Master's thesis

NTNU
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Department of International Business

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Modelling volatility in financial electricity contracts

Applying Stochastic Volatility models to the Nordic/Baltic financial electricity market

Master's thesis in International Business and Marketing
Supervisor: Per Bjarte Solibakke
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Abstract

When the financial market is pricing an option, the only unknown parameter is the future volatility in the price of the underlying asset. The price of the financial asset shifts relatively slowly when market conditions are calm, and the price shifts faster when there is more news, uncertainty, and trading in the market. Hence, the volatility is said to be time-varying. Market actors that understand the time-varying volatility can have more reasonable predictions of the future prices of the asset and the associated risk exposure.

The main objective of this paper is to build and evaluate a two-factor Stochastic Volatility model for the prediction of time-varying volatility in financial contracts for electricity. The paper seeks to answer if the volatility is a process of random information flow to financial markets for electricity, or if it the volatility can be predicted by a Stochastic Volatility model.

The report builds on earlier proven time series methods. Building Semi Nonparametric (G)ARCH models to capture special characteristics of the front year and front quarter futures contracts, including volatility clustering, mean reversion, and asymmetry effects. A two-factor volatility model will be built based on the finding of the GARCH models to do step ahead volatility prediction.

The results confirm several stylised facts from the volatility literature. The contracts show leptokurtosis features, mean reversion effects, volatility clustering and persistence, all contribute to strong data dependency and predictability for the volatility, making volatility not a process of a random walk. The quarter contracts show the strongest volatility clustering and persistence. A positive asymmetry effect where found for the year contracts. The predicted volatility from the SV model is compared with production mix, reservoir levels and temperature to better understand factors contributing to the time-varying volatility. The analysis indicates lower (higher) reservoir levels than the median level coincides with higher (lower) volatility. Less (more) nuclear and hydro power production, and more (less) wind and solar production coincides with higher (lower) volatility. Colder (warmer) temperature than the median coincides with higher (lower) volatility. These comparisons are not tested statistically, rather these are foundations for further research in the field of stochastic volatility models for financial electricity contracts.

Sammendrag

Når finansmarkedene priser en opsjon, er det eneste ukjente parametere den fremtidige volatiliteten til det underliggende verdipapiret. Prisen til et verdipapir svinger relativt sakte når markedsforholdene er rolige, og raskere når det er mer nyheter, usikkerhet og handel i markedene. Markedsaktører som forstår tidsavhenging volatilitet kan oppnå mer realistiske forventninger til prisen på verdipapirene og tilhørende risikoeksponering.

Hovedformålet med denne oppgaven er å bygge og evaluere en to-faktor Stokastisk Volatilitetsmodell for å predikere tidsavhengig volatilitet i finansielle kontrakter for elektrisitet. Oppgaven ønsker å besvare om volatilitet er et produkt av den tilfeldige informasjonsflyten til det finansielle elektrisitetsmarkedet, eller om volatiliteten kan predikeres gjennom en stokastisk volatilitetsmodell.

Oppgaven bygger på tidligere empiriske metoder innen tidsserie økonometri. Det bygges semi-ikke-parametriske (G)ARCH modeller for å fange opp spesifikke karakteristika for front futures års- og kvartalskontrakter, inkludert volatilitetsklynger, reversjons- og asymmetrieffekter. En to-faktor stokastisk volatilitetsmodell bygges på de spesifiserte (G)ARCH modellene for å gjøre volatilitetsprediksjon.

Resultatene bekrefter typiske karakteristika fra volatilitetslitteraturen. Kontraktene har leptokurtosis fordelinger, reversjonseffekter, volatilitetsklynger og volatilitetsutholdenhet. Dette bidrar til sterk dataavhengighet som kan brukes til prediksjon av volatilitet, og volatiliteten er dermed ikke et produkt av tilfeldig informasjonsflyt. Kvartalskontraktene viser sterkest volatilitetsklynger og utholdenhet. En svak positiv asymmetrieffekt ble funnet for årskontraktene. Den predikerte volatiliteten ble sammenlignet med produksjonsmiksen, reservoar nivåer og temperaturdata. Sammenligningen tyder på at lavere (høyere) reservoar nivåer enn medianen sammenfaller med høyere (lavere) volatilitet. Mindre (mer) kjernekraft og vannkraft, og mer (mindre) vind og solkraft sammenfaller med mer (mindre) volatilitet. Kaldere (varmere) temperaturer fra medianen sammenfaller med mer (mindre) volatilitet. Sammenligningene er ikke statistisk testet, men kan danne grunnlaget for videre forskning av finansielle elektrisitetskontrakter og stokastiske volatilitetsmodeller.

Preface

The master thesis marks the end of our studies at Norwegian University of Science and Technology and the Master of Science degree in International Business and Marketing with specialisation in International Business.

Our motivation behind this thesis is three folded. We wanted to learn more about the electricity market as it is a relevant topic of today. Sustainable electricity production is said to be part of the solution to reduce overall CO_2 emissions, and renewable production facilities and cross-border electricity trading is often in the public debate. With only basic quantitative methods and econometric skills we had a desire to learn more advanced technics such as time series analysis. Lastly, we wanted to understand more fundamentals observed in the financial markets and associated market risks. This process has opened our eyes to both the electricity market and the times series methodology, we are left with valuable knowledge and skills transferable to other applications.

The empirical analysis, estimations and modelling has been performed in EViews, SPSS, Microsoft Excel, SNP (a C++ program for nonparametric time series analysis) and EMM (a C++ program for Efficient Methods of Moment estimation).

Due to the ongoing pandemic our work habits have been changed, where physical meetings have been replaced with interactive software like Blackboard Collaborate, Zoom and Microsoft Teams. Relying on interactive software for collaboration, discussions and meeting have been a valuable experience.

We are grateful to our supervisor Professor Per Bjarte Solibakke for introducing us to the electricity market and the volatility literature. Thank you for fruitful discussions, enthusiasm for the topic, and knowledge about the SNP and EMM programs.

A big thank you to our family and friends for support during our studies.

Ålesund, June 2020

Magnus Ofstad Holan | Christian H. Ekornesvåg

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Introduction

Many European countries restructured their electricity sectors around 1990. The restructuring led to a reorganization of the power supply into a competitive part for electricity production and retail, and a monopolistic part for transmission and distribution of the electricity in the grid. The restructuring was the starting point of the Nordic/Baltic Power market which Norway is part of today, where the market participants trade physical electricity at the Nord Pool Spot exchange and financial electricity contracts at the Nasdaq OMX exchange. The Nordic/Baltic Power market is one of the most liquid power derivates market in the world with an objective to reduce and minimize risk, increase transparency, and protect market participants. The risk landscape in the electricity market are more complex than other assets and commodities. There exist no technology enabling electricity to be stored or transferred for longer distances without efficiency losses. Hence, mismatches in supply and demand must be cleared directly, making short spikes in prices and volatility. Due to this non-storability, users of the financial electricity market trade on different contracts to lock in prices for their production or consumption. Hence the financial market for electricity (Nasdaq) is used for price hedging and risk management for the actors in the physical electricity market (Nord Pool Spot).

Many years of international studies of prices in financial data have revealed the presents of stylised facts like skewness, excess kurtosis, volatility clustering and heteroscedasticity (Benoit, 1963) (Fama, 1965), and asymmetry effects (Tversky & Kahneman, 1979) (Barberis, et al., 2001). The price changing process in financial markets are known as volatility. Volatility is a statistical measure of the spread in *returns* around the mean of a given asset or market index. Standard deviation or variance (the squared standard deviation) is notations frequently used for volatility. When the observed price returns are tight around the mean value volatility is low, wider returns imply higher volatility and thus the assets value can be spread out over a larger range of values. Volatility models are used internationally to predict characteristics of future returns, including both absolute magnitude of returns, quantiles and complete densities. Modern portfolio theory (MPT) studies have revealed an increase in volatility leads to increased risk and reduced portfolio returns. Knowledge about the price dynamics and volatility for financial electricity contracts are important for producers, retailers, consumers, and traders in the electricity market.

This paper seeks to build and evaluate a two-factor Stochastic Volatility (SV) model for the prediction of volatility in financial contracts for electricity quoted on Nasdaq OMX market (Nasdaq, 2020). Stochastic volatility models have a simple structure and is useful for explaining common characteristics of returns in assets, commodities, and currencies. The price of a financial asset shifts relatively slowly when market conditions are calm, but the price shifts faster when there is more news, uncertainty, and trading in the market. Hence, the volatility in financial markets are non-constant and frequently changing - the volatility is said to be time-varying. The time-varying volatility in financial markets generates a time-varying risk exposure, making it natural to build stochastic models to understand historical evolution in volatility. The SV implementation seeks to describe how the volatility changes as time goes by. Volatility is an unobserved instrument and is non-traded, hence there exists no perfect estimates of the variable. Rather volatility can be understood as a latent variable modelled from its direct influence on the magnitude of returns. As time-varying volatility is widespread in financial markets, it is an associated risk to the constant changing volatility. Markets actors that understand the time-varying volatility can have more reasonable predictions of the future prices of the asset and the associated risk exposure. A better understanding of factors influencing volatility and precise forecasting of volatility is valuable for practitioners using futures and options for risk management, as higher levels of volatility can imply greater probability of substantial undesirable price changes. Higher (lower) volatility increases (decreases) the derivate prices, an increased knowledge about volatility will thus be beneficial in deciding whether to sell or buy put and call options. In a Black & Scholes option valuation model (Black & Scholes, 1973) the only unknown determinator is the future volatility (σ) in the underlying asset. By revealing factors influencing volatility in a market and establish methods for volatility prediction a more accurate pricing of options can be achieved.

Providing additional knowledge of the volatility in the financial contracts for electricity is important not only for practitioners in the financial electricity market. Volatility in financial electricity contracts could obstruct investment in new and sustainable technologies for electricity productions, hindering companies, nations and the global society reducing CO2 emissions and reaching the UN sustainable goals. Further, in a scenario where electricity prices are co-integrated with other energy and commodity prices, a volatile market can in such a scenario make prediction of raw material costs hard for consumers and businesses outside the electricity market[1]. Hence, the main purpose with this paper is to build a two-factor stochastic volatility model where volatility has its own stochastic process, enabling rational descriptions of the volatility in financial electricity contracts.

The report will build on earlier proven time series methods to identify volatility characteristics and use a Stochastic Volatility model for volatility prediction and forecasting. Quarterly and Yearly Nordic electricity futures traded at Nasdaq OMX will be analysed with 5009 observations in the interval from 3rd of January 2000 to 3rd of January 2020. The contracts are based on the Nordic System price of 1 MWh of electricity according to the daily Elspot system price for the Nordic region which is quoted and published by Nord Pool. For the two series (Quarter and Year), individual Semi-nonparametric (SNP) (G)ARCH models (Enger, 1982) (Bollerslev, 1986) will be created and used to capture special characteristics of the chosen contracts, including volatility clustering and asymmetry effects. A C++ program for SNP models (Gallant & Tauchen, 1990 (Dec 2017)) is applied to specify the optimal model, the set of lag descriptions and model evaluation. The program includes features for prediction, residual analysis, plotting, and simulation used for analysis and interpretation. Shocks will be simulated to identify market behaviour for the two timeseries.

A two-factor volatility model will be built based on the finding of the SNP models to do step ahead volatility prediction and describe relevance for the Nordic Electricity Future Market. The implementation of a two-factor stochastic model uses the Monte Carlo Markov Chain estimator proposed by Chernozhukov and Hong (2003), and the modelling strategy by Gallant and McCulloch (2011), and Gallant and Tauchen (1997) (2016). The implementation method uses Efficient Methods of Moments (EMM) written in a flexible C++ program (Gallant & Tauchen, 2016). Normalized values of the objective function in the optimal specified model is asymptotically χ^2

[1] Several international studies have investigated the argument for co-integration in energy and commodities markets, like gas oil and crude oil (Westgaard, et al., 2011), Gas and Power Spot prices (Jong & Schneider, 2009), and between different European energy exchanges (Veka, et al., 2012)

distributed with a given degrees of freedom enabling model fit evaluation and assessment. When an SV model is correct specified to the data, the applications expand into portfolio management, asset allocation, risk assessment, risk management strategies and derivative pricing purposes. By using the re-projected volatility from the Stochastic Volatility model, a more accurate pricing of option could be established using the Black & Scholes model.

By applying the mentioned methodology, this paper seeks to answer if the volatility is a process of random information flow to financial markets for electricity, or if it the volatility can be predicted by a stochastic volatility model. Thus, is returns and volatility a correlated process, or just a random walk?

In more detail the paper investigates three main topics in the follow order: The first is to identify relevant volatility properties of the front future financial contracts for electricity at Nasdaq OMX, compare quarterly and yearly contracts, capture volatility and simulate shocks. Second is to create and evaluate whether a two-factor stochastic volatility model is appropriate to do step ahead prediction of volatility in financial contracts. Third is to analyse trends in electricity production, reservoir levels and temperature to reveal whether there is some connection to the volatility. By addressing these questions, this paper seeks to help risk managers and other practitioners to better understand the properties of the unobserved volatility and add valuable insights to the Stochastic Volatility literature.

This paper is to our knowledge the first to implement a two-factor stochastic volatility models for the financial contracts in the Nordic/Baltic power market. (G)ARCH models have been implemented in many different markets earlier. Paolella and Taschini (2006) concluded that GARCH models perform well for CO_2 and SO_2 prices. Egeland & Haug (2016) used semi-nonparametric AR(1) GARCH(1,1) models to extract densities and conditional variances for 14 different financial markets. The GARCH-models seems to capture volatility clustering and asymmetry effects. Two-factor SV-models has been implemented in many different markets; Solibakke (2019) built and implemented a two-factor SV model to do step ahead volatility prediction and describe its relevance for equity markets with observations from FTSE100 spot index and the Equinor spot price. Solibakke (2015) used an AR(1) GARCH(1,1) model together with an two-factor stochastic volatility model to forecast and extracting conditional moments for the Brent Oil futures market. Option prices were calculated using re-projected conditional volatility. The same analysis where performed for the European Carbon Markets in Solibakke (2014) and for Front Year Futures Contracts on the European Energy Exchange AG in Solibakke & Dahlen (2012). Earlier research of other energy market prices is voluminous, however the combination of financial contracts for electricity and SV-models are unique, as similar studies have not been conducted on the Nordic Electricity Future contracts traded at Nasdaq OMX. To limit the scope, the paper is written from a Norwegian perspective. This perspective implies that this paper use Norway as an example to elaborate the restructuring of the electricity market in the 1990s. To limit the scope and due to lack of available data, only Norwegian reservoir levels and Oslo temperatures are reported. The production mix are reported from all the Nordic and Baltic countries.

The report is built up the following way: The first chapter is an introduction to the Nordic/Baltic electricity market. First there is an introduction the physical market (Nord Pool) with descriptive information about market characteristics, production in the interval, trends in production mix and reservoir levels. Then an introduction to the financial market (Nasdaq OMX) for electricity is given. The third chapter look at relevant literature of the electricity market and the stochastic volatility (SV) model framework. The fourth chapter elaborate the methodology used, including econometric time series analysis, the Semi-Nonparametric (SNP) models and the two-factor Stochastic Volatility (SV) models. Chapter five gives firstly a description of the two time-series and secondly defines and evaluates the SNP (G)ARCH model used to identify and compare volatility characteristics, and thirdly to simulate shocks. Finally, the chapter end with defining and evaluating the two-factor SV model used for volatility prediction and forecasting. In chapter six the re-projected volatility from the SV model will be compared with variations in production mix, reservoir levels and temperature, to better link the physical and financial electricity market. The purpose of this comparison is to better understand the latent volatility in terms of the observed variables, such as production levels, production mix, reservoir levels and weather. The comparison will not be tested statistically. Chapter seven give implications for further research in the field of financial electricity contracts and SV models. Chapter eight summarizes the findings and conclude.

1. The Electricity Market

The electricity market can be divided into a physical and a financial market. The physical market consists of a wholesale market including professional actors with concession and a retail market for private customers and companies. In the physical market the sellers must deliver the electricity, and the buyers are obligated to receive it. Thus, speculative trading is not possible on this exchange. In the Nordic and Baltic countries, the physical trading takes place on the electricity exchange Nord Pool Spot, while financial trading of power takes place on the Nasdaq OMX. The trading of electricity at Nasdaq is based on the Nordic System Price which is quoted and published by Nord Pool and is an independent auction market.

1.1 Historical development

Many countries in Europe restructured their electricity sectors around 1990. The restructuring led to reorganization of the power supply in a competitive part including generation and consumption and a monopolistic part including transmission and distribution. Open access to the grid is a necessity for efficient competition amongst actors (Wangensteen, 2006). In Norway, the restructuring came into force with the energy act of January 1st in 1991. A free market for electricity trade were introduced and the law reformed the energy sector from purely administration purposes into a more business friendly system. Until 1991 Norway was divided into many local energy markets, where local energy companies had both monopoly and duty to deliver electricity in the local area. A remedy to fulfil the principal objective of the law was to dissolve the bonds between producers and distributors of electricity and it led to important organisational changes. Norwegian Statkraft were divided into a production part, Statkraft SF and a distribution part with responsibility of the grid, Statnett SF (NVE, 2016).

1.2 Actors in a restructured system

In the restructured and liberalized market, electricity becomes a commodity and both commercial and non-commercial actors are involved.

1.2.1 Commercial actors

The commercial actors on Nord Pool Spot are the producers, retailers and traders who choose to trade on the electricity exchange (Nord Pool Spot, 2011). Producers owns, runs, and sell electricity from their production facilities to the electricity exchange. Retailers buy electricity at the exchange and resell it to end users. The traders own the electricity during the trading process and buy electricity from a producer to sell it to a retailer. In addition, brokers can act on behalf of the commercial actors, these play an important role helping to clear the market.

1.2.2 Non-commercial actors

The non-commercial actors operate on a local level and a state level. The grid can be divided into a distribution grid and a transmission grid. At the local level, a local grid operator is handling the low-voltage grid (distribution grid) with distribution to end users. At state level, the high-voltage grid (transmission grid) is handled by the Transmission System Operator (TSO). The TSO is also responsible for the security and supply of electricity in its country. Thus, producers are not responsible for the physical delivery of the electricity to the end user. The TSO must be a non-commercial organization which is independent of commercial players. In Norway, the TSO is Statnett which is state owned (Nord Pool Spot, 2011) . The local grid operator together with the national grid operator (TSO) have a monopoly in transferring electricity from producers to consumers, making grid operators regulated as non-commercial actors.

The TSO is also responsible for keeping the frequency stable at 50 Hz. If the frequency drops below 50 Hz due to an increased consumption, the TSO must ensure that some producers delivers more electricity to the grid by buying excess generation capacity. This is called “up-regulation”. This is also the case if the supply of power excesses the demand, then the TSO sells electricity to the producers, which is called “down- regulation” (Nord Pool Spot, 2011). These trades which the TSO conducts with different market players to keep the grid stable is called regulating power. The price setting in the regulation power market is dependent on the up and down regulation orders.

As an example, imagine a need for a 400 MWh up-regulation in the market. Producers with available generation capacity can then place up-regulating orders. Consumers which can reduce consumption can place down-regulating orders. All the orders are submitted to the TSO which ranks them in an increasing order from the lowest to the highest price. Orders are then activated until 400 MWh is reached starting with the order which has the lowest price. The up-regulation price is set by the price of the last up-regulator order. The orders with a price below the settled up-regulation earn a profit which equals the difference between the offered price and the up-regulating price. If a need for a down-regulation occur the same procedure is used (Nord Pool Spot, 2011).

The electricity is bought and sold hourly and can be divided in a three-step process. First the purchase is made when a retailer places an order of a contracted amount on behalf of a customer to a supplier. Then is the “hour of operation” where the power is delivered and consumed. Finally, after the hour is completed the contract is settled when the retailer pays the supplier for the contracted amount. If the customer is not able to consume the full amount of the contract, the retailer has in practice sold the remaining power to the TSO since the TSO pays the retailer for the remaining power (Nord Pool Spot, 2011).

The trade between the TSO and the retailer is called “balancing power” because it creates a balance between the retailer’s total trade and the customer of the retailer’s consumption. If a need for an up-regulation during the hour of operation occur, the TSO will pay the retailer the up-regulation price for the balancing power which is normally higher than the market price. If the TSO did a down-regulation during the hour of operation, the retailers will normally be paid a price which is below the market price. If the customer is consuming more power than specified in the contract, the TSO will invoice the retailer for the excess power (Nord Pool Spot, 2011).

1.3 The physical electricity market

As mentioned, the trading platform for the physical market is Nord Pool. The typical participators on the exchange are power producers, suppliers, and traders. Some large end users are also buying power on the exchange and not through a supplier (NordPool Group, u.d.). The physical production market includes the Nordic countries Norway, Sweden, Finland and Denmark, and the Baltic countries Latvia, Lithuania, and Estonia.

1.3.1 History

The creation of Nord Pool as we know it today springs from the deregulation of the Norwegian electricity market in 1991. In 1993 the company "Statnett marked AS" (known as Nord Pool today) was established. The total volume in the first operating year was 18.4 TWh. In 1995, the framework of an integrated Nordic power market contract was purposed to the Norwegian parliament. Together with Nord Pool's licence for cross-border trading, this made the foundation for spot trading at Nord Pool. In 1996 a joint Norwegian-Swedish power exchange was established and changed named to Nord Pool ASA. In 1998 Finland joins the exchange followed by Denmark in 2000. Estonia as the first Baltic country opens as bidding area in 2010, followed by Lithuania in 2012 and Latvia in 2013. (Nord pool group, 2019)

1.3.2 Bidding areas

Nord pool spot is divided in 15 bidding areas due to bottlenecks in the transmission and distribution grid. Norway has five different bidding areas while Sweden has four. Denmark has two areas and Latvia, Lithuania, Estonia, and Finland have one each. Bottlenecks occurs due to capacity restrictions in export and import both between bidding areas inside a country and cross border. The TSO in each country determine the trading capacity between the bidding areas and publishes the capacity for the next day at 10:00 AM (Nord Pool Spot, 2019). The exchange capacity between the countries differs amongst the members. For instance, Norway's exchange capacity is around 20% of the installed production capacity (Energifakta Norge, 2019b).

1.3.3 Pricing in the spot market

Due to bottlenecks, two different prices are reported in Nord Pool Spot. The System price which is a theoretical price for the whole marked without bottlenecks on the grid, and the area price which may vary between bidding area due to bottlenecks. When constraints occur electricity will always move from the low-price area to the high price area. This scheme prevents any benefits for market members on the bottlenecks (Nord Pool Spot, 2019). While the TSOs in their respective countries are handling the regulating and balancing power, the electricity in Nord Pool Spot is traded at Elspot which is Nord Pool Spot's day-ahead auction market. Nord Pool calculates a price for each bidding area per hour for the following day. Actors who wants to buy and sell power must send their purchase orders to Nord Pool Spot before noon the day before the power is delivered to the grid. The market is cleared through a double auction process where buyers and sellers submit their supply and demand the coming day. To clear a particular bidding area, a software uses the submitted orders to calculate a market price, hence different prices can occur between bidding areas due to supply and demand conditions. The orders are flexible, and a retailer can for instance place a bid of 100 MWh where the amount purchased on the exchange or produced at his own generation facility can vary with different price levels (Nord Pool Spot, 2011). For two Nord Pool actors located in different bidding areas to be able to trade with each other, they can use the financial market for electricity where no physical delivery takes place. Hence the commercial actors can always trade electricity without taking bidding areas and bottlenecks into account. Nord Pool offers intra-day and day ahead trading. In 2019 a total volume of 494 TWh was traded on the exchange. The largest part was in the Nordic/Baltic day-ahead market with a share of 381.5 TWh (Nord Pool Group, 2020a).

1.3.4 Electricity production in the Nord Pool area from 2000 to 2019

In 2019 the total amount produced reached 403 627 737 MWh in the Nord Pool Spot market (Nord Pool Group, 2020b). Almost 96% of the total production was in the Scandinavian countries. Sweden was the largest producer with a share of 40.31% of total production. Norway followed with 91% of the total amount, the Baltic countries stood for a combined amount of 3,93% where Latvia stood for 1.52%, Estonia 1.51% and Lithuania produced 0.90% (Nord Pool Group, 2020b)

To acquire a better understanding of the Nordic/Baltic electricity market, an overview of each members production in the studied interval has been analysed. The data shows to some extent large fluctuations in yearly production. In addition, an increasing trend in electricity production from fossil fuels where prediction of production is easier, to an increased share of production from renewable sources where variations will occur as often as on an hourly basis is identified. This information will contribute to a better understanding when volatility analysis is conducted later in the report.

Production numbers are reported on the base year of 2018 or 2019 dependent on which countries has released annual reports for 2019. The details in the reporting of each country depends on the information fullness of the published information from the official energy institutions. The numbers are based on electricity production in the country and not electricity consumption since consumption includes imported electricity outside Nord Pool as well. Not all the country's total produced amount is available to trade on Nord Pool, but it gives valuable information regarding the overall production mix and long-term trends in production fluctuations.

Sweden

Sweden's electricity generation mix consist of hydropower, nuclear power, wind power, solar power and different kind of thermal power such as Combined heat and power (CHP), Industrial (CHP) and Gas turbines *(Figure 1)*. In 2018 nuclear power was the largest source with a share of 41.41% while hydropower followed with 38.39%. The others sources followed with wind power 10.47%, thermal power 9.47% and solar Power 0.25% (Energi Føretagen, 2019).

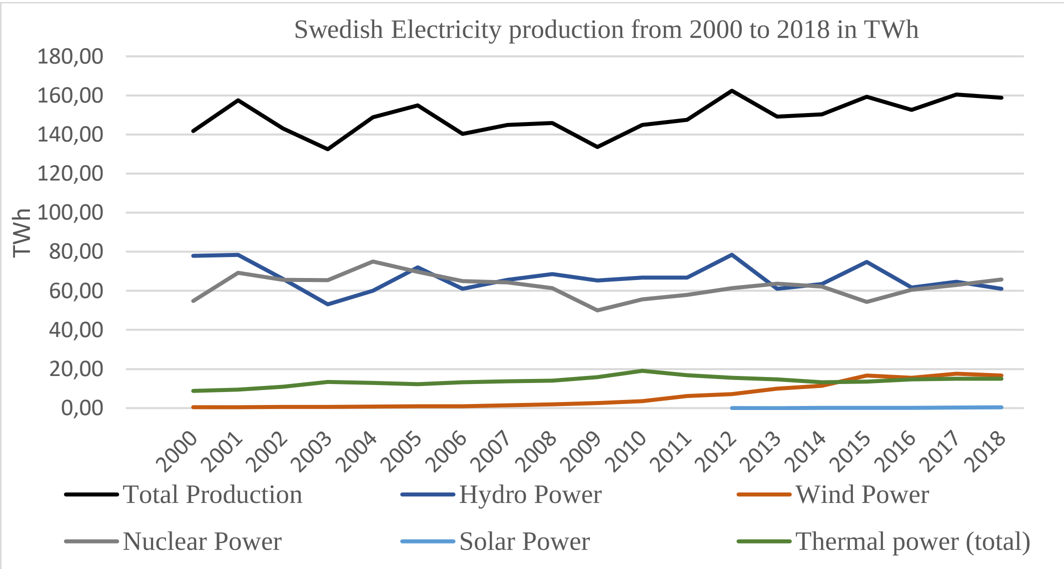

Figure 1: Swedish electricity production from 2000 to 2018 in TWh (Energiföretagen, 2019).

Total production reached the lowest level in 2003 with a production of 132.5 TWh followed by a similar dip in 2009 with a production of 133.6 TWh. In 2003 Hydropower reached the lowest level in the interval with a market share of 40.08% while nuclear power totalled 49.41% of the mix. In 2009 the opposite occurs with a low share of nuclear 37.41% and a high share of hydro 48.86%. The highest level reported in the interval was in 2012 with a production of 162.4 TWh followed by 160.5 TWh in 2017. The average yearly production in the interval is 148.9 TWh. An interesting observation is the large increase in wind power, illustrated by the orange line in *figure 1*. From a level of 2.42 % of the energy mix in 2010 to a level of 10.47 % in 2018.

Norway

The production mix consist of hydropower, wind power and thermal power *(figure 2)*. In 2018, hydropower represented 95% of total production followed by wind power 2.36% and thermal power 2.64% (SSB, 2020). Hence, Norway has the highest amounts of renewables in the production mix. Norway has around 50% of the reservoir capacity in Europe and 75% of the Norwegian production capacity is adjustable (Energifakta Norge, 2019a). High mountains, numerous rivers and big amounts of snow and rain are geographical and metrological conditions giving Norway a comparative advantage in hydropower production. The power plants connected to the reservoirs has a high degree of flexibility and it is easy to adjust the production due to demand and supply conditions (Energifakta Norge, 2019a).

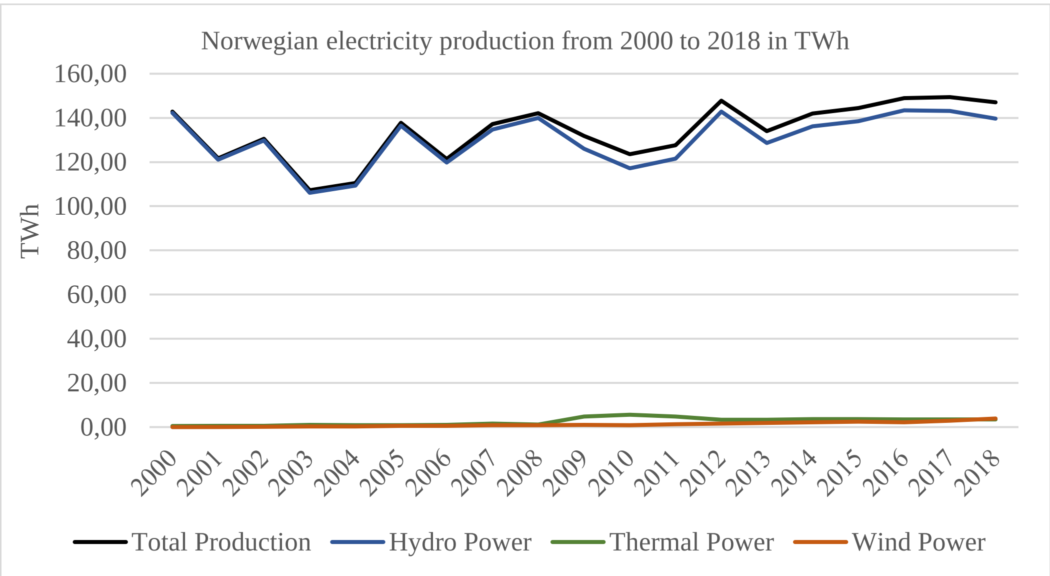

Figure 2: Norwegian electricity production from 2000 to 2018 in TWh (SSB, 2020)

The lowest production level in the interval occurred in 2003 (similar with Sweden) with a low of 107.25 TWh followed by the second lowest production in 2004 with 110.47 TWh. Production reached a maximum in 2017 (while Sweden had the 2nd highest production) with 149.40 TWh and the second-best year was 2016 with a production of 148.99 TWh. The production had a steady increase from 2013 until the peak in 2017. Average production in the period was 134.09 TWh. In 2000 hydro power contributed to 99.63% of total production, and 98.50% in 2008, represented by the blue line in *figure 2.* Wind power has increased a lot from 0.50 TWh (0.02%) in 2000 to 3.88 TWh (2.64%) in 2018, yet it still represents a small amount of annual production. The share of wind power is planned to increase in the future; Total installed capacity of wind power was 1695 MWh at the end of 2018 while in 2019 a total of 1100 MWh of new installed capacity was expected. For 2020 a new record of installed capacity is planned, with a new installation of 1200 MWh meaning the installed capacity will increase around 34% in two years (Vindportalen, u.d.).

