Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Department of Economics

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The Effects of Sector Exclusion

An empirical analysis

Master's thesis in Financial Economics
Supervisor: Snorre Lindset
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Preface

This master thesis is the final piece of the puzzle when completing our master's degree in financial economics at NTNU. As such, it is rounding off several years of higher education within the field of economics and finance. Working on this thesis has been a relatively smooth process, helped by years of experience and cooperation. On the other hand, some technical aspects have provided challenges. We want to give our supervisor, Snorre Lindset, a huge thanks for sound advice and input. Both regarding the selection of topic as well as excellent guidance throughout the project. We would also like to acknowledge the helpful advice and insights on working with such a thesis by our good friend and roommate, Roar Fenne. The value of tips and tricks from others with prior experience is never to be underestimated.

This master thesis is, in its entirety, a collaboration between Håvard Tryland and Vebjørn Jokstad, and any views or errors are entirely attributed to the authors.

Håvard Tryland & Vebjørn Jokstad
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Abstract

In this thesis, we seek to outline the effects on risk and expected returns of sector excluded portfolios using data containing 30 sectors, dated from January 1930, ending in December 2019. In our analysis, we utilize annualized returns and the standard deviation of annualized returns to evaluate our portfolios' level of performance and risk spanning the 90 years. We construct portfolios containing every number of sectors, from one sector up to thirty included sectors. We analyze all four statistical moments as our portfolios increase in size, emphasizing annualized returns. Furthermore, we apply two different investment strategies, a buy-and-hold strategy and a rebalancing strategy, which gave similar results but some discrepancies. We choose to move away from the traditional way of evaluating risk and return where the method used is investigating daily and monthly development. The approach we use in this thesis will yield results that give a better overview of how long-term investments perform compared to each other, the market portfolio, and a 3-month Treasury bill. We find that any single sector or portfolio containing several sectors has yielded returns exceeding that of 3-months Treasury bills. Additionally, in the case where we only exclude one sector, we find that a majority of portfolios obtain annualized returns exceeding the market portfolio. Furthermore, the standard deviation of annualized returns decreases with a numerical value exceeding 90% when we compare portfolios with one sector included with portfolios with one sector excluded.

Sammendrag

I denne oppgaven ønsker vi å kartlegge effekten på risiko og forventet avkastning av sektor eksklusjon, med data som inneholder 30 sektorer, fra januar 1930 til desember 2019. I analysen vår bruker vi annualisert avkastning og standardavviket til annualisert avkastning for å evaluere nivå for prestasjon og risiko på våre porteføljer over et tidsintervall på 90 år. Vi konstruerer porteføljer som inneholder alle kombinasjoner av sektorer, fra en sektor opp til alle 30 sektorene. Vi analyserer alle de fire statistiske momentene for å undersøke utviklingen når vi øker porteføljestørrelsene, med et fokus på annualisert avkastning. Videre, tar vi i bruk to forskjellige investeringsstrategier. En kjøp-og-hold strategi og en rebalanseringsstrategi. De forskjellige strategiene gir lignende resultater, men med noen avvik. Vi velger å tre vekk i fra den tradisjonelle metoden som brukes for å evaluere risiko og avkastning hvor investorer observerer daglig og månedlig bevegelse av avkastning. Den metoden vi velger i vår oppgave vil gi resultater som gir en bedre oversikt over hvordan investeringer med en lang horisont utvikler seg. I tillegg blir det lettere å sammenligne forskjellige porteføljer med hverandre, markedsporteføljen og 3 måneders statsobligasjoner. Våre resultater viser at alle sektorer har produsert annualisert avkastning høyere enn den risikofrie renten over vår tidshorisont. I tillegg finner vi at ved å ekskludere kun en sektor vil et flertall av porteføljene produsere høyere annualisert avkastning enn marketsporteføljen. Videre ser vi at standardavviket til annualisert avkastning reduseres med over 90% når vi sammenligner porteføljer med en sektor inkludert og en sektor ekskludert.

Contents

1 Introduction

Alan Ableson[1] said: "Do you know what investing for the long run but listening to market news every day is like? It's like a man walking up a big hill with a yo-yo and keeping his eyes fixed on the yo-yo instead of the hill." Observing the yo-yo is a viable strategy if an investor's horizon is short. However, as the horizon is extended, any short-term movement should be negligible, rendering this approach redundant. What we believe to be more interesting in an extended timeline is yearly average development. We combine this extended timeline with full sector analysis, forgoing daily and monthly development and instead focusing on annualized returns.

We firmly believe that utilizing annualized returns will provide an easier comparison of the different portfolios we construct on our extended timeline. Both for portfolios containing the same number of sectors and portfolios where we exclude fewer and fewer sectors until we reach the market portfolio. Following modern portfolio theory, we assume rational investors will maximize returns while minimizing variance. At the end of our analysis, we create portfolio frontiers for portfolios with one sector excluded to showcase rational portfolio choices.

Using sectors, rather than individual stocks, in our analysis, some of the idiosyncratic risks are already diversified away within each sector. We believe that the benefits of diversification are visible already from including a few sectors. Our results show the existence of benefits that matches literature on the subject of diversification (Markowitz 1952)(Evans and Archer 1968). We extend our analysis beyond expected returns and variance to skewness and kurtosis to observe how these higher statistical moments vary as our portfolios differ in the number of sectors excluded.

Skewness is a statistical moment that is interesting to analyze. As has been shown by Bessembinder (2018), most stocks obtain an accumulated return of less than the market, resulting in a distribution with negative skewness. Nevertheless, the few stocks that obtain returns exceeding the market do so by a significant amount. As a portfolio increases in size, skewness will move towards a higher negative value as the few stocks that significantly beat the market has a smaller weight in a portfolio. For investors holding a narrow portfolio, picking the stocks that severely outperform expectations results in substantial gains and is crucial to beat the market.

The trade-off between variance and skewness is another interesting aspect in terms of portfolio analysis. Comparing well-diversified portfolios with

[1] Alan Abelson (October 12, 1925 – May 9, 2013) was a veteran financial journalist and longtime writer of the influential Up and Down Wall Street column in Barron's Magazine.

under-diversified portfolios, we can see the effects of the third statistical moment. Our analysis finds that a more diversified portfolio will have a higher value for expected annualized returns. However, it will also have a higher probability of significantly lower returns than the mean expected return relative to the upside. Furthermore, our less diversified portfolios have lower expected annualized returns with higher variances, but the positive skewness increases the probability of returns exceeding the mean expected annualized returns. In these positions, an investor can make a conscious decision to achieve a lower mean value of expected annualized returns, with higher variance to make use of positive skewness, creating a mean-variance-skewed efficient portfolio.

2 Literature

Markowitz (1952) introduced the aspect of diversification for investors. Diversification has become a cornerstone for any investment strategy. Markowitz made a mathematical framework for investors to assess the combined risk and return and thereby create a mean-variance efficient portfolio. The question of how many stocks are required to create an efficient portfolio has been thoroughly examined, but there are still different opinions on the matter. Evans and Archer (1968) determined that a portfolio of ten different stocks is sufficient to obtain all benefits of diversification. Later, Statman (1987) claimed that 30 stocks are the desired amount to achieve all diversification benefits. In our paper, we analyze how diversification evolves when adding sectors instead of adding stocks one by one. When each sector contains a number of stocks greater than one, we expect to see a very steady level of volatility. Figure 5.1 visualizes the steady levels we expected to see.

Todd Mitton and Keith Vorkink (2007) developed a mean-variance-skewed model where investors have a preference for skewness. In their paper, they found support for their model, where investors under diversify to obtain positive skewness. By achieving higher skewness in a portfolio, investors aim for a trade-off between variance and skewness. In our paper, we investigate how skewness varies and how the probability distribution of returns develops for different portfolio sizes.

Even though diversification benefits are well known and documented, investors might find incentives to hold narrow portfolios to increase their chances of returns exceeding the market benchmark. Levy and Livingston (1995) found that with the introduction of transaction costs and superior information of sectors, there are incentives to hold *stronger*[2] securities. Empirically, Kazperczyk, Silam, and Zheng (2005) found that mutual funds

[2] By stronger, we mean securities backed by superior information.

holding narrower portfolios in fewer industries performed better than mutual funds holding more diversified portfolios that more resembled a market portfolio. Portfolio sizes are an interesting aspect of our analysis. We want to look at how portfolios behave differently when adding more sectors to a portfolio, going from a narrow portfolio in few sectors to a portfolio closer aligned with the market portfolio.

We have drawn much inspiration from Bessembinder (2018) and his analysis on single stocks and their performance in comparison to a risk-free rate and the market portfolio. Bessembinder (2018) investigated how single stocks, as opposed to a full portfolio, develop and perform compared to 1-month-Treasury bills and a market portfolio. In his analysis, Bessembinder found that most stocks have a lifetime return less than the risk-free rate. When running similar calculations, but on sector data, we find that all sectors perform above the risk-free rate. Comparing our analysis to Bessembinder, we investigate many of the same statistical moments, with the fundamental difference being single stock versus entire sectors.

Atta-Durkua and Dimson (2018) investigated a strategy where a well-diversified long-term investor applies sector screening. This strategy involves going long the market and short one sector. Their results indicate that industry returns could diverge from the market return because the sector composition will keep changing over time. What Atta-Durka and Dimson utilize in their paper is closely related to our analysis when we exclude one sector from our portfolios. The main difference being Atta-Durka and Dimson going short the excluded sector, while we entirely exclude said sector from our portfolio.

3 Data

In our analysis, we use data provided by Kenneth R. French (2021) through the Tuck School of Business at Dartmouth College website. The data set we use contains every monthly return recorded for 30 different sectors in the period from July 1926 to November 2020. What sector a stock is attributed to is based on its four-digits SIC code at the time. The sector returns are calculated on a month-to-month basis using (4.1). The yearly return is computed as

$$r_y = \prod_{t=1}^{12} (1 + r_m) - 1, \tag{3.1}$$

where r_y denotes yearly return, and r_m denotes monthly return. For simplicity we have decided to use data in the time interval starting in January 1930, ending in December 2019 to obtain a period of 90 years.

At any time, each sector in the data set contains a number of stocks greater than one. The number of firms in a sector changes over time as new firms enter and old firms exit. The financial sector was the largest recorded, containing $1,363$ different firms in July 1996. The coal sector was the smallest recorded, with two firms in 1980. We provide more descriptive statistics in table A.1 in the appendix.

Our data set contains two separate subsets. The first subset calculates returns using a value-weighted approach. The second subset calculates returns using an equally weighted approach. We apply the approach of equally-weighted assets for our constructed portfolios. Thus, we choose to operate with the second subset to apply the same approach to our portfolios and sector returns.

3.1 The Market Portfolio and Risk-Free Rate

The risk-free rate is an investment that carries such a low risk that it becomes negligible. In this analysis, we have constructed the risk-free rate from the 3-month U.S. Treasury bills. As our data is gathered from the U.S. market, we choose to use U.S. Treasury bills. Our U.S. Treasury bill data start from 1934, while we use sector data dating back to 1930. Thus, we face a four-year gap in our data. To circumvent this gap, we invest the risk-free portfolio in the market portfolio during the years between 1930-1934. In 1934, we reinvest in Treasury bills. This strategy gives our risk-free portfolio the same starting date as our other portfolios. We have chosen this method to more easily compare our sector excluded portfolios to the risk-free portfolio.

The market portfolio is a portfolio containing all assets available in the market. We have constructed the market portfolio as an equally weighted portfolio, where every asset has an equally large proportion in the portfolio, regardless of the size of the asset proportionally to the other assets. The market portfolio then becomes the most diversified portfolio we can create, and it projects the overall movement of the economy. The market portfolio is the benchmark to beat for investors as they look to construct portfolios that can potentially net higher returns but with higher risk.

4 Theory and Method

In this section, we will present the methodological framework of our analysis and highlight what metrics we use to find our results. Additionally, we emphasize some important assumptions we make in our analysis.

4.1 Return and Volatility

The data we base our analysis on presents monthly or annual returns in the simple form. The returns for a sector, or portfolio, i in period [3] t is calculated as

$$r_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}. \tag{4.1}$$

Here $P_{i,t}$ denotes the price, or value, of sector i at time t, while $P_{i,t-1}$ indicates the same thing for time $t-1$.

Evaluating the performance of a portfolio with a time horizon of 90 years using day-to-day returns or month-to-month returns is not necessarily the optimal metric. We would rather use a metric of returns that better visualizes our extended timeline. To this end, we use annualized returns, which are computed as the geometric average of one plus the return rate each year

$$R_i = \left(\prod_{t=1}^{T} (1 + r_{i,t}) \right)^{\frac{1}{T}} - 1. \tag{4.2}$$

Here R_i denotes the annualized returns of portfolio i, and $r_{i,t}$ is the return for year t. The annualized returns are the yearly return needed to equal the total return of an investment when also taking compounding of interest into account.

When working with financial data, we have a couple of well-known and helpful statistical tools at our disposal. Firstly, we want to compute the unconditional expected return of an investment. The unconditional expected return is the return we expect from holding a portfolio for one period, based solely on the historical returns of the portfolio. The expectation is calculated as the arithmetic mean of all observations

$$E[r_i] = \frac{1}{T} \sum_{t=1}^{T} r_{i,t}. \tag{4.3}$$

The expectation is known as the first moment of a random variable.

We will also use the expectation operator in combination with the annualized returns, but in this case, we will be summing over the different portfolio choices rather than time. However, the general principles of (4.3) are still the same, in the sense that we are looking for the arithmetic average. Later in the analysis, we compute the expectation of annualized returns based on the number of sectors included in the portfolio. If we now let N denote the

[3] Period, in this case, indicates a given month or a given year.

