2**	-0,22	0,01	0,06	-0,01	0,09	-3,63	0,22	1,43	-0,19	2,63
3	-0,27	0,02	0,06	0,06	0,07	-6,82	0,58	1,41	1,52	1,45	**3**	-0,21	-0,27	-0,06	0,01	0,05	-3,44	-5,09	-1,39	0,16	1,16
4	-0,10	0,18	0,12	0,03	-0,01	-2,43	4,05	2,67	0,66	-0,21	**4**	-0,10	-0,19	-0,01	0,00	0,02	-1,51	-3,72	-0,10	-0,09	0,40
Big	-0,05	0,16	0,14	-0,03	-0,42	-1,92	4,31	3,01	-0,60	-6,19	**Big**	0,06	0,10	-0,01	-0,09	-0,21	1,53	2,44	-0,17	-2,22	-4,13

Table 7. Regressions for 25 value-weight Size-B/M portfolios. Asia Pacific (excluding Japan) and Japan shows data from July 1990-February 2021, 368 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + \varepsilon_{it}}$

Asia Pacific											Japan										
B/M ->	Low	2	3	4	High	Low	2	3	4	High	B/M ->	Low	2	3	4	High	Low	2	3	4	High

Panel A: Three-factor intercepts: RM-RF, SMB, and HML

	α					t(α)				
Small	0,11	-0,04	0,17	0,40	0,62	0,58	-0,29	1,55	3,81	6,10
2	-0,46	-0,25	-0,28	-0,09	-0,10	-3,47	-2,22	-2,62	-0,84	-1,01
3	-0,25	-0,16	0,07	-0,06	-0,25	-1,71	-1,21	0,57	-0,51	-2,01
4	0,21	0,21	-0,01	0,15	-0,24	1,52	1,58	-0,05	1,27	-1,85
Big	0,15	0,11	0,05	-0,18	-0,40	1,46	1,38	0,55	-1,71	-2,89

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

	α					t(α)				
Small	0,23	0,11	0,17	0,44	0,69	1,17	0,82	1,48	4,00	6,53
2	-0,26	-0,13	-0,27	-0,15	0,03	-1,94	-1,12	-2,39	-1,44	0,27
3	-0,06	-0,25	-0,13	-0,15	-0,16	-0,42	-1,78	-1,07	-1,16	-1,24
4	0,18	0,07	-0,23	-0,05	-0,13	1,20	0,47	-1,79	-0,45	-0,96
Big	0,24	0,01	-0,03	-0,17	-0,08	2,21	0,16	-0,26	-1,59	-0,59

	h					t(h)				
Small	-0,76	-0,52	-0,19	-0,11	0,25	-9,07	-8,73	-3,82	-2,28	5,52
2	-0,43	-0,32	-0,12	0,27	0,61	-7,44	-6,31	-2,41	5,81	13,28
3	-0,52	-0,20	-0,01	0,19	0,66	-8,04	-3,25	-0,26	3,30	11,73
4	-0,27	-0,17	-0,02	0,25	0,65	-4,19	-2,82	-0,31	4,90	11,27
Big	-0,47	-0,04	0,05	0,27	0,72	-10,22	-0,99	1,08	5,81	12,37

	r					t(r)				
Small	-0,42	-0,36	-0,08	-0,15	-0,18	-4,27	-5,12	-1,34	-2,69	-3,35
2	-0,26	-0,17	-0,01	0,17	-0,14	-3,80	-2,79	-0,22	3,23	-2,69

Panel A: Three-factor intercepts: RM-RF, SMB, and HML

	α					t(α)				
Small	0,07	0,14	0,19	0,14	0,20	0,44	1,27	1,79	1,92	2,87
2	0,09	-0,19	-0,09	0,05	-0,10	0,58	-2,27	-1,02	0,72	-1,89
3	-0,14	-0,13	-0,13	-0,14	-0,05	-1,17	-1,40	-1,58	-2,00	-0,85
4	-0,14	-0,04	-0,05	-0,05	-0,11	-1,19	-0,41	-0,52	-0,57	-1,34
Big	0,07	0,05	-0,02	0,03	0,10	0,95	0,58	-0,24	0,34	0,71

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

	α					t(α)				
Small	0,06	0,16	0,17	0,15	0,19	0,41	1,37	1,62	2,04	2,69
2	0,04	-0,17	-0,09	0,07	-0,07	0,28	-2,02	-1,04	1,01	-1,40
3	-0,17	-0,11	-0,12	-0,13	-0,03	-1,32	-1,16	-1,46	-1,84	-0,50
4	-0,15	-0,01	-0,03	-0,05	-0,10	-1,30	-0,15	-0,36	-0,56	-1,29
Big	0,09	0,06	-0,02	0,03	0,10	1,15	0,68	-0,23	0,29	0,77

	h					t(h)				
Small	-0,32	-0,18	0,00	0,07	0,30	-4,93	-3,72	-0,03	2,27	10,18
2	-0,53	-0,22	0,05	0,13	0,41	-8,40	-6,20	1,35	4,71	19,50
3	-0,47	-0,23	0,07	0,24	0,48	-9,00	-5,79	1,96	7,95	18,64
4	-0,45	-0,18	0,07	0,32	0,55	-9,42	-4,73	1,94	9,16	16,16
Big	-0,61	-0,11	0,21	0,42	0,79	-18,42	-2,99	5,96	11,28	13,90

	r					t(r)				
Small	0,00	-0,03	0,04	-0,06	0,07	-0,04	-0,33	0,63	-1,26	1,55
2	0,19	-0,04	0,04	-0,05	-0,09	1,91	-0,77	0,69	-1,16	-2,52

3	-0,15	0,16	0,27	0,17	-0,03	-1,95	2,30	4,44	2,52	-0,46	**3**	0,08	-0,10	0,01	0,00	-0,06	1,00	-1,60	0,11	-0,08	-1,51
4	0,11	0,15	0,31	0,32	-0,12	1,43	2,13	4,93	5,24	-1,87	**4**	0,00	-0,07	-0,03	0,06	0,04	0,00	-1,08	-0,42	1,00	0,68
Big	-0,06	0,18	0,07	-0,13	-0,42	-1,05	4,12	1,33	-2,32	-6,20	**Big**	-0,07	0,01	0,01	0,01	-0,09	-1,23	0,12	0,26	0,22	-0,98
			c					**t(c)**						**c**					**t(c)**		
Small	0,48	0,23	0,16	0,19	0,14	5,15	3,55	2,90	3,70	2,81	**Small**	-0,05	0,07	-0,06	-0,07	0,09	-0,51	0,99	-0,94	-1,47	2,02
2	-0,09	-0,04	0,00	-0,13	-0,13	-1,45	-0,74	-0,06	-2,66	-2,66	**2**	0,06	0,12	0,10	0,07	0,06	0,65	2,18	1,76	1,74	1,82
3	-0,30	-0,04	0,08	-0,06	-0,23	-4,27	-0,67	1,45	-0,94	-3,74	**3**	0,00	-0,06	0,12	0,13	0,07	0,04	-1,00	2,37	2,97	1,85
4	-0,10	0,16	0,07	0,01	-0,08	-1,44	2,51	1,12	0,22	-1,33	**4**	-0,15	0,08	0,09	0,18	0,17	-2,06	1,33	1,67	3,49	3,33
Big	-0,16	-0,04	0,12	0,23	-0,15	-3,17	-0,94	2,48	4,55	-2,30	**Big**	0,00	0,12	0,05	-0,01	-0,17	0,01	2,33	0,93	-0,18	-1,99

Table 8. Regressions for 25 value-weight Size-B/M portfolios. For Europe the data is from July 1990-February 2021, 368 months. And for the Emerging Markets the data is from July 1992-February 2021, 344 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

					Europe									**Emerging Markets**			
B/M ->	Low	2	3	4	High	Low	2	3	4	High	B/M ->	Growth	Neutral	Value	Growth	Neutral	Value
Panel A: Three-factor intercepts: RM-RF, SMB, and HML											**Panel A: Three-factor intercepts: RM-RF, SMB, and HML**						
			α					**t(α)**					**α**			**t(α)**	
Small	-0,43	-0,13	-0,07	0,02	0,13	-4,80	-1,75	-1,09	0,39	2,52	**Small**	-0,1	0,2	0,3	-1,4	2,3	6,4
2	-0,18	-0,01	-0,05	0,04	0,06	-2,44	-0,15	-1,01	0,72	1,07	**Big**	0,4	0,2	0,0	7,3	2,7	-0,3
3	-0,04	0,09	-0,05	-0,07	-0,01	-0,50	1,33	-0,75	-1,11	-0,20							
4	0,11	0,05	-0,02	-0,09	-0,10	1,41	0,72	-0,34	-1,28	-1,39							
Big	0,10	0,14	0,04	0,05	-0,23	1,55	2,19	0,64	0,70	-2,66							

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,19	0,00	0,05	0,01	0,12	-2,21	-0,06	0,78	0,27	2,21
2	-0,04	0,06	-0,08	-0,02	0,04	-0,52	0,91	-1,50	-0,36	0,70
3	0,13	0,09	-0,15	-0,14	-0,02	1,64	1,28	-2,05	-2,08	-0,35
4	0,24	0,04	-0,12	-0,13	-0,12	3,00	0,52	-1,70	-1,69	-1,47
Big	0,11	-0,01	0,02	0,07	0,00	1,58	-0,24	0,35	0,97	-0,01
			h					**t(h)**		
Small	-0,54	-0,30	-0,17	0,04	0,26	-10,74	-7,17	-4,54	1,15	8,57
2	-0,54	-0,25	0,04	0,31	0,44	-12,84	-6,61	1,24	9,95	13,94
3	-0,57	-0,20	0,18	0,33	0,50	-12,17	-4,88	4,38	8,37	12,54
4	-0,47	-0,15	0,11	0,37	0,67	-10,12	-3,23	2,93	8,34	14,39
Big	-0,61	-0,26	-0,02	0,32	0,83	-15,82	-7,70	-0,67	7,63	17,88
			r					**t(r)**		
Small	-0,44	-0,23	-0,23	-0,01	-0,01	-6,95	-4,18	-4,90	-0,17	-0,25
2	-0,24	-0,13	0,05	0,11	0,00	-4,54	-2,76	1,32	2,78	0,11
3	-0,30	-0,01	0,19	0,15	0,01	-5,00	-0,17	3,68	2,88	0,10
4	-0,22	0,00	0,18	0,08	0,04	-3,69	0,04	3,53	1,39	0,63
Big	-0,02	0,26	0,01	-0,04	-0,40	-0,50	5,98	0,29	-0,71	-6,78
			c					**t(c)**		
Small	-0,28	-0,15	-0,10	0,10	0,19	-4,39	-2,88	-2,16	2,54	5,13

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA						
		α			**t(α)**	
Small	-0,06	0,18	0,32	-0,82	2,57	6,32
Big	0,37	0,13	-0,01	6,64	2,15	-0,14
		h			**t(h)**	
Small	-0,62	-0,02	0,42	-17,05	-0,6	16,54
Big	-0,4	0,13	0,56	-14,39	4,31	15,74
		r			**t(r)**	
Small	-0,13	-0,03	-0,04	-2,42	-0,6	-1,07
Big	0,05	0,05	-0,04	1,21	1,19	-0,76
		c			**t(c)**	
Small	0,17	-0,11	0,06	4,23	-2,99	2,26

2	-0,29	-0,07	0,06	0,05	0,19	-5,41	-1,40	1,52	1,26	4,77	Big	-0,04	0,03	0,06	-1,47	0,8	1,57
3	-0,40	0,02	0,04	0,02	0,11	-6,64	0,33	0,78	0,37	2,09							
4	-0,31	0,13	0,11	0,01	-0,04	-5,27	2,28	2,23	0,12	-0,70							
Big	0,04	0,32	0,11	-0,06	-0,47	0,83	7,56	2,44	-1,14	-7,99							

4.2.4 Regression: 25 Size-OP portfolios

Panel A in Table 9 (The US, and North America), 10 (Asia Pacific, and Japan), and 11 (Europe, and Emerging Markets) presents intercepts for the 25 Size-OP portfolios for the three-factor model and Panel B presents intercepts and pertinent slopes ($\boldsymbol{\alpha}$, h, r, c) for the five-factor model. Also, as the Size-B/M portfolios, the Size-OP portfolios report the same problem for the three-factor model in the same troublesome extreme growth stock portfolio for the US (Panel A in Table 9), whereas the five-factor model (Panel B) reduces the problem with 16 basispoints. For North America the three-factor model performs poorly with large intercepts in most of the portfolios. The five-factor model improves some of the largest intercepts but leaves behind relative high intercepts for the microcap portfolios in North America for the Size-OP portfolios.