Finland

The domestic production of electricity in Finland reached 67.53 TWh in 2018. The largest share was from renewable energy sources with a total of 46% (hydro 42%, wind 19%, and nearly everything of the remaining from wood-based fuels). The second largest source was nuclear power with a share of around 32%. In third place fossil fuels stood for around 16% and peat around 5% (Statistics Finland, 2020).

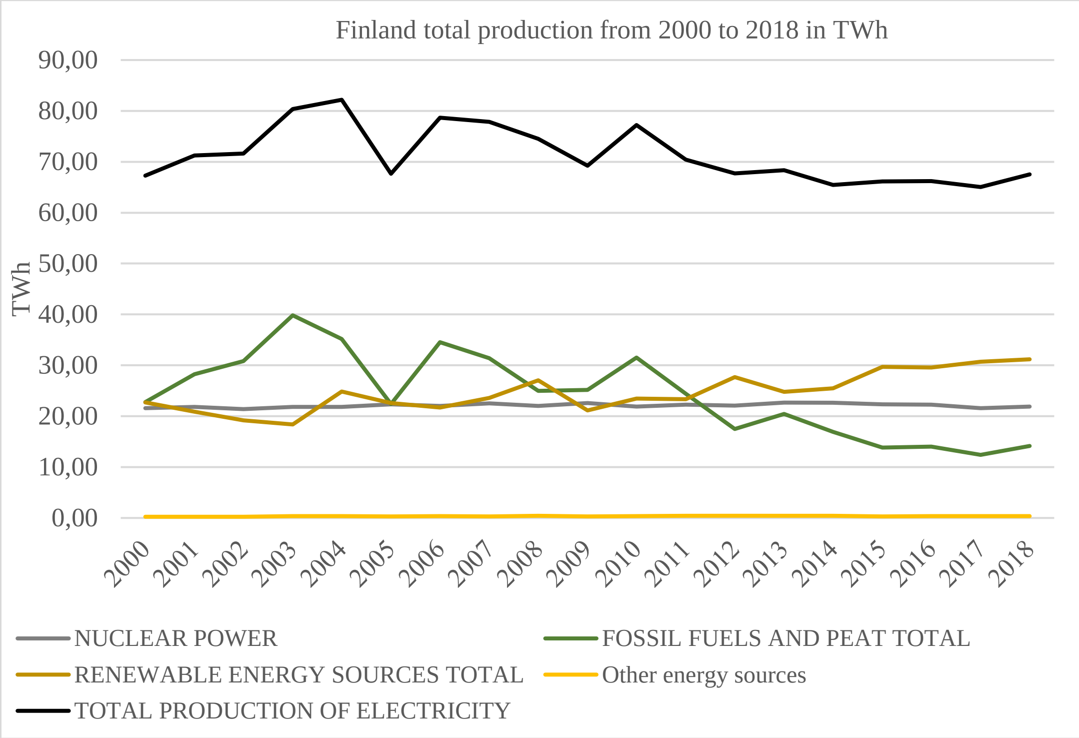

Figure 3: Finland total production from 2000 to 2018 in (TWh) (Statistics Finland, 2020)

Total production *(Figure 3)* in the interval varied from 82.19 TWh in 2004 to 65.04 TWh in 2017. In stark contrast to Sweden and Norway, production in 2003 recorded the 2nd highest production output with a strong peak in Fossil fuels and peat as production source. Nuclear power had a stable production in the whole period while fossil fuels have been reduced in favour for an increase in renewables since 2013 (Statistics Finland, 2019).

Denmark

In 2018 Wind stood for 45.75% of total production, followed by other renewable sources 25.41% including solar, hydro, biomass (straw, wood, biooil, renewable waste) and biogas *(Figure 4)*. The third largest source was coal with a share of 21.63% followed by natural gas at 6.35% and oil in the 5th place with a share of 0.86% (Energistyrelsen, u.d.).

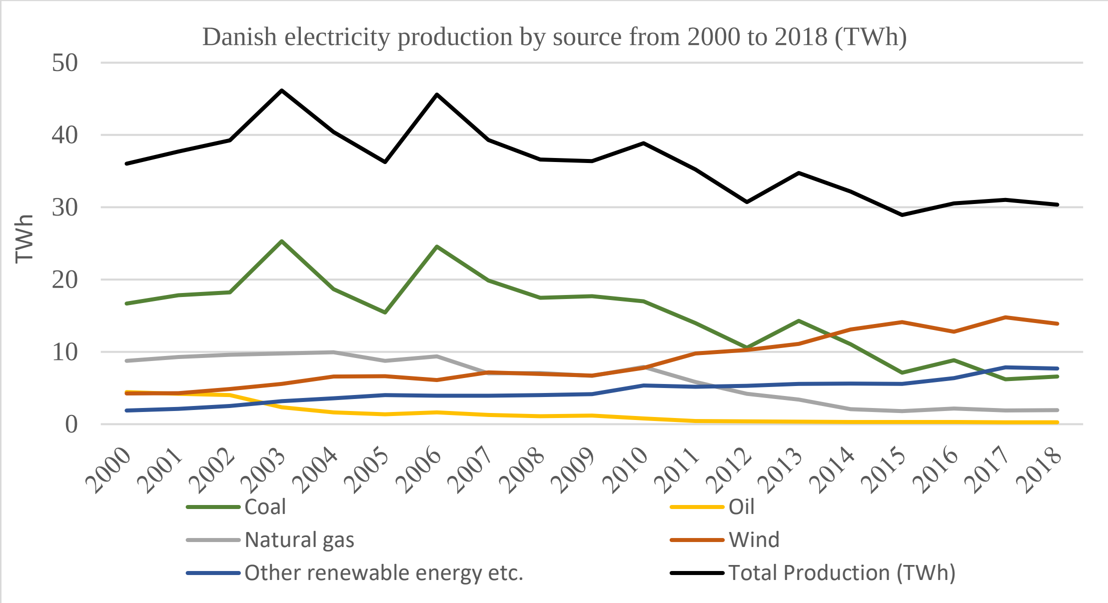

Figure 4: Danish electricity production from 2000 to 2018 (TWh) (Energistyrelsen, u.d.)

The average yearly production in the interval was 36.13 TWh. The lowest production was in 2015 with a production of 28.94 TWh followed by 2018 with a production of 30.38 TWh. In contrast to Sweden and Norway, production peaked in 2003 with a production of 46.16 TWh followed with a new peak in 2006 with a production of 45.60 TWh. Since coal has been an important production source the total production is heavily correlated with the use of coal as an energy source until 2015, represented by the green line in *figure 4*. There have also been large shifts in energy sources in the interval from fossil fuels to renewables. While fossil fuels (Oil, coal and natural gas Oil) totalled to 82.96% of total production in 2000 it was only 28.83% in 2018. Wind power has increased its share from 11.80% in 2000 to 45.76% in 2018 while other renewables has increased from 5.25% in 2000 to 25.41% in 2018. The reduction in fossil full production has exceed the increase in wind production, making overall reduction in production levels in Denmark.

Estonia

Estonia opened as a bidding area in Nord Pool in 2010. In 2018, the electricity production totalled at 12 TWh. The main source is Oil Shale which stood for around 75% of total production, followed by Wood chips and waste 9.67% while shale oil gas and wind energy were around 5%. Estonia has also many other sources in the production mix as listed in the graph below *(Figure 5)*.

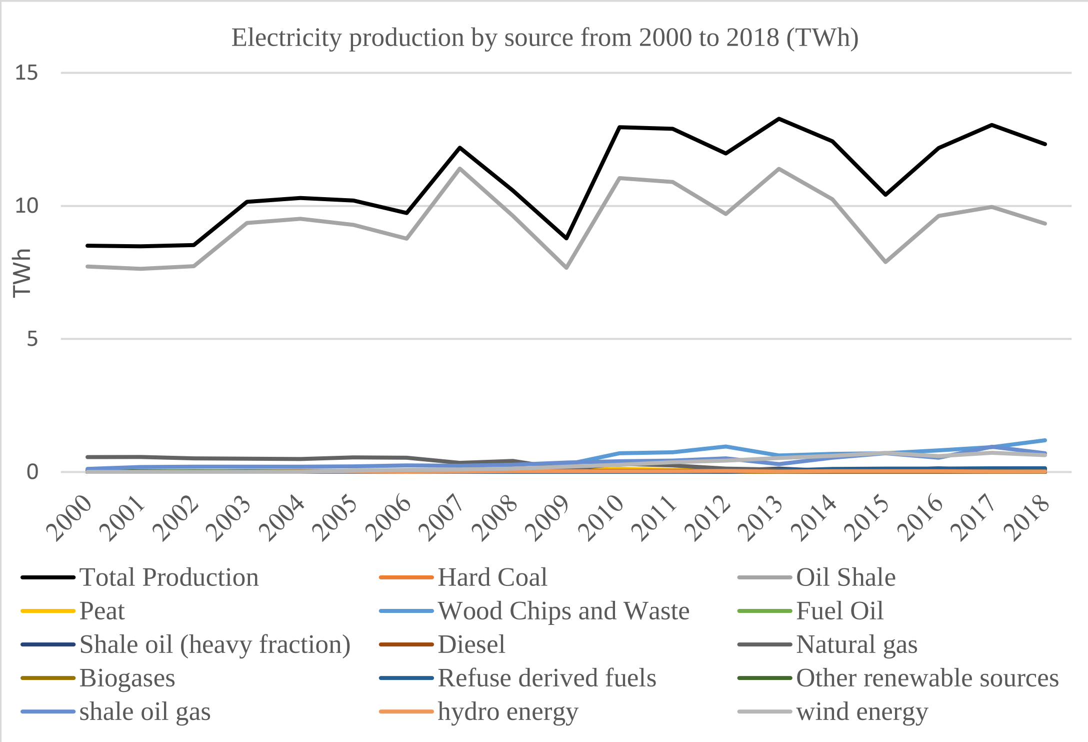

Figure 5: Electricity Production from 2000 to 2018 (TWh) (Statistics Estonia, 2020)

Production varies from a bottom in 2001 with 8.48 TWh to a top in 2017 with a production of 13.05 TWh. Oil shale has remained the main source in the whole period while there has been a steady increase in wood chips and waste, shale oil gas and wind energy from around 2013 (Statistics Estonia, 2020). Trend analysis from *figure 5* may be insufficient due to changing methods for reporting production in the interval. Until 2008 electricity production from renewables included wood, biogas and black liquor. Since 2009 data of electricity and heat produced from wood are shown separately and after 2013 production from biogas are shown separately. Other renewables sources are black liquor, biogas and animal waste.

Latvia

Total production in 2019 was approximately 6.18 TWh. The main sources are thermal and hydropower with a share of 2.82 TWh and 2.09 TWh respectively, indicated by the grey and orange line in *figure 6*. The remaining production mix consists of Biomass 0.39 TWh, Cogeneration 0.38 TWh, Biogas 0.32 TWh and Solar

0.001 TWh. (AST, 2020).

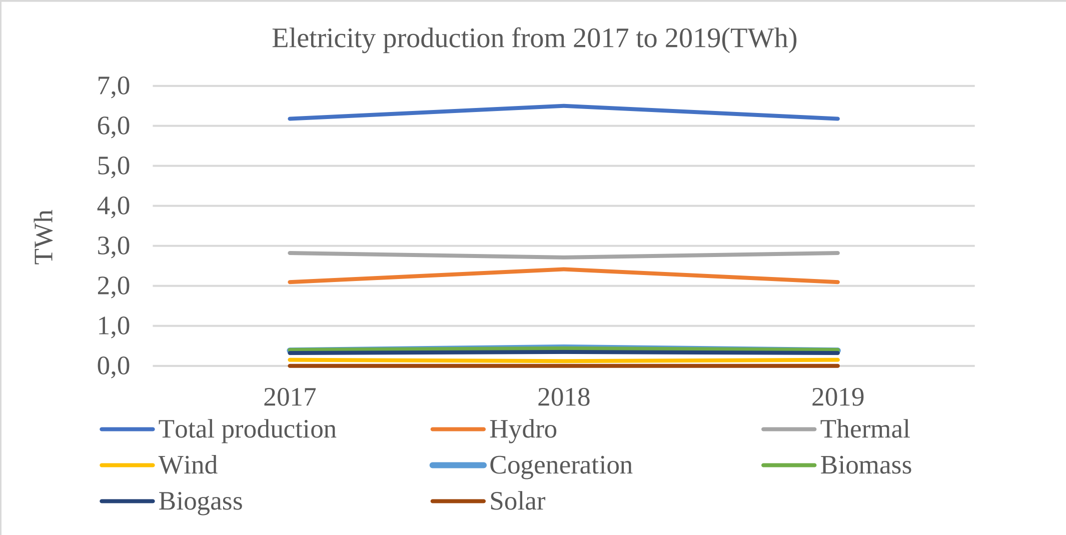

Figure 6: Electricity production from 2017 to 2019 (AST, 2020)

Reliable historical data about production level has only been available from 2017. Latvia's total production in 2019 counts for 0.9% of the total production in Nord Pool.

Lithuania

Lithuania opened as a bidding area in Nord Pool spot in 2012. Electricity generation in Lithuania have dropped drastically the last 20 years. The main reason is the shutdown of nuclear reactor in 2009 which produced 70% of the electricity in the country (World Nuclear Assosiation, 2017). In 2019 the net electricity production amounted to 3,64 TWh. The largest production source was Wind Farms 40%, followed by Thermal power plants 20%, Kruonis HP SP (Pumped storage hydro power plant) 16%. Other renewable energy sources such as power plants operating on biomass and biogas, solar energy plants and waste incineration plants produced 14.7% and Hydroelectric power plants produced 9.4% of the total amount (Litgrid, 2019).

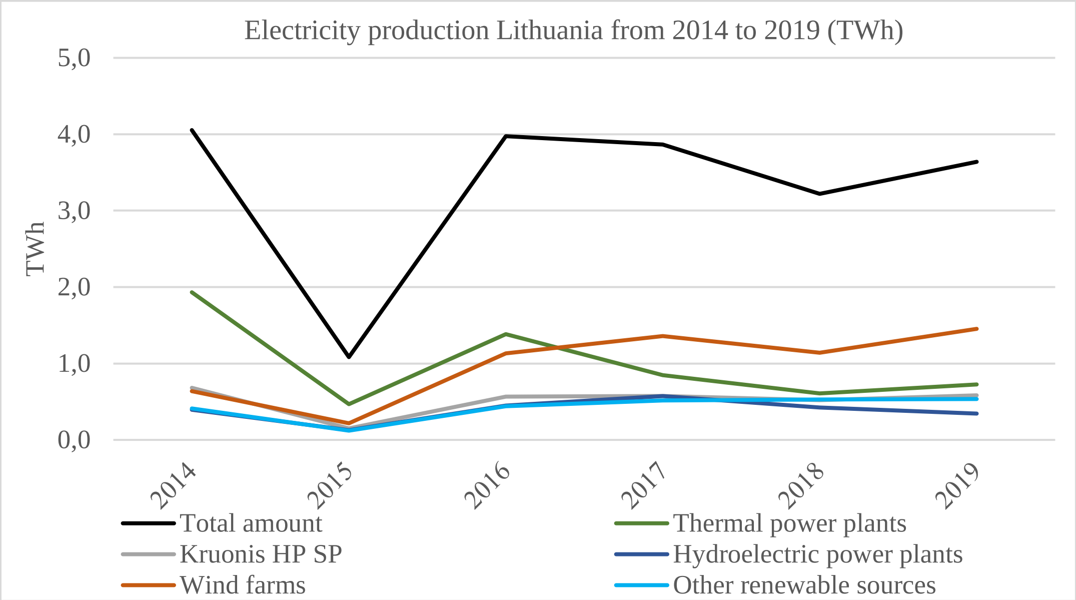

Figure 7: Electricity production from 2014 to 2019 (TWh) (Litgrid, 2019)

Data from the interval shows a peak production in 2014 with 4.05 TWh and the lowest level was in 2015 with a production of 1,08TWh. Over the last five years wind farms have a substantial increase in the production mix from 15.7% in 2014 to 39.9% in 2019, indicated by the orange line in *figure 7*. The overall electricity mix consist of a lot of renewables in Lithuania.

Total production in the Nordic Countries.

The total production in the Nordic countries in the interval varies between 366 TWh and 408 TWh with an average production of 390.41 TWh. Since the Baltics accounts for 4% of the total market, entered the market on different point of time and the historical data is a bit incomplete on these countries, the total production in the interval is reported without the Baltic countries *(figure 8)*. Hydro power is by far the largest production source in the mix followed by Nuclear power. Hydro power production has larger fluctuations than overall total production in the interval indicating substitution effects with other energy sources.

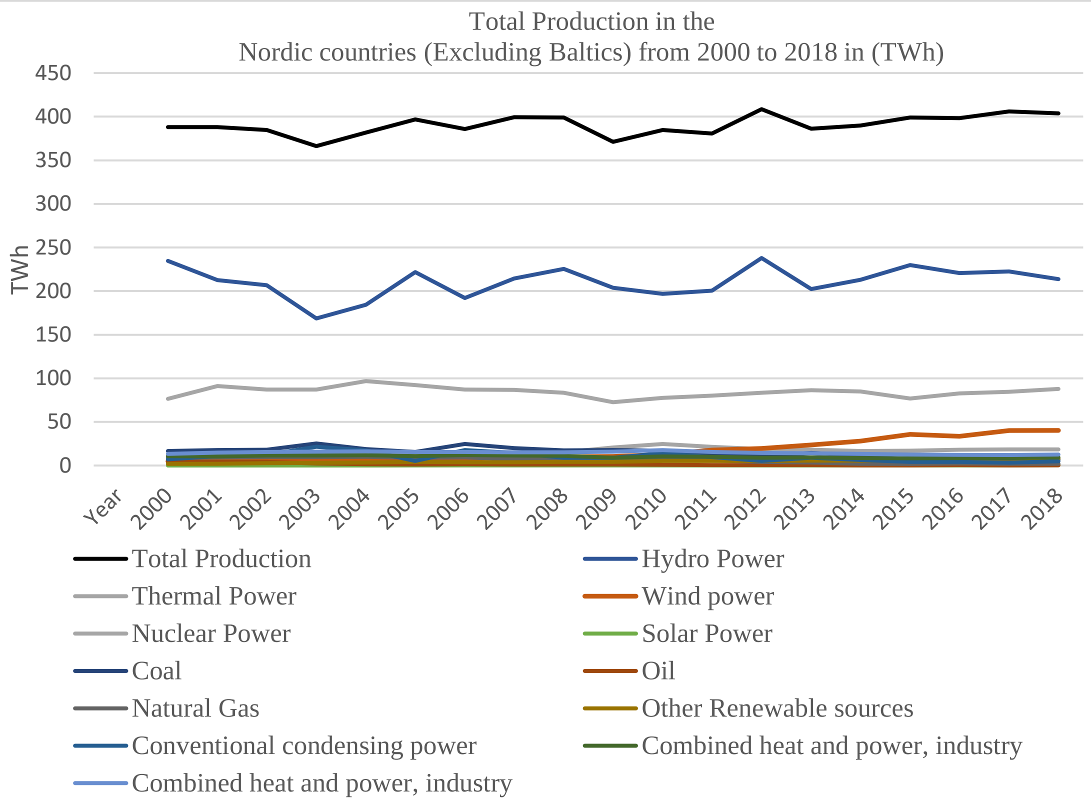

Figure 8: Total Production in the Nordic countries (Excluding Baltics) from 2000 to 2018 in (TWh)

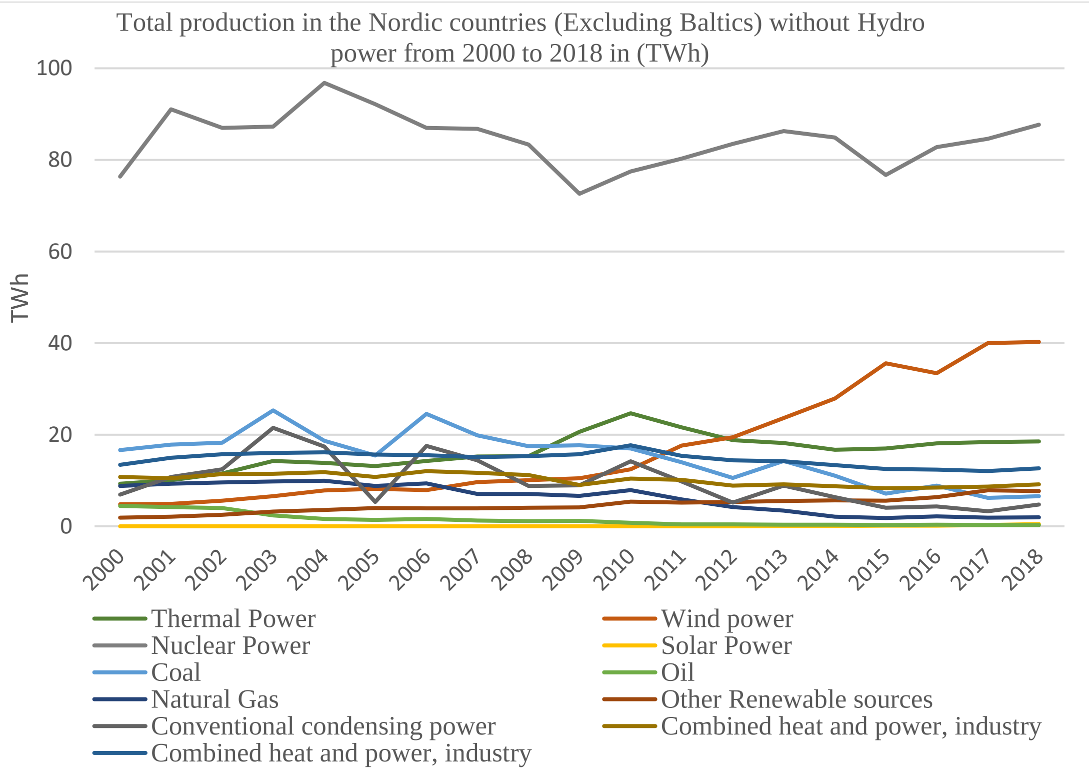

Figure 9: Total production in the Nordic (excluding Baltics) without Hydro power from 2000 to 2018 in (TWh)

In *figure 9* hydro power is removed from the production mix, enabling a better understanding of fluctuations in the other production methods. Wind power production is scaled up while coal and oil are downscaled. In the period from 2000 to 2004 falling hydropower production and increased fossil production indicate a substitution effect. The opposite tendency is observed in 2015.

1.3.5 Reservoir levels in Norway from 2000 - 2019

The reservoir level is a measure of how much water is stored in the hydropower plants in the mountains. Electricity cannot be stored effectively with today's technology, however having water stored in reservoirs enable producers to adjust their electricity production after demand and supply condition. Hence, reservoir levels are a measure of how much electricity suppliers can supply to the market in the future. Information about reservoir levels are accessible for market actors, hence it is reasonable to assume that participants in the financial electricity market take this information into account.

To limit the scope and due to lack of available data, only Norwegian reservoir levels are included. This limitation is supported by the fact that Norway has approximately 50% of the total reservoir capacity in Europe, information about Norwegian reservoir levels can influence their whole market. The maximum reservoir capacity pr. April 2019 is 86.9 TWh (NVE, 2020a). The reservoir levels follow strong seasonal differences due to natural reasons. This trend is easily identified in the statistics. When winter sets in with colder temperatures the rainfall is stored in the mountains as snow reducing inflow to the reservoirs. High electricity consumption and production comes with the cold temperatures making reservoir levels decrease from around week 43 until spring. From around week 17 the spring arrives with warmer weather, snow in the high mountains melts and the water flows into the reservoirs. This leads to a sharp increase in the reservoir levels until it starts to flatten out again in week 35 (late august). Then the levels are quite stable until week 43 before the decline through the winter starts.

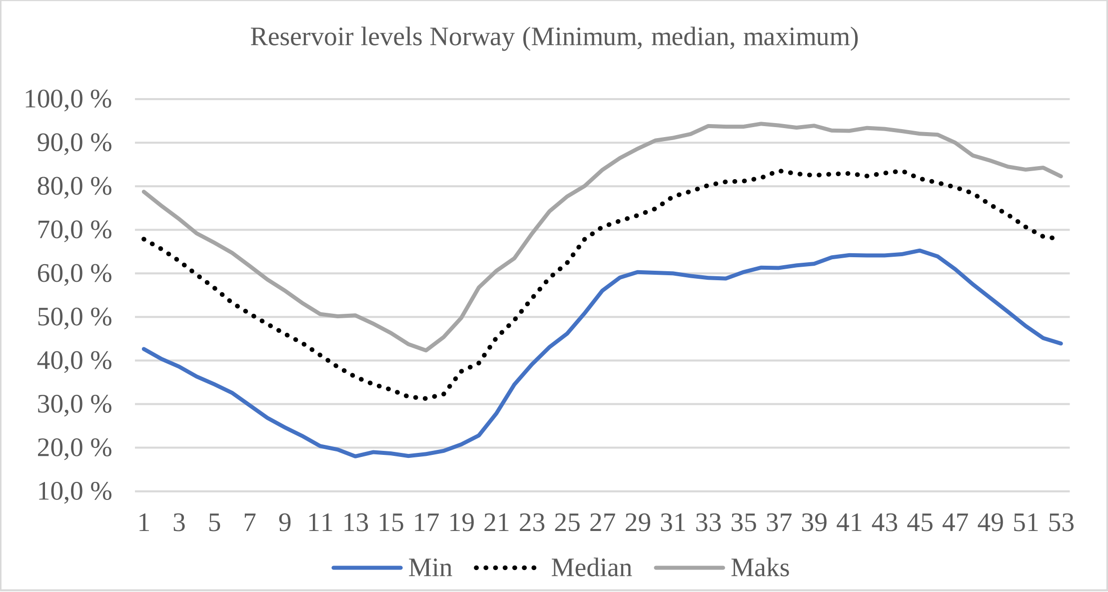

Figure 10: Reservoir levels Norway (NVE, 2020)

The maximum (grey line), minimum (blue line) and median level (dotted line) in *figure 10* is based on the last 20 years of observations. Based on the historical data, the median level reaches a bottom level with 31.3% in week 17 and a maximum level of 83.6% in week 37. Large fluctuations occur due to different weather from year to year, where more (less) snow in the mountains, warmer (colder) winters and wetter (drier) summers will increase (decrease) the reservoir level. The large fluctuations are visible in the dataset since the lowest level reported in week 17 is 18.1% while the highest level is 45.4%.

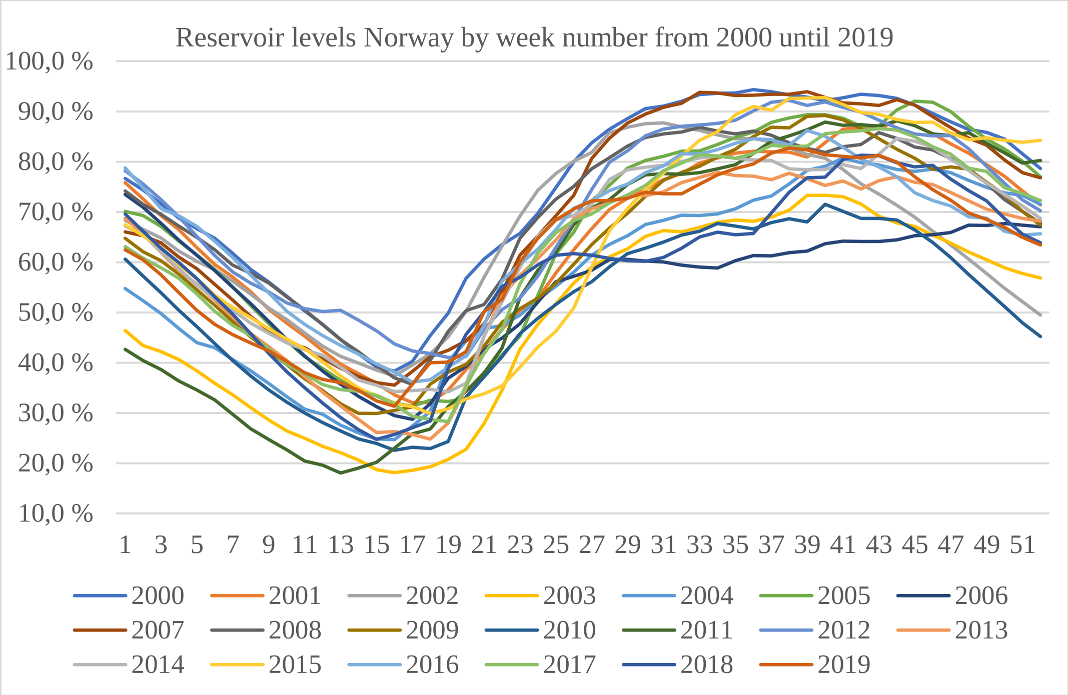

Figure 11: Reservoir levels Norway by week number from 2000 until 2019 (NVE, 2020)

Figure 11 graph reservoir levels for the whole period in total, the seasonal trend is noticeably clear with large fluctuations between the different years. Another interesting observation is the difference shape of the curves regarding how quickly the reservoir levels increase after reaching the bottom level. For instance, year 2006 flattens out at a level around 60% which is well below the median. This observation is also visible in the total production which has a large drop in 2006.

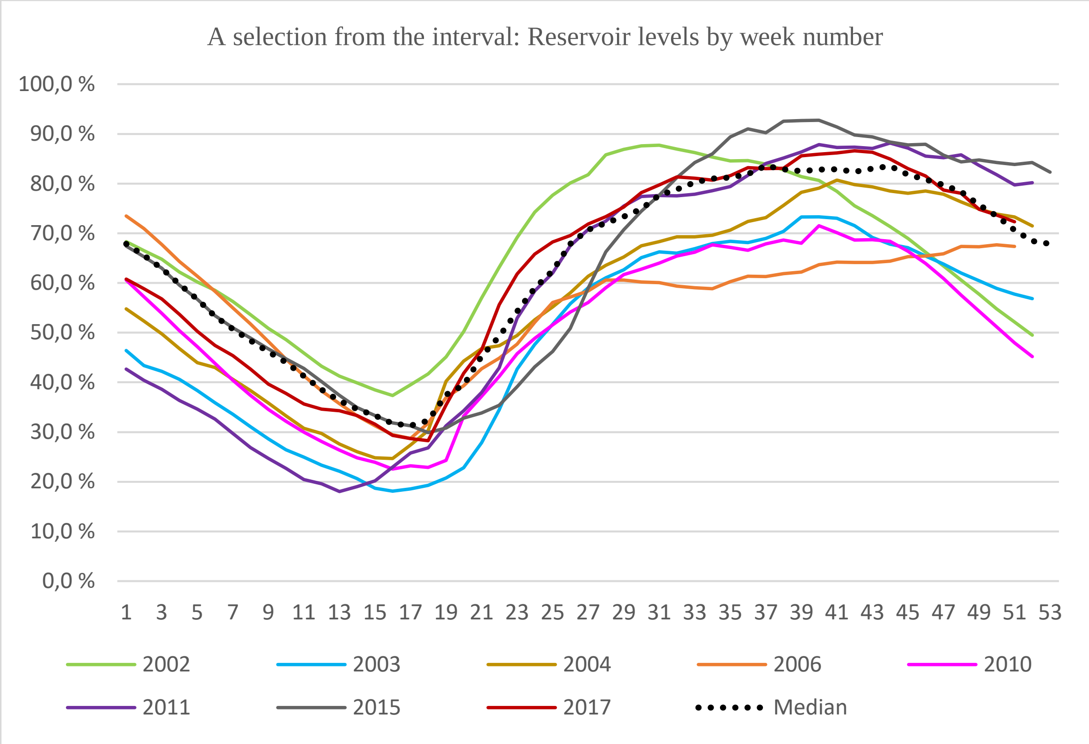

Figure 12: A selection from the interval: Reservoir levels by week number (NVE, 2020)

Figure 12 graph a selection from the interval. The large production decline from late 2002 (green line), during 2003 (blue line) and until 2004 (golden line) is very visible in the series with substantial deviations in reservoir levels from the median level (dotted line). Year 2006 (orange line) is quite unique with an early peak around week 29, with relatively low reservoir that summer and autumn. 2010 (pink line) ended the year with even lower reservoir levels than 2002 and 2003. The low reservoir levels continued into 2011 (purple line), but the year ended with a lot of water in the reservoirs. These periods reveal relatively big deviations from the median reservoir level for the period. 2015 (grey line) follows the median until week 17, lies below until week 31 and above rest of the year. In 2017 (dark red line), the total production reached an all-time high with reservoir levels following the median trend. The selected reservoir levels will be further discussed together with volatility in chapter 6.2

1.3.6 Temperature trend Oslo from 2000 to 2019

Temperatures is one of many factors influencing the electricity price. Temperature has a clear link to electricity spot prices as electricity is used as a primary heating source in many countries and cold weather increase the demand. Haugom *et al.*

(2018) observes a clear negative dependence between temperature and consumption. The news is often reporting record high electricity prices when the cold weather sets in. It is a known fact that the electricity prices follow seasonal trends with low prices in the summer (with lower demand) and high prices in the winter (with higher demand). This relationship permits temperature levels as a proxy for electricity consumption. Temperature information are accessible for market actors, making it reasonable to assume that participants in the financial electricity market take this information into account. Yet this information is restricted by the reliability and duration of the weather forecast.