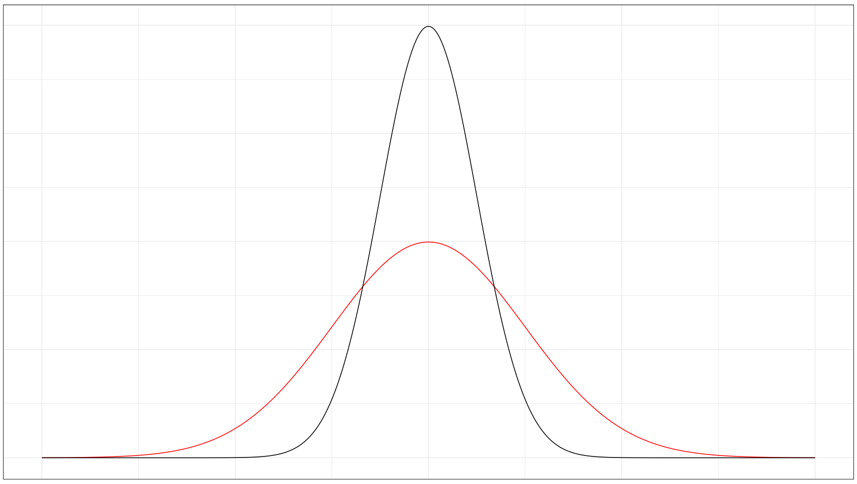


Figure 4.1:
This figure illustrates the second statistical moment, variance, or standard deviation. In this particular case, we use a normal distribution to illustrate, where the graph in red has a higher variance than the black one. Noticeable by the fact that the red graph is substantially more spread out around its mean value.

number of portfolios containing a given number of sectors, the equation will be

$$E[R_i] = \frac{1}{N}\sum_{i=1}^{N} R_i. \tag{4.4}$$

The expected returns is only one piece of the information we would like to compute. We would also like to know how much the returns vary around this point. This spread is measured by the variance, also known as the second moment of a random variable. In figure 4.1 we illustrate two cases of a normal distribution. The distribution in red has a higher variance than the one in black and is more spread out around the mean value. The variance is the expected value of the squared deviation from the mean

$$Var(r_i) = E[(r_{i,t} - E[r_i])^2]. \tag{4.5}$$

The standard deviation, also known as the volatility, is the square root of the variance. We denote the volatility σ and compute it on a sample as

$$\sigma_{r_i} = \sqrt{Var(r_i)} = \sqrt{\frac{\sum_{t=1}^{T}(r_{i,t} - E[r_i])^2}{T}}. \tag{4.6}$$

The volatility and unconditional expectation are the traditional ways of thinking about an investment's risk and return.

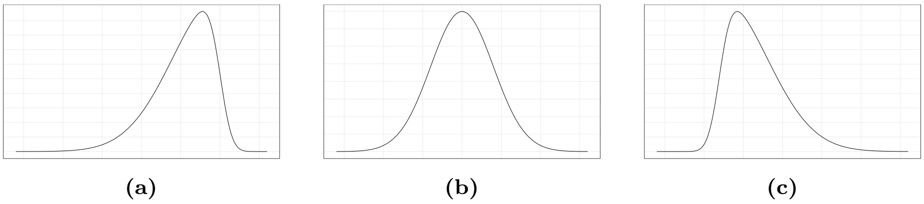


Figure 4.2:
This figure illustrates the concept of skewness, using the normal distribution as an example. It shows a negatively skewed distribution in panel (a), a distribution with zero skewness in panel (b), and a positively skewed distribution in panel (c).

4.2 Skewness and Kurtosis

As Carol Alexander (2008) describes, the variance could be denoted as μ_2, a notation to easier differentiate statistical moments of higher orders. The kth moment is defined by

$$\mu_k = E\left[(r_{i,t} - E[r_i])^k\right]. \tag{4.7}$$

Skewness and kurtosis are known as the third and fourth *standardized* moments. A standardized moment is defined as the kth moment divided by σ^k.

The skewness of the distribution is a measure of the probability distribution's asymmetry, with a positive, negative, or zero value. Visually, skewness can be recognized by a longer tail in either direction. Figure 4.2 visualizes different levels of skewness. To compute the skewness of financial returns, we use the following equation

$$Skewness = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{i,t} - E[r_i]}{\sigma_{r_i}}\right)^3. \tag{4.8}$$

Kurtosis measures how fat the tails of the distribution are. As a benchmark, the standard normal distribution has a kurtosis of three. In our analysis, we report the excess kurtosis as a measure of kurtosis exceeding the normal value. For financial returns, we compute the excess kurtosis as

$$Excess\ Kurtosis = \frac{1}{T}\left(\frac{\sum_{t=1}^{T} r_{i,t} - E[r_i]}{\sigma_{r_i}}\right)^4 - 3. \tag{4.9}$$

In figure 4.3 we show an example of distributions with different levela of excess kurtosis. This particular example is based on a t-distribution, with different degrees of freedom, and as such, the excess kurtosis value is also different.

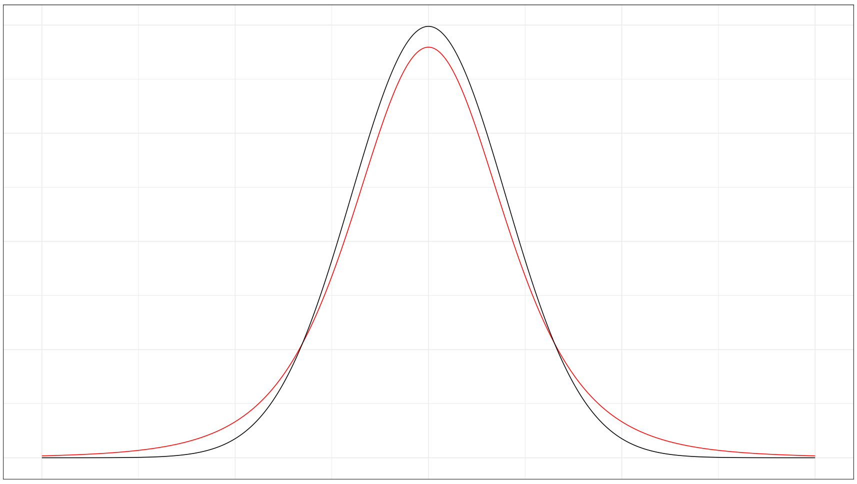


Figure 4.3:
This figure illustrates the fourth statistical moment, kurtosis. A higher level of kurtosis is noticeable by the fact that the distribution has fatter tails. In this case, the distribution in red has a higher value of excess kurtosis than the black one. This particular example is based on a t-distribution with different degrees of freedom.

4.3 Portfolio Construction

We take a systematic approach to construct different portfolios. We construct every possible portfolio containing s sectors. We then get that for s sectors included there are $C(30, s)$ possible portfolios of sector combinations, where

$$C(30, s) = \frac{30!}{s!(30-s)!}. \tag{4.10}$$

From 4.10 we know that the number of possible portfolios lies in the closed interval of 1 to 155,117,520, depending on the number of sectors we are excluding. Having constructed every possible portfolio, we can look into how they have performed historically and compute expectations and volatility as given in expressions (4.3) and (4.6). As mentioned, $C(30, s)$ can become extremely large. To save time while running our simulations and computations, we limit the set of portfolios. For every case where $C(30, s)$ exceeds half a million, we randomly sample 500,000 portfolios and use this sample to compute estimated values for expectation and volatility. A sample size of half a million gives us a good trade-off in computation time versus estimation accuracy.

4.4 The Buy-and-Hold Strategy

One of the investment strategies we use to investigate the effect of sector exclusion is the buy-and-hold strategy. The buy-and-hold strategy is one in which an investor takes a long position in a portfolio consisting of s sectors, and as the position is opened, each sector is equally weighted. However, as time goes on, we do not perform any rebalancing, resulting in continuously updating the weights. Our utilization of annualized returns, given by (4.2), will be key here, particularly as a reliable way of comparing the performance of different portfolios. When comparing different portfolios, we rank them based on the value created. A higher annualized return means a higher value created over the given time span.

In this long-term case, we are not going to compute the risk in the traditional way, as given by (4.6), but rather as the standard deviation of annualized returns. To be more precise, we compute the annualized returns of every portfolio containing s sectors before computing the standard deviation of this subset of data. We define *variability* as the standard deviation of annualized returns and compute it as

$$Variability = \sqrt{\frac{\sum_{i=1}^{N} \left(R_i - \overline{R}\right)^2}{N}}. \tag{4.11}$$

Here, N is the number of different portfolios containing the given number of sectors[4]. As an example, we look at the case where portfolios consist of a single sector, meaning that we have 30 different portfolios, one for each sector, all with a different level of annualized return. The standard deviation of annualized returns will then be calculated as the square root of the expectation of these 30 observations' deviation from the mean. We believe that combining the expectation and standard deviation of annualized returns will give a good indication of the effect of sector exclusion.

Investors using a buy-and-hold strategy are susceptible to market bubbles. This risk is a consequence of the weighting of each sector never being rebalanced, and therefore exposure to sectors that have performed exceedingly well for some time will be rather high. Without rebalancing, the effect of a bubble bursting would be severe, in terms of losses, compared to a case where the overexposure was removed by rebalancing. In their paper, Maeso and Martellini (2020) discuss the average rebalancing strategy compared to the buy-and-hold strategy. After controlling for factor exposures, they find that the average outperformance of the rebalanced strategy with respect to the corresponding buy-and-hold strategy remains substantial at an annualized level above 100 basis points over a five-year time horizon for stocks in the S&P 500 universe. Based on these findings, we also look into a long-term investment strategy where the portfolio weights are rebalanced with regular time intervals.

[4]Meaning that N equals $C(30, s)$.

4.5 The Rebalancing Strategy

The second investment strategy we will be using is a rebalancing strategy. The rebalancing strategy is quite similar to the buy-and-hold strategy, but with one significant difference. In the rebalancing strategy, we assume that portfolios held by investors are rebalanced back to equal weights at regular time intervals. We are using time intervals of ten years, and as we look at data spanning back to 1930, this implies rebalancing at the beginning of every decade. A strategy of this nature can be thought of as opening a buy-and-hold portfolio at the beginning of a decade, and at the beginning of the next decade, closing said position and opening a new one containing the exact same sectors. The use of such a strategy is beneficial to avoid getting overexposed to particular sectors, and by the results of Maeso and Martellini (2020) tends to outperform the buy-and-hold strategy. We would also like to point out that we deliberately set the time intervals between rebalancing to a constant duration rather than attempting to read the market. Additionally, we assume no transaction costs associated with the rebalancing.

To compare the different portfolios using the rebalancing strategy, we will use the same metrics for the buy-and-hold strategy. We evaluate performance by annualized returns, and we assess the level of risk by measuring the variability for specific portfolio sizes.

4.6 The Portfolio Frontier

The idea of the portfolio frontier is based on Markowitz's rule about expectation and variance. As stated in his paper (Markowitz 1952) the mean-variance rule says that an investor would (or should) select portfolios that give the minimum variance for a given expected return or more. Vice versa, the maximum expected return for a given variance or less. Selecting a portfolio means selecting the weights associated with each asset, or in our case, the weights associated with each sector. There are techniques for computing the set efficient portfolios. Markowitz does not go into detail on how, and neither shall we. Nevertheless, some key factors are worth mentioning.

For one, we calculate the expected return of a portfolio as a weighted average of the individual assets' expected return. If we let x_i denote the portion of an investors fortune invested in asset i and $E[r_i]$ be the unconditional expected return on asset i, then the anticipated return of a portfolio would be calculated as

$$E[r_p] = \sum_{i=1}^{S} x_i \cdot E[r_i], \tag{4.12}$$

where S indicates all the different assets available. Additionally. we apply the constraint that

$$\sum_{i=1}^{S} x_i = 1 \tag{4.13}$$

The constraint (4.13) says that the whole investment amount is invested in the s assets that make up the portfolio[5]. Additionally, we also assume $x_i > 0$, meaning that the investor may only take long positions. This makes (4.12) a weighted average of expected returns, with x_i as non-negative weights.

The volatility of the portfolio depends on the weights associated with each different asset. Additionally, the portfolio volatility depends on the volatility of the assets and how the different assets move in relation to each other, their covariance. The covariance of two random variables is computed as

$$\sigma_{r_i r_j} = E\left[(r_i - E[r_i])(r_j - E[r_j])\right] \tag{4.14}$$

Using the previously established notation, we can compute the variance of a portfolios return as

$$Var(r_p) = \sum_{i=1}^{S} x_i^2 \sigma_{r_i}^2 + 2 \sum_{i=1}^{S} \sum_{j>i}^{S} x_i x_j \sigma_{r_i r_j}, \tag{4.15}$$

with the standard deviation of the portfolio return equal to the square root of this variance. These equations are also provided by Markowitz (1952) in his paper. We have only tweaked them to match the notation we have established.

In our analysis, the portfolio frontier is found through a numerical approach, intending to minimize the variance for different levels of expected returns. We assume that ten years worth of data is sufficient to construct the portfolio frontier properly. Our portfolio frontier is then computed starting in the year 1940. We include the idea of the portfolio frontier to visualize how the set of efficient portfolios change when single sectors are excluded.

5 Main Analysis

As discussed in section 4, the standard way of thinking about the risk of an investment is in the form of the investments' volatility. It is well known that the two main components of risk are systematic and unsystematic risk, referring to the risk associated with the entire market and the risk associated with a specific industry or security, respectively. However, in the case examined here, we also have the time effect playing its part, something we

[5] $s \leq S$, meaning the investor invests in some or all sectors available.

will consider in our analysis. We will dive deeper into the actual results of the two strategies for long-term investment we outlined in sections 4.4 and 4.5, and look into the different elements of risk associated with excluding one or more sectors. Finally, we will study the portfolio frontier and look at how the frontier shifts as some sectors are excluded.

5.1 Analysis of Short Term Movement

This section focuses primarily on the traditional way of thinking about a portfolio's risk and return. What is interesting to study is how the expected return and volatility will change as we change the size of a portfolio, starting from one sector and ending up with the market portfolio. We want to investigate whether we can find any odd movement of sector portfolios on a short-term horizon. We do expect sector portfolios to follow the theoretical idea of decreasing volatility as we add more sectors, ending up with a level of volatility equal to the covariance. We are looking at monthly returns for equally weighted portfolios, and we compute their expected return and volatility based on a five-year period of historical data. Our primary focus will be to study the portfolio volatility and comparing it to the standard error[6] of mean portfolio returns, depending on the number of included sectors, and thus getting a better understanding of the risk associated with sector exclusion.

