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Applying Optimization to the Tactical Planning in the Home Health Care

An Iterative Improvement Heuristic to solve the Weekly Routing and Scheduling Problem

Master's thesis in Managerial Economics and Operations Research
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Problem Description

This thesis aims to develop a solution method capable of solving real-life instances of the Weekly Routing and Scheduling Problem (WRSP) in the Home Health Care (HHC). The main goal of the WRSP is to create efficient routes concerning cost, while also considering employee- and user convenience. To evaluate the proposed solution method, the obtained weekly route plan is compared to the initial weekly route plan.

Preface

This master's thesis has been conducted during the spring of 2020 and concludes our Master of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management. The thesis is a continuation of our specialization project completed in the fall of 2019 and was finalized in a home office due to COVID-19.

We would like to thank our supervisor Professor Henrik Andersson for valuable guidance and for showing genuine interest in our work. We would also like to thank Visma Optimization Technologies, especially Carl Andreas Julsvoll, for the contribution of relevant data, domain knowledge, and the opportunity to visit and learn from a Home Health Care provider in Norway.

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Abstract

The Home Health Care (HHC) aims to assist the elderly and people in need so that they can stay at home for as long as possible. As Norway is facing an age wave, the Norwegian HHC providers are experiencing an increased demand for health care services. This challenge could be managed through the application of Operational Research (OR), by identifying and reducing inefficiencies in existing HHC processes. The HHC providers generate routes and scheduling plans based on the needs and preferences of their associated users. Today, most HHC providers base the generation of these routes on the employees' experience, and the routes are manually created on a day-to-day basis. Despite years of experience, the numerous possibilities of combinations in routes make it complicated to manually obtain optimal routes, and inefficiencies concerning driving time or employee and user-related aspects occur. The introduction of OR could, therefore, prove to be interesting for the HHC. This thesis examines the possibility of utilizing OR to solve the complex problem of planning on a tactical level. The aim is to solve the Weekly Routing and Scheduling Problem (WRSP) of the HHC by allocating jobs to days and employees and obtain efficient routes without violating government or patient-related constraining factors. The WRSP in the HHC is an extension of the acknowledged periodic Home Health Care Routing and Scheduling Problem (HHCRSP).

A literature study related to the daily and weekly problem of routing and scheduling in the HHC is conducted. This thesis is concerned with planning on a tactical level and focuses on solution approaches capable of solving realistic size problems. A Mixed Integer Linear Program (MILP) model is developed to solve the WRSP. The MILP is formulated as a multi-objective optimization problem comprising six terms, aiming to minimize driving time while also minimizing employee and user inconvenience. The hindmost is ensured by implementing a fair distribution of workload, minimize overtime, and minimize time spent performing overqualified work. Also, minimizing violation of the time window and facilitating visit continuity is included.

An Iterative Improvement Heuristic (IIH) is proposed to solve the WRSP. The IIH is a matheuristic that identifies and adds sub-optimal allocated jobs from an initial Weekly Route Plan, to an optimization problem which is iteratively solved by the MILP. The initial Weekly Route Plan (WRP) is compounded by a set of daily optimized routes. A function is introduced to identify suboptimal allocated jobs concerning different objectives of the WRSP. The MILP solves the optimization problem by reallocating jobs simultaneously, which reduces the risk of reaching a local optimum.

The IIH is configured and validated using real-life test instances provided by Visma Optimization Technologies. The performance of the matheuristic is validated against the initial WRP, being a composition of daily optimized routes, exposing that the IIH consistently produces more efficient routes concerning all objectives of the WRSP. The IIH is also validated against the exact solution of the MILP, revealing that the IIH outperforms the MILP when solving realistic instances. On average, the matheuristic obtains a 67% reduction in gap compared to the MILP.
Additionally, a study of how the implementation of the IIH may affect the Weekly Route Plan is conducted. The study investigates the trade-off between improvement of- and impact on routes. It is proven that slight alterations of existing routes may produce relative significant improvements in routes, as 44.6% of the total improvement only requires 23% of the total changes in routes, for a given instance.

Lastly, the IIH is applied to the WRSP to produce efficient routes with respect to different aspects of the problem, to reveal trade-offs between the stakeholders' interests. This thesis finds that employee- and user-related objectives can be included and considered without going at the extent of cost-related objectives like driving time. The acquired solution decreases the share of the workweek spent on driving time by 9.7% compared to the initial weekly routes, without worsening the remaining objectives. This implies that the IIH can be utilized to solve realistic size WRSPs and achieve more efficient routes, while still taking user and employee convenient aspects into account. This increases the capacity of the HHC providers and points out the potential for cost-reductions in the HHC.

Sammendrag

Hjemmetjenesten har som mål å tilrettelegge for mennesker med psykiske eller fysiske behov, slik at de kan bo hjemme så lenge som mulig. Norge står overfor en eldrebølge, og den norske hjemmesyketjenesten opplever en økt etterspørsel etter helsetjenester. Denne utfordringen kan håndteres gjennom anvendelse av optimering, ved å identifisere og redusere ineffektivitet i eksisterende prosesser. Hjemmetjenesten generer ruter basert på lister som definerer behovene og preferansene til sine brukere. I dag lager de fleste hjemmetjenesteavdelinger rutene manuelt, basert på de ansattes erfaring med pasientene. Til tross for mange år med erfaring, gjør de utallige mulighetene for kombinasjoner i ruter det krevende å oppnå optimale ruter, når dette gjøres manuelt. Denne oppgaven tar utgangspunkt i dagsoptimerte ruter for å løse det ukentlige ruteplanleggingsproblemet (Weekly Routing and Sceduling Problem, WRSP), på et taktisk nivå.

Et litteraturstudie relatert til det daglige og ukentlige ruteplanleggingsproblemet i hjemmetjenesten er gjennomført. Denne oppgaven fokuserer på planlegging på et taktisk nivå og løsningsmetoder som er i stand til å løse problemer av realistisk størrelse. Et lineært blandet heltallsproblem (Mixed Integer Linear Program, MILP) er modellert for å løse WRSP. Heltallsproblemet er formulert som et fler-objektiv optimeringsproblem, som tar for seg seks objektiver. Problemet tar sikte på å minimere kjøretid, og samtidig minimere ulemper for ansatte og brukere. Minimieringen av disse ulempene sikres ved å etterstrebe en rettferdig fordeling av arbeidsmengden, minimere overtid og minimere tiden brukt til å utføre overkvalifisert arbeid. I tillegg etterstreber modellen å minimere overstiging av tidsvinduet til oppgaver, og sikre besøkskontinuitet.

En iterativ forbedrings-heuristikk (Iterative Improvement Heuristic, IIH) foreslås for å løse WRSP. IIH er en matematisk heuristikk som identifiserer og legger til jobber som er suboptimalt allokert til et nytt optimeringsproblem, som iterativt løses av MILP. En initiell ukeplan (Weekly Route Plan, WRP) brukes som utgangspunkt for heuristikken og denne er sammensatt av et sett med daglige optimerte ruter. MILP løser optimeringsproblemet ved å simultant reallokere jobber, noe som reduserer risikoen for å stagnere i et lokalt optimum.

IIH kalibreres og valideres ved hjelp av testinstanser som er generert på bakgrunn av data levert av Visma Optimization Technologies. Løsningen generert av heuristikken sammenlignes med den initielle WRP-en, og valideringen viser at IIH produserer mer effektive ruter for alle interessentene av WRSP. IIH er også validert ved sammenligning med den eksakte MILP-løsningen.

Resultater fra disse testene viser at IIH utkonkurrerer MILP når WRSP er av virkelighetsnær størrelse. I gjennomsnitt oppnår den matematiske heuristikken 67% reduksjon i gap sammenlignet med MILP-en.

Videre gjennomføres en studie av hvordan implementeringen av IIH kan påvirke de ukentlige rutene. Studien undersøker avveiningen mellom forbedring og endring av ruter, hvor målet er å gjøre få endringer for å oppnå mye forbedring i de eksisterende rutene. Resultatene viser at små endringer i de eksisterende rutene kan gi betydelige forbedringer i ruter, da 44.14 % av den totale mulige forbedringen kun krever 23% av de tilhørende totale endringene i de eksisterende rutene, for en gitt instans.

IIH blir også implementert på WRSP for å generere effektive ruter med hensyn til forskjellige interessenter av problemet, for å avdekke avveininger mellom de ulike objektivene. Resultatene fra testene viser at ansatte- og bruker-relaterte objektiver kan inkluderes uten at dette i stor grad går på bekosting av kostnadsrelaterte objektiver, eksempelvis kjøretid. Den optimerte løsningen reduserer nemling andelen av arbeidsuken som blir brukt på kjøretid med 9.7 % sammenlignet med den initielle WRP-en, uten å påvirke de andre objektivene. Dette impliserer at IIH kan brukes til å løse virkelighetsnære WRSP, som oppnår mer kostnadseffektive ruter, og som samtidig tar hensyn til ansatte- og brukerrelaterte objektiver. Dette vil øke kapasiteten til hjemmetjenesten og belyser potensialet for kostnadsreduksjon i hjemmetjenesten.

Table of Contents

List of Tables

List of Figures

Abbreviations

CHS =	Complete Heuristic Search
HHC =	Home Health Care
HHCRSP =	Home Health Care Routing and Scheduling Problem
IIH =	Iterative Improvement Heuristic
MILP =	Mixed Integer Linear Program
MPP =	Multi-Period Problem
OR =	Operational Research
PVRP =	Periodic Vehicle Routing Problem
PVRPTW =	Periodic Vehicle Routing Problem with Time Windows
SHS =	Single Heuristic Search
SPP =	Single-Period Problem
VRP =	Vehicle Routing Problem
VRPTW =	Vehicle Routing Problem with Time Windows
VRPSD =	Vehicle Routing Problem with Split Deliveries
WRP =	Weekly Route Plan
WRSP =	Weekly Routing and Scheduling Problem

Chapter 1

Introduction

The Home Health Care (HHC) system has been a part of the Norwegian welfare state for over 70 years, aiming to provide people the opportunity of staying at home for as long as possible. This has been a goal since the 1950s, with several action plans and granting of economic support dedicated to the HHC (Statistisk Sentralbyrå, 2019b). The HHC providers offer services such as medical treatment, elderly care, and practical assistance at the users' homes instead of in a hospital or nursing home.

Today, population projections expose an imminent age wave that increases the need for the HHC in the years to come. Roksvaag and Texmon (2012) state that at least one out of three students must graduate with a health-related education to manage the increased demand, yet only 22% of Norwegian students graduated with a health-related degree in 2018 (Statistisk Sentralbyrå, 2019c). Therefore, alternative ways to face the upcoming HHC challenge must be explored to prepare the Norwegian welfare state for the predicted future.

The municipalities in Norway are responsible for providing HHC services to those who need it. People receiving help from the HHC are referred to as users, and each user has a care-plan based on the content and frequency of services needed. The aggregation of care-plans becomes a complex database of users and their needs at different times. Based on experience and local knowledge, the administration of the HHC office often manually creates routes and schedules for the employees. The HHC office must also allocate services to days, based on how often the users need visits. Today the routes are rarely revised, only slightly altered when demand changes, still in a manual way. This traditional and manual approach does not provide the most efficient routes concerning driving time and the utilization of scarce human resources. Thus, the application of Operational Research (OR) in the routing and scheduling problem of the HHC may benefit the Norwegian health care system.

The purpose of this thesis is to study and solve the Weekly Routing and Scheduling Problem (WRSP) of the HHC, which concerns the allocation of jobs to employees and days on a tactical level. The WRSP in the HHC is an extension of the acknowledged periodic Home Health Care Routing and Scheduling Problem (HHCRSP), which is derived from the classic Vehicle Routing Problem (VRP). The first goal of this thesis is therefore to develop a solution method to solve real-life instances of the WRSP. The proposed matheuristic, the Iterative Improvement Heuristic (IIH) utilizes an existing weekly route plan when solving the WRSP. Furthermore, the IIH aims to minimize driving time, while still considering user and employee convenience, by ensuring fairness in workload, comply with time windows, and ensure visit continuity. The second goal of this thesis is to investigate the trade-offs between improvement of- and impact on routes, with respect to the HHC provider, the employees, and the users. Additionally, the aim is to analyze how different stakeholders' interests may affect the routes.

This thesis is written in collaboration with Visma, a Norwegian software company with an optimization team aiming to develop a route planner for municipalities in Norway. Visma has provided real-life data from their pilot municipalities, which is used to test the adequacy of the model and the IIH. The work of this thesis is a continuation of previous work, Lynås and Van de Pontseele (2019), conducted in the fall of 2019. Hence, some parts of this thesis are similar to Lynås and Van de Pontseele (2019). In that case, this is stated in the introduction of the prevailing chapter.

A comprehensive background for the problem in this thesis is given in Chapter 2. Furthermore, Chapter 3 presents the reviewed literature, along with positioning this thesis in the field of optimization. Chapter 4 describes the problem at hand, and Chapter 5 presents the mathematical formulation of the WRSP. The Iterative Improvement Heuristic proposed to solve the WRSP, is presented in 6. A description of how the relevant data were extracted from the data set provided by Visma, follows in Chapter 7, and based on this, instances of different sizes are created. The test instances are used to configure and test the proposed IIH and the computational results are found in Chapter 8. Finally, concluding remarks and possible future research is presented in Chapter 9.

Chapter 2

Background

This chapter introduces the necessary background information for this thesis. Section 2.1 presents the increased need for health care caused by the population projection in Norway. A description of the Norwegian Home Health Care (HHC) system is found in Section 2.2. The main content of this chapter builds upon Lynås and Van de Pontseele (2019).

2.1 Population Projection

Population projections in Norway state that in 15 years there will be a larger share of people above 65 years than people below 19, for the first time in Norwegian history (Leknes et al., 2018). The development is likely to continue, resulting in a doubling of the age segment above 70 by 2060, compared to 2018. This shift is referred to as the age wave.

The age wave can be explained by the aftermath of the baby boom after World War II, along with an increased life expectancy as medical services are improving. Additionally, the birth rates are decreasing and the relative difference in the age of the population is enhanced by immigration and migration to central areas. The young generations migrate from rural districts to central areas, leading to a larger share of elderly people and decreasing birth rates in the rural districts. This results in major regional differences in the magnitude of the age wave, and the rural municipalities are facing the greatest challenges related to future elderly care (Leknes et al., 2018).

Naturally, the probability of age-related diseases increases with higher life expectancy, leading to an extensive need for nursing and care. This implicates, among other things, an increased need for skilled workers and the capacity of hospitals, nursing homes, and health care services. However, as elderly people are healthier than before they can stay longer in their homes and receive HHC instead of using hospitals or institutions. The ability to stay at home for as long as possible has been a goal in Norway since the 1950s (Statistisk Sentralbyrå, 2019b). At the same time, capacity at the HHC providers has already reached its limit and the number of people in need of HHC services has increased by 8.8% since 2015 and is continuously rising (Statistisk Sentralbyrå, 2019a). The use of HHC will contribute to the liberation of space in hospitals, institutions, and nursing homes, that should be kept available for more severe cases.

There are several ways to address the issue at hand, and increasing the workforce in the HHC or streamline the existing HHC processes, are some of them. However, it is estimated that one out of three students must educate themselves in health care to endure the rising demand for health care services in Norway (Roksvaag and Texmon, 2012). Streamlining the HHC processes could increase capacity by minimizing time spent on non-user related activities. In the HHC in Norway, travel time between users accounts for 18 to 26% of the working time (Holm and Angelsen, 2014). By minimizing the travel time, each employee could increase the number of visits per day, hence increase the capacity of the HHC provider.

To reduce travel time, a reassessment of the current routes could be conducted, to possibly reveal inefficiencies. However, the planning problem in the HHC is very complex due to the immense number of combinations of employees, users, and days. Finding an optimal allocation of routes and schedules is therefore challenging. The introduction of Operational Research (OR) tools for decision and planning, such as mathematical optimization models, has proven to create more efficient routes and thereby reduce travel time and increase capacity of the HHC. On the other hand, many practical and ethical aspects must be taken into account in the planning problem, such as continuity of care and employee satisfaction, which can be difficult to mimic for a mathematical model.

2.2 Home Health Care in Norway

HHC services are performed at the users' home, and the services provided, ranging from showering or vacuuming twice a week, to distributing medications daily. Even though there are regional differences in the operation of the HHC centers, the overall management and planning are similar. Important elements of the HHC follows.

Users

A user is an individual receiving HHC services at home. Users range from elderly people to people with disabilities, mentally ill people, people in need of medical treatment, or people in need of practical assistance. People who need rehabilitation after hospitalization or palliative treatment can in some cases also be treated as users of the HHC. Different age groups have different needs, for instance, numbers show that younger users receive a weekly average of 25 hours of practical assistance, compared to people over 67 years receiving 2-3 hours per week (Statistisk Sentralbyrå, 2019b).

To receive HHC services, an application must be submitted to the municipality. The municipality assesses the application and the user's need for care and grants the request if the criteria for receiving help are met and there is available capacity. The administration, the user, and often relatives of the user, collaborate to find a suitable care-plan. The care-plan includes the scope of the need in terms of what should be done and how often a visit is required and is continuously assessed and altered if needed.

Employees

An employee is defined as a person who is working part-time or full-time for the HHC provider in the municipality. The variety of services in the HHC requires a wide set of skills, and the HHC staff consists of employees with different levels of education and experience, ranging from doctors to assistants. Generally, employees with higher skill levels prefer doing advanced services, such as medical injections, rather than simple practical assistance.

The majority of employees in the Norwegian health care work shifts, and the same accounts for the employees in the HHC. There are mainly three shifts during a week; day, evening, and night. Most of the advanced or demanding services are performed during the day shift, and employees are staffed accordingly.

During a regular workday and workweek in the HHC, the level of workload naturally fluctuates, as the users' demand varies across times of day, and days of the week. To ensure predictability and fairness for the employees, the HHC provider tries to evenly distribute the workload across employees. Overtime and last-minute changes in the employees' shifts are prevented as far as possible. A service can be characterized as physically or psychologically challenging, and such services should preferably be distributed evenly across employees.

All users are given a primary contact, who is responsible for follow-up and assessment and adjustments of the user's care-plan. The primary contact is often also the main employee visiting the user, aiming to ensure visit continuity for both employee and user. However, visit continuity is challenged by the primary contacts' shifts and the HHC's general capacity. Primary contacts are usually experienced and educated employees.

Services

The employees of the HHC provide services for users according to their care-plan or perform administrative jobs at the HHC main office. Medical treatment and giving practical assistance at the users' homes are examples of user-related services, while user documentation and ordering of medicine are administrative jobs.

The skill level required to perform a job varies according to its medical complexity, ranging from practical assistance to advanced medical treatment. Practical assistance may be vacuuming, preparing meals, or assisting in dressing, while medical treatment is injections and medical dosage. Some services may require the presence of two employees, due to the physical or psychological strain. In these cases, at least one of the employees must have the skill level required for performing the service. Some services may also require to be performed in a specific order, such as giving insulin and preparing a meal. Generally, the scope of service differs according to the required effort by the assigned employee, hence some services are more demanding than others, for instance moving a paralyzed user or treating a mentally ill user.

The care-plan contains information about the duration of services and the users' preferred time of visit during the day. Some services are naturally performed at a certain time of the day, e.g. getting up in the morning, while others are more flexible, such as vacuuming.

The care-plan also includes information about how often the HHC is needed. Some services must be performed daily, for instance getting out of bed in the morning. Others need to be performed with a given frequency, preferably evenly spread out over a period. Some examples of frequent services are vacuuming once a week and giving medicine twice a week with a gap of a few days in between. A user may also need several services on a single day and such services can be performed consecutively during one visit. In the previous example, this can be illustrated by assembling and performing the two services, distribution of medicine and vacuuming, on the same day during the same visit.

The HHC also offers the service of a safety alarm, being a triggering alarm for the user to use if they require immediate help. If an alarm is triggered, the employees at work are notified and must respond to the urgent need.

Routes

To provide services to the users, the employees travel between the users' houses according to a predefined path commonly called routes. Since not all users require daily visits, the routes differ from day to day. The distances between the users' houses vary greatly both among and within the municipalities, and this affects the employees' means of transport. In central areas, employees use walking, biking, and public transport as a mode of transport to complete their routes, whilst in rural districts with longer distances driving is preferred. At many HHC centers, a composition of the above transportation modes is used. The routes give the order of the visits and take into account the geographical distances between the users, duration, and preferred times of visits, as well as the employees' and users' preferences. Every day, the employees receive one or several routes to complete during their shift. To reduce travel time, an employee is often primary contact to users within the same geographical area.

The routes begin and end at the HHC center, as the employees receive their daily routes at the HHC center in the morning and perform user documentation back at the HHC center at the end of the shift. All employees eat lunch at the HHC center if possible, preferably at the same time. In that way, the beginning and end of a shift, as well as the lunch break is a part of all employees' route, in addition to user-related and administrative jobs.

Generation of Routes in the Home Health Care Today

The age wave is inevitable, and more efficient routes will contribute to an increase in the HHC capacity. Today, most HHC providers create the routes of the HHC employees manually, based on domain knowledge from years of experience and incorporated habits. The routes are mainly generated on a daily basis, and disruptions and revision of routes are also handled manually.

As mentioned, it is challenging to manually assess the numerous possibilities of routes and advanced mathematical optimization approaches are currently being explored as an alternative. Some HHC providers in Norway are at present testing a mathematical Route Planning system provided by the software company Visma. The Visma Route Planner obtains efficient routes by solving the daily routing problem mathematically, resulting in a disruption of previously manually created routes. This disruption in incorporated working habits and visiting patterns could negatively affect the everyday life of employees and users in the HHC, however, driving time is reduced. The challenge is therefore to balance the ratio between efficiency and consistency in routes.

Both the manual and the mathematical approach to route generation today mainly focus on the day-to-day problem. However, the total demand for care repeats itself on a weekly basis, and to find efficient routes, the perspective could be shifted to consider the week as a whole and thereby allow services to be moved between days.

Chapter 3

Literature Survey

Chapter 2 illustrates an increasing demand for Home Health Care (HHC) services, and along with this, Operations Research (OR) has in recent years increased its attention towards HHC relevant problems. There is a substantial amount of literature on the field, and this chapter presents the literature survey conducted for this thesis. The academic search portals Oria, owned by the Norwegian University of Science and Technology, and Google Scholar have been used to retrieve relevant articles.

The literature refers to the challenge of making efficient routes in the HHC as the Home Health Care Routing and Scheduling Problem (HHCRSP), which is derived from the known Vehicle Routing Problem (VRP). The Weekly Routing and Scheduling Problem can be seen as an extension of the HHCRSP, which is the focus of this thesis.