Table 10 shows large intercepts for Asia Pacific both for the three- and five-factor model. The three-factor model reports highest intercepts in microcap portfolios and for extreme OP portfolios. The five-factor model improves over the three-factor model, however, most of the large intercepts survives in the five-factor model. Japan does not report any troublesome areas and shows that the five-factor model gives improved intercepts.

Table 11 shows 25 Size-OP portfolios for Europe and 6 Size-OP portfolios for Emerging Markets. The three-factor model performs poorly in Europe and reports high intercepts in the lowest OP quantile and for microcap portfolios. The five-factor model improves the intercepts in most of the portfolios and is the superior model for the Size-OP portfolios. The five-factor model in Emerging Markets shows lower intercepts than the three-factor model on average, but there are still large pricing errors across the board.

Table 9. Regressions for 25 value-weight Size-OP portfolios. The US shows data from July 1963–February 2021, 692 months. For North America the data is from July 1990-February 2021, 368 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

The US

OP ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,25	0,11	0,05	0,14	-0,09	-2,98	1,92	0,80	0,96	-1,13
2	-0,18	-0,04	0,05	0,02	0,09	-2,43	-0,77	1,02	0,30	1,25
3	-0,14	0,00	0,05	0,03	0,15	-1,54	-0,01	0,93	0,50	1,96
4	-0,09	-0,02	0,04	0,05	0,15	-0,99	-0,24	0,69	0,86	2,28
Big	-0,28	-0,15	-0,01	0,06	0,18	-3,11	-2,54	-0,17	1,45	3,71
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,09	0,08	-0,05	-0,02	-0,20	-1,19	1,40	-0,82	-0,16	-2,83
2	0,00	-0,09	-0,03	-0,09	-0,06	-0,04	-1,80	-0,68	-1,63	-0,98
3	0,13	-0,03	0,00	-0,09	0,00	1,85	-0,42	0,02	-1,60	-0,03
4	0,15	0,04	-0,03	-0,09	0,04	1,98	0,62	-0,44	-1,53	0,60
Big	0,01	-0,08	0,02	0,02	0,07	0,18	-1,37	0,39	0,49	1,63
			h					**t(h)**		
Small	-0,13	0,28	0,30	0,18	0,18	-3,91	10,13	11,64	2,65	5,47
2	-0,15	0,19	0,24	0,25	0,19	-5,70	7,90	10,79	10,09	6,87

North America

OP ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,03	0,28	0,18	0,19	0,13	-0,28	4,08	2,47	2,14	1,24
2	-0,39	0,02	0,10	0,24	0,19	-4,61	0,32	1,52	2,88	1,78
3	-0,25	-0,02	0,04	0,09	0,12	-2,34	-0,26	0,47	1,01	1,23
4	-0,26	0,05	0,13	0,07	0,22	-2,08	0,76	1,69	0,88	2,34
Big	-0,50	-0,22	-0,11	0,17	0,19	-4,47	-2,74	-1,79	2,98	2,67
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	0,19	0,27	0,10	0,11	0,04	2,11	3,80	1,44	1,33	0,44
2	-0,13	-0,03	0,01	0,10	-0,04	-2,46	-0,45	0,15	1,36	-0,57
3	0,07	-0,03	-0,09	-0,08	-0,06	0,89	-0,36	-1,16	-1,13	-0,69
4	0,04	0,08	0,05	-0,09	0,09	0,40	1,24	0,59	-1,30	0,99
Big	-0,10	-0,04	-0,06	0,16	-0,02	-1,22	-0,53	-0,93	2,65	-0,27
			h					**t(h)**		
Small	-0,18	0,34	0,28	0,35	0,28	-4,25	10,31	8,25	8,97	5,96
2	-0,25	0,22	0,16	0,25	0,26	-10,08	7,44	4,98	7,68	7,07

3	-0,09	0,20	0,22	0,22	0,12	-2,85	7,09	8,93	8,56	3,99	**3**	-0,15	0,15	0,25	0,25	0,20	-3,94	4,48	7,13	7,39	5,21
4	-0,02	0,15	0,21	0,17	-0,02	-0,58	5,45	8,16	6,53	-0,67	**4**	-0,17	0,17	0,16	0,19	0,04	-3,47	5,46	4,41	5,52	1,05
Big	0,23	0,18	0,13	-0,01	-0,15	7,36	7,15	5,31	-0,59	-7,85	**Big**	0,25	0,21	0,18	-0,03	-0,29	6,60	6,15	6,03	-0,99	-10,96
			r					**t(r)**						**r**					**t(r)**		
Small	-0,60	0,11	0,28	0,35	0,39	-17,24	3,97	10,10	4,73	11,38	**Small**	-0,61	0,05	0,22	0,30	0,35	-13,83	1,56	6,22	7,46	7,13
2	-0,59	0,12	0,24	0,38	0,56	-20,82	4,74	10,16	14,87	18,77	**2**	-0,66	0,09	0,21	0,44	0,69	-25,06	2,78	6,31	12,64	17,93
3	-0,76	0,04	0,15	0,38	0,55	-22,67	1,39	5,88	13,80	17,80	**3**	-0,68	0,02	0,35	0,53	0,56	-16,83	0,65	9,66	14,59	14,17
4	-0,74	-0,22	0,17	0,38	0,35	-19,91	-7,28	5,98	13,84	11,37	**4**	-0,70	-0,06	0,21	0,38	0,38	-13,86	-1,70	5,55	10,76	8,55
Big	-0,76	-0,27	-0,11	0,13	0,35	-22,61	-10,38	-4,20	6,97	17,79	**Big**	-0,75	-0,36	-0,08	0,06	0,41	-18,96	-10,16	-2,41	2,13	14,78
			c					**t(c)**						**c**					**t(c)**		
Small	0,11	-0,03	0,03	0,27	-0,04	2,19	-0,83	0,69	2,60	-0,88	**Small**	0,10	-0,04	-0,05	-0,17	-0,20	1,77	-1,00	-1,02	-3,27	-3,25
2	0,06	0,08	0,04	-0,08	-0,11	1,34	2,17	1,25	-2,18	-2,63	**2**	0,02	0,05	0,03	-0,13	-0,18	0,70	1,20	0,76	-3,00	-3,60
3	-0,11	0,06	0,00	0,01	-0,11	-2,33	1,35	0,05	0,22	-2,50	**3**	-0,17	0,00	-0,07	-0,14	-0,19	-3,21	-0,09	-1,47	-3,15	-3,70
4	-0,05	0,07	0,06	0,09	0,02	-0,87	1,76	1,40	2,36	0,46	**4**	-0,04	-0,04	0,00	0,03	-0,07	-0,60	-0,93	-0,03	0,71	-1,26
Big	-0,23	0,08	0,02	-0,02	0,00	-4,71	2,03	0,61	-0,71	0,11	**Big**	-0,36	-0,11	-0,08	-0,04	0,12	-7,18	-2,52	-1,89	-1,04	3,45

Table 10. Regressions for 25 value-weight Size-OP portfolios. Asia Pacific (excluding Japan) and Japan shows data from July 1990-February 2021, 368 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

			Asia Pacific											**Japan**							
OP ->	Low	2	3	4	High	Low	2	3	4	High	OP ->	Low	2	3	4	High	Low	2	3	4	High

Panel A: Three-factor intercepts: RM-RF, SMB, and HML

	α					t(α)				
Small	0,19	0,67	0,43	0,50	0,59	1,46	5,69	4,17	4,36	5,33
2	-0,55	-0,36	0,04	0,07	0,27	-4,28	-3,20	0,38	0,61	2,51
3	-0,62	-0,18	-0,12	0,27	0,30	-4,92	-1,28	-1,13	2,23	2,42
4	-0,50	0,07	0,09	0,20	0,42	-3,36	0,55	0,58	1,66	3,20
Big	-0,55	-0,24	0,04	0,28	0,27	-3,88	-2,11	0,39	2,77	2,69

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

	α					t(α)				
Small	0,45	0,70	0,35	0,33	0,42	3,62	5,66	3,24	2,82	3,68
2	-0,16	-0,27	-0,09	-0,05	0,12	-1,35	-2,29	-0,85	-0,45	1,12
3	-0,32	-0,18	-0,20	-0,01	0,09	-2,55	-1,25	-1,74	-0,09	0,73
4	-0,24	0,04	-0,08	-0,11	0,21	-1,58	0,25	-0,50	-0,93	1,59
Big	0,07	0,12	-0,01	-0,05	0,05	0,68	1,16	-0,07	-0,55	0,48

	h					t(h)				
Small	-0,45	0,07	0,32	0,36	0,11	-8,39	1,41	6,89	7,12	2,21
2	0,00	0,45	0,26	0,28	-0,11	0,05	8,89	5,76	5,25	-2,39
3	-0,04	0,44	0,12	0,07	0,03	-0,71	7,06	2,40	1,36	0,52
4	0,15	0,13	0,08	0,18	-0,08	2,26	2,15	1,23	3,59	-1,39
Big	0,12	0,17	0,15	-0,02	-0,29	2,56	3,83	3,35	-0,60	-6,88

	r					t(r)				
Small	-0,56	-0,10	0,08	0,25	0,25	-9,10	-1,58	1,48	4,27	4,47
2	-0,57	-0,06	0,25	0,24	0,27	-9,58	-0,95	4,74	3,92	4,79

Panel A: Three-factor intercepts: RM-RF, SMB, and HML

	α					t(α)				
Small	0,01	0,19	0,15	0,14	0,42	0,14	2,91	1,99	1,69	3,87
2	-0,23	-0,07	0,02	0,02	0,07	-2,84	-1,05	0,23	0,25	0,76
3	-0,22	-0,14	-0,08	-0,04	0,01	-2,42	-1,70	-0,96	-0,43	0,13
4	-0,29	-0,11	0,05	0,07	-0,05	-2,88	-1,11	0,62	0,78	-0,46
Big	-0,18	-0,05	0,13	0,05	0,14	-1,20	-0,48	1,71	0,56	1,51

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

	α					t(α)				
Small	0,08	0,20	0,13	0,12	0,34	1,11	3,07	1,68	1,41	3,23
2	-0,11	-0,03	0,02	-0,05	-0,01	-1,76	-0,55	0,21	-0,58	-0,13
3	-0,11	-0,11	-0,08	-0,09	-0,06	-1,39	-1,35	-1,04	-0,99	-0,51
4	-0,18	-0,09	0,05	0,04	-0,10	-1,95	-0,94	0,57	0,43	-1,06
Big	0,09	0,07	0,12	-0,04	-0,01	0,74	0,74	1,59	-0,55	-0,10

	h					t(h)				
Small	0,06	0,15	0,17	0,09	0,10	2,01	5,63	5,37	2,64	2,24
2	0,09	0,17	0,09	0,11	0,06	3,13	6,38	2,12	3,08	1,42
3	0,10	0,12	0,15	0,15	0,01	2,94	3,64	4,62	3,84	0,20
4	0,04	0,13	0,09	0,15	0,00	0,93	3,34	2,43	3,87	-0,05
Big	-0,13	-0,03	0,00	0,08	-0,06	-2,55	-0,77	-0,09	2,49	-1,82

	r					t(r)				
Small	-0,24	-0,04	0,06	0,05	0,28	-4,86	-0,93	1,28	0,83	4,01
2	-0,39	-0,11	0,03	0,28	0,35	-8,93	-2,57	0,50	4,78	5,61