We compute expected monthly returns, monthly standard deviations, skewness, and excess kurtosis. All these are calculated as mean values, meaning for instance, that we compute the volatility of all portfolios with two sectors included, taking the average and plotting it, as shown in figure 5.1. In this particular figure, we plot the results of these calculations based on historical data spanning the time period from November 1958 to October 1963. We have carried out the exact same computation for several time periods, each randomly sampled from the 90 year timeline. As a result of this, we have seen that the values for expected return and volatility depend on the time period sampled, but the development of these metrics as more sectors are included stays consistent. The choice of November 1958 to October 1963 is simply arbitrary, and our focus will be more directed to relative changes, as this is consistent across sub-periods, whereas the actual number may vary. In other words, the key parts of figures 5.1 and 5.2 are not the values on the y-axis, but rather the shape of the graph. From these figures we see that including more sectors does indeed reduce the portfolio volatility, however, the diversification benefits are rather small, but the marginal effect of including more sectors is larger when the initial portfolio is small. One particular case

[6] Standard error being the normal notation for the standard deviation for the distribution of means.

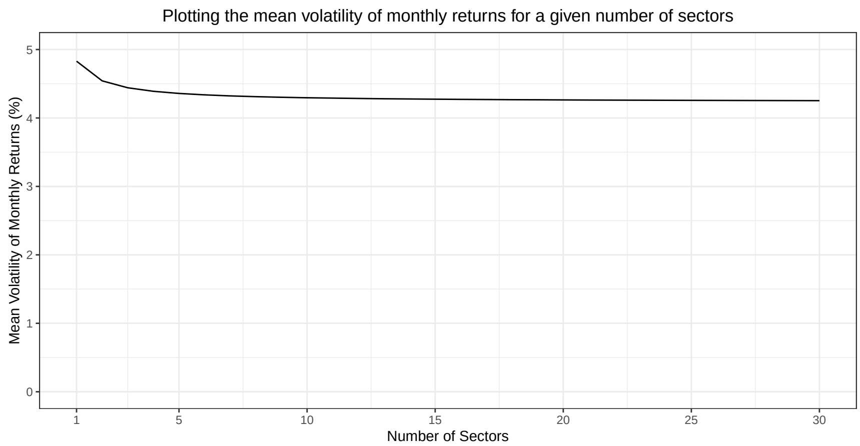


Figure 5.1:
This figure shows the mean volatility of monthly portfolio returns depending on the number of sectors is included in the portfolios. This particular case is computed based on monthly returns from November 1958 to October 1963, which is a randomly sampled time period from the 90 years of data.

we noted, which will come back to later, is the difference in volatility between 20 and 25 included sectors. The results show that the volatility only decreases by 0.149% when including an additional five sectors, from 4.263% to 4.257%.

We expect the expectation of monthly returns to be constant at the level of the expected return for a randomly selected sector. The reasoning comes from the fact that we take the expectation over a set of expected values. From figure 5.2 we can see that this does in fact hold for our sample time period, and having run several other simulations we can also say that this is consistent across different time periods. Sampling different time periods may have an impact on the level of mean expected return, but this level does not change depending on the number of included sectors.

For skewness and kurtosis, we follow the same method as for return and volatility. Calculating the average for every portfolio of a given size. We made two noticeable observations. First, the numerical values of these moments do not appear to vary much depending on the size of the portfolio. Second, unlike volatility, skewness and kurtosis develop in different ways depending on what time period we sample. In figures 5.3 and 5.4 we show the development of skewness and excess kurtosis, depending on portfolio size for two different time periods. In one case we look at the same time period as we did for volatility and expectation, while the other period is September 2012 to August 2017, again being randomly selected. The results of these figures show no clear pattern in the development of skewness and kurtosis. We carried out the same computation for several randomly selected time periods,

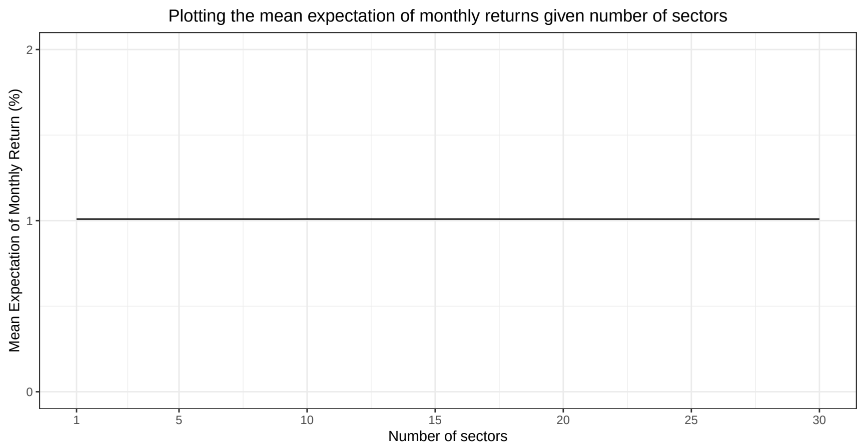


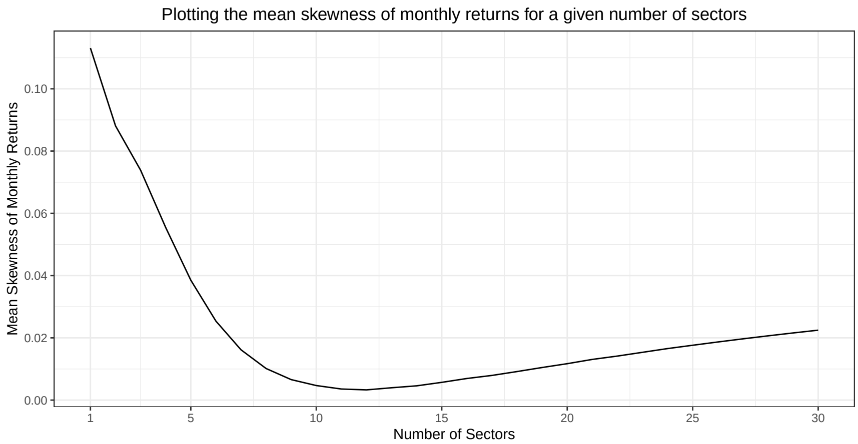
Figure 5.2:
This figure shows the mean expectation of monthly portfolio returns depending on the number of sectors included in the portfolios. This particular case is computed based on monthly returns from November 1958 to October 1963, which is a randomly sampled time period from the 90 years of data.

each resulting in different values and development patterns for skewness and kurtosis.

As we have already seen, the variance of a portfolio does not change too much when one includes more sectors. However, it is reasonable to expect that a limited number of sectors perform exceedingly well, or exceedingly bad over a given time period. This particular idea could lead to some inherent risk when excluding sectors. Under the assumption of an efficient market, no one would be able to consistently pick the high-yield sectors. With this assumption in mind, we will look closer into how much the expected returns for portfolios of a given size vary.

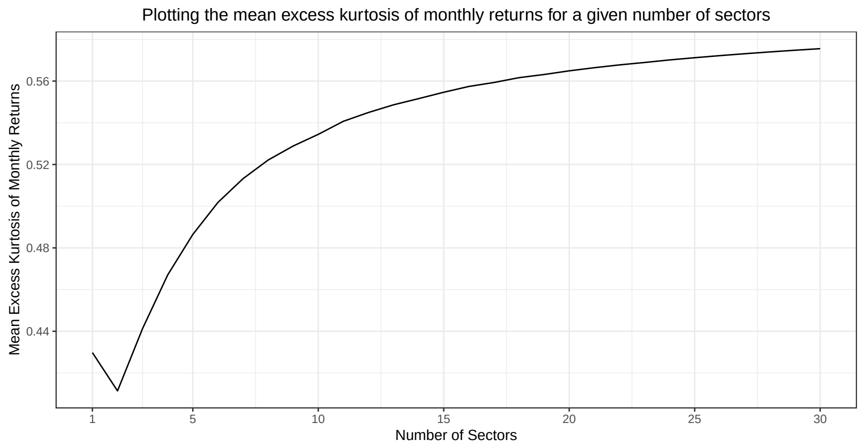
In figure 5.5 we show two histograms where we plot the density for different levels of expected returns. Figure 5.5a plots the density of expected returns for portfolios where we exclude 10 sectors. Figure 5.5b plots density expected returns for portfolios where we exclude 5 sectors. We calculate the expected return of a portfolio using the mean of the expected returns for the included sectors. We do not know the distribution for the expected return of the included sectors, but we do not need to. The central limit theorem tells us that the distribution of expected portfolio returns is approximately normal. There are two key observations we want to point out regarding figure 5.5. First, we can see that the mean of the distribution is the same, which corresponds nicely with what we discussed in context with figure 5.2. Secondly, and more importantly, looking at the scaling of the x-axis we can see a noticeable difference in the variance for the two distributions. To be more precise, in the case of portfolios of size 20, the standard error of monthly

(a)



(b)

Figure 5.3:
This figure shows how the mean skewness of monthly returns develops as more sectors are added to a portfolio. In panel (a) the data runs from November 1958 to October 1963, while in panel (b) the data is from September 2012 to August 2017. Note the difference in development patterns for the two periods.



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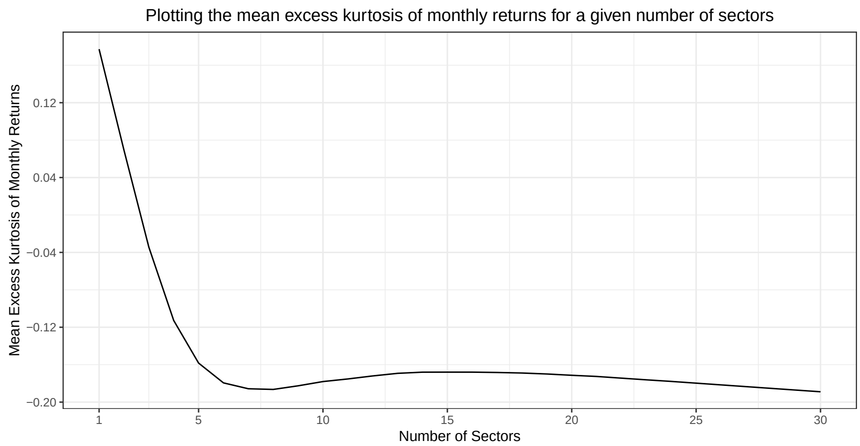
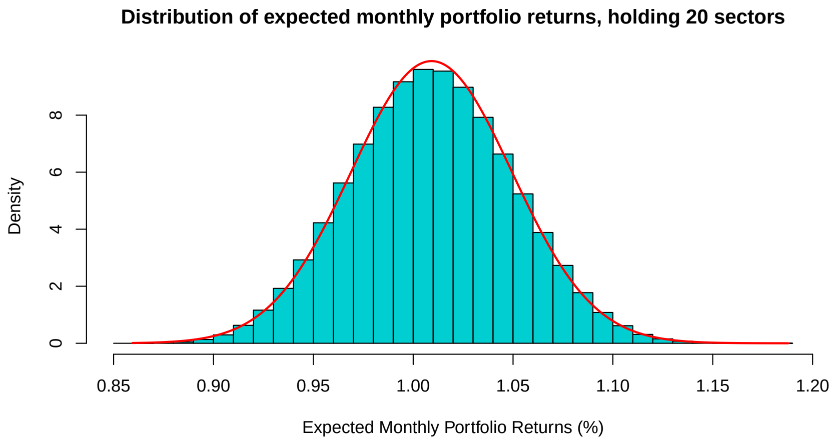


(b)

Figure 5.4:
This figure shows how the mean excess kurtosis of monthly returns develops as more sectors are added to a portfolio. In panel (a) the data runs from November 1958 to October 1963, while in panel (b) the data is from September 2012 to August 2017. Note the difference in development patterns for the two periods.



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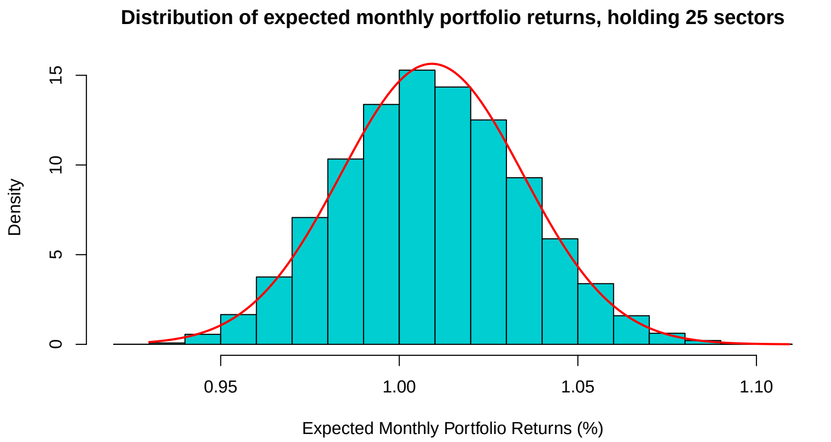


(b)

Figure 5.5:
This figure shows the distribution of expected monthly returns, in percent, for portfolios of size 20 and 25. In panel (a) we have excluded ten sectors, while in panel (b) five sectors are excluded. We calculate the expected returns over the period starting from November 1958, ending in October 1963. We have also superimposed a normal curve with the appropriate mean and variance.

portfolio returns is found to be 0.0403%, while for a portfolio of size 25 it is found to be 0.0255%[7]. Previously we showed that the volatility of a portfolio decreased by as little as 0.149% when increasing portfolio size from 20 to 25, and from what we have just discussed we can find that the standard error of monthly portfolio returns decreases as much as 36.75% when increasing portfolio size in the same manner. A decrease in standard error does not only have a basis in financial theory but also from a purely mathematical standpoint. The standard error is known to decrease when the sample size increase, as a matter of fact, the standard error is inversely proportional to the square root of the sample size.

If we use the fact that the distribution of expected portfolio returns, by the central limit theorem, is approximately normal we can compute confidence intervals for expected monthly returns. Knowing that, for a normal distribution, a 95 percent confidence interval contains every output within two standard deviations away from the mean, we can say that 95% of all portfolios of size 20 have an expected monthly return in the interval $[0.929\%, 1.090\%]$. Similarly, for portfolios of size 25 we have that the expected monthly return for 95% of the portfolios lies in the interval $[0.958\%, 1.060\%]$. Running these calculations shows that the 95 percent confidence interval is 1.6 times larger when excluding ten sectors compared to five. Running the same simulations and computations for several different, randomly selected, time periods we find that these results are persistent across the time periods.