Section 3.1 presents four hierarchical levels of control, introducing the managerial aspects of the HHCRSP. The basic model for this thesis, namely the general VRP is presented in Section 3.2, while Section 3.3 explains the search strategy used. The key aspects of the HHCRSP found in existing literature are presented in Section 3.4, and in Section 3.5 existing solution methods are presented. Section 3.2 and Section 3.4 build upon previous work, conducted in Lynås and Van de Pontseele (2019). Lastly, in Section 3.6 we motivate the scope of our thesis.

3.1 Hierarchical Levels of Control

HHC managers are facing the highly complex task of allocating employees to services and constructing efficient routes for each employee. The formulation of the model depends on the planning horizon of the problem, as input factors range from fixed to flexible, based on whether the plan is short-term or long-term. The choice of time horizon for the HHCRSP also defines the hierarchical level of control. To assess the implications of different planning horizons, a framework from Hall et al. (2012) is introduced. The framework focuses on how decision making and planning in health care organizations is done, given different time horizons. Considering the HHC, four hierarchical levels of control are presented, each related to a time horizon:

- *Strategic Level.* The strategic level addresses the distribution of core resources, such as the number of employees working, and decision making for achieving high-level goals, e.g. reducing the number of employees working on a given day. The planning horizon is typically long-term, e.g. a year or more.

- *Tactical Level.* The tactical level addresses capacity over a medium-term, typically with a planning horizon of several weeks. The actual health care demand is used as input, and routes may be modified at this level, while still preserving the structure. For instance, employees may swap services internally or services may switch days in a period.
- *Offline Operational Level.* The offline operational level addresses the scheduling of specific users to employees and typically involves a planning horizon of a week. Short-term decisions regarding the execution of health care processes, e.g. adding a user to one of the routes, are addressed at this level.
- *Online Operational Level.* This level addresses the monitoring and control of the day-to-day activities in health care and all uncertainty has to be dealt with immediately.

Section 3.6 discusses how the choice of Hierarchical Level of Control has shaped this thesis.

3.2 Vehicle Routing Problem and its Most Common Extensions

The most common model used to solve the HHCRSP is the classical Vehicle Routing Problem (VRP), both with and without extensions. VRP was first introduced by Dantzig and Ramser over 60 years ago and has ever since been the foundation of many operational research problems (Laporte, 2009). The classical VRP defined by Laporte (2007) is reformulated to suit the HHC problem in (3.1)-(3.6).

$$min \sum_{n \in \mathcal{N}} \sum_{(i,j) \in \mathcal{J}} T_{ij} x_{ij}^n \tag{3.1}$$

$$\sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{J}} x_{ij}^n = 1, \qquad j \in \mathcal{J} \tag{3.2}$$

$$\sum_{j \in \mathcal{J}} x_{0j}^n = 1, \qquad n \in \mathcal{N} \tag{3.3}$$

$$\sum_{i \in \mathcal{J}} x_{i0}^n = 1, \qquad n \in \mathcal{N} \tag{3.4}$$

$$\sum_{i \in \mathcal{J}} x_{ji}^n - \sum_{i \in \mathcal{J}} x_{ij}^n = 0, \qquad n \in \mathcal{N}, j \in \mathcal{J} \tag{3.5}$$

$$x_{ij}^n \in \{0, 1\}, \qquad (i, j) \in \mathcal{J}, n \in \mathcal{N} \tag{3.6}$$

A vehicle and a customer corresponds to an employee and a user respectively. The problem consists of determining a set of routes for n employees, starting and ending at the depot (3.3)-(3.4), such that each user is visited by exactly one employee (3.2), and the total travel cost, T_{ij}, is minimized (3.1). Constraints (3.5) ensure flow in and out of a node, while constraints (3.6) are the binary constraints.

The basic VRP describes the routing as a single-period problem (SPP) in a simple manner that is not directly applicable to real life. Ever since the introduction of VRP, there has been an immense amount of extensions added to the problem, some of which have become standard models in the literature. An example of this is the Periodic Vehicle Routing Problem (PVRP), modeling the routing problem over a period as a multi-period problem (MPP). Furthermore, the Vehicle Routing Problem with Time Windows (VRPTW) assesses the situation where customers have a predefined time window indicating their preferred time of visit, while the Vehicle Routing Problem with Split Deliveries (VRPSD) allows several vehicles to visit one customer. Depending on the field of research, many other aspects can be included. For the HHC, break regulations, even distribution of workload among employees, synchronized services requiring the presence of two employees simultaneously, temporal dependencies, and incorporation of employee preferences in the generation of routes may be included.

3.3 Literature Search Strategy

To review relevant literature for this thesis, the general terms presented in Table 3.1 are investigated and the search words are used in conjunction with VRP. The search strategy is directed both towards possible formulations of the mathematical problem, and solution methods.
The search strategy for obtaining relevant literature is influenced by the article of Fikar and Hirsch (2017), providing a review of the existing literature for the HHCRSP. The article presents an overview of the most common objectives, constraints, and solution methods used in both SPP and MPP HHC.

Table 3.1: Search Words Used in Conjunction with the Term VRP

General Terms	Objective Function	Constraints	Solution Methods
PVRP	Objective function	Skill level	Exact
VRPTW	Multi-objective	Working time	Metaheuristics
PVRPTW	Weights	Break	Matheuristics
HHC	Cost	Pattern	Evolutionary Algorithm
	Fairness	Continuity	Neighbourhood Search
		Dependencies	Integer Programming

Only the most relevant articles from the literature search are discussed in this review, mainly being the multi-period HHCRSP. However, since the majority of previous research is related to SPPs and the aspects of SPPs are relatable for MPPs, applicable SPP research is also included.

3.4 Routing and Scheduling in the Home Health Care

This section introduces the most frequent modeling elements of the HHCRSP. Section 3.4.1 and Section 3.4.2 present general aspects of the SPP and MPP respectively, considering both the problem objectives and constraints. A single-period HHC problem comprises the routing and scheduling of a set of services on one day. The multi-period HHC problem is related to the allocation of services to days over a period, and routing and scheduling within each day of the period.

3.4.1 Objectives and Constraints in Single-Period Problems

In the following section objectives and constraints of the single-period HHC problems are presented.

Objectives
A weighted multi-objective function, weighing the objectives according to their importance, is used in the majority of the literature for SPPs. The objectives can be divided into two main

categories, the first being monetary objectives and the second being objectives related to the user- and employee convenience.

The monetary goal of the SPP in the HHC is to minimize costs, more specifically minimize traveling costs. In Liu et al. (2017) and Rasmussen et al. (2012), the actual cost of traveling is minimized, however, traveling cost can be measured by different manners, which is reflected in the articles. For instance Mankowska et al. (2014) and Nickel et al. (2012) minimize total traveling distance, and Trautsamwieser and Hirsch (2011) and Bredström and Rönnqvist (2008) minimize total travel time. Cost minimization is also motivated by other aspects of the HHC problem, such as cost concerning overtime. Minimization of overtime worked by employees is implemented by several articles, e.g. Braekers et al. (2016) and Hiermann et al. (2015). Overqualified staff is considered an expensive workforce and can be minimized in the objective function, which is suggested by Trautsamwieser and Hirsch (2011). Long waiting times between visits may extend the workday unnecessarily, and minimizing waiting time is, therefore, a cost-minimizing measure, implemented by Hiermann et al. (2015) and Mısır et al. (2015).

In addition to the most common monetary objectives, the literature reviewed strives to include other aspects that are considered relevant for the HHC problem, such as employees' and users' preferences. Introducing time windows as an extension to the VRP, as described in Section 3.2, is a way of taking users' preferences into account, by penalizing the objective function for late starting times. Braekers et al. (2016) implement this by introducing soft time windows and linearly penalizing for time exceeding the soft time window. Mankowska et al. (2014) propose a different approach, by penalizing the tardiness of services. Another way of taking employee and user preferences into account is by considering visit combination preferences. A mathematical model tends to treat all elements as homogeneous objects, however, the objects in the HHC problem are employees and users, which in no sense are identical. Therefore, preferred combinations of employees and users can be rewarded or penalized in the objective function. Trautsamwieser and Hirsch (2011) illustrate this by introducing a penalty if a user does not receive service from the preferred employee. Rasmussen et al. (2012), on the other hand, uses a general employee-patient visit preference, which is added as a positive measure in the objective function.

Constraints

The three most common constraints are time window restrictions, working time restrictions, and temporal dependencies. All of the reviewed articles naturally include the time window extension and a selection restricts the service to be performed within a hard time window, as presented in Rasmussen et al. (2012) and Trautsamwieser and Hirsch (2011). Working time restrictions are introduced as limits to how much an employee is allowed to work as done by Braekers et al. (2016) and Rasmussen et al. (2012). Temporal dependencies in the HHC refer to a set of services that are either classified as synchronized, meaning that they require the presence of two employees simultaneously or temporal dependent, indicating that the order of execution matters. Bredström and Rönnqvist (2008) implement both synchronized and temporal dependent services. Eveborn et al. (2006) account for synchronized services by duplicating the original service and fix both starting times to be equal.

3.4.2 Additional Objectives and Constraints in the Multi-Period Problems

Compared to the SPP, the MPP has received less attention due to the increased complexity. Factors of low importance in the SPP become significant when aggregated to an MPP as elements can be measured over time, such as the total difference in workload across days. The multi-period HHCRSP, therefore, constitutes some of the fundamental modeling choices from the SPP, e.g. time window, with an additional number of objectives and constraints. A review of articles regarding MPPs related to the HHC problem follows, emphasizing the distinctions from the SPP.

Objectives

The MPP manages the routing and scheduling problem over a time horizon, and depending on the planning hierarchy level, capacity and demand may no longer be static. This increases the number of variations of objectives compared to the SPP. Barrera et al. (2012) and Yuan et al. (2015) addresses possible actuality that the HHC is overstaffed and aims to optimize the number of employees working each shift, hence minimize the size of the workforce and thereby also the costs of the HHC providers. This can only be accounted for in a long-term perspective on a strategic level of planning for MPPs. Furthermore, while the reduction of travel cost is the main focus in single-period HHC problems, MPPs additionally put a greater emphasis on staffing and service-related factors. These factors comprise service coverage, the fairness of work schedule, and continuity of care.

Many articles in the literature of MPPs aim to perform all services, and the literature found presents three main ways of formulating this objective, starting with Carello and Lanzarone (2014). Carello and Lanzarone (2014) require all services to be performed at any cost, resulting in penalization for overtime. Bennett and Erera (2011) on the other hand, implement the objective by maximizing the number of services that are served. Thirdly, Nickel et al. (2012) aim to perform all services by penalizing for the non-performed services.

A fair work schedule for employees is of high importance for multi-period HHC problems and is included in the majority of the articles reviewed. Employee satisfaction depends on the degree of even distribution of the number of services, working time, and the heaviness of services. Different approaches have been assessed to take this into account. Hertz and Lahrichi (2009) associate a heaviness weight to every service and traveling distance, to evaluate the total heaviness worked by each employee and minimizing this. Barrera et al. (2012) aim to maintain fair work schedules by minimizing the difference in total working times on a day, for each employee. Lastly, Cappanera and Scutellà (2013) measure the utilization factor for each employee, based on working hours compared to shift hours, and penalizes for great differences among employees.

Continuity of care is a significant factor in the MPPs, which incorporates the importance of human relations in home health care. The simplest way of ensuring continuity is by minimizing the total number of unique employees visiting users during a period, which is presented by Nickel et al. (2012) and Bowers et al. (2015). Similarly, a minimization of the number of times a user receives services from a new employee is formulated by Wirnitzer et al. (2016). Carello and Lanzarone (2014) distinguish between users who need hard, partial, or no continuity of care and prioritizes thereafter, ensuring continuity for those who need it the most.

Constraints

As mentioned above, continuity of care is given extra attention in the MPPs, but instead of punishing for lack of continuity in the objective function the model may also enforce continuity by restricting visits by the same employee, or a limited amount of employees. Among others, this is implemented by Cappanera and Scutellà (2013) and Duque et al. (2015).

The introduction of visit patterns becomes relevant when addressing multi-period problems. Visit patterns provide information about predefined service combinations, which define the service frequency related to a user. Begur et al. (1997) ensure these patterns by restricting services to be performed on a fixed set of days. A more flexible approach is proposed by both Bennett and Erera (2011) and Nickel et al. (2012), allowing services to be performed on different days, but restricting the frequency of services with a predefined number of days in between, such as every other day.

3.5 Solution Methods for the HHCRSP

The ability to solve the HHCRSP mathematically depends on the selection of solution method, and numerous approaches are mentioned in the articles reviewed. Simultaneously, computational comparisons are difficult to derive, due to great differences in objectives and constraints, and the field seems to be highly heterogeneous in terms of focus areas. Section 3.5.1 and 3.5.2 present some of the variety in solution methods used, concerning SPPs and MPPs respectively.

3.5.1 Solution Methods for SPPs

The single period HHCRSP is known to be complex even in its simplest form, nevertheless, there are plenty of studies applying an exact solution procedure, e.g. in Hansen et al. (2009). This paper solves an assigning problem by applying column generation in a branch-and-price framework. Similarly, both Rasmussen et al. (2012) and Yuan et al. (2015) apply a branch-and-price algorithm, where the problem is modeled as a set partitioning problem with side constraints.

However, the majority of research found concerns solution approaches applying metaheuristics to the problem. Population-based algorithms are highly popular, and are among others introduced by Akjiratikarl et al. (2007), Mutingi and Mbohwa (2014) and Yalçındağ et al. (2016). Akjiratikarl et al. (2007) introduces the population-based heuristic technique "Particle Swarm Optimization" in the HHC context. This is an optimization procedure based on the social behavior of groups of organizations, where the algorithm reallocates a population rather than a single individual while looking for a solution. A fuzzy simulated evolution algorithm is presented in Mutingi and Mbohwa (2014), using multi-objective fuzzy evaluation techniques based on the principles of fuzzy set theory for capturing the uncertainties inherent in a system.

Another widely used procedure within the field of metaheuristics is the application of search-based algorithms. Mankowska et al. (2014) proposes neighborhood structures along with local search procedures for an HHCRSP, comprising a Variable Neighborhood Search. Furthermore, Braekers et al. (2016) introduces a large neighborhood search heuristic in a multi-directional local search framework. It is based on the idea that in order to find new efficient solutions that are neighbors of a solution x, it is sufficient to start a search from x in the direction on one objective at a time only.

A combination of the two approaches, that is exact solution methods and metaheuristics, are also developed for the single period HHCRSP, comprising matheuristics. It is most commonly in this type of heuristic to use set partitioning procedures to generate problem clusters, while linear programming techniques optimize start times and enable synchronization. Allaoua et al. (2013) proposes an integer linear programming formulation but develops a metaheuristic by decomposing the problem into two problems, to deal with larger instances. The first problem presents the pre-assigning of nurses to shifts as a set partitioning problem, while the second problem is concerned with routing. In Fikar and Hirsch (2015) the solution method is also a two-fold, where stage 1 uses set partitioning, while stage 2 applies a unified tabu search. Similar approaches are also studied in Bertels and Fahle (2006) and Bredström and Rönnqvist (2008).

3.5.2 Solution Methods for MPPs

As mentioned, the multi-period HHCRSP is less covered in literature, and the scope of reviewed solution approaches for the problem is smaller, while the variety causing computational complexity for the SSPs certainly applies to the MPPs as well. The planning horizon increases from one day to most commonly five days, leading to a massive increase in solution space. This may require other solution approaches than the ones utilized for SPPs.

The majority of articles found focus on the application of either exact solution methods or metaheuristics, to solve the multi-period HHCRSP. Exact solution procedures focusing on a Branch and Price and Cut (BPC) procedure are reviewed in among others Bard et al. (2014b). A BPC algorithm was developed and proved capable of finding near-optimal solutions within 50 minutes for small instances. Another approach is presented by Trautsamwieser and Hirsch (2014), where a BPC algorithm is developed to include a wide range of mandatory break and work regulations, while still finding the global optimal solutions. Additionally, a Variable Neighbourhood based solution approach is used to provide good initial upper bounds for the algorithm.

Moreover, integer programming is employed in several articles. Bachouch et al. (2011) propose an integer linear program for deciding which employee to be allocated to a service, and when to execute the service during the planning horizon. Cappanera and Scutellà (2013) and Cappanera and Scutellà (2015) emphasize a fair workload distribution across employees. Cappanera and Scutellà (2015) use pattern generation policies within an exact optimization problem to balance the workload of employees, where the concept of pattern is introduced as a key tool to jointly address assignment, scheduling, and routing decisions, where a pattern specifies a possible schedule for visits. Two balancing functions are studied; maximizing the lowest employee utilization factor and minimizing the highest employee utilization factor.

Considering metaheuristics, construction-based procedures, and local search-based procedures prevail in the literature concerning MPPs. This is in contrast to SPPs where population-based algorithms are frequently used. Shao et al. (2012) use a construction-based procedure and solves the problem in two levels. At the first level patterns are assigned to patients, and at the second level patients are repeatedly assigned to therapists. The algorithm used in the second level constructs a set of candidate patients for a therapist, calculates the cost, builds the therapist-patient assignment problem, and adds patients at their current location to a route. This algorithm is designed to avoid time infeasibility while minimizing overtime worked. The solution method studied in Bard et al. (2014a) is also a construction-based two-phased method, but the variables are assigned in a different order. In the first phase, feasible solutions are constructed by stringing individual routes together over the period, and in the second phase, a local optimum is found by exploring neighborhoods around the solution constructed in the first phase.

Hertz and Lahrichi (2009) present a search-based algorithm, where a mixed-integer programming model with some non-linear constraints and a non-linear objective is solved using a Tabu Search algorithm with various neighborhoods. Furthermore, Milburn and Spicer (2013) present a metaheuristic solution approach that searches for the Pareto optimal frontier for travel cost, nurse consistency, and balanced workload objectives.

Lastly, to tackle stochasticity in the MPPs, Rodriguez et al. (2015) introduce stochastic programming, while Carello and Lanzarone (2014) present exact robust optimization techniques. Dealing with dynamic events is most frequently solved by combining specific techniques with exact or heuristic solution approaches. E.g. Koeleman et al. (2012) studies the dynamic assignment of clients by modeling the problem as a Markov decision process, and the objective is to minimize the joint costs of rejecting and serving clients. Other techniques may be Monte Carlo simulations, studied by Carello and Lanzarone (2014) and discrete event formulations, as presented by Bowers et al. (2015).

3.6 Motivation for this Master's Thesis

The field of HHC has received increasing attention by OR in recent years, however, the majority of the research is related to the single-period problem. This thesis addresses the challenge of allocating services to employees and days, and thereby solve the multi-period routing and scheduling problem on a tactical level. Solution methods to the multi-period problem have been reviewed, revealing the massive computational complexity related to a multi-period problem. To handle this computational complexity, we aim to utilize existing, presumably good, single-period routes, to form improved multi-period routes.

Several single-period papers reviewed present solution methods that successfully obtain more efficient daily routes, and the SPP in the HHC is continuously being studied with progressive results. An assumption is therefore that single-period routes can be handily obtained, either manually or by optimization, and compiled to form the foundation of initial sub-optimal multi-period routes. However, we hypothesize that the initial sub-optimal multi-period routes can be improved using a matheuristic. The mat heuristic preserves parts of the routes while wisely selecting remaining parts of the routes to be solved exact in an iterative manner. On a tactical level, this may contribute to improved routes with respect to efficiency, without causing major disruptions for the HHC providers.

The articles reviewed considering MPPs maximum account for three objectives at the same time. In this thesis, we wish to account for a greater share of the objectives, to comply with the complexity of the HHC problem, by including objectives related to fairness in workload, overqualified work and visit continuity.

This thesis addresses the multi-period problem of the HHC, hereafter referred to as the Weekly Routing and Scheduling Problem (WRSP). Moreover, single-period routes are defined as daily routes, and the multi-period routes are referred to as a weekly route plan.

Chapter 4

Problem Description

This chapter describes the Weekly Routing and Scheduling Problem (WRSP) in the Home Health Care (HHC). The WRSP is defined as the tactical problem of optimizing the Weekly Route Plan (WRP) in the HHC by allocating services to employees and days while taking certain restrictions into consideration.

The problem is simplified to only comprise user-related services, hereafter referred to as tasks. Every task has a duration, constituting the workload of the task and a time window, specifying when the task should begin and end, according to the care-plan. A task may not start before the beginning of the time window but is allowed to exceed it, which is referred to as a time window violation. Lastly, all tasks have an associated required skill level to ensure that the task is performed by a competent employee. Users have different needs and may require help several times during a shift, or only a few times a week. Either way, this is represented as a set of tasks in their care-plan. The number of repetitions of a task across a week is defined as a frequency. A task with a frequency of twice a week may, for instance, be executed on Monday and Wednesday or Tuesday and Thursday, representing two possible patterns. Every task is given a pattern according to its frequency. The gap between every execution may vary, but repeating tasks should preferably be spread out evenly across the week. Thus, a task can be seen as an overall term, consisting of several executions across the week, hereafter referred to as jobs. An example of a task is the distribution of medicine, with the related activity of distributing medicine twice a week, being two jobs.

An HHC provider is responsible for serving a set of users according to the needs registered in the associated care-plan. The tasks are performed by an HHC employee at the users' home address. Users may have preferences when it comes to which employees they are visited by, and these are taken into account to ensure follow-up and ensure visit continuity. HHC employees have a certain level of education and experience which corresponds to a skill level. Employees are not allowed to perform tasks with requirements exceeding their skill level but can execute lower-level tasks. Execution of lower-level tasks is referred to as overqualified work. Employees are working shifts with a start- and end time. The current problem is limited to include day-shift only. The actual end time of a shift may vary, and overtime is registered if the actual end time is later than the planned end time. All employees must start and end their shifts at the HHC center. For simplicity, all employees are assumed to use cars as a mode of transportation. Additionally, synchronized jobs, requiring the presence of two employees at a time, and jobs with temporal dependencies, are not included.

The primary objective of this problem is to create efficient routes across a period of one week, by minimizing the total driving time between tasks. Additionally, several sub-objectives related to employee- and user convenience, are included. With respect to employees, the amount of overtime worked and overqualified work should be minimized. The routes should also be fair in terms of workload distribution between employees across the week. This is accounted for by minimizing the total of absolute greatest difference in workload on each day of the week. User convenience is considered by striving to perform tasks within the related time window by increasingly penalizing time window violations. Users have preferences regarding visiting combinations and respecting these are rewarded to ensure visit continuity.

Chapter 5

Mathematical Model

As the Home Health Care (HHC) problem can be interpreted as a Weekly Routing and Scheduling Problem (WRSP), this chapter presents a mathematical model formulated as a Mixed Integer Linear Program (MILP). Model assumptions are found in Section 5.1. Section 5.2 presents the necessary definitions of the model and Section 5.3 and 5.4 present the objective function and constraints respectively. Lastly, Section 5.5 comprises subtour elimination constraints to strengthen the formulation of the model while 5.6 describes the preprocessing of feasible arcs. The mathematical model is a result of further development and modification of the mathematical model presented in Lynås and Van de Pontseele (2019). Appendix A shows the compressed model.

