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Green investment under time-dependent subsidy retraction risk

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ABSTRACT

Driven by ambitious targets to reduce greenhouse gas emissions many countries have introduced support schemes to accelerate investments in renewable energy. However, in recent years experience showed that, over time, retraction of support schemes becomes more likely. This has a severe effect on investment behaviour. In this paper we study the effect of a potential subsidy retraction of a feed-in tariff (FIT) on investment in renewable energy capacity, where we explicitly account for the fact that the likelihood of policy retraction may change over time. We show that the range of FITs, for which it is optimal to invest immediately, decreases the longer a subsidy has been in place. If the FIT offered is too small and/or the subsidy has been installed too long ago, it is optimal for the firm to wait with investment until the subsidy is eventually retracted and free market conditions prove to be profitable enough. Furthermore, we show that whether a policy maker aiming at accelerating investment prefers investors to consider retraction risk to be time-dependent or not, depends on how much time has passed since the subsidy has been introduced at the moment the investor considers investment for the first time.

1. Introduction

Increasing the share of renewable energy (RE) production to the overall energy mix is recognized as critical in reaching ambitious targets to reduce greenhouse gas emissions (European Commission, 2015). Due to the deregulation of the majority of electricity markets worldwide, it is private investors with an objective of maximizing profit that decide whether RE projects are built or not (Abadie and Chamorro, 2014). Many countries have introduced different types of support schemes aimed at accelerating investments in RE. Governments therewith, want to ensure competitiveness of RE production and encourage investments. By 2019 nearly all countries worldwide have employed RE support policies and targets (REN21, 2019). Feed-in pricing policies, i.e. feed-in tariffs (FITs) and feed-in premiums (FIPs), have been instrumental in encourag-

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ing RE projects worldwide, since they provide a stable income to generators and help increase the bankability of projects (IRENA et al., 2018). By 2019 at least 111 countries have employed some type of feed-in policies (REN21, 2019).

Policy uncertainty in the form of sudden revisions or retractions of renewable support schemes, however, had a dramatic impact on investments in RE projects during recent years. This type of policy risk represents a significant challenge for actors in the RE sector currently (White et al., 2013) and hinders needed investments. Policy uncertainty is frequently mentioned as (one of) the key reasons for the lack of investments in RE (White et al., 2013, Climate Finance Leadership Initiative, 2019)[1] One country where the effect of policy risk recently had dramatic impact on RE investments is Australia. In 2019 investment in RE projects across the country "has slowed dramatically" after a record-breaking two years" according to the Clean Energy Council, Australia's RE association. Investors were citing national policy uncertainty as one of the two key factors for pulling back from investments (The Sydney Morning Herald, 2019). Another recent example comes from the solar industry in China, where on June 1, 2018 authorities suddenly and very unexpectedly, strictly limited new solar installations that would qualify for FITs. Analysts estimated, that this sudden policy change set a stop to at least 20GW of solar projects that were expected to be built in China in 2018 (The Economist, 2018).

From an investor's perspective experience shows that over time, retraction or revision of support schemes become more likely. One of the main reasons for this is technological development. The main motivation for policy makers to grant support to renewable electricity projects in the form of subsidies is to ensure their competitiveness compared to traditional power sources, like coal or gas. As renewable electricity technologies mature, investment in these resources becomes profitable even without generous support from governments. Therefore, firms may expect that governments eventually want to terminate these support schemes or revise them in ways that make them less generous. A recent example is China, that significantly cut the total size of its renewable power subsidies for 2020 compared to prior years[2]. China's government also announced to stop subsidising onshore wind capacity by 2021 and is reportedly considering to scale back or even abolish subsidies for the offshore wind power sector in 2022 "as the costs of renewables continue to fall and the industry becomes more economically competitive" (CXTech (2020)). Other reasons for increasing likelihood of policy retraction over time are diminishing public funds, changes of governments or the fact that RE capacity goals have been reached. There are several cases where budget constraints led regulators to retract or significantly reduce provided FITs. One prominent example is Spain, where a new government announced a drastic retroactively implemented cut of subsidies to electricity suppliers in 2013 as a consequence of a EUR 26bn tariff deficit, which was besides the effects of the financial crisis also a result of a too generous system of subsidies provided by the previous government (Financial Times, 2013). Similarly, Italy provided generous subsidies funded by power consumers leading to significant investment in RE from 2005 to 2012 but had to curtail their renewable support schemes substantially in 2015 in order to relieve consumers, who were struggling with high electricity bills in the aftermath of the economic crisis (The Wall Street Journal, 2014). Also regulators in the UK were forced to drastically cut too generous subsidies in 2015 (Department of Energy & Climate Change, 2015).

While FITs have long been considered to be the most effective scheme for accelerating development of RE sources (Couture and Gagnon, 2010, del Rio and Mir-Artigues, 2012, Ritzenhofen and Spinler, 2016), we currently can observe a shift from tariff-based instruments to competitive auctions (REN21, 2019). This may lead investors to perceive an increase in the risk of retraction of currently installed FIT schemes. Therefore we propose to study the effect of a potential subsidy retraction on investment in RE capacity assuming that the likelihood of policy retraction in not constant over time.

We develop a model in which a profit maximising investor has the option to invest in an RE project. The current subsidy scheme provides investors in RE projects with a fixed FIT for each unit produced, therewith shielding them from market uncertainty. However, investments are threatened by a potential subsidy retraction, which is assumed to become more likely the longer the subsidy scheme has been provided to investors. After subsidy retraction electricity produced must be sold on the spot or futures market at a price that varies over time. Some of the examples of such retroactive subsidy revisions have occurred in several countries in the last decade (REN21, 2015). Examples are Spain, Belgium, the Czech Republic, Bulgaria and Greece (Boomsma and Linnerud, 2015). In order to relate our results to a realistic case of green investment we present a case study considering investment in an onshore wind project using the most recent available data for the case of Europe. FITs and electricity price data is taken from Germany, where different types of FIT support schemes have been applied since 1991[3] Finally, we present extensions to our model, that account for different types of limits for the subsidy period. We first consider the case that the regulator announces the subsidy to be provided for a limited time period. Second, we analyse the case, where subsidy is announced to be retracted in case electricity market price raises to a certain level.

Applying a real options approach allows us to account for important characteristics of RE investments. RE investments entail large sunk investment costs, which are often specific to the considered project. Second, the project value depends on uncertain future framework conditions, like fluctuating electricity prices or changing support schemes. Third, investors have the option to postpone investments if current framework conditions do not justify immediate investment.

Our results show that the range of FITs, for which it is optimal to invest immediately, decreases the longer a subsidy has been in place. If the FIT offered is too small and/or the subsidy has been installed for too long, it is optimal for the firm to

[1] Recent examples are, for instance, The Climate Finance Leadership Initiative, a group of banks and asset managers assembled by Michael Bloomberg, that released a report outlining that one of the problems of securing investment for low-carbon energy in emerging-market countries is the unpredictability of government policies (Climate Finance Leadership Initiative, 2019).

[2] https://oilprice.com/Latest-Energy-News/World-News/China-Slashes-Renewable-Subsidies.html.

[3] The German *Electricity Feed-in Act* (1991) was, in fact, the first green electricity feed-in tariff scheme in the world.

wait with investment until the subsidy has been retracted and free market conditions are profitable enough. Specifically, we derive the optimal electricity market price threshold that triggers investment in this case. We find that increasing market price volatility (drift) discourages (encourages) investment, also for the cases that investment is made when the subsidy is still provided.

Furthermore, we show that considering subsidy retraction risk to be time-dependent can significantly affect the optimal investment strategy compared to the case when subsidy retraction risk is considered constant over time. While we can confirm earlier research in that policy risk discourages investment (see, e.g., Dalby et al., 2018, Boomsma and Linnerud, 2015 or Fuss et al., 2008 for similar results), it is not straightforward to conclude whether a policy maker, who aims to accelerate investment, would prefer investors to consider subsidy retraction risk to be time-dependent or not. In fact this depends on the FIT provided and the time passed since the subsidy has been introduced.

Finally, we conclude that setting a limit for the period during which the subsidy is provided both in the form of time or a specific market price, discourages investment. The best from a policy makers point of view with the main objective to accelerate investment is to announce a support scheme for an unlimited period of time.

Real options theory has been increasingly applied to investment problems in the energy sector under market uncertainty and policy change in recent years. Early contributions to work on policy and investment from a real options perspective have focused on tax policy uncertainty ((Dixit and Pindyck, 1994, Chapter 9), Hassett and Metcalf, 1999, Pawlina and Kort, 2005). Another group of papers studied the effect climate change policy uncertainty represented as exogenous event that creates uncertainty in the carbon price on investment in RE sources (Yang et al., 2008, Fuss et al., 2008). A rather recent strand of literature has analysed the effect of policy uncertainty in the form of random provision, revision or retraction of a subsidy (see, e.g., Boomsma et al., 2012, Boomsma and Linnerud, 2015, Ritzenhofen and Spinler, 2016, Eryilmaz and Homans, 2016 and Chronopoulos et al., 2016). Similar to our case (Ritzenhofen and Spinler, 2016) and (Boomsma and Linnerud, 2015) study the effect of a potential future retroactively applied subsidy retraction. Ritzenhofen and Spinler (2016) consider a case where regulators may decide to switch from a fixed-price FIT to a free-market regime. They find that uncertainty regarding future regulatory regimes delays or even reduces investment activity for FIT levels near electricity market prices and high probabilities of an imminent regime switch. Similarly, Boomsma and Linnerud (2015) conclude that the prospects of subsidy termination slows down investments if subsidy it is retroactively applied, but speeds up investments if it is not. We confirm their result that potential retroactive subsidy retraction reduces investment activity.

However, all of the aforementioned papers analysing policy uncertainty in the form of subsidy retraction consider that the likelihood of subsidy revisions is constant over time. We contribute to this strand of literature by allowing the probability of subsidy retraction to depend on the time since the support scheme was originally introduced. Specifically, we look at the case that investors perceive subsidy retraction to become more likely the longer subsidy has been provided.

There is also a growing body of empirical work studying policy risk in the power sector. The majority of this work focuses on analyzing how policy changes and/or policy uncertainty impact investments in renewables (see, for example, Eyraud et al. (2011), Lüthi and Wüstenhagen (2012), Linnerud et al. (2014), Karneyeva and Wüstenhagen (2017) and Sendstad et al. (2020)). Here, Sendstad et al. (2020) specifically, investigates the effect of retroactive policy changes on the investment decisions in renewable energy and finds that sudden unexpected policy changes deter further investment activity in renewables. We are, however, not aware of any paper that attempts to measure investors' perceived policy risk in the power sector nor attempts to measure policy risk for the power sector in general. A recent strand of literature, initiated by the work of (Baker et al., 2016), attempts to measure policy uncertainty in a more general sense referring to the probability of political decisions, events, or conditions significantly affecting current and future business conditions. Baker et al. (2016) developed a new index of economic policy uncertainty based on newspaper coverage frequency for the United States. Using firm-level data they find that policy uncertainty is associated with greater stock price volatility and reduced investment and employment in policy-sensitive sectors like defense, health care, finance, and infrastructure construction. We are however, not aware of any work using this methodology to measure policy uncertainty for the power sector.

From a more technical point of view, policy uncertainty is usually modelled in the literature by a homogeneous stochastic process with different states, each one representing a different level of subsidy (or the retraction of it). Technically, our contribution to the state of the art is to consider an non-homogeneous stochastic process to model policy uncertainty. In this case the investment problem relies on the resolution of a time-dependent optimal stopping problem where different regimes are considered. From a pure mathematical point of view, this type of optimal stopping problems are studied by Oliveira and Perkowski (2019). In this theoretical paper, the system of Hamilton-Jacobi-Bellman (HJB) equations is derived and technical conditions satisfied by value functions are presented. Since our set up fits in the one presented in the previous paper, we take into account some of the technical results of Oliveira and Perkowski (2019). Our paper contributes to the literature by presenting the specific solution of such a stopping problem and derives explicit expressions for the value function and threshold curves for the investment problem considered. This has, to the best of our knowledge, not been done before.

The remainder of this paper is organised as follows. Section 2 introduces a benchmark model, for which we present the solution procedure in Section 3. Comparative statics results for the benchmark model are presented in Section 4. We then check the robustness of our model in Section 5, relaxing a key assumption we make in the benchmark model. The solution for the robustness model is presented in Section 5.1, and the results are analysed in Section 5.3, based on a case study introduced in Section 5.2. We finally present relevant extensions in Section 6, and conclude in Section 7.

2. Model

We study a profit maximising firm that considers the option to invest in a RE project. In order to accelerate investment in new RE projects regulators are currently providing a subsidy in the form of a fixed feed-in-tariff (FIT) that offers a subsidy payment of $(p+F)$ per MWh of electricity produced during the contract. Investors are aware of the possibility that, at some random point in time, ν, regulators may retract the current subsidy scheme, which would, as a consequence, expose the firm to market risk. Similar to Boomsma et al. (2012), Boomsma and Linnerud (2015), Ritzenhofen and Spinler (2016) and Dalby et al. (2018), we are considering a single subsidy revision. After the subsidy has been retracted the firm will sell the electricity at market price per MWh of electricity produced.

We consider that retraction of the subsidy support scheme will occur at an exponentially distributed random time, denoted by ν, with intensity $\lambda(t)$, that is assumed to be increasing with time. Therefore, retraction of the subsidy support scheme becomes more likely as time goes by. For illustration purposes we assume the following time-dependent intensity $\lambda(s) = \lambda_0 s$. Then

$$F_\nu(u) = 1 - e^{-\int_u^\infty \lambda_0 s ds} = 1 - e^{-\lambda_0 u^2/2}.$$

Most of the qualitative results that we show in the paper carry over for other intensity functions, as long as they are non-decreasing in t.

In what follows, the stochastic process θ provides us the information regarding the state of the subsidy. Specifically, $\theta = 1$ while the subsidy is still active and $\theta = 0$, otherwise. Therefore,

$$\theta_s = \begin{cases} 1, & s < \nu \\ 0, & s \geq \nu \end{cases}$$

Due to the structure of ν, θ is not, a priori, a Markov process, unless one takes into account the information regarding the moment when the support scheme was implemented and the current time. Therefore, from now on, we consider the bi-dimensional pair (t, θ), where t represents the time elapsed since the moment when the support scheme was implemented. This new process is, in fact a Markov process.