This difference in marginal volatility versus marginal standard error when excluding five extra sectors could be an indicator that the classical measure of risk might not be the best course of action for the case of sector exclusion. We also provide, in figure 5.6, a graph showing the sample standard error of monthly returns for all different portfolio sizes. In table A.2 we provide an in-depth overview of the mean expectation, volatility, skewness, excess kurtosis, and standard error of monthly returns for the time period November 1958 to October 1963.

[7]These values for the standard error is rounded to four decimal places.

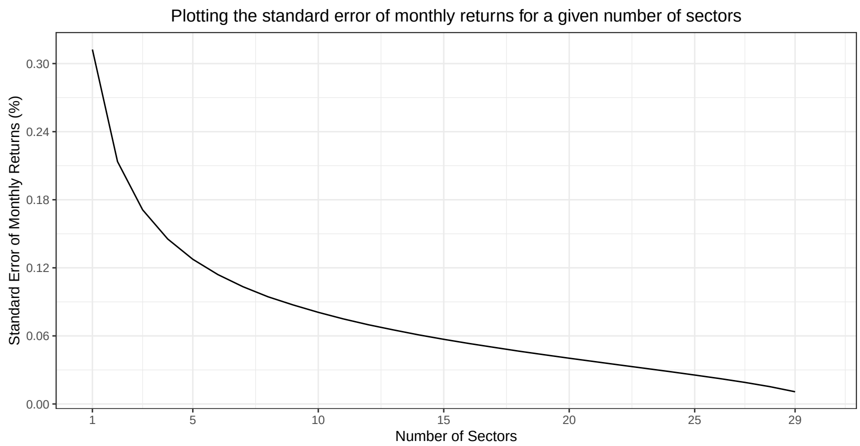


Figure 5.6:
This figure shows the standard error of monthly returns, computed over a five-year period starting in November of 1958 and ending in October 1963. We ran several simulations, each sampling a random time period, and found that the shape of the graph was persistent in all cases.

5.2 Market Portfolio and Risk Free Rate

Imagine the idea of investing a single dollar in the market at the beginning of 1930 and track the value of the dollar over the 90 years until the end of 2019. By tracking this dollar we can observe how the market develops and this will give us an indication of how we can expect our portfolios to develop. Figure 5.7 shows the dollar invested using both the buy-and-hold and the rebalancing strategy. Additionally, we include the risk-free rate as a point of comparison to a risk-free investment.

Figure 5.7 shows that the value created from investing in the market portfolio is quite substantial. The value of the single dollar raises to $91,259.69 and $66,247.73 for the rebalancing- and the buy-and-hold strategy respectively, corresponding to 13.53% and 13.13% annualized returns. The difference in value created from the rebalancing strategy and the buy-and-hold strategy indicates that rebalancing to avoid overexposure to any one sector provides higher returns in the long run. This is in line with the results of Maeso and Martellini (2020). For the risk-free investment, on the other hand, we notice a significantly lower payoff. The value of a single dollar grows to $17.77, which corresponds to 3.49% annualized returns. The fact that investing in the market outperforms a risk-free investment is not news.It is in fact a very well-studied field, known as the equity premium puzzle. However, what would be of greater interest to us is to compare the value created by investing in portfolios containing a different number of sectors. We will analyze these portfolios using both the buy-and-hold strategy and the rebalancing strategy.

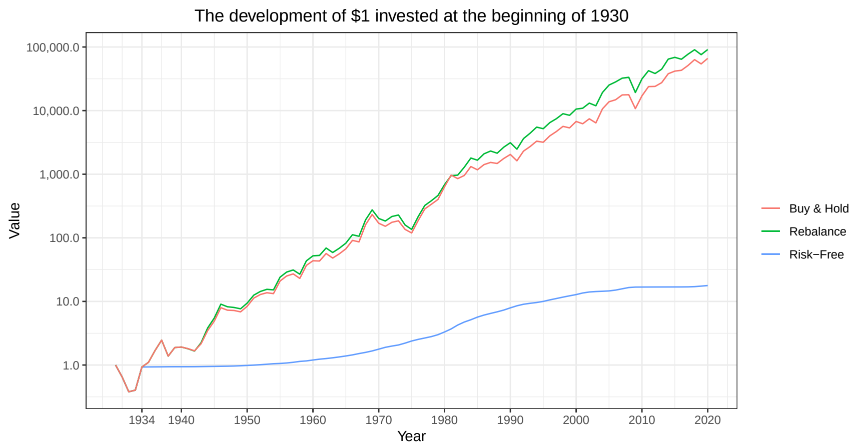


Figure 5.7:
This figure shows the development of one dollar invested at the beginning of 1930, following three investment strategies. The green line shows a rebalancing strategy, where the initial investment is in an equally weighted market portfolio, and weights are rebalanced back to equal every decade. The red line shows the buy-and-hold strategy, where the initial investment is in an equally weighted market portfolio, and the portfolio is never rebalanced. The blue line shows an investment in the risk-free asset. The y-axis is logarithmically-scaled to make the plot more readable.

5.3 Applying The Buy-and-Hold Strategy

First, in figure 5.8, we look at a scenario where portfolios consist of only a few sectors. In this figure, we plot the value of the top three performing and the bottom three performing portfolios[8]. The main idea here is to visualize the gap in value created, depending on what sector or sectors one is exposed to. In figure 5.8a portfolios contain only a single sector, and in figure 5.8b each portfolio contains five sectors. The figure shows quite clearly that holding a single sector over such a long period of time is a rather volatile strategy, with the top-performing sector, tobacco products, providing annualized returns of 16.47% and being valued at $909,939.60 at the end of the period. By comparison, the worst-performing sector, coal, provided only 6.30% annualized returns, for a final value of $244.01. We notice that even the worst-performing portfolio, in this case, does outperform the risk-free investment, by almost three percentage points when measuring annualized returns. As we noted in section 3, each sector contains several different stocks, the coal sector in particular contained, on average, a little over nine stocks during this time span. Following the indication of Evans and Archer (1968) where 10 stocks are enough to obtain all diversification benefits, having an average of 9 stocks in the coal sector we can assume that the potential for significant losses is diversified away. With the di-

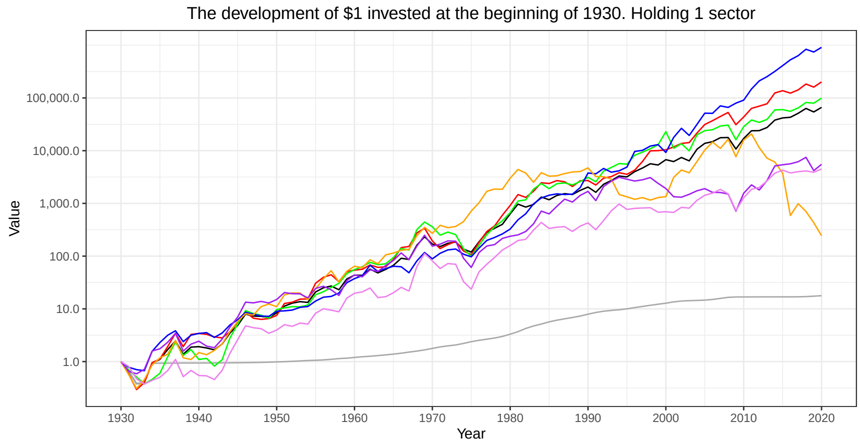
[8]The ranking of different portfolios performance is based on the portfolios annualized returns.

versification already present in each sector, it helps explain why even the worst-performing portfolios yield higher annualized returns than the risk-free rate. On the other hand, we also note the upside of picking the right sector, as the tobacco products section is valued at $\$909,695.59$ more than the coal sector, or in other words, it is worth $3,728.11\%$ of the coal sector. Comparing the different portfolio investments to an investment in the buy-and-hold market portfolio we find that the upside potential of single sector portfolios is huge, with the tobacco products sector being valued at 13.74 times the buy-and-hold market portfolio at the end of 2019.

Expanding the portfolio sizes to 5 sectors appears to have a significant effect on the variability of our portfolio population. We find that when we increase the portfolio size the standard deviation of annualized returns is reduced by 0.77 percentage points[9]. The top-performing portfolio with a value of $277,122.50, which corresponds to a value 4.18 of the market at the end of 2019. The difference between the top and bottom-performing portfolios is $272,178.45. We further expand the portfolio size to 25 and 29 included sectors. A natural assumption when we increase the portfolio size is that the trend of reduced variability will continue. In figure 5.9 we observe our one dollar investment, but this time the portfolio size is increased to 25 and 29. As expected the trend does continue. In the case of five sectors excluded, the gap between top and bottom has decreased to $54,435.68, and the top-performing portfolio being valued at 1.19 times the buy-and-hold market portfolio. For the single sector excluded case, the corresponding numbers are $31,368.82, and 1.03 times the buy-and-hold market. In table 5.1 we provide a summary of the numbers discussed so far in this section.

As an end note to this section, we point to the interesting observation that as many as 80% of portfolios containing 29 sectors have higher annualized returns than the market portfolio. Our takeaway from this observation is that there are a few sectors that have had a very low rate of return, and therefore many portfolios that exclude these low-performance sectors grant returns above market levels. In figure 5.10 we show what percentage of portfolios create more value than the market, for a given number of sectors included.

[9]See A.3 for full descriptive statistics.



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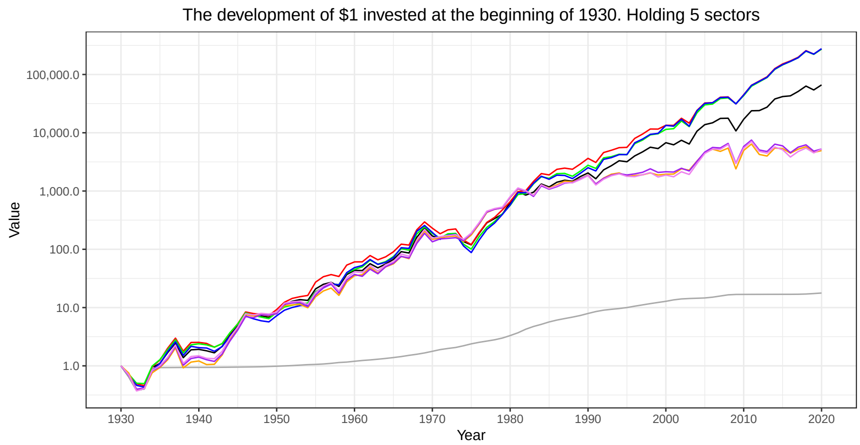
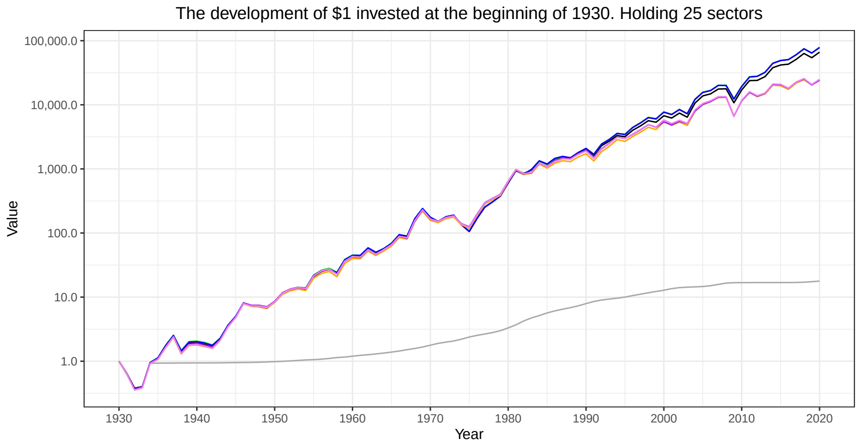


(b)

Figure 5.8:
This figure shows the result of applying the buy and hold strategy for portfolios consisting of one and five sectors, opening the position at the beginning of 1930 and closing at the end of 2019. In panel (a) we hold only a single sector in each portfolio, and in panel (b) we hold portfolios of five sectors. The graph shows the top three performing and the bottom three performing portfolios, with the market as a whole plotted in black and risk-free investment in dark grey, as guidelines. The y-axis of the graph is logarithmically-scaled to make the graph more readable.



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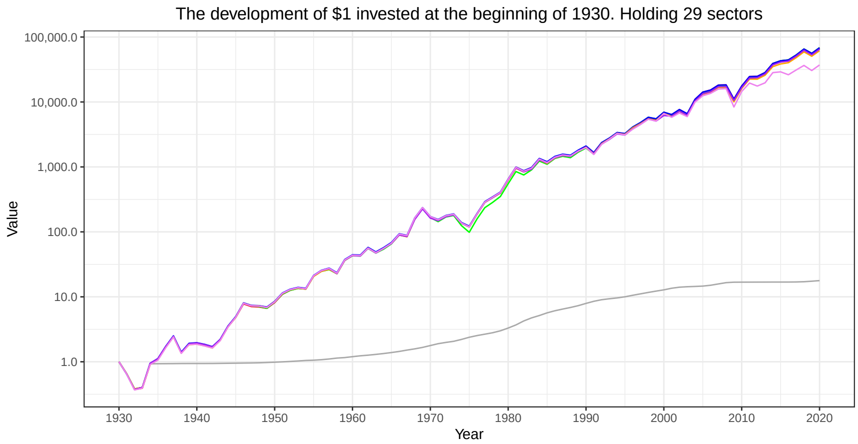


(b)

Figure 5.9:
This figure shows the result of applying the buy and hold strategy for portfolios excluding one and five sectors, opening the position at the beginning of 1930 and closing at the end of 2019. In panel (a) we exclude five sectors from our portfolios, and in panel (b) we exclude one sector. The graph shows the top three performing and the bottom three performing portfolios, with the market as a whole plotted in black and risk-free investment in dark grey, as guidelines. The y-axis of the graph is logarithmically-scaled to make the graph more readable.

Table 5.1:
This table summarizes the results of investing one dollar in different sized portfolios at the beginning of 1930, when applying the buy and hold strategy. We tabulate the end value and the annualized returns for the top- and bottom-performing portfolios of different sizes. We also show the end value as a percentage of the buy and hold market portfolio's end value and risk-free investment. All figures are rounded to two decimal places.