5.1 Model Assumptions

A fundamental assumption of the mathematical model is that the HHC problem can be interpreted as a network graph consisting of nodes and arcs. Each node corresponds to a job on a day of the week, and an arc expresses an employee performing a job directly after another. The path between the nodes forms the routes for each day in the period. The HHC office is referred to as the depot and is given by node 0. All other jobs are given by unique nodes, however, at the same location, if they are related to the same user.

5.2 Definitions

This section introduces the sets and indices, parameters, and decision variables used in the mathematical model.

Sets and Indices

$\mathcal{D}$	-	set of days, $d \in \mathcal{D}$
$\mathcal{N}^d$	-	set of employees n working day $d \in \mathcal{D}$
$\mathcal{T}$	-	set of tasks $t \in \mathcal{T}$
$\mathcal{J}$	-	set of all jobs $j \in \mathcal{J}$
$\mathcal{J}_t^{\mathcal{T}}$	-	set of jobs j in task $t \in \mathcal{T}$
$\mathcal{A}^{nd}$	-	set of feasible arcs between jobs (i, j), on day d, for employee $n \in \mathcal{A}^{nd}$
$C_t^{\mathcal{T}}$	-	set of feasible patterns for task t, $c \in C_t^{\mathcal{T}}$

Parameters

A_{jc}^{dt} - $\begin{cases} 1, & \text{if job } j \text{ in task } t \text{ is assigned to day } d \text{ in pattern } c \\ 0, & \text{otherwise} \end{cases}$

$Q_j^{\mathcal{J}}$ - skill level required at job j

$Q_n^{\mathcal{N}}$ - skill level of employee n

T_{ij} - driving time between job i and job j

D_j - duration of job j

$\underline{T}_j$ - earliest starting time of job j

$\overline{T}_j$ - latest starting time of job j

$\underline{S}$ - start time of any employee n on any day d

$\overline{S}$ - end time of any employee n on any day d

P_j^n - preference score on combination between job j and employee n

$\overline{L^1}$ - maximum value of time window violation for stage one

$\overline{W}$ - maximum waiting time between two jobs

Decision Variables

x_{ij}^{nd} - $\begin{cases} 1, & \text{if employee } n \text{ performs job } j \text{ directly after job } i \text{ on day } d \\ 0, & \text{otherwise} \end{cases}$

y_c^t - $\begin{cases} 1, & \text{if pattern } c \in C_t^{\mathcal{T}} \text{ is chosen for task } t \\ 0, & \text{otherwise} \end{cases}$

s_j - start time of job j

e^{nd} - end time for employee n on day d

o^{nd} - duration of overtime worked by employee n on day d

l_j^1 - minutes violating time window for job j, 0 - $\overline{L^1}$

l_j^2 - minutes violating time window for job j, exceeding $\overline{L^1}$

$\overline{w}_d$ - the greatest workload on day d for an employee

$\underline{w}_d$ - the least workload on day d for an employee

5.3 Objective Function

$$min \qquad w_1 \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} T_{ij} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N}^d} x_{ij}^{nd} \tag{5.1}$$

$$+w_2 \sum_{j \in \mathcal{J}} (p_1 l_j^1 + p_2 l_j^2) \tag{5.2}$$

$$+w_3 \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N}^d} o^{nd} \tag{5.3}$$

$$+w_4 \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N}^d | Q_j^{\mathcal{J}} < Q_n^{\mathcal{N}}} D_j x_{ij}^{nd} \tag{5.4}$$

$$+w_5 \sum_{d \in \mathcal{D}} (\overline{w}_d - \underline{w}_d) \tag{5.5}$$

$$-w_6 \sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N}^d} P_j^n x_{ji}^{nd} \tag{5.6}$$

The objective function is minimized and given by expressions (5.1)-(5.6). The function consists of one main objective (5.1), minimizing total driving time, and several sub-objectives, (5.2)-(5.6). All of the objectives are given a weight w to set their relative importance in the model.

The sub-objective (5.2) encourages jobs to be performed within their time window, and expression (5.3) penalizes overtime. The variables l_j^1 and l_j^2 represent time window violation in minutes, and the associated parameters p_1 and p_2 set the size of the penalty. Expression (5.4) penalizes the function for every minute performed of overqualified work. The sub-objective (5.5) strives for evenness in workload during a day by penalizing the absolute greatest difference in workload for an employee on each day. Consistency in employee-user combination is rewarded in term (5.6), encouraging the users' preferred employee to perform the job.

5.4 Constraints

$$\sum_{c \in C^{\mathcal{T}}} y_c^t = 1, \qquad t \in \mathcal{T} \tag{5.7}$$

$$\sum_{i \in \mathcal{J}} \sum_{n \in \mathcal{N}^d} x_{ij}^{nd} - \sum_{c \in C^{\mathcal{T}}} A_{jc}^{dt} y_c^t \leq 0, \qquad t \in \mathcal{T}, j \in \mathcal{J}_t^{\mathcal{T}}, d \in \mathcal{D} \tag{5.8}$$

$$\sum_{i \in \mathcal{J}} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N}^d} x_{ij}^{nd} = 1, \qquad t \in \mathcal{T}, j \in \mathcal{J}_t^{\mathcal{T}} \tag{5.9}$$

$$\sum_{j \in \mathcal{J}} x_{0j}^{nd} = 1, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.10}$$

$$\sum_{j \in \mathcal{J}} x_{j0}^{nd} = 1, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.11}$$

$$\sum_{j \in \mathcal{J}} x_{ji}^{nd} - \sum_{j \in \mathcal{J}} x_{ij}^{nd} = 0, \qquad i \in \mathcal{J} \backslash \{0\}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.12}$$

Constraints (5.7) assign one pattern to each task. Constraints (5.8) ensure that each job in a task is performed on a day according to the chosen pattern for that task and constraints (5.9) ensure that each job is performed only once during the period.

Constraints (5.10) and (5.11) state that all employees must start from, and return to node 0, which represents the HHC center. The flow in and out of a node is controlled by constraints (5.12).

$$\underline{S} + T_{0j} \leq s_j + M_j^{1nd}(1 - x_{0j}^{nd}), \qquad j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.13}$$

$$s_i + D_i + T_{ij} \leq s_j + M_j^{2nd}(1 - x_{ij}^{nd}), \qquad i, j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.14}$$

$$\underline{T}_j \leq s_j \leq \overline{T}_j + l_j^1 + l_j^2, \qquad j \in \mathcal{J} \tag{5.15}$$

$$l_j^1 \leq \overline{L^1}, \qquad j \in \mathcal{J} \tag{5.16}$$

$$s_j + D_j + T_{j0} - M_j^{3nd}(1 - x_{j0}^{nd}) \leq e^{nd}, \qquad j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.17}$$

$$e^{nd} - \overline{S} \leq o^{nd}, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.18}$$

Constraints (5.13) ensure that the starting time of the first job performed on a day is consistent with the employee's starting time and the time it takes to drive from depot to the job. Constraints (5.14) guarantee that every subsequent job has a starting time respecting the previous sequence of jobs and driving times.

Constraints (5.15) set the value of l_j^1 and l_j^2, penalizing late starting times in the objective function. Constraints (5.16) ensure that the maximum value of l_j^1 is $\overline{L^1}$, to preserve two-stage linear penalization of the violation of time window.

Constraints (5.17) obtain the value of an employee's actual ending time on a day, and constraints (5.18) calculate the minutes of overtime worked by an employee on a day. Parameters M_j^{1nd} - M_j^{3nd} in (5.13), (5.14) and (5.17) are sufficiently large coefficients.

$$\sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} (D_j + T_{ij}) x_{ij}^{nd} \leq \overline{w}_d, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.19}$$

$$\sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} (D_j + T_{ij}) x_{ij}^{nd} \geq \underline{w}_d, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.20}$$

Constraints (5.19) and (5.20) create an upper and lower boundary for the workload of an employee on a day, by adding together the duration of jobs and driving times between jobs.

$$x_{ij}^{nd} \in \{0, 1\}, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d, (i, j) \in \mathcal{A}^{nd} \tag{5.21}$$

$$y_c^t \in \{0, 1\}, \qquad t \in \mathcal{T}, c \in \mathcal{C}_t^{\mathcal{T}} \tag{5.22}$$

$$s_j, l_j^1, l_j^2 \geq 0, \qquad j \in \mathcal{J} \tag{5.23}$$

$$\overline{w}_d, \underline{w}_d \geq 0, \qquad d \in \mathcal{D} \tag{5.24}$$

$$e^{nd}, o^{nd} \geq 0, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{5.25}$$

Constraints (5.21) - (5.22) ensure binary variables and constraints (5.23) - (5.25) ensure non-negativity for the remaining decision variables.

5.5 Strengthening the Formulation of the Model

Subtour Elimination (SE) constraints (5.26) are formulated to decrease the run time of the model by eliminating subtours in the network. These are added to the model for testing in Chapter 8.2.1

$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} x_{ij}^{nd} \leq |\mathcal{S}|, \qquad \mathcal{S} \subset \mathcal{J}, n \in \mathcal{N}, d \in \mathcal{D}, |\mathcal{S}| \geq 2 \tag{5.26}$$

5.6 Preprocessing of Feasible Arcs

A set of all feasible arcs, $\mathcal{A}^{nd}$, was presented in Section 5.2 and the search space is comprised of all the feasible arcs. Variables x_{ij}^{nd} are created based on the feasible arcs, leaving out infeasible or unnecessary variables, by taking into consideration the restrictions presented in Chapter 4. Preprocessing the arcs by removing illegal combinations based on these restrictions reduces the number of feasible arcs, thus reducing the search space and the run time of the MILP model.

Sufficient Skill Level
Skill requirements related to a task must be respected. No arcs are created for employees with insufficient skill levels for a given task.

Task and Jobs
Jobs within the same task can not be performed on the same day, i.e no arcs are created for these instances. Also, no arcs between $i = j$ are created, as it is not possible to travel from one job back to the same job.

Possible Patterns
Each task is given a pattern, and the jobs are to be performed according to its task's pattern. The patterns are constructed with a predefined gap between two jobs. For example for tasks with two jobs, a legal pattern requires a gap of one day between the execution of the jobs. Due to symmetry in the problem some jobs should not be performed on certain days, hence no arcs are created. This is the case for tasks with jobs performed two, three, four, and five times a week, considering a period of five days.

- **Two times a week**
  The first of the two jobs is restricted to not being performed on the two latest days of the period. Similarly the second of the two jobs cannot be performed on the first two days of the period. In both cases, no arcs are created.
- **Three times a week**
  Due to the one-day gap requirement, a task with three jobs must be performed on the first, third, and fifth day of the period. Hence arcs are only created for the first job on the first day, the second job on the third day, and the third job on the fifth day of the period.
- **Four times a week**
  Using the same argument as for tasks with frequency two times a week, the first job in the task can only be performed on the first two days of the week. Similarly, the second job in the task can only be performed on the second or third day of the week. The same pattern goes on for the third and the fourth job in the task.

- **Five times a week**
  A task with five jobs means that one job should be performed each day. Therefore the first job is assigned to the first day, the second to the second day, and so on.

Waiting Time and Time Window

Each task has a related starting time and ending time. If the earliest ending time of one task is later than the latest starting time of some other tasks, jobs in these tasks cannot be performed consecutively and no arcs are created. An arc from i to j is therefore only created if:

$$\underline{T}_i + D_i + T_{ij} \leq \overline{T}_j$$

To comply with the waiting time restriction, no arcs are created if waiting time is greater than the maximum waiting time. By way of explanation, the following must hold to create an arc:

$$\overline{T}_j - (\underline{T}_i + D_i + T_{ij}) < \overline{W}$$

Chapter 6

Solution Method

This chapter presents the proposed solution method for the Weekly Routing and Scheduling Problem (WRSP) of the Home Health Care (HHC). Section 6.1 introduces an overview of the solution method. Section 6.2 presents the solution representation and describes how initial solutions are obtained, while Section 6.3 describes the process of the Iterative Improvement Heuristic (IIH). Lastly, the implementation of the matheuristic is included in Section 6.4.

6.1 Overview of the Solution Method

The proposed solution method is a matheuristic, illustrated in Figure 6.1, consisting of two stages: constructing an initial Weekly Route Plan (WRP) and the Iterative Improvement Heuristic. The initial WRP in stage one is constructed using a set of daily routes, and this is used as input in the IIH in stage two. The IIH in stage two aims to reduce the complexity of the WRSP by iteratively solving parts of the problem and continuously updating the routes accordingly.

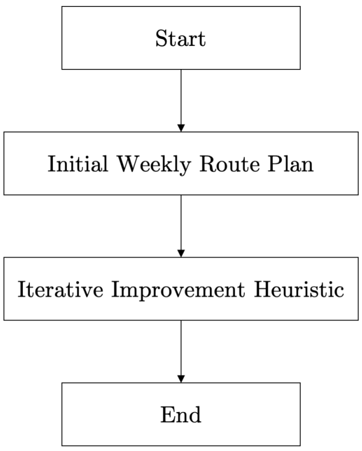


Figure 6.1: Overview of Solution Method

6.2 Solution Representation and Construction of Initial Solution

A solution comprises one route for each employee n on a day d of the week. To imitate that all routes start and end at the HHC office, the first and last job of each route is 0, as the HHC office is represented by job 0. The order of jobs is given by index k. The routes are dynamic in the solution method, and jobs may be added or removed due to improvements in the routes. Figure 6.2 illustrates the solution representation for a selection of a week. For simplicity, the notation d_i is used for day i, and n_i for employee i. In this example, employee n_1 has a route on day d_2 comprising six jobs in the given order of 0 - 12 - 2 - 36 - 24 - 23 - 33 - 0, decided by $k = 1$ to $k = 6$. This representation is chosen to make the IIH easier to implement, as it permits a part of the solution to be optimized, without majorly affecting the remaining part of the solution.

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
d_1	n_1	3	5	11	15	22	30	24	
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	31	40		
d_3	n_1	13	4	7	32	16	34		
	n_2	6	25	38	39	17	28	27	20

Figure 6.2: Solution Representation

An initial solution is used as a starting point for the IIH. The initial solution consists of daily routes compounded into an initial Weekly Route Plan (WRP). Each daily route is created from a daily perspective, based on a set of jobs given by a predefined allocation to days.

6.3 The Iterative Improvement Heuristic

The IIH optimizes a part of the WRP in each iteration by allowing reallocations of jobs. This section presents possible reallocations of jobs by defining a set of operators and outlines the process of the IIH.

6.3.1 The Operators of the Iterative Improvement Heuristic

The initial WRP is assumed to be relatively efficient on a daily basis, however, changes to the routes may prove to be favorable when considering the weekly problem. To investigate potential changes and utilize the existing routes, four operators are proposed to define possible reallocations of jobs; the Intra Swap Operator, the Inter Swap Operator, the Intra Move Operator, and the Inter Move Operator.

Intra Swap Operator
The Intra Swap Operator defines swaps of two jobs within the same route, i.e. the jobs swap indices while the rest of the route is unchanged. An example of an intra swap is illustrated in Figure 6.3, where job 5 and 22 swap index 2 and 5.

		1	2	3	4	5	6	7	8
d_1	n_1	3	5	11	15	22	30	24	
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	31	40		
d_3	n_1	13	4	7	32	16	34		
	n_2	6	25	38	39	17	28	27	20

Figure 6.3: Example of the Intra Swap Operator

Intra Move Operator

The Intra Move Operator defines a move within the route the job currently belongs to, resulting in a change in the index of the job. An example of an intra move is illustrated in Figure 6.4, as job 5 moves from index 2 to index 4, resulting in a change in indices, k-1, for jobs 11 and 15, as shown by the stippled square.

		1	2	3	4	5	6	7	8
d_1	n_1	3	5	11	15 5	22	30	24	
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	31	40		
d_3	n_1	13	4	7	32	16	34		
	n_2	6	25	38	39	17	28	27	20

Figure 6.4: Example of the Intra Move Operator

Inter Swap Operator

The Inter Swap Operator defines two jobs swapping routes, meaning exchange in the employee, day, or both. Figure 6.5 shows an example of an inter swap between job 5 and job 21. In this example, job 5 swaps with job 21 from index 2 to 4, from day 1 to 2 and employee 1 to 2. The remaining jobs are unaffected.

		1	2	3	4	5	6	7	8
d_1	n_1	3	5	11	15	22	30	24	
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	31	40		
d_3	n_1	13	4	7	32	16	34		
	n_2	6	25	38	39	17	28	27	20

Figure 6.5: Example of the Inter Swap Operator

Inter Move Operator

The Inter Move Operator defines the move of a job to a different route, implying a different employee, day, or both. An example of an inter move across both day and employee is shown in Figure 6.6. The example shows how job 5 with index 2 from the route belonging to employee n_1 on day d_1, moves to the route belonging to employee n_2 on day d_2 with index 5. The figure also marks the jobs on d_1 and d_2 affected by the operator with a stippled square, having changed indices k to $k-1$ and k+1 respectively.

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
d_1	n_1	3	5	11	15	22	30	24	
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	5	31	40	
d_3	n_1	13	4	7	32	16	34		
	n_2	6	25	38	39	17	28	27	20

Figure 6.6: Example of the Inter Move Operator

6.3.2 The Process of the Iterative Improvement Heuristic

This section outlines the process of the IIH. The IIH optimizes parts of the WRSP while continuously updating the routes to generate improved current solutions, based on an initial WRP.

The WRSP is solved using a mathematical model that allows the reallocation of jobs according to the operators describes in Section 6.3.1. As the mathematical model permits several reallocations simultaneously, the risk of reaching a local optimum is decreased. The starting point is an initial Weekly Route Plan, which is set to be the Current Solution. At the beginning of the IIH, the entire Current Solution is fixed. The fixed Current Solution comprises fixed jobs, meaning that the related preceding job, employee, and day is fixed accordingly. The set of fixed jobs is denoted $\mathcal{J}^F$. Parts of this fixed Current Solution is iteratively opened up and added to the optimization problem, which is solved by the mathematical model. The optimization problem then comprises a set of open jobs $\mathcal{J}^O$, and a set of jobs $\mathcal{R}^O$ in open routes, representing the only jobs allowed to be reallocated. The process of the IIH is presented in Algorithm 1. The algorithm consists of two steps, step one comprising lines 3-4 and step two lines 5-8.

In step one, a set of open jobs $\mathcal{J}^O$ of the Current Solution s^C, is chosen by applying the function $OpenJobSelection(s^C)$ to the Current Solution. The mathematical model solves the problem comprising only the set of open jobs $\mathcal{J}^O$, allowing reallocations according to any of the moves and swaps defined by the operators. The output from this step is a new Current Solution s^C containing updated routes. This is the input of step two of the IIH.

In step two, the search space is enlarged through increasing the number of possible reallocations of the set of open jobs, by opening routes in the Current Solution. The $OpenRouteSelection(s^C, d, n)$ function is applied to the Current Solution and used to select jobs in an open route, denoted as the set $\mathcal{R}^O$. The open route is not defined as an entire route, but may also be only a selection of jobs in a route. The jobs in the open route are still fixated to a route with a given employee n on a given day d in the Current Solution, but not to the preceding job. The model now solves the optimization problem comprising the open jobs and jobs in open routes, being the set of $\mathcal{J}^O$ and $\mathcal{R}^O$. The reallocations of the open jobs are still allowed according to any of the operators. In addition, the open jobs may interact with the open route by performing inter moves to the open route. Jobs in the open routes are only permitted to be reallocated according to the intra swap and intra move operator, as these jobs are still fixated to an employee and a day. This process continues until all routes have been explored as open routes for the reallocation of the open jobs. The Current Solution, s^C, is updated every time a new route is opened, and the output of step two is an improved Current Solution.

Algorithm 1: The Iterative Improvement Heuristic

Input: Initial Solution, s^I
Input: Maximum Run Time T^{total}
Output: Current Solution, s^C

```
1 s^C := s^I
2 while run time ≤ T^total do
3     J^O := OpenJobSelection(s^C)
4     s^C ← solve mathematical model with J^O
5     for d ∈ D, n ∈ N^d do
6         R^O := OpenRouteSelection (s^C, d, n)
7         s^C ← solve mathematical model with J^O and R^O
8     end
9 end
```

One round of the heuristic search consists of multiple iterations of solving the mathematical model until all routes of the week have been explored. The while-loop starting on line 2 illustrates that the heuristic search may be iteratively conducted by choosing a new set of open jobs. In that case, the output of step two will be the input of step one, and the open jobs are chosen by applying $OpenJobSelection(s^C)$, to the new Current Solution.

Figure 6.7 illustrates how a set of open jobs and a set of jobs in open routes can be chosen from a solution. In this example, the open jobs are jobs 5, 10, 18, and 20, and the open route is the route belonging to employee n_1 on day d_3, comprising jobs 13, 4, 7, 32, 16 and 34.

		1	2	3	4	5	6	7	8
d_1	n_1	3	5	11	15	22	30	24	
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	31	40		
d_3	n_1	13	4	7	32	16	34		
	n_2	6	25	38	39	17	28	27	20

Figure 6.7: Example of Two Open Jobs and an Open Route

Figure 6.8 illustrates how the mathematical model may choose a combination of moves and swaps defined by the operators to improve the solution. This example shows an inter move of job 5 and an intra swap of job 4 and 34.

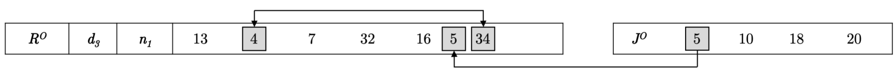


Figure 6.8: Example of the Intra Swap Operator and the Inter Move Operator

The updated Current Solution is presented in Figure 6.9, illustrating the affected parts by stippled squares. Jobs 11, 15, 22, 30, and 24 change indices to k-1 due to the inter move of job

5. Job 5 changes index, day, and employee, while jobs 34 and 4 change indices only.

		1	2	3	4	5	6	7	8
d_1	n_1	3	11	15	22	30	24		
	n_2	1	10	29	26	18	9	37	
d_2	n_1	12	2	36	24	23	33		
	n_2	8	14	19	21	31	40		
d_3	n_1	13	34	7	32	16	5	4	
	n_2	6	25	38	39	17	28	27	20

Figure 6.9: Example of Changes in Routes due to the Operators

6.4 Implementing the Iterative Improvement Heuristic

This section demonstrates how the IIH is implemented to solve the WRSP. To utilize the solution representation and explore the possible reallocations defined by the operators, adjustments to the mathematical model in Chapter 5 have been made. A description of these adjustments follows in Section 6.4.1 and Section 6.4.2.