In light of the information introduced in the previous paragraph, one can update the distribution of ν taking into account that $\nu > t$. We therefore, introduce a new random variable, represented by ν_t. The distribution function for ν_t can be computed as the conditional distribution of the random variable ν, as we show in the next equation:

$$P(\nu_t > t + s) = P(\nu > s + t \mid \nu > t) = e^{-\int_t^{t+s} \lambda_0 u du}.$$

The random variable ν_t denotes the (random) time remaining until retraction of the subsidy, given that it has been active in the last t units of time. Additionally, one can notice that the expected time of ν_t is equal to

$$E[\nu_t] = \frac{\sqrt{2\pi}\left(1 - \Phi\left(t\sqrt{\lambda_0}\right)\right)e^{\lambda_0 t^2/2}}{\sqrt{\lambda_0}}, \tag{1}$$

where Φ denotes the distribution function of a normal distribution, with mean 0 and variance 1. We note that $E[\nu_t]$ is a decreasing function of t and verifies $E[\nu_t] \to 0$. This highlights that the subsidy retraction is likely to occur sooner when t increases.

We furthermore assume that the electricity market price, hereafter denoted by $P = \{P_s : s \geq 0\}$, follows the following stochastic differential equation:

$$dP_s = \mu(\theta_s) P_s ds + \sigma(\theta_s) P_s dW_s, \quad P_0 = p \tag{2}$$

where

$$\mu(\theta) = \begin{cases} 0, & \theta = 1 \\ \mu, & \theta = 0 \end{cases}, \quad \sigma(\theta) = \begin{cases} 0, & \theta = 1 \\ \sigma, & \theta = 0 \end{cases} \tag{3}$$

and $W = \{W_s : s \geq 0\}$ is a Brownian motion. By construction, the process P incorporates the information regarding the activeness of the subsidy. Therefore, as long as the FIT is provided, the unit price is fixed and equal to p. In this period the firm earns $p + F$ for each unit produced by unit of time. We interpret this FIT as a premium F offered on top of the electricity price at the beginning of the planning horizon, i.e. $p + F$. Upon retraction of the subsidy, the firm earns the electricity market price, which is assumed to follow a geometric Brownian motion (as in, for example, Fleten et al. (2007), Ritzenhofen and Spinler (2016), Boomsma and Linnerud (2015) and Chronopoulos et al. (2016)), with drift μ and volatility σ. We furthermore, assume that the project is sufficiently small not to affect long-term electricity prices, i.e. the producer is a price-taker, following among others (Boomsma et al., 2012), (Ritzenhofen and Spinler, 2016) and (Boomsma and Linnerud, 2015).

Given the definition of $\mu(\theta)$ and $\sigma(\theta)$, we make the simplifying assumption that at the moment of subsidy retraction, the market price is equal to p, independently of the time elapsed since the beginning of the planning horizon. In practice however, once the subsidy is retracted, the company will sell the electricity at the market price, which is not known to the firm beforehand (as the market price evolves randomly over time). Therefore, in that case, the price of the electricity at the moment of the investment will be a random variable, with a density function that we, in general, cannot compute explicitly. This means that we cannot find closed-form expressions for the waiting region and the value function. Making this simplifying assumption allows us to derive analytical results for the problem, which will serve as an important benchmark case.

In Section 5 we relax this assumption, allowing the electricity market price to evolve according to a geometric Brownian motion from the beginning of the considered planning horizon on. In this case, we are only able to find the waiting region and the value function conditional on the market price at the retraction time. Then, to solve the (unconditional optimisation) problem, one needs to compute the expected value of the value function, which is a highly non-linear function. For that reason, these computations are, in general, only possible to do from a numerical point of view. However, we will show that the qualitative results of the benchmark model are robust to this assumption.

Furthermore, in the special case that the intensity of retraction is constant, the involved expressions and expectations turn out to be simpler, and we are able to provide an analytical result for the value function even when the price is evolving from the beginning (see Section 5.1 for the derivations of this case).

The profit function of the firm, hereby denoted $\Pi_\theta(.)$, is given by

$$\Pi_\theta(P_s) = \begin{cases} K(p+F) - C(K), & \theta = 1 \\ KP_s - C(K), & \theta = 0 \end{cases} \tag{4}$$

where $K > 0$ represents the annual production, $C(.)$ denotes the production costs, and $p + F$ represents the FIT. Upon investment the firm needs to pay a sunk cost of I.

The investment problem of the firm can then be formulated as the following optimal stopping problem:

$$V_\theta(p,t) = \sup_\tau E_{\theta,p,t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(P_s)ds - e^{-r\tau}I(K)\right], \tag{5}$$

where we use the index $\theta \in \{0, 1\}$ to denote the current state of the subsidy (non-active or active, respectively), and r is the exogenously given discount rate. Furthermore, due to the time-dependence of the intensity rate $\lambda_0 t$, at which the subsidy is retracted, we need to take time into account in the optimisation problem (5). For that reason, V_θ has an explicit dependency on the variable t, which we include notation-wise.

From the mathematical point of view, this is an optimal stopping problem, where both the profit function and the price dynamics are affected by a switching regime. In (Oliveira and Perkowski, 2019), the authors consider a more general set-up, where they deduce the system of Hamilton-Jacobi-Bellman (HJB) equations that characterise the value function and they prove existence and uniqueness of solution to the equations. Since in that case the authors deal with an exit problem, we re-write our problem (5) in a more convenient way as follows

$$\begin{aligned} V_\theta(p,t) =& \sup_\tau E_{\theta,p,t}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds - \int_0^\tau e^{-rs}\Pi_\theta(P_s)ds - e^{-r\tau}I(K)\right] \\ =& E_{\theta,p,t}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] + \sup_\tau E_{\theta,p,t}\left[\int_0^\tau e^{-rs}(-\Pi_\theta(P_s))ds - e^{-r\tau}I(K)\right]. \end{aligned} \tag{6}$$

One may notice that the computations regarding the first expected value in the second line of (6) are straightforward since it does not depend on the stopping time τ. Therefore we are left to find the solution of the second term in the second line of (6), which we denote by

$$v_\theta(p,t) = \sup_\tau E_{\theta,p,t}\left[\int_0^\tau e^{-rs}(-\Pi(P_s))ds - e^{-r\tau}I(K)\right]. \tag{7}$$

We next present the Hamilton-Jacobi-Bellman (HJB) equations that characterise $v_\theta(p, t)$. In this case, as we have two regimes (that correspond to two possible states of the subsidy: active or inactive) that influence the dynamics of the price and profit function of the firm, we need to solve two sets of HJB equations:

$$\min\left(rv_0(p,t) - \mathcal{L}_0 v_0(p,t) + \Pi_0(p), v_0(p,t) + I(K)\right) = 0, \tag{8}$$

$$\min\left(rv_1(p,t) - \mathcal{L}_1 v_1(p,t) + \Pi_1(p), v_1(p,t) + I(K)\right) = 0, \tag{9}$$

where $\mathcal{L}_i$ (for $i = 0, 1$) is the infinitesimal generator of the bi-variate process $(\theta, P) = \{(\theta_s, P_s), s > 0\}$. In view of the assumptions about θ and P, the following holds:

$$\mathcal{L}_0 v_0(p,t) = \mu p \frac{\partial v_0(p,t)}{\partial p} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 v_0(p,t)}{\partial p^2},$$

$$\mathcal{L}_1 v_1(p,t) = \frac{\partial v_1(p,t)}{\partial t} + \lambda(t)(v_0(p,t) - v_1(p,t)).$$

Finally, we note that in Oliveira and Perkowski (2019) the authors do not consider a specific profit function nor specific price dynamics. Therefore, all the computations and the analysis necessary to obtain the value functions will be presented in the next sections.

3. Model solution

In this section we present the value function of the firm depending on whether the subsidy is still present or not. We first derive the value function for the case that the subsidy has been retracted already (see Section 3.1) and then continue with deriving the value function for the case that the subsidy is still active (see Section 3.2). We then present the optimal investment strategy of the firm.

3.1. Solution when the subsidy has been retracted

In case the subsidy has been retracted, we have a standard investment problem, for which the solution is already known; see, for instance, Dixit and Pindyck (1994). Moreover, we know that for such a case the investment decision is a threshold decision, meaning that investment is optimal for large values of price ($p \geq p_0^*$) while, for small values of price ($p < p_0^*$), it is optimal to wait. Upon investment, the firms earns a perpetual value given by:

$$E_{\theta,p,t}\left[\int_0^\infty e^{-rs}(KP_s - C(K))ds\right] - I(K) = \frac{K}{r-\mu}p - \frac{C(K)}{r} - I(K).$$

Before investment (ie., $p < p_0^*$), the firm does not earn any profits. Therefore, the value of the firm is equal to the value of the option to invest. Furthermore, as the subsidy is not active, there is no longer the need for the time dependency. Therefore, we may drop the explicit dependency on t of the value function V_0.

In view of these considerations, and application of results provided by Dixit and Pindyck (1994), we end up with the result presented in the following proposition. The proofs all propositions can be found in the appendix.

Proposition 1. *The value function when the subsidy is no longer available, i.e. $V_0(p)$, is given by:*

$$V_0(p,t) \equiv V_0(p) = \begin{cases} Ap^{d_1}, & p \leq p_0^* \\ \frac{K}{r-\mu}p - \frac{C(K)}{r} - I(K), & p > p_0^* \end{cases}$$

where

$$p_0^* = \frac{d_1}{d_1 - 1}(I(K) + \frac{C(K)}{r}) \times \frac{r-\mu}{K}, \tag{10}$$

$$A = \frac{K}{r-\mu}\frac{1}{d_1}{p_0^*}^{1-d_1}, \tag{11}$$

with $d_1 = \frac{\sigma^2 - 2\mu + \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8r\sigma^2}}{2\sigma^2} > 1.$

3.2. Solution when the subsidy is still active

When the subsidy is still active, the firm's revenues would be constant upon investment. Given the electricity price p at the moment the firm decides to invest, the firm will receive by unit of electricity produced p with a top-up payment of an amount F during the life of the subsidy. Once the subsidy will be retracted, the firm will be exposed to output price uncertainty. Thus, we need to take into account the possible changes in the state process θ.

As discussed above, in order to find V_1 one has to compute the function v_1, which satisfies

$$rv_1(p,t) - \frac{\partial v_1(p,t)}{\partial t} - \lambda_0 t(v_0(p) - v_1(p,t)) + K(p+F) - C(K) = 0,$$

in the continuation region. Here we omit the time dependency of v_0 on t, as the problem is time-homogeneous in this case (see Section 3.1).

The solution of this problem is not straightforward. We present the solution in the following proposition.

Proposition 2. *Assume that F is such that it satisfies the following condition*

$$\underline{F} \equiv -\frac{\sigma^2}{2}\frac{1}{K}\frac{C(K) + rI(K)}{r}d_1 < F < \frac{1}{K}\frac{rI(K) + C(K)}{1 - E[e^{-r\nu_0}]} \equiv \overline{F}.$$

Then the value function for the optimal stopping problem presented in (6) is given by:

$$V_1(p,t) = \begin{cases} V_{1,1}(p,t), & p < c_1^* \\ V_{1,2}(p,t), & (p,t) \in [c_1^*, p_0^*) \times [t_1^*(p), \infty) \\ V_{1,3}(p,t), & (p,t) \in [c_1^*, p_0^*) \times [0, t_1^*(p)) \\ V_{1,4}(p,t), & p > p_0^* \end{cases}$$

where,

$$V_{1,1}(p,t) = V_{1,2}(p,t) = Ap^{d_1}E\left[e^{-r\nu_t}\right],$$

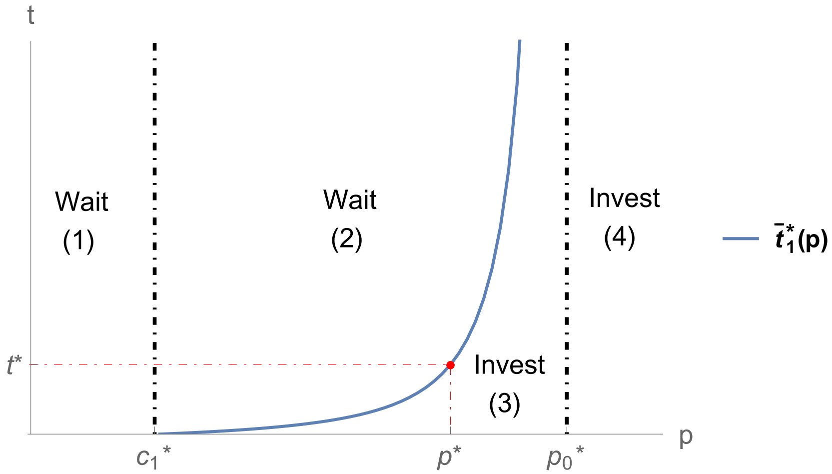

Fig. 1. Investment and continuation regions in the (p, t)-plane.

$$V_{1,3}(p,t) = V_{1,4}(p,t) = \left(\frac{K(p+F)}{r} - \frac{C(K)}{r} - I(K) \right) + \left(\frac{Kp}{r-\mu} - \frac{K(p+F)}{r} \right) E[e^{-r\nu_t}],$$

and p_0^* *is given by (11). Finally* $t_1^*(p)$ *is uniquely defined by the equation:*

$$Ap^{d_1} E\left[e^{-r\nu_{t_1^*(p)}}\right] = \left(\frac{K(p+F)}{r} - \frac{C(K)}{r} - I(K) \right) + \left(\frac{Kp}{r-\mu} - \frac{K(p+F)}{r} \right) E[e^{-r\nu_{t_1^*(p)}}]. \tag{12}$$

Additionally, $c_1^* \equiv c_1^*(F)$ *verifies the following conditions:*

(i) $c_1^*(F) \in (0, p_0^*)$ and $t_1^*(c_1^*) = 0$, if $\underline{F} < F < \overline{F}$

(ii) $\lim_{F \nearrow \overline{F}} c_1^*(F) = 0$, and $\lim_{F \searrow \underline{F}} c_1^*(F) = p_0^*$.