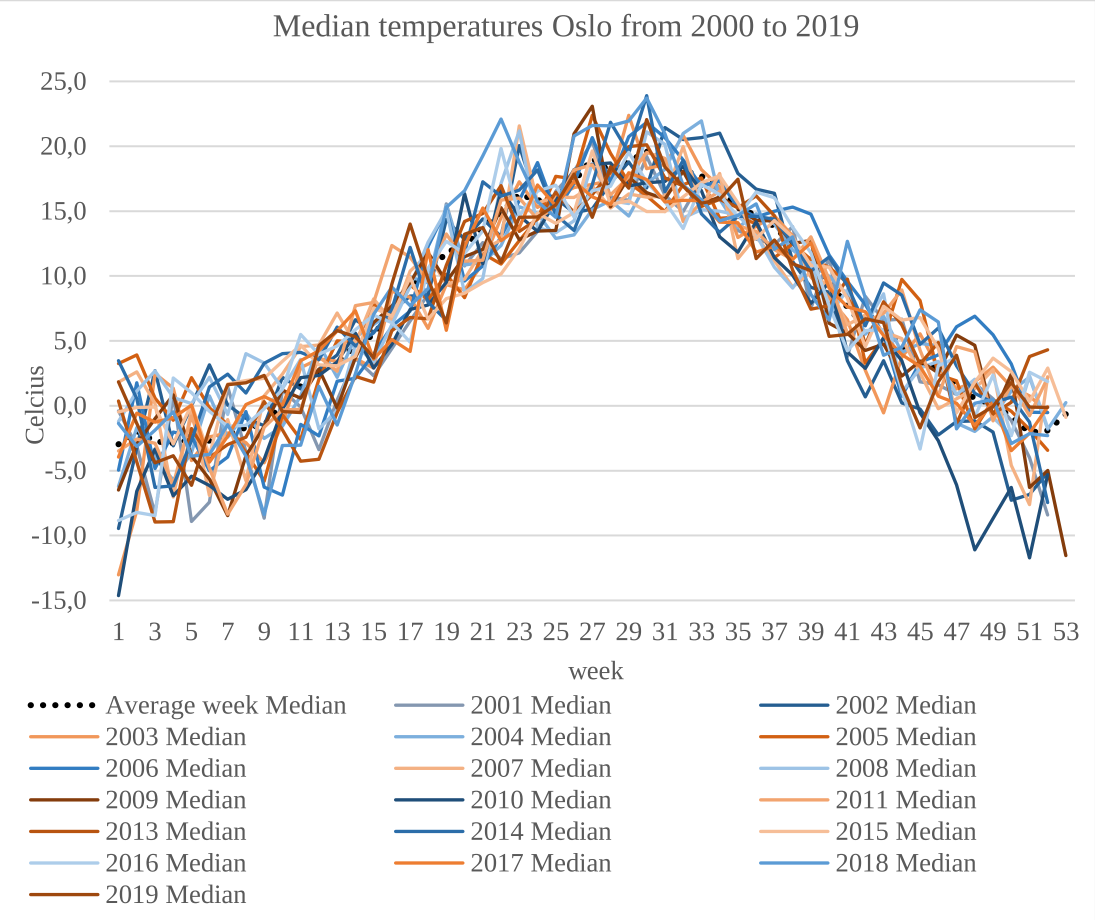

Figure 13: Median Temperatures Oslo from 2000 to 2020 (Norsk klimaservicesenter, 2020)

Figure 13 shows median temperatures in Oslo from 2000 to 2020 week by week. The median temperature is an arithmetic average of the temperatures between 00:00 and 24:00. The "Average Week Median" used as a basis is an average temperature based on the observed temperatures from 2000 until 2020. Interesting candidates for further analysis are year 2001 with a strong deviation from the median from week 49 through 52 continuing with large deviations from the mean into 2002. Further, 2007 also has an interesting spike between week 49 and 52. 2010 also has a strong deviation from the mean continuing in the start of 2011.

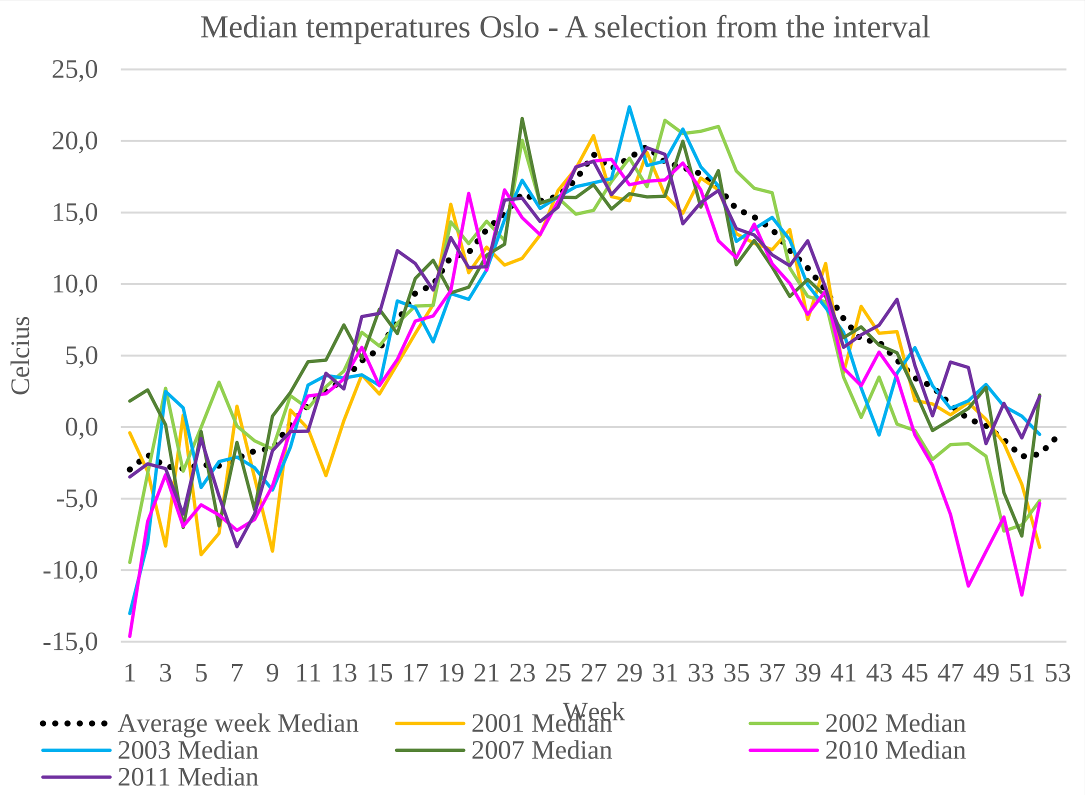

Figure 14: Median temperatures Oslo from 2000 to 2020 – A selection from the interval (Norsk klimaservicesenter, 2020)

Figure 14 graph a selection from the interval which will be further discussed together with volatility in chapter 6.3.

1.3.7 Producers market adjustment factors

Different technics to produce electricity react differently to supply and demand conditions in the electricity market. Here we will briefly discuss how producers react and adjust their production after supply and demand conditions.

Renewables

Most renewable production methods have low variable costs as the input factors such as water, wind and sun are free of charge. In the same way as electricity, these input factors cannot be stored, and most producers are therefore willing to produce when the price is above zero. In general, production occurs regardless of price but depends on weather conditions, hence most renewables are said to be non-adjustable energy sources. For producers which has the possibility to store water in reservoirs, other conditions must be considered. Shall they produce today, or hope for a better price in the future? The decision is based on the difference between todays price and the expected price in the future. It is a challenging exercise since it is difficult to predict water inflow which can vary locally, further it is difficult to predict consumption and development in the market (Energifakta Norge, 2019a). A thinkable worst-case scenario for a reservoir owner is when production is forced due to full reservoirs in a combination with bad market conditions such as low price (Energifakta Norge, 2019a). Producers can use the financial market for electricity to lock in their production, making production nearly unaffected of fluctuations in the spot price. We will discuss the financial markets in the next chapter.

Thermal power

Thermal power uses sources such as natural gas, coal, nuclear power, and waste. Thermal power production has different production assumption then renewables. An advantage with thermal power is the possibility to produce independent of weather conditions. Production is profitable when the electricity price is above the production cost. The cost of production depends on which production source is used, but is mainly dependent on the price of coal, gas and emission allowances (Energifakta Norge, 2019a), implying higher variable costs. Nuclear power plants are expensive to build, but quite cheap to run due to relatively low variable costs. The economics of nuclear power involves capital costs, plant operating costs, external costs, and other costs such as system costs and nuclear specific taxes. On a lifetime basis, nuclear plants are an economic source of electricity production. Some of the advantages with nuclear power are high reliability and low greenhouse gas emissions.

1.3.8 Market changing factors

The power market has gone through large changes in the near past and is still facing different challenges ahead. The agenda of the energy industry is heavily affected by climates changes, regulatory and safety factors. Many industries (customers of the exchange) are opting to reduce emissions and the market needs to take that into account. It requires more use of renewable sources which can be challenging for the market. In earlier times where traditional energy sources where a larger part of the mix, it was easier to predict, forecast and decide where to produce energy. Now a days, when renewables are a larger part of the mix it is more demanding to forecast and predict the production. Due to the changing weather conditions and forecasting problems, there can be large production fluctuations within the hour. The changing environment affects the Transmission system operators who is responsible for balancing the grid with a constant flow at 50 Hz. The trend from the Nord Pool members energy mix reveal large increases of renewables as production sources. Since electricity cannot be stored, and most renewables are non-adjustable new theoretical aspects arise. In periods with a lot of wind, rain and sun combined with low demand for electricity clearing the market suggests lower electricity prices. In periods with less wind, rain and sun combined with high demand for electricity clearing the market suggest higher electricity prices. If these aspects occur combined with fewer adjustable (nuclear, coal, natural gas, storable hydro) and more non-adjustable (wind, sun, un-storable hydro) sources the volatility in the spot price might increase. This information is available for market participants, but to what extent it gives implications for the financial market for electricity is to our knowledge unidentified.

1.4 The financial market for electricity

In this chapter we will elaborate the financial markets for electricity contracts and different types of contract.

1.4.1 Introduction to the financial market for electricity.

The Nordic/Baltic power market is one of the most liquid power derivates markets in the world. All contracts are settled financially, therefor no power is physical delivered. To be granted permit to trade on Nasdaq OMX, actors must provide security for its exposure. Hence, the market is not public but limited mostly to institutional investors which can provide this security. Private persons who desires access to the market face stricter financial requirements than companies. The financial contracts are traded with a longer time span than the physical day-ahead contracts. The users of the financial electricity market trade on different contracts to lock in their price – hence the market is used for price hedging and risk management for actors in the Nordic/Baltic electricity market (Wangensteen, 2006). By locking a specific future price, actors may reduce risk and reduce fluctuations in margins. A power producer can secure some of the production in the financial market to reduce fluctuations in margins. How big proportion of production secured in the financial market will vary with risk appetite and other financial factors in the focal firm. Retailers can use the financial market to secure operations after signing contracts of electricity delivery to end users. Large consumers operating in an energy intensive industry can use the financial market for risk management since the electricity price is a primary risk exposure impacting profitability of operations. A typical consumer participating in the financial electricity market are the aluminium producers in Norway. Traders and speculators participate in the financial market to catch price differences between spot prices and different contracts. These participators have an important role to increase liquidity and effective market clearing.

A various number of contracts are available for trade at Nasdaq OMX such as Futures, DS Futures[2] and Options. The length of the contract varies from day to week, month, quarter and year. Options in a combination with futures offers valuable strategies for managing risk associated with power trading. The benefits of using options for risk management is to limit the exposure to downside risk (loss) while maintaining the exposure to upside potential (profit). In this paper quarterly and yearly front future contracts are studied.

1.4.2 Nordic electricity base futures

A future contract is an agreement to buy (long) or sell (short) an asset at a specified future delivery date for a fixed price which is specified at the present moment (forward price).

The contract type is a standardized electricity future contract with cash settlement. The contract is based on the Nordic System price of 1 MWh of electricity according to the daily Elspot system price for the Nordic region which is quoted and published by Nord Pool Spot. The contract base size is 1 MWh and the number of delivery hours for each series is specified in the trading system and the product calendar and may vary with the applicable delivery period. The contract size, which is expressed

[2]Future product with no settlement during the trading period prior to the expire day. https://www.nasdaq.com/solutions/power-ds-futures

in MWh, will be a function of the applicable number of delivery hours and the lot size. The base load years normally vary between 8760 and 8784 hours. The trade lot is 1 MWh and the bank day calendar follow the bank days in Norway. The contracts are traded in Euro with a minimum ticker size of 0,01 EUR thus the contract price is expressed in EUR/MWh. The base load covers all hours of all days in the delivery period (Nasdaq, 2018). Settlement of future contracts involves both a daily mark-to market settlement and a final spot reference cash settlement, after the contract reaches its expiry date. Mark-to market settlement covers profit or loss from day to day changes in the daily closing price of each contract. Final settlement, which begins at delivery, covers the difference between the final closing price of the future contract and the system price in the delivery period (Nasdaq, u.d.). The financial market cannot be used to trade one single kWh, as mentioned earlier. Instead, the financial market is utilized to manage prices and risks. We will illustrate this by an example:

A producer and a retailer have agreed on a quarter future contract for electricity. The contract price is 45 Euro per MWh, and the contract size is 8 MWh. The delivery period is set to October 2020. Two scenarios can occur:

1) Where the average system price for third quarter was 46 Euro per MWh. This higher price in the market is initially a disadvantage for the retailer and an advantage for the producer. However, since they have a future contract, the producer will compensate the retailer by 1 €/MWh * 8 MWh = 8 Euro is transferred from producer to retailer.
2) Where the average system price for third quarter was 44 Euro per MWh. This lower price in the market is initially an advantage for the retailer and a disadvantage for the producer. However, since they have a future contract, the retailer will compensate the supplier by 1 €/MWh * 8 MWh = 8 euro is now transferred from retailer to producer.

When clearing the contract, the contract price is compared to the average system price for the contract period. The money to be transferred between the actors is found by multiplying the price difference by the contract size.

1.4.3 Nordic Electricity Base Options

An option differs from a future since it gives the holder a right, but not an obligation to buy or sell a specified amount of electricity, at a specified price by a stated expiration or maturity date. A buyer of an option has a right but not an obligation to buy, while the seller has a duty to sell. Options can either bought or sold (written). There are two kind of options, a call option and a put option.

A call option gives the buyer a right, but not an obligation to buy a specified amount of electricity, at a specified price by a stated expiration or maturity. A duty is imposed on the seller to sell a specified amount of electricity, at a specified price by a stated expiration or maturity date.

A put option gives the buyer a right, but not an obligation to sell a specified amount of electricity, at a specified price by a stated expiration or maturity. A duty is imposed on the seller to buy a specified amount of electricity, at a specified price by a stated expiration or maturity date.

The type of contract is a standardized option contract on corresponding Contract base of "Nordic Electricity Base Quarterly Electricity Future Contract". The contract base is quarterly/yearly future contracts with the same base and lot size as the futures. The option style is European option meaning the option can only be exercised on expiration date.

1.4.4 Pricing of futures and options (Black Scholes model)

The Black and Scholes (Black & Scholes, 1973) model is used for determining the market value of an option. The model is based on the following assumptions (Lumby & Jones, 2015): The option are European calls, no taxes or transaction costs are involved with option trading, option investors can lend and borrow at an interest rate equal to risk free rent, the underlying shares can be freely bought and sold even in fractional amounts, there are no dividends payable on the shares before the option's expiry date and both the Risk free interest rate and the shares standard deviation remain constant over the life of the option. Even though some of the assumptions in the model is quite unrealistic, the basic model is a good predictor of option values and can be adjusted to apply for more realistic scenarios.

The following notation is used by Lumby & Jones (2015):

C = The market value of a call option (i.e. the option premium).

S = Current market price of the shares.

X = Options exercise price.

Rf= Risk-free rate of interest

T= Time in years, until the option expires.

σ = Volatility (as measured by the standard deviation= of the share price.

$\log_e$ = Natural log.

e = The mathematical constant 2.71828

N = Cumulative area under the normal curve.

The Black and Scholes Option valuation model:

$$C = [\,S \times N(d_1)] - [\,X \times e^{-Rf \times T} \times N(d_2)]$$

Where the two adjustments factor are:

$$(d_1\,) = \frac{\log_e(\frac{S}{X}) + (Rf \ \times T)}{\sigma \times \sqrt{T}} + (0.5 \times \sigma \times \sqrt{T})$$

and:

$$(d_2) = (d_1) - \sigma \times \sqrt{T}$$

The determinants of the market value of an option in the Black and Scholes model is based on a combination of the exercise price (X), the current market share price (S), the time to expiry (T), the volatility of the share price(σ) and the annual time value of money ($e^{-Rf \times T}$). By assessing the determinants, it is necessary to highlight the only unknown determinator is the volatility of the underlying asset. With improved knowledge of the uncertainty regarding volatility in the financial contracts of electricity, practitioners undertaking risk management can make better decision based on the improved knowledge. The re-projected volatility numbers from our findings in chapter 5.3 can be inserted in the Black and Scholes model to calculate the option price.

As every parameter except volatility is known, the Black and Scholes model can be reversed to calculate the implied volatility (Latane & Rendleman, 1976). This method uses the markets expectations in the options prices to calculate a volatility index, hence the method sets practicality over precision. More about Implied Volatility in the Nordic Power Market in Birkelund & Opdal (2014).

2. Literature review

This chapter will look at different relevant literature for the research topic. The first part will look at relevant research of the electricity market, and the second part will look at research relevant for stochastic volatility models.

2.1 References to the electricity market:

Like in many other commodities markets, the research on electricity market is comprehensive. Many disciplines have done research trying to understand the dynamics in especially the system price in an electricity market. Research have been done to both understand and describe market characteristics, volatility dynamics, drivers behind price fluctuations and shock responses.

Solibakke (2016) analysed the NordPool Spot system price volatility with an SNP (G)ARCH specification of 14.1.1.1.12.0.0.0. The system price volatility showed characteristics of mean-reversion effects with seasonal changes and volatility clustering. Further Solibakke (2016) report that large price changes from shocks gives high conditional volatility reaction. Whereas smaller price changes from shocks give fairly small responses in the volatility. Asymmetry is reported as low for small price changes (-5%< and <5%), but becomes big under big price changes (<-10% and 10%<) (Solibakke, 2016). The persistence in the System Price is reported to approximately 12 days.

Haugom, et al., (2017) analysed the forward premium in the Nordpool power market. Results concluded that the average spot price and deviation of water inflow from its usual level have significant positive impacts on the forward premium. A negative relationship between electricity consumption and temperature was also reported.

Nevertheless, as the system spot prices is set through a closed double auction system once a day, modelling the system price is challenging. The auction system set the system prices based on the reported demand and supply of electricity in the different bidding areas to clear the market within and between the bidding areas. This process is conducted by the TSO, and the market participants i.e. the producers, the retailers and the consumers, act as a price taker. From as risk management point of view, looking at the financial markets for electricity contracts makes more sense. As the research of volatility in Nordic futures market for electricity contracts are rather inadequate, looking at the methodology used in other markets seems appropriate.

When looking at volatility in time series, the ARCH and GARCH framework is interesting. Paolella and Taschini (2006) looked at different forecasting methods, including analysis of supply and demand factors and spot-future parity. Such methods where proven to give misleading conclusions, due to market complexity. More effectively where statistical models relying on historical price information. The authors evaluated the performance of different statistical GARCH models for the prices of CO_2 and SO_2 certificates, both tail thickness for the unconditional distribution and conditional distribution where evaluated. This work where strengthen by Banz & Truck (2009), who used an AR(1) GARCH(1,1) to evaluate prices of CO_2 certificates. Both analyses observe heteroskedasticity in the returns and found efficient model fit with conditional variances.

Egeland & Haug (2016) used semi-nonparametric AR(1) GARCH(1,1) models to extract densities and conditional variances for 14 different financial markets, including stocks, stocks indices and commodities. The GARCH-models seems to capture volatility clustering and asymmetry effects. The paper found good evidence for asymmetry effects, where price decreases gave higher volatility than price increases. The strongest asymmetry effects were found in the stock indices, but where clearly present both in CO_2 and Brent Oil financial market.

2.2 References to stochastic volatility models:

A stochastic volatility (SV) model has its own stochastic process and is therefore useful to model time varying volatility in financial markets. SV model implementation has been done to different equities and commodities. To what we have found, no one has implemented GARCH- and SV models for det Nordic futures market for electricity contracts.

The stochastic volatility stream started with Andersen *et al. (2002)*, where an SV diffusion process for an observed stock price S_t is provided by $\frac{dS_t}{S_t} = (\mu + cV_t)dt + \sqrt{V_t}dW_t$, where the unobserved volatility process V_t is either log linear or squared root. Andersen *et al* (2002) estimated both versions of the SV model with S&P500 data, however both versions where sharply rejected.

Later Chernov *et al.* (2003) added a jump component to the basic SV model from Andersen *et al (2002),* which improved det model fit radically. This refinement gave characteristics of tick non-Gaussian tails and persistent time-varying volatility (volatility clustering). A two-factor volatility model out performed one-factor models, as one of the volatility factors (V_{1t}) are extremely persistent to capture volatility clustering, and the other (V_{2t}) is strongly mean-reverting to fatten tails. Another extension is to enable correlation between the mean (w_{1t}) and the two SV factors (w_{2t}, w_{3t}). This extension is crucial to enable the asymmetry effect (the correlation between return innovations and volatility innovations).

Solibakke (2015) used an AR(1) GARCH(1,1) model together with a two-factor stochastic volatility model to forecast and extracting conditional moments for the Brent Oil futures market. The paper report risk measures, conditional one-step-ahead moments, forecasts of one-step-ahead conditional volatility and evaluate shocks from conditional variance functions. Option prices were calculated using re-projected conditional volatility. The paper gave insights how to build up valid scientific commodity market models. The same analysis where performed for the European Carbon Markets in Solibakke (2014) and for Front Year Futures Contracts on the European Energy Exchange AG in Solibakke & Dahlen (2012). In Solibakke (2019) a two-factor volatility model where built and implemented to do step ahead volatility prediction and describe its relevance for equity markets. The paper used observations from nine years of the FTSE100 spot index and the Equinor spot price. The paper outlined the stylised facts from the volatility literature, like density tails, persistence, mean reversion, asymmetry and long memory, all contributing to systematic data dependencies. State vectors, conditional distribution and step ahead predictions where outlined as well. The stochastic volatility models performed well and where fruitful to understand more of the price processes in these financial markets.

3. Data

As the electricity system prices is set through a closed double auction system once a day, modelling the system price is challenging in the short run. Due to bottlenecks and bidding areas, different system prices can occur, making modelling even harder. On the other hand, the financial contracts for electricity is traded continuously in one market with one price for the whole Nord Pool area, making it more convenient for modelling. In this paper we are interested in the longer financial contracts as these contracts are mainly used for hedging and risk management. The raw data consist of observations of both front quarter and front years futures contract's prices, spanning from 3. January 2000 to 3. January 2020 traded at the Nasdaq OMX exchange. The data set contains 5009 observations of each contract. The raw data prices will be transformed into returns, an explanation why will be provided in the next section. Returns are a logarithmic transformation of the change in price from day to day. By using front year and front quarter contracts, we do not mix up the different maturity of the contracts. In other words, we cannot take the returns (logarithmic price change) between 30.09.2019 and 01.10.2019 for quarter contracts. The contract traded 30.09.2019 is a 2019Q4 contract, whereas the contract traded 01.10.2019 is a 2020Q1 contract – the two contracts are different products. By using front contracts, we avoid the problem with mixing the price of different products. In the analysis a dummy variable will be added to simulate different shocks on the markets. Hereafter, the data set for the front quarter futures contracts is referred to as QUARTER, and front year futures contracts as YEAR.

4. Methodology

This chapter will start by defining the basic definitions for time series analysis in econometrics. A time series is generated when having repeated observations of the same variable over a given time interval (Bjørnland & Thorsrud, 2015). Highly all data in macroeconomics and finance can be described as time series, it exists time series of GDP, stock prices, exchange rates, commodity prices, and interest rates. In notations, when a variable is denoted y_t, the subscript *t* referred to the period for the observation of the variable y.

The behaviour of a time series is the sum of four additive factors: *trend*, *cycle*, *seasonal components* and *noise* (Bjørnland & Thorsrud, 2015). The *trend* refers to if the timeseries trend upwards or downwards over time (non-stationary), the *cycle* the series follow some cyclic patterns, the *seasonal components* the time series follow some seasonal structure. Example of a seasonal trend is the power consumption, and thus the prices, tend to be lower in the summer than in the winter. Lastly, the *noise* or white noise is an important feature in econometric models, denoted as ε_t, which is a sequence of independent and random variables with a distribution

$$u_t \sim i.i.d.N(0, \sigma^2)$$

That is, u_t has mean of zero and a constant variance of σ^2, denoted as *normal distribution.* (Bjørnland & Thorsrud, 2015). The term *iid* refer to u_t as *identical and independent distributed.* When $N(0,1)$ then u_t is said to be standardized normal distributed denoted $Z_t \sim N(0,1)$

4.1 Normality

Normality, or normal distribution, is important in statistics in order to test hypothesis. In a least squared regression, the regressor y_t partly depends on the error term u_t, then it can be stated if u_t is normally distributed, then y_t will also be normally distributed (Brooks, 2008). Further, the least squared estimators are linear combinations of random variables, i.e. $\hat{\beta} = \sum w_t y_t$, here w_t are weights. As the weighted sum of a normal random variable is also normally distributed, it can be stated that the coefficient estimates are also normally distributed.

To identify if a random variable is normally distributed a few factors are controlled. First the two moments of the distribution are checked – the *mean* and the *variance* as stated earlier should be zero and constant (i.e. $u_t \sim i.i.d. N(0, \sigma^2)$). Further the third and fourth moments known as the *skewness* and *kurtosis* are checked. The skewness measure whether the distribution is symmetrical around the mean, if the random variable is normal the skewness has a value around zero. The kurtosis measure how fat the tails of the distribution are, and a value of three represent normality. The curve of the normal distribution is said to be *mesokurtic*. In financial data a *leptokurtic* distribution is often found, this implies fatter tails and more peakness in the mean compared to the normal distribution – i.e. kurtosis is larger than three.

A test used to check whether the coefficients of skewness and kurtosis are jointly zero are the Jarque-Bera test (Jarque & Bera, 1980). The test statistic follows a χ^2 distribution under the null hypothesis that the variable is symmetric and mesokurtic. Opposite properties of the variable result in a rejection of the null with a conclusion of non-normality.

4.2 Stationarity

As mentioned earlier, the price in a time series can have different behaviours related to its trend. The time series can be stationary or non-stationary. A stationary time series will over longer intervals move around its mean – known as mean-reverting. The opposite of a stationary time series is a non-stationary time series, here the price can follow a positive or negative *trend* over time.
In many statistical analyses an important property is stationary time series. Whether a time series is stationary or not is important due to several motives (Brooks, 2008):

- Stationarity or not can strongly impact the time series behaviour and properties. To illustrate this, a shock in the time series is used. With a stationary timeseries, a shock will gradually fade out, implying when a shock in time *t* occurs, one will see a smaller effect in time *t+1,* and a smaller effect again in time *t+2* and so on. In a non-stationary time- series, the shock in time *t* will be as big in the periods *t+1, t+2* and so on. Hence the shock is persistence and the effect will stay in the time series longer.
- Regressing non-stationary variables which are independent and random of each other can with standard technics give significant coefficients and high R^2. These

results are useless as the two variables are independent of each other, such a regression will be called *spurious regression.*

- With non-stationary variables, the standard assumption for asymptotic analysis will not be applicable, implying that *t*-statistics will not follow *t*-distributions, *F*-statistics will not follow *F*-distribution and so on. Hence, the outcome from the hypothesis testing is not valid when regressing non-stationary variables with standard regression technics.

To make time series stationary, this paper uses *returns* instead of prices in the data set. Returns are found by taking todays price of the contract, divided by yesterday's price of the contract. This ratio is taken logarithmic and multiplied by 100:

$$returns = y_t = ln\,(\frac{p_t}{p_{t-1)}}) \cdot 10$$

To test if the dataset is a stationary time series, the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979), the Phillip-Perron (PP) test (Phillips & Perron, 1986) and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (Kwiatkowski, et al., 1992) are applied. The null-hypothesis in the ADF and PP-test is that the time series is unit-root, i.e. non-stationary. A rejection of the null hypothesis conclude that the time series is stationary. The KPSS-test applies opposite of the ADF and PP-test, i.e., the null is that the time series is stationary – a fail to reject the null confirm stationary time series.

4.3 Autocorrelation

The next step is to define whether it exists autocorrelation or not in the time series. A stationary time series will have a constant variance in the interval, known as no-autocorrelation. In case of a shock, the prices will move significant (due to the shock), but after short time the price will revert to its long-term mean value. In stationary time series, stochastic shocks only have a temporary effect on the price.

When a time series is non-stationary (not mean reverting), the time series will have a non-constant variance and autocorrelation properties. Such properties imply that after a stochastic shock in the price, the price will move from this new level – known as autocorrelation.

If autocorrelation is present, there are some dependency in the time series. The use of lags makes it possible to integrate this dependency and to build a good model that explain the time series. When the residuals in our well specified models show no autocorrelation, it confirms that the models we build managed to integrate the dependency that exist in the dataset. In other words, the residuals in the models are roughly white noise. The Ljung Box test (Q) is applied (Ljung & Box, 1978) to test the null hypothesis of no autocorrelation. A rejection of the null hypothesis conclude that the variables are not autocorrelated.

4.4 Independence

The BDS test (Brock, et al., 1996) is another test for goodness of fit. The test detects nonlinear structures and serial dependence in the estimated residuals from the specified model. The null hypothesis is that residuals are independent and identically distributed (iid), in other words there are no repeated patterns in the residuals and the model is well specified. Rejection of the null hypothesis indicate presence of structures in the time series not included in the fitted model. Such structures can be nonlinear or serial dependency; consequently, the model is not optimal specified when the null is rejected.

4.5 ARMA Models

When doing statistical analysis of time series, autoregressive-moving-average (ARMA) models are useful for describing stochastic processes. The ARMA-model consist of two polynomials:

1) **The autoregression (AR):** Involves regressing the variable on its own lagged (past) values, in other words it seeks to explain the momentum and mean reversion effects in the time series. Letting u_t be a white noise term, an autoregressive model of p orders can be expressed with sigma notations as

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i \, y_{t-i} + u_t$$

   This shows that in an AR process the current value of y_t depends on previous periods value of y_t plus an error term.