3	-0,41	0,06	0,14	0,39	0,45	-6,65	0,83	2,51	6,53	7,46	**3**	-0,37	-0,06	0,03	0,18	0,22	-7,01	-1,04	0,59	2,82	3,04
4	-0,34	0,07	0,23	0,39	0,33	-4,51	0,99	3,02	6,76	4,96	**4**	-0,41	0,00	0,07	0,10	0,24	-6,60	0,07	1,18	1,68	3,64
Big	-0,86	-0,54	0,09	0,47	0,34	-15,64	-10,23	1,70	10,16	6,76	**Big**	-1,08	-0,48	0,01	0,33	0,62	-13,72	-7,53	0,23	6,30	12,16
			c					**t(c)**						**c**					**t(c)**		
Small	0,33	0,10	0,09	0,01	0,02	5,66	1,69	1,82	0,25	0,46	**Small**	0,11	0,02	-0,08	-0,14	-0,12	2,45	0,62	-1,65	-2,68	-1,77
2	-0,06	-0,18	-0,10	-0,09	-0,07	-1,05	-3,29	-2,00	-1,48	-1,34	**2**	0,18	0,05	0,08	-0,02	0,04	4,46	1,18	1,25	-0,36	0,74
3	-0,13	-0,11	-0,05	0,10	-0,25	-2,19	-1,55	-0,95	1,75	-4,30	**3**	0,14	0,22	0,01	-0,08	-0,16	2,91	4,34	0,15	-1,39	-2,37
4	-0,13	-0,02	0,05	0,18	-0,02	-1,83	-0,30	0,62	3,28	-0,32	**4**	0,02	0,23	0,16	-0,06	0,03	0,31	3,94	2,86	-1,06	0,56
Big	-0,23	-0,06	-0,03	0,10	0,02	-4,47	-1,27	-0,65	2,32	0,48	**Big**	-0,13	-0,03	-0,06	-0,07	0,18	-1,74	-0,51	-1,25	-1,36	3,69

Table 11. Regressions for 25 value-weight Size-OP portfolios. For Europe the data is from July 1990-February 2021, 368 months. And for the Emerging Markets the data is from July 1992-February 2021, 344 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}$ and the five-factor regression equation is: $R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}$

Europe

OP ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,38	0,11	0,19	0,34	0,21	-5,59	1,98	3,75	6,43	3,18
2	-0,36	0,00	0,01	0,18	0,38	-5,57	-0,03	0,11	2,98	6,03
3	-0,34	0,01	0,14	0,05	0,23	-4,75	0,17	2,13	0,66	3,21
4	-0,34	-0,04	0,03	0,18	0,18	-3,79	-0,60	0,42	2,54	2,54
Big	-0,47	-0,04	0,05	0,02	0,24	-4,30	-0,65	0,85	0,34	3,87

Emerging Markets

OP ->	Growth	Neutral	Value	Growth	Neutral	Value
Panel A: Three-factor intercepts: RM-RF, SMB, and HML						
		α			**t(α)**	
Small	-0,01	0,38	0,29	-0,21	8,32	4,17
Big	-0,12	0,1	0,47	-1,82	2,07	8,7

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,15	0,13	0,15	0,28	0,13	-2,32	2,29	2,82	5,11	1,85
2	-0,15	0,04	-0,05	0,03	0,27	-2,38	0,67	-0,83	0,57	4,09
3	-0,12	0,02	0,04	-0,06	0,09	-1,80	0,34	0,60	-0,77	1,29
4	-0,07	-0,03	-0,07	0,10	0,09	-0,81	-0,45	-0,91	1,26	1,22
Big	0,07	0,16	0,09	-0,18	0,01	0,81	2,56	1,31	-3,02	0,09
			h					**t(h)**		
Small	-0,21	0,06	0,06	0,04	0,02	-5,81	1,92	1,91	1,41	0,40
2	-0,06	0,11	0,14	0,14	0,04	-1,77	3,22	4,05	4,05	1,02
3	-0,09	0,16	0,16	0,12	0,10	-2,16	4,38	4,14	2,86	2,48
4	-0,01	0,24	0,24	0,06	-0,04	-0,18	5,49	5,53	1,40	-0,93
Big	0,15	0,13	-0,14	0,03	-0,15	3,09	3,59	-3,67	0,73	-4,70
			r					**t(r)**		
Small	-0,47	-0,07	0,07	0,11	0,18	-10,11	-1,73	1,76	2,63	3,61
2	-0,43	-0,10	0,09	0,29	0,26	-9,61	-2,39	2,04	6,69	5,55
3	-0,45	-0,03	0,22	0,22	0,31	-8,85	-0,71	4,54	4,13	5,87
4	-0,58	-0,02	0,22	0,18	0,21	-9,05	-0,40	3,92	3,25	3,91
Big	-1,08	-0,42	-0,10	0,43	0,50	-17,54	-9,26	-2,08	9,73	11,91
			c					**t(c)**		
Small	-0,07	0,13	0,06	0,03	-0,06	-1,53	3,06	1,65	0,77	-1,24

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA						
		α			**t(α)**	
Small	0,19	0,33	0,01	4,58	6,94	0,17
Big	0,12	0,13	0,30	2,23	2,55	6,12
		h			**t(h)**	
Small	0,05	0,10	0,10	2,62	4,05	3,41
Big	-0,03	0,11	-0,08	-1,20	4,00	-3,10
		r			**t(r)**	
Small	-0,46	0,12	0,59	-15,70	3,62	14,62
Big	-0,57	-0,05	0,38	-14,30	-1,41	10,84
		c			**t(c)**	
Small	0,05	-0,03	0,12	2,30	-1,23	4,01

2	-0,12	0,09	0,16	0,03	-0,11	-2,81	2,12	3,68	0,67	-2,28	Big	0,06	-0,06	-0,01	1,97	-2,15	-0,52
3	0,00	0,05	-0,10	-0,04	-0,15	-0,10	1,16	-2,16	-0,70	-2,87							
4	0,05	0,03	-0,05	0,01	-0,13	0,82	0,47	-0,91	0,25	-2,47							
Big	-0,30	0,00	0,18	-0,01	0,02	-4,89	-0,06	3,74	-0,25	0,40							

4.2.5 Regression: 25 Size-Inv portfolios

Also, for Size-Inv portfolios, the US in Panel B in Table 12, shows a high intercept in the lowest Inv quantile for the microcap portfolio (0,18% (t=2,08)) for the five-factor model. However, as for the other Size, B/M and OP portfolios the five-factor model performs well. North America (Table 12) also reports lower intercepts for the five-factor model but seems to struggle with the microcap portfolios as they are high for both the three-factor and the five-factor model.

For Asia Pacific, the five-factor model has performed well, and the intercepts has improved when adding the RMW and CMA factors. Interestingly, Table 13 shows that the five-factor model performs worse than the three-factor model. Almost all intercepts worsens when the five-factor model is applied. This is in line with my results from the GRS test statistics in Table 5, whereas the four-factor model that excluded HML gave the lowest GRS test statistics. For the 25 Size-Inv portfolios it is difficult to see whether the five-factor model adds value in explaining expected stock returns for Japan. In other words, the three- and five-factor model seem to give similar results for Japan in Size-Inv portfolios.

Table 14 shows 25 Size-Inv portfolios for Europe and 6 Size-Inv portfolios for Emerging Markets. Unlike the intercepts in Size-OP portfolios in Table 11, the five-factor model performs well for Europe in Size-Inv portfolios of Table 14. This is reflected by very low intercepts close to zero, which also goes for the three-factor model. Emerging Markets continues the pattern by reporting high intercepts, but again, the five-factor model shows lower intercepts and is thereby a better model explaining expected stock returns for the Size-Inv portfolios.

Table 12. Regressions for 25 value-weight Size-Inv portfolios. The US shows data from July 1963–February 2021, 692 months. For North America the data is from July 1990-February 2021, 368 months. Panel A of the table shows three-factor intercepts (**α**) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (**α**), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

The US

Inv ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	0,10	0,15	0,16	0,07	-0,44	1,04	2,94	2,82	1,21	-6,73
2	0,00	0,10	0,15	0,08	-0,26	0,00	1,75	2,92	1,53	-4,94
3	0,09	0,18	0,08	0,08	-0,16	1,23	3,15	1,46	1,36	-2,55
4	0,01	0,04	0,07	0,12	-0,03	0,10	0,61	1,40	2,05	-0,49
Big	0,14	0,07	0,00	0,07	0,05	1,84	1,31	0,04	1,45	0,73
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	0,18	0,12	0,12	0,04	-0,31	2,08	2,33	2,13	0,78	-4,92
2	-0,03	0,00	0,08	0,03	-0,13	-0,50	0,05	1,65	0,67	-2,68
3	0,04	0,10	-0,01	0,06	-0,02	0,56	1,78	-0,28	0,99	-0,38
4	-0,10	-0,06	0,01	0,08	0,13	-1,51	-0,96	0,26	1,39	2,00
Big	-0,04	-0,05	-0,06	0,04	0,19	-0,68	-1,04	-1,52	0,91	3,50
			h					**t(h)**		
Small	-0,12	0,20	0,23	0,16	-0,02	-3,08	8,53	8,92	5,97	-0,81
2	0,05	0,27	0,19	0,28	-0,12	2,18	10,71	7,97	12,55	-5,46

North America

Inv ->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	0,31	0,31	0,24	0,14	-0,39	2,77	4,25	3,47	2,09	-3,89
2	-0,03	0,19	0,09	0,02	-0,49	-0,39	2,58	1,36	0,40	-5,70
3	0,01	0,09	0,12	-0,05	-0,27	0,17	1,29	1,66	-0,65	-2,60
4	0,09	0,20	0,12	0,11	-0,21	1,22	2,79	1,69	1,42	-1,83
Big	0,10	0,01	0,00	-0,01	-0,14	1,48	0,16	0,08	-0,08	-1,34
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	0,36	0,27	0,23	0,22	-0,14	3,58	3,74	3,28	3,10	-1,54
2	-0,08	0,11	0,02	0,02	-0,21	-1,44	1,48	0,33	0,40	-3,14
3	-0,09	-0,06	0,04	-0,01	0,05	-1,20	-0,92	0,60	-0,08	0,60
4	0,00	0,06	0,04	0,12	0,07	-0,06	0,89	0,61	1,53	0,64
Big	0,00	-0,13	-0,03	0,09	0,11	-0,03	-2,42	-0,56	1,30	1,40
			h					**t(h)**		
Small	-0,13	0,16	0,14	0,24	0,00	-2,69	4,58	4,26	7,21	0,08
2	0,01	0,09	0,16	0,12	-0,11	0,34	2,73	4,93	4,04	-3,44