The results presented in Proposition 2 show that we have to distinguish between four regions for the value function in case the subsidy is still active. Fig. 1 illustrates the shapes of the different regions, assuming that the intensity function, $\lambda(t)$, at which the subsidy is retrieved is non-decreasing in t. If the initial value of the process P is strictly smaller than a threshold c_1^* the firm is in the continuation region. We note that the threshold c_1^* is defined implicitly by the equation $t_1^*(c_1^*) = 0$, as stated in Proposition 2. We indicate this continuation region by "Wait (1)" in Fig. 1. For such low values of p it is in fact never optimal for the firm to invest, neither before nor after the subsidy is retracted, independently of how much time has passed since the subsidy has been installed. Once the subsidy has been retracted the firm will invest as soon as the price hits the level p_0^*. The value function in this region, $V_{1,1}(.,.)$, is therefore equal to the value of the option to invest at p_0^*, scaled by the discount factor $E[e^{-r\nu_t}]$, which accounts for the expected time it takes until the subsidy is retracted, for a given t.

If the current value of the process P is large enough, specifically $p \geq p_0^*$, it is optimal for the firm to invest immediately independent on whether the subsidy is still provided or not. We indicate this region by "Invest (4)" in Fig. 1. The value function in this region, $V_{1,4}(.,.)$ consists of two terms. The first term represents the discounted expected future cash flows of the project in case the firm would receive the FIT forever minus the investment cost. The second term accounts for the additional revenues earned in the open market as soon as the subsidy is retracted, scaled by a factor accounting for the expected time until the subsidy is retracted.

For intermediate values of p between c_1^* and p_0^*, whether the firm is in the continuation region or in the stopping region depends on how much time has passed since the subsidy was installed (represented by t). Here we distinguish two cases. If p is such that $[c_1^*, p_0^*]$ and sufficiently much time has passed since the introduction of the subsidy, i.e. $t > t_1^*(p)$, it is never optimal for the firm to invest before the subsidy has been retracted. Once the subsidy has been retracted it is optimal for the firm to invest at the moment the price hits the level p_0^*. This is similar to the case in the first continuation region (Wait (1)). The value function in this continuation region, indicated by "Wait (2)" in Fig. 1, denoted by $V_{1,2}(.,.)$, is therefore equal to the value of the option to invest once the subsidy has been retracted scaled by a discount factor accounting for the expected time until subsidy retraction. If, however, relatively little time has passed since the subsidy has been introduced, specifically $t \leq t_1^*(p)$, it is optimal for the firm to invest immediately. The value function for this case, $V_{1,3}(.,.)$, is equal to the value in the other investment region. This investment region is indicated by "Invest (3)" in Fig. 1. Note that the boundary curve separating the investment and continuation regions in this case increases with the value of p, as well as the time t, for which the subsidy has been active already. The particular shape of the boundary curve depends on the particular intensity

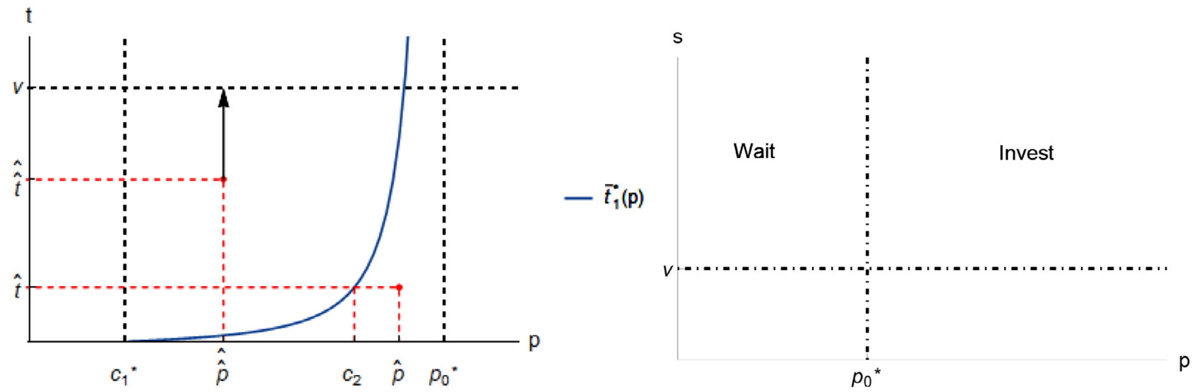

Fig. 2. Investment and continuation regions for the case that the subsidy is active (left graph) and that the subsidy has been retracted already (right graph).

rate at which the subsidy is expected to be retrieved. However, as long as $\lambda(.)$ is a non-decreasing function, the retraction of the subsidy becomes more likely as time passes, and therefore, one may expect that, also in this case, the boundary curve is upward sloping in the (p, t)-plane[4]

Note that the investment strategy (given the subsidy is still present) depends on how much time has passed since the subsidy introduction at the moment the firm considers the investment problem for the first time. In case the firm considers investment at the moment the subsidy has been introduced, i.e. $t = 0$, then the horizontal line at $t = 0$ in Fig. 1 represents the decision rule as a function of p. For $t = 0$ the firm will invest if $p > c_1^*$. If the firm considers the investment problem for the first time when $t > 0$ time has already passed since the subsidy introduction, then the threshold curve $t_1^*(p)$ determines the investment strategy. For $t = t^*$, for example, the firm invests immediately if $p > p^*$, as indicated in Fig. 1. Otherwise, it is optimal for the firm to wait with investment until the subsidy has been retracted and the price has reached the threshold p_0^*.

Fig. 2 illustrates the investment and waiting region for both cases. The left graph shows the different regions for the case that subsidy is active and the right graph illustrates the investment and waiting regions for the case that subsidy has been already retracted. Consider the point $(\hat{t}, \hat{p})$ indicated in the left graph of Fig. 2. If the firm considers investment for the first time at $\hat{t}$ for a value of $p = \hat{p}$, then it is optimal for the firm to invest immediately. The time period it takes until subsidy is eventually retracted then only affects the project revenues earned but not the investment strategy. Now consider the point $(\hat{\hat{t}}, \hat{\hat{p}})$ inside the continuation region. If the firm considers investment for the first time after $\hat{\hat{t}}$ time units have passed since the introduction of the subsidy, the optimal decision of the firm is to wait with investment. In fact, the firm will first wait until the subsidy has been retracted. This movement in time is indicated by the vertical arrow towards the retraction time v. Once the subsidy has been retracted the optimal investment strategy is illustrated in the right graph of Fig. 2. The firm will then continue waiting until the price process $P(s)$ hits the investment threshold p_0^*, at which it is optimal to invest.

The following remark comments on the investment strategy that results when a too small or too large FIT is offered to the firm.

Remark 1.

(a) If the decision-maker offers a FIT equal to $p + \overline{F}$ for each unit produced, then it holds that $c_1^* = 0$. This means that it is optimal for the firm to invest for any initial price p given investment is considered at $t = 0$. As a consequence, the optimal strategy illustrated in Fig. 1 changes in the sense that the waiting region (1) disappears; Moreover, when $F > \overline{F}$, the waiting region (1) will not exist either and the investment region (regions (3) plus (4)) will be larger than the one obtained when $F = \overline{F}$ as for this case it holds that $t_1^*(p) > 0$ for every $p \in [0, p_0^*)$. This means, that it is optimal for a firm to invest for every price p as long as less than $t_1^*(p = 0)$ time has passed since the subsidy has been provided.

(b) If $F = \underline{F}$, then both the waiting region (2) and the investment region (3) depicted in Fig. 1 will disappear because $\lim_{F \searrow \underline{F}} c^*(F) = p_0^*$, which implies that the boundary t_1^* turns into a vertical line in p_0^*. In this case, the firm does not have any additional incentive to make the decision earlier than in the case that subsidy is not provided. It is also important to note that when $F < \underline{F}$, the investment decision will be optimal only for prices greater than p_0^*.

Remark 1 shows that if the FIT offered is sufficiently smaller than the current market price, specifically, if the FIT offered is smaller or equal to $p + \underline{F}$ (note that $\underline{F} < 0$), then it is more profitable for the firm to invest in the free electricity market

[4] Indeed, one may confirm that the proof of Proposition 2 in Appendix A.2 only rely on the monotony of the function $\lambda(t)$ and not on its specific expression.

where no subsidy is provided. This means that policy makers cannot speed up investment by offering a FIT below this level. In the following we will focus on analysing the cases that F is set such that the condition presented in Proposition 2 holds.

In the next proposition, we provide the expected time to undertake the investment decision given it is not optimal to invest immediately.

Proposition 3. *In case the current price is p, the time elapsed since the introduction of the subsidy is equal to t, the subsidy is still active and it is not optimal for the firm to invest immediately, the expected time to undertake the investment decision, which we denote by* $E_{1,t}[\tau_I]$, *is given by*

$$E_{1,t}[\tau_I] = \left[E[\nu_t] + \frac{\ln \frac{p_0^*}{p}}{\mu - \frac{1}{2}\sigma^2} \right] 1_{\{t > t_1^+(p), \mu > \frac{1}{2}\sigma^2\}}, \tag{13}$$

where $E[\nu_t]$ *is given by Equation (1), and* 1_A *denotes the indicator function of proposition A*[5]

The expression (15) for the expected investment time stated in Proposition 3 should be understood as follows. If the time t when the firm considers investment is such that $t < t_1^*(p)$, it is optimal for the firm to invest immediately. This means that in this case the expected time to investment is equal to zero. In case $t > t_1^*(p)$, however, it is not optimal for the firm to invest immediately. Then the firm first waits until the subsidy is retracted (which will take $E_{1,t}[\nu_t]$ time, on average). Once the subsidy is retracted the firm will wait until the market price increases up to p_0^* (which will take $\left[\frac{\ln \frac{p_0^*}{p}}{\mu - \frac{1}{2}\sigma^2} \right]$ time, on average), and then take the investment decision. Note that the expected time for the price to hit p_0^* is finite if and only if $\mu > \frac{1}{2}\sigma^2$ (see Willmott et al., 1995).

4. Comparative statics results

In this section we will first present some analytical results related to the behaviour of the investment boundary in key parameters. We then compare how the optimal investment strategy and the expected time of investment differs when the firm considers the retraction risk to be constant compared to time-dependent. In order to allow comparison we also present the analytical results for the case of constant retraction risk, i.e, we consider the case $\lambda(t) = \hat{\lambda}$. Note that we assume here that the intensity rate of retraction increases linearly with time, i.e., $\lambda(t) = \lambda_0 t$, order to derive the expressions for the relevant quantities explicitly.

We note again that for the case that the subsidy is no longer active, the problem is a standard one. Detailed results regarding the comparative statics of this case can be found in the literature (see, for example, Dixit and Pindyck (1994)) and can be easily derived analytically. Indeed, one can prove that p_0^* increases with σ and decreases with μ. This means the higher the market price uncertainty the larger the investment threshold as the value of the option to invest increases. The higher the trend of the price process, the smaller the investment threshold as the value of the project increases and the firm is more eager to undergo investment and therewith, receive the project value. In the following we now focus on the results related to the investment strategy when the subsidy is still active. Proposition 4 presents results on the behavior of the investment threshold curve in key parameters.

Proposition 4. *The investment threshold curve,* $t_1^*(p)$, *increases with* μ *and F and decreases with* σ *and* λ_0, *i.e.*

$$\frac{\partial t_1^*(p)}{\partial \mu} > 0, \ \frac{\partial t_1^*(p)}{\partial \sigma} < 0, \ \frac{\partial t_1^*(p)}{\partial \lambda_0} < 0, \ \frac{\partial t_1^*(p)}{\partial F} > 0.$$

The boundary c_1^* *decreases with* μ *and F and increases with* σ *and* λ_0, *i.e.*

$$\frac{\partial c_1^*}{\partial \mu} < 0, \ \frac{\partial c_1^*}{\partial \sigma} > 0, \ \frac{\partial c_1^*}{\partial \lambda_0} > 0, \ \frac{\partial c_1^*}{\partial F} < 0.$$

The results presented in Proposition 4 show that keeping all other parameters constant, increasing the volatility (drift) decreases (increases) the optimal investment boundary, therewith decreasing (increasing) the investment region for the case that the subsidy is active. This is not surprising as higher electricity price uncertainty increases the value of the firm's investment opportunity as it will receive the electricity price once the subsidy has been retracted. Therefore, the firm demands a higher FIT (i.e. larger p, as we keep F constant for this case) to undergo investment immediately. A higher electricity price drift, however, increases the value of the project. Therefore, for higher drift the firm has a larger incentive to invest immediately, which means that the range of prices p for which it is optimal to invest immediately at the first time the firm considers investment, is larger. Increasing F and therewith, the FIT offered, has the same effect. Proposition 4 also shows that increasing the likelihood of policy retraction, i.e. increasing λ_0, decreases the investment region, and therewith the range of FITs for which the firm is willing to invest immediately. The reason for that is that increasing λ_0 decreases the project value, as the firm expects to receive the FIT over a shorter period of time. Therefore, it demands higher levels of the FIT

[5] If $\mu < \frac{1}{2}\sigma^2$, then this expected time is infinite, as follows from standard results of stochastic processes; see, for instance, Kulkarni (2016).

to justify immediate investment when subsidy is (still) provided. From a policy makers perspective that aims to accelerate investment, policy risk is damaging as it reduces the firm's willingness to invest. If investors perceive policy risk to be larger, they will be more hesitant to invest. In that case regulators would need to offer a higher FIT in order to trigger the same amount of investment.

We also note that the result of Proposition 4 shows that in case the price p lies in (c_1^*, ∞) and μ or F increase, the firm may still decide to invest even if more time elapsed since the introduction of the subsidies. On the contrary, if σ increases, it may be too late for a firm to invest in that range of prices. In that case the investment will only occur after the subsidy is removed and the market price reaches the value p_0^*. This means that our results are coherent with the standard real options result that increasing the volatility postpones the investment decision.

We now analyze how the investment strategy differs when subsidy retraction risk is assumed to be time-dependent compared to the case that the firm assumes it to stay constant over time. Therefore, we provide the equivalent of Proposition 2 for the case $\lambda(t) = \hat{\lambda}$ in the following proposition. This means that the probability that the subsidy will be retracted is assumed to be constant with a rate $\hat{\lambda}$. In this case the time until retraction is exponentially distributed, and we are dealing with a homogeneous process, as opposition of the time-dependent case, that leads to a non-homogeneous process.