2) **The moving average (MA)**: This part seeks to model the error term as a linear combination of error terms occurring simultaneously and at various times in the past. In other words, it tries to explain the effect of a shock observed. Letting u_t $(t = 1, 2, 3 \ldots n)$ be a white noise process with $\mathrm{E}(u_t) = 0$ and $\mathrm{var}(u_t = \sigma^2)$. Then an qth order moving average model, denoted MA(*q)* can be expressed by sigma notations as:

$$y_t = \mu + \sum_{i=1}^{q} \theta_i \, u_{t-i} + u_t$$

This shows that a MA process is a linear combination of white noise processes, making that y_t depends on both present and previous values of a white noise disturbance term (Brooks, 2008).

By merging the AR(*p)* and the MA(*q)* models, an ARMA(*p*,*q)* model with of *p* orders of the AR part, and *q* orders of the MA part is obtained. The ARMA model states that today's value of the time series y_t depends linearly on its own previously values and the combination of today's and previously values of a white noise error term.

4.6 A step into non-linearity land

In most qualitative methods and econometrics classes taught in undergraduate and graduate level the focus is at linear models. In a linear model, there is one parameter that is multiplied by each variable in the model (Brooks, 2008). Many non-linear models can be made linear by transforming the data (using e.g. logarithms).

However, not all relationship in finance are necessary linear. Campbell *et al.* (1997) highlighted that options payoff non-linear in some input variables, and that investors accept the trade-off between risk and returns non-linear. These points motivate for non-linear models, in addition financial data has some common features that require non-linear models (Brooks, 2008):

- **Leptokurtosis***:* The observed returns from financial markets tend to not fit well with the normal distribution. Often the return distribution exhibit properties of additional peakness at the mean and fatter tails compared to the normal distribution, making the kurtosis higher than three.

- **Volatility clustering**: In financial markets the volatility tends to occur in clusters. One can expect to see large returns are followed by large returns, and small returns are followed by small returns – of both signs (Benoit, 1963) (Fama, 1965). A possible explanation behind this dynamic is the information flow to the market, which drive price changes, tend to appear in clusters rather than being evenly spread out in time.

  When testing for volatility clustering, there are two main test-statistics to use, the Ljung-Box test statistics for squared returns (Q^2) and the ARCH statistics. In case of significant statistics, there are some autoregressive conditional heteroskedasticity in the data implying that the variance of the error term today is a function of previous error terms – namely volatility clustering. When volatility clustering is present one says that the volatility exhibits persistence, Autocorrelation in residuals is a sign of misspecification of the model, whereas autocorrelation in volatility is a sign of data dependency which makes a foundation for volatility forecasting.

- **Asymmetry***:* the phenomena in financial markets known from the prospect theory (Tversky & Kahneman, 1979) (Barberis, et al., 2001) where volatility tend to rise more after a large price drop than after a price jump of the same size. Negative price shocks have a larger impact on the volatility than positive price shocks, i.e. positive and negative shocks might not have the same impact on volatility. This asymmetry is called both a leverage effect and a risk premium effect (Engle & Patton, 2000). As the price of an asset decline, the companies become more leveraged since the relative value of their debt rises relative to that of their equity. As a result, it is expected that the stock becomes riskier and more volatile. This is referred to as the leverage effect (Ait-Sahalia, et al., 2011). The risk premium effect is that due to risk aversion, the demand for a stock will decrease when news of increased volatility occurs. The effects of asymmetry can be different in markets with different characteristics. Negative (positive) asymmetry effects can be indicated by negative (positive) skewness.

To test for non-linear relationships the Ramsey Regression Equation Specification Error (RESET) test (Ramsey, 1969) is applied. The test is used for linear regression models, as it tests if non-linear combinations of the explanatory variables help explaining the response variable. If the model rejects the null hypothesis then the model has some non-linear relationships not considered, i.e. the model is mis-specified.

4.7 The (G)ARCH techniques

In financial time series, volatility clustering is a well-established phenomenon, implying that volatility one day tend to correlate positively with the volatility the day after. To model time series with volatility clustering, ARCH and GARCH models are widespread. ARCH/GARCH models can be shown to be an ARMA model for the conditional variance function (Brooks, 2008)).

The ARCH (Autoregressive Conditional Heteroscedastic) (Enger, 1982) model uses earlier observations to estimate the variance one period ahead. The model has a lag configuration where the squared residual (ε_{t-j}^2) of the last observation is used to find volatility (σ_t^2) the next day.

ARCH (p):

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_t \, \varepsilon_{t-j}^2$$

An expansion of the ARCH framework is the GARCH (Generalized Autoregressive Conditional Heteroscedastic) model (Bollerslev, 1986), where both the squared residual (ε_{t-j}^2) of the last observation and the last period forecast is included (σ_{t-j}^2), when finding next period volatility (σ_t^2).

GARCH (p,q):

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_t \, \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_t \, \sigma_{t-j}^2$$

Where an ARCH model only takes the squared residual from last period when modelling the next period’s variance, the GARCH model include the last period squared residual <u>and</u> the last period forecast of the volatility. The effect of this expansion makes the GARCH model handle the persistence in a shock better than an

ARCH model. A GARCH(*p,q)* model consist of *p* orders of last periods residual observations and *q* orders of last period forecast observations.

GARCH-models is useful for forecasting volatility (Brooks, 2008) as it describes the changes in the conditional variance of the error term u_t. It can be proven that

$$var(y_t|y_{t-1}, y_{t-2}, \dots) = var(u_t|u_{t-1}, u_{t-2}, \dots)$$

This shows the conditional variance of y_t, according to its prior values, is the same as the conditional variance of u_t according to its prior values. This relationship its quite useful, by modelling σ_t^2 we will have models and forecast for the variance of y_t as well. When doing a regression where the dependent variable y_t is the return of an asset, a forecast of σ_t^2 will in fact be a forecast of future variance of y_t (Brooks, 2008). Forecasting volatility is usefull in pricing of financial options as volatility is part of the pricing model.

4.8 SNP Model fitting.

When selecting the model, including to many parameters will cause overfitting of the model. The Bayesian Information Criterion (BIC) (Schwarz, 1978) indicate the best fitted model given the data set when using the maximum likelihood function $f(y|\hat{\theta}k)$. When the number of parameters increases, the penalty term $(k \ln n)$ increases. As models with lower BIC value is preferred over higher BIC values (Schwarz, 1978), the BIC criterion reduces the risk of overfitting the model.

The BIC is specified as:

$$BIC = -2 \ln f(y|\hat{\theta}k) + k \ln\ n$$

Here y is the observed data set, θk is the parameter value that maximizes the likelihood function, *k* is number of parameters used in the estimation, and *n* is total number of observations.

4.9 The semi-nonparametric method for nonparametric time series analysis

When working with stationary and multivariate time series, the one-step-ahead conditional density represents the process of the time series. The conditional density captures a number of properties from the time series, like conditional heteroskedasticity, non-normality, time irreversibility and other forms of nonlinearities found in financial data (Gallant & Tauchen, 1990 (Dec 2017)).

The SNP method is parametric, however it includes properties from nonparametric models, making it referred to as semi-nonparametric. The semi-nonparametric method is an extension of Hermite functions, used for estimating the conditional density in time series analysis. This expansion makes the SNP model a method that nets the Gaussian VAR model, the semiparametric VAR model, the Gaussian ARCH model, the semiparametric ARCH model, the Gaussian GARCH model and the semiparametric GARCH model. Fitting of the SNP model is solved by conventional maximum likelihood combined with a model selection strategy to set the correct order of expansion.

The SNP model is written as a C++ program which include features for prediction, residual analysis, plotting, and simulation used for analysis and interpretation. Predicted values and residuals, are useful for diagnostic analysis and measurements of fit. Density plots are useful for visualising asymmetries and heavy tails. Simulations, like Monte Carlo analysis is used for bootstrapping. Reprojection, a way of Kalman filtering, is useful for forecasting the volatility process of a continuous-time stochastic volatility model (Gallant & Tauchen, 1990 (Dec 2017)). The SNP program thus enabling efficient model specification and shock simulation relevant when analysing the time-series.

Letting z refer to an M-vector, the Hermite density has the form $h(z)\alpha[P(z)]^2\emptyset(z)$, where $P(z)$ is a multivariate polynomial of degree K_z and $\emptyset(z)$ is the density function of the multivariate Gaussian distribution with a mean of zero. The conditional density, which is given by the entire past, depends only on the L lags from the past. The tuning parameter K_z is used to control to what degree the model separates from normality – the degree of polynomials in z, and the K_x is used to control to what degree these separations vary during the history of the process (Gallant & Tauchen, 1990 (Dec 2017)).

The SNP method include a set of distinct lag descriptions, where total number of lags is denoted as L. The model includes the parameters I_z and I_x, but those has no effect in univariate timeseries (Gallant & Tauchen, 1990 (Dec 2017)). The following notation is used for lags:

L_u: Number of lags in VAR

L_g: Number of lags in GARCH

L_r: Number of lags in ARCH

L_p: Total number of lags in the x part of the polynomial $p(z, x)$

L_v: Lags in the leverage effect in GARCH

L_w: Lags in additive level effect

If one or several of the lag operators above is set to zero, the model will give strong restrictions to the process of y_t, as given below:

Table 1: Restrictions when choosing lag operators (Gallant & Tauchen, 1990 (Dec 2017))

Restrictions when choosing lag operators						
Parameter setting						Classification of y_t,
$L_u = 0$	$L_g = 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z = 0$	$K_x = 0$	Iid Gaussian
$L_u > 0$	$L_g = 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z = 0$	$K_x = 0$	Gaussian VAR
$L_u > 0$	$L_g = 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z > 0$	$K_x = 0$	Semi-parametric VAR
$L_u \geq 0$	$L_g = 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z = 0$	$K_x = 0$	Gaussian ARH
$L_u \geq 0$	$L_g = 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z > 0$	$K_x = 0$	Semiparametric ARCH
$L_u \geq 0$	$L_g > 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z = 0$	$K_x = 0$	Gaussian GARCH
$L_u \geq 0$	$L_g > 0$	$L_r \geq 0$	$L_p \geq 0$	$K_z > 0$	$K_x = 0$	Semi-parametric GARCH
$L_u \geq 0$	$L_g \geq 0$	$L_r \geq 0$	$L_p > 0$	$K_z > 0$	$K_x > 0$	Nonlinear nonparametric

The process of building an SNP model start by adding one lag in the VAR model, later an extension with two lags are included. Further, the best fitted ARCH model is set, followed by fitting the GARCH model. The last steps are to control if the asymmetry effect and additive level are significant (Gallant & Tauchen, 1990 (Dec 2017)).

4.10 Stochastic volatility

In the previous section we described the procedure for fitting a statistical model to explain our time series. The next step is to build and implement a scientific stochastic volatility (SV) model for the same time series, using a Bayesian Markov Chain Monte Carlo Simulation method for estimation and assessment of an SV model, as proposed by Chernozhukov and Hong (2003). Under time-varying volatility in financial markets, the Stochastic Volatility is the main way of modelling such a property. This part seeks to introduce how to build scientific model where the volatility has its own stochastic process, which well-specify volatility in electricity forward contracts.

The stochastic volatility (SV) and (G)ARCH models have several parallels and illuminate many of the same facts, however the main advantages of a direct volatility modelling are convenience and a more natural presentation. The SV model has its own stochastic process, without connections to the implied one-step-ahead distribution draw from an arbitrary yet convenient time interval used in the (G)ARCH estimation.

We start by looking at an SV diffusion process by Andersen *et al. (2002)* for an observed stock price S_t is provided by $\frac{dS_t}{S_t} = (\mu + cV_t)dt + \sqrt{V_t}dW_t$, where the unobserved volatility process V_t is either log linear or squared root. Andersen *et al* (2002) estimated both versions of the SV model with S&P500 data, however both versions where sharply rejected. Later Chernov *et al.* (2003) added a jump component to the basic SV model which improved the model fit radically. This refinement gave characteristics of tick non-normal tails and persistent time-varying volatility (volatility clustering). A two-factor volatility model outperform one-factor models, as one of the volatility factors (V_{1t}) are extremely persistent to capture volatility clustering, and the other (V_{2t}) is strongly mean-reverting to fatten tails. Another extension is to enable correlation between the mean (w_{1t}) and the two SV factors (w_{2t}, w_{3t}). This extension is crucial to enable the asymmetry effect (the correlation between return innovations and volatility innovations). A logarithmic SV model with two stochastic volatility factors for the Nordic electricity forward contracts is specified as:

$$
\begin{aligned}
y_t &= a_0 + a_1(y_{t-1} - a_0) + \exp(V_{1t} + V_{2t}) \cdot u_{1t} \\
V_{1t} &= b_0 + b_1\left(V_{1,t-1} - b_0\right) + u_{2t} \\
V_{2t} &= c_0 + c_1\left(V_{2,t-1} - c_0\right) + u_{3t} \\
u_{1t} &= W_{1t} \\
u_{2t} &= s_1(r_1 \cdot W_{1t} + \sqrt{1 - r_1^2} \cdot W_{2t} \\
u_{3t} &= s_2(r_2 \cdot W_{1t} + \frac{r_3 - (r_2 \cdot r_1)}{\sqrt{1 - r_1^2}} \cdot W_{2t} + \sqrt{1 - r_1^2 - \left(\frac{r_3 - (r_2 \cdot r_1)}{\sqrt{1 - r_1^2}}\right)^2} \cdot W_{3t}
\end{aligned}
$$

where $W_{it}, i = 1, 2, 3$ are basic Brownian motions (random variables). The parameter vector is $\rho = (a_0, a_1, b_0, b_1, s_1, s_0, c_0, c_1, r_1, r_2, r_3)$, where the r's are correlation coefficients (Gallant & Tauchen, 2016).

4.11 Motivation for Stochastic volatility models

Applying an SV model for a financial asset is motivated from the assumption in the SV model that the volatility at day t is partly given by unpredicted events the same day. As the amount of news items is continuously changing from one time to another, the volatility will continuously change. With a stochastic stream of information to the market, a stochastic volatility model seems appropriate. From the SV model a volatility forecast can be estimated. The volatility forecast cannot be assorted with the implied volatility, as the latter is the market actors excepted volatility calculated via the Black and Scholes model. In this paper the volatility forecast is referred to as the *re-projected conditional volatility*, as we i) estimate an GARCH model for the volatility, ii) simulate an SV model from the findings of the GARCH model, and iii) re-project the volatility from the SV model back to the GARCH model. The *re-projected volatility* at time t is a forecast estimated from the data series up to time $t-1$, using only information available at time $t-1$. This estimation method let us obtain no look-ahead bias in the estimation of the predicted volatility. The last available volatility forecast can be plotted directly into the Black & Scholes model to get more precise option prices for the contracts. The calculated option price can be compared with the option prices found in the market, making a starting point for innovative risk and portfolio management strategies

4.12 SV computational methodology

Efficient Method of Moments (EMM) is the computational method for statistical analysis of an SV model, projected by Gallant & Tauchen (2016), and Gallant & McCulloch (2011) and designed as a flexible C++ program. By applying the Metropolis-Hastings (M-H) algorithm, parallel computing and Bayesian Markov Chain Monto Carlo (MCMC) simulation the EMM calibrate the volatility innovations in an SV model against the return innovations from a statistical model. To calibrate the model, we use the statistical model from the SNP framework, i.e., the (G)ARCH models. The EMM matches the SV model by using a score generator from the statistical (G)ARCH model. EMM is a simulation-based moment matching procedure, where the moments matched are the scores from the statistical model – the score generator. If the score generator approximate the distribution of the data well, the estimated parameters in the SV-model are also efficient (Gallant, et al., 1997). The output of the EMM method is a volatility simulation for forecasting, which with a volatility filter can be re-projected back to the original data. This can be implemented in the (G)ARCH model, giving us a new scientific model, which uses observations in prices today to predict something about where volatility goes tomorrow. The EMM implementation can be summed up the following way:

The SNP model, the Metropolis-Hasting algorithm, together with parallel computing is applied to estimate the stochastic volatility model and the parameter estimates: $\theta = (a_0, a_1, b_0, b_1, c_0, c_1 s_1, s_2, r_1, r_2, r_3)$. The by-product is a 250 000 simulated realization of the vector. From the 250 000 vector simulations, a reduced form auxiliary SNP model is established with a likelihood function. The SNP gives a useful description of the one-step ahead conditional variance. At the end, moving backwards to understand the unobserved state vector from the observed process as implied by the model. The Nonlinear Kalman filter generate a new SNP model for $\hat{y}_t$ (the SV-model simulated data) and obtain $\hat{\sigma}_t^2$ of $\hat{y}_{t+1}$ given $\{\tilde{y}_\tau\}_{\tau=1}^{250\,000}$. The next is to run an Ordinary Least Squared regression of the volatility factors $\hat{V}_{i,t}, i = 1{,}2$ on $\hat{\sigma}_t^2$, $\hat{y}_t$ and $|\hat{y}_t|$. The last step is to evaluate the SV-model function on the observed data series $\{\tilde{y}_\tau\}_{\tau=1}^{t}$ which gives predictive values for $\tilde{V}_{i,t}, i = 1{,}2$ at the actual observed data points.

4.13 SV Model Fitting and Evaluation.

When evaluating the SV model fit, two aspects are evaluated. The first one is the individual posterior score from the simulations, and the second is to evaluate the normalized mean SNP score vector with associated standard deviations and quasi-t-ratios.

When running the 250 000 simulations, each simulation received its individual posterior score. The objective is to pick the simulation which receive the highest score, thus represent the lowest distance between the model and what we observed. The highest posterior score is found through an iteration process. The frequently changing factor score indicate that the simulations are searching for the optimal solution, a stationary simulation chain would indicate misconfiguration of the model. The highest posterior score is put into a Chi squared test (Pearson, 1900). Under correct specification of the structural model, the normalized value of the optimized EMM objective function follows the asymptotically $\chi 2$ distribution with the degrees of freedom equal to the length of θ minus the length of p minus one (the last account for the SNP normalization rule that A(1,1)=1) (Gallant & Tauchen, 2016) . The purpose is to test whether there is a statistically significant difference between the expected frequencies and the observed series. Hence, the null hypothesis is no difference between the distribution and the alternative is a difference between the distributions. If the test is not significant, the null hypothesis is not rejected, i.e. there are no statistical difference between the expected and observed series. The SV model is therefore an acceptable approximation of the score statistics (SNP model).

The EMM report the normalized mean SNP score vector (parameter), and the associated unadjusted standard deviations are the squared roots of the diagonal elements. The quasi-t-ratios are the normalized mean score divided by the unadjusted standard deviations. Because the quasi-t-ratios only take the arbitrariness in $\hat{\theta}_n$, while treating $\hat{\rho}_n$ as if it were a fixed value of p_0, the quasi-t-ratios are not asymptotically N(0,1). Despite that, the quasi-t-ratios are helpful to evaluate the model fit and the underlying causes of a statistically significant chi-squared statistics. A quasi-t-statistic over 2 indicate failure to fit the corresponding score (parameter) (Gallant & Tauchen, 2016). Hence, t-statistics under 2 implies that the SV model manage to match the respective parameter from the SNP model.

5. Empirical Results

This chapter will present empirical results. Starting with a description of the two time-series. Next is to present the findings from the SNP modelling following by the findings from the SV modelling.

5.1 Description of the time series

In this section an evaluation of the descriptive properties of the two time-series will be performed individually.

5.1.1 Front Year Futures Contracts (YEAR)

The dataset consists of daily returns of the front year futures contracts spanning from 3. January 2000 until 3. January 2020. As contracts are only traded during Norwegian bank days, there are 5009 observations. Characteristics of the YEAR dataset is reported in *table 2*.

Table 2: Statistics for Front Year Futures Contracts

Statistics for Front Year Future Contracts (YEAR)								
Mean / Mode	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Jarque-Bera test	Q(12)	$Q^2(12)$
0.0260	0.0000	16.354	6.8945	0.1769	8.8524	9899.7	58.203	2727.1
0.0000	1.5916	-12.262	-0.0509	0.0528	0.0120	{0.0000}	{0.0000}	{0.0000}
BDS-Z-statistic (e = 1) m=2	m=3	m=4	m=5	KPSS (i+trnd)	PP (i+trnd)	Augmented DF-test	ARCH (12)	VaR 2.5% CVaR2.5%
15.658	19.830	23.658	27.790	0.1235	-66.831	-51.222	1072.9	-0.0327
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.11}	{0.0000}	{0.0000}	{0.0000}	-0.0466

The numbers in braces are P-values for statistical significance
Rejection rate are 5%, implying P-values less than 5% reject the null hypothesis.

The mean is positive, and the standard deviation is 1.59. It is reported a maximum (minimum) value of 16.35 (-12.26). The data reports a small skewness (0.05), indicating a small asymmetry effect. The kurtosis is 9.89 – where values over 3 is named leptokurtosis and is characterised by heavy tails and peakness above the mean. The Jarque-Bera test clearly conclude by rejected the null-hypothesis of normality, the time series is non-normal distributed. The KPSS accept the null hypothesis of stationarity and the Augmented-Dickey-Fuller (ADF) and Phillips-Perron (PP) reject the null hypothesis of non-stationarity. These tests confirm that transforming the prices to returns made the time-series stationary. The BDS Z-statistics reject the null hypothesis of independence in the data, clearly there are some dependency and structure in the data, and the Q and Q^2 both rejected the null hypothesis of no autocorrelation and volatility clustering. The ARCH-test rejected the null hypothesis of constant variance – ARCH effects. By that we confirm presence of data dependence in the time series, i.e. volatility clustering. *Figure 15* shows the price and the returns (price change), the former is appearing non-stationary and the latter is clearly stationary and show tendency to volatility clustering during several periods of the time series. *Figure 16* graph the Kernel density distribution of the returns, indicating some peakness above the mean.

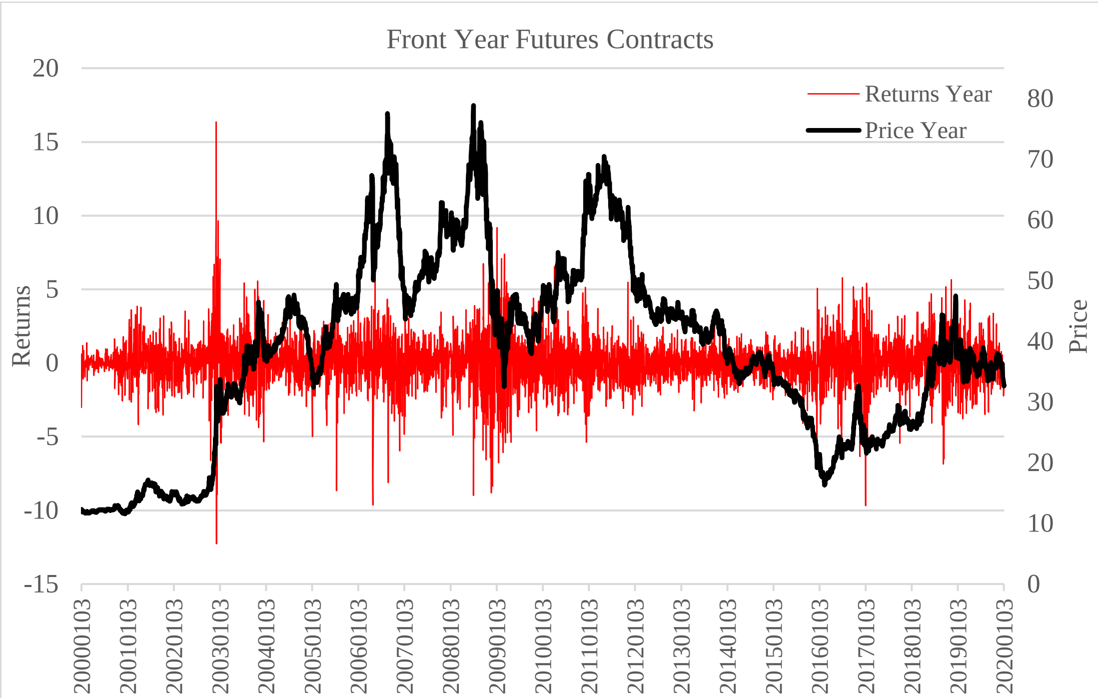

Figure 15: Front Year Futures Contract returns and prices 03.01.2000 – 03.01.2020

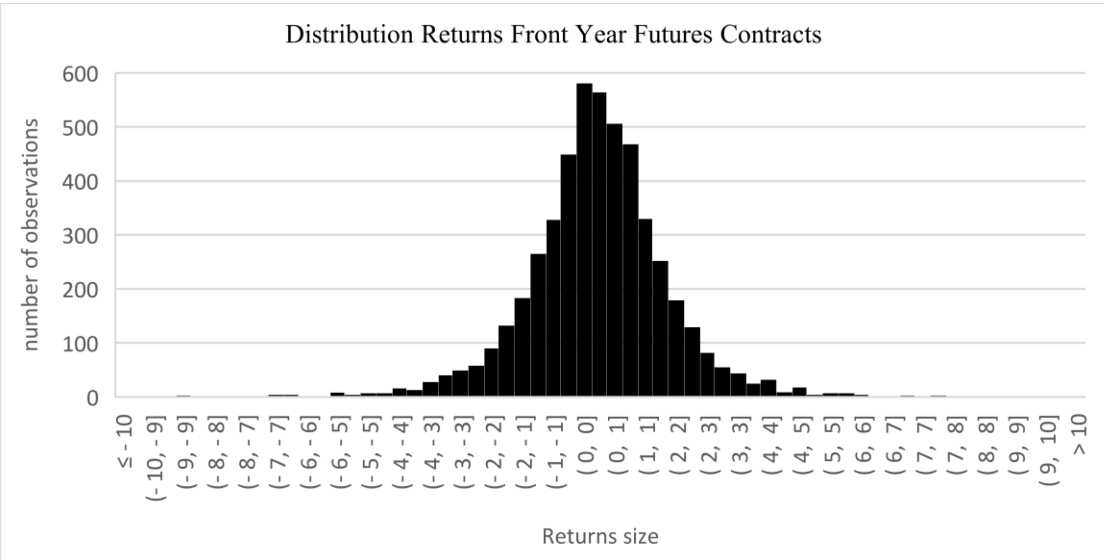

Figure 16: Distribution Returns Front Year Futures Contracts from 2000 to 03.01.2020

5.1.2 Front Quarter Futures Contracts (QUARTER)

The dataset consists of daily returns of the front quarter futures contracts spanning from 3. January 2000 until 3. January 2020. As contracts are only traded during Norwegian bank days there are 5009 observations. Characteristics of the QUARTER dataset is reported in *table 3*.

Table 3: Statistics for Front Quarter Future Contracts

Statistics for Front Quarter Future Contracts (QUARTER)								
Mean / Mode	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Jarque-Bera test	Serial dependence Q(12)	$Q^2(12)$
-0.0047	0.0000	23.028	5.6643	0.16290	5.5415	6680.2	42.929	1673.3
0.0000	2.4480	-15.649	0.0167	-0.00188	{0.0626}	{0.0000}	0.0000	0.0000
BDS-*Z*-statistic (*e* = 1)				KPSS	PP	Augmented	ARCH	VaR 2.5%
m=2	m=3	m=4	m=5	(i+trnd)	(i+trnd)	DF-test	(12)	CVaR 2.5%
8.5040	10.243	11.466	12.883	0.1526	-66.412	-66.412	667.64	-0.0496
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.07}	{0.0000}	{0.0000}	{0.0000}	-0.0716

The numbers in braces are P-values for statistical significance. Rejection rate are 5%, implying P-values less than 0.05 reject the null hypothesis.

The mean is negative and the standard deviation 2.45. It is reported a maximum (minimum) value of 23.028 (-15.649). The data reports a skewness closed to zero (0.02), indicating a marginal positive asymmetry effect. The kurtosis is 5.66, where values over three is named leptokurtosis and is characterised with heavy tails and peakness above the mean. The Jarque-Bera test clearly conclude by rejecting the null-hypothesis of normality, the time series is non-normal distributed. The KPSS accept the null hypothesis of stationarity and the Augmented-Dickey-Fuller (ADF) and Phillips-Perron (PP) reject the null hypothesis of non-stationarity. These tests confirm that transforming the prices to returns made the time-series stationary. The BDS Z-statistics reject the null hypothesis of independence in the data, clearly there are some dependency and structure in the data, and the Q and Q^2 both rejected the null hypothesis of no autocorrelation and volatility clustering. The ARCH-test rejected the null hypothesis of constant variance – ARCH effects. By that we conclude that there exists some data dependence in the time series, i.e. volatility clustering. *Figure 17* shows the prices and the return (price change) series, the former is seeming non-stationary and the latter is clearly stationary and show tendency to volatility clustering during several periods of the time series. *Figure 18* shows the Kernel density distribution of the returns, indicating some heavy tails and peakness above the mean.

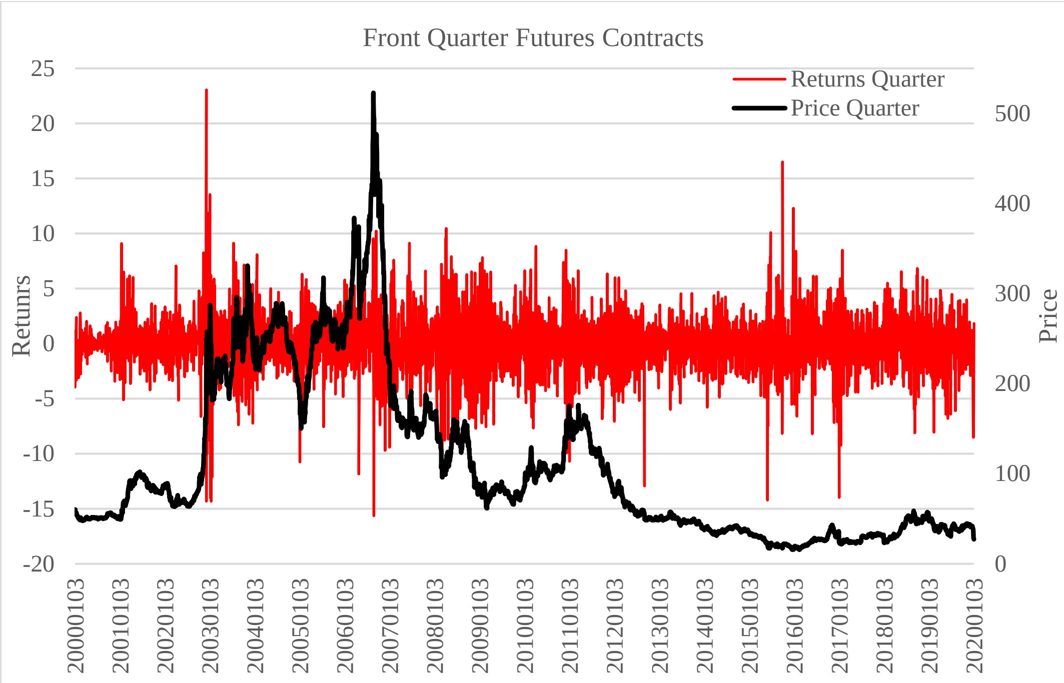

Figure 17: Front Quarter Futures Contracts returns and prices 03.01.2000 – 03.01.2020

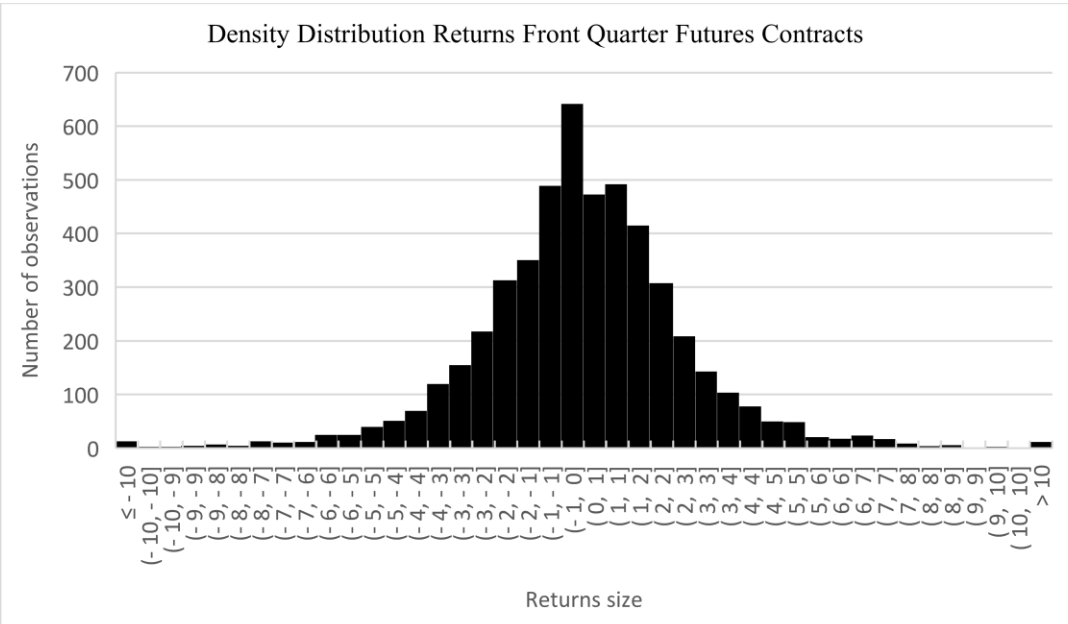

Figure 18: Distributions Returns Front Quarter Futures Contracts from 2000 to 2020

5.2 SNP Model Evaluation

In this section the SNP estimation and model fitting will be explain, followed by an evaluation of the two SNP models individually.