3	0,12	0,22	0,22	0,20	-0,08	3,75	8,45	9,31	7,63	-3,00	**3**	0,17	0,20	0,21	0,22	-0,17	4,61	6,17	6,22	6,57	-4,07
4	0,12	0,28	0,20	0,10	-0,16	3,80	10,40	8,24	3,54	-5,44	**4**	0,12	0,21	0,20	0,05	-0,19	3,36	6,56	5,93	1,24	-3,74
Big	-0,13	-0,01	0,12	0,00	-0,07	-4,39	-0,33	6,28	-0,15	-2,76	**Big**	-0,09	0,04	0,05	0,00	-0,03	-3,33	1,45	1,90	-0,02	-0,79
			r					**t(r)**						**r**					**t(r)**		
Small	-0,53	-0,01	0,08	0,06	-0,21	-12,43	-0,48	2,77	2,23	-6,93	**Small**	-0,42	-0,04	-0,06	-0,10	-0,29	-8,42	-1,26	-1,83	-3,01	-6,37
2	-0,21	0,19	0,10	0,23	-0,15	-8,05	7,04	4,16	9,69	-6,45	**2**	-0,15	-0,02	0,07	0,08	-0,25	-5,49	-0,61	2,03	2,67	-7,62
3	-0,07	0,07	0,22	0,16	-0,12	-1,90	2,57	8,56	5,91	-4,19	**3**	0,09	0,24	0,17	0,13	-0,34	2,25	7,14	4,81	3,62	-7,92
4	0,01	0,15	0,11	0,13	-0,25	0,27	5,13	4,14	4,51	-7,91	**4**	0,01	0,18	0,20	0,06	-0,31	0,28	5,34	5,77	1,64	-5,95
Big	0,06	0,05	0,14	0,15	-0,01	1,78	2,16	6,67	6,78	-0,44	**Big**	-0,09	0,12	0,02	-0,03	-0,05	-3,28	4,69	0,66	-0,80	-1,21
			c					**t(c)**						**c**					**t(c)**		
Small	0,43	0,16	0,05	0,01	-0,27	7,01	4,42	1,37	0,35	-6,06	**Small**	0,42	0,18	0,12	-0,11	-0,47	6,62	3,91	2,68	-2,54	-8,16
2	0,44	0,17	0,15	-0,13	-0,38	11,97	4,37	4,15	-3,72	-11,00	**2**	0,43	0,33	0,15	-0,12	-0,62	11,84	7,37	3,41	-3,09	-14,64
3	0,33	0,27	0,11	-0,14	-0,47	6,74	6,82	3,11	-3,60	-11,62	**3**	0,26	0,21	0,02	-0,34	-0,67	5,26	4,86	0,50	-7,56	-12,38
4	0,50	0,22	0,13	-0,02	-0,41	10,89	5,33	3,38	-0,46	-8,87	**4**	0,33	0,22	-0,02	-0,13	-0,54	7,09	5,27	-0,35	-2,66	-8,22
Big	0,81	0,47	0,11	-0,11	-0,67	17,74	14,97	3,67	-3,30	-17,71	**Big**	0,49	0,32	0,10	-0,30	-0,82	14,05	9,41	2,87	-6,95	-16,18

Table 13. Regressions for 25 value-weight Size-Inv portfolios. Asia Pacific (excluding Japan) and Japan shows data from July 1990-February 2021, 368 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}$ and the five-factor regression equation is: $R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}$

			Asia Pacific												**Japan**						
Inv ->	Low	2	3	4	High	Low	2	3	4	High	Inv ->	Low	2	3	4	High	Low	2	3	4	High

Panel A: Three-factor intercepts: RM-RF, SMB, and HML

	α					t(α)				
Small	0,53	0,60	0,58	0,40	-0,06	4,05	5,48	4,52	3,55	-0,44
2	-0,18	0,14	-0,13	-0,19	-0,54	-1,70	1,30	-1,10	-1,61	-4,45
3	-0,22	0,22	-0,11	0,01	-0,43	-1,66	1,72	-0,94	0,10	-3,52
4	-0,10	0,04	0,23	0,08	-0,09	-0,79	0,31	1,96	0,59	-0,57
Big	0,18	-0,02	0,01	-0,01	-0,12	1,35	-0,21	0,11	-0,09	-0,73

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

	α					t(α)				
Small	0,65	0,63	0,60	0,43	0,00	5,20	5,78	4,46	3,68	0,01
2	-0,08	0,12	-0,20	-0,18	-0,31	-0,79	1,06	-1,67	-1,43	-2,64
3	-0,39	0,14	-0,22	0,02	-0,32	-2,86	1,06	-1,75	0,13	-2,63
4	-0,38	-0,13	0,10	0,09	0,04	-3,25	-1,05	0,77	0,62	0,25
Big	-0,11	-0,14	0,03	0,10	0,28	-0,92	-1,32	0,32	0,96	2,05

	h					t(h)				
Small	-0,29	-0,08	0,17	0,11	-0,08	-5,43	-1,70	2,90	2,17	-1,30
2	0,05	0,15	0,30	0,45	-0,06	1,06	3,06	5,78	8,50	-1,17
3	0,22	0,13	0,29	0,14	-0,06	3,73	2,20	5,56	2,42	-1,25
4	0,09	0,25	0,16	0,24	-0,14	1,84	4,48	3,06	3,73	-2,04
Big	-0,01	-0,11	-0,02	0,14	-0,10	-0,11	-2,57	-0,53	3,17	-1,78

	r					t(r)				
Small	-0,41	-0,20	-0,08	-0,07	-0,09	-6,55	-3,63	-1,22	-1,25	-1,41
2	-0,24	-0,06	0,14	0,12	-0,17	-4,48	-1,12	2,29	1,96	-2,85

Panel A: Three-factor intercepts: RM-RF, SMB, and HML

	α					t(α)				
Small	0,06	0,11	0,22	0,12	0,28	0,74	1,44	2,32	1,33	2,81
2	-0,10	-0,06	-0,04	0,04	-0,09	-1,21	-0,79	-0,55	0,40	-1,07
3	-0,13	-0,08	-0,13	-0,14	-0,06	-1,46	-0,91	-1,63	-1,44	-0,61
4	-0,09	-0,10	0,03	-0,14	0,00	-0,90	-1,12	0,33	-1,53	0,01
Big	-0,04	-0,10	-0,11	0,08	0,07	-0,31	-1,04	-1,31	0,94	0,71

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

	α					t(α)				
Small	0,11	0,15	0,23	0,11	0,25	1,44	2,07	2,40	1,18	2,60
2	-0,02	-0,01	0,00	0,06	-0,16	-0,34	-0,07	0,02	0,66	-2,00
3	-0,07	-0,05	-0,09	-0,13	-0,09	-0,95	-0,62	-1,22	-1,28	-0,98
4	-0,05	-0,08	0,04	-0,13	-0,04	-0,49	-0,95	0,40	-1,46	-0,40
Big	0,03	-0,09	-0,11	0,05	0,03	0,34	-1,11	-1,25	0,58	0,39

	h					t(h)				
Small	0,08	0,14	0,12	0,14	-0,05	2,38	4,45	3,03	3,73	-1,12
2	0,10	0,11	0,21	0,08	-0,01	3,59	3,76	6,32	2,12	-0,39
3	0,06	0,16	0,20	0,16	-0,07	1,94	4,60	6,26	3,74	-1,94
4	0,06	0,12	0,15	0,15	-0,05	1,48	3,24	4,10	3,82	-1,25
Big	-0,21	0,04	0,07	0,14	-0,05	-6,29	1,11	2,09	4,34	-1,53

	r					t(r)				
Small	-0,10	-0,13	-0,04	0,05	0,02	-1,89	-2,72	-0,63	0,75	0,35
2	-0,17	-0,17	-0,15	-0,09	0,21	-3,87	-3,54	-2,96	-1,44	3,80

3	0,13	0,03	0,24	0,12	0,02	1,94	0,46	3,88	1,75	0,41	**3**	-0,08	0,00	-0,11	-0,13	0,00	-1,61	0,03	-2,19	-1,92	-0,03
4	0,24	0,20	0,22	0,11	-0,05	4,13	3,19	3,58	1,51	-0,61	**4**	-0,06	0,04	0,01	-0,07	0,06	-0,97	0,76	0,12	-1,14	1,02
Big	0,09	-0,08	-0,05	0,07	-0,18	1,60	-1,65	-1,03	1,39	-2,66	**Big**	0,07	0,16	0,00	0,01	-0,06	1,21	2,97	-0,02	0,12	-1,01
			c					**t(c)**						**c**					**t(c)**		
Small	0,46	0,28	0,09	0,03	0,01	7,79	5,44	1,46	0,50	0,15	**Small**	0,28	0,11	0,00	-0,01	-0,33	5,98	2,41	0,02	-0,16	-5,64
2	0,19	0,20	-0,04	-0,30	-0,39	3,70	3,71	-0,71	-5,23	-6,97	**2**	0,36	0,12	0,06	0,02	-0,19	8,74	2,70	1,16	0,40	-3,82
3	0,26	0,19	-0,16	-0,26	-0,41	4,15	3,01	-2,71	-4,01	-7,13	**3**	0,43	0,35	0,04	-0,20	-0,36	9,61	6,94	0,77	-3,15	-6,41
4	0,41	0,13	-0,01	-0,25	-0,32	7,46	2,22	-0,21	-3,64	-4,12	**4**	0,33	0,39	0,10	-0,15	-0,26	5,50	7,34	1,86	-2,59	-4,52
Big	0,73	0,53	0,03	-0,49	-0,90	13,59	10,94	0,73	-10,12	-14,12	**Big**	0,96	0,61	0,04	-0,43	-0,63	19,18	12,18	0,74	-8,78	-12,45

Table 14. Regressions for 25 value-weight Size-Inv portfolios. For Europe the data is from July 1990-February 2021, 368 months. And for the Emerging Markets the data is from July 1992-February 2021, 344 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

			Europe											**Emerging Markets**			
Inv ->	Low	2	3	4	High	Low	2	3	4	High	Inv ->	Growth	Neutral	Value	Growth	Neutra l	Value
Panel A: Three-factor intercepts: RM-RF, SMB, and HML											**Panel A: Three-factor intercepts: RM-RF, SMB, and HML**						
			α					**t(α)**					**α**			**t(α)**	
Small	-0,03	0,14	0,16	0,10	-0,32	-0,47	2,54	2,94	1,79	-4,02	**Small**	0,24	0,30	0,02	3,98	5,45	0,26
2	-0,08	0,14	0,16	0,04	-0,19	-1,28	2,47	2,66	0,83	-2,85	**Big**	0,24	0,15	0,23	3,15	2,84	3,01
3	-0,02	0,01	0,10	-0,07	-0,17	-0,28	0,21	1,59	-1,15	-2,20							
4	-0,02	-0,03	0,10	0,07	-0,13	-0,22	-0,37	1,53	0,97	-1,47							

Big	0,00	0,11	-0,03	-0,02	0,02	-0,03	1,69	-0,48	-0,24	0,19

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

			α					**t(α)**		
Small	0,05	0,11	0,13	0,11	-0,14	0,74	1,95	2,15	1,87	-1,83
2	-0,10	0,08	0,11	0,00	-0,03	-1,61	1,37	1,78	0,02	-0,47
3	-0,07	-0,02	0,05	-0,06	-0,05	-0,90	-0,38	0,67	-0,81	-0,72
4	-0,08	-0,06	0,07	0,05	0,00	-0,97	-0,89	0,97	0,72	0,03
Big	-0,04	0,05	-0,03	0,02	-0,01	-0,56	0,85	-0,52	0,38	-0,10
			h					**t(h)**		
Small	-0,07	0,08	0,15	0,03	-0,26	-1,96	2,52	4,51	0,79	-6,15
2	0,12	0,15	0,17	0,14	-0,15	3,26	4,69	4,83	4,36	-4,53
3	0,15	0,22	0,22	0,09	-0,16	3,41	5,88	5,68	2,22	-3,85
4	0,15	0,12	0,16	0,15	-0,13	3,36	2,96	3,98	3,59	-2,85
Big	-0,12	-0,02	-0,05	-0,02	0,15	-3,37	-0,53	-1,45	-0,71	3,88
			r					**t(r)**		
Small	-0,21	0,02	0,08	-0,03	-0,30	-4,72	0,47	1,77	-0,67	-5,53
2	-0,02	0,08	0,08	0,11	-0,25	-0,51	1,96	1,81	2,85	-5,95
3	0,05	0,04	0,10	-0,01	-0,15	0,98	0,82	2,07	-0,27	-2,87
4	0,07	0,02	0,06	0,06	-0,15	1,21	0,33	1,23	1,12	-2,66
Big	-0,05	0,04	-0,02	0,00	0,16	-1,05	1,02	-0,36	0,00	3,26
			c					**t(c)**		

Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA

		α			**t(α)**	
Small	0,23	0,30	0,13	4,67	5,16	1,88
Big	0,17	0,14	0,28	2,80	2,39	4,58
		h			**t(h)**	
Small	0,03	0,16	0,06	1,00	5,43	1,88
Big	-0,03	0,10	-0,07	-1,12	3,59	-2,39
		r			**t(r)**	
Small	-0,08	-0,01	-0,09	-2,34	-0,25	-1,81
Big	0,02	0,02	0,02	0,39	0,44	0,48
		c			**t(c)**	

Small	0,31	0,26	-0,01	0,03	-0,45	7,01	6,49	-0,15	0,64	-8,37
2	0,37	0,27	0,07	-0,14	-0,47	8,06	6,58	1,55	-3,57	-11,19
3	0,26	0,23	0,09	-0,15	-0,60	4,73	4,82	1,76	-3,05	-11,72
4	0,31	0,34	0,03	-0,17	-0,68	5,36	6,68	0,58	-3,21	-12,14
Big	0,67	0,39	0,13	-0,45	-0,66	14,55	9,00	3,02	-10,41	-13,39

Small	0,39	0,03	-0,58	14,41	0,88	-15,91
Big	0,51	0,09	-0,52	15,38	2,84	-15,87

5 Discussion

Comparing to existing work, also Fama and French (2015) in their tests on the US return find that portfolios of small stocks, whose returns behave like those of firms that invest a lot despite low profitability, have low average returns that cause problems for the three-factor and five-factor model. These problems are also present in markets I have examined outside the US, especially for North America, Asia Pacific (excluding Japan), and Europe. Although the five-factor model captures some of the troublesome average returns, it is still persistent in the model. The low average return on these stocks and the problems they create for asset pricing models are challenging for future research.

As found in Fama and French (2015), who report that the value factor becomes redundant in explaining average stock returns in the US, I similarly show the same evidence in the replication part of the thesis, but also in the extension part for when additional observations are added. Also, for North America when adding profitability and investment factors, the value factor seems to be redundant in explaining average stock returns. Interestingly, at the same time, this is not the case for Asia Pacific, Japan, Europe, and Emerging Markets, where the value factor is central for these markets. My findings for the significance of the value factor for markets outside the US and North America are backed by Foye (2018), which reports that Eastern Europe, Latin America, and Asia, all exhibit a strong value premium. Also, Fama and French (2017) states that North America, Europe, and Asia Pacific increase with the book-to-market ratio. Like Fama and French (2015), I find that the CMA factor is prominent in explaining average return for the US, but for all other markets examined outside the US indicates little to no investment patterns in my sample. Opposite findings are shown outside the US for Fama and French (2017), where they report that in North America the investment factor adds unique information, whereas the CMA factor is redundant for Europe and Japan. Findings from their paper show that average returns for small stocks of the three regions are substantially lower for the highest investment portfolios and less consistent for big stocks, except for North America where there is no other systematic investment pattern in average returns for small or big stocks. Further, they state that the role of the investment factor of the five-factor model may largely be to absorb the low average returns of high investment small stocks. Lin (2017) found that the investment factor is redundant in explaining average return in China, as well as evidence for a value effect. These results are in line with my finding for markets examined outside the US and North America. The author suggests two reasons behind these patterns. “First, a bank-oriented market (like many emerging markets) such as China makes firm profitability more reliable than investment as a proxy to predict a firm’s future performance. Second, managers are more likely to be pressured to overinvest and pursue the private benefits for controlling shareholders in developing countries where firms are characterized by more severe ownership concentration which weakens the predictive power of past investment. Therefore, past investment (total asset growth) is not an appropriate proxy for future investment growth rates.” (Lin, 2017, p. 159-160). Although this is the result for Lin (2017) in China, it could also be helpful in explaining differences for other markets with similar patterns such as Emerging Markets and specific countries in developed market portfolios.

Japan is the only market in this thesis that passes the GRS test for the five-factor model and subsets of its factors. Despite this, the five-factor model is not able to offer an improved explanation over the three-factor model. Findings from Fama and French (2017) confirm my results and concerns for the sample. Their findings also reports that the five-factor model in Japan passes the GRS test. Further they conclude that aside from a strong value effect, there is not enough variation in average returns for Japan for the years of 1990 to 2015 to challenge the three-factor model or any other model they considered. Moreover, the reasons behind such important differences in factor significance across different markets are unclear and difficult to elucidate. There could be several causes, such as measurement error in explanatory variables, missing data, heteroskedasticity, and other errors which would result in biased and inefficient estimators, especially for Emerging Markets. Further, for a more extensive research one should use different sorts of portfolios (which is not done in this thesis) for a more comprehensive analysis.

6 Conclusion

Table 15 below summarizes the main findings of this thesis. The five-factor asset pricing model performs well in the US, but HML seems to be redundant in explaining average stock returns. Similar conclusions can be drawn for North America, which also reports somewhat weak relation towards the SMB factor. Also, for Size-B/M and Size-OP portfolios the CMA factor seems to be redundant but adds value in Size-Inv portfolios. For Asia Pacific the five-factor model performs well under the GRS test. There are however little to no size premium (SMB is not statistically significant) and the SMB factor has a negative correlation with the market portfolio, which is not in line with financial evidence. Also, the CMA factor seem to be redundant in explaining average stock return. From summary statistics Japan shows no market, size, profitability, nor investment effect. This is also reflected in the GRS test whereas the five-factor model never was superior to the three or four-factor model. In other words, the five-factor model does not offer an improved explanation over the three-factor model in Japan, and RM-RF, SMB, RMW, and CMA factors seems to be redundant in explaining average stock returns. Further, the five-factor model is the superior model compared to the three-factor model in Europe. However, the SMB and CMA factors seem to be redundant in explaining average stock returns for Europe. Moreover, Emerging Markets have huge cross-sectional pricing errors for the five-factor model and models that include subsets of its factors. Also, from summary statistics the size factor is insignificant for Emerging Markets. Despite of this, the five-factor model offers an improvement over the three-factor model, but the SMB and CMA factors seems to be redundant in explaining average stock returns for Emerging Markets.

Table 15. Shows which factor of RM-RF, SMB, HML, RMW, and CMA that adds value or is redundant in explaining average return for each market included in the thesis. "✔" indicates that the factor for the specific market adds value, and "Redundant" indicates that the factor for the specific market is redundant. Also, "Small" indicates that the factor for the specific market adds little value in explaining average return.

	The US	**North America**	**Asia Pacific**	**Japan**	**Europe**	**Emerging Markets**
RM-RF	✔	✔	✔	Redundant	✔	✔
SMB	✔	Redundant/Small	Redundant/Small	Redundant	Redundant	Redundant/Small
HML	Redundant	Redundant	✔	✔	✔	✔
RMW	✔	✔	✔	Redundant	✔	✔

CMA	✔	Redundant/Small	Redundant	Redundant	Redundant	Redundant

So, to answer my research question: "*Does the five-factor model of Fama and French replicate across the world?*". I can conclude that the five-factor model does *not* replicate across the globe, and different markets need to account for different set of factors.

Factor models are often used and are the best in evaluating portfolio performance. In this thesis, my tests conclude that the five-factor model adds value in explaining average returns in most markets. In applications where the sole interest is abnormal returns (measured by regression intercepts), my tests suggest that a four-factor model that drops HML for the US and North America performs as well as the five-factor model. Whereas for Asia Pacific, Europe, and Emerging Markets the four-factor model that drops CMA performs as well as the five-factor model. In Japan, the five-factor model does not offer an improvement over the three-factor model. But if one is interested in portfolio tilts toward size, value, profitability, and investment premiums, the five-factor model is the choice. A common critique of how the five-factor model is evaluated is based on the fact that test portfolios are formed on the same characteristics as the factors, causing high correlation between the dependent variable and independent variables, and thereby artificially small pricing errors and high R^2-values.

Furthermore, factor models are not often used to estimate the cost of equity capital because it is difficult to provide meaningful and precise estimates. Fama and French (2017) state that estimates of the cost of equity capital from asset pricing models are imprecise, and so arguably useless. For instance, if the CAPM is the correct model and we have been given returns for the true market portfolio to calculate the cost of equity for a firm, we then need estimates for its market beta and the expected market premium. Also, if we ignore the evidence that the expected market premium and market beta vary trough time (Fama and French, 2017) we will achieve large standard errors of the estimates. So, because the trade-off between simplicity and precision varies across sectors and markets, practitioners must apply the techniques and factors that best suit their circumstances. Combining my findings with Fama and French (2017) indicates that national rather than regional factor models may be a better choice in the markets I have examined outside the US.

7 References

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8 Appendix

8.1 Summary statistics (Replication)

Panel A of Table A1 shows average monthly excess returns (returns in excess of the one-month US Treasury bill rate) for 25 VW portfolios from independent sorts of stocks into Size and B/M groups. Panel B of Table A1 shows average excess returns for 25 VW portfolios from independent sorts of stocks into Size and profitability quintiles, and Panel C shows average excess returns for 25 VW portfolios into Size and Investment groups.

Table A1. Average monthly percent excess returns for 25 portfolios formed on Size and B/M, Size and OP, and Size and Inv: July 1963–December 2013, 606 months. On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part.

	Fama and French						**Replication**				
	Low	2	3	4	High		Low	2	3	4	High
	Panel A: Size-B/M portfolios						**Panel A: Size-B/M portfolios**				
Small	0,26	0,81	0,85	1,01	1,15	**Small**	0,26	0,80	0,80	1,03	1,11
2	0,48	0,72	0,94	0,94	1,02	**2**	0,46	0,75	0,89	0,93	1,03
3	0,50	0,78	0,79	0,88	1,07	**3**	0,50	0,78	0,77	0,89	1,06
4	0,60	0,57	0,71	0,85	0,86	**4**	0,59	0,59	0,66	0,88	0,88
Big	0,46	0,51	0,48	0,56	0,62	**Big**	0,45	0,48	0,54	0,51	0,62
	Panel B: Size-OP portfolios						**Panel B: Size-OP portfolios**				
Small	0,56	0,94	0,90	0,95	0,88	**Small**	0,56	0,95	0,87	0,96	0,85
2	0,59	0,78	0,84	0,81	0,98	**2**	0,58	0,78	0,82	0,82	0,99
3	0,53	0,77	0,72	0,78	0,94	**3**	0,54	0,74	0,76	0,77	0,94
4	0,57	0,65	0,63	0,70	0,82	**4**	0,57	0,64	0,68	0,69	0,81
Big	0,39	0,33	0,43	0,47	0,57	**Big**	0,34	0,37	0,47	0,50	0,54
	Panel C: Size-Inv portfolios						**Panel C: Size-Inv portfolios**				
Small	1,01	0,98	0,99	0,89	0,35	**Small**	1,00	0,99	0,98	0,88	0,35
2	0,92	0,91	0,92	0,90	0,48	**2**	0,92	0,90	0,91	0,90	0,47
3	0,90	0,93	0,81	0,82	0,50	**3**	0,90	0,94	0,81	0,81	0,50
4	0,79	0,72	0,71	0,75	0,54	**4**	0,79	0,72	0,73	0,75	0,55
Big	0,71	0,52	0,49	0,48	0,42	**Big**	0,71	0,53	0,49	0,48	0,42

25 Size-B/M Portfolios

From Table A1 we can see from the tables of Fama and French (2015) on the left-hand side, and for my replication on the right-hand side, that the datapoints are very similar. Even though we used both the same sources for collecting the data, there are minor differences in our results. In each Book-to-Market equity (B/M) quantile of Panel A of Table A1 for both Fama and French, and replication, average returns typically fall from small stocks to big stocks. We refer to this as "The size effect". The only exception is from the first quantile "Low Book-to-market equity" (extreme growth stocks), where there is no clear relationship between size and average returns. The relationship between average returns and Book-to-Market equity is what we call "The value effect". From Table A1, it is clear that average returns increase from low to high Book-to-market equity, indicating a strong value effect.