Proposition 5. *Assume that $\lambda(t) = \hat{\lambda}, \forall t$, and F is such that*

$$\underset{\sim}{F} \equiv \frac{C(K) + rI(K)}{K} - p_0^* < F < \frac{r+\hat{\lambda}}{K}\left(I(K) + \frac{C(K)}{r}\right) \equiv \tilde{F}.$$

Then, the value function for the optimal stopping problem presented in (5) is given by:

$$V_1(p) = \begin{cases} \frac{Ap^{d_1}\hat{\lambda}}{r+\hat{\lambda}} & p < p^* \\ \frac{K(p+F)}{r+\hat{\lambda}} - \frac{C(K)}{r} + \frac{Kp}{r-\mu}\frac{\hat{\lambda}}{r+\hat{\lambda}} - I(K) & p \geq p^*, \end{cases}$$

where p^* is the smallest solution of the following equation:

$$\frac{A(p^*)^{d_1}\hat{\lambda}}{r+\hat{\lambda}} = \frac{K(p^*+F)}{r+\hat{\lambda}} - \frac{C(K)}{r} + \frac{Kp^*}{r-\mu}\frac{\hat{\lambda}}{r+\hat{\lambda}} - I(K). \tag{14}$$

The expected time to take the decision to invest, in case $p < p^*$, is given by

$$E_1[\tau_I] = \left[\frac{1}{\hat{\lambda}} + \frac{\ln\frac{p^*}{p}}{\mu - \frac{1}{2}\sigma^2}\right] 1_{\{\mu > \frac{1}{2}\sigma^2\}}.$$

For the case that subsidy has been retracted already the results are the same as presented in Proposition 1.

In case the retraction probability is considered to stay constant over time, it is optimal for the firm to undergo investment immediately the first time it considers investment, if p is larger or equal than the threshold p^*. Otherwise, it is optimal to wait until the subsidy is retracted and the electricity market price hits the threshold p_0^* (given by equation (11)). The optimal decision in this case is independent of how much time has passed since the subsidy has been introduced.

We now compare the cases of constant versus time-dependent retraction probability. Note that if the following relation between the intensity parameters λ_0 and $\hat{\lambda}$ holds:

$$\frac{\hat{\lambda}^2}{\lambda_0} = \frac{2}{\pi},$$

then the expected time to retraction at time $t = 0$ is the same for the two cases, i.e. $E[\nu_0] = \frac{1}{\hat{\lambda}}$. As $E[\nu_t]$ is a decreasing function of t, it follows that

$$E[\nu_t] < \frac{1}{\hat{\lambda}}, \text{ for } t > 0,$$

which means that as time since the introduction of the subsidy passes, the expected time until retraction in the non-homogeneous case (i.e., when $\lambda(t) = \lambda_0 t$) decreases and the difference between this situation and the homogeneous one increases. For the following analysis we will assume that the expected time to retraction at time $t = 0$ is the same for the two cases.

Fig. 3 illustrates how the investment strategies of the cases of constant and time varying retraction probability differ. The blue solid line illustrates the investment boundary $t_1^*(p)$ for the case of time-dependent retraction risk, while the blue dashed line represents the boundary between the waiting and investment region for the constant case. Note that our numerical results indicate that $c_1^* < p^*$ always holds. As depicted in Fig. 3, one may define $t^* \equiv t_1^*(p^*)$, which represents the last moment since subsidy introduction, at which it is still optimal to invest for a FIT scheme paying $p^* + F$ per unit produced if the firm considers retraction risk to be time-dependent.

We find that if $t \leq t^*$ the range of FITs, $p + F$, for which the firm will invest immediately is larger if it considers retraction probability to be time-dependent. If however, $t > t^*$, the opposite holds. A firm that considers retraction probability to be

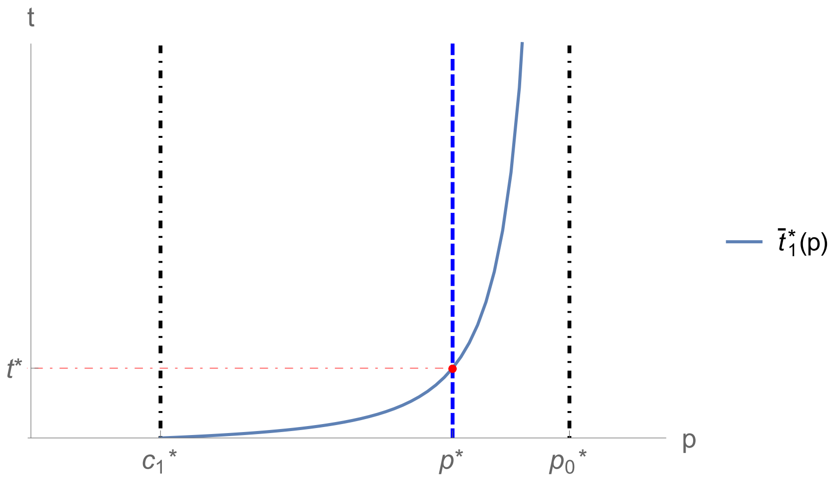

Fig. 3. Optimal investment regions for the case of constant and time-dependent subsidy retraction probability assuming that the expected time of retraction at $t = 0$ is equal for the two cases.

time-dependent will invest immediately, for a smaller range of FITs than in the case it considers retraction probability to be constant. From a policy makers point of view that aims to speed up investment, it is therefore, not straightforward to conclude whether it would prefer investors to consider retraction probability to be constant or increase over time. In fact, at the moment of subsidy introduction, it would prefer investors to consider retraction probability to be time-dependent. If, however, subsidy has been introduced at least a time period of t^* ago, it would prefer that investors consider retraction probability to be constant.

5. Robustness

In this section, we relax the assumption that the electricity market price starts to evolve stochastically only after the subsidy has been retracted. We will show that the comparative statics results for the benchmark model remain for this more general case. In the following we first state the model solution in Section 5.1, and then show results in Section 5.3, based on a case study introduced in Section 5.2.

5.1. Model solution

We now assume that the electricity price follows a geometric Brownian motion from time zero on. Therefore, the process P defined by Equations (2)-(3) is now replaced by

$$dP_s = \mu P_s ds + \sigma P_s dW_s, \quad P_0 = p. \tag{15}$$

We note that in this case P_{ν_t} is a random variable, distinct from P_t, for a general deterministic t. Although both are random, their density function is not the same. Moreover, $E[P_{\nu_t}] \neq E[P_t] = pe^{\mu t}$, as we show in the following proposition.

Proposition 6. *The expected value of the electricity price at the time of retraction of the subsidy, $E[P_{\nu_t}]$, when $\lambda(t) = \lambda t$, is given by*

$$E[P_{\nu_t}] = p + p\mu\sqrt{\frac{2\pi}{\lambda_0}}e^{\frac{(\mu-\lambda_0 t)^2}{2\lambda_0}}\Phi\left(\frac{\mu-\lambda_0 t}{\sqrt{\lambda_0}}\right),$$

where Φ denotes the distribution function of a standard normal random variable.

Assuming that all other model characteristics presented in Section 2 remain unchanged, then the value function, now denoted by $\overline{V}_\theta$, is still defined as in equation (5). As in the benchmark case, the decomposition in (6) is still valid for $\overline{V}_\theta$. The second term in the second line of (6) is now defined by $\overline{v}_\theta$.

Relaxing the assumption regarding the electricity price development does not change the value function when the subsidy is not active, i.e. $\overline{V}_0 \equiv V_0$. For this case, the investment region remains the same as for our benchmark model presented in Proposition 1. If the subsidy has been already retracted, it is optimal to invest once the selling price rises above p_0^* as given in equation (11), and wait for investment, otherwise.

When the subsidy is still active, the value function $\overline{V}_1$ can be computed noticing that

$$E_{1,p,t}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] = \frac{K(p+F)}{r} - \frac{C(K)}{r} + E_{1,p,t}\left[e^{-r\nu_t}\left(\frac{KP_{\nu_t}}{r-\mu} - \frac{K(p+F)}{r}\right)\right],$$

and $\bar{v}_1$, which is the solution to the stopping problem stated in equation (7), satisfies the HJB equation

$$\min\left(r\bar{v}_1(p,t) - \tilde{\mathcal{L}}_1\bar{v}_1(p,t) + \Pi_1(p), \bar{v}_1(p,t) + I(K)\right) = 0,$$

where $\tilde{\mathcal{L}}_1$ is the operator defined by

$$\tilde{\mathcal{L}}_1\bar{v}_1(p,t) = \frac{\partial \bar{v}_1(p,t)}{\partial t} + \mu p\frac{\partial \bar{v}_1(p,t)}{\partial p} + \frac{1}{2}\sigma^2 p^2\frac{\partial^2 \bar{v}_1(p,t)}{\partial p^2} + \lambda(t)(v_0(p) - \bar{v}_1(p,t)). \tag{16}$$

In order to introduce a representation of the value function, one may notice that the main difference between the benchmark model and the robustness model is the fact that the market price at the retraction moment is unknown for this case. Thus, the relationship between the initial price p and the price when the subsidy retraction occurs, P_ν, is also unknown. In case P_ν is known, although different from p, an analytical representation for the value function $\overline{V}_1$ would be possible to find. To facilitate the explanation, let us assume that $P_\nu = p_1$ is deterministic and known. The investment problem in this case would be given by

$$V_1^A(p,p_1,t) = \sup_\tau E_{1,p,t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(P_s)ds - e^{-r\tau}I(K)\,\middle|\,P_\nu = p_1\right], \tag{17}$$

where P is defined as in (20), and therefore $V_1^A(p,p_1,t)$ represents the value function if we would know the market price at the moment that the subsidy is retracted. The value function V_1^A for that case can be derived along the same lines as in Proposition 2. The following proposition states the solution for this problem.

Proposition 7. *Assume that F is such that it satisfies the following condition $\underline{F} < F < \overline{F}$, where $\underline{F}$ and $\overline{F}$ are defined in Proposition 1. Then the value function V_1^A is given by:*

$$V_1^A(p,p_1,t) = \begin{cases} V_{1,1}^A(p,p_1,t), & p < \tilde{p} \\ V_{1,2}^A(p,p_1,t), & (p,p_1) \in [\tilde{p},\infty) \times (0, c_{1,A}^*(p)) \\ V_{1,3}^A(p,p_1,t), & (p,p_1,t) \in [\tilde{p},\infty) \times [c_{1,A}^*(p), p_0^*) \times [t_{1,A}^*(p,p_1),\infty) \\ V_{1,4}^A(p,p_1,t), & (p,p_1,t) \in [\tilde{p},\infty) \times [c_{1,A}^*(p), p_0^*) \times [0, t_{1,A}^*(p,p_1)) \\ V_{1,5}^A(p,p_1,t), & (p,p_1) \in [\tilde{p},\infty) \times [p_0^*,\infty) \end{cases}$$

where,

$$V_{1,1}^A(p,p_1,t) = V_{1,2}^A(p,p_1,t) = V_{1,3}^A(p,p_1,t) = Ap_1^{d_1}E\left[e^{-r\nu_t}\right],$$

$$V_{1,4}^A(p,p_1,t) = V_{1,5}^A(p,p_1,t) = \left(\frac{K(p+F)}{r} - \frac{C(K)}{r} - I(K)\right) + \left(\frac{Kp_1}{r-\mu} - \frac{K(p+F)}{r}\right)E[e^{-r\nu_t}],$$

p_0^* is given by (11) and $t_{1,A}^*(p,p_1)$ is uniquely defined by the equation

$$Ap_1^{d_1}E\left[e^{-r\nu_{t_{1,A}^*(p,p_1)}}\right] = \left(\frac{K(p+F)}{r} - \frac{C(K)}{r} - I(K)\right) + \left(\frac{Kp_1}{r-\mu} - \frac{K(p+F)}{r}\right)E\left[e^{-r\nu_{t_{1,A}^*(p,p_1)}}\right].$$

Additionally, $c_{1,A}^*(p) \equiv c_{1,A}^*(p;F)$ verifies the following conditions:

(i) For $p \in (0,p_0^*)$, $c_{1,A}^*(p;F) \in (0,p_0^*)$ and $t_{1,A}^*(p,c_1^*) = 0$, if $\underline{F} < F < \overline{F}$

(ii) $\lim_{(p;F)\to(0;\overline{F})} c^*(p;F) = 0$, and $\lim_{(p;F)\to(p_0^*;\overline{F})} c^*(p;F) = p_0^*$.

Next we show that the value function of the robustness model can be stated in terms of this value function V_1^A.

As P_{ν_t} is in fact unknown, then, by using the strong Markov property, one can obtain the value function, $\overline{V}_1$, taking the expected value of V_1^A with respect to P_{ν_t}. Moreover, using similar arguments, the corresponding threshold boundary, that we denote by $\bar{t}_1^*(p)$, is equal to $\bar{t}_1^*(p) = E_p[t_1^*(p,P_{\nu_{t_1^*}})]$, where $t_1^*(p,P_{\nu_t})$ is the boundary splitting the waiting and continuation regions when P_{ν_t} is unknown. Additionally, $\bar{c}_1^*$ is the price level such that $\bar{t}_1^*(\bar{c}_1^*) = 0$.

The value function for the robustness model can now be presented as in the following proposition.

Proposition 8. *The solution of the optimal stopping problem (5), when the process P satisfies (20), is given by*

$$\overline{V}_1(p,t) = E_{1,p,t}\left[V_1^A(p,P_{\nu_t},t)\right].$$

Since P_{ν_t} is unknown, it is not possible to recover the value of the firm from Proposition 7 as $V_1^A(p,P_{\nu_t},t)$ is a random variable. Proposition 8 states that the value of the firm is in fact equal to the expected value of $V_1^A(p,P_{\nu_t},t)$.

In the special case that the intensity rate of retraction is constant with time, i.e., if $\lambda(t) = \hat{\lambda}$, we may derive analytical results, as we show next. In this case, due to the memoryless property of the exponential distribution, it is possible to find analytical expressions for both the expected value of the market price at the retraction time ν_t and for the value function when the subsidy is active.

Table 1
Parameter values for the base case.