5.2.1 SNP estimation and model fitting

This section will present the specification of the best fitted model to our time series. For model selection and fitting the BIC-criterion is used. The first SNP run is done with polynomial of degree 4. We run the SNP again with polynomial degree of 6 and 8. The best fitted model is the SNP model with the lowest BIC. After running the SNP model with several polynomial degrees, the best fitted model has det configuration of

$$\left(L_u, L_g, L_r, L_p, K_z, I_z, K_x, I_x\right) = (11116000)$$

This model has 1 lag in Var (L_u), GARCH (L_g), ARCH (L_r) and x part of the polynomial $p(z, x)$ (L_p). The optimal degree of polynomials in z (K_z) is 6. The SNP model tends to set K_z unreasonably high in some applications, and models for financial markets with a $K_z > 6$ is recommended to be avoided (Gallant & Tauchen, 1990 (Dec 2017)). This supports the result of $K_z = 6$ as correct. I_z, K_x, I_x are all set to zero. In addition, we use 1 lag in both the leverage effect of GARCH (L_v) and the additive level effect (L_w). This model, a GARCH(1,1)
(L_r,L_g) with one lags in VAR(L_v) and six Hermite polynomials (K_z) yields the lowest BIC for both time series, reported in *table 4*.

Table 4: Optimal SNP Model Specifications

Time series	L_u	L_g	L_r	L_p	K_z	I_z	K_x	I_x	L_v	L_w	BIC
QUARTER	1	1	1	1	6	0	0	0	1	1	1.265
YEAR	1	1	1	1	6	0	0	0	1	1	1.241

The SNP model report a semi-nonparametric GARCH(1,1) model with one lag in VAR as the best fitted model for both the QUARTER and YEAR series, BIC scores are 1.265 and 1.241.

5.2.2 SNP model evaluation YEAR

As stated earlier, we choose to use a semi-nonparametric (SNP) GARCH(1,1) model. The residual statistics for the optimal SNP GARCH model are reported in *table 5*. When an optimal model is specified, the residual should be normally distributed N[0,1] and non-significant. When these properties are fulfilled the model capture the structure in the data efficiently, making data left in the residual roughly white noise.

Table 5: Residual Statistics for Front Year Future Contracts

Residual Statistics for Front Year Future contracts								
Mean / Mode	Median / Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Jarque-Bera	Serial dependence Q(12)	$Q^2(12)$
0.0003	0.0003	4.8669	4.4725	-0.0015	0.0249	460.73	20.927	10.831
	0.9999	-6.1134	-0.1132	0.0064	{0.9876}	{0.0000}	{0.0510}	{0.5430}
BDS-statistic (*e*=1) m=2	m=3	m=4	m=5	ARCH (12)	RESET (12;6)			
-0.7262	-1.6060	-1.8307	-1.4485	10.977	22.548			
0.4677	0.1083	0.0672	0.1475	{0.5308}	{0.001}			

The numbers in braces are P-values for statistical significance
Rejection rate are 5%, implying P-values less than 5% reject the null hypothesis

Table 5 report that in the residual statistics the mean is approximately zero (0.0003) and the standard deviation is close to one (0.99), referring to the standardized normal distribution $N(0, 1)$. Further, the Kurtosis has fallen from 6.89 to 4.47 and the Jarque-Bera statistic confirms that the residual statistics are closer to a normal distribution with a reduction from 9899.7 to 460.7. The Q^2 and the ARCH-test has p-values of 0.54 and 0.53, both fail to reject the null hypothesis about no volatility clustering in the residuals (no ARCH-effects). The Q and BDS-test fail to reject the null hypothesis for all lags, i.e. there are no dependency in the residuals. For the RESET test we reject the null hypothesis regarding linear relationship, i.e. the fitted model's residuals show some non-linear relationships. There seem to be some parameters that is not completely stable, and that the SNP GARCH model has minor structures in the data not considered. Specifying the model perfectly with normally distributed residuals is demanding, an implication further studies is the obtain a perfect specified model.

Table 6 report the statistical SNP Model parameters for YEAR. The first 6 parameters are the 6 polynomials used. Parameter 8 are a non-significant constant. Parameter 9 are the parameter for the first lag. Parameter 10 is the constant for the variance term, 11 is for the ARCH term which capture volatility clustering, parameter 12 capture that historical volatility has a strong impact at today's observations. 13 is for the asymmetry effect, where a positive value indicates a positive effect, i.e. the volatility shows stronger responses to positive than negative shocks. Parameter 14 is used to configurate how strong the volatility influences the parameters. Parameter 9 to 14 are all statistically significant, where parameter 12 stands out with a very strong value. The eigenvalue of the variance function is 0.9996.

Table 6: Statistical SNP Model parameters for YEAR

Index	theta	std error	t-statistic	descriptor
1	0.01409	0.01718	0.81975	a0[1] 1
2	-0.06631	0.23226	-0.28551	a0[2] 2
3	-0.00856	0.01400	-0.61145	a0[3] 3
4	0.08081	0.04560	1.77206	a0[4] 4
5	-0.00459	0.00994	-0.46195	a0[5] 5
6	-0.00371	0.09018	-0.04112	a0[6] 6
7	1.00000	0.00000	0.00000	A(1,1) 0 0
8	-0.01835	0.02333	-0.78637	b0[1]
9	0.05122	0.01502	3.40906	B(1,1)
10	0.05264	0.02223	2.36805	R0[1]
11	0.27970	0.11582	2.41507	P(1,1) s
12	0.95990	0.00381	252.24270	Q(1,1) s
13	0.18746	0.08429	2.22391	V(1,1) s
14	0.21637	0.10938	1.97806	W(1,1) s

Largest eigenvalue of variance function P & Q companion matrix = 0.999639

Figure 19 graph the projected conditional volatility for the time series as an index. It seems like the volatility change randomly, but there seems to be periods with more volatility than others – i.e. volatility clustering. There are periods with more volatility; winter 2003, in 2006, from 2008 to 2010, in 2016 and 2018. Other periods have lower volatility like 2000, in 2004, in 2007, from 2012 to 2016 and 2017. The volatility seems to be mean reverting to a slightly increased long-term value around 25.

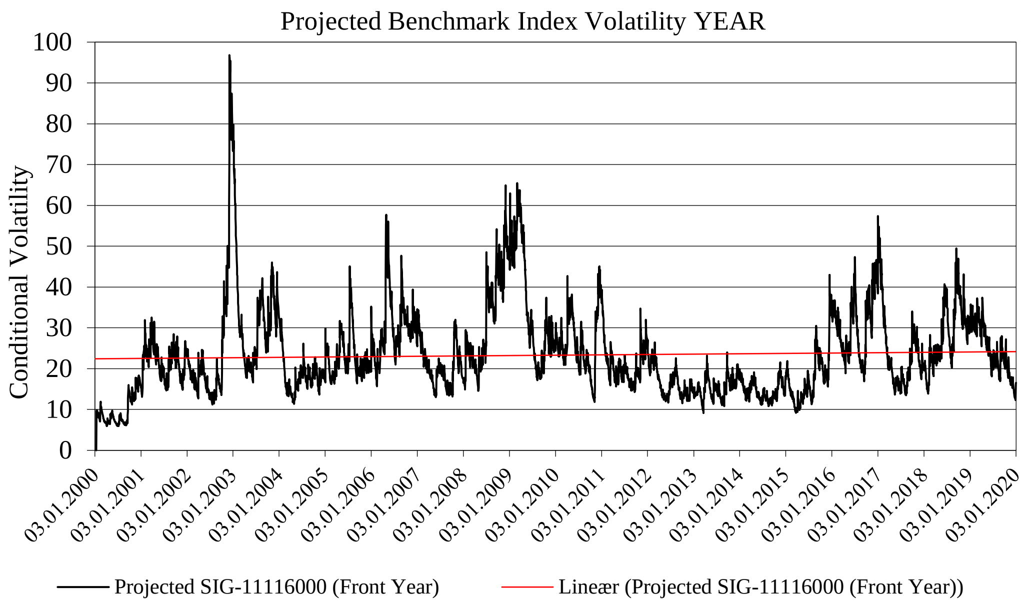

Figure 19: Projected Benchmark Index Volatility YEAR

Figure 20 graph the projected conditional volatility together with a moving average for m=4 and m=15 lags for the squared residuals of an AR(1) model for the returns. There seems to be a good fit between the projected volatility and the moving average processes for 4 and 15 lags.

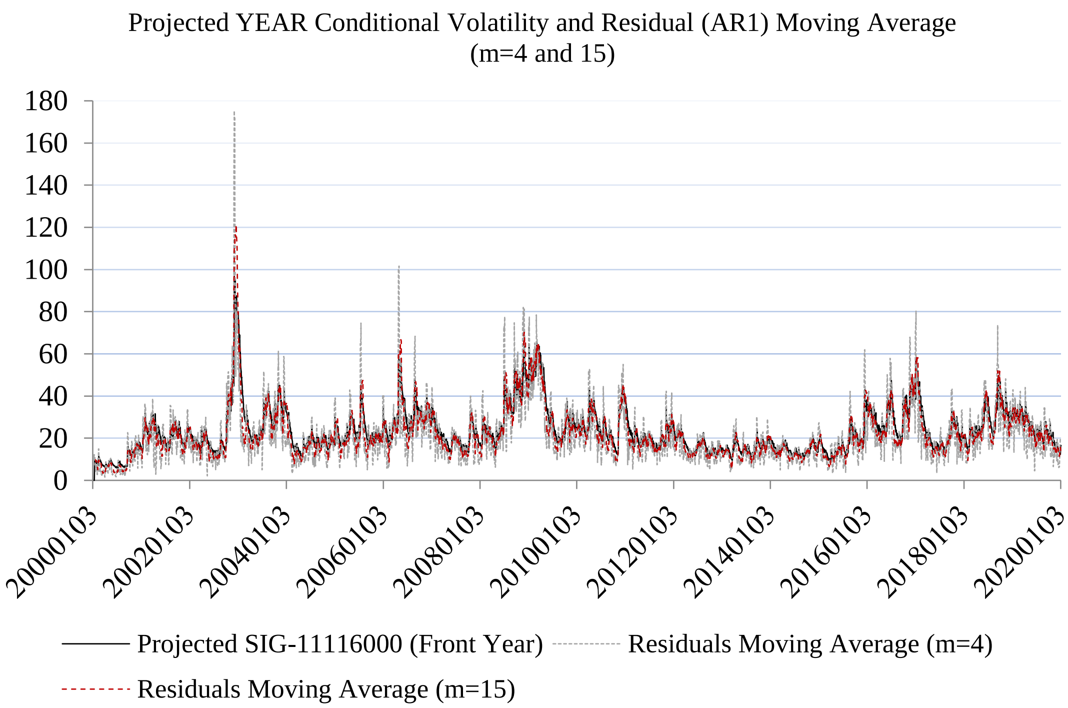

Figure 20: Projected YEAR Conditional Volatility and Residuals (AR1) Moving Average (m=4 and 15)

Figure 21 graph an index for the one-step-ahead densities for the volatility at the mean, conditional to the values for x_{t-1} (unconditional mean). In other words, given yesterday volatility were at the mean (0.0), where will todays volatility (conditional mean) be? The conditional densities are compared to a normal distribution. The figure shows the conditional densities for YEAR time-series have non-normal features; including peakness in the interval $-\frac{1}{2}$ and $+\frac{1}{2}$ standard deviation from the mean, a smaller distribution in the interval $\pm\frac{1}{2}$ to ± 2 standard deviations from the mean, and fatter tails from ± 2 standard deviations. Such properties are known as leptokurtosis and are frequently found when analysing financial data. This confirms correctness of using Hermite polynomials to move away from the normal distribution when describing the densities for the time-series.

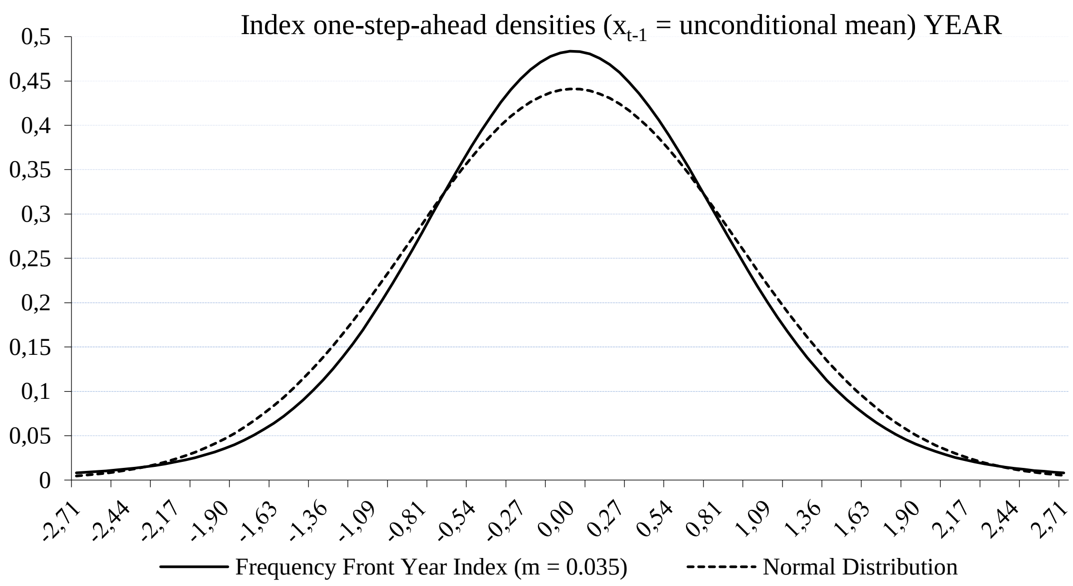

Figure 21: Index one-step-ahead densities (xt-1 = unconditional mean) YEAR

Figure 22 shows the one-step-ahead densities when adding a shock to the time series. The shock is added to yesterday's mean ($x_{t-1} = unconditional\ mean$), and the graph show the frequency distribution for the conditional mean today ($x_t = condtional\ mean$). Shocks are added ranging from -20% to $+20\%$ and the baseline profile showing the mean of the densities ($m = 0.035$). When evaluating the densities after different shocks to the baseline profile, we see that the densities after the shock is clearly wider. The widest densities are found for the biggest shocks

($\pm$20%), and the densities becomes smaller with smaller shocks. An interesting observation is that there are hard to see any clear differences in the width between the positive and the negative shocks. This is in contrasts with the findings in Egeland & Haug (2016) which found clearer difference in the width of positive and negative shocks, however the differences where biggest in the stock indexes, and smaller in the commodities (especially for Brent oil front month futures contrast). When having a closer look at the figure, the densities for the negative shocks tend to be more peaked than the positive shocks, indicating wider densities for the positive shocks. This indicate positive shocks imply a higher degree of uncertainty to the volatility than negative shocks.

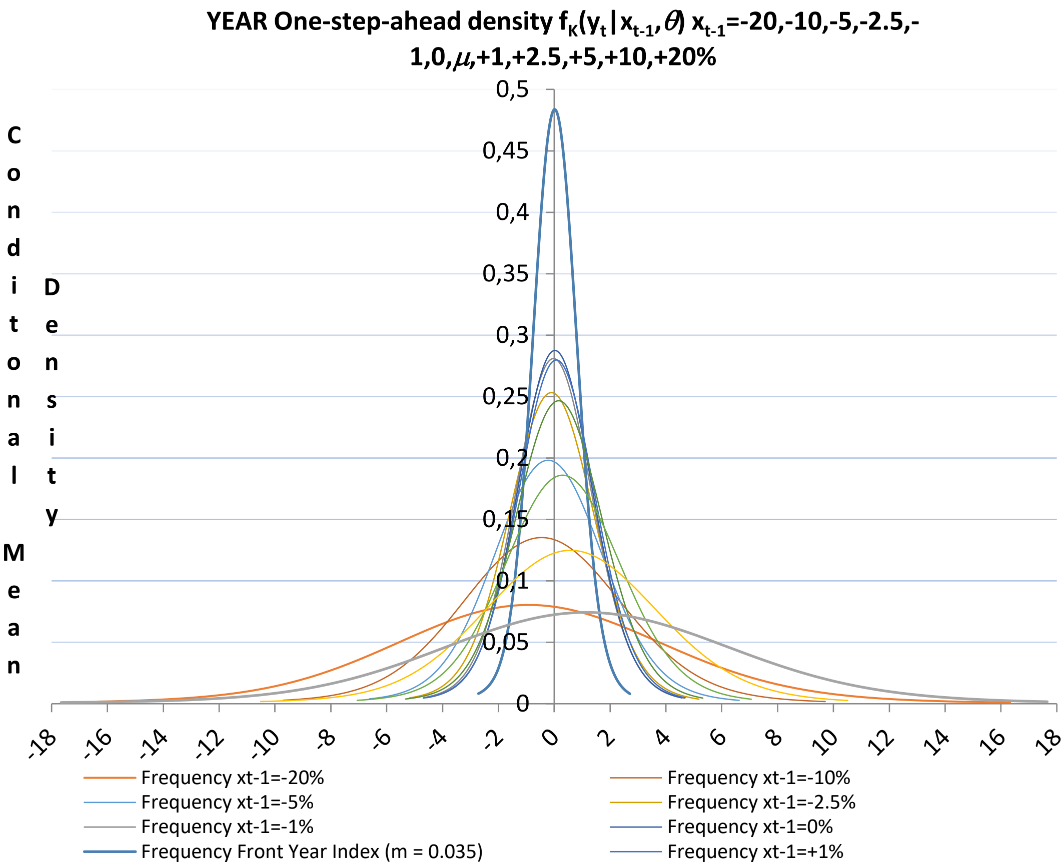

Figure 22: YEAR One-step-ahead density $f_K(y_t|x_{t-1},q)$ x_{t-1} = -20,-10,-5,-2.5,-1,0,m,+1,+2.5,+5,+10,+20%

Figure 23 visualize the connection between the one-step-ahead conditional variance and the percentage change in the unconditional mean. This connection is referred to as the *asymmetry effect*, some disciplines uses "*leverage effect*" and "*risk premium effect*". We can interpret the graph as showing how the conditional variance function reacts to a surprisingly shock to the system. The figure confirms what the previous figure indicated, i.e. that positive shocks tend to giver bigger effects than negative shocks to the volatility. However, this asymmetry effect is opposite and clearly weaker than the effects found in the financial markets for stocks (S&P500, DJIA+) and commodities (oil, carbon, salmon) in Egeland & Haug (2016).

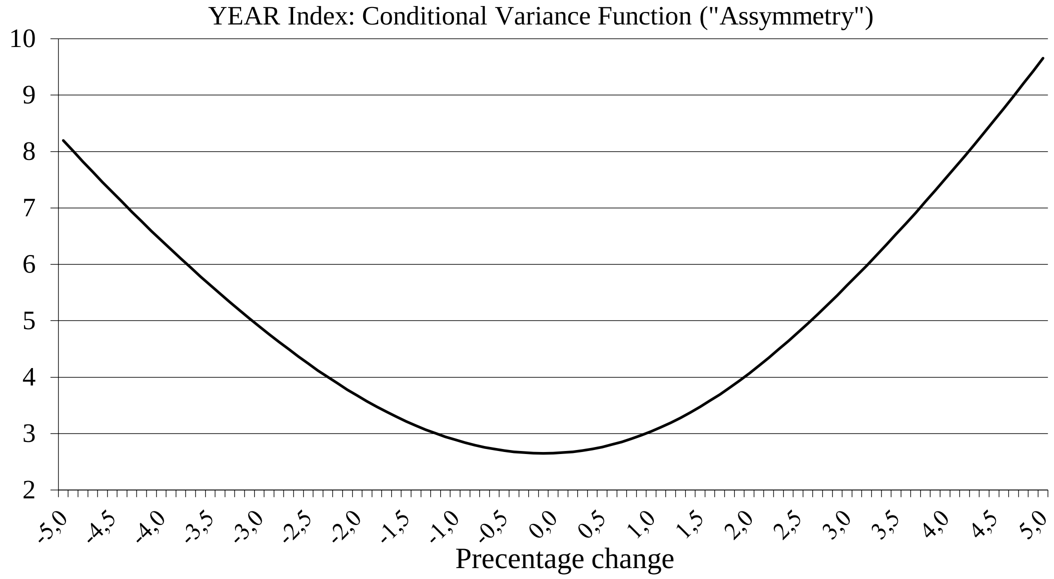

Figure 23: Front YEAR Index: Conditional Variance Function ("Asymmetry")

5.2.3 SNP model evaluation QUARTER

As stated earlier, we choose to use a semi-nonparametric (SNP) GARCH(1,1) model. The residual statistics for the optimal SNP GARCH model are reported in *table 7*. When an optimal model is specified, the residual should be normally distributed N[0,1] and non-significant. When these properties are fulfilled the model capture the structure in the data efficiently, making data left in the residual roughly white noise.

Table 7: Residual Statistics for Front Quarter Future Contracts

Residual Statistics for Front Quarter Future contracts								
Mean / Mode	Median / Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Jarque Bera test	Serial dependence Q(12)	$Q^2(12)$
-0.0024	-0.0121	5.1960	5.6334	0.0762	1.9034	1480.2	28.231	18.531
	1.0001	-7.5447	-0.2184	0.0289	{0.3861}	{0.0000}	{0.0050}	{0.1010}
BDS-statistic (*e*=1)				ARCH	RESET			
m=2	m=3	m=4	m=5	(12)	(12;6)			
-1.0864	-2.2393	-2.3173	-2.1462	19.2877	7.2235			
{0.2773}	{0.0251}	{0.0205}	{0.0319}	0.0818	{0.3007}			

The numbers in braces are P-values for statistical significance
Rejection rate are 5%, implying P-values less than 5% reject the null hypothesis

The table report that in the residual statistics the mean is approximately zero (0.0024) and the standard deviation is approximately one (1.0001), referring to the normal distribution $N(0,1)$. The Kurtosis has marginally declined from 5.66 to 5.63 and the Jarque-Bera statistic confirms that the residual statistics are closer to a normal distribution with a reduction from 6680.2 to 1480.2. The Q^2 and the ARCH-test has p-values of 0.10 and 0.08, both fails to reject the null hypothesis about no volatility clustering in the residuals (no ARCH-effects). The Q and BDS-test only fail to reject the null hypothesis for the first lag, there are no dependency in the time-series. Rest of the lags in the BDS test reject the null hypothesis and conclude that there is some dependency in lags 2 to 5. The RESET test fails to reject the null hypothesis regarding no non-linear relationships. There seem to be some parameters that is not completely stable, and that the SNP GARCH model has minor structures in the data not considered. Specifying the model perfectly with normally distributed residuals is demanding. An implication further studies would be to expand the GARCH(1,1) model to a GARCH(1,2) model. Including several lags could give significant values in the Q and BDS tests, giving normally distributed residuals and a well-specified model.

Table 8 give the statistical SNP Model parameters for QUARTER. The first 6 parameters are the 6 polynomials used. Parameter 8 are a non-significant constant. Parameter 9 is the parameter for the first lag. Parameter 10 is the constant for the variance term, 11 is for the ARCH term which capture volatility clustering, parameter 12 capture that historical volatility has a strong impact at today's observations. 13 is for the asymmetry effect, where a positive value indicates a positive effect, i.e. the volatility shows stronger responses to positive than negative shocks. Parameter 14 is used to configurate how strong the volatility influences the parameters. Parameter 9 to 12 are all statistically significant, where parameter 12 stands out. Parameter 13 and 14 is not significant. The eigenvalue of the variance function is 1.04769.

Table 8: Statistical SNP Model Parameters QUARTER

Statistical SNP Model Parameters QUARTER

Index	theta	std error	t-statistic	descriptor
1	0.0239	0.0130	1.8356	a0[1] 1
2	-0.1337	0.0149	-8.9801	a0[2] 2
3	-0.0048	0.0077	-0.6294	a0[3] 3
4	0.0549	0.0070	7.8664	a0[4] 4
5	0.0199	0.0076	2.6198	a0[5] 5
6	-0.0865	0.0080	-10.8497	a0[6] 6
7	1.0000	0.0000	0.0000	A(1,1) 0 0
8	-0.0314	0.0184	-1.7068	b0[1]
9	0.0541	0.0145	3.7344	B(1,1)
10	0.0900	0.0120	7.4902	R0[1]
11	0.3847	0.0177	21.6981	P(1,1) s
12	0.9485	0.0036	266.8388	Q(1,1) s
13	0.0008	282382.9000	0.0000	V(1,1) s
14	0.0000	4500.6163	0.0000	W(1,1) s

Largest eigenvalue of variance function P&Q companion matrix = 1.04769

Figure 24 show the projected conditional volatility for the time series as an index. It seems like the volatility change randomly, but there seems to be periods with more volatility than others – i.e. volatility clustering. There are periods with more volatility; winter 2003, winter 2004, in 2006, from 2008 to 2010, 2015-2016 and 2018. Other periods have lower volatility like 2000, summer 2004, from 2012 to 2016 and 2017. The volatility seems to be mean reverting to an increased long-term value around 38. An interesting finding is the volatility seems to be more volatile for the QUARTER series than YEAR – as there are more and bigger changes. The long-term mean reverting value for quarter seem to be higher as well.

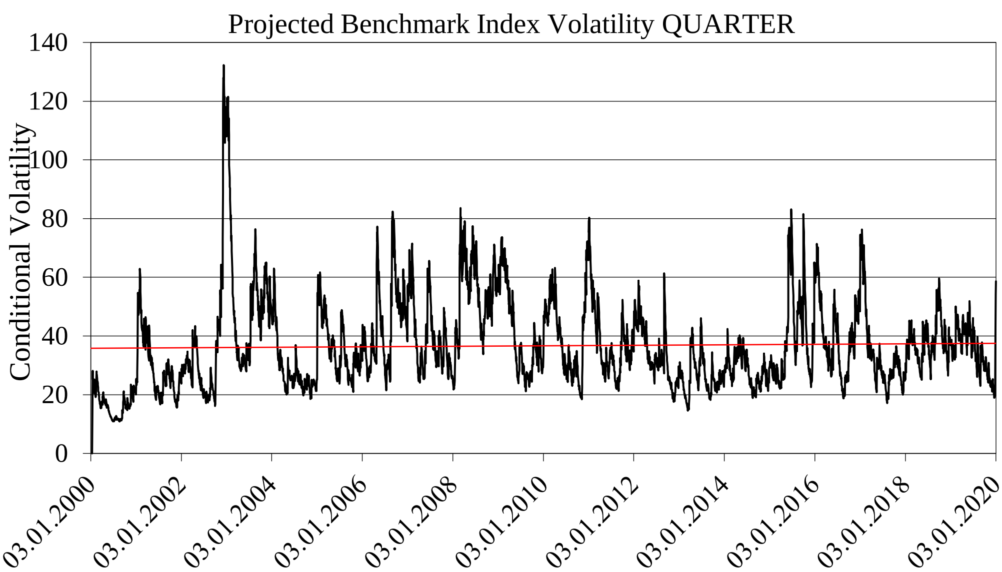

Figure 24: Projected Benchmark Index Volatility QUARTER

Figure 25 graph the projected conditional volatility together with a moving average for m=4 and m=15 lags for the squared residuals of an AR(1) model for the returns. There seems to be a good fit between the projected volatility and the moving average processes for 4 and 15 lags.

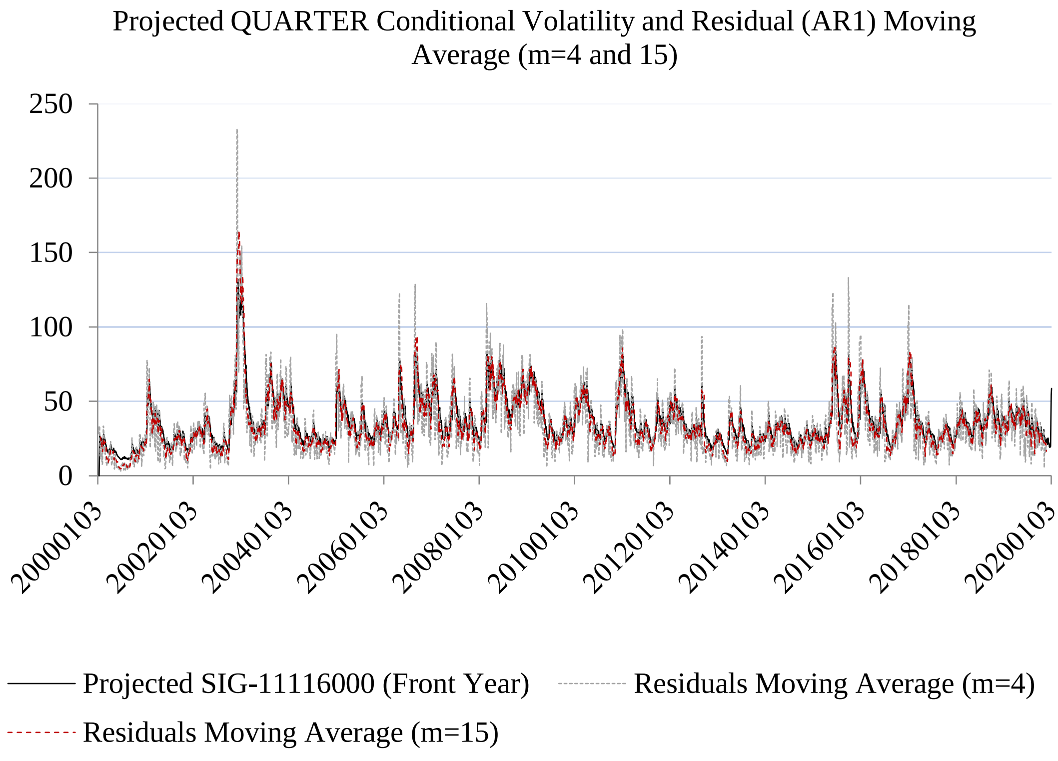

Figure 25: Projected QUARTER Conditional Volatility and Residual (AR1) Moving Average (m=4 and 15)

Figure 26 show an index for the one-step-ahead densities for the volatility at the mean, conditional to the values for x_{t-1} (unconditional mean). In other words, given that yesterday's volatility where at the mean (0.0), where will todays volatility (conditional mean) be. The conditional densities are compared to a normal distribution. The figure show that the conditional densities for QUARTER time-series have non-normal features; including peakness in the interval -2 and $+2$ standard deviation from the mean, a smaller distribution in the interval ± 2 to ± 7 standard deviations from the mean, and fatter tails from ± 8 standard deviations. Such properties are known as leptokurtosis and are frequently found when analysing financial data. This confirms correctness of using Hermite polynomials to move away from the normal distribution when describing the densities for the time-series. The conditional one-step-ahead densities for the QUARTER time-series have a clearly wider density than the YEAR, indicating that tomorrows volatility is more unknown for Quarter.