6.4.1 Adjusting the Mathematical Model

To adjust the mathematical model to manage the definitions of open jobs, open routes, and fixed jobs, the following parameters are introduced:

$$F_{ij}^{nd} \quad - \quad \begin{cases} 1, & \text{if job } j \text{ in } \mathcal{J}^F\text{, and is performed by employee } n \text{ on day } d \text{ directly after job } i \text{ in the} \\ & \text{Current Solution} \\ 0, & \text{otherwise} \end{cases}$$

$$O_{j}^{nd} \quad - \quad \begin{cases} 1, & \text{if job } j \text{ in } \mathcal{R}^O\text{, and is performed by employee } n \text{ on day } d \text{ in the Current Solution} \\ & \text{or job } j \text{ is performed by employee } n \text{ on day } d \text{ directly after job } i \text{ in } \mathcal{J}^O \text{ in the} \\ & \text{Current Solution} \\ 0, & \text{otherwise} \end{cases}$$

An important assumption of the mathematical model is that a job j is performed if there exists an arc between i and j, where i is the preceding job of j. In the fixed Current Solution, all jobs j have a parameter F_{ij}^{nd} equal to 1 and a fixed starting time s_j. Furthermore, the parameter O_j^{nd} is set to 0 for all jobs j in the fixed Current Solution. A practical illustration of this is shown in Figure 6.11, based on the solution presented in Figure 6.10. The nodes correspond to jobs and both routes start at the HHC office, however, the routes are only for illustration and the relative location is fictive. The two routes are performed by different employees on different days, and as this illustration shows the fixed Current Solution, all jobs j have a corresponding F_{ij}^{nd} equal to 1. Table 6.12 summarizes the values for F_{ij}^{nd} and O_j^{nd} in the fixed Current Solution.

		1	2	3	4	5	6	7	8
d_1	n_2	1	10	29	26	18	9	37	
d_3	n_1	13	4	7	32	16	34		

Figure 6.10: A Compressed Version of the Solution Representation

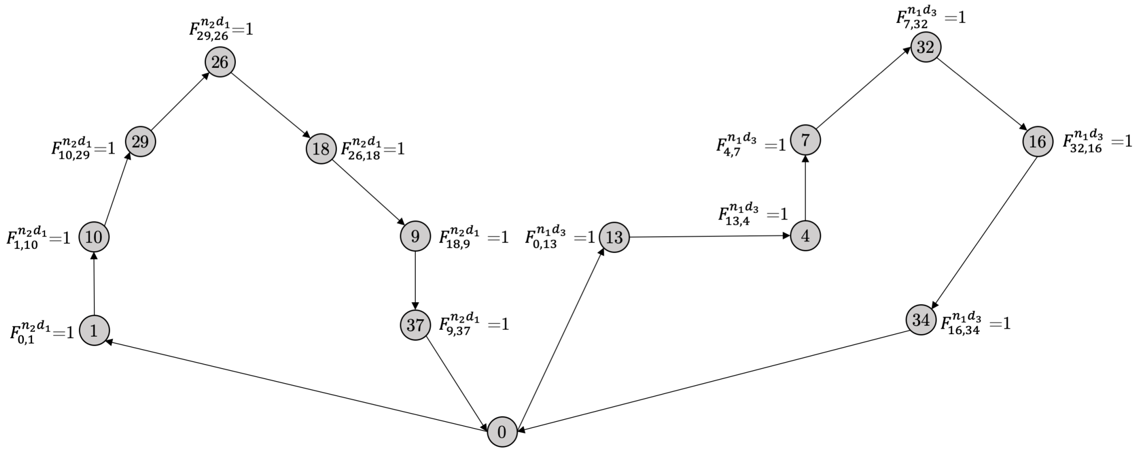


Figure 6.11: A Practical Illustration of the Solution Representation

		1	2	3	4	5	6	7	8
$F^{n_2d_1}_{ij}$		1	1	1	1	1	1	1	
$O^{n_2d_1}_{j}$		0	0	0	0	0	0	0	
$F^{n_1d_3}_{ij}$		1	1	1	1	1	1		
$O^{n_1d_3}_{j}$		0	0	0	0	0	0		

Figure 6.12: A Summary of the Values for F^{nd}_{ij} and O^{nd}_{j} in the Fixed Current Solution

In the mathematical model, the variable x^{nd}_{ij} related to each pair of jobs (i, j) must be set according to the introduced parameters, and this is ensured by constraints 6.1 and 6.2. Constraints 6.1 ensure that the set of fixed jobs in $\mathcal{J}^F$ is set to be performed according to the routes in the Current Solution. Constraints 6.2 fixate the set of jobs $\mathcal{R}^O$ in the open route to its respective employee n and day d, according to the routes in the Current Solution. This allows the preceding job i to be decided by the model.

$$F^{nd}_{ij} - x^{nd}_{ij} \leq 0, \qquad i \in \mathcal{J}, j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \;\middle|\; F^{nd}_{ij} = 1 \tag{6.1}$$

$$O^{nd}_{j} - \sum_{i \in \mathcal{J}} x^{nd}_{ij} \leq 0, \qquad j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \;\middle|\; O^{nd}_{j} = 1 \tag{6.2}$$

The jobs are iteratively added to the optimization problem, by setting the parameter F^{nd}_{ij} related to open jobs or jobs in an open route to 0 and the starting time s_j is relaxed. In addition, the parameter O_j^{nd} is set to 1 for jobs in the open routes and jobs subsequent to open jobs. If O_j^{nd} for a job j is set to 1, the related F_{ij}^{nd} is always set to 0 and the starting time is relaxed. This is illustrated by an example in Figure 6.13 and Figure 6.14. The example shows two open jobs, 10 and 18. Using the new notation, this implies that the parameters $F_{1,10}^{n_2 d_1}$, and $F_{26,18}^{n_2 d_1}$ are set to 0. In addition $O_{29}^{n_2 d_1}$ and $O_9^{n_2 d_1}$ are set to 1, as they are directly subsequent to the open jobs. Figure 6.14 also shows an open route comprising six jobs, 13, 4, 7, 32, 16, and 34, illustrated by a route with stippled lines. The parameter $O_j^{n_1 d_3}$ for these jobs is set to 1. The starting times s_j of the affected jobs are determined by the mathematical model. A summary of the values of F_{ij}^{nd} and O_j^{nd} is found in Table 6.15.

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
d_1	n_2	1	10	29	26	18	9	37	
d_3	n_1	13	4	7	32	16	34		

Figure 6.13: A Compressed Version of the Solution Representation, Illustrating Two Open Jobs and an Open Route

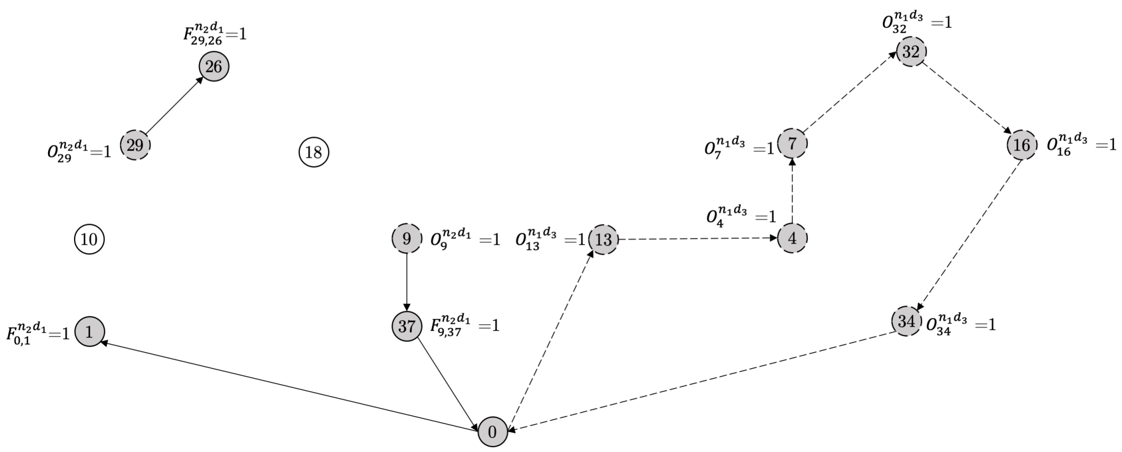


Figure 6.14: A Practical Illustration of the Solution Representation with Two Open Jobs and an Open Route

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
$F_{ij}^{n_2 d_1}$		1	0	0	1	0	0	1	
$O_j^{n_2 d_1}$		0	0	1	0	0	1	0	
$F_{ij}^{n_1 d_3}$		0	0	0	0	0	0		
$O_j^{n_1 d_3}$		1	1	1	1	1	1		

Figure 6.15: A Summary of the Values for F_{ij}^{nd} and O_j^{nd}, Illustrating Two Open Jobs and an Open Route

6.4.2 Adjusting the Implementation of the Operators

This section describes how the operators are realized through the adjustments of the mathematical model. An explanation of how the mathematical model allows the reallocation of jobs according to each of the operators follows.

Intra Swap Operator and Intra Move Operator

The mathematical model allows reallocation of jobs according to the Intra Swap Operator and the Intra Move Operator by setting F_{ij}^{nd} to 0 for the open jobs, and O_j^{nd} to 1 for the jobs in the open route. The example in 6.16 illustrates that an intra swap is performed between job 4 and 34, by setting $O_4^{n_1 d_3}$, $O_7^{n_1 d_3}$ and $O_{34}^{n_1 d_3}$ to 1. In addition, job 16 performs an intra move by setting $O_{16}^{n_1 d_3}$ and $O_{34}^{n_1 d_3}$ equal to 1, allowing job 16 to change index from $k = 5$ to $k = 6$.

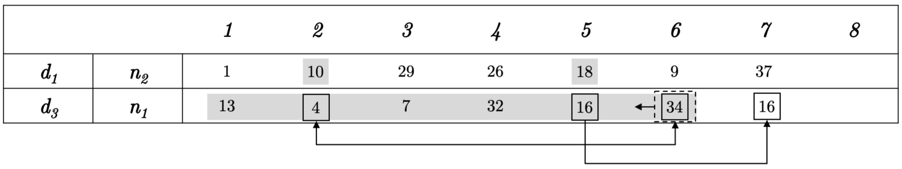


Figure 6.16: A Compressed Version of the Solution Representation, Illustrating an Intra Swap and an Intra Move

Inter Swap Operator and Inter Move Operator

The reallocations according to the Inter Swap Operator and the Inter Move Operator is allowed in the mathematical model by setting the parameter F_{ij}^{nd} related to the open jobs j to 0, and O_j^{nd} to 1 for the jobs in the open route. In the example illustrated in Figure 6.17, the open job 10 performs an inter move to an open route. Similarly to the previous examples, this is allowed by setting $F_{1,10}^{n_2 d_1}$ to 0, and the parameter $O_{29}^{n_2 d_1}$ of the subsequent job 29 to 1. As all jobs j in the open route have a parameter of $O_j^{n_1 d_3}$ equal to 1, job 10 may be reallocated anywhere within the open route, for instance to $k = 4$.

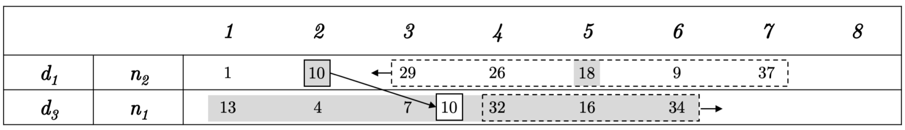


Figure 6.17: A Compressed Version of the Solution Representation, Illustrating an Inter Move

Chapter 7

Case Study

This chapter describes the Case Study in focus of this thesis. Section 7.1 describes the collaboration with Visma and Section 7.2 presents the data provided by Visma. Section 7.3 introduces a new data structure more appropriate for this problem, as well as explaining how necessary parameters were extracted and created from the given data. Lastly, Section 7.4 summarizes how test instances are generated based on these constructed parameters and how initial routes are created. The test instances and initial routes are used to conduct a computational study of the problem in Chapter 8.

7.1 Collaboration with Visma

Visma is a Norwegian software company that in recent times has established an optimization team working with delivering automated systems to the government and other public institutions. One of these automated systems is a daily route planner based on optimization algorithms, for HHC providers. In the past three years, Visma has collaborated with students at NTNU to explore new aspects of the HHC problem and further develop the Visma Route Planner. This thesis is written as a part of that collaboration, with guidance from the Visma optimization team during the fall of 2019 and the spring of 2020.

Visma is working with several municipalities in Norway and has provided the necessary data for this thesis. Visma also facilitated a day-visit to one of their pilot municipalities, for us to gain insight into many of the aspects of the HHC. This thesis is limited to consider the day-shift only, as the day-shift was found to be the busiest shift of the day with the most demanding jobs (Lynås and Van de Pontseele, 2019). The given data comprises one anonymized day-shift of the HHC in Oslo. As the data only includes one single day, the frequency distribution of jobs over the course of a week is extracted from the analysis by Lynås and Van de Pontseele (2019).

7.2 Description of Data

This section presents the contents of the data provided, which is summarized in Table 7.1. The table shows that the data provided comprises 40 employees and 300 jobs, distributed across 200 users.

Table 7.1: Characteristics of the Data

	Days	Employees	Users	Jobs
No. in Data	1	40	200	300

Employees and Shifts

In the provided data, employees are assigned a unique employee ID and a skill level according to their experience and education. Skill levels range from one to four, corresponding to an assistant, health worker, nurse, and doctor. The data comprises one day with one shift, lasting from 09:00 until 17:00.

Users and Jobs

According to the data provided by Visma, every user has a unique user ID with an associated address given in coordinates. Furthermore, every job has a unique job ID with associated details. These details are starting time, latest starting time, duration, the level of skill requirement needed, and a user ID to identify who is serviced by the job.

The starting time related to a job indicates the earliest point of time when the job can be performed, while the latest starting time expresses the latest point of time the job should start. The difference between the earliest-and latest point of starting time plus duration is referred to as the time window of the job. Figure 7.1 illustrates how jobs are distributed based on their related starting time. The figure shows that a large fraction of the jobs during a day have a starting time between 11:00 and 13:00, indicating that the workload is highest in the middle of the day.

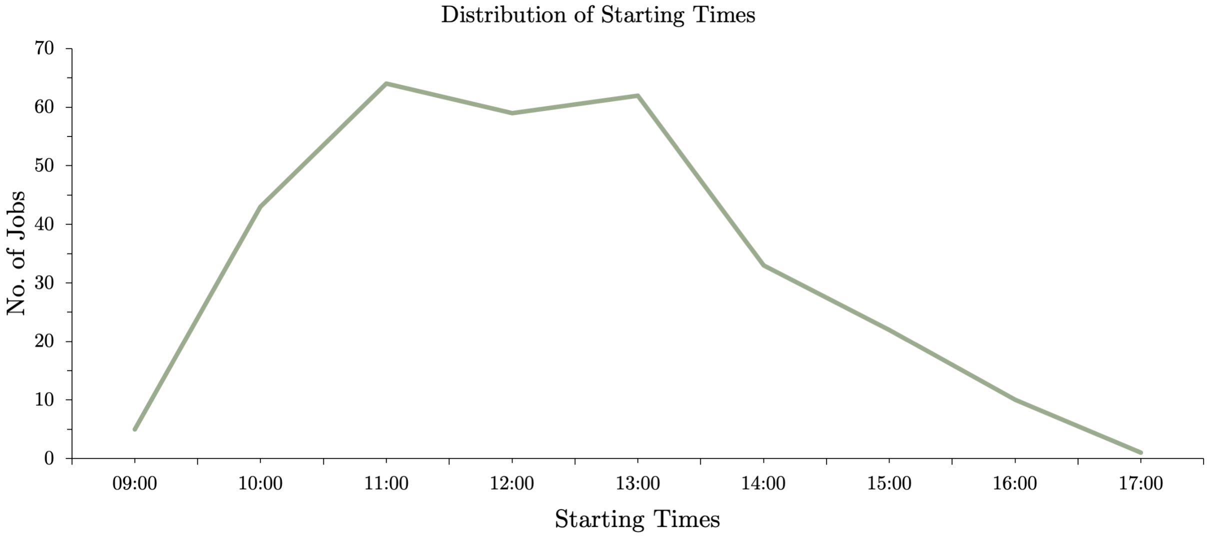


Figure 7.1: Distribution of Jobs' Earliest Starting Time

The duration of a job is the estimated time it takes to perform the job. The duration is given in minutes, ranging from 5 to 60, and 42% of jobs during the day shift have a duration of more than or equal to 45 minutes. Table 7.2 presents the distribution of jobs across durations.
By simplifying and assuming that the duration of a job can be distributed evenly across the related time window, Figure 7.2 shows the distribution of workload across the day. This is a gross simplification because a job's duration obviously cannot be distributed over the period

Table 7.2: Distribution of Jobs across Durations

Minutes	5	10	15	20	25	30	45	60
Distribution of Jobs	9.7%	10.7%	10.0%	8.3%	10.3%	9.0%	29.3%	12.7%

it can be performed, however, it illustrates the busiest hours of the day. According to this, the busiest hour is between 13:00 and 14:00, requiring 1609 minutes of work. As mentioned in Chapter 2, an employee spends somewhere between 18 and 26% of their working time on driving between users (Holm and Angelsen, 2014). This results in approximately 46 minutes of work per hour of the day. To meet the demand of 1609 minutes, at least 35 employees must be at work between 13:00 and 14:00. The need for employees ranges from six to 35 within the same shift, emphasizing that certain times of the day are very busy compared to others.

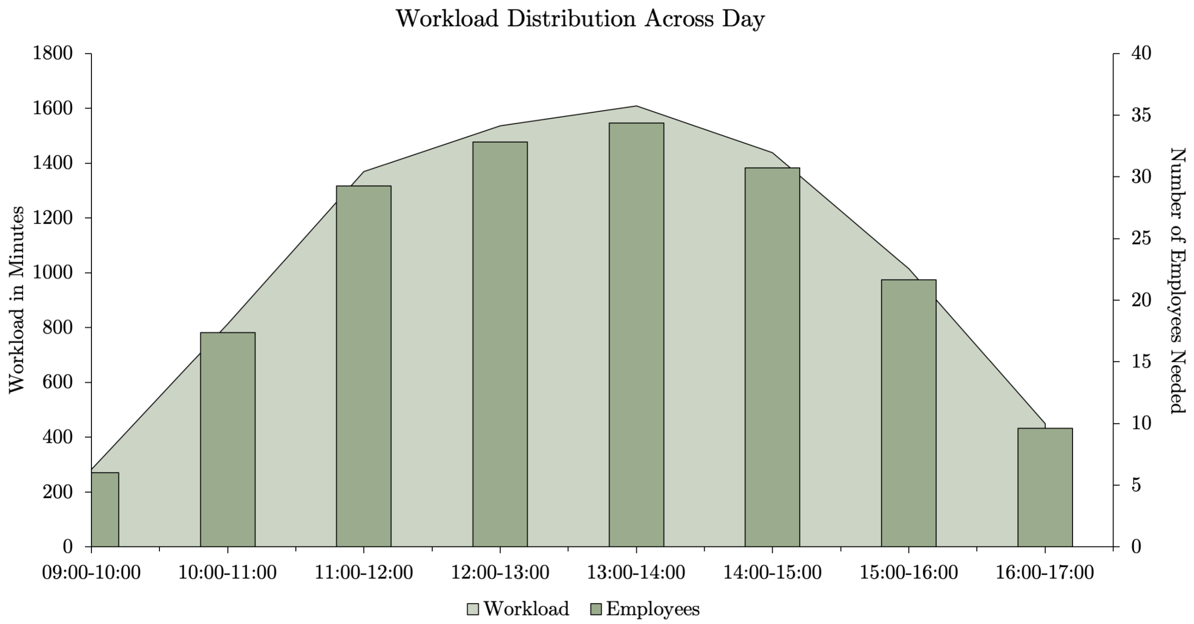


Figure 7.2: Simplified Illustration of Workload in Minutes and Employees Needed Across the Day

The number of jobs concerning a specific user varies as users have different needs. The data reveals that the number of jobs concerning a specific user varies within a day, and previous studies by Lynås and Van de Pontseele (2019) shows that the number of jobs concerning a user also varies across the week. Lynås and Van de Pontseele (2019) calculate a frequency distribution of jobs by using weekly HHC data and identifying the frequency of jobs based on jobs with the same user ID, duration, starting time, and ending time. This is referred to as frequent jobs. Examples of jobs that are frequent are the distribution of medicine, assistance in getting out of bed, or other jobs that have to be performed either several times during a day or every day at a specific time. 24.8% of the jobs are repeated every day while 53.8% only occur once during the five days. The remaining 21.4% have a frequency of two, three, or four occurrences over the period of five days, and the total distribution of frequency of jobs is shown in Table 7.3.

Table 7.3: Distribution of Jobs across Frequencies

Frequency	**1**	**2**	**3**	**4**	**5**
Distribution of Jobs	53.8%	5.7%	4.3%	11.4%	24.8%

7.3 Generating Parameters for the Test Instances

To run tests and perform a thorough analysis of the HHC problem, multiple test instances of appropriate sizes must be generated based on the provided data from Visma. To ensure the relevance of the analysis, the structure of the data must be preserved and relative sizes have been used to construct parameters. In cases where the data is insufficient or simply lacking, parameters have been created based on common values from the literature and conversation with the employees in one of the pilot-municipalities. Based on the characteristics described in Section 7.2, this section presents how parameters have been generated to create test instances.

7.3.1 Transition to a New Data Structure

To generate suitable test instances for the model, an appropriate data structure must be created. In the data set provided, all details are related to a JobID e.g. starting times and duration, but to solve the Weekly Routing and Scheduling Problem (WRSP) efficiently and reduce the complexity of the problem, a new way of structuring the data is introduced. As mentioned, jobs related to a user can be frequent, meaning that certain jobs should be performed a certain amount of times during a week. These jobs can be viewed as completely identical, e.g. distribution of medicine twice a week. Instead of having a set of identical jobs, we create a parenting task, where jobs are instances of the task, and Table 7.4 shows an example of the relation between a task and its user. This example illustrates how the distribution of medicine is a task related to a user, consisting of two identical jobs to be performed on Monday and Wednesday.

Table 7.4: Example of Relation Between Task, User and Jobs

Example: Task 10, related to user 9, consisting of two jobs				
Monday	Tuesday	Wednesday	Thursday	Friday
Job 4		Job 5		

To generate these parenting tasks, jobs are set as instances of tasks, based on the shares of frequent jobs in Table 7.3. Therefore, the only input in generating the test instances is the number of jobs. The other parameters are either calculated by using a distribution from the data or chosen randomly from the data. The basic idea is that all details previously related to a job, now belong to the overall task and the data is connected in the following way:

$$Task = TaskID,\ JobIDs,\ UserID,\ DT,\ P,\ ST,\ LST,\ \overline{LST},\ D,\ SR,\ EU$$

TaskID=*ID*, ***JobID***=*JobID*, ***UserID***=*UserID*, ***Driving Time***=*DT*, ***Pattern***=*P*, ***Starting Time***=*ST*, ***Latest Starting Time***=*LST*, ***Absolute Latest Starting Time***=$\overline{LST}$, ***Duration***=*D*, ***Skill Requirement***=*SR*, ***Employee-User score***=*EU*,

An elaboration of how the parameters are created and connected to a task follows.