Parameter	Value	Unit	Description
I	2 600 000	EUR	Investment cost
K	5200	MWh	Annual production of power plant
C_0	8.44	EUR/MWh	Marginal production cost
$C(K)$	43,900	EUR/year	Total production cost
p	35.08	EUR/MWh	Base-load electricity start price
$p+F$	59.4	EUR/MWh	Feed-in-Tariff
λ_0	$\frac{\pi}{200}$	-	
σ	0.185	-	Electricity price volatility
μ	0	-	Electricity price drift
r	0.03	-	Discount rate

Proposition 9. *In the particular case that $\lambda(t) \equiv \hat{\lambda}$, we have:*

$$E[P_\nu] = E[P_{\nu_t}] = \frac{\hat{\lambda}}{\hat{\lambda} - \mu}, \quad \text{for } \hat{\lambda} > \mu.$$

Since in this particular case the infinitesimal generator of the process (P, θ) is not time dependent

$$\mathcal{L}_1 v_1(p) = \mu p \frac{\partial v_1(p)}{\partial p} + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2 v_1(p)}{\partial p^2} + \hat{\lambda}(v_0(p) - v_1(p)), \tag{18}$$

the value functions are time independent ($\bar{V}(p,t) \equiv \bar{V}(p)$) and can be obtained solving the HJB equations (8) and (9) (considering now (24)). As explained above, the value function $\bar{V}_0(p) = V_0(p)$. Therefore it remains to present the value function $\bar{V}_1$ for this case.

Proposition 10. *The value function for the constant intensity rate case when the subsidy is still present is given by:*

$$\overline{V}_1(p) = \begin{cases} Bp^{\beta_1} + Ap^{d_1} & p < \bar{p}^* \\ \frac{K(p+F)}{r+\hat{\lambda}} - \frac{C(K)}{r} + \frac{Kp}{r-\mu}\frac{\hat{\lambda}}{r-\mu+\hat{\lambda}} - I(K) & p > \bar{p}^* \end{cases}$$

where d_1 is defined in Proposition 1 and

$$\beta_1 = \frac{1}{2}\left[1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + \frac{8(r+\hat{\lambda})}{\sigma^2}}\right].$$

Additionally, B and $\bar{p}^*$ satisfy the following system of equations:

$$B(\bar{p}^*)^{\beta_1} + A(\bar{p}^*)^{d_1} = \frac{K(p+F)}{r+\hat{\lambda}} - \frac{C(K)}{r} + \frac{Kp}{r-\mu}\frac{\hat{\lambda}}{r-\mu+\hat{\lambda}} - I(K), \tag{19}$$

$$B\beta_1(\bar{p}^*)^{\beta_1-1} + Ad_1(\bar{p}^*)^{d_1-1} = \frac{K}{r+\hat{\lambda}} + K\frac{\hat{\lambda}}{r-\mu+\hat{\lambda}}. \tag{20}$$

In the following we introduce a case study, which will present the basis for the comparative statics analysis.

5.2. Case study

Our case study considers an investment in a single onshore wind turbine using the most recent available data for Europe. The parameter values used in our calculations below are summarized in Table 1.

The parameters are based on an average onshore wind turbine in Europe of 2.7 MW ((Wind Europe, 2018, Section 1.6)). Given an average onshore wind capacity factor for a wind turbine in Europe of 22% ((Wind Europe, 2018, Section 1.5)), the annual production quantity of the power plant is set to 5200 MWh[6] The average CapEx per MW was approximately equal to 1.3 MEUR/MW in 2018 (Wind Europe, 2018). Following Ritzenhofen and Spinler, 2016 in assuming initial investment cost are given by about 75% of CapEx, while 25% constitute variable production cost, we set the investment cost equal to 2 600 000 EUR and assume marginal production cost of 8.44 EUR/MWh.

[6] Note that we, by applying the profit function presented in Eq. 4, assume that the annual production quantity is constant. Renewable electricity generation is generally largely dependent on weather conditions, making production highly variable both in the short and medium term. However, production is more predictable and less variable on the long run, e.g. on yearly time scales. In the context of long term investment decisions we, therefore, do not consider variability in renewable energy generation for our analysis (see Boomsma et al., 2012, Dalby et al., 2018 and Bigerna et al., 2019 for similar assumptions).

Regarding the other parameters we consider the case of Germany, where a FIT support scheme was employed through the *Renewable Energy Sources Act 2000* (Germany) and came into force on April 1, 2000 (IEA, 2016)[7] Since then the support scheme has been revised several times. In 2014 the scheme was adapted in terms of the level of FITs offered for the last time so far. From 2014 on, Germany was supporting onshore wind turbine projects with an average of 89.00 EUR/MWh for the first five years of the project lifetime and 49.50 EUR/MWh, thereafter (German Federal Ministry of for Economic Affairs and Energy, 2014)[8] Given this information we set the FIT used for our case study equal to 59.4 EUR/MWh[9] Note that in 2017 the support scheme was revised again affecting projects commissioned from 2019 on[10]

Similar to Fleten et al. (2007), Boomsma et al. (2012) and Ritzenhofen and Spinler (2016) we use forward and futures prices to estimate the electricity spot price process. We use weekly one-year base-load electricity future prices in Germany between January 1, 2009 and Dec 27, 2018 stated at the European Energy Exchange to estimate the drift and volatility of the electricity price process as well as current prices. The electricity price drift and volatility are estimated and rounded to 0% and 18.5%, respectively.[11] As base-load electricity start price we set p equal to 35.08 EUR/MWh, which is equal to the average 1-year future base load price in 2014. The risk-adjusted discount rate of our analysis is calculated as the sum of the risk-free rate plus a risk premium reflecting the risk embedded in the project. The risk-free rate is calculated using the average observed 10-year German government bond yields over the last 10 years (ending January 1, 2019). This results in a rate of 1.428%. For the risk premium we follow the suggestion of Egli and Schmidt (2018), who state that the best indicator for project specific risk is the debt margin offered to the onshore wind projects. Using a risk premium of 1.73%[12] we round our discount rate to 3%. Finally, we set λ_0 for the base case equal to $\frac{\pi}{200}$ which results in an expected retraction time of 10 years (i.e. $E[\nu] = 10$).

5.3. Comparative statics

In this section we will first present the results for the robustness model using the base case parameter set and then check whether the qualitative results derived for the benchmark model hold for the robustness model. For the robustness model we use `Python` in order to run numerical simulations, used to compute the expected value defined in Proposition 8.

Concerning the robustness model, the investment regions were obtained computationally according to the following algorithm[13]:

Using the parameter values of the case study we derive that $\bar{c}_1^* = 23.1661$[14] and $p_0^* = 49.0536$[15] Fig. 4 illustrates the specific results for the base case parameter set. The blue line indicates the resulting investment boundary, $\bar{t}_1^*(p)$. Assuming that the subsidy has been introduced (in the considered revised form) in 2014 and that the firm considers investment in the wind project for the first time in 2018, i.e. 4 years after subsidy introduction, it is optimal to undertake investment if $p \geq 24.65$[16], or equivalently the FIT provided is larger or equal to 48.97 EUR/MWh. This means that the FIT of 59.4 EUR/MWh provided in Germany for projects commissioned before 2019, would be more than sufficient to incentivize immediate investment in the project. Note that the corresponding NPVs at the beginning of the time horizon, i.e. in 2014, for the benchmark and the robustness cases assuming the firm invests in 2018 for the boundary price of $p = 24.65$, are both equal to 972, 815.[17]

Note that we assumed that the expected retraction time at $t = 0$ (i.e. in 2014) is assumed to be equal to 10 years. Fig. 4 also indicates the investment boundary (by the orange curve) for the case that the expected retraction time at $t = 0$ is set to 5 years. This leads to a significantly larger value of p, for which the firm considers investment to be optimal in 2018. Specifically, we find that in that case the firm would undergo investment if $p \geq 32.32$ or equivalently the FIT is larger or equal to 56.64 EUR/MWh.

Fig. 5 illustrates that the shape of the investment boundary $\bar{t}_1^*(p)$, when we assume that the electricity price evolves from time zero on, is very similar to the investment curve when the electricity price process is limited to start evolving

[7] The *Renewable Energy Sources Act 2000* succeeded the German *Electricity Feed-in Act* (1991–2001), which was the first green electricity feed-in tariff scheme in the world.

[8] Note that the specific FIT levels paid for a project depend on several factors including the size of the project and actual production compared to a reference level (see German Federal Ministry of for Economic Affairs and Energy, 2014 for details).

[9] The parameter is set equal to the weighted average of the offered FIT with an assumed 20-year-lifetime, i.e. $[(5 years) * (89.00 EUR/MWh) + (15 years) * (49.50 EUR/MWh)]/(20 years) \approx$ 59.4 EUR/MWh.

[10] While the FIT level for onshore wind projects was set by the state before this revision, from 2019 on it is set through auctioning in the market.

[11] The volatility (σ) is calculated through the sample variance of a sequence of the log-differences of the weekly prices. The drift (μ) is calculated as the sum of the average of the sequence and 0.5* volatility.

[12] Egli (2019) find these rates for Italy, Germany and the UK for 2016–2017 to be equal to 1.1%, 2.3% and 1.8%, respectively. We use the average of these for the risk premium.

[13] Note that, from Propositions 7 and 8, one can notice that an alternative way to represent the value function is given by: $\overline{V}_1(p,t) = \max\left(\frac{K(p+F)}{r}E_{1,p,t}[1-e^{-r\nu_t}] + E_{1,p,t}\left[e^{-r\nu_t}\frac{KP_{\nu_t}}{r-\mu}\right] - I(K) - \frac{C(K)}{r}, E_{1,p,t}[V_0(P_{\nu_t})e^{-r\nu_t}]\right)$.

[14] Note that the thresholds $\bar{c}_1^*$ and c_1^* are defined implicitly by the equations $\bar{t}_1^*(\bar{c}_1^*) = 0$ and $t_1^*(c_1^*) = 0$. Therefore, in the latter case, one has to run Algorithm 1 for $t = 0$ and find the level of p that makes $InvWin = WaitWin$, while in the former case, one has to solve Eq. (14), for $t = 0$.

[15] Note that we consider F to lay in the interval $\underline{F} = -25.6113 < F < 93.6509 = \bar{F} = 93.6509$.

[16] This value is calculated as the solution of p to the equation $\bar{t}_1^*(p) = 4$.

[17] The calculation of the NPV in the benchmark case follows from $V_{1,3}$, as defined in Proposition 2, whereas for the robustness case it follows from $V_{1,4}^A$ and from Proposition 6. For this particular set of parameters, both NPVs are equal as $\mu = 0$, and therefore $E[P_{\nu_t}] = p$.

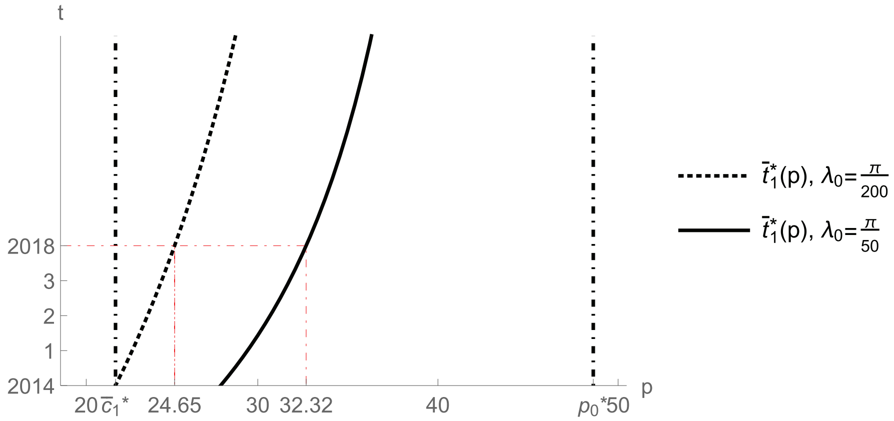

Fig. 4. Investment boundary for the robustness model ($\bar{t}_1^*(p)$) using the base case parameter set, assuming that the time horizon starts at 2014.

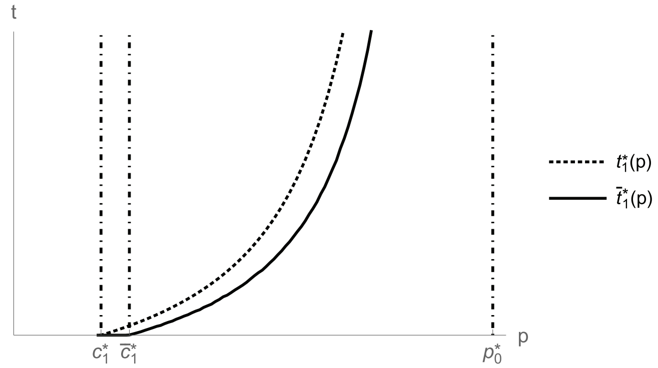

Fig. 5. Investment boundary for the benchmark ($t_1^*(p)$) and the robustness model ($\bar{t}_1^*(p)$) for the base case parameter set.

Table 2
Threshold results of the benchmark and robustness models for the case of constant retraction risk for the base case parameter set.

threshold	value
p^*	22.11
$\bar{p}^*$	32.65

once the subsidy has been retracted, i.e. $t_1^*(p)$. For the latter case the investment boundary lies farther to the right in the $(p,\ t)$-plane, i.e. $\bar{t}_1^*(p) > t_1^*(p)$, which means that the investment region when the subsidy is still active is slightly smaller. As the price process is assumed to evolve over a longer time horizon, the uncertainty faced by the firm about the project revenues after the subsidy has been retracted increases. This leads to a higher option value and therefore, the firm is more hesitant to invest immediately when the subsidy is still provided.

The plots of Fig. 6 show that the analytical comparative statics results we presented for the benchmark model still hold when we allow electricity prices to evolve from time zero on. The investment boundary $\bar{t}_1^*(p)$ decreases in σ and λ_0 and increases in μ and F.

Furthermore, in Fig. 7 we plot the boundary curves for the benchmark and robustness case for different values of the drift μ and volatility σ, showing that the qualitative results hold for several values of these parameters. In particular, it is evident that, regardless the drift and volatility values, investment in the benchmark case occurs, on average, earlier than in the robustness model. Moreover, the difference in the exercise boundary for both models decrease with decreasing volatility.

As in the case that the retraction rate is constant in time, we are able to derive explicit expressions for the investments thresholds, we check whether the qualitative results also hold in this case. In Table 2 we state the investment thresholds for the benchmark and the robustness cases for constant retraction risk, using the base case parameter set.