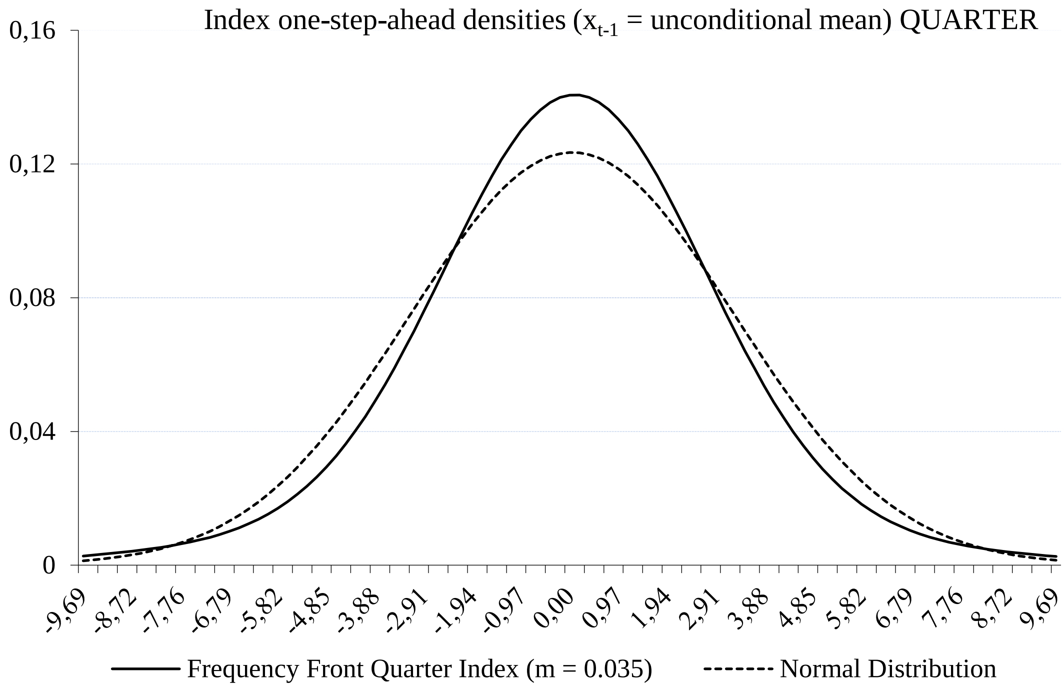

Figure 26: Index for the one-step-ahead densities for the volatility at the mean, conditional to the values for x_{t-1} (unconditional mean)

Figure 27 graph the one-step-ahead densities when adding a shock to the time series. The shock is added to yesterday's mean ($x_{t-1} = unconditional\ mean$), and the graph show the frequency distribution for the conditional mean today ($x_t = condtional\ mean$). There are added shocks ranging from -20% to $+20\%$ and the baseline profile showing the mean of the densities ($m = 0.035$). When evaluating the densities after different shocks to the baseline profile, we see that the densities after the shock is clearly wider. The widest densities are found for the biggest shocks ($\pm 20\%$), and the densities becomes smaller with smaller shocks. An interesting observation is that there are hard to see any clear differences in the width between the positive and the negative shocks. This is in contrasts with the findings in Egeland & Haug (2016) which found clearer difference in the width of positive and negative shocks, however the differences where biggest in the stock indexes, and smaller in the commodities (especially for Brent oil front month futures contrast). When having a nearer look at the figure, one cannot see any difference in peakness for the positive and negative shocks either, which indicate the same degree of uncertainty for the volatility after both positive and negative shocks.

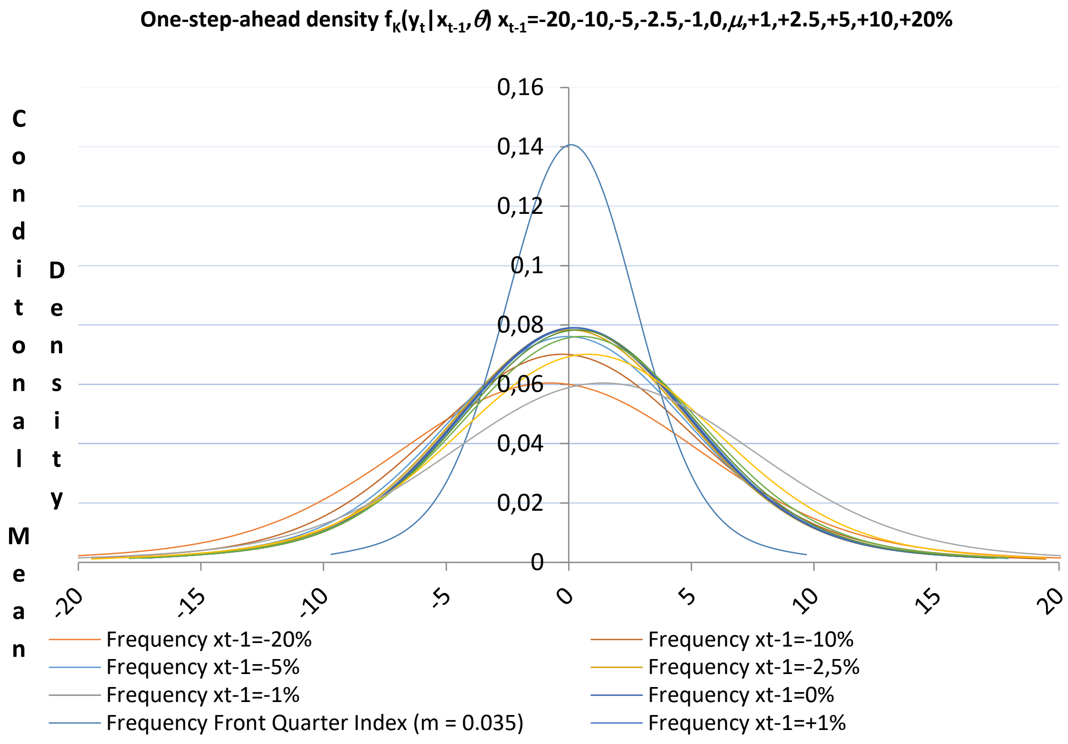

Figure 27: One-step-ahead density $f_K(y_t|x_{t-1},q)$ x_{t-1} = -20,-10,-5,-2.5,-1,0,m,+1,+2.5,+5,+10,+20%

When comparing the one-step-ahead densities for the different shocks between QUARTER (*figure 27)* and YEAR (*figure 22)*, there are some clear differences. First, the densities for quarter are wider and less peaked than the year, indicating more uncertainty after shocks for the quarter-contracts than the year-contracts. The second difference are related to the difference in peakness between the positive and negative shocks for the two contracts. Where the year-contracts had wider and flatter densities for the positive shocks, indicating more uncertainty in volatility after positive shocks than negative shocks, no such difference where found for the quarter-contracts.

Figure 28 visualize the connection between the one-step-ahead conditional variance and the percentage change in the unconditional mean. This connection is referred to as the *asymmetry effect*, some disciplines uses "*leverage effect*" and "*risk premium effect*". We can interpret the graph as showing how the conditional variance function reacts to surprisingly shocks to the system. The figure confirms what the previous findings for QUARTER has indicated, i.e. that neither positive (negative) shocks tend to giver bigger effects than negative (positive) shocks to the volatility - there are no asymmetry effects for the time-series.

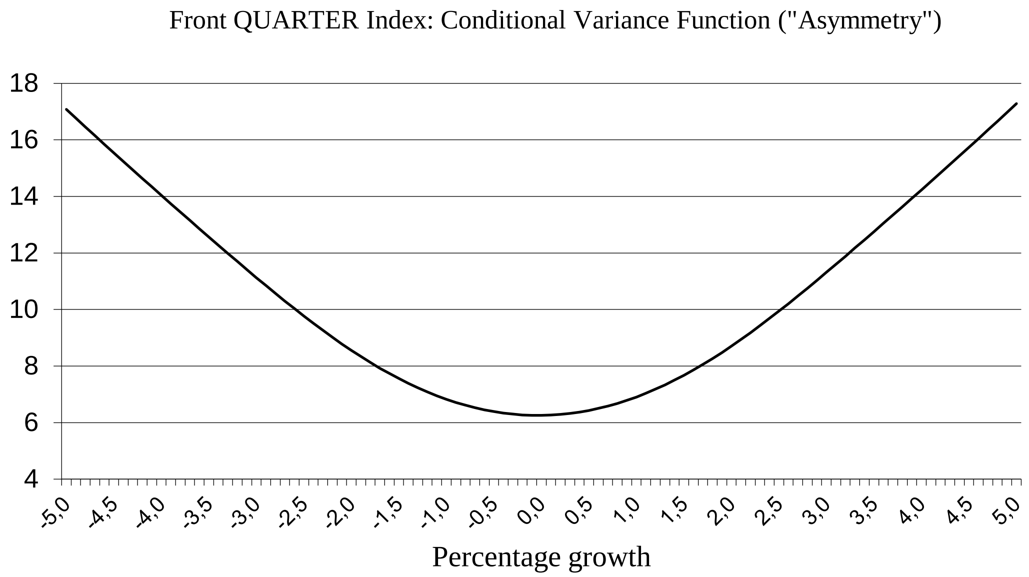

Figure 28: Front QUARTER Index: Conditional Variance Function ("Asymmetry")

Where the analysis found a small positive asymmetry effect for the YEAR (*figure 23)*, no asymmetry effects where found for QUARTER *(figure 28)*. These results stand in sharp contrast to the findings in research for other energy markets, equities and commodities (Solibakke, 2014) (Egeland & Haug, 2016) (Solibakke, 2020)

5.3 Stochastic Volatility model Evaluation

The stochastic volatility model is estimated using the Efficient Methods of Moments (EMM), and the estimated model makes a connection between the statistical and scientific model. For model evaluation, the number of observations and simulations are logged, and the optimal model is found by the BIC-criterion and the lowest posterior score which is tested with a chi squared test. The method gives a reprojection of the latent volatility which is split into two volatility factors; V_1 which capture volatility clustering and V_2 which capture mean reverting effects.

5.3.1 SV model evaluation YEAR

Figure 29 illustrate the iterative factor process leading to the optimal iterative posterior score for the SV model. The frequently changing factor score indicate that the simulations are searching for the optimal solution, a stationary simulation chain would indicate misconfiguration of the model. The optimal model produces the highest iterative posterior score of -3.7628 with the associated Chi-square test statistic of 0.2882 at 3 degrees of freedom. The degrees of freedom are found by taking the 12 parameters from the SNP model minus the 8 parameters in the SV model minus 1. The null hypothesis fails to be rejected, hence there are no statistical difference between the expected and observed series. The SV model is therefore an acceptable approximation of the score statistics (SNP model).

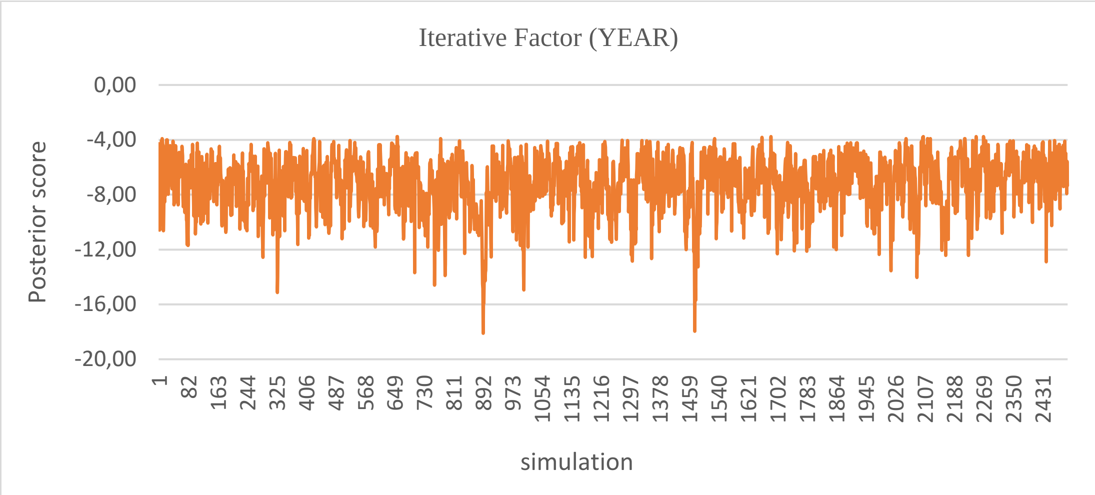

Figure 29: Iterative Factor (YEAR)

The parameters for the optimal model from the EMM estimations is reported in *table 9*. We recall from earlier that a quasi-t-statistics below two implies that the SV model manage to match the respective parameter from the SNP model. We notice that every parameter is significant with quasi t-ratios less than 2, however parameter 2 is barely over with 2.05. The first 6 parameters are the 6 polynomials used, parameter 8 is a constant, parameter 9 are the parameter for the first lag and parameter 10 is the constant for the variance term. Parameter 11 for the ARCH term which capture volatility clustering and parameter 12 that capture historical volatility having a strong impact on today's observations. Parameter 13 is for the asymmetry effect, where a positive value indicates a positive effect, i.e. the volatility shows stronger responses to positive than negative shocks. The positive asymmetry coefficient suggest that the YEAR contracts show higher volatility from large price increases. This is in line with findings from the SNP-model for YEAR, and are opposite of research in other energy markets, equities and commodities (Solibakke, 2014) (Egeland & Haug, 2016) (Solibakke, 2020). Parameter 14 is used to configurate how strong the volatility influences the parameters.

Table 9: Score Diagnostics: Parameters Scientific Model Front Year Future Contracts

Score diagnostics: Parameters Scientific Model Front Year Future Contracts				
Index	φ	Mean	unadjusted standard errors	quasi-t-ratios
1.00	a0[1]	-0.93016	1.98295	-0.46908
2.00	a0[2]	4.08756	1.98546	2.05875
3.00	a0[3]	-0.22982	1.94380	-0.11823
4.00	a0[4]	-0.80993	2.02666	-0.39964
5.00	a0[5]	2.45620	3.18444	0.77131
6.00	a0[6]	-0.71330	7.90094	-0.09028
7.00	A(1,1)	0.00000	0.00000	0.00000
8.00	b0[1]	-0.65248	1.61105	-0.40500
9.00	B(1,1)	-0.21183	0.94321	-0.22458
10.00	R0[1]	3.30761	2.49069	1.32799
11.00	P(1,1)	5.35768	4.47406	1.19750
12.00	Q(1,1)	25.21551	22.61630	1.11493
13.00	V(1,1)	1.82583	1.83584	0.99455
14.00	W(1,1)	0.33892	0.38106	0.88941

Distributed Chi-square (no.of.freedom)	χ^2 (3)
Posterior at the mode	-3.7628
Chi-squared test statistics	{0.2882}

The numbers in braces are P-values for statistical significance
Rejection rate are 5%, implying P-values less than 5% reject the null hypothesis

The next is to run a regression of V_i at $\hat{\sigma}_t^2$, $\hat{y}_t$ and $|\hat{y}_t|$, this gives us predicted values of $V_{it}|\{y_\tau\}_{\tau=1}^{t}$ on the observed dataset. *Figure 30* report the two latent volatility factors for the time series of YEAR. The plot indicate that V_1 is a slowly moving factor showing volatility clustering and persistence. Volatility factor V_2 is moving faster and shows strong mean reverting characteristics, i.e. it absorbs shocks fast. The leptokurtosis feature of the return distribution is due to the two factors; V_1 is moving around the long term mean (~0.5), making it responsible for the many observations around the mean of the distribution, and V_2 is jumping far away from the mean (~0.0) making it responsible for the fatter tails in the distribution. For the YEAR, V_1 is traversing frequently between 0.4 and 0.8 and V_2 is traversing between 0.04 and 0.2. The ordinary least square R^2 is 0.845 for V_1 and 0.14 for V_2, by that we conclude that the slowly moving and persistence volatility factor V_1 is the main contributor to the volatility.

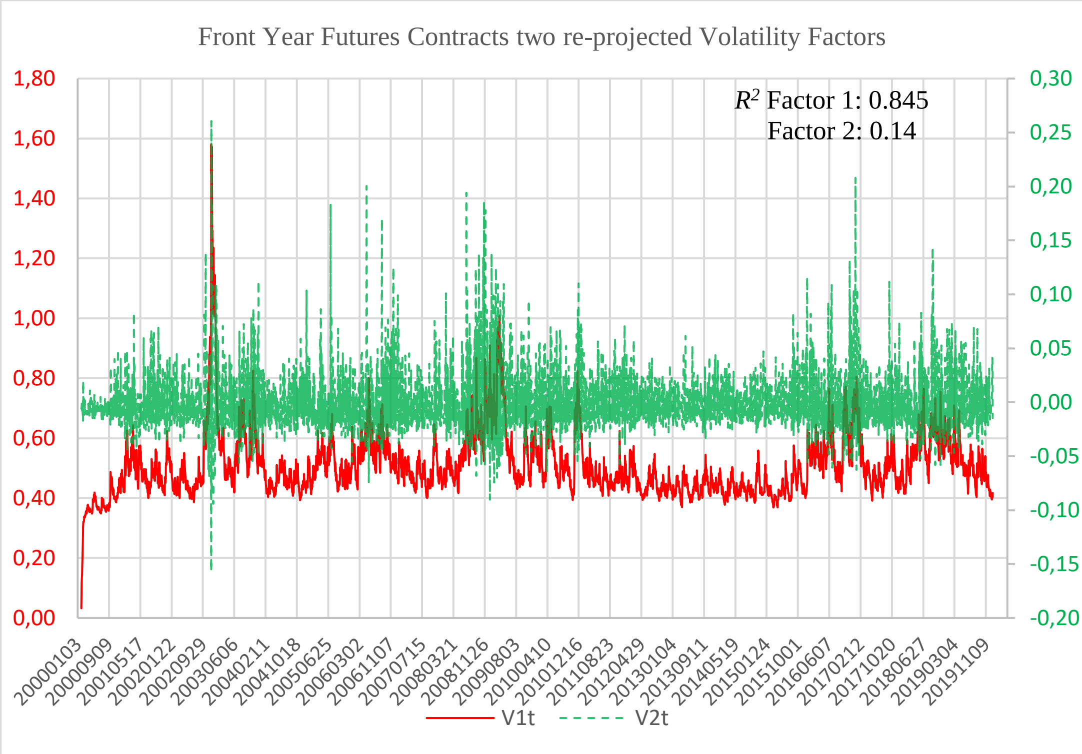

Figure 30: Front Year Futures Contracts two re-projected Volatility Factors

Figure 31 reports the returns and the re-projected conditional volatility together. The re-projected volatility has a long term mean around 20 and seem to fit the returns well. There seems to be peaks in the volatility in the end of 2002, end of 2008, early 2017 and late 2018. Calmer periods around 2000, between 2004 and 2006, summer 2007, between 2011 and 2016, 2017 and autumn 2019

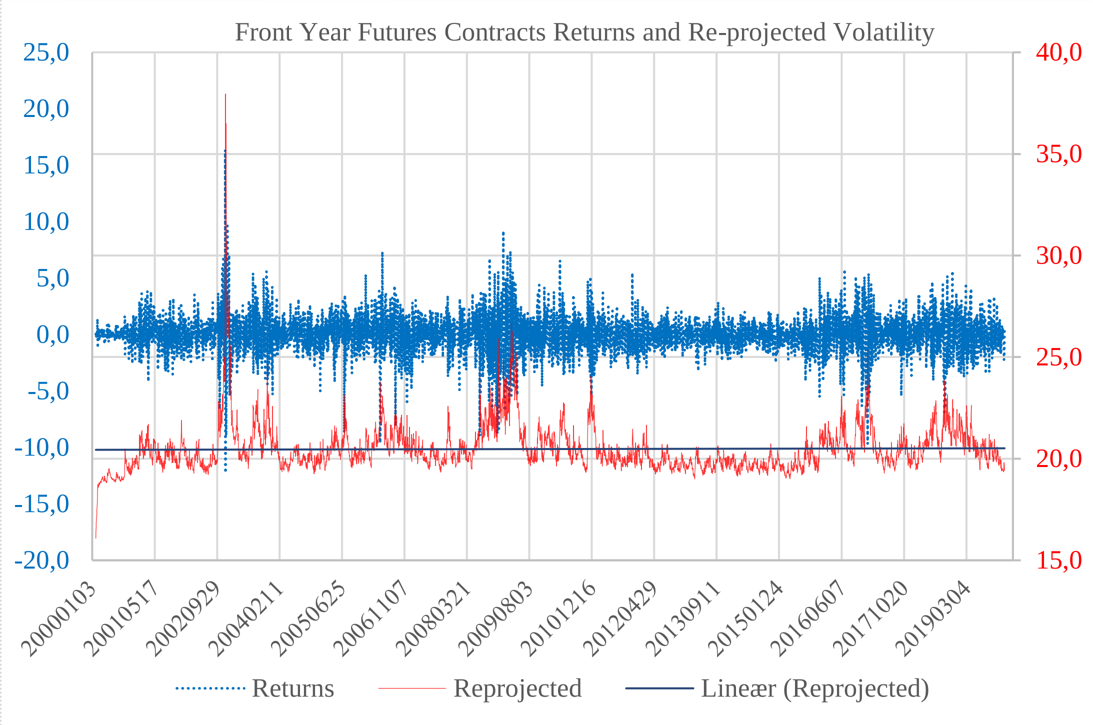

Figure 31: Front Year Futures Contracts Returns and Re-Projected Volatility

Figure 32 graph the re-projected volatility from the SV model together projected volatility from the SNP 11116000 model and the residual (AR1) moving average (m=4 and 15). The re-projected volatility at time t is a forecast estimated from the data series up to time $t-1$, using only information available at time $t-1$ obtains no look-ahead bias in the estimation of the predicted volatility. The last available volatility forecast can be plotted directly into the Black & Scholes model to get more precise option prices.

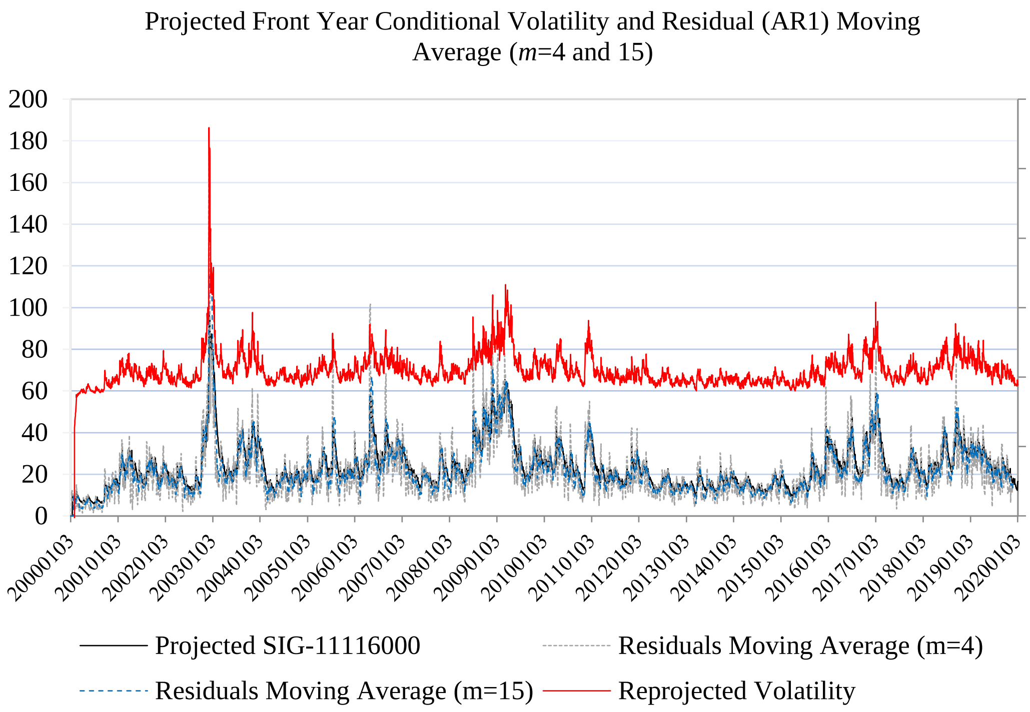

Figure 32: Projected Front Year Conditional Volatility and Residual (AR1) Moving Average (m=4 and 15)

Figure 33 reports the autocorrelation plot for the re-projected volatility with 40 lags. In day 25 the autocorrelation falls below 0.5. The plot reveals the strong data dependency, YEAR shows substantial persistence in volatility.

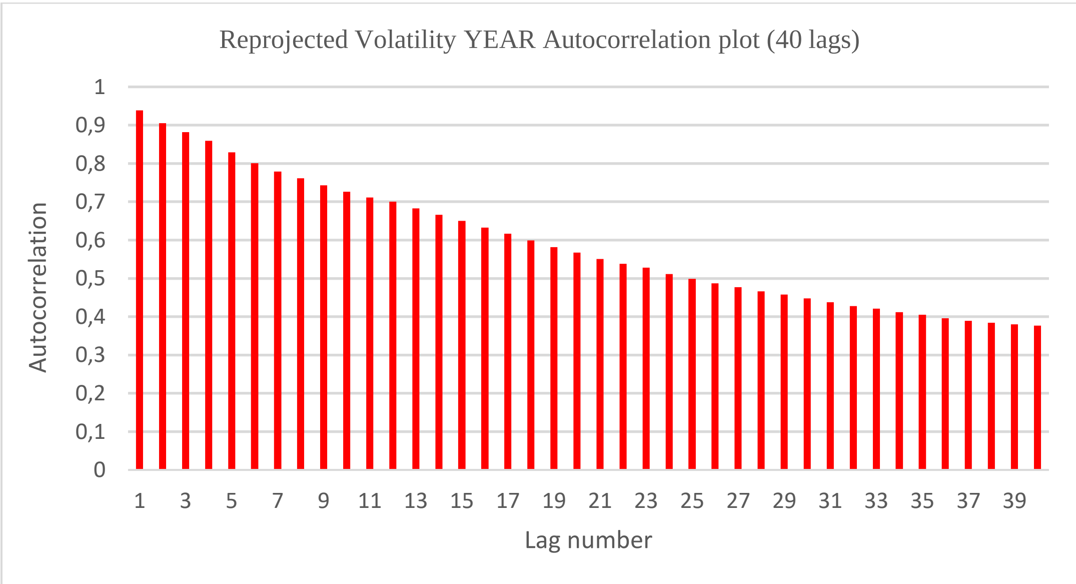

Figure 33: Reprojected Volatility YEAR Autocorrelation plot (40 lags)

5.3.2 SV model evaluation QUARTER

Figure 34 illustrate the iterative factor process leading to the optimal iterative posterior score for the SV model. The frequently changing factor score indicate that the simulations are searching for the optimal solution, a stationary simulation chain would indicate misconfiguration of the model. The optimal model produces the highest iterative posterior score of -1.1679 with the associated Chi-square test statistic of 0.79 at 3 degrees of freedom. The degrees of freedom are found by taking the 12 parameters from the SNP model minus the 8 parameters in the SV model minus 1. There is a failure to reject the null hypothesis, hence there are no statistical difference between the expected and observed frequencies. The SV model is an acceptable approximation of the score statistics (SNP model).

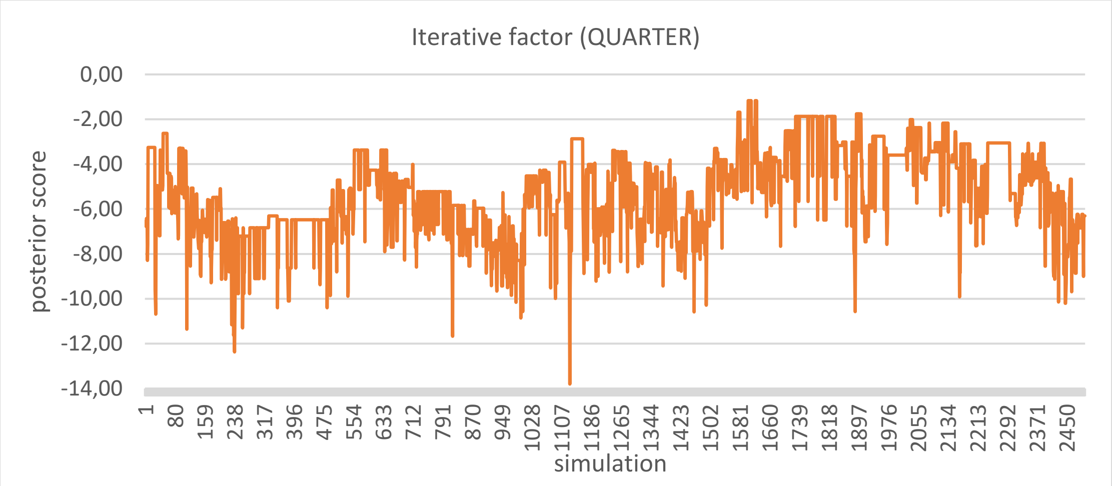

Figure 34: Iterative factor (QUARTER)

The parameters for the optimal model from the EMM estimations is reported in *table 10*. We recall from earlier that a t-statistics under 2 implies that the SV model manage to match the respective parameter from the SNP model. Every parameter is significant with quasi t-ratios less than 2. We recall that the first 6 parameters are the 6 polynomials used, parameter 8 is a constant, parameter 9 are the parameter for the first lag and parameter 10 is the constant for the variance term. Parameter 11 for the ARCH term which capture volatility clustering and parameter 12 that capture historical volatility having a strong impact on today's observations. Parameter 13 is for the asymmetry effect, a value of 0 indicate no asymmetry effects. This is in line with findings from the SNP-model for QUARTER, and are opposite of research in other energy markets, equities and commodities (Solibakke, 2014) (Egeland & Haug,

2016) (Solibakke, 2020). Parameter 14 is used to configurate how strong the volatility influences the parameters.

Table 10: Score diagnostics: Parameters Scientific Model Front Quarter Future Contracts

Score diagnostics: Parameters Scientific Model Front Quarter Futures Contracts				
Index	Φ	mean	unadjusted standard errors	quasi-t-ratios
1.00	a0[1]	-0.22838	1.93717	-0.11789
2.00	a0[2]	-0.01312	1.96802	-0.00667
3.00	a0[3]	-1.98503	2.15286	-0.92204
4.00	a0[4]	1.65459	2.36876	0.69850
5.00	a0[5]	-0.77219	2.37633	-0.32495
6.00	a0[6]	1.68834	2.28501	0.73888
7.00	A(1,1)	0.00000	0.00000	0.00000
8.00	b0[1]	0.35841	1.99092	0.18002
9.00	B(1,1)	-0.86979	1.28088	-0.67906
10.00	R0[1]	-1.78391	6.48042	-0.27528
11.00	P(1,1)	-1.59358	9.38754	-0.16975
12.00	Q(1,1)	-6.55925	38.54152	-0.17019
13.00	V(1,1)	0.00000	0.00000	0.29446
14.00	W(1,1)	0.00000	0.00001	-0.05470
Distributed Chi-square (no.of.freedom)				χ^2 (3)
Posterior at the mode				-1.1679
Chi-squared test statistics				{0.7607}
The numbers in braces are P-values for statistical significance				
Rejection rate are 5%, implying P-values less than 5% reject the null hypothesis				

The next step is to run a regression of V_i at $\hat{\sigma}_t^2$, $\hat{y}_t$ and $|\hat{y}_t|$, this gives us predicted values of $V_{it}|\{y_\tau\}_{\tau=1}^{t}$ on the observed dataset. *Figure 35* shows the two latent volatility factors for the time series QUARTER. The plot indicate that V_1 is a slowly moving factor showing volatility clustering and persistence. Volatility factor V_2 is moving faster and shows strong mean reverting characteristics, i.e. it absorbs shocks fast. The leptokurtosis feature of the return distribution is due to the two factors; V_1 is moving around the long term mean (~0.6), making it responsible for the many observations around the mean of the distribution, and V_2 is jumping far away from the mean (~0.0) making it responsible for the fatter tails in the distribution. V_1 is traversing between 0.4 and 1.0 and V_2 between 0.05 and 0.5. The ordinary least square R^2 is 0.98 for V_1 and 0.074 for V_2, by that we conclude that the slowly moving and persistence volatility factor V_1 is the main contributor to volatility.