Driving Times

Each task has a specific user, and every user has a home address given in coordinates. These coordinates are used to calculate driving times in minutes between all users, and thereby tasks. If two tasks belong to the same user, such as showering and distribution of medicine, these can be performed consecutively with a driving time of zero between the two. Driving Time between two tasks may be asymmetric due to, for instance, one-way roads, as the calculations respect feasible driving patterns.

Patterns

To ensure that the jobs within a task are evenly spread over the course of five days, according to the associated frequency, patterns are created. A pattern gives a list of days when the jobs can be performed. There are many possible patterns, for instance, there are ten ways of choosing two days within a five day period for a task with frequency two. To reduce the complexity of the problem and ensure a gap of one day between each job, if attainable, the number of patterns is limited to 15. A description of possible patterns related to frequencies is found in Table 7.5, and a more detailed illustration is found in Appendix B.1.

Table 7.5: Possible Patterns

Frequency	No. of Patterns	Description
5	1	Every day of the week
4	5	Four times a week
3	1	Mon, Wed and Fri
2	3	Mon and Wed, Tue and Thu or Wed and Fri
1	5	Any day of the week

Starting Times and Duration

Every task is given a Starting Time, a Latest Starting Time, and Duration by randomly selecting a job's associated starting time, latest starting time, and duration from the data. Starting Time and Latest Starting Time are given in minutes after midnight, and Duration is given in minutes. The difference between the Starting Time and the Latest Starting Time is referred to as a Time Window, given by:

$$\textit{Time Window} = \textit{Latest Starting Time - Starting Time + Duration}$$

Starting after the Latest Starting Time is allowed at an extra cost related to overtime, however only until the Absolute Latest Starting Time or one hour after the shift ends at 17:00. Absolute Latest Starting Times are given in minutes after midnight, and given by the following:

$$\textit{Absolute Latest Starting Time} = \textit{min}\Big\{\textit{Latest Starting Time + 60, 1080}\Big\}$$

Maximum waiting time is set to two hours between two jobs and the maximum driving time between two tasks is set to 15 minutes.

A summary of the relations between the different points of time concerning a task is found in Figure 7.3.

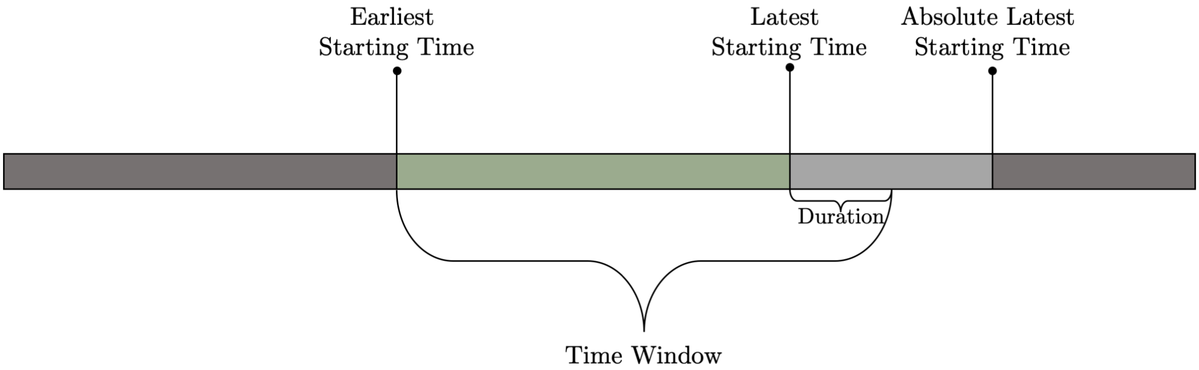


Figure 7.3: Relations Between Time Parameters

Skill Requirement

All tasks are given a level of skill requirement needed to perform the task. The skill requirements are based on the distribution of skill requirements in the data given in Table 7.6.

Table 7.6: Distribution of Tasks across Skill Requirements

Level	**1**	**2**	**3**	**4**
Distribution of Tasks	26.0%	24.0%	24.0%	26.0%

7.3.2 Generating Parameters Related to Employees

The number of employees per day is determined based on the relationship between the number of jobs and the number of employees in the data set provided, as shown in Table 7.1. The initial job-employee ratio of 7.5 is rounded up to 8.

Employees are given a skill level related to their education and experience, based on the distribution from the data, given in Table 7.7. To ensure that the skill level of the employees is sufficient to serve the skill requirements of tasks on a certain day, at least one employee with sufficient skill level must work each day.

Table 7.7: Distribution of Employees Across Skill Levels

Level	**1**	**2**	**3**	**4**
Distribution of Employees	25.0%	25.0%	25.0%	25.0%

Compliance of user convenience is done by generating a fictive Employee-User score. The Employee-Job preference defined in the mathematical model in Chapter 5 now belongs to a user instead of a job, but the result is the same. The scale of convenience ranges from 0 to 1. The combination of user and employee is chosen randomly in the creation of the test instance by giving all tasks one preferred employee with a score of 1. Lastly, it is ensured that all employees have approximately the same number of tasks they are preferred for.

7.4 Generating Test Instances and a Weekly Route Plan

Based on the previous discussion, test instances are created. The characteristics of an instance are given in Appendix B.1. All instances are constructed based on the same set of relative sizes and parameters, and the number of jobs is the only input.

In this thesis, the Visma Route Planner is utilized to generate daily routes based on the test instances. A test instance comprises five days that can be divided into individual days and used as input in the Visma daily Route Planner. These daily routes can thereafter be compounded into a Weekly Route Plan of five days, which are further studied for potential improvement in Chapter 8.

Chapter 8

Computational Study

This chapter presents how the Iterative Improvement Heuristic (IIH) is applied to the Weekly Routing and Scheduling Problem (WRSP) to generate an improved Weekly Route Plan (WRP) with respect to different objectives. Section 8.1 introduces the hardware and software used in this study. Section 8.2 forms the basis for the study, by testing and strengthening the Mixed-Integer Linear Programming (MILP) model and preparing the configuration of the heuristic parameters. Furthermore, Section 8.3 configures the heuristic parameters and validates the proposed IIH. Finally, a practical study of the impact on routes relative to improvement, when considering different stakeholders, is found in Chapter 8.4.

8.1 Test Environment

The mathematical model described in Chapter 5 is implemented using the commercial optimization software Xpress-Workbench. The test instances are generated using Python and are exported as .txt-files that are used as input in the MILP, and into the Visma Route Planner. Python is also used to implement the proposed solution method. All instances are solved on Apple MacBook. Minor deviations may occur due to external factors, such as additional software running on the MacBook. The software and hardware used are summarized in Table 8.1.

Table 8.1: Hardware and Software Used in Testing

Processor	1.4 GHz Dual-Core Intel Core i5
Memory	4,0 GB
Operating System	macOS Catalina Version 10.15.4
Xpress-Workbench	5.0.2 64 bit
Python Version	3.8.0

8.2 Preliminary Study

This section presents the preliminary study for the IIH. First, Section 8.2.1 illustrates the complexity of the WRSP and proposes a strengthening formulation of the model. Second, Section 8.2.2 presents the preparations for the configuration of the heuristic parameters.

8.2.1 Studying the Mixed Integer Linear Programming Model

To test and strengthen the MILP presented in Chapter 5 a Base Case Weight set is needed to solve the WRSP. The objective function consist of six terms, each with a respective weight, w, as Table 8.2 describes. The weights represent the valuation of the associated objective term and routes greatly depend on the size of the weights relative to one another. In addition, the unit of the objective terms plays an important role as the sizes of the units differ. The objectives Driving Time, Overtime, and Violation of Time Window, are set to be the most important as these are directly related to cost for the HHC providers, employee- or user convenience. These objective terms are given a weight of 1, while the remaining are set to 0.1 to still generate realistic routes. The Base Case Weight set is presented in Table 8.3.

Table 8.2: Objectives with Weighting Variables and Units

Weight	Objective	Abbreviation	Unit
w_1	Driving Time	DT	Minutes
w_2	Time Window	TW	Minutes
w_3	Overtime	OT	Minutes
w_4	Overqualified Work	OW	Minutes
w_5	Absolute Difference in Workload Across Days	WD	Minutes
w_6	Employee-User Preferences	EU	# of Jobs

Table 8.3: Base Case Weight Set

	DT	TW	OT	OW	WD	EU
Base Case Weights	1	1	1	0.1	0.1	0.1

Instances of different sizes, based on the number of jobs, are created to identify the greatest size the model can solve exact within a reasonable time. Each size was solved twice based on two separate test instances generated as described in Chapter 7. The maximum run time was set to one hour. Table 8.4 summarizes the results, showing that instances up until the size of 20 jobs can be solved using the model described in Chapter 5. It is therefore evident that solving real-life instances is extremely challenging for the MILP and the need for a heuristic approach to solve the WRSP is exposed.

Subtour Elimination (SE) constraints with a subset of size limited to two and three are added to the model to strengthen the formulation. Table 8.4 reveals that the SE constraints (5.26) on average reduce the run time of the smaller size instances of 20 and 25 jobs, and reduce the gap when solving most of the instances greater than 25 jobs. The SE constraints are therefore included in the MILP model throughout the rest of the computational study.

Table 8.4: Testing and Strengthening of the MILP

			With SE Constraints	
# of Jobs	**Run Time**	**Gap**	**Run Time**	**Gap**
20	50	0.00%	4	0.00%
25	>3600	0.33%	142.5	0.00%
30	>3600	8.37%	>3600	5.99%
35	>3600	16.26%	>3600	18.05%
40	>3600	28.09%	>3600	25.12%

8.2.2 Preparing the Configuration of the Heuristic Parameters

As mentioned in Chapter 7, the Visma Route Planner is utilized to obtain optimized daily routes, and an initial Weekly Route Plan (WRP) is constructed based on these. To be able to identify the improvement caused by the IIH compared to an initial WRP, the objective weights are adjusted to imitate the Visma Route Planner. This adjusted objective weight set is referred to as the Visma Weight Set and is presented in Table 8.5. This set of weights is used throughout Section 8.4.2.

Table 8.5: Visma Weight Set

	DT	TW	OT	OW	WD	EU
Visma Weight Set	0.5	0.5	0.33	0	0	0

The parameters needed for the IIH must be configured using instances of appropriate sizes to imitate the real-life challenges of the HHC. Therefore, three instance sizes are introduced, consisting of 20, 40, and 80 jobs, referred to as micro, small, and large instances respectively. The instances are constructed based on the discussion in Chapter 7 and the characteristics of the instances are summarized in Table 8.6. As previously described, a task consists of one or several identical jobs. The small and large instances are introduced to investigate whether the size of the instance affects the choice of parameter set. The micro instances are only included in the validation of the heuristic in Section 8.3.6.

Table 8.6: Characteristics of the Instances

	Characteristics		
Instance Size	**Micro**	**Small**	**Large**
Jobs	20	40	80
Jobs per skill level: 1-2-3-4	5-5-5-5	10-10-10-10	20-20-20-20
Employees	1	1	2
Employees per skill level: 1-2-3-4	0-0-0-1	0-0-0-1	0-0-1-1
Preferred Tasks per employee	14	26	26-26
Number of days	5	5	5
Tasks per frequency: 1-2-3-4-5	11-1-1-1-0	20-2-1-2-1	41-3-2-3-3

Selecting Open Jobs

The concept of the IIH is to decrease the solution space by iteratively solving an optimization problem consisting of only a selection of jobs from the Current Solution, as described in Chapter 6. This selection of jobs is previously referred to as open jobs and jobs in open routes. The optimization problem is concerned with the reallocation of the selected jobs and must be of a size that the mathematical model can solve within reasonable time. To increase the probability of obtaining an improved solution with few heuristic searches, the selection of jobs should be conducted wisely. The hypothesis is that some jobs may be sub-optimal allocated in the initial WRP and that these should be identified and added to the optimization problem to generate more efficient routes. By adding jobs to the optimization problem, it is referred to solving the optimization problem comprising these identified jobs, as previously described in Chapter 6.

The identification of open jobs is executed by the $OpenJobSelection(s^C)$ function presented in Chapter 6. The function aims to identify the jobs in the Current Solution causing the most negative impact on the routes concerning different objectives. Examples are jobs contributing to long driving times, overtime, or violation of time window. Throughout Section 8.3, the function $OpenJobSelection(s^C)$ identifies jobs contributing to long driving times in the Current Solution. This is retrieved by calculating the number of minutes saved if the job is removed from the route it currently belongs to, illustrated in Figure 8.1. The example shows the saving of driving time if job 2 is removed, calculated as $DT_{12} + DT_{23} - DT_{13}$. The jobs with the largest savings are selected to be added to the optimization problem, and the number of jobs chosen is the parameter J, configured in the next section.

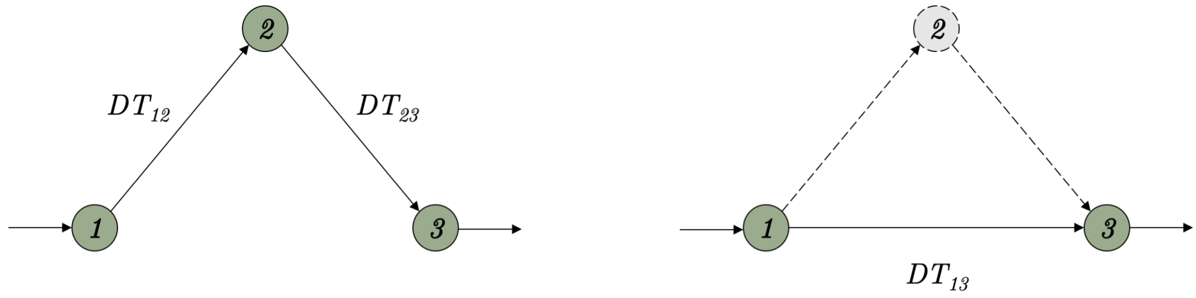


Figure 8.1: Example of Calculating Saved Driving Time

The structure of the problem implies that the number of possible reallocations of jobs differs across tasks. In Chapter 7, a task was introduced as a set of one or several jobs, with a set of possible patterns. A pattern restricts the jobs in a task to be allocated to certain days, and the set of patterns for a given frequency, therefore, restricts the number of possible reallocations of the job. For example, tasks with frequency 3 and 5 have only one pattern, implying that jobs belonging to these tasks cannot be reallocated across days. Furthermore, the jobs in tasks with frequency 2 or 4 are dependent on other jobs, as these jobs cannot be reallocated if this is not consistent with one of the allowed patterns. Jobs belonging to tasks with frequency 1, however, may be performed on any day of the week and the allocation is not dependant on other jobs. This implies that the potential for reallocation is greater for jobs belonging to frequency 1 tasks, compared with other frequencies, and makes this a wise choice for the $OpenJobSelection(s^C)$.

The number of jobs capable of reallocation also directly affects the potential for improvement, as a greater number increases the probability of improvement. This supports the selection of jobs in frequency 1 tasks, as they constitute 53.8% of all jobs in the test instances. Therefore, in the configuration of the parameters, and the validation of the heuristic, the $OpenJobSelection(s^C)$ conditionally selects jobs belonging to frequency 1 tasks and having the greatest saved driving time potential.

8.3 Configuration and Validation of the Iterative Improvement Heuristic

This section presents the configuration of the parameter set (J,R) needed in the IIH. Section 8.3.1 configures the heuristic parameter J denoting the number of open jobs, while Section 8.3.2

configures the heuristic parameter R, denoting the number of jobs in an open route, in combination with J. Both parameters are configured for small and large instances, based on the average results from five instances of each size. Finally, Section 8.3.6 validates the IIH against the MILP and the initial WRP generated by Visma.

In the configuration, the maximum run time of the IIH is set to 30 minutes, and the registered objective value is the best solution found within this time limit. As explained in Chapter 6, the IIH comprises several rounds of heuristic searches, until a maximum run time is reached. One round of the heuristic search consists of multiple iterations of solving the mathematical model until all routes of the week have been explored as open routes.

8.3.1 Introducing and Testing the Parameter J

The parameter J in the heuristic decides the number of jobs that are added to the optimization problem, to be reallocated by the mathematical model. The jobs are selected by the function $OpenJobFunction(s^C)$ based on the number of jobs given by J. A suitable range for this parameter indicates an appropriate range for testing of the parameter set (J,R), which is used in the IIH. The study of J is completed by iteratively performing only step 1 of the IIH until the maximum run time is reached. Preliminary testing found that values of the parameter J between 4 and 18 is a reasonable range for testing and the ranges vary slightly between the small and large instances.

Table 8.7 presents the results from testing the IIH with different values for parameter J, for five small and five large instances. The table shows the average improvement of the objective value compared to the initial WRP, obtained from Visma. Furthermore, the minimal objective value improvement, as well as the maximum objective value improvement is presented, to reveal the ranges across the test instances. Lastly, the standard deviation of the improvement is registered, where the unit is percentage points, but for simplicity, this is written as %. The IIH applied to small and large instances is in most cases capable of solving the optimization problem up until J=18 and J=14, respectively. On the other end, the solution method with a $J < 8$ for small instances and a $J < 4$ for large instances does not provide significant improvement.

Table 8.7: Configuration of Heuristic Parameter J

	Improvement of O.V. (%)							
	Small Instances				Large Instances			
J	**Min.**	**Avg.**	**Max.**	**Std.**	**Min.**	**Avg.**	**Max.**	**Std.**
4					1.12	2.61	3.51	0.99
6					2.43	3.38	4.49	0.94
8	2.25	7.52	12.56	4.27	2.43	3.35	4.08	0.77
10	3.02	9.26	14.03	4.86	2.34	4.45	6.53	1.84
12	2.78	9.34	13.51	5.06	2.61	5.05	7.31	1.96
14	0.00	8.58	18.41	6.71	0.00	2.65	5.49	2.51
16	0.00	9.13	20.21	7.33				
18	0.00	3.73	9.70	5.12				

In general, the results exhibit a notable improvement of the objective values, compared to the initial WRP, for both the small and large instances. The average improvement is greater for the small instances compared to the large instances, which can be explained by the number of iterations conducted within the maximum run time. The small instances are relatively easy for the model to solve, and several heuristic searches are performed within the run time of 30 minutes. This is not the case for the large instances, where the heuristic search is completed only a few times or not completed at all. It can also be argued that the average improvement is greater for the smaller instances as a given J constitutes a larger portion of the jobs in the smaller instances, compared to the large. Furthermore, the degree of average improvement increases as J increases for both instance sizes, which is rather instinctive as the optimization problem shifts towards the exact optimization problem.

For both the small and large instances, $J = 12$ yields the best average improvement in objective value. For $J > 12$, some of the initial WRPs related to the small and large instances are not improved as the problem becomes too hard to solve within the time limit, resulting in a minimal objective value change of 0%. This is also the reason for the descending average improvement in objective value, as J increases. The standard deviation measures the degree of variation from the average improvement, in the test instances. In general, the standard deviation is greater in the small instances compared to the large instances, and the standard deviation increases as J increases. Hence, an increase in J increases the average improvement of the objective value, but also generates an increase of the standard deviation, as some instances are not solved within the maximum run time.

The improvement potential is identified, however, only a limited number of possible reallocations of the open jobs have been explored, as only step 1 of the algorithm has been conducted. The following describes how the additional parameter R permits additional reallocations to occur.

8.3.2 Introducing and Testing the Parameter R

Parameter R is introduced to allow the open jobs to move to different routes, as described as step 2 of the algorithm, in Chapter 6. R is the maximum number of jobs in an open route, i.e. the maximum length of the open route. As explained in Chapter 7, the average length of a route is 8, and testing of R combined with different values of J revealed that R=8 yielded the best results. R=8 implies that the heuristic selects open routes of size up until 8. Shorter routes were explored and improvement was registered, however, the improvement was consistently poorer. The extensive results are found in Appendix C.1.

Figure 8.11 illustrates an example with three open routes, where R=8. An open route belongs to an employee n on a day d, and this implies that the entire route of an employee may be divided into several open routes, where the maximum length is equal to 8. This is conducted by applying the function $OpenRouteSelection(s^C, d, n)$ to the Current Solution. In the figure, the route belonging to n_2, consisting of 11 jobs, is divided into two open routes of size 8 and 3, and these are added to the optimization problem in two separate iterations. The mathematical model solves the optimization problem in each iteration. The size of R also affects the number of iterations needed to conduct one full round of the heuristic search, directly influencing the run time of the solution method, suggesting that greater values of R are preferred. R=8 is therefore chosen, and further configuration of the parameter set (J,R) is conducted by combining R=8 with the seemingly best values for J, found in the previous testing for the small and large instances as shown in Table 8.7.

		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
d_1	n_2	1	10	29	26	18	9	37	32	3	8	6
					One Iteration						One Iteration	
d_3	n_1	13	4	7	32	16	34					
				One Iteration								

Figure 8.2: Example of Open Routes when R=8

8.3.3 Configuring the Parameter Set for Small Instances

Table 8.8 displays the results from the configuration of the parameter set (J, R) on small instances. The numbers concerning parameter (J, 0), are presented previously in Table 8.7, while the remaining parameter sets where $R = 8$, represent the augmenting of open routes to the optimization problem. The results show that the augmenting of $R = 8$ leads to an additional increase in the average improvement of the objective value compared to previous tests. The only exception is for $J = 16$, where the average improvement is greater when not opening any routes. This may be because the optimization problem becomes larger when the jobs in the open routes are added, hence solving the mathematical model takes longer time, and the number of iterations within the maximum run time is decreased.

For most parameter sets with a value of R greater than zero, even the minimal value of the improvement is higher than the average improvement when $R = 0$. This proves that opening routes to reallocate the open jobs is significantly enhancing the improvement compared to only considering the open jobs. It also appears that a low value of J combined with $R = 8$ yields better results than increasing J and setting R to 0, as (J,R) = (8,8) gives a greater average improvement than any of the parameter sets (J,0). The parameter set (12,8) yields the greatest average improvement, with an average objective value increase of 15.23%. Furthermore, the standard deviation related to this parameter set is the lowest, indicating that there are smaller differences between the improvements of the instances for this parameter set, compared to the other parameter sets.