Panel A: Holding One Sector			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$909,939.60	16.47%	$244.01	6.30%
% of Market Portfolio Value		% of Market Portfolio Value	
1,373.54%		0.37%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
5,120,028.18%		1,372.99%	

Panel B: Holding Five Sectors			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$277,122.50	14.94%	$4,944.05	9.91%
% of Market Portfolio Value		% of Market Portfolio Value	
418.31%		7.46%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
1,559,306.80%		27,819.07%	

Panel C: Excluding Five Sectors			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$78,508.46	13.34%	$24,072.78	11.86%
% of Market Portfolio Value		% of Market Portfolio Value	
118.51%		36.34%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
441,749.68%		135,452.19%	

Panel D: Excluding One Sector			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$68,523.72	13.17%	$37,154.90	12.40%
% of Market Portfolio Value		% of Market Portfolio Value	
103.44%		56.08%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
385,567.76%		209,062.38%	

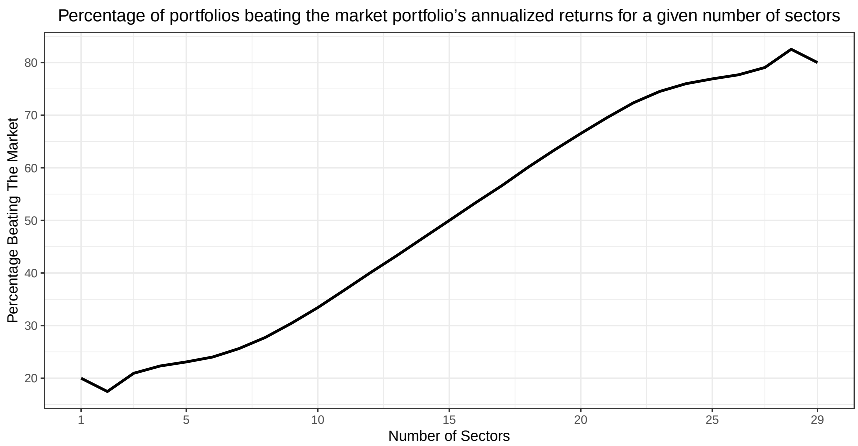


Figure 5.10:
This figure shows the percentage of portfolios that create more value than the market portfolio over 90 years, depending on the number of sectors included in the portfolios.

5.3.1 Expectation and Variance

So far, we have focused our effort solely on comparing the performance of a few selected portfolios in the presence of sector exclusion. In this section, however, we will look more closely into the expected portfolio returns in an annualized form. Furthermore, we investigate the development of the standard deviation of the annualized returns, or variability, for varying portfolio sizes.

Under the assumption that an investor has no expertise in any one sector, and as such picks sectors to his portfolio arbitrarily, what kind of future annualized returns could he expect based on these 90 years of historical data? One might expect that the expectation of annualized returns should follow a trend similar to the expectation of simple returns, as presented in figure 5.2. However, as a consequence of the diversification benefits, this is not the case. We noted in section 5.1 that the volatility of monthly returns for an investment in sectors did not decrease much when more sectors were included, but the diversification benefits were still present, and it is for this reason that the expectation of annualized returns follows the development shown in figure 5.11. As the figure shows, an investor picking sectors at random would always increase his expected annualized return by including more sectors in his portfolio, until it reaches the market portfolio level of annualized returns. This result is an interesting effect, and it stems from the decreasing volatility when increasing portfolio size, and can be shown through the use of a Taylor expansion. Even though the following explanation is for annualized returns over a two-year time period, the explanation holds also for longer time periods. We show it using a two-year approach in an attempt to make it easier to follow.

Letting $f(x) = \sqrt{x}$ we can approximate this as a second order Taylor polynomial around the point of $x = a$ as

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$
$$f(x) \approx \sqrt{a} + \frac{1}{2\sqrt{a}}(x-a) - \frac{1}{4a^{3/2}}(x-a)^2.$$

If we let x be a stochastic variable giving the total return of an investment[10] over the course of two years, with an expected value of $E[x]$, then we can compute the value of $f(x)$ around this expectation as

$$f(x) \approx \sqrt{E[x]} + \frac{1}{2\sqrt{E[x]}}\left(x - E[x]\right) - \frac{1}{4(E[x])^{3/2}}\left(x - E[x]\right)^2.$$

We can find the expectation of $f(x)$ taking advantage of the fact that the expectation operator is a linear operator

$$\begin{aligned} E[f(x)] &\approx \sqrt{E[x]} + \frac{1}{2\sqrt{E[f(x)}}\left(E[x] - E[x]\right) - \frac{1}{4(E[x])^{3/2}}E\left[(x - E[x])^2\right] \\ &= \sqrt{E[x]} - \frac{1}{4(E[x])^{3/2}}E\left[(x - E[x])^2\right]. \end{aligned} \tag{5.1}$$

We know that $E\left[(x - E[x])^2\right] = Var(x)$, and we have already showed that the expected return of a portfolio is constant while the variance (volatility) decreases as more sectors are added. All of this put together explains why the expectation of annualized returns increase when portfolio size increase, as shown in figure 5.11.

As we discuss in section 4 we chose to evaluate the risk of an investment with a horizon of 90 years differently than an investment with a five-year horizon. For us to compare the different investments we then need to use economic intuition rather than solely looking at numbers. There are some interesting points we have found in our calculations. Comparably, the variance of any investment, long or short term, will have reduced variance when a portfolio's size is increased. Markowitz (1952) made diversification benefits very clear. Going further, we investigate the standard deviations of annualized returns for any given portfolio size. Figure 5.12 shows the downward trend for the variability as we add more sectors to our portfolio. If we compare this figure to figures 5.6 and 5.1, we see a very different form on our graphs. Our graphs for a five-year period are very similar in form. They have a marginally large reduction when we add sectors to an already small portfolio. As the portfolio

[10] And as such $\sqrt{x}$ would be the annualized returns over a two year period.

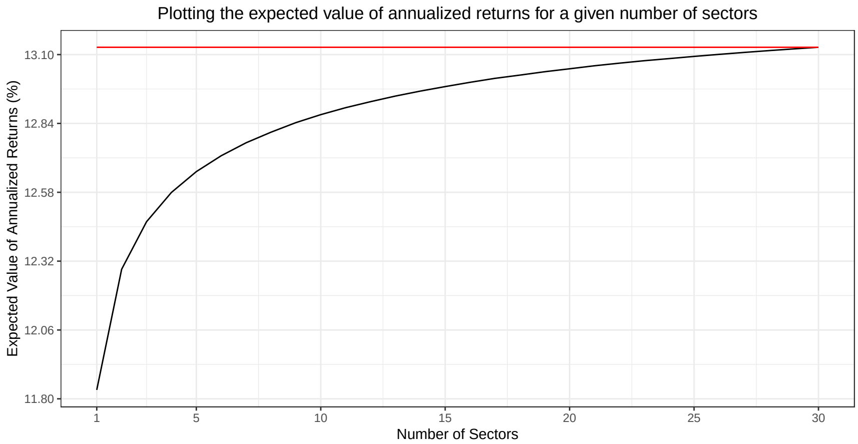


Figure 5.11:
This figure shows how the expectation of annualized returns develops, depending on the number of sectors included in a portfolio. The red line at the top of the graph shows the annualized returns of the buy-and-hold market portfolio.

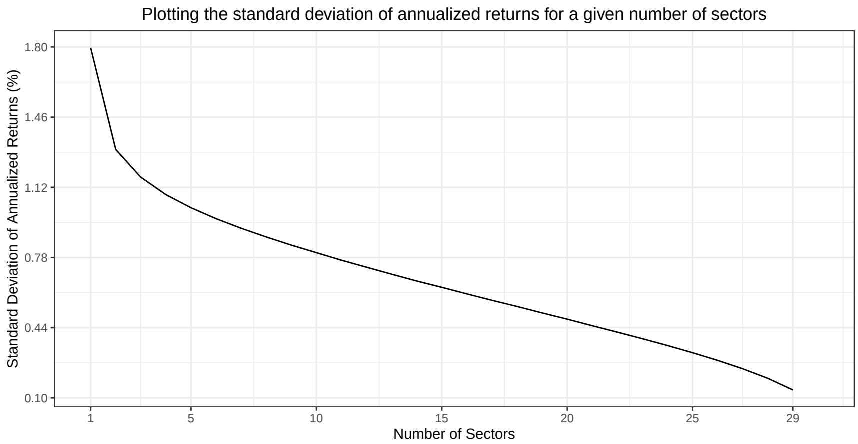


Figure 5.12:
This figure shows the standard deviation of annualized returns, or variability, computed across the population of portfolios of any given size. The standard deviation is reported in percent.

grows the marginal change is reduced. When we do the same for a 90 year period we can see that our graph does not follow the same movement pattern. Figure 5.12 shows that the marginal reduction in variability is not following a pattern of decreased value with an increase in the number of sectors. We also notice that the reduction in variability is, relatively speaking, much higher than for the volatility. When moving from one sector included in a portfolio to 29 sectors included the variability decreases by 92.30%, while the corresponding decrease in volatility is only 11.92%. Previously we also looked at the relative decrease in volatility, as well as the standard error of expected monthly returns when comparing portfolios of size 20 and 25. Revisiting that idea for this particular case shows a decrease of 33.77% for the standard deviation of annualized returns, which is way above the relative decrease of volatility and is more comparable to the relative decrease in standard error for expected monthly returns.

Lastly, it seems unlikely that figure 5.11 shows that the expected annualized returns for portfolios with only one sector excluded are lower than the expected annualized returns of the market portfolio. Even more so when we have shown that 80% of these portfolios produce annualized returns exceeding the market portfolio. However, by looking closer at our calculations we find some numbers that can help us explain why this statistic makes sense. On average, the portfolios that obtain annualized returns exceeding the market portfolio, do so by as little as 0.028 percentage points. For the portfolios that obtain annualized returns less than the market portfolio, we record an average of 0.14 percentage points below the benchmark. The worst performing portfolio records a difference as big as 0.72 percentage points below the benchmark while the best performing portfolio is only differencing 0.042 percentage points above the market.

5.3.2 Skewness and Kurtosis

We just showed that the portfolios that perform worse than the market portfolio tend to do so by a greater margin than the portfolios that exceed the market portfolio. From this observation, we can assume that the distribution of annualized returns is negatively skewed for larger portfolios. In an attempt to get a more complete overview of how the distribution of annualized returns looks for different portfolio sizes, we also compute the skewness and kurtosis. The outcome of these computations does support what we have just discussed. In particular, we note a skewness value of -4.66 when we exclude a single sector. A key takeaway here is that both the skewness and excess kurtosis hold values close to zero when we exclude many sectors. In figure 5.13 we show the skewness values for the population of annualized returns, given the number of sectors included in the portfolio. For the excess kurtosis, on the other hand, the value shot upwards as one passes 20

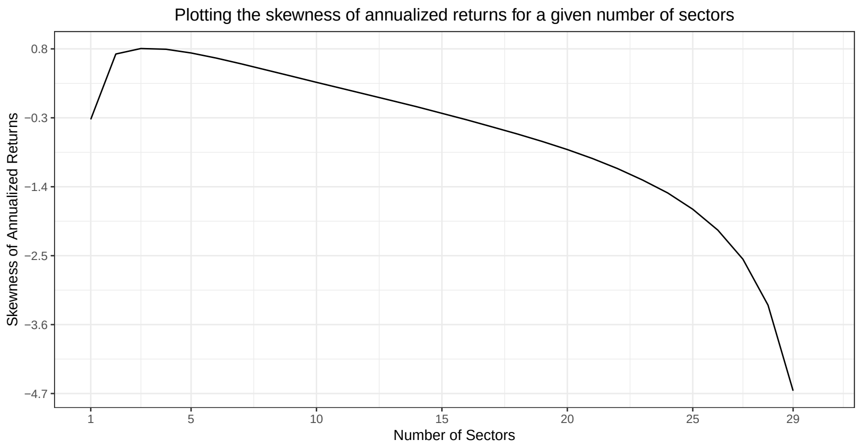
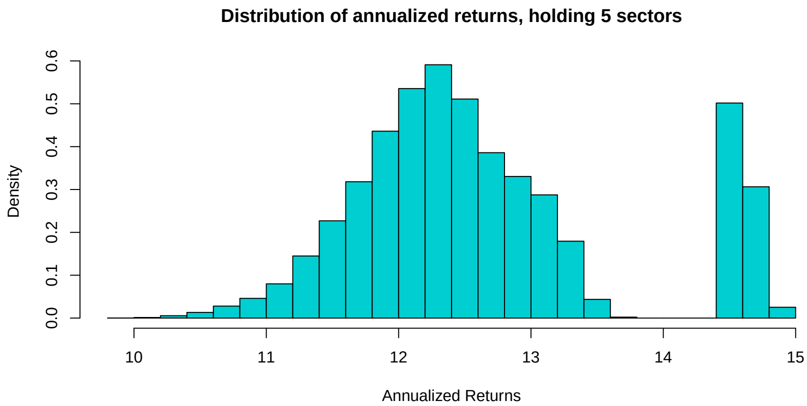


Figure 5.13:
This figure shows the development of skewness of annualized returns as one includes more sectors per portfolio, when applying the buy-and-hold strategy.

included sectors, reaching its maximum at 29 included sectors with a value of 21.22. The development of excess kurtosis is less relevant to us, but we show it in figure A.1 in the appendix.

What the figures we have just discussed show is noticeable in the distribution of annualized returns as well. To show this we provide two histograms showing the density of annualized returns for portfolios of size 5 and 25. Regarding figure 5.14 we point out that the decrease in the standard deviation of annualized returns is noticeable as the scaling of the x-axis changes, with the spread of annualized returns for portfolios of size 25 being considerably smaller. Additionally, we note the change from slightly positive skewness values when holding five sectors, to negative skewness values when holding 25 sectors.