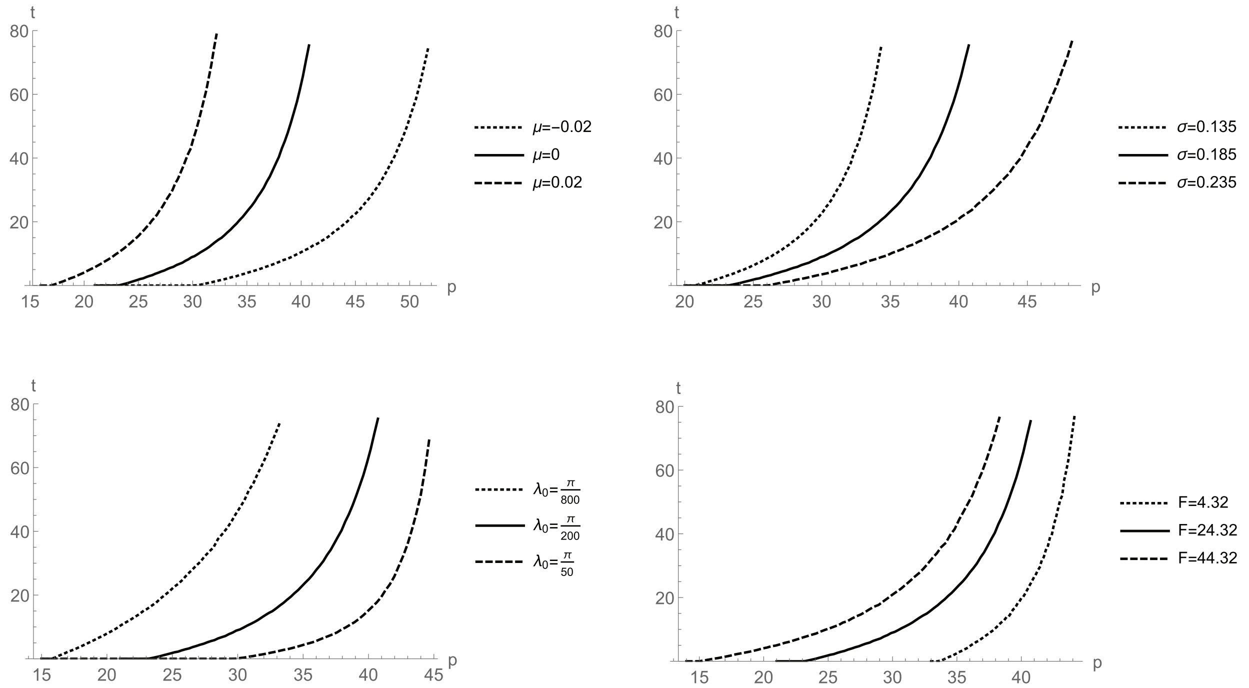

Fig. 6. Sensitivity analysis of the investment boundary $\bar{t}_1^*(p)$ w.r.t to the drift μ (upper left graph), volatility σ (upper right graph), λ_0 (lower left graph) and F (lower right graph), respectively.

Table 3
Threshold results of the benchmark and robustness models for constant retraction risk for different values of σ.

threshold	$\sigma = 0.185$	$\sigma = 0.100$	$\sigma = 0.05$	$\sigma = 0.005$
p^*	22.11	19.32	18.17	17.83
$\bar{p}^*$	32.65	23.73	20.07	18.00

Similar to the time-dependent case, we conclude from these results that the investment threshold for the benchmark case, 22.11, is lower than for the robustness case, 32.65,[18] which means that investment when the market price at retraction is known beforehand occurs earlier than in the case, where this price is stochastic, given the subsidy is still available. Moreover, as the results in Table 3 indicate, the difference between these thresholds in the benchmark and the robustness models decrease with decreasing volatility σ, which agrees with the fact that the impact of the assumption of fixed prices until subsidy retraction, decreases when the volatility of the prices decreases as well. For this case, the corresponding NPV's are equal to 1, 182, 133.[19]

6. Extensions

We now extend our model to account for two different features relevant for support policies in the electricity sector. Note that we will extend the model presented in the robustness section that allows for the electricity process to develop according to a GBM from the beginning of the planning horizon on. In Section 6.1 we consider the case that the period where subsidy is provided is announced to be of a finite length. We then look at the case where the subsidy is retracted in case the electricity market price hits a certain level in Section 6.2.

6.1. Time limit

We will now account for an additional boundary condition: a time boundary. Although our approach accounts for the fact that the probability of the subsidy being retracted increases as time goes by, it is still possible that the subsidy is active forever. As many support schemes are announced to be offered for a specific time period by regulators, we will now extend

[18] The values for the investment threshold for the benchmark constant case, p^*, can be obtained using equation (18), whereas for the robust case, $\bar{p}^*$, they are derived from equations (26) and (27).

[19] For the same reason as in the time-dependent case, the NPV's for the benchmark and for the robustness cases are equal.

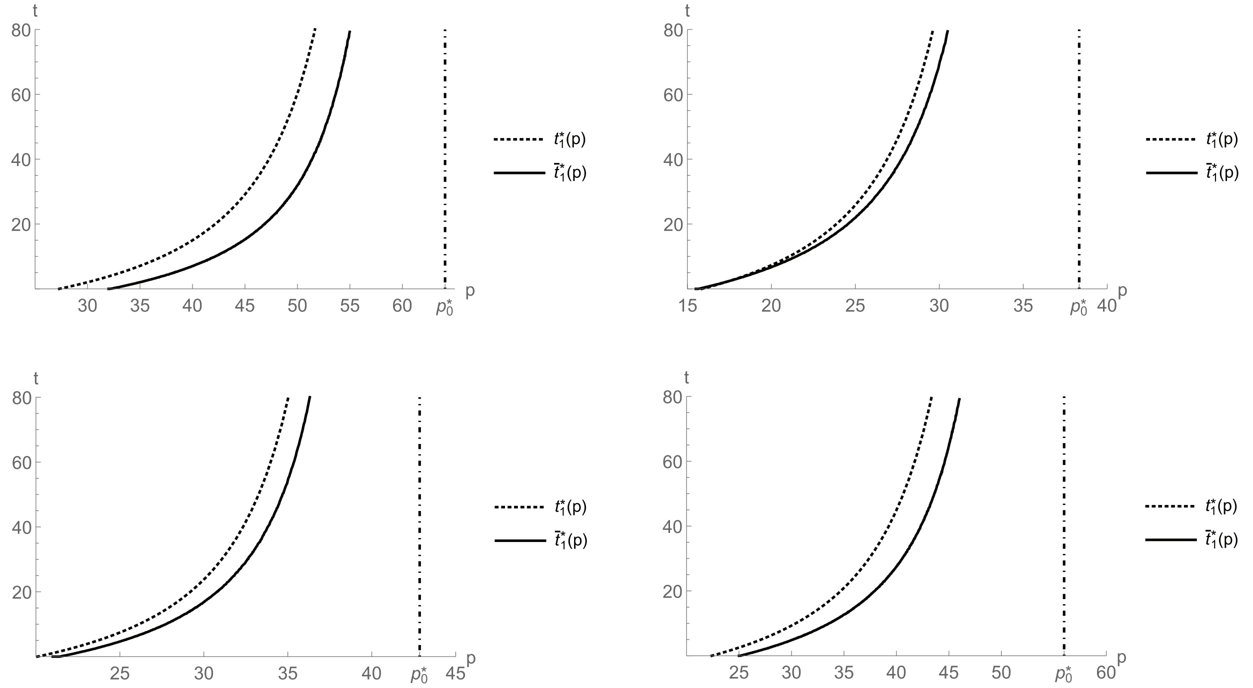

Fig. 7. Numerical comparison of the boundary curves for the benchmark and robustness cases, for different values of μ and σ. Upper panel: same volatility, $\mu = -0.025$ (left plot) and $\mu = 0.025$ (right plot). Lower panel: same drift, $\sigma = 0.15$ (left plot) and $\sigma = 0.22$ (right plot).

our model to account for a maximum length of the period, denoted by T, where the subsidy is active. This means that the subsidy retraction will occur according to a random variable defined as

$$\nu_{t,T} = \min(\nu_t, T - t).$$

The value functions of this model are analogous to the ones of the robustness model presented in Proposition 7 replacing ν by $\nu_{t,T}$. We denote the investment boundary by $\hat{t}^*_{1,T}(p)$ for this case.

In order to compute the investment regions and the value function for this model, the random variable $\nu_{t,T}$ is simulated as follows:

We note that the only difference between Algorithm 1 and 2 is in the computation of the time at which the subsidy is retracted. In Algorithm 1 we take that time is the result of a generation of a random variable (step 6), whereas in Algorithm 2 we check if this value is larger than T; if larger, we set it equal to T (steps 6 and 7). All other steps are the same in both algorithms.

Fig. 8 illustrates the corresponding waiting and investment regions for this case. It shows that the investment curve $\hat{t}^*_{1,T}(p)$ tends as expected towards the upper boundary T. Otherwise, the waiting and continuation regions are similar to the benchmark case.

Fig. 9 illustrates how the investment boundary is affected when the time limit of the subsidy period increases from 5, to 10 and 20 years. The longer the announced time period for the subsidy scheme, the closer the investment curve lies to the boundary without a time limit (i.e. $\hat{t}^*_1(p)$), for small values of p. For relatively large values of p the curve bends towards the time limit T. The investment region decreases when regulators announce a shorter time period during which subsidy is provided. As the subsidy is expected to be provided for a shorter period of time, the firm requires a higher FIT to justify investment. From this we can conclude, that if a policy maker's main objective is to accelerate investment, it should not announce a time limit for the subsidy scheme.

6.2. Price limit

We now assume that the retraction of the subsidy will happen latest after a certain electricity market price level has been hit. This means that we extend the model presented in Section 5 by a new boundary condition that depends on the electricity market price rather than on time passed (as in the previous section).

From a regulators point of view, it makes sense to retract the subsidy when the electricity market price reaches a "high enough" level. If the electricity price is relatively high, the market conditions should be profitable enough for firms to undertake investments without any support. Therefore, we study now how such a price limit affects investment behavior.

Algorithm 1 Robustness Model.

procedure INVESTMENT DECISION
    $\forall$ pairs *(p,t)*
    Generate N values, u_i from a distribution $U \sim Uniform(0,1)$
    Generate N values, n_i from a distribution $X \sim Normal(\mu t, \sigma^2 t)$
    For each u_i, compute ν_i using the Inverse Transform Sampling Theorem:
        $\nu_i = -t + \sqrt{t^2 - \frac{2}{\lambda_0} \log u_i}$
    For each ν_i generate the price at time ν_i from a GBM:
        $p_{\nu_i} = pe^{n_i}$
    Approximate the expected values using the sample:
        $E_{1,p,t}[1 - e^{-r\nu_t}] = \frac{1}{N} \sum_{i=1}^{N} 1 - e^{-r\nu_i}$
        $E_{1,p,t}\left[e^{-r\nu_t} \frac{K}{r-\mu} \frac{1}{d_1} p_0^* \left(\frac{p_{\nu_t}}{p_0^*}\right)^{d_1}\right] = \frac{1}{N} \sum_{i=1}^{N} e^{-r\nu_i} \frac{K}{r-\mu} \frac{1}{d_1} p_0^* \left(\frac{p_{\nu_i}}{p_0^*}\right)^{d_1}$
        $E_{t,p}\left[e^{-r\nu_t} V_0(p_{\nu_t})\right] = \sum_{i=1}^{N} e^{-r\nu_i} V_0(p_{\nu_i})$
    $InvWin \leftarrow \frac{K(p+F)}{r} E_{1,p,t}[1 - e^{-r\nu}] + E_{1,p,t}\left[e^{-r\nu} \frac{K}{r-\mu} \frac{1}{d_1} p_0^* \left(\frac{p_\nu}{p_0^*}\right)^{d_1}\right] - I(K) - \frac{C(K)}{r}$
    $WaitWin \leftarrow_{1,p,t} \left[e^{-r\nu_t} V_0(p_{\nu_t})\right]$
    if $InvWin > WaitWin$ **then**
        The company decides to invest
    else
        The company decides to wait

Algorithm 2 Simulate $\nu_{t,T}$.

procedure GENERATE VALUES OF $\nu_{t,T}$
    $\forall$ pairs *(p,t)*
    Generate N values, u_i from a distribution $U \sim Uniform(0,1)$
    For each u_i, compute $\nu_{it,T}$ using the Inverse Transform Sampling Theorem:
        $\nu_{it,T} = -t + \sqrt{t^2 - \frac{2}{\lambda_0} \log u_i}$
    if $\nu_{it,T} > T - t$ **then**
        $\nu_{it,T} \leftarrow T - t$
    else
        $\nu_{it,T}$ remains the same

Algorithm 3 Simulate $\nu^{\tilde{p}}$.

procedure GENERATE VALUES OF $\nu^{\tilde{p}}$
    $\forall$ pairs *(p,t)*
    Generate $N\$values, \$u_i$ from a distribution $U \sim Uniform(0,1)$
    For each u_i, compute ν_i using the Inverse Transform Sampling Theorem:
        $\nu_i = -t + \sqrt{t^2 - \frac{2}{\lambda_0} \log u_i}$
    For each i, generate a GBM P starting in (p,t) until time reaches ν
    Compute $\tilde{T} = \min\{s : P_s \geq \tilde{p}\}$
    if $\tilde{T}$ exists, **then**
        $\nu_i \leftarrow \tilde{T}$
    else
        ν_i remains the same

Let $\tilde{p}$ denote the fixed price threshold at which the subsidy will be retracted if it has not been already retracted and let

$$\tilde{T} = \min\{s : P_s = \tilde{p}\}.$$

Then, the random variable that represents the time at which the change of regime takes place can be defined as

$$\nu_t^{\tilde{p}} = \min(\nu_t, \tilde{T}).$$

The value functions of this model are analogous to the ones of the robustness model replacing ν by $\nu_t^{\tilde{p}}$. We denote the investment boundary by $\hat{t}^*_{1,\tilde{p}}(p)$ for this case.

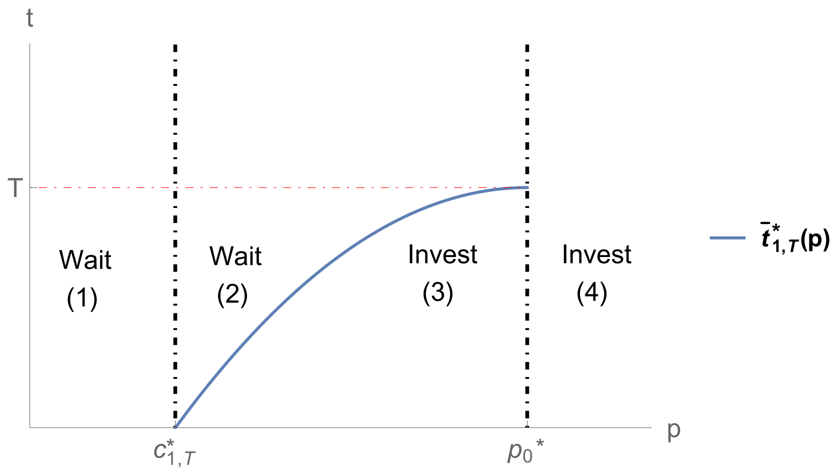

Fig. 8. Investment and continuation regions when subsidy is provided a maximum of T years.