Table 8.8: Configuration of Parameter Set (J,R) on Small Instances

		Small Instances Improvement of O.V. (%)			
J	***R***	**Min.**	**Avg.**	**Max.**	**Std.**
8	**0**	2.25	7.52	12.56	4.27
10	**0**	3.02	9.26	14.03	4.86
12	**0**	2.78	9.34	13.51	5.06
14	**0**	0.00	8.58	18.41	6.71
16	**0**	0.00	9.13	20.21	7.33
18	**0**	0.00	3.73	9.70	5.12
8	**8**	8.9	15.10	20.37	4.96
10	**8**	9.26	15.00	19.62	4.29
12	**8**	11.49	15.23	18.20	2.92
14	**8**	0.00	12.40	21.37	7.85
16	**8**	0.00	8.31	15.57	7.72
18	**8**	0.00	4.77	14.90	6.86

To further evaluate the performance of the parameter sets, an analysis of the improvement in objective value across time and number of heuristic searches, is conducted. The seemingly best parameter sets, (10,8), (12,8), and (14,8), are therefore tested on one instance, and the improvement in objective value is registered. Figure 8.3 and Figure 8.4 illustrate the improvement in objective value across time and number of heuristic searches respectively. Figure 8.3 shows that the objective value for the parameter set (12,8) linearly decreases during the first 600 seconds and then stagnates, not obtaining any improvement in the remaining 1200 seconds. The parameter set (10,8) decreases towards its minimal objective value at 900 seconds, and parameter set (14,8) shows no improvement in objective value at all. The latter indicates that a feasible solution is not found within the maximum run time, and the objective value is therefore equal to the objective value in the initial WRP obtained by Visma. After 30 minutes, parameter set (12,8) provides the best solution for this test instance.

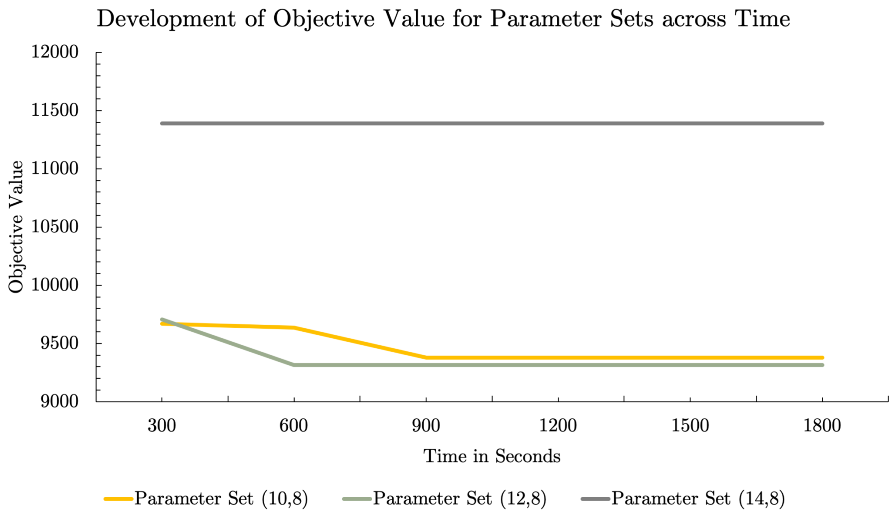


Figure 8.3: Development of Objective Value for Parameter Sets Across Time

Figure 8.4 shows the improvement of the objective value, considering the number of heuristic searches. In this figure, the parameter set (14,8) is not present, as not even a single heuristic search is conducted. For parameter sets (10,8) and (12,8) the solution is improved up until the third heuristic search, and then it stagnates. The figure illustrates that the parameter set (10,8) provides the best solution after one heuristic search, but is outperformed by parameter set (12,8) as of the second heuristic search. Furthermore, results show that for the considered parameter sets and the given test instance, three heuristic searches are sufficient to find the optimal solution provided by the IIH. Both parameter sets perform eight heuristic searches within the maximum run time, implying that the difference between $J = 10$ and $J = 12$ is of little significance concerning the number of iterations needed to perform one heuristic search, indicating that the largest J is preferred.

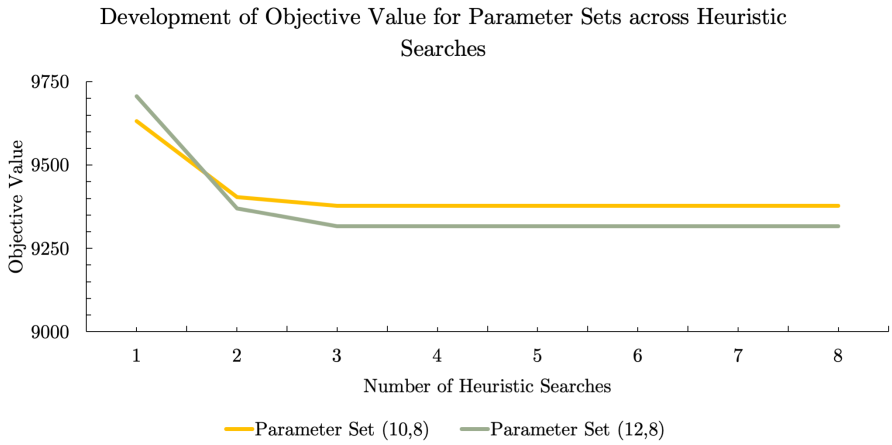


Figure 8.4: Development of Objective Value for Parameter Sets Across Heuristic Searches

Due to the largest average objective value improvement, as well as the lowest associated standard deviation, parameter set $(J, R) = (12,8)$ is chosen for small instances, for the IIH. This is supported by the study of the development of the objective value considering time and number of heuristic searches.

8.3.4 Configuring the Parameter Set for Large Instances

The performance of the parameter set may be dependant on the size of the test instances, and this is investigated by also configuring the parameter set for large instances. The results are presented in Table 8.9, for different values of J in combination with $R = 0$ and $R = 8$. The results concerning parameter sets $(J, 0)$ are the same as in Table 8.7, and are only repeated for comparison with the augmenting of open routes. Once again, the average improvement of the objective value for most of the large instances increases when the open route R is added to the optimization problem. It is shown that the parameter set with the lowest value of J, being $(J,R) = (4,8)$, yields a higher average improvement in objective value, than any of the parameter sets $(J, 0)$. Hence, including open routes to the optimization problem is beneficial, compared to only including open jobs.

The parameter set generating the largest average improvement in objective value is $(J, R) = (8,8)$, yielding an average improvement of 8.36 %. However, the standard deviation, being equal to 2.91%, is the third-highest across all the parameter sets. Similar to the small instances, the standard deviation tends to increase as J increases.

Table 8.9: Configuration of Parameter Set (J,R) on Large Instances

		Large Instances Improvement of O.V.(%)			
J	R	Min.	Avg.	Max.	Std.
8	**0**	2.43	3.35	4.08	0.77
10	**0**	2.34	4.45	6.53	1.84
12	**0**	2.61	5.05	7.31	1.96
14	**0**	0.00	2.65	5.49	2.51
4	**8**	4.90	6.39	8.64	1.40
6	**8**	5.02	7.19	10.67	2.23
8	**8**	5.17	8.36	12.58	2.91
10	**8**	6.33	8.33	14.04	3.22
12	**8**	2.91	4.95	8.03	2.15
14	**8**	0.00	2.98	8.20	3.39

For the parameter sets, (14,0) and (14,8), the minimal value of the improvement of the objective function is 0%. The lack of improvement indicates that the optimization problem for at least one of the large instances is too complex to find a feasible solution within the maximum run time, and not even step 1 of the IIH has been conducted. In general, the average improvement in objective value is decreasing for parameter sets when $J > 12$, and this can be explained by Figure 8.5. The figure illustrates that the average number of heuristic searches conducted decreases when J increases. For (J,R) = (12,8) and (J,R) = (14,8), the IIH is not even able to solve a single heuristic search within the maximum run time of 30 minutes, probably because of the execution time of each iteration. Furthermore, this implies that there are open routes in the problem that have not been considered for reallocation of open jobs, and this obviously affects the improvement of the solution for parameter sets (12,8) and (14,8). To address this challenge, an investigation of the implementation of a time limit on the execution time of each iteration is conducted in the following section.

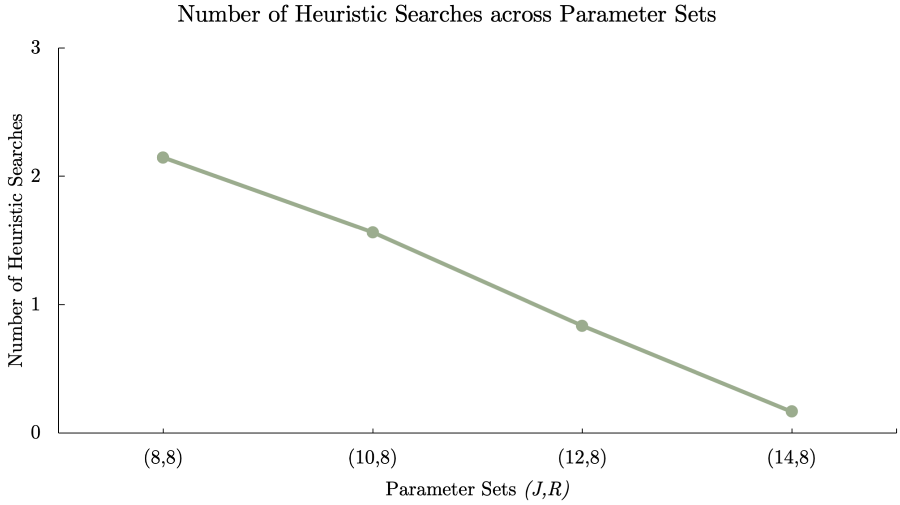


Figure 8.5: Average Number of Heuristic Searches Across Parameter Sets in Large Instances

8.3.5 Adjusting the Time Limit of Each Iteration

The testing of large instances reveals the challenges related to large instances, as not even one single heuristic search is conducted within the maximum run time of 30 minutes, for large values of J. Furthermore, when solving the optimization problem with the MILP, the objective value improvement is often greater at the beginning of the execution, implying that interrupting the execution and retrieving the best solution found at that time may be beneficial as it would save time. Therefore, to ensure an increase in the number of iterations of the optimization problem, as well as exploiting the fact that improved solutions are usually found early on, a time limit of each iteration is enforced. The parameter sets of $R = 8$ and J ranging from 8 to 14 is tested again with time limits of 1 minute and 4 minutes for each iteration.

The results are found in Table 8.10, revealing an increase in the average objective value improvement for all parameter sets, for both time limits. This implies that a higher number of heuristic searches yields a greater average improvement of the objective value, as a larger portion of the solution space is explored.

Table 8.10: Configuration of Parameter Set (J,R) with Time Limit

			Large Instances Improvement of O.V. (%)			
J	***R***	**Time Limit**	**Min.**	**Avg.**	**Max.**	**Std.**
8	**8**	No limit	5.17	8.36	12.58	2.91
10	**8**	No limit	6.33	8.33	14.04	3.22
12	**8**	No limit	2.91	4.95	8.03	2.15
14	**8**	No limit	0.00	2.98	8.20	3.39
8	**8**	1 min.	6.75	8.50	12.58	2.52
10	**8**	1 min.	7.05	8.96	13.19	2.66
12	**8**	1 min.	7.55	8.77	12.62	2.16
14	**8**	1 min.	4.63	7.25	10.25	2.15
8	**8**	4 min.	6.51	8.63	12.58	2.59
10	**8**	4 min.	6.42	9.11	12.93	2.96
12	**8**	4 min.	4.95	6.90	8.19	1.31
14	**8**	4 min.	2.80	5.23	7.83	1.92

The largest average improvement of objective value is registered for parameter set (10,8) with a time limit of 4 minutes, giving an average improvement of 9.11%. However, the standard deviation using this parameter set is the highest across all parameter sets with time limits, being 2.96%. By setting the time limit to 1 minute for this parameter set, the average improvement decreases to 8.96%, being the second-largest average improvement of objective value. The standard deviation also decreases, however not significantly. The third-largest average improvement of objective value is 8.77%, generated by the parameter set (12,8) and 1-minute time limit. The standard deviation with this parameter set is 2.16%, being significantly lower than 2.96%. When comparing (10,8) with 4-minute time limit and (12,8) with 1-minute time limit, the relative improvement in standard deviation is larger than the relative reduction in the average improvement of the objective value. Hence, it can be argued that the parameter set (J,R)=(12,8) is preferred as the standard deviation is smaller, which implies that the probability of deviation in the improvement of a solution is lower.

Figure 8.6 presents the average number of heuristic searches conducted within the maximum run time of 30 minutes, where each iteration has a time limit of 1 or 4 minutes. As expected, it shows that the average number of heuristic searches conducted is higher for all parameter sets, when the time limit is set to 1 minute, compared to 4 minutes.

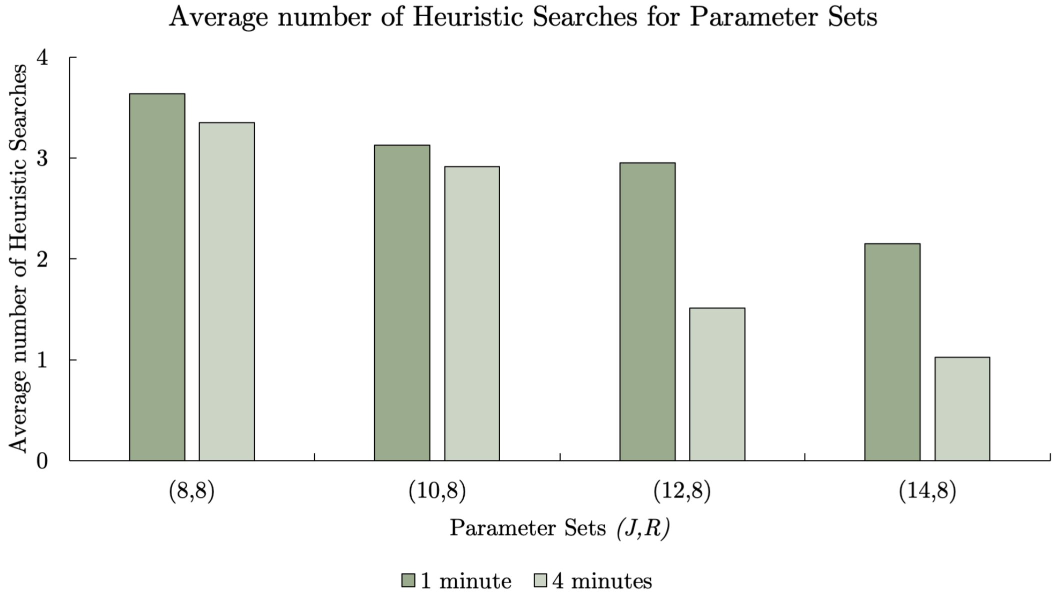


Figure 8.6: Average Number of Heuristic Searches Across Parameter Sets

Table 8.10 shows that for parameter sets (8,8) and (10,8) the 4 minutes time limit provides the best results. However, when J increases, the 1-minute time limit outperforms the 4-minute time limit, indicating that the extra 3 minutes of solving each mathematical model does not justify the lack of iterations, which are conducted when the time limit is equal to 1 minute. As previous results of the configuration of the parameter sets favor larger values of J, and the 1-minute time limit outperforms the 4-minute time limit as J increases, the 1-minute time limit is implemented in the IIH.

This section reveals the potential for improvement from the initial WRP when applying the IIH. The parameter set of $(J,R) = (12,8)$ is chosen for both small and large instances. This implies that $J = 12$ is used in step 1 of the algorithm, and that $(J,R) = (12,8)$ is used in step 2 of the algorithm. A time limit of 1 minute for each iteration is implemented when solving large instances. The chosen parameter set and time limit for the IIH is summarized in Table 8.11.

Table 8.11: Chosen Parameter Set and Time Limit for Large Instances

Chosen Parameter Set		
J	***R***	**Time Limit**
12	**8**	1 min.

8.3.6 Validation of the Iterative Improvement Heuristic

In this section, the IIH is validated against the results obtained by the MILP model presented in Chapter 5. Three instance sizes, micro, small and large, are used for testing along with the chosen parameter set presented in Table 8.11. Table 8.12 presents the average results from running the IIH and the MILP model on five instances of each size, with a maximum run time of one hour. The gap presented is calculated using the lower bound determined by Xpress.

Table 8.12: Validation of the Heuristic Against the MILP

	MILP Model		IIH	
	Run Time	**Gap**	**Run Time**	**Gap**
Micro	12	0.00%	>3600	1.92%
Small	>3600	44.02%	>3600	16.42%
Large	>3600	95.55%	>3600	31.43%

The results in Table 8.12 from testing the IIH against the MILP reveals that the MILP proves to be better than the IIH for the micro instances, however, the gap of the solution obtained by the IIH is only 1.92%. The MILP solves the micro instances on an average of 12 seconds, while the IIH is permitted to run for one hour, indicating that the MILP solves the problem more rapidly. Despite this, analysis of the average improvement in objective value across time for the IIH reveals that the IIH obtains the best solution within 120 seconds, indicating that the remaining 1680 seconds of run time is excessive. For small and large instances, the IIH consistently provides better solutions than the MILP, as the average reduction in gap between the two, for small and large instances, is 63% and 67%, respectively.

The MILP and the IIH are also evaluated against the initial WRP, obtained by compiling a set of Visma's optimized daily routes. Table 8.13 presents the average change in objective value of the solutions obtained by the MILP and the IIH, compared to the initial WRP. For micro instances, the average improvement obtained by the MILP model compared to the initial WRP is 20.37%, which is 8.1% better than the IIH. However, for large instances, the MILP model performs disastrous, while the IIH can obtain an average improvement in the WRP of 7.56%.

Table 8.13: Validation of the Heuristic Against the initial WRP

	MILP Model	IIH
	Change (%)	**Change (%)**
Micro	20.37↓	18.73↓
Small	40.42↑	15.95↓
Large	1653.09↑	7.56↓

This indicates that the IIH, in opposition to the MILP, is indeed capable of solving realistic size instances of the WRSP, and consistently produces improved routes compared to the initial WRP. The IIH is therefore utilized with the configured parameter set and a time limit of 1 minute, to assess trade-offs between the improvement of- and impact on the initial WRP, when considering different aspects of the HHC problem.

8.4 Impact on the Weekly Route Plan

To provide practical insight, this section aims to analyze the impact on weekly routes, when focusing on different aspects of the WRSP. The IIH is implemented to solve the WRSP of one large instance comprising 80 jobs and generates improved routes from the perspective of different stakeholders. Section 8.4.1 and Section 8.4.2 analyze how routes may change when focusing on reducing driving time and violation of time window respectively. These two sections investigate the trade-off between potential improvements of and changes in routes, by comparing the initial WRP obtained by Visma, with the improved WRP, obtained by the IIH. Lastly, Section 8.4.3 reveals how different stakeholders are affected when generating monetary routes, employee convenient routes, and user convenient routes.

8.4.1 Focusing on Reducing Driving Time

There is a general need for increased capacity in the HHC. Driving time constitutes 18% to 26% of the employees' workday, hence, a reduction in driving time may contribute to the liberation of resources (Holm and Angelsen, 2014). The HHC desires to obtain more efficient routes by reducing driving time with as few changes to the existing routes as possible. This study reveals the possibility of improvement in routes concerning the driving time when using the proposed IIH with the configured parameter set (12,8) on a large test instance. Additionally, the trade-off between changes in routes and reduction of driving time is evaluated by identifying and reallocating the jobs with most savings potential with respect to driving time.

To analyze the impact on the WRP when implementing the IIH and focusing on driving time, a set of open jobs for reallocation must be chosen. As the aim is to reduce driving time, the function $OpenJobSelection(s^C)$ identifies and lists jobs belonging to tasks with frequency 1, and large savings potential. Thereafter, all of these are attempted to be reallocated in sets of size 12 as J = 12, considering one set of jobs in each round of the heuristic search. Throughout this section, the IIH is implemented in two ways: a complete heuristic search (CHS) and a single heuristic search (SHS). A complete heuristic search refers to solving the problem without a maximum run time, allowing the heuristic to conduct multiple heuristic searches until the solution stagnates and all jobs belonging to frequency 1 tasks have been attempted reallocated. A single heuristic search means that only the first 12 jobs with the largest savings potential are attempted to be reallocated.

Due to the process of the heuristic search, the number of jobs changing starting time relative to other changes might seem excessively large. This is because the heuristic updates the starting times of jobs in the open route every time an improved solution is found and thereafter fixates this solution while moving on in the search. Ultimately, this means that if a job is reallocated twice during the heuristic search, the starting times in both routes are changed, instead of only the latter where the job is lastly allocated. Also, it is emphasized that a change in day or employee always requires a change in the preceding job for at least two jobs.

Complete Heuristic Search

Table 8.14 exhibits the results from solving the WRSP with the CHS and the SHS. The change in the objective value and driving time is compared to the initial WRP and all changes in routes are registered, reviewing the number of jobs changing days, employees, preceding jobs, and starting times.

Table 8.14: Improvement of- and Impact on Routes When Focusing on Driving Time

	Change		Changes in Routes			
	OV	**DT**	**Day**	**Employee**	**Preceding Job**	**Starting Time**
Complete Heuristic Search	16.05%	15.70%	26	15	55	63
Single Heuristic Search	6.75%	7.00%	6	4	19	44

The solution obtained by using the CHS yields an improvement in the objective value of 16.05% compared to the initial WRP. This improvement is due to a 15.70% reduction in driving time and 74.9% reduction of time window violation, implying that several jobs are sub-optimal allocated in the initial WRP. This is also supported by the registered changes, revealing that 26 jobs move to a different day and 15 jobs change employee. Of the 26 jobs changing day, 12 also change employee. These types of changes are referred to as major changes to the routes, while changes in preceding job and starting time are referred to as minor changes. The minor changes do not significantly affect employees nor users. These changes only facilitate the major changes to the routes, and it is argued that a change in the preceding job does not negatively affect employees nor users. The same accounts for a change in starting time, as long as the starting time still respects the time window of the job.

The preceding job of a job is changed for 55 of the jobs. For 22 of these jobs, the change in the preceding job is due to a change in day, or both day and employee. This is illustrated by the green portions in Figure 8.7. The remaining 33, comprising 60% of the jobs changing preceding jobs, are minor changes that facilitate the open jobs to be reallocated to the existing routes. 50.9% of the jobs changing preceding job also change starting time, while only 9.1% of jobs changing preceding job keep their initial starting time.