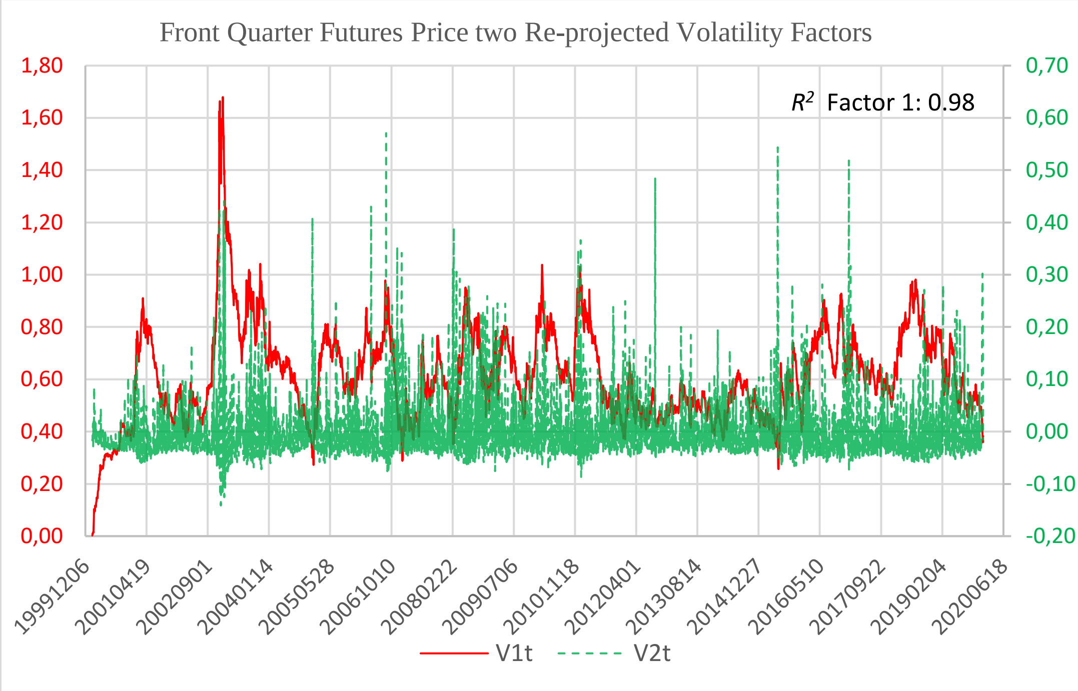

Figure 35: Front Quarter Futures Price two Re-projected Volatility Factors

Figure 36 report the returns and the re-projected conditional volatility together. The re-projected volatility has a long term mean around 22 and seem to fit the returns well. There seems to be peaks in the volatility in the end of 2002, autumn 2006, summer 2008, winter 2010, winter 2011, autumn 2016 and summer 2018. The conditions where calmer summer 2002, winter 2005, summer 2007, autumn 2010, between 2011 and 2015, autumn 2018 and autumn 2019.

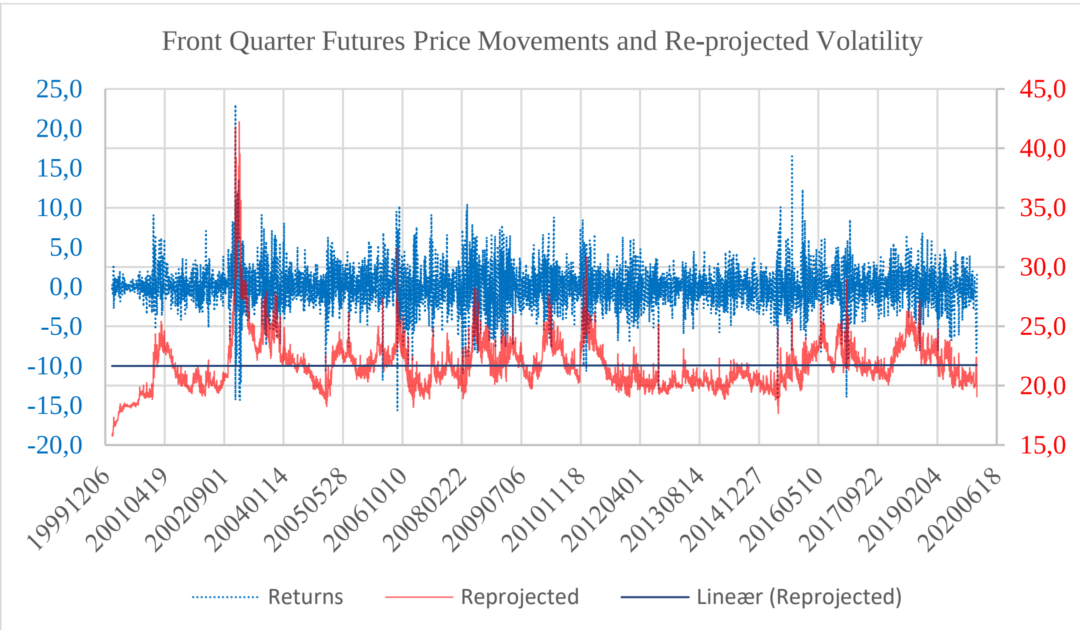

Figure 36: Front Quarter Futures Price Movements and Re-projected Volatility

Figure 37 shows the re-projected volatility together projected volatility from the SNP 11116000 model and the residual (AR1) moving average (m=4 and 15). The re-projected volatility at time t is a forecast estimated from the data series up to time $t-1$, using only information available at time $t-1$ obtains no look-ahead bias in the estimation of the predicted volatility. The last available volatility forecast can be plotted directly into the Black & Scholes model to get more precise option prices.

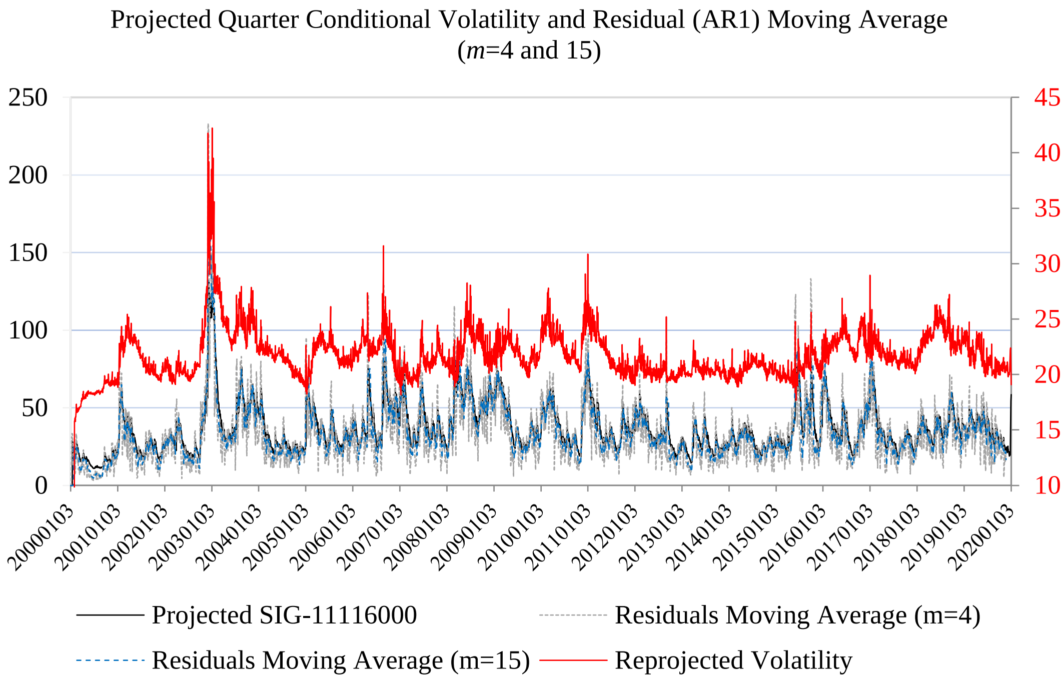

Figure 37: Projected QUARTER Conditional Volatility and Residual (AR1) Moving Average (m=4 and 15)

Figure 38 shows the autocorrelation plot for the Re-projected Volatility with 40 lags. After 40 days, the autocorrelation is over 0.6. The plot reveals the strong data dependency, QUARTER shows substantial persistence in volatility.

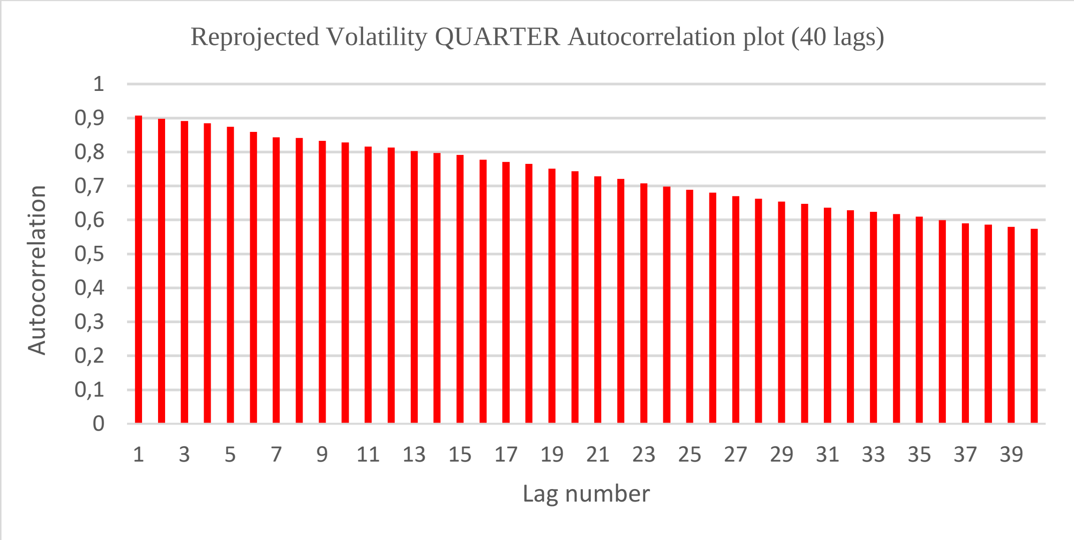

Figure 38: Re-projected Volatility QUARTER Autocorrelation plot (40 lags)

Both series indicates when returns becomes wider (narrower) volatility increases (decreases). When comparing the two series and the associated volatility factors, we see that the slowly moving and persistence volatility factor V_1 is the main contributor to the volatility with the ordinary least square R^2 of 0.845 for YEAR and 0.98 for QUARTER. Volatility factor V_2 has R^2 of 0.14 and 0.074 respectively. The long term mean of V_1 is approximately 0.5 and 0.6 for YEAR and QUARTER. Both volatility factors seem the be more volatile for the QUARTER time series than the YEAR time series. For the re-projected volatility figures, the YEAR series is traversing around a mean of 20 and the QUARTER around 22, There is a clear difference in the re-projected volatility movement seen by the graphs. QUARTER have more frequent and bigger movements than YEAR, where YEAR has 4 spikes above 25, the QUARTER has 13. In addition, QUARTER has 4 spikes over 30. The autocorrelation plots captured the strong volatility clustering and high persistence in the re-projected volatility, where QUARTER shows the strongest autocorrelation. Overall, we can conclude that the re-projected time series captures the more volatile volatility in the QUARTER than the YEAR.

6. Volatility and market factors.

The output from last chapter was an optimal SV-model for modelling the latent volatility. The volatility is said to be latent as it is not a directly observed instrument, rather it can be understood as a variable modelled from its direct influence on the magnitude of returns. From the SV-modelling a conditional volatility forecast were generated, named as the re-projected volatility. The literature regarding factors affecting volatility in the financial electricity contracts are quite sparse. The purpose of this chapter is to analyse observable variables influencing the electricity spot price in the light of the modelled volatility forecast. There is a consensus that supply and demand-side variables influencing the spot electricity price, but can changes (shocks) in these supply and demand-side variables be identified in the conditional volatility forecast?

Re-projected volatility forecast will be compared with production mix, reservoir levels and temperature. The production mix is chosen since it represents the supply side in the spot electricity market. In addition, the production mix has been in a change last 20 years. Now we see less adjustable production sources (like coal and nuclear) and more un-adjustable renewables (like solar and wind), making it natural to look for any observable effects in volatility. Reservoir levels are chosen since the production mix has a large share of hydropower, and unlike other energy sources a large part of the hydropower is storable in reservoirs. Reservoir levels is a supply-side variable. It is thinkable that the adjustable production reserve will affect volatility. Temperature is another factor which has a direct effect on the spot price. Temperature can be used as a proxy for electricity consumption, making it a demand-side variable.

As market actors take information from the real life into account when pricing contracts, will the latent volatility react to this information flow? We look for outliers or deviations from some long-term median level in the supply and demand-side variables, and if these correspond to outlier values for the re-projected volatility forecast in the same period. The analysis is based on the graphical observations of the variables, making the results not empirical evidences. One should bear in mind some challenges throughout this analysis. Firstly, the financial worlds are complex, we know certain markets and events are inter-connected, yet not all connections are established. Secondly, many events in the timespan is not filtered out, such as the financial crisis in 2008. Thirdly, one should have in mind look-ahead biases in the sample as it is challenging to know which information was available at a certain time. Despite some weaknesses in the sample and the methodology, these analyses can be a starting point for further research in the financial electricity market.

6.1 Production mix in Nord Pool and Volatility

In this section we will look at how the total production and the production mix align with the volatility for YEAR and QUARTER. We use data from chapter 2.3.5, spanning from 2000 to 2018. We use one graph for Total production, including the different methods used to produce electricity. As Hydro power stands for around 50% of the total production, another graph without hydro power is included. For volatility, we plot in returns and re-projected volatility (Repro) into a graph, presenting their development in the interval. By this, we can look at how changes in production mix align with the re-projected volatility. Total production is interesting because it represents the supply side in the economy, where cut in production can lead to increased prices due to inelastic demand for electricity in the short run. In longer run the demand is more elastic as consumers can substitute electricity consumption to other energy sources. More about this in Halvorsen (2012).

Figure 39 indicate a dip in hydro production (orange line) in 2002 and 2003, this is traced by an increase in re-projected volatility for both contracts to an all-time high, where YEAR (*figure 41*) peaked at 38 and QUARTER *(figure 42)* peaked at 42. In 2006 the hydro production had another dip, followed by new spikes in volatility. In the period between 2008 and 2011 the hydro power where overall at a relatively low level, the same period had clearly a lot of volatility, with several spikes. (One should remember the global financial crisis of 2008 as a possible driver to some of this volatility). The dip in hydro production from 2012 to 2013 is not easy to see in the volatility. For the period from 2013 to 2015 the hydro power production rose, and the volatility where calm for the period. From 2015 the hydro power production had a small reduction in the same period we see several periods with volatility clustering and spikes.

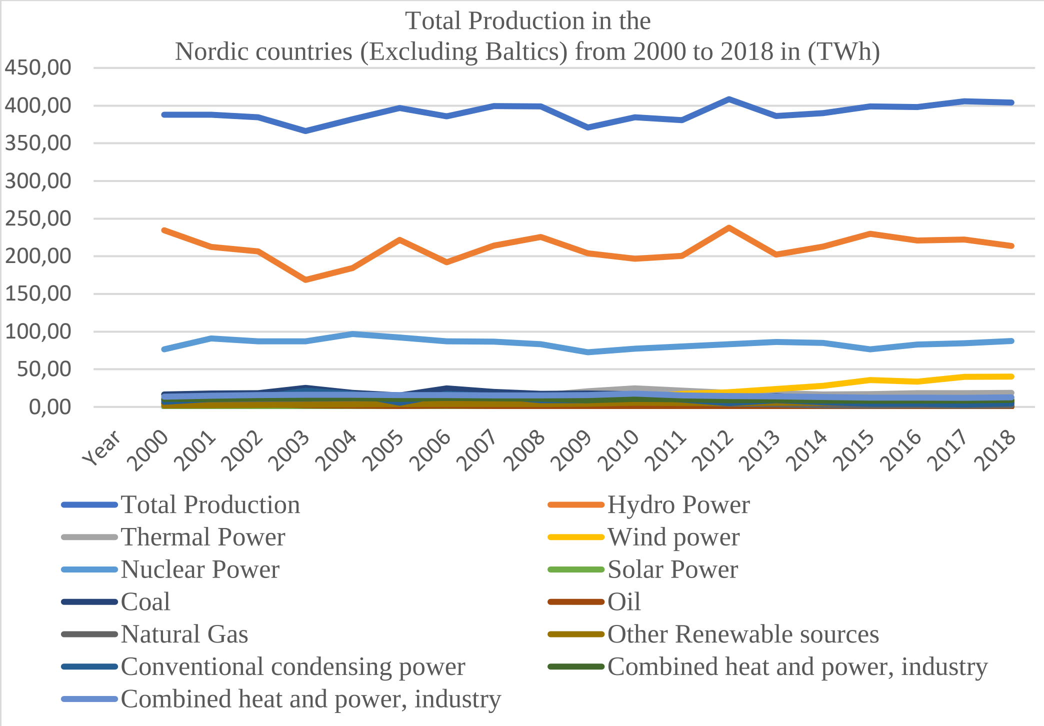

Figure 39: Total Production in the Nordic countries (Excluding Baltics) from 2000 to 2018 in (TWh)

In *figure 40* we exclude hydro power to better visualize changes in the other production technologies. The nuclear production (grey line) had a dip in 2002 and 2003, just as the hydro power and the volatility where all-time high. From 2007 to 2009 there where a large decrease in nuclear production, and the volatility where high the same period. Further the hydro production had a dip in 2009. (one should remember the global financial crisis of 2008 as a possible driver to this volatility). The nuclear power had a new dip in 2015, the time after 2015 had several periods with volatility clustering and spikes. An interesting observation is that 2015 is the year where wind production in Nord Pool reaches over 30 TWh.

Where hydro reservoir production and nuclear production can be adjusted due to market condition like supply and demand, the wind and solar production is fully controlled by meteorology. A hypothesis for further research is to investigate if the increased wind production the last years have led to an increase in volatility for financial contracts for electricity.

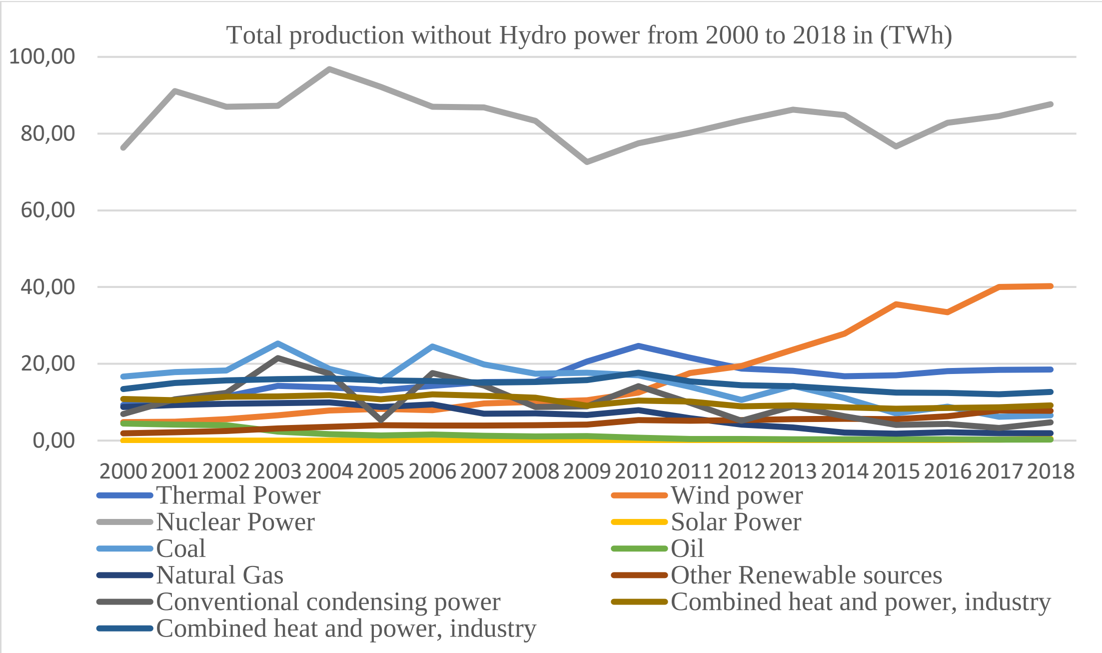

Figure 40: Total production without Hydro power from 2000 to 2018 in (TWh)

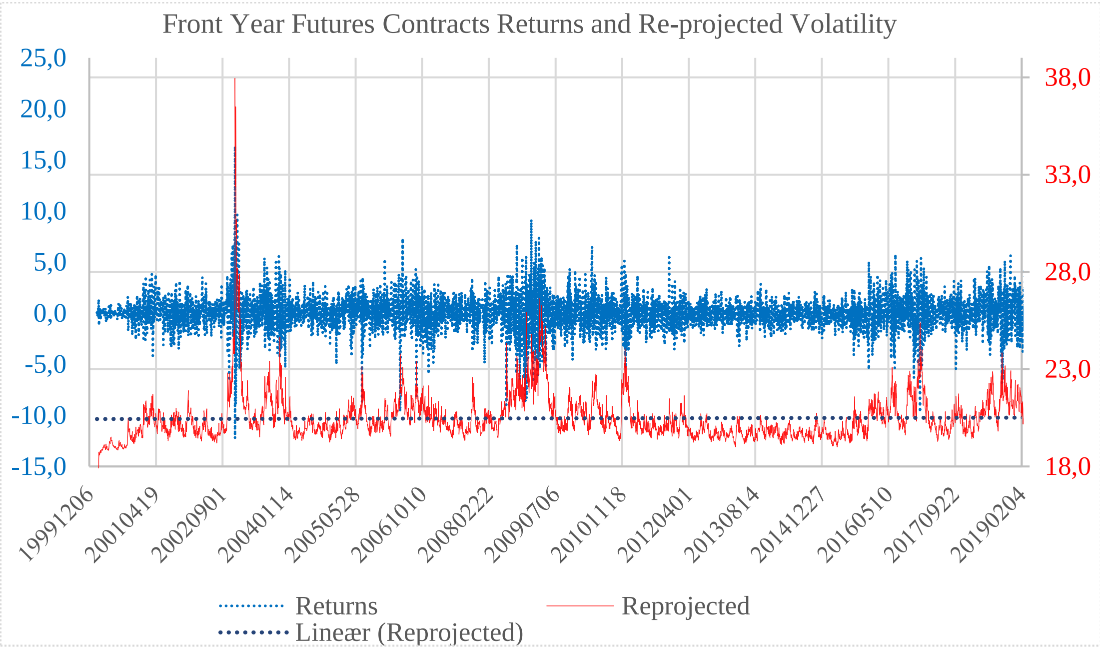

Figure 41: Front Year Futures Contracts Returns and Re-projected Volatility

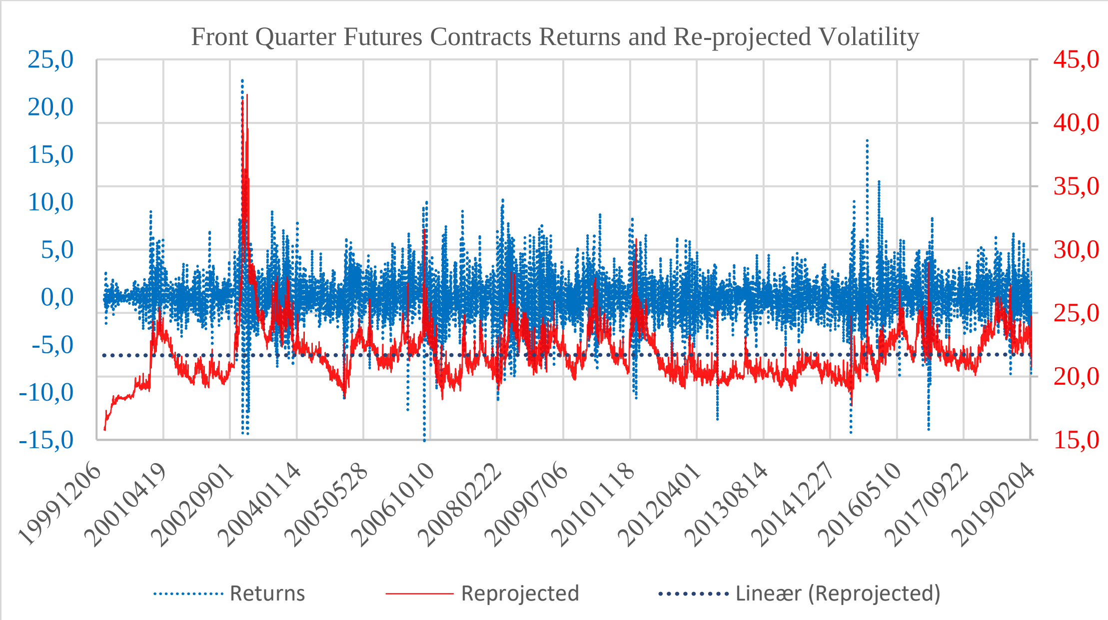

Figure 42: Front Quarter Futures Contracts Returns and Re-projected Volatility

6.2 Reservoir level in Norway and Volatility.

In this section we will look at how the Norwegian reservoir level for eight different years *(Figure 43)* align with the volatility for YEAR *(Figure 44)* and QUARTER *(Figure 45)*. The water in the reservoirs are related to the supply side in the economy, as water is an input factor when producing electricity. When there exists scarcity in a resource the price tends to change, both for the input and for the final good. When there are low reservoir levels and thus little supply of water to produce electricity, the electricity price (the system price) and the financial contracts rises in prices – and thus the volatility. When the supply of water in the reservoir are rich, the prices and thus volatility drops.

We use reservoir levels from chapter 2.3.5. Norway's has 50% of Europe's reservoir capacity, making Norwegian reservoir levels substantial in the market. Sweden and Finland also have reservoir capacity, however this paper if written from a Norwegian perspective partly to set limitations and partly due to lack of open sources to reservoir levels outside Norway. The different years in the interval are chosen because they deviate from the long run median revoir level. Whereas 2017 is chosen because the overall electricity production in Nord Pool reached an all-time high that year. For volatility, we plot in returns and re-projected volatility (Repro) into a graph, presenting their development during the year. By this, we can look at how deviation from the seasonal patterns for reservoir levels align with the re-projected volatility.

2002, represented by the green line, started with high reservoir levels in Norway, reaching around 88% in week 31. The re-projected volatility (green line) were at a very low value in the same period, this is especially clear for the QUARTER series which spanned around 22. The end of 2002 ended significantly different than the other years, where the reservoir levels fell earlier and steeper. The same period show dramatically increases in volatility both for YEAR and QUARTER, reaching all-time high levels at 38 (41) for YEAR (QUARTER). The low reservoir levels and high volatility continued into 2003 (light blue line).

2004 (golden line) show no clear trend. 2006 (orange line) is another interesting year, it started with very high reservoir levels and low volatility. During the summer the water disappeared fast from the reservoirs, and in week 34 the level is very low and far away from the median level (dotted line). The volatility (orange line) picked up drastically in the same period, particularly visible in the QUARTER contracts reaching volatility over 30. The reservoir levels picked up and reached the median level in the end of 2006, the volatility went down simultaneously.

Year 2010 (pink) ended with the lowest reservoir level, and the volatility is clearly present both in YEAR and QUARTER contracts. The low reservoir levels and relatively high volatility continued into the beginning of 2011 (purple line). During the spring, the reservoirs level strengthens, and reached the median level in week 23, then followed and increased to the end of the year. The volatility level (purple) had a negative trend from the beginning to the end of the year, ending below 20 for both contracts at the end of 2011. 2015 (grey line) follows the median until week 17, lies below until week 31 and above rest of the year. Some volatility spikes are observed in the spring and summer time in 2015. One single spike is observed in the autumn 2015 when reservoir levels where high. In 2017 (dark red), the reservoir levels were relatively close to the median level, with falling volatility (dark red).

When looking at reservoir levels, a reasonable hypothesis could be that there is not the actual amount of water in the reservoir at a particular time that generates volatility. Reservoir levels follows seasonal trends, often with lowest level in March/April, and highest level in September/October. Rather, one can think that a deviation from the long-term median level (the black dotted line) of reservoir level is what contributes to volatility in the financial markets for electricity contracts. Interesting implications for further studies could be test empirically if there are a correlations and causality between deviations in reservoir levels and volatility.