25 Size-OP Portfolios

For Panel B in Table A1, I observe the same size effect as in Panel A for each quantile for operating profitability (OP), whereas average returns typically fall from small stocks to big stocks. The profitability effect is evident for the extreme low OP and extreme high OP quantile, whereas the middle three quantiles show little to no patterns, but typically increases with higher OP.

25 Size-Inv Portfolios

Also, in Panel C in Table A1, there is evident size effect in most of the size quantiles. The only exception is in the last quantile where investment is high. In this column, average returns increase as stocks goes from small to big for the first four size quantiles (0,35% to 0,55%), and then drops at the last quantile to 0,42%. For the investment effect, I observe that in every size quintile the average return on the portfolio in the lowest investment quintile is much higher than the return on the portfolio in the highest investment quintile.

Table A2. Average percent returns, standard deviations (Std. dev.), and t-statistics for the average return for the portfolios used to construct SMB, HML, RMW, and CMA; July 1963–December 2013, 606 months. The first is small (S) or big (B), the second is the B/M group, high (H), neutral (N), or low (L), the OP group, robust (R), neutral (N), or weak (W), or the Inv group, conservative (C), neutral (N), or aggressive (A). On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part. These are the portfolios used to create the factors.

Fama and French							Replication						
2x3 Sorts							**2x3 Sorts**						
Size-B/M	**SL**	**SN**	**SH**	**BL**	**BN**	**BH**	**Size-B/M**	**SL**	**SN**	**SH**	**BL**	**BN**	**BH**
Mean	0,93	1,31	1,46	0,89	0,94	1,10	Mean	0,92	1,30	1,44	0,88	0,95	1,12
Std dev.	6,87	5,44	5,59	4,65	4,34	4,68	Std dev,	6,87	5,45	5,61	4,64	4,31	4,89
t-Statistic	3,32	5,93	6,44	4,69	5,36	5,78	t-Statistic	3,29	5,88	6,31	4,68	5,45	5,65
Size-OP	**SW**	**SN**	**SR**	**BW**	**BN**	**BR**	**Size-OP**	**SW**	**SN**	**SR**	**BW**	**BN**	**BR**
Mean	1,02	1,27	1,35	0,81	0,87	0,98	Mean	1,02	1,27	1,36	0,78	0,88	0,99
Std dev.	6,66	5,32	5,96	4,98	4,38	4,39	Std dev,	6,62	5,28	5,98	5,17	4,32	4,39
t-Statistic	3,77	5,87	5,60	4,00	4,91	5,50	t-Statistic	3,79	5,90	5,58	3,73	5,01	5,53
Size-Inv	**SC**	**SN**	**SA**	**BC**	**BN**	**BA**	**Size-Inv**	**SC**	**SN**	**SA**	**BC**	**BN**	**BA**
Mean	1,41	1,34	0,96	1,07	0,94	0,85	Mean	1,40	1,34	0,95	1,07	0,94	0,85
Std dev.	6,12	5,22	6,59	4,38	4,08	5,18	Std dev,	6,10	5,22	6,59	4,37	4,08	5,19
t-Statistic	5,66	6,35	3,59	5,99	5,69	4,03	t-Statistic	5,64	6,33	3,57	6,02	5,68	4,03

Table A3. Averages, standard deviations, and t-statistics for monthly returns. Summary statistics for monthly factor percent returns; July 1963–December 2013, 606 months. On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part.

Fama and French						Replication					
2x3 Factors											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Mean	0,50	0,29	0,37	0,25	0,33	Mean	0,50	0,28	0,38	0,27	0,33
Std dev.	4,49	3,07	2,88	2,14	2,01	Std dev,	4,49	3,05	2,83	2,24	2,03
t-Statistic	2,74	2,31	3,20	2,92	4,07	t-Statistic	2,74	2,26	3,31	2,98	4,01

Table A2 shows the 6 VW portfolios formed on Size and B/M, Size and OP, and Size and Inv. These portfolios are used to create the factors we initially explained earlier in the introduction part of this section. Panel A, B, and C in Table A3 displays the five factors created by the portfolios in Panel A, B, and C in Table A2. Average return for the market portfolio (Rm-Rf) is 0,50% per month, 0,28% for SMB, 0,38% for HML, 0,27% for RMW, and 0,33% for CMA. The average return, standard deviation and the t-statistics displays approximately the same results for the replication when comparing with the results for Fama and French (2015).

Table A4. Correlation between the factors. On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part.

	Fama and French					**Replication**					
2x3 Factors											
	Rm-Rf	SMB	HML	RMW	CMA		Rm-Rf	SMB	HML	RMW	CMA
Rm-Rf	1,00					Rm-Rf	1,00				
SMB	0,28	1,00				SMB	0,28	1,00			
HML	-0,30	-0,11	1,00			HML	-0,27	-0,09	1,00		
RMW	-0,21	-0,36	0,08	1,00		RMW	-0,23	-0,34	0,07	1,00	
CMA	0,39	-0,11	0,70	-0,11	1,00	CMA	-0,39	-0,12	0,70	-0,04	1,00

Table A4 displays the correlation between factors and the replication part of my thesis. As mentioned in the thesis, financial theory states that there is a positive relation between stocks and betas, whereas small stocks tend to have higher market betas than big stocks, as they are considered to be riskier. Since there is a positive correlation between profitability and investment, it is surprising that the correlation between the profitability and investment factors is negative for the US, however, slightly less negative for the replication part in the table. Also, there seems to be a high correlation between HML and CMA factors which makes sense because it is known that high B/M value firms tend to be low investment firms.

8.2 Regression (Replication)

For more insight in model performance, I examine the regression details of the 25 Size-B/M, 25 Size-Op, and the 25 Size-Inv portfolios. From the regression output I am mostly interested in intercepts and pertinent slopes. Results from Fama and French and Replication in Table 2 in Panel A, B and C, shows that HML is a redundant factor for describing average returns and so, to simplify the task, one could drop the HML factor. But because of the interest in value premium on the B/M, OP and Inv portfolios we still include the factor. Fama and French (2015) substituted the HML factor with HMLO (orthogonal HML), which define as the sum of the intercept and residual from the regression of HML on RM−RF, SMB, RMW, and CMA. Using HMLO over HML in a five-factor model, provide the same intercepts and residuals, so the two regressions are equivalent for judging model performance. Because of this, I will further present regression details using the HML factor over HMLO.

Table A5. Regressions for 25 value-weight Size-B/M portfolios; July 1963 to December 2013, 606 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

Fama and French

B/M->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,49	0,00	0,02	0,16	0,14	-5,18	0,07	0,40	2,88	2,37
2	-0,17	-0,04	0,12	0,07	-0,02	-2,75	-0,80	2,24	1,40	-0,38
3	-0,06	0,06	0,02	0,06	0,12	-0,98	0,92	0,33	0,96	1,66
4	0,14	-0,10	-0,04	0,07	-0,08	2,24	-1,46	-0,55	1,05	-0,94
Big	0,17	0,02	-0,07	-0,11	-0,18	3,53	0,40	-0,95	-1,86	-1,92
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,29	0,11	0,01	0,12	0,12	-3,31	1,61	0,17	2,12	1,99
2	-0,11	-0,10	0,05	0,00	-0,04	-1,73	-1,88	0,95	-0,04	-0,64
3	0,02	-0,01	-0,07	-0,02	0,05	0,40	-0,10	-1,06	-0,25	0,60
4	0,18	-0,23	-0,13	0,05	-0,09	2,73	-3,29	-1,81	0,73	-1,09
Big	0,12	-0,11	-0,10	-0,15	-0,09	2,50	-1,82	-1,39	-2,33	-0,93
			h					**t(h)**		
Small	-0,43	-0,14	0,10	0,27	0,52	-10,11	-4,38	3,90	10,12	17,55
2	-0,46	-0,01	0,29	0,43	0,69	-15,22	-0,45	11,77	16,78	24,44
3	-0,43	0,12	0,37	0,52	0,67	-14,70	3,71	12,28	17,07	18,75
4	-0,46	0,09	0,38	0,52	0,80	-15,18	2,76	11,03	15,88	20,26
Big	-0,31	0,03	0,26	0,62	0,85	-14,12	1,09	7,54	21,05	18,74
			r					**t(r)**		
Small	-0,58	-0,34	0,01	0,11	0,12	-13,26	-10,56	0,31	3,89	3,95
2	-0,21	0,13	0,27	0,26	0,21	-6,75	4,89	10,35	9,86	7,04

Replication

B/M->	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: RM-RF, SMB, and HML										
			α					**t(α)**		
Small	-0,49	0,01	-0,02	0,18	0,14	-5,05	0,13	-0,43	3,22	2,31
2	-0,18	-0,01	0,09	0,08	0,00	-2,94	-0,17	1,54	1,62	0,06
3	-0,06	0,07	0,02	0,09	0,11	-0,93	1,10	0,32	1,39	1,42
4	0,13	-0,07	-0,07	0,11	-0,05	1,94	-0,98	-0,91	1,55	-0,63
Big	0,17	-0,01	0,01	-0,23	-0,21	3,52	-0,09	0,19	-3,36	-1,97
Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA										
			α					**t(α)**		
Small	-0,28	0,16	-0,01	0,18	0,11	-3,09	2,34	-0,26	3,13	1,85
2	-0,11	-0,05	0,00	0,02	-0,01	-1,66	-0,87	-0,05	0,31	-0,10
3	0,05	0,00	-0,08	0,00	0,01	0,74	0,00	-1,21	0,06	0,10
4	0,18	-0,21	-0,20	0,07	-0,09	2,79	-2,94	-2,85	0,95	-1,00
Big	0,11	-0,11	-0,05	-0,24	-0,08	2,42	-1,92	-0,71	-3,54	-0,77
			h					**t(h)**		
Small	-0,41	-0,14	0,13	0,25	0,49	-9,52	-4,14	4,70	9,28	16,86
2	-0,46	-0,02	0,27	0,40	0,66	-15,18	-0,65	10,18	16,47	24,81
3	-0,41	0,04	0,33	0,48	0,66	-14,00	1,44	10,75	16,38	17,99
4	-0,44	0,07	0,34	0,50	0,73	-14,24	2,05	10,07	14,84	17,34
Big	-0,31	0,04	0,24	0,73	1,03	-14,36	1,35	6,91	22,42	20,12
			r					**t(r)**		
Small	-0,49	-0,40	-0,05	-0,05	0,01	-11,54	-12,34	-2,00	-1,90	0,47
2	-0,15	0,08	0,24	0,12	0,03	-4,90	3,27	9,17	4,93	1,00

3	-0,21	0,22	0,33	0,28	0,33	-6,99	6,77	10,36	8,98	8,88	3	-0,13	0,15	0,23	0,16	0,21	-4,70	5,13	7,63	5,47	5,93
4	-0,19	0,27	0,28	0,14	0,25	-6,06	7,75	7,99	4,16	6,14	4	-0,13	0,24	0,27	0,06	0,07	-4,17	7,28	8,12	1,84	1,67
Big	0,13	0,25	0,07	0,23	0,02	5,64	8,79	2,07	7,62	0,49	Big	0,18	0,20	0,09	0,07	-0,13	8,74	7,15	2,47	2,26	-2,62
			c					**t(c)**						**c**					**t(c)**		
Small	-0,57	-0,12	0,19	0,39	0,62	-12,27	-3,46	6,59	13,15	19,10	Small	-0,14	-0,03	0,05	0,09	0,09	-2,28	-0,59	1,31	2,37	2,17
2	-0,59	0,06	0,31	0,55	0,72	-17,76	1,94	11,27	19,39	22,92	2	-0,12	0,04	0,02	0,11	0,00	-2,80	0,99	0,49	3,12	-0,10
3	-0,67	0,13	0,42	0,64	0,78	-20,59	3,64	12,52	18,97	19,62	3	-0,26	0,07	0,07	0,11	0,11	-6,02	1,45	1,59	2,65	2,09
4	-0,51	0,31	0,51	0,60	0,79	-15,11	8,33	13,35	16,41	18,03	4	-0,06	0,24	0,18	0,08	0,05	-1,22	4,82	3,70	1,66	0,74
Big	-0,39	0,26	0,41	0,66	0,73	-16,08	8,38	10,80	19,88	14,54	Big	-0,06	0,17	0,17	-0,04	-0,38	-1,81	4,17	3,30	-0,78	-5,05