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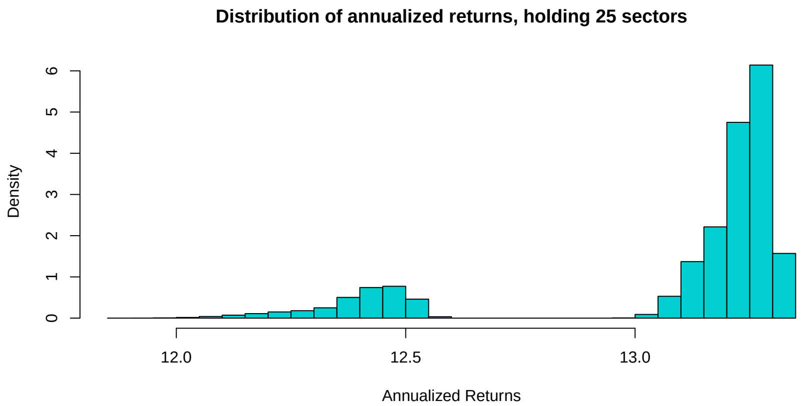


(b)

Figure 5.14:
This figure shows the distribution of annualized returns across the possible portfolios. We compute annualized returns for every possible portfolio in the cases where we hold five or twenty-five sectors in each portfolio. The y-axis measures the density, so we scale the axis as a probability distribution. Note that the scaling of the x-axis is different for the two plots.

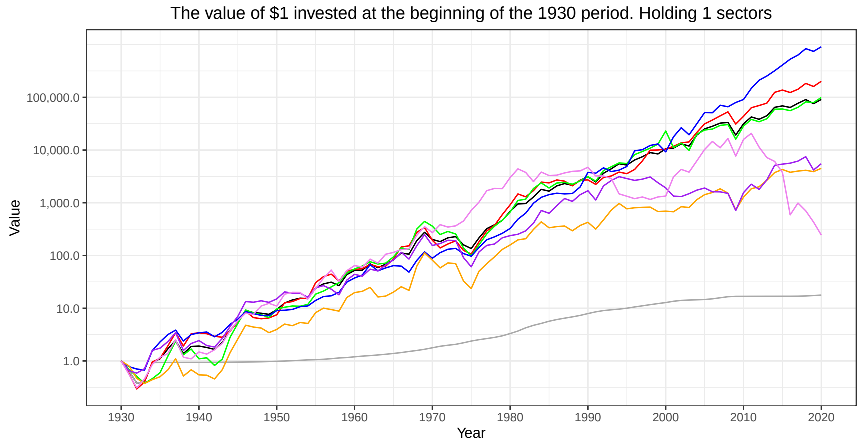
5.4 Applying The Rebalancing Strategy

In this section, we will change our investment strategy from buy-and-hold to rebalancing. We will proceed with the same quantitative methods as in the last section. We are looking for similarities and discrepancies between the two strategies. We start with the same experiment where we invest one dollar in different portfolios at the beginning of 1930 and we look at the development of that dollar. In figure 5.15 we compute the development of the dollar for portfolios consisting of one and five sectors. We note that the development of a dollar that is invested into one sector will have the same pattern using both strategies, as there is no rebalancing possible for one sector. The difference between figure 5.8a and figure 5.15a lays in the market portfolio. It is being rebalanced in the same manner as our portfolios.

When we apply the rebalancing strategy we still observe a significant difference between the top-performing portfolios and the bottom-performing portfolios. Figure 5.15b shows the development of our 1 dollar invested in the three top-performing portfolios and the bottom three performing portfolios consisting of five sectors. The best performing portfolio obtains an accumulated value of $811,300.90 at the end of 2019, corresponding to a value of 8.89 times the value accumulated by the market portfolio. The worst performing portfolio is valued at $9,861.64. If we compare these numbers to our buy-and-hold strategy numbers, the best performing rebalanced portfolio has a value in excess of $500,000 over the best performing buy-and-hold portfolio. These results complement Maeso and Martellini's (2020) findings.

This trend continues as our portfolios grow in size. Percentage-wise, all portfolios accumulate higher values than their buy-and-hold counterparts. The top-performing rebalanced portfolio with 5 sectors excluded accumulates a value almost 1.5 times bigger than the top-performing buy-and-hold portfolio. When we exclude one sector, the rebalanced portfolio accumulating the most value is valued at one-third more than the buy-and-hold counterpart. In table 5.2 we provide a complete overview of best and worst portfolios for different portfolio sizes, as well as a comparison to the market portfolio and risk-free rate.

We have observed that our 1 dollar invested using the rebalancing strategy accumulates more value than when we invest the 1 dollar using the buy-and-hold strategy. When we use the buy-and-hold strategy, we find that 80% of portfolios with one sector excluded obtains higher annualized returns than the market portfolio. Doing the same calculations when we apply the rebalancing strategy we observe a very different situation. To obtain a 50% chance of randomly picking a portfolio that performs better than the market we need portfolios containing 23 sectors, as opposed to the 15



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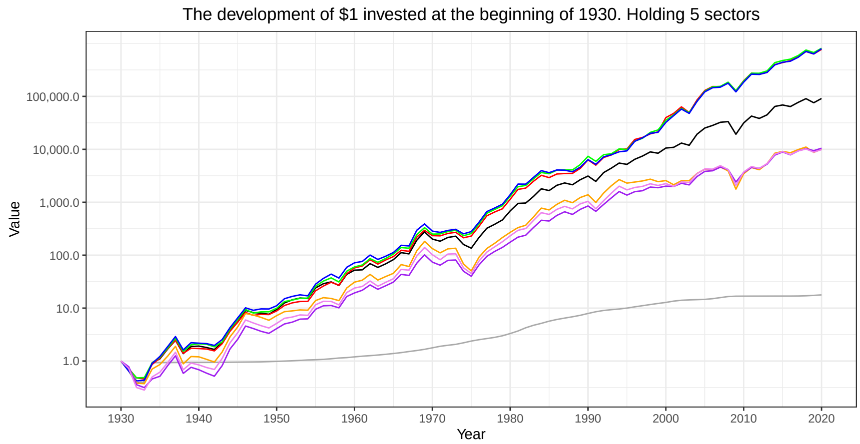
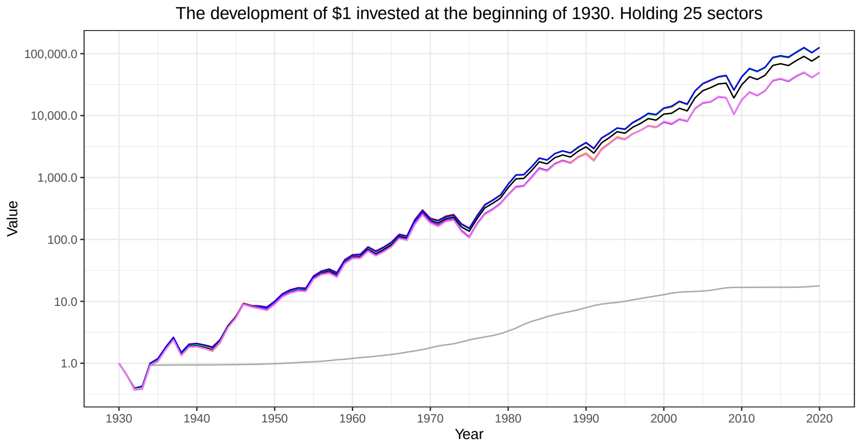


(b)

Figure 5.15:
This figure shows the result of applying the rebalancing strategy for portfolios holding one and five sectors, opening the position at the beginning of 1930 and closing at the end of 2019. In panel (a) we hold one sector in each portfolio, and in panel (b) we hold five. The graph shows the top three performing and the bottom three performing portfolios, with the market as a whole plotted in black and risk-free investment in dark grey, as guidelines. The y-axis of the graph is logarithmically-scaled to make the graph more readable.



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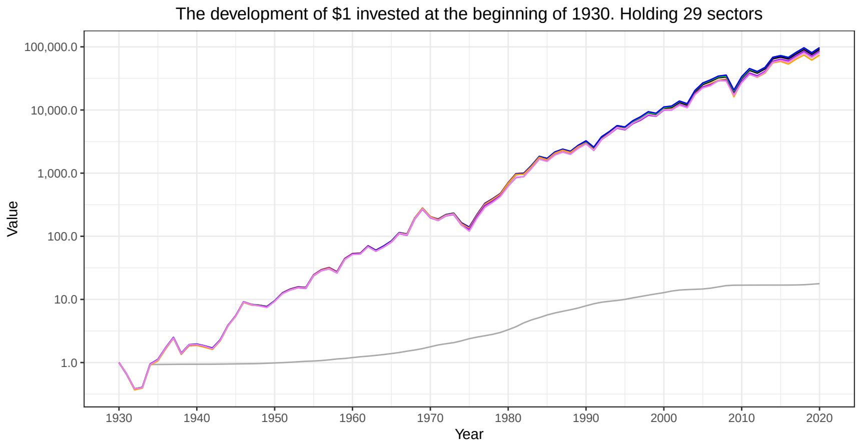


(b)

Figure 5.16:
This figure shows the result of applying the rebalancing strategy for portfolios excluding one and five sectors, opening the position at the beginning of 1930 and closing at the end of 2019. In panel (a) we exclude five sectors from our portfolios, and in panel (b) we exclude one. The graph shows the top three performing and the bottom three performing portfolios, with the market as a whole plotted in black and risk-free investment in dark grey, as guidelines. The y-axis of the graph is logarithmically-scaled to make the graph more readable.

Table 5.2:
This table summarizes the results of investing one dollar in different sized portfolios at the beginning of 1930 when applying the rebalancing strategy. We tabulate the end value an the annualized returns for the top- and bottom-performing portfolios of different sizes. We also show the end value as a percentage of the rebalancing market portfolio's end value and risk-free investment. All figures are rounded to two decimal places.

Panel A: Holding One Sector			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$909,939.60	16.47%	$244.01	6.30%
% of Market Portfolio Value		% of Market Portfolio Value	
997.08%		0.27%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
5,120,028.18%		1,372.99%	

Panel B: Holding Five Sectors			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$811,300.90	16.32%	$9,861.64	10.76%
% of Market Portfolio Value		% of Market Portfolio Value	
889.00%		10.81%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
4,565,010.11%		55,489.26%	

Panel C: Excluding Five Sectors			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$126,091.50	13.94%	$49,388.80	12.76%
% of Market Portfolio Value		% of Market Portfolio Value	
138.17%		54.12%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
709,488.89%		277,899.82%	

Panel D: Excluding One Sector			
Top-performing portfolio		Bottom-performing portfolio	
Value	Annualized Returns	Value	Annualized Returns
$96,976.58	13.61%	$74,081.23	13.27%
% of Market Portfolio Value		% of Market Portfolio Value	
106.26%		81.18%	
% of Risk-Free Asset Value		% of Risk-Free Asset Value	
545,665.690,4%		416,838.64%	

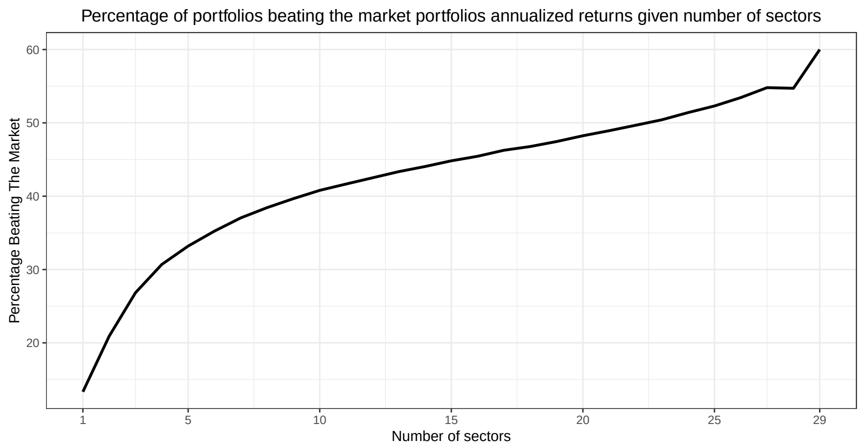


Figure 5.17:
This figure shows the percentage of portfolios that create more value than the market portfolio over 90 years, depending on the number of sectors included in the portfolios.

sectors needed when we use a buy-and-hold strategy. Additionally, we note that if we exclude one sector there are 60% of the portfolios that beat value accumulated by the market portfolio, as opposed to the 80% for the buy-and-hold strategy. In figure 5.17 we show what percentage of portfolios create more value than the market for a given number of sectors included.

5.4.1 Expectation and Variance

We have previously shown that the expected value of annualized returns is increasing when the variance of the underlying asset's returns decrease, and as such we would expect it to converge towards the return of the market portfolio as portfolio size increase. In addition to this, we would expect, based on the development shown in figures 5.15 and 5.16, that the standard deviation of annualized returns should decrease as portfolio size increases. Figures 5.18 and 5.19 shows exactly this, with the expected value of annualized returns growing fast when portfolio size is small, while the marginal increase is decreasing as the portfolio size grows. In terms of the variability, we note that the value coincides with the buy-and-hold case for single sector portfolios, as the two strategies are identical at this point, but the variability of the rebalancing strategy drops quicker and to lower values than its counterpart. In fact, the standard deviation of annualized returns drops as much as 95.76% when moving from a single sector to 29 sectors included[11].

More similarities with the buy-and-hold strategy can be drawn when we look into the fact that the expected value of annualized returns for any sector

[11] A comprehensive overview of descriptive statistics for the rebalancing strategy is provided in table A.4.