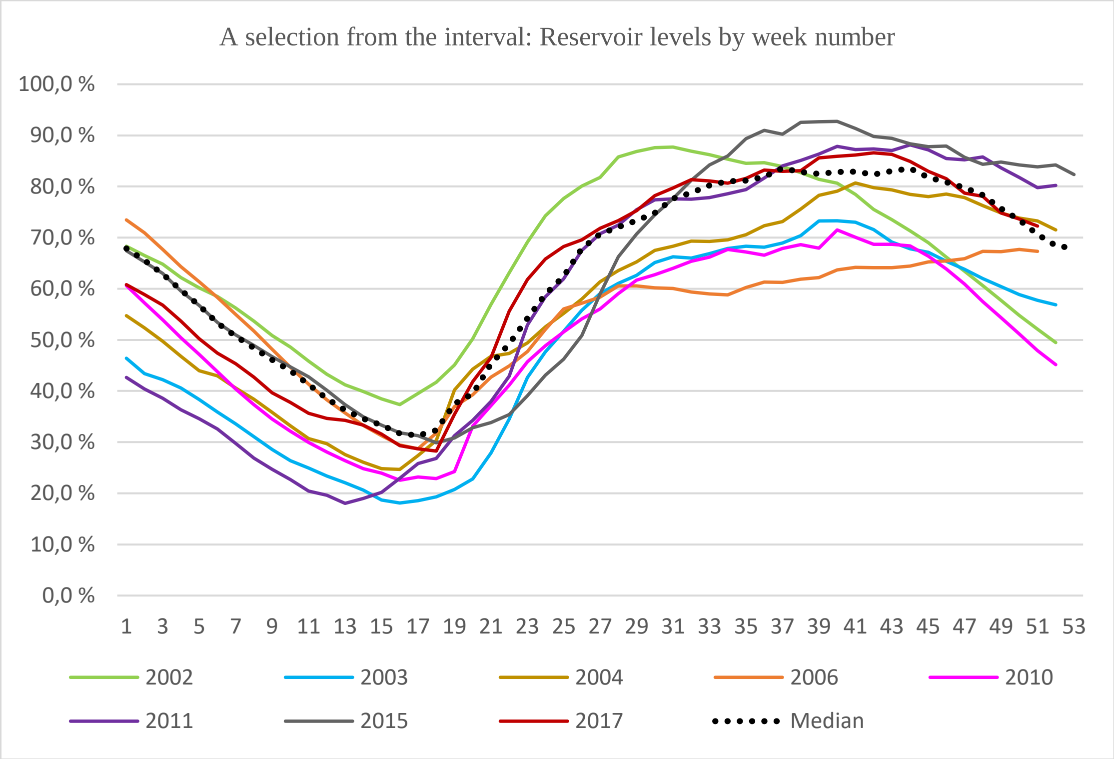

Figure 43: A selection from the interval: Reservoir levels by week number (NVE, 2020b)

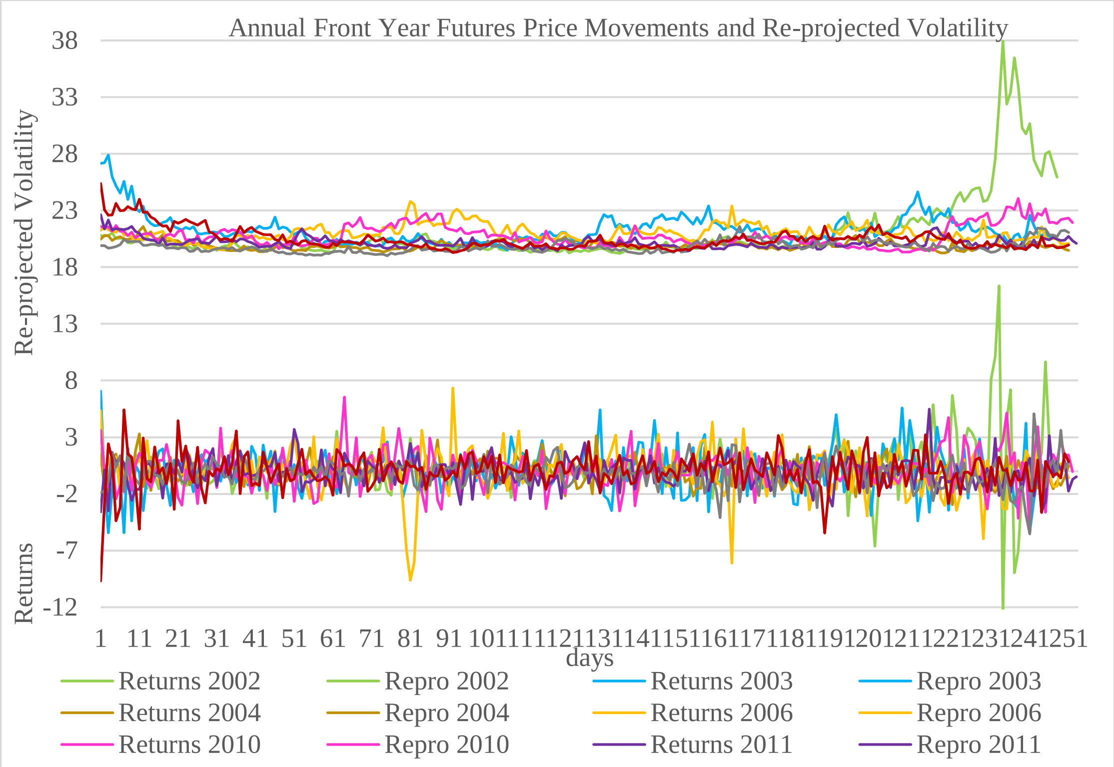

Figure 44: Annual Front Year Futures Price Movements and Re-projected Volatility

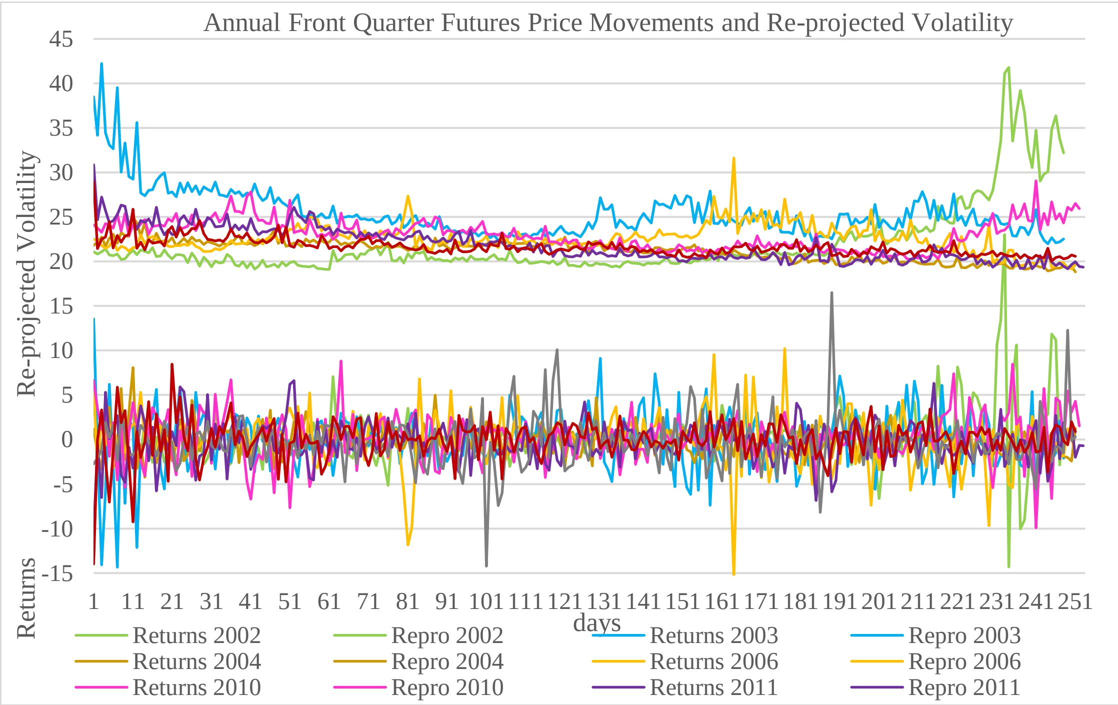

Figure 45: Annual Front Quarter Futures Price Movements and Re-projected Volatility

6.3 Temperature and Volatility

As mentioned in chapter 1.3.6 temperature is one of many factors influencing spot prices. Electricity is used as primary heating source in many countries and the electricity price is usually following seasonal trends where cold weather increases the demand and spot price for electricity. Temperature can be a proxy for electricity consumption, and thus reflect the demand side of electricity. The news is often reporting "record high prices" when cold periods occur. A visual inspection of *Figure 46* and *Figure 47* which report the re-projected volatility for the yearly and quarterly contracts will be compared with temperature to look for indications of connections between temperature and volatility.

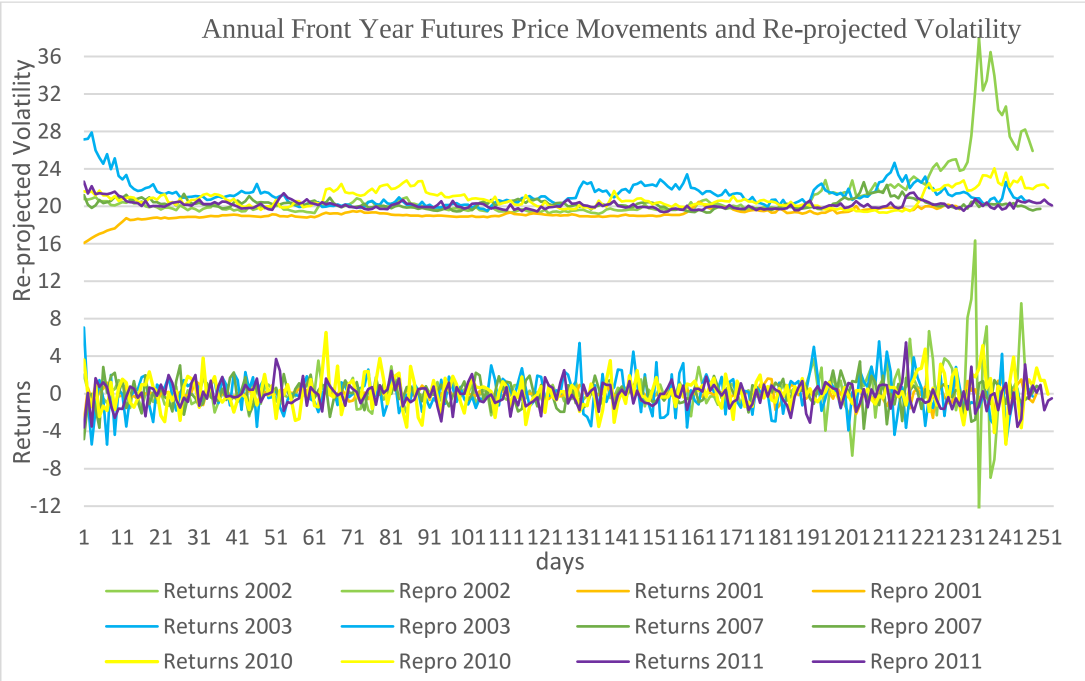

Figure 46: Annual Front Year Futures Price Movements and Re-projected Volatility

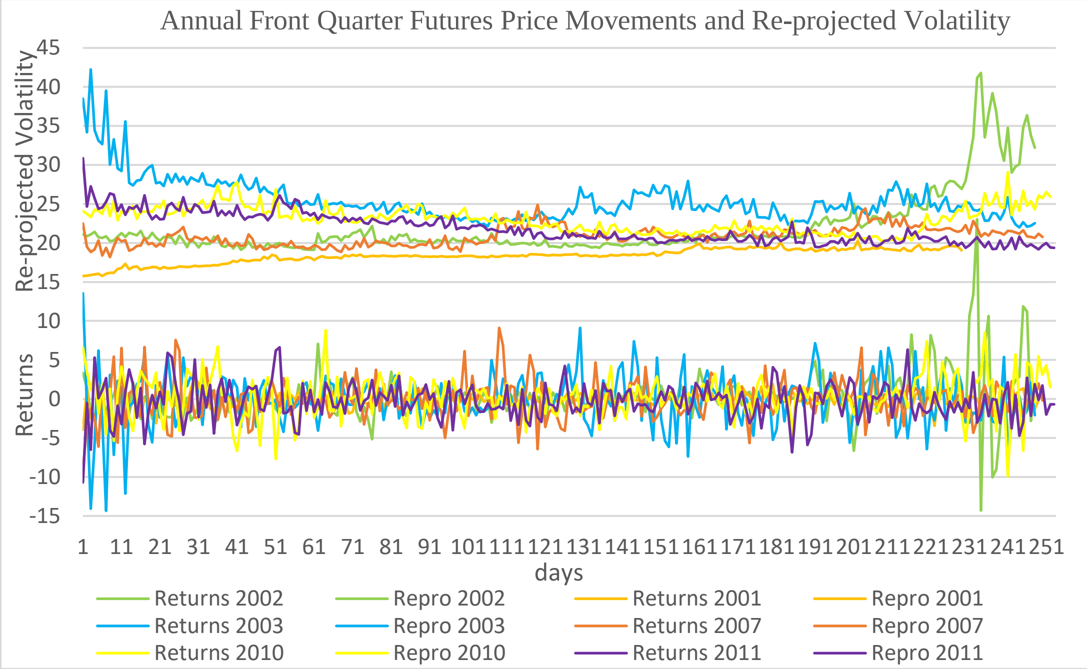

Figure 47: Annual Front Quarter Futures Price Movements and Re-projected Volatility

The median temperature in 2001 (*Figure 48*) is balancing around the average median until week 50 where it has a sharp decline and the year ends with a few cold weeks. In week 52 the temperature is 6.5 degrees below the median in the interval. The volatility is very calm in the end of the year and a visual connection between temperature and volatility is not possible to observe. As we learned from the last section the reservoir level is ending at the 6th highest level in the interval.

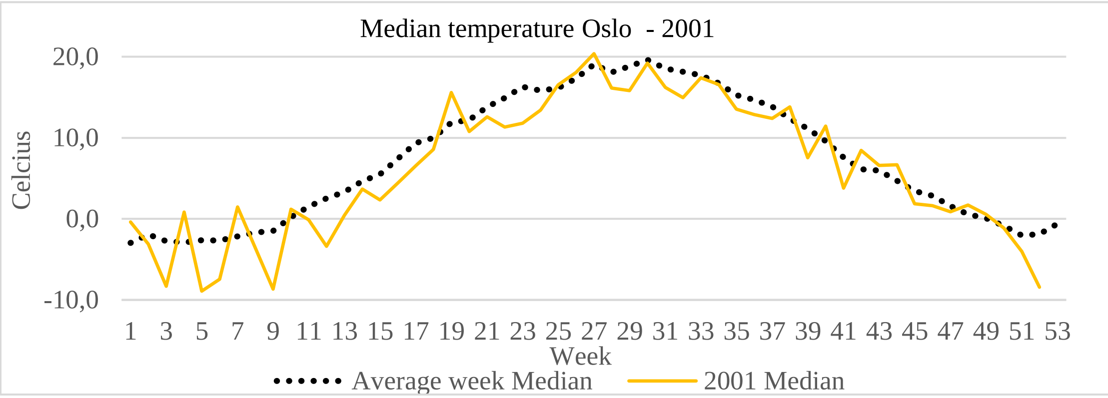

Figure 48: Median temperature Oslo – 2001 (Norsk klimaservicesenter, 2020)

2002 is the year with the largest volatility spikes. The temperatures also have interesting features moving from around the median to well below the median at the end of the year *(Figure 49)*. The temperature has a large shift from week 37 until the end of the year with a temperature below the average median from week 38 with especially large variations between week 40 until 42 and 50, 51 and 52. The volatility is quite stable both in the YEAR and QUARTER until 03.10.2020 (week 40) which is around the same time as the temperatures starts deviating from the median and the reservoir level drops below average. The largest spike occurs on the 5th of December (end of week 49) which is corresponding with the largest temperature drop from the median. It is interesting to observe both large deviations in temperature and reservoir level in a year with very high volatility

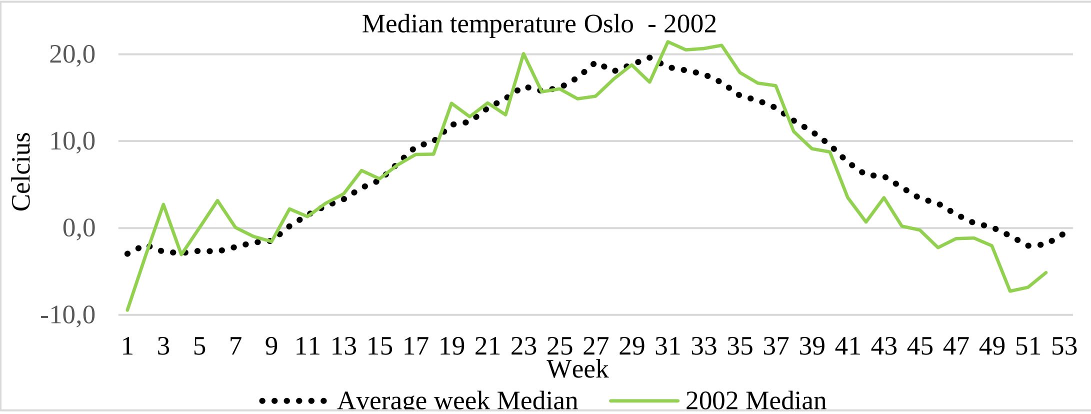

Figure 49: Median temperature Oslo – 2002 (Norsk klimaservicesenter, 2020)

In 2003, the temperatures in week one and two is 10 and 6.2 degrees below the median, while moving to 5.2 and 4.3 above the median in week 3 and 4 *(Figure 50)*. Volatility in the beginning of the year is the highest in the interval when temperatures are also low compared to the median. The volatility has a sharp decline to 27 at point 13 (start of week 4) which is similar to when the temperature starts to normalize around the mean. It is also interesting to observe a more stable volatility period in the year contracts through the year as the temperature are moving around the median until a new spike builds up with a top at point 212 (mid week 45) with volatility at 24.64 which is similar to large temperature deviations. The quarter contract also has a spike at this observation; However, it has many similar spikes at this level through the year. It is also interesting to observe another year with a large volatility spikes in a combination with low temperatures and low reservoir level.

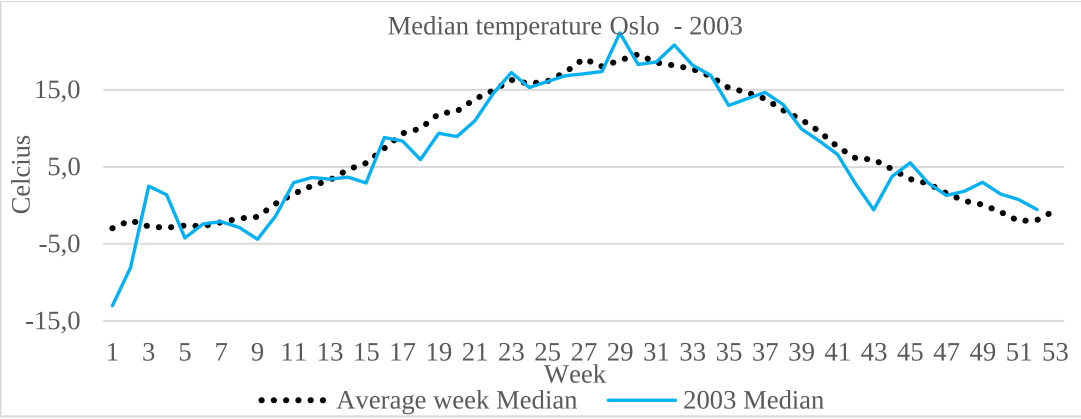

Figure 50: Median temperature Oslo – 2003 (Norsk klimaservicesenter, 2020)

In 2007, the temperature has large variations in the end of the year with a few cold weeks with median temperature of -7.6 in week 51 which is 5.6 degrees below the average median temperature in the interval *(Figure 51)*. The volatility in both QUARTER and YEAR is stable and even declining at the end of the year indicating the large temperature swings is not observable in the volatility for this year. In contrast to the previous years the reservoir level is the 5th highest at the end of the year.

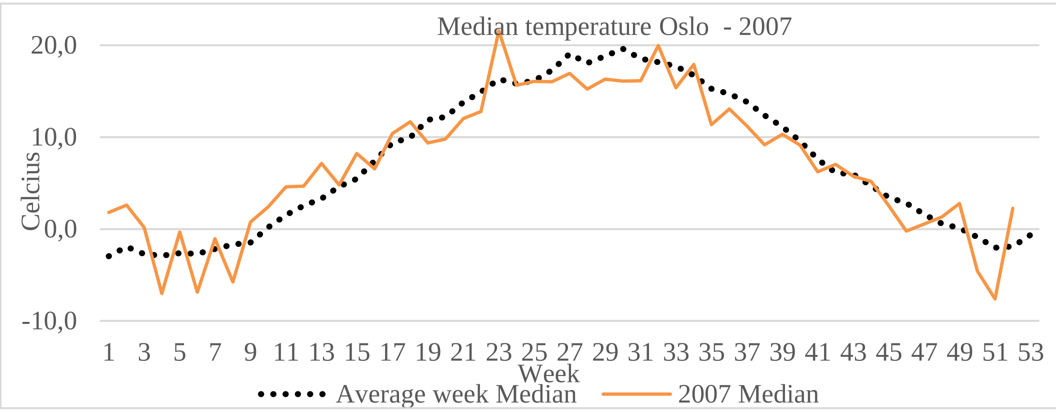

Figure 51 - Median temperature Oslo – 2007 (Norsk klimaservicesenter, 2020)

In 2010, the year is both starting and ending with low temperatures *(Figure 52)*. The volatility at the start of the year is around 25 with a peak at observation 40 of the quarter forward contracts with a volatility of 27. Observation 40 is equal to week 9 where the cold weather is coming to an end. At the end of the year the volatility is increasing from observation 216 (week 45) with a volatility of approximately 20 until it tops out at observation 241 with a volatility of 29 (week 50). It is interesting to observe volatility is starting to increase as the temperatures starts deviating from the average median in the interval. Further it is another year with large temperature deviations and low reservoir level.

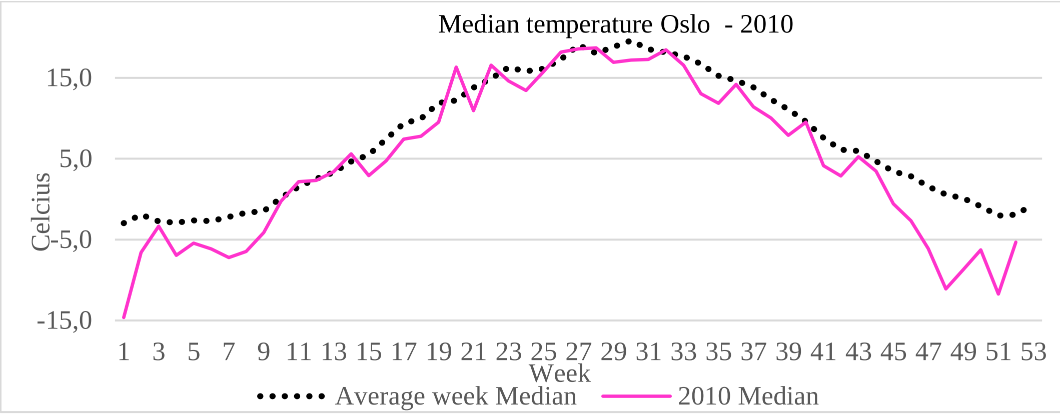

Figure 52: Median temperature Oslo – 2010 (Norsk klimaservicesenter, 2020)

2011 starts with the continuing cold temperature from 2010 *(Figure 53)*. From week 5 to 9 the temperature is well below the average and many volatility spikes are visible in the quarter contracts. The largest spike is observation 52 (start of week 11) with a volatility of 26 and the reservoir level in the start of this year is at the lowest value in the interval.

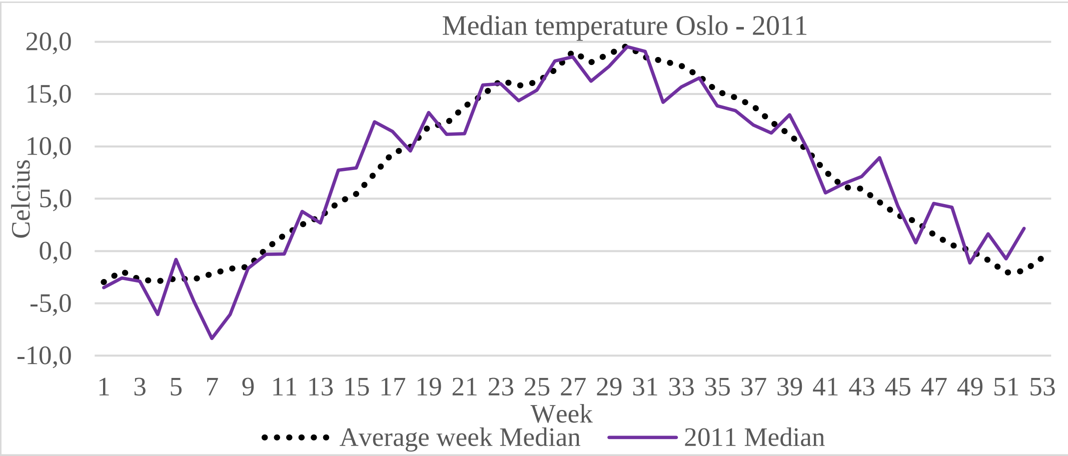

Figure 53: Median temperaure Oslo – 2011 (Norsk klimaservicesenter, 2020)

The visual inspections have led to a few interesting observations where low temperatures have corresponded with volatility spikes in both QUARTER and YEAR. Such results can make sense as temperatures and consumption are negatively correlated in the Spot market (Haugom, et al., 2018). These observations have been especially visible when the reservoir levels are low. It would be interesting to do empirical regression studies on these connections to find out if they are significant or just spurious.

7. Implications for further studies

This study is restricted to financial market for electricity contracts, namely quarter and year front futures contracts in the Nordic/Baltic market. The main result from this study is that two-factor Stochastic Volatility models perform well for the chosen market, and thus gives valuable results to the discipline of Stochastic Volatility modelling. Suggestion for further research is to apply SV models to other financial markets for equities and commodities.

We recall from empirical analysis of the GARCH (1,1) model, where RESET test (YEAR) and BDS test (QUARTER) show significant residuals. An implication for further research is to add more lags in the model to incorporate more information from the time series. A good result from such a study would be if all tests shows non-significant residuals.

This study only applied graphical analysis when considering the re-projected volatility against production mix, reservoir levels and temperature. A topic of interest is to do empirical analysis of the relationship between volatility and these market factors. If doing such an analysis, one should consider using moving average to calculate mean values of production mix, reservoir levels and temperatures to make sure no look-ahead bias occur. Future research can also investigate whereas the reservoir levels influence the effect from temperature on the volatility in financial electricity contracts.

One highly relevant research problem is to consider the growth in renewable energy sources (mainly solar power and windmills) the last years have led to increased volatility in financial electricity contracts. This comes from the fact that solar power and windmills are non-adjustable production technics where the energy cannot be stored to periods with higher prices – in contrast to hydro power production with reservoirs where energy can be stored. If such a problem is confirmed it can give implications to future policy making and production mix investments.

8. Summary

The Norwegian society is in a change, fossil energy sources could be replaced with greener and more sustainable sources to reduce Norway's CO_2 emissions. Electricity is part of the solution, where petrol cars, diesel ferries, and wood and oil for indoor heating are getting replaced with alternatives running on renewables. However, to reduce the CO_2 emissions through electrification, Norway should continue to invest in sources for sustainable electricity production. Hydro power plants, windmills and solar power are some of the sources to a more renewable electricity production. Volatility in prices for electricity is a challenge for producers, retailers, consumers, and investors involved in the Nordic/Baltic power market. In short run volatility can complicate prediction of raw material costs. In the long run volatility and/or increased prices can hinder investment and economic growth to occur. Adding information about the price dynamics behind financial contracts for electricity can be useful when scaling up both consumption, production, and investment in electricity.

This paper started with an overview to the physical electricity market. Due to bottlenecks and different bidding areas in the physical market, trading and risk management of the spot price between actors is not possible. The financial electricity market enables market actors located in different bidding areas to trade with each other to do risk management operations. Market speculators contributes to increasing market liquidity in the futures, DS-futures and options contracts in the financial electricity market.

From the introduction, we recall that the main purpose of this paper was to build a two-factor stochastic volatility model where volatility has its own stochastic process, enabling rational descriptions of the volatility in financial electricity contracts. The paper seeks answer if the volatility is a process of random information flow to financial markets for electricity, or if it the volatility can be predicted by a stochastic volatility model. Thus, is returns and volatility a correlated process, or just a random walk? In more detail the paper has investigated three main topics in the following order.

The first was to identify relevant volatility properties of the front future financial contracts for electricity at Nasdaq OMX, compare quarterly and yearly contracts, capture volatility and simulate shocks. Both series had leptokurtosis features with excess kurtosis and heavy tails - properties often found in financial time series. This non-normality where confirmed by the Jarque-Bera test. The series was converted from prices into returns to make stationary time series, confirmed by the KPSS, ADF and PP tests. By the BDS, Q, Q^2 and ARCH testes the time series for quarter and year show data dependency and volatility clustering. Further on an optimal SNP GARCH model were specified by the framework of Gallant & Tauchen (1990 (dec 2017)). Despite some non-linear relationship in YEAR and some lag dependency in QUARTER, most residuals were normally distributed and non-significant, i.e. the models were optimal specified. The QUARTER time series tend to be more volatile than YEAR, with more and higher spikes and a higher long-term mean reverting value at 38 against 22. When analysing the unconditional one-step-ahead densities, QUARTER show clearly wider densities than YEAR, indicating that tomorrows volatility is more unknown for QUARTER than YEAR. In the next we looked at the Conditional One-step-ahead densities, where different shocks were added to the yesterdays (unconditional) mean to see the density distribution in todays (conditional) mean. When comparing the one-step-ahead densities for the different shocks between QUARTER and YEAR, there were some clear differences. First, one can see that the densities for QUARTER are wider and less peaked than YEAR, indicating more uncertainty after shocks for the quarter-contracts than the year-contracts. The second difference were related to the difference in peakness between the positive and negative shocks for the two contracts. Where the year-contracts had wider and flatter densities for the positive shocks, indicating more uncertainty in volatility after positive shocks than negative shocks, no such difference where found for the quarter-contracts. These results where strengthen by the Conditional Variance Function, where the purpose was to highlight any asymmetry effects. Where the analysis found a positive asymmetry effect for the YEAR, implying that positive price shocks increases volatility, no asymmetry effects where found for QUARTER. This result stands in sharp contrast to research in other energy markets, equities and commodities (Solibakke, 2014) (Egeland & Haug, 2016) (Solibakke, 2020) where negative asymmetry effects where found. Differences in asymmetry effects might come from who participate in these financial markets, and how these actors react to price changes.

The second and main topic was to create and evaluate whether a two-factor stochastic volatility model is appropriate to do step ahead prediction of volatility in financial contracts. From the optimal SNP models, a two-factor stochastic volatility model where calibrated after the EMM method of Gallant & Tauchen (1990) (2016). Optimal models where found through an iterative factor process, tested with the Chi squared test and quasi-t-ratios. When comparing the two series and the associated volatility factors, we saw that the slowly moving and persistence volatility factor V_1 was the main contributor to the volatility with the ordinary least square R^2 of 0.845 for YEAR and 0.98 for QUARTER. Volatility factor V_2 which is rapidly mean reverting to fatten tails has R^2 of 0.14 and 0.074 respectively, i.e. it is absorbing shocks fast. The leptokurtosis feature of the return distribution is due to the two factors; V_1 is moving around the long term mean, making it responsible for the many observations around the mean of the distribution, and V_2 is jumping far away from the mean making it responsible for the fatter tails in the distribution. Both volatility factors seem the be more volatile for the QUARTER time series than the YEAR time series. This result is strengthened by the re-projected volatility graph, QUARTER has more frequent and bigger movements than YEAR. Further, YEAR has 4 spikes above 25, the QUARTER has 13 spikes. In addition, QUARTER has 4 spikes over 30. The autocorrelation plot shows more persistence and data dependency for QUARTER. Overall, we can conclude that the re-projected time series captures the more volatile volatility in the QUARTER than the YEAR. The series shows that when returns become wider (narrower) volatility increases (decreases).

Third topic was to analyse trends in electricity production, reservoir levels and temperature from the Nordic and Baltic region to reveal whether there is some connection to the re-projected volatility. There seem to be some connection between production levels and mix and the volatility levels, where high volatility levels arise in the same periods as low production volumes occur. Interesting application for further studies is to look for correlation and causality between the increase in non-adjustable production methods for electricity (mainly windmills and solar panels) last 6-7 years and the increased volatility in financial electricity contracts the last 5-6 years. Further, we saw indication of connection between Norwegian reservoir levels and the re-projected volatility. In periods where the reservoir levels had larger negative deviations from the long-run median level, the financial electricity contracts had more volatility, this was strongest in the QUARTER contracts. The visual inspections of temperature have led to a few interesting observations where low temperatures have corresponded with volatility spikes in both the quarterly and yearly contracts. These observations have been especially visible when the reservoir levels are low. However, all these findings should be tested empirically before conclusions are made.

This paper intended to answer whether the latent volatility is a process of random information flow to financial markets for electricity, or if it the volatility can be predicted by a stochastic volatility model. Thus, is returns and volatility a correlated process, or just a random walk? The main objective for this paper was to implement optimal two-factor stochastic volatility model with the capability to predict and capture stylized features of financial markets. Such features include serial correlation in the mean, asymmetry effects and volatility clustering. When such features are found in a time series it indicates significant data dependency in the volatility, which is a foundation for forecasting. All these features where found in the re-projected volatility for QUARTER and YEAR, the data dependency indicate that the latent volatility can be predicted by a stochastic volatility model. This result seems remarkable; yet price processes are barely predictable as prices respond differently to the information flow and unpredictable events in the market. However, the variance of the forecast error is time dependent and can be estimated by the means of past observed variations. Regardless of markets and contracts, observed volatility clustering suggest that unconditional return distributions are not normally distributed, a result at odds with the hypothesis of normally distributed price changes. By this we can conclude that returns and volatility is a correlated process, and not a process of random work. For empirical financial data analysis, stochastic volatility models perform well as a practical descriptive and forecasting device for risk managers and other practitioners. This result applies as much for participant in the financial market for electricity as in other financial markets. The SV method adds information about conditional mean and volatility, forecasting conditional volatility (through filtering), conditional variance functions (asymmetries) and mean reversion (persistence) analysis.

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