Table A6. Regressions for 25 value-weight Size-OP portfolios; July 1963–December 2013, 606 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

	Fama and French											Replication									
OP->	Low	2	3	4	High	Low	2	3	4	High	**OP->**	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Three-factor intercepts: RM-RF, SMB, and HML											**Panel A: Three-factor intercepts: RM-RF, SMB, and HML**									
			α					**t(α)**						**α**					**t(α)**		
Small	-0,30	0,10	0,05	0,09	-0,02	-3,25	1,54	0,85	1,30	-0,30	Small	-0,30	0,10	0,03	0,11	-0,03	-3,28	1,63	0,45	1,57	-0,38
2	-0,24	-0,03	0,05	0,04	0,16	-3,16	-0,55	0,94	0,58	2,08	2	-0,24	-0,03	0,05	0,04	0,18	-3,10	-0,47	0,91	0,63	2,28
3	-0,21	0,07	0,01	0,05	0,20	-2,27	1,04	0,14	0,79	2,51	3	-0,20	0,02	0,07	0,05	0,21	-2,11	0,31	1,32	0,75	2,49
4	-0,11	-0,02	-0,05	0,06	0,18	-1,15	-0,24	-0,73	0,96	2,43	4	-0,11	-0,03	0,02	0,05	0,18	-1,04	-0,42	0,29	0,67	2,38
Big	-0,17	-0,20	-0,03	0,05	0,22	-1,90	-2,94	-0,58	1,20	4,03	Big	-0,25	-0,17	0,00	0,08	0,19	-2,57	-2,64	0,03	1,84	3,47
	Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA											**Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA**									
			α					**t(α)**						**α**					**t(α)**		
Small	-0,10	0,04	-0,05	-0,05	-0,15	-1,28	0,64	-0,80	-0,80	-2,05	Small	-0,10	0,05	-0,09	-0,02	-0,14	-1,30	0,86	-1,39	-0,38	-1,95
2	-0,05	-0,11	-0,03	-0,11	0,00	-0,83	-1,86	-0,64	-1,92	0,02	2	-0,05	-0,10	-0,06	-0,09	0,00	-0,72	-1,75	-1,12	-1,50	-0,04
3	0,08	0,04	-0,06	-0,07	0,03	1,15	0,67	-1,05	-1,23	0,43	3	0,11	-0,03	0,00	-0,10	0,02	1,50	-0,39	-0,02	-1,69	0,33
4	0,16	0,02	-0,12	-0,09	0,05	1,91	0,26	-1,97	-1,52	0,76	4	0,17	0,02	-0,08	-0,13	0,04	2,06	0,27	-1,26	-2,16	0,53
Big	0,14	-0,11	-0,03	0,02	0,08	2,08	-1,67	-0,57	0,42	1,85	Big	0,08	-0,07	0,02	0,03	0,06	1,09	-1,17	0,41	0,70	1,34
			h					**t(h)**						**h**					**t(h)**		
Small	-0,14	0,24	0,26	0,28	0,21	-3,82	8,05	9,32	9,31	6,17	Small	-0,11	0,25	0,30	0,32	0,14	-3,12	8,33	10,37	10,66	4,23
2	-0,12	0,17	0,23	0,18	0,15	-3,96	5,84	9,51	6,38	5,08	2	-0,12	0,15	0,22	0,22	0,13	-4,09	5,62	8,75	8,10	4,21
3	0,00	0,14	0,21	0,19	0,09	0,11	4,36	7,68	6,74	2,93	3	-0,02	0,15	0,15	0,19	0,10	-0,66	4,85	5,83	6,77	3,12
4	0,03	0,15	0,21	0,10	0,02	0,72	4,80	7,19	3,60	0,69	4	-0,02	0,11	0,20	0,15	-0,01	-0,54	3,36	6,83	5,15	-0,38
Big	0,22	0,16	0,04	0,00	-0,13	6,70	5,33	1,42	-0,19	-6,13	Big	0,22	0,17	0,08	-0,02	-0,13	6,31	6,22	2,87	-1,12	-5,85
			r					**t(r)**						**r**					**t(r)**		
Small	-0,67	0,21	0,30	0,47	0,45	-17,70	6,98	10,59	15,08	12,95	Small	-0,59	0,13	0,29	0,38	0,34	-16,61	4,42	10,27	13,01	10,16
2	-0,60	0,21	0,29	0,45	0,55	-19,94	6,90	11,32	15,76	17,91	2	-0,56	0,13	0,25	0,38	0,54	-19,23	4,97	10,36	14,12	17,78

3	-0,76	0,03	0,24	0,38	0,57	-21,06	0,93	8,33	13,12	17,19	3	-0,77	0,06	0,16	0,39	0,56	-22,44	1,88	6,09	13,78	16,98
4	-0,75	-0,15	0,23	0,39	0,37	-18,94	-4,54	7,49	12,95	11,09	4	-0,74	-0,21	0,19	0,41	0,36	-19,59	-6,71	6,70	14,09	11,21
Big	-0,71	-0,26	-0,08	0,12	0,35	-21,05	-8,41	-2,82	5,66	15,54	Big	-0,76	-0,30	-0,10	0,14	0,35	-21,96	-11,01	-3,70	7,02	16,57
			c					**t(c)**						**c**					**t(c)**		
Small	-0,06	0,25	0,34	0,31	0,14	-1,42	7,58	10,89	9,08	3,76	Small	0,09	0,00	0,03	-0,03	-0,07	1,75	-0,06	0,76	-0,79	-1,38
2	-0,09	0,29	0,26	0,23	0,05	-2,65	8,94	9,52	7,44	1,56	2	0,05	0,12	0,08	-0,06	-0,07	1,06	2,97	2,07	-1,47	-1,63
3	-0,17	0,26	0,24	0,23	0,02	-4,41	7,31	7,89	7,49	0,65	3	-0,15	0,12	0,08	0,05	-0,09	-2,96	2,60	2,04	1,12	-1,76
4	-0,02	0,30	0,30	0,26	0,02	-0,41	8,56	9,08	8,12	0,48	4	0,00	0,15	0,12	0,15	0,03	0,01	3,18	2,79	3,41	0,59
Big	-0,03	0,23	0,19	-0,04	-0,12	-0,83	6,82	6,16	-1,82	-5,22	Big	-0,24	0,06	0,07	-0,01	-0,02	-4,57	1,54	1,74	-0,32	-0,57

Table A7. Regressions for 25 value-weight Size-Inv portfolios; July 1963–December 2013, 606 months. Panel A of the table shows three-factor intercepts (α) produced by RM-RF, SMB, and HML. Panel B shows five-factor intercepts (α), slopes for HML (h), RMW (r), and CMA (c), and t-statistics for each coefficient. On the left-hand side, we can see Fama and French data from the paper, "A five-factor asset pricing model". On the right-hand side we can see data for the replication part. The three-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + \varepsilon_{it}}$ and the five-factor regression equation is: $\boldsymbol{R_{it} - R_{Ft} = \alpha_i + \beta_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it}}$

Fama and French

Inv->	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Three-factor intercepts: RM-RF, SMB, and HML									
			α					**t(α)**		
Small	0,09	0,15	0,17	0,06	-0,48	1,01	2,74	2,76	1,00	-7,19
2	0,01	0,10	0,15	0,08	-0,26	0,14	1,72	2,74	1,45	-4,71
3	0,09	0,19	0,10	0,11	-0,17	1,11	3,15	1,80	1,73	-2,50
4	0,02	0,01	0,04	0,14	-0,03	0,24	0,19	0,66	2,09	-0,38
Big	0,15	0,07	0,02	0,07	0,05	1,86	1,18	0,39	1,43	0,75
	Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA									
			α					**t(α)**		
Small	0,21	0,11	0,09	0,02	-0,35	2,66	1,93	1,47	0,32	-5,30
2	-0,01	-0,01	0,06	0,02	-0,14	-0,14	-0,21	1,12	0,30	-2,59
3	0,03	0,10	-0,01	0,09	-0,02	0,40	1,74	-0,21	1,37	-0,33
4	-0,09	-0,09	-0,04	0,08	0,15	-1,20	-1,42	-0,73	1,22	2,05
Big	-0,04	-0,07	-0,06	0,04	0,20	-0,49	-1,42	-1,31	0,90	3,33
			h					**t(h)**		
Small	-0,10	0,17	0,16	0,12	0,00	-2,67	6,53	5,50	4,35	0,14
2	0,06	0,26	0,14	0,25	-0,11	2,33	9,11	5,26	10,24	-4,36
3	0,13	0,21	0,21	0,18	-0,04	3,53	7,26	7,99	6,12	-1,40
4	0,15	0,29	0,25	0,08	-0,19	4,34	9,41	8,63	2,50	-5,57
Big	-0,10	-0,04	0,10	0,00	-0,06	-2,94	-1,86	4,47	-0,18	-2,04
			r					**t(r)**		
Small	-0,55	0,04	0,15	0,11	-0,19	-14,42	1,52	5,13	3,79	-5,93
2	-0,18	0,27	0,17	0,30	-0,15	-6,54	9,37	6,02	11,72	-5,86

Replication

Inv->	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Three-factor intercepts: RM-RF, SMB, and HML									
			α					**t(α)**		
Small	0,08	0,16	0,16	0,06	-0,48	0,89	2,91	2,7	1,06	-7,18
2	0,02	0,11	0,15	0,09	-0,26	0,24	1,77	2,65	1,56	-4,75
3	0,09	0,2	0,11	0,1	-0,16	1,21	3,31	1,89	1,63	-2,4
4	0,02	0,02	0,07	0,13	-0,03	0,3	0,29	1,18	2,05	-0,41
Big	0,15	0,07	0,02	0,07	0,06	1,82	1,23	0,4	1,28	0,8
	Panel B: Five-factor coefficients: RM-RF, SMB, HML, RMW and CMA									
			α					**t(α)**		
Small	0,22	0,13	0,11	0,03	-0,36	2,80	2,34	1,81	0,58	-5,39
2	0,01	-0,01	0,07	0,02	-0,14	0,11	-0,14	1,27	0,37	-2,64
3	0,04	0,11	-0,01	0,06	-0,02	0,52	1,77	-0,12	0,99	-0,30
4	-0,09	-0,10	-0,01	0,07	0,14	-1,26	-1,62	-0,14	1,01	1,98
Big	-0,04	-0,05	-0,07	0,03	0,20	-0,54	-1,06	-1,44	0,66	3,36
			h					**t(h)**		
Small	-0,07	0,18	0,20	0,13	0,00	-1,84	6,94	6,79	4,65	-0,09
2	0,05	0,24	0,16	0,25	-0,12	2,06	8,68	6,02	10,12	-4,78
3	0,10	0,19	0,21	0,18	-0,07	2,86	6,58	7,75	5,97	-2,29
4	0,12	0,26	0,18	0,07	-0,17	3,47	8,42	6,65	2,29	-5,14
Big	-0,12	-0,04	0,09	0,01	-0,06	-3,41	-1,58	4,29	0,53	-2,04
			r					**t(r)**		
Small	-0,54	-0,02	0,07	0,06	-0,18	-14,75	-0,73	2,59	1,98	-5,84
2	-0,21	0,19	0,10	0,24	-0,13	-7,96	7,04	3,99	9,60	-5,27