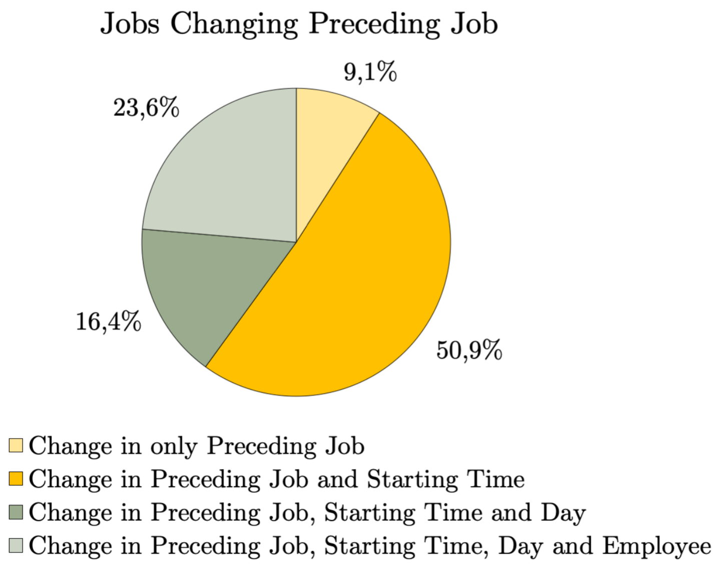
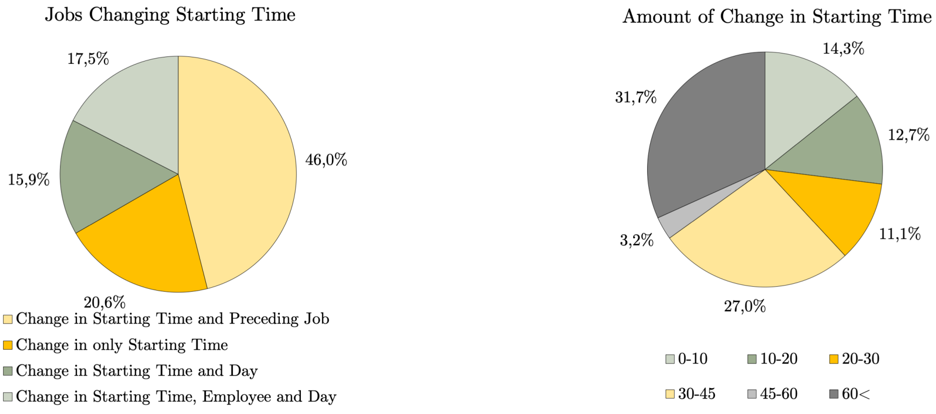


Figure 8.7: Jobs Changing Preceding Job

The highest number of changes is related to jobs changing starting time, affecting 63 jobs. As previously mentioned, some of these changes only occur due to the process of the heuristic, however, the majority is due to the improved solution. Figure 8.8a illustrates that 20.6% of jobs changing starting time, only change starting time. 46% of jobs changing starting time, change the preceding job without changing day or employee, indicating that the order within routes is affected, however, this is categorized as a minor change. Only 33.4% of the changes in starting time are due to major changes, like a change in day or day and employee, illustrated by the green portions of the figure.

A distribution of jobs across the amount of change in starting time is illustrated in Figure 8.8b. The figure reveals that 38.1% of jobs changing starting time are affected by less than 30 minutes, which can be argued to be of minor significance for both users and employees of the HHC. 31.7% of the jobs changing starting time experience a change of more than 60 minutes, which might seem like a significant change. At the same time, it is found that these specific changes in starting time are related to the jobs already experiencing major changes, such as a change in day or employee. It can, therefore, be concluded that the high number of changes to starting time does not indicate a majority of major changes to the routes, as the analysis of the changes provides a more nuanced picture.



(a) Jobs Changing Starting Time

(b) Minutes Change in Starting Time

Figure 8.8: Jobs Changing Starting Time

Single Heuristic Search

In the case of the SHS, the improvement of the objective value is solely due to the reduction in driving time of 7.0%, as time window violation increases by 24% and thereby aggravates the objective value, as presented in Table 8.14. The number of changes in routes when implementing the SHS is significantly lower compared to the solution of the CHS. This is natural as the CHS has attempted to reallocate all the jobs belonging to tasks of frequency 1, while the SHS only attempts to reallocate the 12 jobs with the largest savings potential, considering driving time.

In total, six jobs change employee, day, or both, which in other words means that six jobs change route. This affects the order within the routes. The preceding job is changed for 19 jobs, and 44 jobs have changed starting time. Again, the change in starting time is only a minor practical change. 31.6% of the jobs changing preceding jobs happen due to a change in day or employee, while the remaining 68.4% of the jobs change preceding job to allow other jobs to move to their respective route.

The relation between the degree of improvement and the number of changes exposes the magnitude of impact required to obtain more efficient routes. This relative improvement between the CHS and the SHS is illustrated in Figure 8.9. The figure shows that the SHS obtains 7% improvement in driving time, constituting 44.6% of the obtained improvement by the CHS of 15.7%. At the same time, only 23% of the required changes occur, as six instead of 26 jobs experience major changes. By simplification, if the improvement is distributed equally across the number of jobs changing routes, the solution obtained by the SHS yields better results than the CHS. The relative number of improvement in driving time, per job changing route, is lower for the solution obtained by the CHS, than for the SHS. This implies that a significant reduction of driving time can be obtained by only a few alterations of the initial WRP.

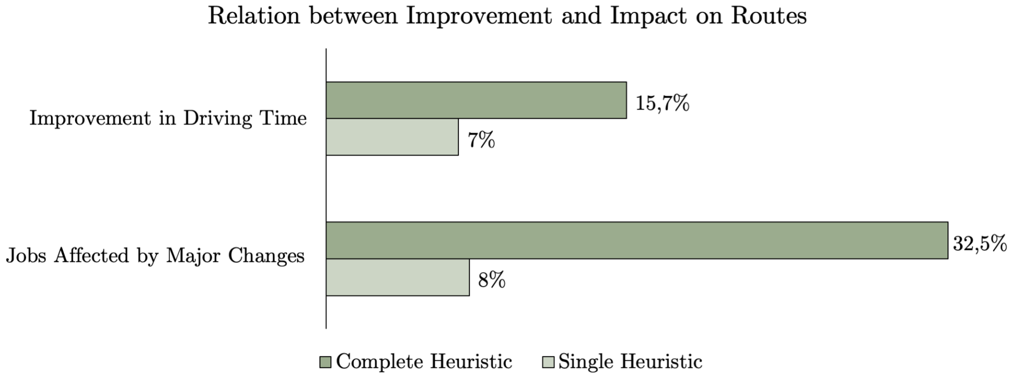


Figure 8.9: Relation Between Improvement of- and Impact on Routes Obtained by the Complete and Single Heuristic Search

Reallocation of Individual Jobs

To further investigate the possibility of reducing the impact on routes while still obtaining more efficient routes, we attempt to identify how each reallocation of a job contributes to a reduction in driving time. This is studied by conducting the SHS for each of the 12 jobs with most savings potential, individually. This implies that the parameter set is equal to (1,8). For these tests, the SHS does not conduct step 1 of the IIH, as there is only one open job. The Current Solution is reset to the initial WRP every time the SHS is conducted with a new open job. This implies that the solutions of the 12 heuristic searches are not additive.

Results are exhibited in Table 8.15 for a selection of the jobs, starting with the job with the largest savings potential, considering driving time. An extensive list of results is found in Appendix C.2. Improvement of the objective value and the reduced driving time compared to the initial WRP is calculated. Furthermore, changes in routes concerning jobs changing day, employee, preceding job, and starting time is registered for each of the iterations.

Table 8.15: Improvement of- and Impact on Routes when Focusing on Driving Time, reallocating Jobs Individually

	Change		Changes in Routes			
	OV	**DT**	**Day**	**Employee**	**Preceding Job**	**Starting Time**
Job 27	2.13%	2.14%	1	1	2	28
Job 4	2.09%	2.10%	1	1	6	27
Job 16	0.00%	0.00%	0	0	0	0
Job 18	1.24%	1.25%	1	1	2	28

It appears that job 27 and job 4 are sub-optimal allocated in the initial WRP, as the reallocation of these jobs to new routes reduces driving time by 2.14% and 2.10% respectively. Furthermore, it can be concluded that large savings potential does not imply that improvement is possible to obtain, when only reallocating the respective job. This is shown by the non-improved solution when attempting to reallocate job 16 while reallocating job 18 improves driving time by 1.25%.

A reduction in driving time of 2.14% when reallocating only job 27 constitutes 30.8% of the improvement of driving time obtained by the SHS for all 12 jobs simultaneously. This is illustrated by the relative difference between the improvement in driving time obtained by reallocating job 27, and the SHS for all 12 jobs, in Figure 8.10. The figure also illustrates how both implementations of the SHS impact the routes, revealing that the reallocation of job 27 only requires major changes to 3.8% of the jobs in the instance. This implies that some jobs contribute greatly to reducing driving time, and as few changes are required to reallocate job 27, significant improvement can be obtained by few alterations.

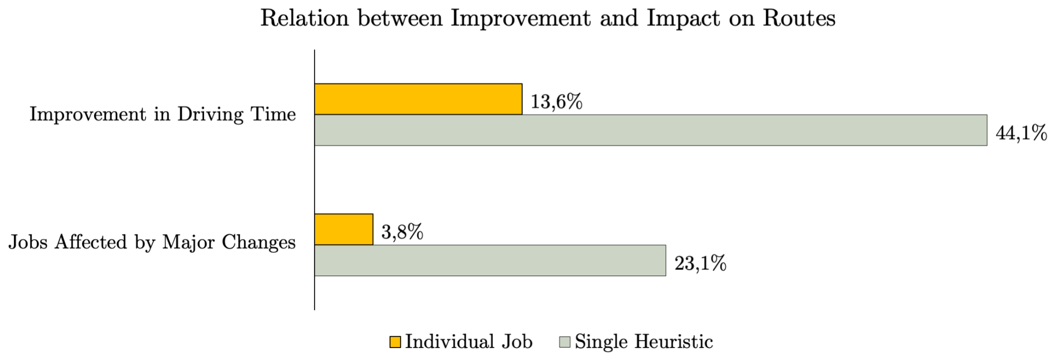
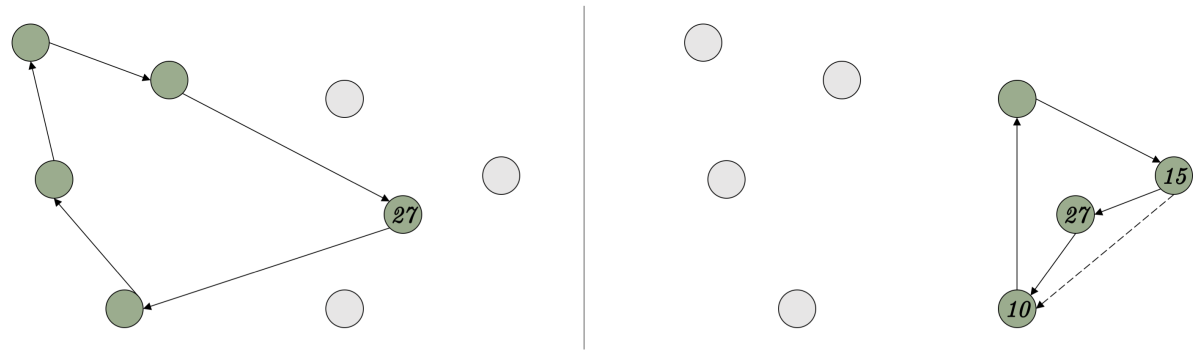


Figure 8.10: Comparing Improvement in Driving Time and Changes to the Routes

The savings potential decreases from job 27 to 18, hence the potential for reducing driving time by reallocation also decreases, as shown by the reduced improvement of driving time. Job 27 and job 4 stand out as they cause a significant amount of driving time in the initial WRP. Compared to the job with the third most savings potential, the potential for saving is 38.5% and 22.3% greater for job 27 and 4, respectively.

The improved routes obtained by the SHS with job 27, reallocate job 27 from Tuesday to Friday and from employee 1 to employee 2, as illustrated in the example in Figure 8.11. Job 27 is reallocated to being between job 15 and job 10, resulting in a change in preceding jobs for job 27 and 10, and the change of starting time for 28 jobs. Preceding job and starting time is also changed for the affected jobs in the route on Tuesday for employee 1. By ignoring the changes in starting times solely due to the heuristic process, in reality, 13 jobs change starting time. Of these jobs, the average change in starting time is 29 minutes. Most of the jobs changing starting time are either in the same route as job 27 initially was or in job 27's new route.



Tuesday, Employee 1 Friday, Employee 2

Figure 8.11: Example of Job Changing Route

8.4.2 Focusing on Reducing Time Window Violation

An important aspect of the problem, from the perspective of the users of the HHC, is the compliance of the time window. The HHC, therefore, strives to perform jobs within a related time window, as this increases user convenience. A violation of a time window can also result in overtime for the employees, causing inconvenience for employees and additional cost for the HHC providers. Focusing on reducing time window violations therefore indirectly also reduces overtime for employees. This section studies how routes are affected when aiming to reduce time window violation, in the same manner as the previous section. The initial WRP is compared to the solution obtained by the CHS and the SHS. Similarly, the changes in routes generating the greatest reduction in time window violation are identified to evaluate the trade-off between changes in routes and reduction in time window violation.

Different implementations of the IIH are applied to one large instance of 80 jobs, where six jobs causing time window violation in the initial WRP, have been identified by $OpenJobSelection(s^C)$, and added to the optimization problem to be reallocated. The total time window violation is 13 minutes in the initial WRP, distributed across the six jobs. This is hardly a violation of any significance, however, the concept of allowing these types of jobs to be reallocated is still relevant.

Complete Heuristic Search and Single Heuristic Search
The results in Table 8.16 reveal that the CHS obtains the same solution as the SHS, implying that the best possible solution is found during the first heuristic search. The solution acquires an improvement in the objective value of 0.83%, and a reduction in time window violation of 20.72%. This implies that reducing time window violation by letting the violating jobs be reallocated by the IIH, is successful.

Table 8.16: Improvement of- and Impact on Routes when Focusing on Time Window Violation

	Change		Changes in Routes			
	OV	**TW**	**Day**	**Employee**	**Preceding Job**	**Starting Time**
Complete Heuristic Search	0.83%	20.72%	2	1	12	32
Single Heuristic Search	0.83%	20.72%	2	1	12	32

By further analysis of the solution, it is found that the improvement in time window violation is caused by changing days for only two out of six of the violating jobs. This suggests that the remaining four jobs are not worth reallocating with respect to the other objectives, and as the violation in minutes is rather minor, this is not surprising. For these jobs, it might be beneficial to identify and add preceding jobs of the job with time window violation to the optimization problem, as the reallocation of a preceding job might liberate time in the current route, and diminish the time window violation of the job.

The reallocation of the two jobs causes a change in the preceding job for 12 jobs, and the starting time is changed for 32 jobs. This indicates that relatively major changes in routes must be conducted to reduce the time window violation, even though the violation is only caused by a few number of jobs. However, a distribution of jobs across types of changes, given that the starting time has changed, is found in Figure 8.12, and provides a more nuanced picture. The figure reveals that 65.6% of the jobs with a changed starting time, only change starting time, implying that the route itself is intact. In fact, it can, therefore, be argued that the amount of changes needed to reduce the time window violation is only minor. Additionally, it is found that by ignoring the change in starting time only due to the heuristic process, the real number of jobs changing starting time is only 16.

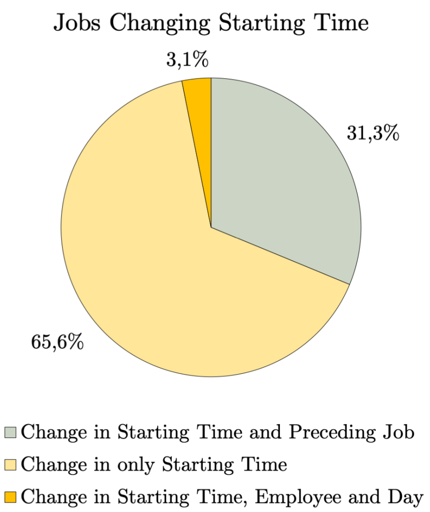


Figure 8.12: Jobs Changing Starting Time

Reallocation of Individual Jobs
Attempting to identify whether the reduction in time window violation can be obtained by few alterations of the initial WRP, a study of allocating each of the jobs individually is conducted. Similar to the study in Chapter 8.4.1, the six jobs violating time windows are added to the optimization problem individually and an SHS is applied. Results are found in Table 8.17 for a selection of the six jobs causing time window violation. An extensive list of results is found in Appendix C.2, revealing that only job 2 is reallocated when using this approach. This suggests that improvement in time window violation requires the opening of more than one job, i.e. larger impacts on the initial WRP. The reallocation of job 2 reduces the time window violation by 2.59%, which constitutes only 12.5% of the possible improvement of time window violation if solving the optimization problem by applying the CHS. At the same time, the difference in the number of changes to the routes when performing the CHS compared to the reallocation of job 2 only, is rather small. One additional job changes day, two more jobs change starting

time and there is a doubling in the number of jobs changing preceding jobs when conducting the CHS compared to the results for job 2 in Table 8.17. This implies that the improvement in time window violation relative to the number of changes is greater when conducting the CHS than the SHS on individual jobs.

Table 8.17: Improvement of- and Impact on Routes when Focusing on Time Window Violation, reallocating Jobs Individually

	Change		Changes in Routes			
	OV	**TW**	**Day**	**Employee**	**Preceding Job**	**Starting Time**
Job 62	0.00%	0.00%	0	0	0	0
Job 2	0.45%	2.59%	1	1	7	30
Job 38	0.00%	0.00%	0	0	0	0

8.4.3 Focusing on Maximizing Employee and User Convenience

The users and employees of the HHC are important stakeholders in the WRSP, and it should be in the HHC providers' interest to maximize these stakeholders' satisfaction. As previously described by the formulation of the MILP in Chapter 5, employee inconvenience may be diminished by minimizing Overtime, difference in Workload, and Overqualified Work. Similarly, user inconvenience can be minimized by reducing Time Window violation and ensure visit continuity by maximizing the preferred Employee-User combination. This section investigates how the shift of focus from the HHC provider's monetary perspective to the perspective of employees and users, may impact the routes.

It is challenging to use a function to identify exactly which jobs are sub-optimal allocated concerning user and employee convenience. Therefore, instead, improved routes with respect to employees and users are obtained by adjusting the objective weight sets in the IIH. The IIH is applied to the problem with the configured parameter set (12,8) and a time limit of 1 minute for each iteration. Furthermore, $OpenJobSelection(s^C)$ identifies jobs having large savings potential, considering driving time. As studies in the previous sections have not considered all objectives of the problem, the Base Case Weight set presented in Section 8.2.1 is utilized and modified to mimic the generation of monetary routes, and user convenient and employee convenient routes. The modified weight sets presented in Table 8.18 are based on preliminary sequential testing and adjusting the initial Base Case Weights for the different stakeholders.

Table 8.18: Adjusted Weight Sets

	DT	**TW**	**OT**	**OW**	**WD**	**EU**
Visma Weight Set	0.5	0.5	0.33	0	0	0
Base Case Weight Set	1	1	1	0.1	0.1	0.1
User Convenient Weight Set	0.1	1	0.1	0.1	0.1	900
Employee Convenient Weight Set	0.1	0.1	1	1	1	0.1

To analyze the impact on routes, one large test instance comprising an initial WRP is used. The WRSP is solved by the IIH with the Visma Weight Set to obtain improved routes, and thereafter by the IIH with the different adjusted weight sets to obtain monetary, employee convenient and user convenient routes. In all cases, the IIH uses the initial WRP obtained by Visma as an initial solution.

Initial Weekly Route Plan

The characteristics of the Initial WRP is found in Table 8.19. The table states a total driving time of 647 minutes in the Initial WRP. This means that driving time constitutes 14.4% of the employees' workweek. A workweek consists of 7.5 hours for each employee on each day of the week. The Initial WRP also generates a total of 25 minutes of time window violation distributed between two jobs, which can be seen as rather insignificant. The employees work 21 minutes of overtime during the week. The total overqualified work in minutes is calculated from the Initial WRP and found to be 1185 minutes. As 50% of the jobs have a skill requirement lower than the lowest skill level of the employees working, this is only 135 minutes higher than the lowest possible value. The calculated difference in workload between employees based on the Initial WRP is 321 minutes, resulting in an average of 64.2 minutes difference between the employees, each day of the week. This is a significant difference, however, not surprising as distribution of workload is not considered in the generation of the Initial WRP. The workload is calculated as the sum of duration of jobs and driving time between jobs in a route, and does not include waiting time. It is emphasized that this simplification might be misleading, as the workload distribution is calculated as the absolute difference, ultimately meaning that the real difference in workload might be close to zero. Lastly, the employee-user combination score is 55, which is relatively high only by coincidence as neither this objective is considered in the creation of the Initial WRP. The maximum value for the employee-user score objective is 80 since each job has exactly one preferred employee.

Table 8.19: The Initial WRP

WRP	DT	TW	OT	OW	WD	EU
Initial	647	25	21	1185	321	55

Improved Weekly Route Plan

The IIH with the Visma Weight Set is applied to the problem to obtain an Improved WRP, which is presented in Table 8.20. The table presents the characteristics of the solution and a comparison of objective values, by calculating the change in objectives between the Initial and Improved WRP.

Both the Initial WRP and the Improved WRP only consider driving time, time window violation, and overtime, as these objectives are considered to be cost-related for the HHC provider. This means that the solutions to the WRSP presented in the table are not optimized concerning overqualified work, absolute difference in workload across days nor employee-user preferences, and the values for these objectives are simply a result of the optimized routes, considering the relevant objectives.

Table 8.20: Comparing the Initial WRP and the Improved WRP

WRP	DT	TW	OT	OW	WD	EU
Initial	647	25	21	1185	321	55
Improved	585	20	21	1185	314	56
Change in Objective	↓ 9.5%	↓ 20.6%	0.0%	0.0%	↓ 2.2%	↑ 1.8%

When comparing the Improved WRP with the Initial WRP, an improvement in driving time, time window violation, workload distribution, and employee-user score, is found. The two latter are by coincidence, however, the first two objectives improve due to the reallocation of jobs belonging to tasks with frequency 1, which are sub-optimal allocated in the Initial WRP. Total driving time is reduced by 9.5%, yielding a value of 585 minutes a week, constituting 13% of the workweek. The share of total work hours spent on driving has thereby been reduced by 9.7%. Total time window violation has decreased by 20.6%, implying great improvements, however, this only makes up 5 minutes in total across all routes of the week.

Monetary Weekly Route Plan

The Monetary WRP is found by using the Base Case Weight Set in the IIH. The monetary objectives are still the most important in these routes, as the Base Case Weight set emphasizes the same objectives as the two route plans described in the previous. However, unlike the previously discussed route plans, the Monetary WRP considers the remaining objectives to some extent, to reflect reality. To account for the new objectives added to the WRSP, an impairment of the monetary objectives is expected, compared to the Improved WRP. Objective values related to the result and a comparison with the Improved WRP, is presented in Table 8.21.