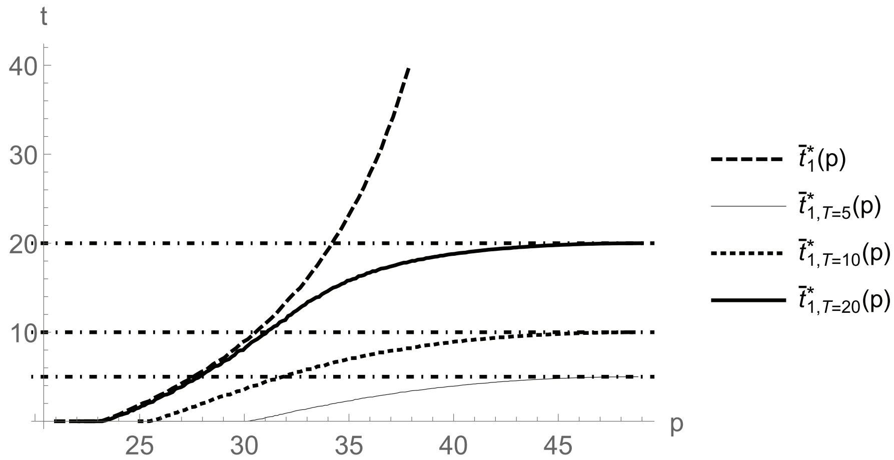

Fig. 9. Effect of increasing time limit T on the investment boundary. [$T \in$ 5, 10, 20].

In order to compute the investment regions and the value function for this model, the random variable $\nu_t^{\tilde{p}}$ is simulated according to Algorithm 3:

For this case, the optimal strategy is illustrated in Fig. 10. Indeed, it is expected that the investment region decreases when compared with the optimal strategy described in Section 5. This happens because

$$\nu_t^{\tilde{p}} \leq \nu_t,$$

with probability 1. The most interesting, however, is the shape of the curve $\hat{t}^*_{1,\tilde{p}}(p)$ (when compared with $\bar{t}^*_1(p)$) and how the optimal strategy changes when one increases the price limit.

Fig. 10 shows that the investment boundary, $\hat{t}^*_{1,\tilde{p}}(p)$, illustrated by the blue curve first increases for relatively small (but larger than $c^*_{1,\tilde{p}}$) values of p and then decreases towards the retraction level $\bar{p}$ for larger values of p. In this case the investment region is bounded by the price limit $\bar{p}$. Note that as p denotes the starting value for the electricity market price, the probability of subsidy retraction increases the closer p is to $\bar{p}$. If p is close to $\bar{p}$ the firm expects the subsidy to be retracted soon and therewith, to receive the subsidy over a too short time period to justify immediate investment. Fig. 11 shows that the smaller the price threshold $\bar{p}$ is set the smaller the range of FITs for which it is optimal for the firm to undergo investment immediately. For $t = 0$ the investment region is limited from above by $\bar{p}$ and from below by $c_{1,\bar{p}}(p)$. The lower boundary $c_{1,\bar{p}}(p)$ increases in $\bar{p}$, therefore, decreasing the investment region. Note that the investment strategy for the case when subsidy has been retracted, is not affected by $\bar{p}$. We can conclude that setting a price limit for the support period may drastically discourage investment.

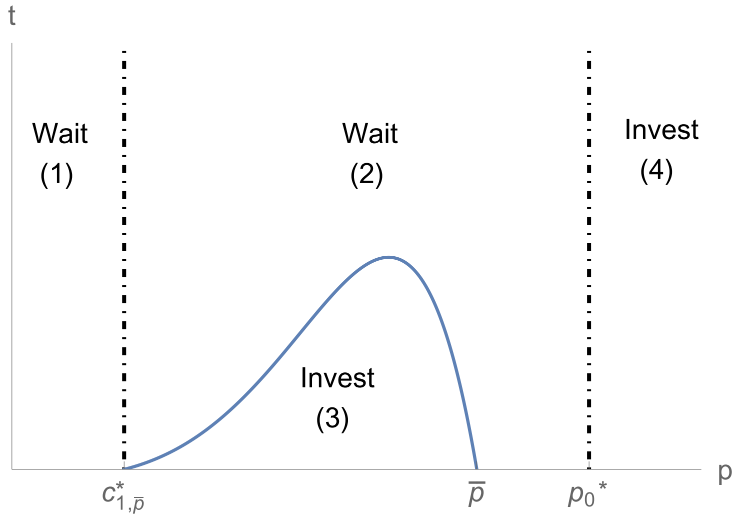

Fig. 10. Investment and continuation regions when subsidy retraction is triggered by a certain market price level $\bar{p}$.

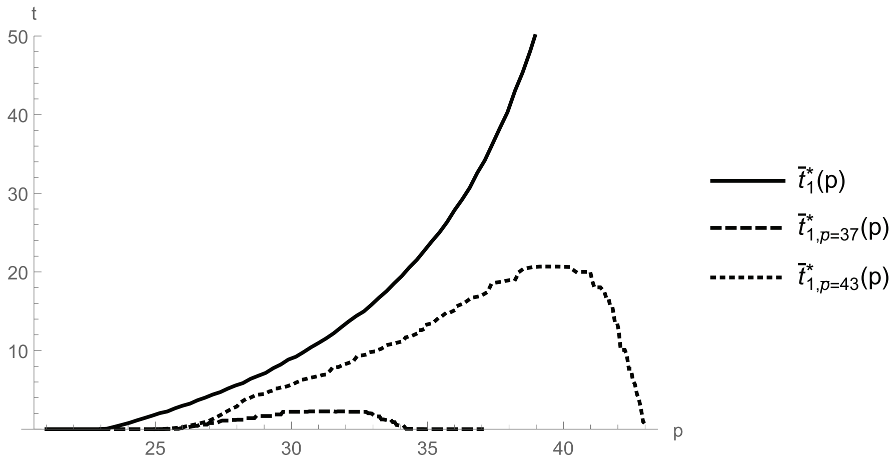

Fig. 11. Effect of increasing price threshold $\bar{p}$ on the investment boundary. [$\bar{p} \in \{37, 43\}$].

7. Conclusions

In this paper we study the effect of policy risk in the form of a potential future retroactive retraction of a subsidy on green investment when subsidy retraction risk is considered time-dependent. We consider a profit maximising investor who has the option to invest in a RE project. Subsidy is currently provided in the form of a fixed FIT. However, the longer the current support scheme is implemented the likelier the investor considers it to be retracted soon. After retraction of the subsidy investors must operate in the free electricity market, receiving the electricity market price for each unit produced.

We find that it is optimal for the firm to either invest immediately for a certain range of FIT prices when it considers the investment decision for the first time. This FIT price range decreases the longer ago the subsidy support has been provided. Otherwise, it is optimal for the firm to wait until the subsidy is retracted and the free market conditions are profitable enough. We provide a sensitivity analysis of the optimal investment strategy with respect to key parameters. Comparing our results to the investment strategy if retraction risk is considered constant over time, we conclude that it is not clear per se whether the investment incentive is higher for one or the other case. It depends on how much time has passed since the subsidy has been introduced.

The investment analysis in our paper may not only provide insight for investors, but not less importantly, also for policy makers who aim to design efficient policy measures. We confirm earlier research in that policy risk is harmful to investors‘ willingness to invest and therefore, diminishes the effect of subsidies. Furthermore, we find that subsidy schemes that are announced for an unlimited time period are most effective in accelerating investment.

In this paper we consider that the probability of subsidy retraction increases linearly in time passed since the support scheme has been introduced. One can however, apply the suggested model framework straightforwardly to consider different functional dependencies. A more challenging question would be to consider the intensity parameter itself as a random variable. In that case, the time dependency would also mean that as time goes by, the firm gets some information, which may then be taken into account in the distribution of the intensity parameter. This would in fact present a Bayesian setting. We leave this to future research. It would also be interesting to consider the relationship of subsidies with the declining investment costs.

Our framework is flexible enough to consider different support schemes, e.g. feed-in premiums or green certificates, or compare the effect of retroactive versus non-retroactive subsidy retraction.

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Appendix A. Appendix: Proofs

A1. Proposition 1

Firstly we can recall that the solution of the HJB equation when the subsidy is no longer available is not time-dependent, and, consequently, we will may drop t, i.e. $v_0(p, t) \equiv v_0(p)$). Additionally, v_0 satisfies

$$\min\left\{rv_0(p) - \mu p v_0'(p) - \frac{1}{2}\sigma^2 p^2 v_0''(p) + Kp - C(K), v_0(p) + I(K)\right\} = .0$$

Based on the real options literature, e may guess that the solution satisfies the following conditions:

(a) $rv_0(p) - \mu p v_0'(p) - \frac{1}{2}\sigma^2 p^2 v_0''(p) + Kp - C(K) = 0,$ and $v_0(p) \geq -I(K),$ for $p \leq p_0^*$ (corresponding to the "waiting region");
(b) $rv_0(p) - \mu p v_0'(p) - \frac{1}{2}\sigma^2 p^2 v_0''(p) + Kp - C(K) \geq 0,$ and $v_0(p) = -I(K),$ for $p > p_0^*$ (corresponding to the "investment region").

In the continuation region ($p < p_0^*$), the solution to the Euler-Cauchy equation

$$rv_0(p) - \mu p v_0'(p) - \frac{1}{2}\sigma^2 p^2 v_0''(p) + Kp - C(K) = 0,$$

is given by $v_0(p) = Ap^{d_1} + Bp^{d_2} - \frac{Kp}{r-\mu} + \frac{C(K)}{r}$, where

$$d_1 = \frac{\sigma^2 - 2\mu + \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8r\sigma^2}}{2\sigma^2} > 0, \text{ and } d_2 = \frac{\sigma^2 - 2\mu - \sqrt{4\mu^2 - 4\mu\sigma^2 + \sigma^4 + 8r\sigma^2}}{2\sigma^2} < 0.$$

since v_0 is a supersolution to the HJB equation, we have that $v_0(p) + I(K) \geq 0, \forall p > 0$, which implies that $B \geq 0$. Additionally, we want that $v_0(p)$ be a function satisfying the property that $\{v_0(P_\tau)\}_{\tau\in\mathcal{S}}$ is a uniformly integrable family of random variables, which is only possible when $B = 0$. Consequently,

$$v_0(p) = Ap^{d_1} - \frac{K}{r-\mu}p + \frac{C(K)}{r}, \quad \forall p < p_0^*$$

By using the smooth-pasting and value matching conditions, we get

$$\begin{cases} A{p_0^*}^{d_1} - \frac{K}{r-\mu}p_0^* + \frac{C(K)}{r} = -I(K) \\ Ad_1{p_0^*}^{d_1} - \frac{K}{r-\mu}p_0^* = 0 \end{cases} \Leftrightarrow \begin{cases} p_0^* = (I(K) + \frac{C(K)}{r})\frac{d_1}{d_1-1} \times \frac{r-\mu}{K} \\ A = \frac{K}{r-\mu}\frac{1}{d_1}{p_0^*}^{1-d_1} \end{cases}$$

which proves Eqn 11 and Eqn 12. To end the proof, we still need to verify that (i) when $p \leq p_0^*$, the value function satisfies $v_0(p) \geq -I(K)$, and, (ii) when $p > p_0^*$, $rv_0(p) - \mu p v_0'(p) - \frac{1}{2}\sigma^2 p^2 v_0''(p) + Kp - C(K) \geq 0$. Since $v_0(0) = \frac{C(K)}{r}$, and $v_0(p_0^*) = -I(K)$, the result follows if v_0 is decreasing in $(0, p_0^*$. It is a matter of computations to see that

$$v_0'(p) = Ad_1p^{d_1-1} - \frac{K}{r-\mu}, \quad v_0'(0) = -\frac{K}{r-\mu} < 0 \quad \text{and} \quad v_0'(p_0^*) = 0.$$

Since $v'(p)$ is an increasing function, $v_0'(p) < 0, p \in [0, p_0^*)$, which proves the (i). To prove (ii), we notice that when $v_0(p) = -I(K)$, then

$$rv_0(p) - \mu p v_0'(p) - \frac{1}{2}\sigma^2 p^2 v_0''(p) + Kp - C(K) \geq 0 \Leftrightarrow p \geq \frac{C(K) + rI(K)}{K}.$$

Taking into account that $v_0(p) = -I(K)$ for $p \geq p_0^*$, the result holds true, if and only if $p_0^* > \frac{C(K)+rI(K)}{K} \Leftrightarrow \frac{d_1}{d_1-1} \times \frac{r-\mu}{r} > 1$. By using the re-parametrization provided in Guerra et al. (2017), we have that

$$\frac{d_1}{d_1-1} \times \frac{r-\mu}{r} = \frac{d_1}{d_1-1} \times \frac{d_1d_2 + 1 - d_1 - d_2}{d_1d_2} = \frac{d_2 - 1}{d_2} > 1.$$

Thus, the function $v_0(p)$ is well defined as

$$v_0(p) = \begin{cases} Ap^{d_1} - \frac{K}{r-\mu}p + \frac{C(K)}{r}, & p \le p_0^* \\ -I(K), & p > p_0^* \end{cases} \tag{21}$$

In order to obtain the value function $V_0(p)$, it still need to be computed $E_p[\int_0^\infty e^{-rs}\Pi_0(P_s)]$. Thus,

$$V_0(p) = v_0(p) + E_p\left[\int_0^\infty e^{-rs}\Pi_0(P_s)\right].$$

Since the expected total payoff is given by

$$\mathbb{E}_p\left[\int_0^\infty e^{-rs}\Pi_0(P_s)\right] = \frac{K}{r-\mu}p - \frac{C(K)}{r},$$

the value function when the subsidy is no longer available is given by:

$$V_0(p) = \begin{cases} Ap^{d_1}, & p \le p_0^* \\ -I(K) + \frac{K}{r-\mu}p - \frac{C(K)}{r}, & p > p_0^* \end{cases}.$$

A2. Proof of proposition 2

We start this section stating an auxiliary result that will be useful in the proof of Proposition 2.