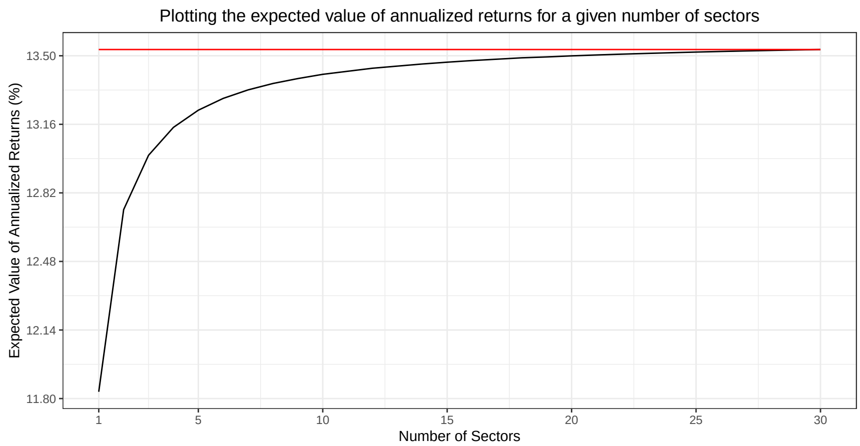


Figure 5.18:
This figure shows how the expectation of annualized returns develops, depending on the number of sectors included in a portfolio. The red line at the top of the graph shows the annualized returns of the rebalancing market portfolio.

excluded portfolio is below the market benchmark, even though the majority of portfolios outperform the market when only a single sector is excluded. We see that on average the winners perform 0.046 percentage points above this benchmark, whilst the losers perform 0.074 percentage points below. The single best-performing portfolio provided annualized returns of 0.077 percentage points above market levels, while the worst performer provided 0.26 percentage points below the market benchmark.

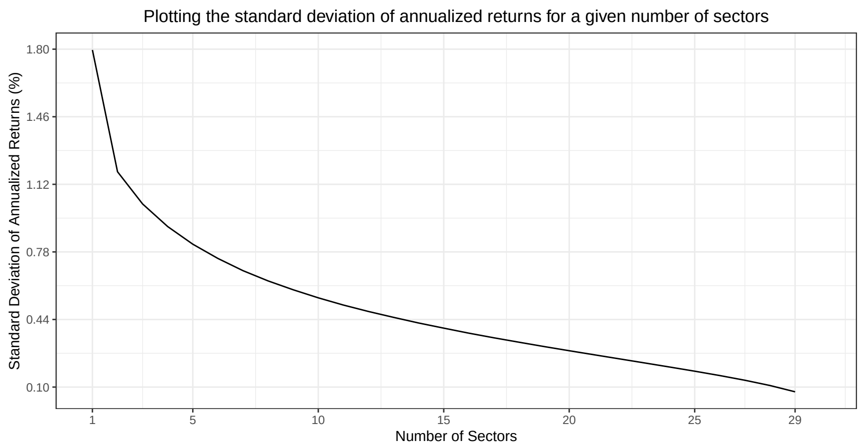


Figure 5.19:
This figure shows the standard deviation of annualized returns, or variability, computed across the population of portfolios of any given size. The standard deviation is reported in percent.

5.4.2 Skewness and Kurtosis

Our observations in section 5.4.1 indicate that we will have less negative skewness in annualized returns when we use the rebalancing strategy as opposed to the buy-and-hold strategy. Figure 5.20 shows the skewness for the population of annualized returns for the different portfolio sizes. In terms of skewness, we can see that the numerical values are noticeably lower than for the buy-and-hold case, with a skewness value of -1.47 compared to -4.66 when a single sector is excluded. However, the overall trend is the same. The distribution of annualized returns is positively skewed when holding only a few sectors, but turns negative as portfolio size increases.

The kurtosis, on the other hand, indicates normal levels as shown in figure A.2. By normal values, we mean that for most cases, besides the outermost points[12], the excess kurtosis is close to zero. Combining this with what we have discussed so far indicates that the risk associated with excluding many sectors is rather significant, however, there is a large presence of upside risk. Excluding few sectors on the other hand comes with less risk, but in this case, the upside risk is less prevalent, and the downside risk is larger, relatively speaking.

Taking a look at our histograms in figure 5.21, showing the distribution of annualized returns for portfolios of size five and 25, we can see all of what we have discussed so far. We note that the mean value is larger for the 25 portfolio case, and looking at the scaling of the x-axis we also note the difference in the variance of these annualized returns. Additionally, we note the change from a longer right tail in the first case, to a longer left tail in the second case, which shows the transition from positive to negative skewness. Finally, we note that the distributions for annualized returns under the rebalancing strategy provide a much smoother distribution, compared to the buy-and-hold strategy of figure 5.14.

[12] By outermost points, we mean holding a single sector, or excluding a single sector.

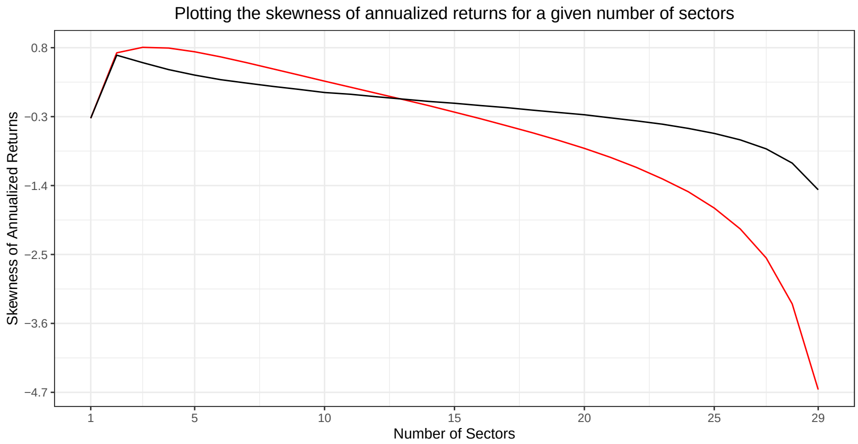
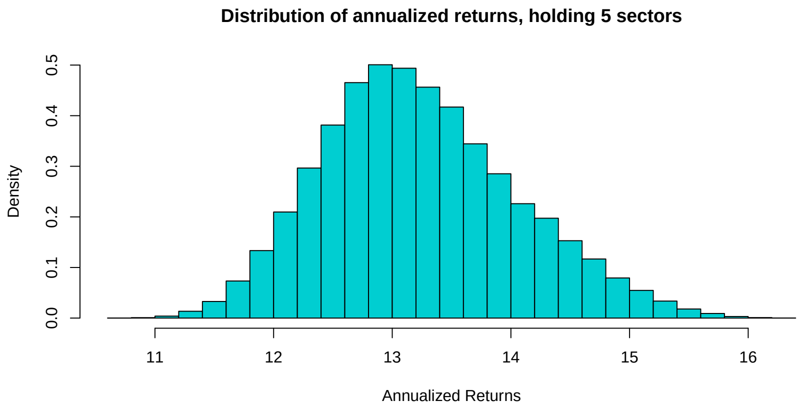
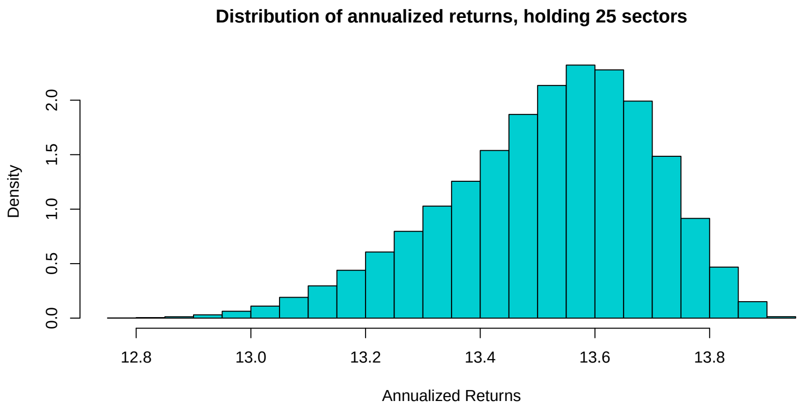


Figure 5.20:
This figure shows the skewness value for the population of annualized returns depending on the portfolio size. The black graph measures the skewness using the rebalancing strategy, while the red graph is for the buy-and-hold strategy as a comparison.



(a)



(b)

Figure 5.21:
This figure shows the distribution of annualized returns across all possible portfolios when applying the rebalancing strategy. We compute annualized returns for every possible portfolio in the cases where we hold five or twenty-five sectors in each portfolio. The y-axis measures the density, so we scale the axis as a probability distribution. Note that the scaling of the x-axis is different for the two plots.

5.5 The Portfolio Frontier

As a final point, we will take a look at the effect of sector exclusion in the scenario where the investor actively tries to minimize the risk of an investment. Here, we apply the mean-variance idea of Markowitz (1952) as presented in section 4.6. Additionally, as discussed in the aforementioned section, the frontiers require historical data, to be computed, and as such the frontiers we provide are assumed to be the 1940 portfolio frontiers based on ten years of historical returns. To be precise, we compute the different frontiers by minimizing the portfolio variance, for a given level of expected return[13], through a numerical approach.

In figure 5.22 we have selected and plotted a few different frontiers, where a single sector is excluded. When comparing the different frontiers in this figure, we note that the market portfolio is less risky than the sector excluded ones, noticeable by the fact that it in general lies above and to the left of the others. Of course, the expected return and volatility associated with a portfolio, be it the market or any other, is the same at the extremities, meaning where the expected return is at a minimum or a maximum. This is due to our assumption of no short selling, meaning that the only way of achieving these levels of expected returns is to hold the single sector that provides it. Additionally, we point out that we have not included every single frontier in this figure, as most cases bang on overlaps with the market portfolio. In table A.5 we provide an overview of the weights associated with each sector for different points on the market frontier and note that several sectors are not weighted at all. It is due to this fact that so many portfolio frontiers completely overlap the market frontier, as excluding these (never weighted) sectors would have zero effect. In fact, there are as many as 22 sectors that are never weighted in a mean-variance market portfolio.

Finally, we look into how a long-term, passive investment based on the portfolio frontier would have turned out. Assume now that an investor invested a single dollar in an equally weighted market portfolio at the beginning of 1930. At the beginning of 1940, he rebalances his portfolio weights, intending to minimize volatility based on the previous ten years of data. This can be thought of as excluding sectors based on the risk associated with each sector, and investing in the minimum variance portfolio on the market frontier. What kind of value would this minimum variance portfolio create if held until the end of 2019, assuming the investor did not rebalance it again. The results are shown in figure 5.23, and we can see that such an investment strategy performed better than the buy-and-hold market portfolio but worse than the rebalancing market portfolio. We find that the end value of

[13]Expected return for each sector computed as the mean monthly return from 1.Jan 1930 to 31.Dec 1939

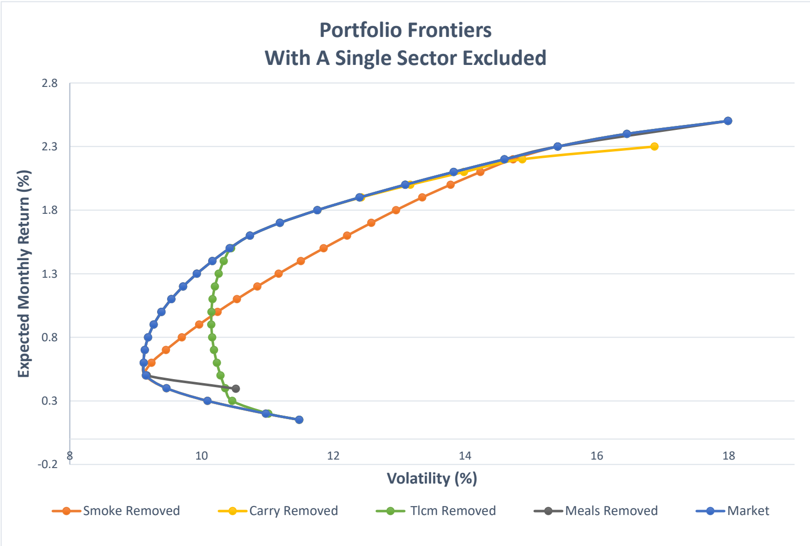


Figure 5.22:
This figure shows the frontier for different portfolios when we exclude single sectors. Additionally, we show the frontier for the market portfolio as a guideline. The expected return (y-axis) and volatility (x-axis) are calculated monthly from 1.Jan 1930 to 31.Dec 1939.

such an investment would have been $81,084.14. Additionally, the yearly volatility for an investment strategy based on the frontier is higher than for both the buy-and-hold and rebalancing market portfolios, indicating that a long-term, passive, investment based solely on minimizing the variance based on a few years of historical data is not necessarily a good idea.

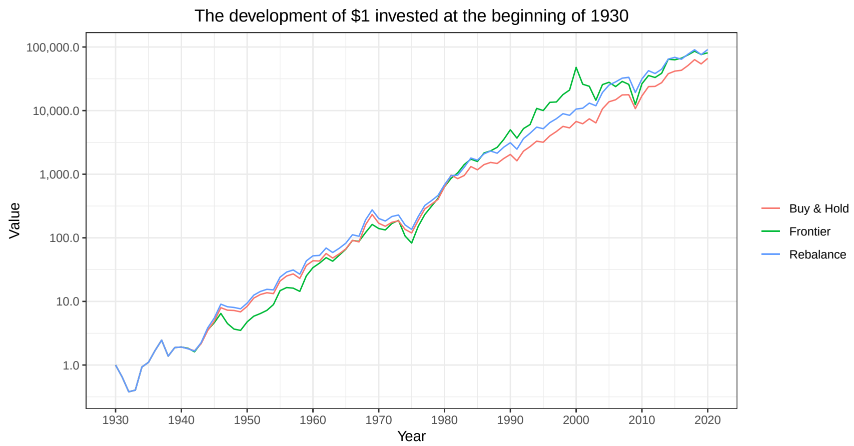


Figure 5.23:
This figure shows the value created from a single dollar invested at the beginning of 1930, when investing it in the market portfolio using the buy-and-hold strategy and the rebalancing strategy. Additionally we plot the development for an investment based on the frontier.

6 Conclusions

Our findings indicate that the effect of sector exclusion does not become apparent when we measure using the traditional approach of expectation and volatility of monthly (or yearly) returns. In section 5.1, the measure of unconditional expectation for the return of a portfolio is unaffected when portfolio size increases and is constant at the level of expected return for a randomly selected sector. At the same time, we find that the diversification benefits are marginally low, as figure 5.1 shows the volatility of an investment only drops 11.94% when increasing from a single sector per portfolio to the market portfolio. On the other hand, however, how much the expected returns for different portfolios vary around this constant mean value does indeed change when portfolios increase in size. When our portfolios contain 20 sectors, compared to 25, the 95 percent confidence interval for expected returns is 1.6 times bigger while still centered at the same value.