Table 8.21: Comparing the Improved WRP and the Monetary WRP

WRP	DT	TW	OT	OW	WD	EU
Improved	585	20	21	1185	314	56
Monetary	586	17	21	1140	90	57
Change in Objective	↑ 0.1%	↓ 12.9%	0.0%	↓ 3.8%	↓ 71.2%	↑ 1.8%

It can be argued that the changes in the objective values of all the monetary objectives are insignificant, meaning that driving time still constitutes approximately 13% of the workweek. Even though results reveal an improvement of 12.9% in total time window violation, this only makes up 3 minutes, which is a minor improvement. The inclusion of the last three objectives in the model results in an improvement for all of the objectives, especially considering the total absolute difference in workload between employees. The objective is improved by 71.2%, yielding an average absolute difference in workload equal to 18 minutes per day of the week, between the two employees. The improvement of the objective is a result of 14 jobs changing days, and Figure 8.13 illustrates the comparison of the difference in workload between the Improved WRP and the Monetary WRP. The figure reveals the unfairness concerning workload distribution in the Improved WRP. As the objective is reduced without majorly negatively affecting the remaining objectives in the Monetary WRP, a more fair route seems to be within reach without major changes to the routes. An analysis of the routes reveals that the real difference in workload between the two employees across the week is 65 minutes. This is significantly less than in the Improved WRP, generating a real difference in workload of 347 minutes, i.e. almost six hours.

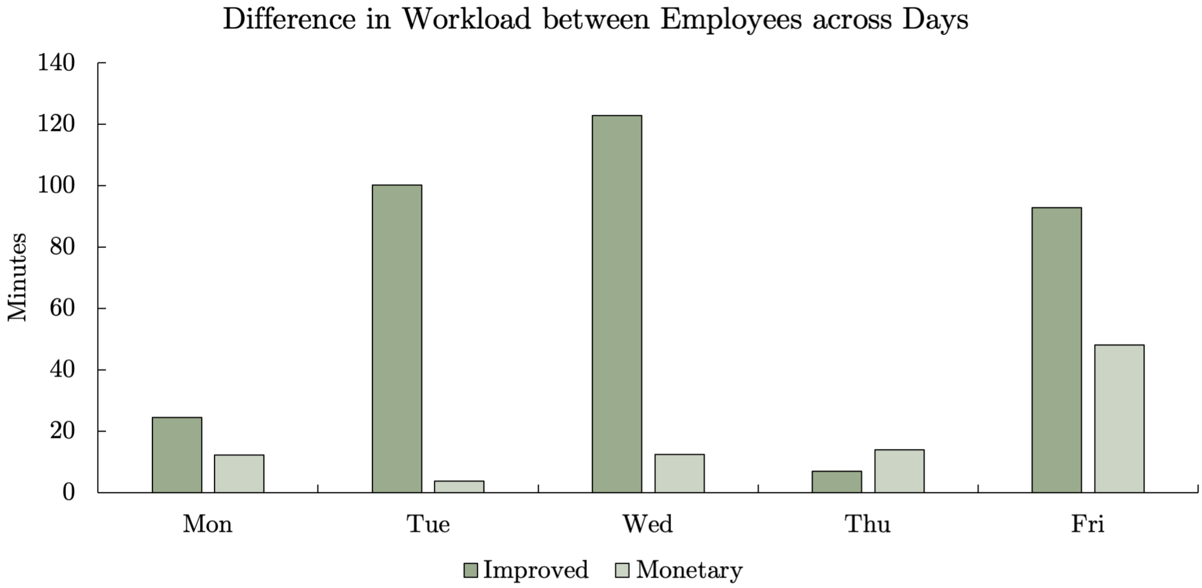


Figure 8.13: Difference in Workload Between Employees Across Days

In total, it can be argued that the Monetary WRP provides better routes concerning the HHC providers, the employees, and the users, as all objectives are improved or almost unchanged compared to the Initial WRP. The IIH is capable of finding a better solution that improves the monetary objectives, without negatively affecting the user and employee-related objectives. As the Monetary WRP is almost consistently performing better than the Improved WRP, the Monetary WRP is used for comparison in the following generation of User- and Employee Convenient Weekly Route Plans.

User Convenient Weekly Route Plan

The Monetary WRP is compared to the User Convenient WRP in Table 8.22. The table shows the objective values for both solutions obtained by the IIH, and the User Convenient WRP is found by adjusting the objective weights to reflect the importance of user-related objectives, being time window violation and employee-user combination, presented in Table 8.18.

Table 8.22: Comparing the Monetary WRP and the User Convenient WRP

WRP	DT	TW	OT	OW	WD	EU
Monetary	586	17	21	1140	90	57
User Convenient	675	15	21	1140	321	61
Change in Objective	↑ 15.1%	↓ 12.5%	0.0%	0.0%	↑ 255.3%	↑ 7.0%

The User Convenient WRP yields an improvement in the user-related objectives, as expected. Again, it is emphasized that the reduction in time window violation only constitutes 2 minutes, in reality, however, the improvement implies that the heuristic is capable of obtaining more user convenient routes. The employee-user score increases by 7.0%, resulting in four more users to be visited by their related preferred employee. These improvements come at a cost, and both total driving time and the total difference in workload between employees is significantly increased in the User Convenient WRP. As we now strive to comply with time windows, the importance of driving time is downsized, and routes become more inefficient as driving time increases by 89 minutes across the week. This is mainly due to the four extra jobs being performed by their preferred employee. This change also results in workload distribution across employees becoming largely unfair, increasing by 255.3%. An analysis of the routes finds that the real difference in workload between the two employees is equal to the absolute difference of 321 minutes, meaning that one employee consistently works longer hours than the other. In conclusion, more user convenient routes may be obtained, but at the expense of driving time and workload distribution.

Employee Convenient Weekly Route Plan

Furthermore, to assess how more employee convenient routes may affect the WRSP, the Base Case Weight set is modified to reflect the importance of employee satisfaction. The employee relevant objectives are overtime, overqualified work, and fair workload distribution, and the weight set used is presented in Table 8.18. The IIH is then applied to the WRSP, and the solution obtained is presented in Table 8.23 as the Employee Convenient WRP. A comparison with the Monetary WRP is found, and a discussion of how the routes are affected follows.

Table 8.23: Comparing the Monetary WRP and the Employee Convenient WRP

WRP	DT	TW	OT	OW	WD	EU
Monetary	586	17	21	1140	90	57
Employee Convenient	756	59	21	1140	8	58
Change in Objective	$\uparrow$ 29.0%	$\uparrow$ 244.2%	0.0%	0.0%	$\downarrow$ 91.6%	$\uparrow$ 1.8%

The Employee Convenient WRP yields an improvement in the absolute difference in workload objective of 91.6%, leading to a total of only 8 minutes across the whole week. This has a major impact on total driving time and total time window violation, as both are significantly negatively affected. Total driving time increases by 29.0% to a number of 756 minutes, constituting 16.8% of the workweek in the Employee Convenient WRP. Even though these routes may seem fairer concerning workload distribution, in reality, the Employee Convenient WRP comprise longer days than the Monetary WRP. The increase in the importance of the distribution of workload objective has generated fair routes by increasing driving time. Total time window violation has a seemingly significant increase of 244.2%, but again it is pointed out that the increase in actual value is only 42 minutes across the entire week. Overqualified work remains the same at a value of 90 minutes higher than the least possible value of 1050 minutes. A reduction in overqualified work could occur if the employee with the lowest skill level, performed all the lower-level jobs. However, this would strongly affect the workload distribution objective, implying that when considering employee-convenient routes in this specific case, these two objectives seem to be working against each other. Lastly, overtime has remained unchanged in all of the discussed WRPs, including the Employee Convenient WRP, even though the objective is emphasized. Analyzing the test instance we find that the job causing overtime has a time window at the end of the shift. Furthermore, since this period of the day is already busy, the heuristic is not successful in reallocating this job to minimize the overtime in the solution. In total, it can be concluded that what may seem like fair routes concerning employees, could in reality prove to be a poorer solution for the employees, as the driving time objective is increased unnecessarily.

Practical insight concludes that improvement in routes is attainable, however, the degree of improvement is dependant on the degree of changes allowed. Improved routes with respect to all objectives may be obtained by applying the IIH with the Base Case Weights, acquiring the Monetary Weekly Route Plan. Alterations to the Monetary Weekly Route Plan may improve efficiency with respect to other stakeholders of the problem, however at the extent of the monetary objectives.

Chapter 9

Concluding Remarks and Future Research

This chapter highlights the findings of this thesis and outlines future research opportunities. A summary of the findings and concluding remarks are presented in Section 9.1. Possible extensions of the problem and potential future exploration concerning solution methods are discussed in Section 9.2.

9.1 Concluding Remarks

This thesis examines the Weekly Routing and Scheduling Problem (WRSP) of the Home Health Care (HHC). The problem concerns the tactical planning of allocating jobs to employees and days, over the course of a week, while still considering aspects of the problem related to users and employees. The aim of the WRSP is therefore to minimize driving time, while also reducing inconvenience for users and employees, and this is formulated as a multi-objective optimization problem consisting of six terms. The monetary objectives are related to driving time and overtime, while employee inconvenience is accounted for by minimizing minutes of overqualified work and absolute difference in workload across employees. User convenience is ensured by minimizing violation of time window and maximize the employee-user score, striving to assure visit continuity.

The first goal of this thesis is to develop a solution method able to solve realistic size instances of the WRSP. A matheuristic, the Iterative Improvement Heuristic (IIH), is proposed to solve the WRSP by utilizing an existing Weekly Route Plan (WRP) and perform reallocations of jobs. The matheuristic iteratively selects jobs to be reallocated and adds these to an optimization problem which is solved by a MILP implemented in Xpress. The MILP allows multiple reallocations simultaneously, avoiding the risk of stagnating in a local optimum. Validating the IIH against the initial WRP reveals that the IIH consistently outperforms the initial Weekly Route Plan by an average of 7.56% and the MILP by 67% when solving large instances.

The second goal of this thesis is to investigate the trade-offs between improvement of- and impact on routes, with respect to different stakeholders. This is conducted by identifying sub-optimal allocated jobs in the initial WRP and attempting to reallocate these jobs by implementing the IIH. It is proven that the IIH obtains significant improvement without majorly impacting the existing routes. Additionally, the IIH is able to find improved routes compared to the initial WRP, without aggravating any of the objectives. Results uncover that more efficient routes with respect to the HHC providers' monetary objectives may be obtained while still improving the user and employee related objectives. The obtained solution decreases time spent on driving time during the workweek by 9.7% without negatively affecting the other aspects of the problem.

In conclusion, the proposed Iterative Improvement Heuristic proves capable of solving the Weekly Routing and Scheduling Problem for realistic size instances. This is supported by results revealing that the matheuristic performs significantly better than the exact solution method given by the MILP when solving large instances. Additionally, the solution obtained consistently yields more efficient routes than the initial Weekly Route Plan, with respect to all stakeholders of the HHC problem.

9.2 Future Research

In this section, future research opportunities for the WRSP are highlighted. Section 9.2.1 discusses possible extensions of the WRSP, which would improve the realism of the problem. Section 9.2.2 addresses how uncertainty could be handled by considering robustness. Finally, further development of the proposed solution method is suggested in Section 9.2.3.

9.2.1 Improving Realism of the WRSP

Throughout this thesis, several assumptions have been made for simplification, e.g. only considering one shift, one transportation mode, and a limited number of possible patterns. Increasing the number of possible patterns and shifts would increase the flexibility of the model by enabling jobs to switch days and shifts, which could be considered in a future study. In reality, the HHC in Norway uses several means of transport, including driving, walking, biking, public transport and carpooling. An extension to the model including an increased number of transportation modes to reduce travel time could, therefore, be interesting to investigate. User preferences are handled in a simple manner in the current model, and more advanced approaches such as visit history or reduction in the number of unique employees visiting the same patient could be extensions included in future research. Lastly, considering synchronized jobs and jobs with temporal dependencies, are also possibilities for future research of the WRSP, to improve the realism of the problem.

9.2.2 Handling Uncertainty in the WRSP

Like every real-life problem, uncertainty plays an important role in the WRSP. There may be sudden changes in the duration of a job or an unforeseen event may occur at a user's home, such as a triggering alarm often used in the HHC, requiring the immediate presence of an employee. Several methods can be used to tackle this aspect in the WRSP. One of these involves the ability to cope with unpredictable events by constructing a more robust model. Robustness can be interpreted as the capability to manage minor changes in demand or capacity, such as the addition of an extra job or reduction in the number of employees due to illness. An interesting research opportunity is to explore whether the weekly routes could be created with robustness in mind to minimize the impact of disruptions in routes. This could be conducted by creating a simulation model imitating disrupting events and iterative solve the disrupted problem and use a feedback loop to create more robust initial routes. Robust routes might be overly conservative and lead to poor initial routes, but prove cost-efficient in the long term when considering disruptions.

9.2.3 Further Developing the Solution Method

The solution method proposed in this thesis is a matheuristic where parts of the problem are wisely selected, and solved, iteratively. Further research of alternative algorithms for selecting a part of the problem could be explored, for instance, a selection based on geographical sectors or modes of transport. In addition, more advanced algorithms based on Artificial Intelligence for selecting parts of the problem could be interesting to research. This thesis applies simple operators to the WRSP, and alternative heuristic operators could be examined to improve the search. Lastly, other heuristic approaches, such as genetic algorithms, could be investigated for the WRSP, as this has proven to be successful for similar problems.

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Appendix A

Appendix - Compressed Model

A.1 Definitions

Sets and Indices

$\mathcal{D}$	-	set of days, $d \in \mathcal{D}$
$\mathcal{N}^d$	-	set of employees n working day $d \in \mathcal{D}$
$\mathcal{T}$	-	set of tasks $t \in \mathcal{T}$
$\mathcal{J}$	-	set of all jobs $j \in \mathcal{J}$
$\mathcal{J}_t^{\mathcal{T}}$	-	set of jobs j in task $t \in \mathcal{T}$
$\mathcal{A}^{nd}$	-	set of feasible arcs between jobs (i, j), on day d, for employee $n \in \mathcal{A}^{nd}$
$C_t^{\mathcal{T}}$	-	set of feasible patterns for task t, $c \in C_t^{\mathcal{T}}$

Parameters

A_{jc}^{dt}	-	$\begin{cases} 1, & \text{if job } j \text{ in task } t \text{ is assigned to day } d \text{ in pattern } c \\ 0, & \text{otherwise} \end{cases}$
$Q_j^{\mathcal{J}}$	-	skill level required at job j
$Q_n^{\mathcal{N}}$	-	skill level of employee n
T_{ij}	-	driving time between job i and job j
D_j	-	duration of job j
$\underline{T}_j$	-	earliest starting time of job j
$\overline{T}_j$	-	latest starting time of job j
$\underline{S}$	-	start time of any employee n on any day d
$\overline{S}$	-	end time of any employee n on any day d
P_j^n	-	preference score on combination between job j and employee n
$\overline{L^1}$	-	maximum value of time window violation for stage one
$\overline{W}$	-	maximum waiting time between two jobs

Decision Variables

x_{ij}^{nd}	-	$\begin{cases} 1, & \text{if employee } n \text{ performs job } j \text{ directly after job } i \text{ on day } d \\ 0, & \text{otherwise} \end{cases}$
y_c^t	-	$\begin{cases} 1, & \text{if pattern } c \in \mathrm{C}_t^{\mathcal{T}} \text{ is chosen for task } t \\ 0, & \text{otherwise} \end{cases}$
s_j	-	start time of job j
e^{nd}	-	end time for employee n on day d
o^{nd}	-	duration of overtime worked by employee n on day d
l_j^1	-	minutes violating time window for job j, 0 - $\overline{L^1}$
l_j^2	-	minutes violating time window for job j, exceeding $\overline{L^1}$
$\overline{w}_d$	-	the greatest workload on day d for an employee
$\underline{w}_d$	-	the least workload on day d for an employee

A.2 Objectives

$$min \qquad w_1 \sum_{i\in\mathcal{J}} \sum_{j\in\mathcal{J}} T_{ij} \sum_{d\in\mathcal{D}} \sum_{n\in\mathcal{N}^d} x_{ij}^{nd} \tag{A.1}$$

$$+w_2 \sum_{j\in\mathcal{J}} (p_1 l_j^1 + p_2 l_j^2) \tag{A.2}$$

$$+w_3 \sum_{d\in\mathcal{D}} \sum_{n\in\mathcal{N}^d} o^{nd} \tag{A.3}$$

$$+w_4 \sum_{i\in\mathcal{J}} \sum_{j\in\mathcal{J}} \sum_{d\in\mathcal{D}} \sum_{n\in\mathcal{N}^d | Q_j^{\mathcal{J}} < Q_n^{\mathcal{N}}} D_j x_{ij}^{nd} \tag{A.4}$$

$$+w_5 \sum_{d\in\mathcal{D}} (\overline{w}_d - \underline{w}_d) \tag{A.5}$$

$$-w_6 \sum_{i\in\mathcal{J}} \sum_{j\in\mathcal{J}} \sum_{d\in\mathcal{D}} \sum_{n\in\mathcal{N}^d} P_j^n x_{ji}^{nd} \tag{A.6}$$

A.3 Constraints

$$\sum_{c \in C^{\mathcal{T}}} y_c^t = 1, \qquad t \in \mathcal{T} \tag{A.7}$$

$$\sum_{i \in \mathcal{J}} \sum_{n \in \mathcal{N}^d} x_{ij}^{nd} - \sum_{c \in C^{\mathcal{T}}} A_{jc}^{dt} y_c^t \leq 0, \qquad t \in \mathcal{T}, j \in \mathcal{J}_t^{\mathcal{T}}, d \in \mathcal{D} \tag{A.8}$$

$$\sum_{i \in \mathcal{J}} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N}^d} x_{ij}^{nd} = 1, \qquad t \in \mathcal{T}, j \in \mathcal{J}_t^{\mathcal{T}} \tag{A.9}$$

$$\sum_{j \in \mathcal{J}} x_{0j}^{nd} = 1, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.10}$$

$$\sum_{j \in \mathcal{J}} x_{j0}^{nd} = 1, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.11}$$

$$\sum_{j \in \mathcal{J}} x_{ji}^{nd} - \sum_{j \in \mathcal{J}} x_{ij}^{nd} = 0, \qquad i \in \mathcal{J} \backslash \{0\}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.12}$$

$$\underline{S} + T_{0j} \leq s_j + M_j^{1nd}(1 - x_{0j}^{nd}), \qquad j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.13}$$

$$s_i + D_i + T_{ij} \leq s_j + M_j^{2nd}(1 - x_{ij}^{nd}), \qquad i, j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.14}$$

$$\underline{T}_j \leq s_j \leq \overline{T}_j + l_j^1 + l_j^2, \qquad j \in \mathcal{J} \tag{A.15}$$

$$l_j^1 \leq \overline{L^1}, \qquad j \in \mathcal{J} \tag{A.16}$$

$$s_j + D_j + T_{j0} - M_j^{3nd}(1 - x_{j0}^{nd}) \leq e^{nd}, \qquad j \in \mathcal{J}, d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.17}$$

$$e^{nd} - \overline{S} \leq o^{nd}, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.18}$$

$$\sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} (D_j + T_{ij}) x_{ij}^{nd} \leq \overline{w}_d, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.19}$$

$$\sum_{i \in \mathcal{J}} \sum_{j \in \mathcal{J}} (D_j + T_{ij}) x_{ij}^{nd} \geq \underline{w}_d, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.20}$$

$$x_{ij}^{nd} \in \{0, 1\}, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d, (i, j) \in \mathcal{A}^{nd} \tag{A.21}$$

$$y_c^t \in \{0, 1\}, \qquad t \in \mathcal{T}, c \in C_t^{\mathcal{T}} \tag{A.22}$$

$$s_j, l_j^1, l_j^2 \geq 0, \qquad j \in \mathcal{J} \tag{A.23}$$

$$\overline{w}_d, \underline{w}_d \geq 0, \qquad d \in \mathcal{D} \tag{A.24}$$

$$e^{nd}, o^{nd} \geq 0, \qquad d \in \mathcal{D}, n \in \mathcal{N}^d \tag{A.25}$$

A.4 Strengthening the Formulation of the Model

Subtour Elimination Constraints:

$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}} x_{ij}^{nd} \leq |\mathcal{S}|, \qquad \mathcal{S} \subset \mathcal{J}, n \in \mathcal{N}, d \in \mathcal{D}, |\mathcal{S}| \geq 2 \tag{A.26}$$

Appendix B

Appendix - Generation of Test Instances

B.1 Input Parameters

Days: 5
Jobs per Employee: 8
Starting time Employee: 540 minutes
Ending time Employee: 1020 minutes
MaxWaiting: 120 minutes
Time Window = Latest Starting Time - Starting Time + Duration
Absolute Latest Starting Time = min(Latest Starting Time + 60, 1080)

Frequency	**1**	**2**	**3**	**4**	**5**
Distribution of Jobs	53.8%	5.7%	4.3%	11.4%	24.8%

Level	**1**	**2**	**3**	**4**
Skill Level Employees	25%	25%	25%	25%
Skill Requirement Jobs	26%	24%	24%	26%

Pattern	Frequency	Mon	Tue	Wed	Thu	Fri
1	5					
2	4					
3	4					
4	4					
5	4					
6	4					
7	3					
8	2					
9	2					
10	2					
11	1					
12	1					
13	1					
14	1					
15	1					

Appendix C

Appendix - Computational Results

C.1 Configuration of R

Instance Size	J	R	Average Improvement
40	8	4	10.90%
40	8	8	11.52%
40	10	4	11.27%
40	10	8	11.63%
40	12	4	10.88%
40	12	8	11.15%
40	14	4	4.52%
40	14	8	5.05%
40	16	4	4.82%
40	16	8	5.19%
80	8	4	7.44%
80	8	8	9.87%
80	10	4	7.49%
80	10	8	10.78%
80	12	4	9.03%
80	12	8	5.91%
80	14	4	2.66%
80	14	8	4.10%
80	16	4	2.88%
80	16	8	3.16%

C.2 Results from Single Heuristic Search

Results from re-allocating Individual Jobs based on Saving Driving Time

	Change in %		Changes in Routes			
	OV	DT	Day	Employee	Preceding Job	Starting Time
Job 27	2.13%	2.14%	1	1	2	28
Job 4	2.09%	2.10%	1	1	6	27
Job 39	0.69%	0.69%	1	1	3	25
Job 14	0.00%	0.00%	1	1	6	27
Job 16	0.00%	0.00%	0	0	0	0
Job 18	1.24%	1.25%	1	1	2	28
Job 26	1.40%	1.41%	0	1	2	35
Job 15	0.00%	0.00%	0	0	0	0
Job 40	0.00%	0.00%	0	0	0	0
Job 23	0.19%	0.18%	1	1	6	31
Job 31	0.00%	0.00%	0	0	0	0
Job 19	0.47%	0.47%	0	1	2	25

Results from re-allocating Individual Jobs based on Reducing Violation of Time Window

	Change in %		Changes in Routes			
	OV	TW	Day	Employee	Preceding Job	Starting Time
Job 62	0.00%	0.00%	0	0	0	0
Job 66	0.00%	0.00%	0	0	0	0
Job 48	0.00%	0.00%	0	0	0	0
Job 2	0.45%	2.59%	1	1	7	30
Job 62	0.00%	0.00%	0	0	0	0
Job 49	0.00%	0.00%	0	0	0	0

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