3	-0,01	0,11	0,29	0,18	-0,13	-0,36	3,71	10,65	5,99	-4,20	3	-0,05	0,08	0,24	0,17	-0,12	-1,50	2,68	8,95	5,78	-3,92
4	0,05	0,21	0,21	0,16	-0,31	1,51	6,68	7,29	5,02	-8,77	4	0,02	0,17	0,11	0,15	-0,25	0,69	5,53	4,06	5,01	-7,39
Big	0,05	0,07	0,17	0,15	-0,02	1,50	2,74	7,20	6,05	-0,71	Big	0,07	0,05	0,15	0,16	0,00	2,04	2,01	6,82	6,59	-0,01
			c					**t(c)**						**c**					**t(c)**		
Small	0,22	0,38	0,34	0,18	-0,31	5,27	13,11	10,50	5,69	-8,78	Small	0,29	0,18	0,11	0,03	-0,28	5,36	4,56	2,66	0,81	-6,16
2	0,47	0,47	0,36	0,17	-0,51	15,85	15,12	12,21	6,28	-18,17	2	0,41	0,21	0,18	-0,09	-0,38	10,62	5,11	4,73	-2,56	-10,19
3	0,47	0,53	0,37	0,06	-0,56	11,59	16,71	12,66	1,83	-16,72	3	0,36	0,33	0,15	-0,10	-0,48	6,64	7,88	3,79	-2,37	-10,91
4	0,64	0,56	0,39	0,11	-0,60	16,64	16,55	12,46	3,10	-16,03	4	0,52	0,30	0,17	0,05	-0,39	10,13	6,66	4,21	1,03	-7,90
Big	0,69	0,48	0,25	-0,12	-0,76	18,03	18,80	10,27	-4,59	-24,15	Big	0,80	0,51	0,15	-0,12	-0,69	15,92	14,59	4,62	-3,33	-16,77

25 Size-B/M Portfolios

Panel A in Table A5 shows that when comparing the Replication with Fama and French (2015), the three-factor model intercepts and their associated t-statistics are approximately the same. The extreme growth stocks (left column of the intercept matrix) are a huge problem for the three-factor model. In the top left corner, we see a negative return of -0,49% per month for both Fama and French and the Replication, and a slightly lower t-ratio (-5,05) for the Replication. Panel B in Table A5 shows the slopes and their associated t-statistics for the five-factor model. The slopes shown in the table are the intercept (α), HML (h), RMW (r), CMA (c), and their associated t-statistics. If we compare the intercepts for the three-factor versus the five-factor model we see that the intercepts are as expected, lower on average for the five-factor model. This implies that the five-factor model has less cross-sectional pricing errors. Moreover, the pattern in the extreme growth intercepts (negative for small stocks and positive for large) survives in the five-factor model for both Fama and French and for the Replication.

The RM-RF and SMB slopes are not shown in the tables. Fama and French (2015) claims the market slopes are always close to 1.0, and the slope for the SMB factor is strongly positive for small stocks and slightly negative for big stocks, which is also confirmed in my research. So, to save space, I only include the HML, RMW, and CMA slopes. I can report the same findings as Fama and French (2015) in the portfolios of small B/M growth stocks where almost all returns are strongly negative, and strongly positive for high B/M value stocks. Moreover, the RMW and CMA slopes shows that the portfolio is dominated by microcaps whose returns behave like those of unprofitable firms that grow rapidly. Interestingly, I can report a lower return than Fama and French (2015) in the high B/M value stocks for the RMW slopes. For the CMA slopes I can also report a much higher return on all low B/M growth stocks. For instance, top left corner for the left table for CMA slope has a return of -0,57% (t=-12,27) versus the right table of 0,14% (t=-2,28). Further, the CMA slope from column 3 to high is also on average lower than Fama and French (2015).

25 Size-OP Portfolios

The five-factor intercepts for the portfolios in Panel B of Table A6 show no patterns and most of them are close to zero. This is in line from the GRS test in Table 2, that average absolute intercepts are smaller for the Size-OP portfolios than for the other examined portfolios. The intercepts for the three-factor model on the other hand is a bit more of a problem, whereas the intercept for the microcap portfolio in the lowest profitability quintile is up to -30% (t=1,28) for both the Fama and French and the Replication. The regression details on the 25 Size-OP portfolios tell us that for small and big stocks, low profitability is not a five-factor asset pricing problem. When comparing Size-OP with Size-B/M portfolios, we find that Size-OP portfolios are far less challenging for the five-factor model as they are for the Size-B/M portfolios. Fama and French (2015) claims this is because the Size-OP portfolios do not isolate small stocks whose returns behave like those of firms that invest a lot despite low profitability. When comparing the tables, we can see results that are quite similar as Fama and French (2015), but for the CMA slopes we find the same pattern as for 25 Size-B/M portfolios. In the extreme growth stocks (left column of the intercept matrix) portfolio, we observe a difference of 0,15% between the tables. For column 2 towards high for the CMA slopes, we observe much lower returns than Fama and French (2015).

25 Size-Inv Portfolios

Panel A of Table A7 shows that the big problems of the three-factor model are strong negative intercepts for the portfolios in the three smallest Size quintiles and the highest Inv quintile, which means that the three-factor model performs poorly in these portfolios. Moreover, moving towards the five-factor intercepts we can see the model perform better than the three-factor, but has still strong negative intercepts for the portfolios in the two smallest Size quintiles and the highest Inv quintile. The CMA slopes for the 25 Size-Inv portfolios show the pattern I had expected, which is positive for low investment portfolios and negative for high investment portfolios. This goes for both the tables above, despite some differences in the portfolios in the four smallest Size quintiles and the middle (2,3,4) Inv quintile. For those, the differences are the same as for the 25 Size-OP portfolios and 25 Size-B/M portfolios, which is smaller returns for these slopes.

8.3 Summary statistics (Extension)

Table A8: Average percent returns, standard deviations (Std. dev.), and t-statistics for the average return for the portfolios used to construct SMB, HML, RMW, and CMA. The US shows data from July 1963–February 2021, 692 months. For North America, Asia Pacific (excluded Japan), Japan and Europe the data is from July 1990-February 2021, 368 months. For the Emerging Markets the data is from July 1992-February 2021, 344 months. The first is small (S) or big (B), the second is the B/M group, high (H), neutral (N), or low (L), the OP group, robust (R), neutral (N), or weak (W), or the Inv group, conservative (C), neutral (N), or aggressive (A). These are the portfolios used to create the factors.

			The US							North America			
2x3 Sorts							**2x3 Sorts**						
Size-B/M	SL	SN	SH	BL	BN	BH	**Size-B/M**	SL	SN	SH	BL	BN	BH
Mean	0,98	1,27	1,37	0,95	0,93	1,09	Mean	0,82	1,03	1,14	1,00	0,92	0,84
Std dev,	6,82	5,49	5,73	4,58	4,31	5,02	Std dev,	7,01	5,54	5,37	4,62	4,12	4,85
t-Statistic	3,79	6,07	6,27	5,43	5,70	5,72	t-Statistic	2,25	3,56	4,07	4,17	4,28	3,32
Size-OP	SW	SN	SR	BW	BN	BR	**Size-OP**	SW	SN	SR	BW	BN	BR
Mean	1,04	1,24	1,32	0,81	0,90	1,02	Mean	0,91	1,15	1,24	0,74	0,91	1,02
Std dev,	6,56	5,34	6,12	5,14	4,31	4,36	Std dev,	6,28	5,15	5,68	5,37	4,28	4,12
t-Statistic	4,17	6,10	5,65	4,16	5,48	6,18	t-Statistic	2,77	4,30	4,20	2,64	4,07	4,76
Size-Inv	SC	SN	SA	BC	BN	BA	**Size-Inv**	SC	SN	SA	BC	BN	BA
Mean	1,35	1,30	0,99	1,07	0,94	0,92	Mean	1,21	1,11	0,82	0,97	0,91	0,93
Std dev,	6,15	5,27	6,56	4,34	4,08	5,13	Std dev,	5,75	4,99	6,55	4,06	4,15	5,42
t-Statistic	5,79	6,49	3,95	6,49	6,07	4,71	t-Statistic	4,05	4,25	2,42	4,60	4,19	3,31
			Asia Pacific							**Japan**			
2x3 Sorts							**2x3 Sorts**						
Size-B/M	SL	SN	SH	BL	BN	BH	**Size-B/M**	SL	SN	SH	BL	BN	BH
Mean	0,33	0,72	1,08	0,87	0,89	1,13	Mean	0,31	0,40	0,53	0,28	0,36	0,49

Std dev,	6,74	6,43	6,63	5,87	5,82	6,98	Std dev,	7,56	6,50	6,44	6,04	5,47	6,02
t-Statistic	0,94	2,15	3,13	2,85	2,93	3,12	t-Statistic	0,79	1,18	1,57	0,88	1,27	1,56
Size-OP	SW	SN	SR	BW	BN	BR	**Size-OP**	SW	SN	SR	BW	BN	BR
Mean	0,64	0,93	1,04	0,75	1,01	0,96	Mean	0,37	0,46	0,54	0,21	0,41	0,36
Std dev,	7,06	6,13	6,24	6,96	6,05	5,54	Std dev,	6,82	6,26	6,86	6,64	5,48	5,49
t-Statistic	1,73	2,92	3,21	2,06	3,20	3,33	t-Statistic	1,05	1,40	1,52	0,61	1,44	1,27
Size-Inv	SC	SN	SA	BC	BN	BA	**Size-Inv**	SC	SN	SA	BC	BN	BA
Mean	0,98	0,95	0,55	0,98	0,94	0,85	Mean	0,47	0,43	0,45	0,36	0,30	0,32
Std dev,	6,33	6,13	7,11	5,53	5,82	6,87	Std dev,	6,93	6,24	7,01	6,03	5,48	5,85
t-Statistic	2,98	2,97	1,48	3,41	3,11	2,36	t-Statistic	1,29	1,31	1,22	1,13	1,05	1,06
			Europe							**Emerging Markets**			
2x3 Sorts							**2x3 Sorts**						
Size-B/M	SL	SN	SH	BL	BN	BH	**Size-B/M**	SL	SN	SH	BL	BN	BH
Mean	0,60	0,76	0,94	0,65	0,78	0,75	Mean	0,39	0,89	1,31	0,79	0,90	1,02
Std dev,	5,46	5,10	5,25	4,65	5,04	5,97	Std dev,	6,37	6,19	6,20	6,04	6,44	6,66
t-Statistic	2,10	2,87	3,43	2,70	2,98	2,43	t-Statistic	1,13	2,67	3,92	2,43	2,60	2,83
Size-OP	SW	SN	SR	BW	BN	BR	**Size-OP**	SW	SN	SR	BW	BN	BR
Mean	0,56	0,91	1,07	0,54	0,77	0,78	Mean	0,89	1,16	0,98	0,72	0,84	0,98
Std dev,	5,26	5,06	5,30	5,75	5,09	4,71	Std dev,	6,30	6,13	6,02	6,71	6,29	6,05
t-Statistic	2,04	3,45	3,89	1,81	2,88	3,18	t-Statistic	2,63	3,52	3,03	1,98	2,48	3,02
Size-Inv	SC	SN	SA	BC	BN	BA	**Size-Inv**	SC	SN	SA	BC	BN	BA
Mean	0,84	0,92	0,64	0,71	0,74	0,70	Mean	1,11	1,13	0,78	0,94	0,89	0,82
Std dev,	5,16	4,87	5,61	4,86	5,01	5,43	Std dev,	6,20	5,96	6,63	6,03	6,22	6,78
t-Statistic	3,12	3,64	2,19	2,82	2,85	2,48	t-Statistic	3,30	3,51	2,18	2,89	2,65	2,25