Our analysis finds that any long-term investment in sectors, be it a single sector or a combination of several, has yielded returns exceeding that of 3-month Treasury bills. One key factor for this result is that sectors are already somewhat diversified and contain more than a single stock at any point in time. However, the most notable effect of sector exclusion in our analysis is the difference in value created by different portfolios. In particular, we note the difference in value created by a single sector for the 90 year period, where picking the right sector could have provided a gain of $909,695.59 over the worst option. Our contribution indicates that the standard deviation of annualized returns could be a good measure of the risk associated with sector exclusion, in the long run, indicating how much the annualized returns vary around a mean value. We show that the variability decreases by 92.30% when moving from a single included sector to 29, when applying the buy-and-hold strategy. For the rebalancing strategy, the corresponding value is 95.76%. By making use of figures 5.12 and 5.19 an investor can choose a variability level he is comfortable with, and invest in a certain number of sectors based on that.

We show in section 5.3 that the expected value of annualized returns for a portfolio increases with the size of the portfolio because the volatility of the portfolio decreases when including more sectors.[14] This discovery is interesting when viewed in relation to the risk associated with a portfolio, measured by the variability. Combining these shows that larger portfolios have both a higher expectation and a lower standard deviation for their annualized returns, indicating that sector exclusion is not beneficial. However, this is where our investigation into the skewness of annualized returns is an interesting factor. The fact that the skewness moves from positive values for small portfolios to negative values for larger portfolios shows that there is an argument for holding narrower portfolios, as the upside risk is significantly higher. Figure 5.21 shows the movement from a relatively large upside risk for narrow portfolios to relatively large downside risk when portfolios become larger.

Finally, we show that investors who exclude sectors to minimize their portfolio volatility, meaning they exclude sectors based on a minimum variance portfolio, do not necessarily obtain returns above the level of the market portfolio. At the same time, the yearly volatility of this sector excluded portfolio in the long-run is, in fact, larger than for the market portfolios. Our takeaway from this is that basing a long-term passive investment on a short history of returns is not a good approach neither in terms of value created nor volatility of the investment.

[14]The decrease in volatility is limited, but never the less present.

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A Appendix

Table A.1:
In this table, we present an overview of all the different sectors included in the dataset we use, in addition to some descriptive statistics about each sector. The original data is monthly data starting in July 1926 and ending in November of 2020, however, we compute descriptive statistics only over the time period of interest to us, January 1930 to December 2019. The table shows the mean, median, minimum, and maximum number of stocks associated with each sector over the 90 year period of time. All data provided by Kenneth R. French (2021) through the Tuck School of Business at Dartmouth College website.

Sector	**Mean**	**Median**	**Min**	**Max**
Food Products	87.14	78	47	164
Beer & Liquor	12.72	12	4	26
Tobacco Products	9.87	10	3	17
Recreation	65.15	63	9	212
Printing & Publishing	37.56	28	7	100
Consumer Goods	65.54	51	11	160
Apparel	49.61	54	14	116
Healthcare, Medical Equipment & Pharmaceutical Products	231.15	123	4	737
Chemicals	64.93	71	19	108
Textiles	33.31	29	7	96
Construction & Construction Materials	124.64	103.5	28	303
Steel Works Etc	69.18	71	27	104
Fabricated Products & Machinery	125.97	121	25	278
Electrical Equipment	51.85	48	13	191
Automobiles & Trucks	60.50	56	41	109
Aircraft, Ships, & Railroad Equipment	32.51	33	17	49
Precious Metals, Non-Metallic, & Industrial Metal Mining	31.99	26	13	80
Coal	8.79	9	2	15
Petroleum & Natural Gas	126.46	129	36	394
Utilities	111.85	106	21	205
Communication	55.19	40	4	188
Personal & Business Services	245.05	190	3	1,030
Business Equipment	285.68	283	7	862
Business Supplies & Shipping Containers	54.32	50	13	113
Transportation	89.77	82	63	150
Wholesale	97.30	93	6	318
Retail	162.01	159	48	354
Restaurants, Hotels, & Motels	50.72	55	6	159
Banking, Insurance, Real Estate, & Trading	491.08	610.5	18	1,363
Everything Else	133.70	77	4	764

Table A.2:
This table shows an overview of the computed values for mean expected return, volatility, skewness, excess kurtosis, and standard deviation of expected returns for portfolios of all different sizes. This particular data set is calculated based on the November 1958 to October 1963 time period. The actual numerical value may vary across different time periods, but the relative changes when we increase the portfolio size stays consistent in the case of expectation, volatility, and standard deviation of expected returns. All values are rounded to two decimal places.

Sectors	Expectation (%)	Volatility (%)	Skewness	Excess Kurtosis	SE of Monthly Returns (%)
1	1.01	4.83	0.06	0.43	0.31
2	1.01	4.54	-0.05	0.41	0.21
3	1.01	4.44	-0.09	0.44	0.17
4	1.01	4.39	-0.11	0.47	0.15
5	1.01	4.36	-0.13	0.49	0.13
6	1.01	4.34	-0.14	0.50	0.11
7	1.01	4.32	-0.15	0.51	0.10
8	1.01	4.31	-0.15	0.52	0.09
9	1.01	4.30	-0.16	0.53	0.09
10	1.01	4.30	-0.16	0.53	0.08
11	1.01	4.30	-0.16	0.54	0.07
12	1.01	4.29	-0.16	0.54	0.07
13	1.01	4.28	-0.17	0.55	0.07
14	1.01	4.28	-0.17	0.55	0.06
15	1.01	4.27	-0.17	0.55	0.06
16	1.01	4.27	-0.17	0.56	0.05
17	1.01	4.27	-0.17	0.56	0.05
18	1.01	4.27	-0.17	0.56	0.05
19	1.01	4.27	-0.17	0.56	0.04
20	1.01	4.26	-0.17	0.56	0.04
21	1.01	4.26	-0.18	0.57	0.04
22	1.01	4.26	-0.18	0.57	0.03
23	1.01	4.26	-0.18	0.57	0.03
24	1.01	4.26	-0.18	0.57	0.03
25	1.01	4.26	-0.18	0.57	0.03
26	1.01	4.26	-0.18	0.57	0.02
27	1.01	4.26	-0.18	0.57	0.02
28	1.01	4.25	-0.18	0.57	0.02
29	1.01	4.25	-0.18	0.57	0.01
30	1.01	4.25	-0.18	0.58	N/A

Table A.3:
This table shows the expected value and standard deviation of annualized returns calculated across all portfolios of a given size when applying the buy and hold strategy. Additionally, we tabulate the skewness and excess kurtosis of annualized returns for the same portfolios. All figures are rounded to two decimal places.

Sectors	Expectation (%)	Standard Deviation (%)	Skewness	Excess Kurtosis
1	11.83	1.80	-0.32	1.90
2	12.29	1.30	0.72	0.87
3	12.47	1.17	0.81	0.58
4	12.58	1.08	0.79	0.17
5	12.66	1.02	0.73	-0.22
6	12.72	0.97	0.65	-0.55
7	12.77	0.92	0.56	-0.82
8	12.81	0.88	0.46	-1.04
9	12.84	0.84	0.37	-1.21
10	12.87	0.80	0.27	-1.35
11	12.90	0.77	0.17	-1.44
12	12.92	0.73	0.07	-1.51
13	12.94	0.70	-0.03	-1.55
14	12.96	0.67	-0.12	-1.56
15	12.98	0.64	-0.23	-1.55
16	13.00	0.60	-0.33	-1.51
17	13.01	0.57	-0.44	-1.44
18	13.02	0.54	-0.56	-1.33
19	13.04	0.51	-0.68	-1.19
20	13.05	0.48	-0.81	-1.00
21	13.06	0.45	-0.95	-0.77
22	13.07	0.42	-1.11	-0.43
23	13.08	0.39	-1.29	0.01
24	13.08	0.35	-1.50	0.59
25	13.09	0.32	-1.76	1.46
26	13.10	0.28	-2.09	2.78
27	13.11	0.24	-2.56	4.99
28	13.11	0.19	-3.29	9.40
29	13.12	0.14	-4.66	21.22
30	13.13	N/A	N/A	N/A

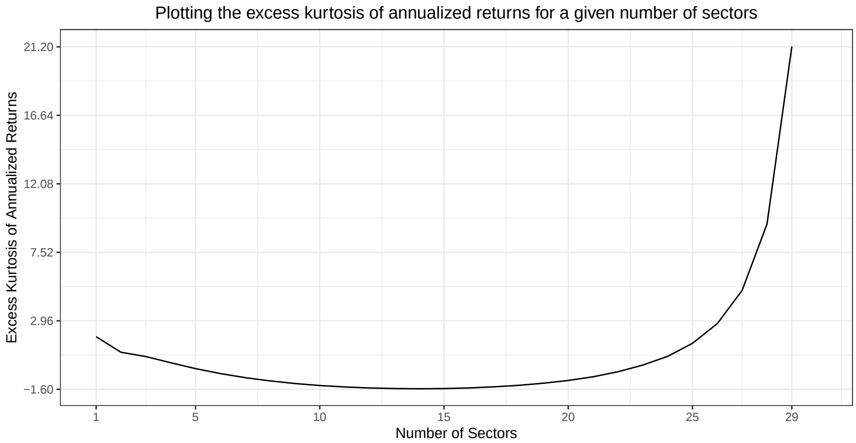

Figure A.1:
This figure shows the excess kurtosis for the distribution of annualized returns, depending on the portfolio size. In this particular case, we have applied the buy and hold strategy.

Table A.4:
This table shows the expectation and standard deviation of annualized returns for all portfolios of different sizes when applying the rebalancing strategy. Additionally, we tabulate the skewness and excess kurtosis of annualized returns for the same portfolios. All figures are rounded to two decimal places.

Sectors	Expectation (%)	Standard Deviation (%)	Skewness	Excess Kurtosis
1	11.83	1.79	-0.32	1.90
2	12.74	1.18	0.68	0.60
3	13.01	1.02	0.56	0.24
4	13.14	0.91	0.45	0.00
5	13.23	0.82	0.36	-0.14
6	13.29	0.75	0.29	-0.23
7	13.33	0.69	0.24	-0.29
8	13.36	0.63	0.18	-0.32
9	13.39	0.59	0.14	-0.35
10	13.41	0.55	0.08	-0.37
11	13.42	0.51	0.06	-0.37
12	13.44	0.48	0.02	-0.39
13	13.45	0.45	-0.02	-0.39
14	13.46	0.42	-0.06	-0.38
15	13.47	0.40	-0.09	-0.38
16	13.48	0.37	-0.12	-0.36
17	13.48	0.35	-0.16	-0.35
18	13.49	0.33	-0.20	-0.33
19	13.49	0.30	-0.23	-0.31
20	13.50	0.28	-0.27	-0.30
21	13.50	0.26	-0.32	-0.26
22	13.51	0.24	-0.37	-0.21
23	13.51	0.22	-0.42	-0.15
24	13.52	0.20	-0.49	-0.07
25	13.52	0.18	-0.57	0.04
26	13.53	0.16	-0.67	0.21
27	13.52	0.13	-0.81	0.49
28	13.52	0.11	-1.04	1.03
29	13.53	0.08	-1.47	2.36
30	13.53	N/A	N/A	N/A

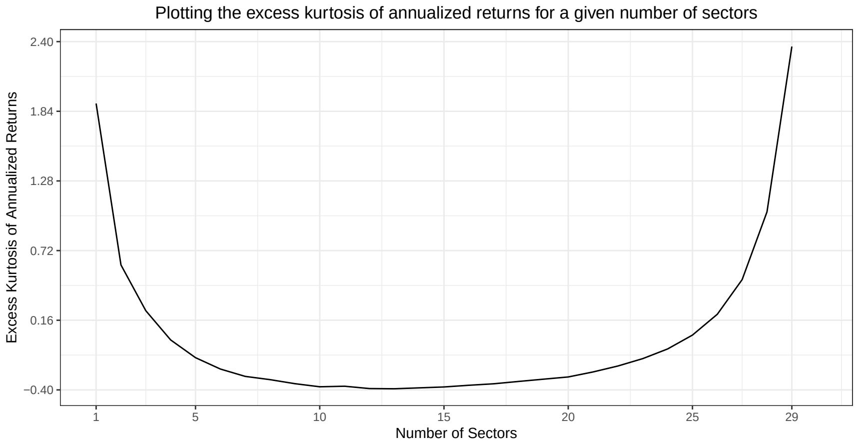

Figure A.2:
This figure shows the excess kurtosis for the distribution of annualized returns, depending on the portfolio size. In this particular case, we apply the rebalancing strategy.

Table A.5:
This table shows how every sector is weighted at the beginning of 1940 for the market portfolio at the time to achieve minimum variance for different levels of expected return. These are monthly expectations computed based on ten years of historical data, and the weights for each sector are computed numerically. Note that the outermost points, of 0.15% and 2.50% returns, are obtained by holding a single sector. Meals and Carry respectively. In addition, several sectors only have zero weights, meaning they are never included in the minimum variance portfolio. All returns, deviations, and weights are rounded to two decimal places.

	Expected Return									
	0.15%	0.40%	0.6%	0.70%	1.00%	1.30%	1.60%	1.90%	2.20%	2.50%
	Volatility									
	11.48%	9.47%	9.12%	9.14%	9.39%	9.93%	10.74%	12.40%	14.60%	17.99%
	Weights									
Food	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Beer	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Smoke	0.00	0.00	**0.07**	**0.16**	**0.45**	**0.75**	**0.93**	**0.49**	**0.11**	0.00
Games	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Books	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hshld	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Clths	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hlth	0.00	**0.05**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Chems	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Txtls	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cnstr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Steel	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FabPr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ElcEq	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Autos	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Carry	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.08**	**0.27**	**1.00**
Mines	0.00	0.00	0.00	0.00	0.00	0.00	**0.07**	**0.34**	**0.47**	0.00
Coal	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oil	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Util	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Telcm	0.00	**0.64**	**0.91**	**0.81**	**0.52**	**0.24**	0.00	0.00	0.00	0.00
Servs	0.00	0.00	**0.03**	**0.03**	**0.02**	**0.02**	0.00	0.00	0.00	0.00
BusEq	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Paper	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.10**	**0.16**	0.00
Trans	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Whlsl	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Rtail	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Meals	**1.00**	**0.31**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fin	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Other	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

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