Proposition 11. *Let v_0 be the function defined in (29). Then the integral equation*

$$\int_{t_1^*(p)}^\infty (-\lambda_0\ s v_0(p) + K(p+F) - C(K))e^{\frac{\lambda_0}{2}t_1^{*2}(p)+rt_1^*(p)-\frac{\lambda_0}{2}s^2-rs}ds = I(K) \tag{22}$$

defines implicitly the function t_1^* in the interval $[c_1^*, p_0^*)$, where $\tilde{p} < c_1^* < p_0^*$ and $\tilde{p} \equiv \frac{C(K)-KF+rI(K)}{K}$. Additionally, $c_1^* \equiv c_1^*(F)$ verifies the following conditions:

$$(i) \quad c^*(F) \in (0, p_0^*) \ \text{ and } \ t_1^*(c_1^*) = 0, \quad \text{if } \ \underline{F} \equiv -\frac{\sigma^2}{2}\frac{1}{K}\frac{C(K)+rI(K)}{r}d_1 < F < \frac{1}{K}\frac{rI(K)+C(K)}{1-E_{1,0}(e^{-r\nu})} \equiv \overline{F}$$

$$(ii) \quad c^*(\overline{F}) = 0, \quad \text{and} \quad \lim_{F \searrow \underline{F}} c^*(F) = p_0^*.$$

When $F > \overline{F}$, then t_1^* is defined for $p \in (0, p_0^*)$ and $t_1^*(p) > 0$ for every $p \in (0, p_0^*)$.

Proof. Let $f(p)$ be a function such that

$$f(p) = \int_0^\infty (-\lambda_0\ s v_0(p) + K(p+F) - C(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K)$$

The value of c_1^* such that $f(c_1^*) = 0$, is the one that satisfies $t_1^*(c_1^*) = 0$. Without loss of generality, we assume that $F > 0$. It is a matter of calculations to see that

$$\begin{aligned} f(p_0^*) &= \int_0^\infty (-\lambda_0\ s v_0(p_0^*) + K(p_0^*+F) - C(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K) \\ &= \int_0^\infty (\lambda_0\ sI(K) + K(p_0^*+F) - C(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K) \\ &> \int_0^\infty (\lambda_0\ sI(K) + rI(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K) = 0, \end{aligned}$$

while

$$\begin{aligned} f(\tilde{p}) &= \int_0^\infty (-\lambda_0\ s v_0(\tilde{p}) + K(\tilde{p}+F) - C(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K) \\ &= \int_0^\infty (-\lambda_0\ s v_0(\tilde{p}) + rI(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K) \\ &< \int_0^\infty (\lambda_0\ sI(K) + rI(K))e^{-rs-\frac{\lambda_0}{2}s^2}ds - I(K) = 0 \end{aligned}$$

Thus,as f is a continuous function, there is $c_1^* \in (\tilde{p}, p_0^*)$ such that $f(c_1^*) = t_1^*(c_1^*) = 0$.

Let us consider now the function

$$\begin{aligned} g(a) &= \int_a^\infty (-\lambda_0\ s v_0(p) + K(p+F) - C(K))e^{ra+\frac{\lambda_0}{2}a^2-rs-\frac{\lambda_0}{2}s^2}ds - I(K) \\ &= \int_a^\infty (-\lambda_0\ s(v_0(p) + I(K)) + K(p+F) - C(K) - rI(K))e^{ra+\frac{\lambda_0}{2}a^2-rs-\frac{\lambda_0}{2}s^2}ds \end{aligned}$$

for a fixed $p \in (c_1^*, p_0^*)$. Then, combining the arguments above with the fact that $p \to -\lambda_0\ sv_0(p) + K(p+F) - C(K)$ is an increasing function, we get that

$$g(0) \geq f(c_1^*) = 0. \tag{23}$$

Additionally, it is easy to see that the function $s \to -\lambda_0\ s(v_0(p) + I(K)) + K(p+F) - C(K) - rI(K)$ is an affine function verifying $\lim_{s\to\infty}(-\lambda_0\ s(v_0(p) + I(K)) + K(p+F) - C(K) - rI(K)) = -\infty$. Therefore, there is $N > 0$ such that, for $a > N$ we have $g(a) < 0$. Combining these facts with Equation (34) and the continuity of the function $p \to -\lambda_0\ sv_0(p) + K(p+F) - C(K)$ we get the existence of the function t_1^*.

The second part of this proof relies on studying c_1^* as a function of F. On the one hand, if F is sufficiently large, then we may guess that it is optimal to invest for every price p (which implies that $t_1^*(0) = 0$). On the other one, if F is sufficiently small, perhaps negative, then investment will never be profitable as long as the subsidy is active. One may formalise such idea as below.

(i) For $t_1^*(0) = 0$:

$$\begin{aligned}\int_{t_1^*(0)}^{\infty} e^{-\frac{\lambda_0}{2}s^2 - rs}(-\lambda_0\ sv_0(0) + kF - C(K))ds &= \int_0^{\infty} e^{-\frac{\lambda_0}{2}s^2 - rs}\left(-\lambda_0\ s\frac{C(K)}{r} - C(K) + KF\right)ds \\ &= \frac{1}{r}\int_0^{\infty} e^{-\frac{\lambda_0}{2}s^2 - rs}(-(\lambda_0 s + r)C(K) + rKF)ds \\ &= -\frac{C(K)}{r} + KFe^{\frac{r^2}{2\lambda_0}}\sqrt{\frac{2\pi}{\lambda_0}}\left(1 - \Phi\left(\frac{r}{\sqrt{\lambda_0}}\right)\right) \\ &= -\frac{C(K)}{r} + \frac{KF}{r}(1 - E_{1,0}(e^{-\nu})).\end{aligned}$$

Consequently, $-\frac{C(K)}{r} + \frac{KF}{r}(1 - E_{1,0}(e^{-\nu})) = I(K)$ and

$$F = \frac{1}{K}\frac{rI(K) + C(K)}{1 - E_{1,0}(e^{-\nu})}.$$

(ii) For $t_1^*(p_0^*) = 0$:

$$\begin{aligned}&\int_0^{\infty}(-\lambda_0\ sv_0(p_0^*) + k(p_0^* + F) - C(K))e^{-\frac{\lambda_0}{2}s^2 - rs}ds = I(K) \\ &\Leftrightarrow \int_0^{\infty}(k(p_0^* + F) - C(K) - rI(K))e^{-\frac{\lambda_0}{2}s^2 - rs}ds = 0,\end{aligned}$$

that is true when

$$0 = k(p_0^* + F) - C(K) - rI(K) \Leftrightarrow F = \frac{d_1}{K}\frac{-\sigma^2}{2}\left(I(K) + \frac{C(K)}{r}\right).$$

The proof is complete due to the monotony of t_1^* with respect to F, which is stated in Proposition 4. □

The next result follows immediately from the proof of Proposition 11.

Corollary 1. *Let t_1^* be the function defined by equation (30). Then,*

$$\lambda_0 t_1^*(p) \leq \frac{K(p+F) - C(K) - rI(K)}{v_0(p) + I(K)}.$$

Proof of Proposition 2: The proof of Proposition 2 will be split in two steps: firstly, we will obtain the function v_1, defined in (7), and secondly we will compute V_1 by using the expression in (6). We start by solving the HJB equation (22).

Taking into account that

$$\min\{rv_1(p,t) - \frac{\partial v_1}{\partial t}(p,t) - \lambda_0 t(v_0(p) - v_1(p,t)) + K(p+F) - C(K), v_1(p,t) + I(K)\} = 0,$$

the following conditions have to be verified:

$$\begin{cases} rv_1(p,t) - \frac{\partial v_1}{\partial t}(p,t) - \lambda_0 t(v_0(p) - v_1(p,t)) + K(p+F) - C(K) \geq 0 \\ v_1(p,t) + I(K) \geq 0. \end{cases} \tag{24}$$

Additionally, in the investment region the auxiliary function $v_1(p,\ t)$ verifies $v_1(p,t) = -I(K)$. Therefore, one may conclude that the set of points $(p,\ t)$, where

$$-rI(K) - \lambda_0 t(v_0(p) + I(K)) + K(p+F) - C(K) < 0$$

has to be contained in the continuation region. Taking into account the expression of v_0, we can conclude that the region

$$\left\{(p,t): p < \tilde{p} \wedge \lambda_0 t > \frac{K(p+F) - C(K) - rI(K)}{v_0(p) + I(K)}\right\},$$

where $\tilde{p} \equiv, \frac{C(K) - KF + rI(K)}{K}$ belongs to the waiting region. In accordance with the comments above, we present the value function in the four regions presented in Proposition 2.

(a) $p > p_0{}^*$
Having proved that it is optimal to invest, when the subsidy is no longer available, for $p > p_0^*$, then one may guess that, under the existence of a FIT support scheme, it is also optimal to invest for $p > p_0^*$. Then, one may suppose that $v_1(p,t) = -I(K)$ and, afterward, verify that the HJB equation is satisfied.
In light of the comments above, we have to prove that

$$-rI(K) - \lambda_0 t(v_0(p) + I(K)) + K(p+F) - C(K) \geq 0.$$

Since it was already proved that $v_0(p) = -I(K)$, for $p > p_0^*$, and $p_0^* > \tilde{p}$, it is straightforward to conclude that

$$-rI(K) - \lambda_0 t(-I(K) + I(K)) + K(p+F) - C(K) > -rI(K) + K(\tilde{p}+F) - C(K) + KF = 0,$$

as required.

(b) $(p,t) \in [c_1^*, p_0^*] \times [0, t_1^*(p)]$
From proposition 11, we know that

$$\lambda_0 t_1^*(p) < \frac{K(p+F) - C(K) - rI(K)}{v_0(p) + I(K)},$$

which implies that (35) are satisfied for $(p,t) \in [c_1^*, p_0^*] \times [0, t_1^*(p)]$.

(c) $(p,t) \in [c_1^*, p_0^*] \times [t_1^*(p), \infty)$
A linear ordinary differential equation can be written as

$$x' = a(t)x + b(t) \tag{25}$$

Then, for every $(t_0, x_0) \in \mathbb{R}^2$, the unique solution of (37), with $x(t_0) = x_0$ is given by

$$x(t) = e^{\int_{t_0}^t a(s)ds}\left(\int_{t_0}^t b(s)e^{-\int_{t_0}^s a(u)du}ds + x_o\right), \quad \text{for } t \in \mathbb{R}.$$

So, the solution of $rv_1(p,t) - \partial_t v_1(p,t) - \lambda_0 t(v_0(p) - v_1(p,t)) + K(p+F) - C(K) = 0$ can be determined using the previous expression. Firstly, the ODE is rewritten as

$$\partial_t v_1(p,t) = (r + \lambda_0 t)v_1(p,t) - \lambda_0 t v_0(p) + K(p+F) - C(K),$$

which means that we have $a(t) = r + \lambda_0 t$ and $b(t) = -\lambda_0 t v_0(p) + K(p+F) - C(K)$.
For every $p \in [c_1^*, p_0^*)$ and $t > t_1^*(p)$ we have that the solution for $v_1(p, t)$ must be given by

$$v_1(p,t) = e^{\int_{t_1^*(p)}^t r+\lambda_0 s ds}\left(\int_{t_1^*(p)}^t (-\lambda_0\ s v_0(p) + K(p+F) - C(K))e^{-\int_{t_1^*(p)}^s r+\lambda_0 u du}ds - I(K)\right)$$

Although it is not possible to find an explicit expression for $t_1^*(p)$, an implicit representation can be found. When $t \to \infty$, we must have

$$\int_{t_1^*(p)}^t (-\lambda_0\ s v_0(p) + K(p+F) - C(K))e^{-\int_{t_1^*(p)}^s r+\lambda_0 u du}ds - I(K) = e^{-\int_{t_1^*(p)}^t r+\lambda_0\ s ds}v_0(p).$$

Computing the limit, we get

$$\int_{\tilde{t}_1(p)}^\infty (-\lambda_0\ s v_1(p) + K(p+F) - C(K))e^{\frac{\lambda_0}{2}t_1^{*2}(p) + rt_1^*(p) - \frac{\lambda_0}{2}s^2 - rs}ds = I(K).$$

In Proposition 11 we proved existence and uniqueness of solution for this equation, for each $p \in [c_1^*, p_0^*)$.
We still need to prove that $v_1(p,t) \geq -I(K)$ in the considered region. Let us fix $(p, t_1^*(p))$, with $p \in (c_1^*, p_0^*)$. Then, in this point,

$$rv_1(p, t_1^*(p)) - \partial_t v_1(p, t_1^*(p)) - \lambda_0 t_1^*(p)(v_0(p) - v_1(p, t_1^*(p))) + K(p+F) - C(K) = 0 \Leftrightarrow$$
$$\Leftrightarrow \partial_t v_1(p, t_1^*(p)) = -rI(K) + K(p+F) - C(K) - \lambda_0 t_1^*(p)(v_0(p) + I(K))$$

Since $\lambda_0 t_1^*(p) < \frac{K(p+F) - C(K) - rI(K)}{v_0(x) + I(K)}$, then

$$\partial_t v_1(p, t_1^*(p)) \geq -rI(K) + K(p+F) - C(K) - Kp + C(K) - KF + rI(K) = 0.$$

Therefore, $\partial_t v_1(p, t_1^*(p)) \geq 0$.
Now, fix p and let (p, t_1) be such that $\partial v_1(p, t_1) < 0$ in order to get a contradiction. We know that $\lim_{t\to\infty} v_1(p,t) = v_0(p)$.
Then, we have 2 possible scenarios:

- If $\exists t_1$ such that $v_1(p, t_1) > v_0(p)$, we define t' as $t' = \inf\{t > t_1^*(p) : \frac{\partial v_1}{\partial t}(p,t) = 0 \wedge v_1(p,t) > v_0(p)\}$. Then:

$$\begin{aligned} 0 = \partial_t v_1(p,t') &= rv_1(p,t') - \lambda_0 t'(v_0(p) - v_1(p,t')) + K(p+F) - C(K) \\ &> rv_1(p,t_1^*(p)) - \lambda_0 t'(v_0(p) - v_1(p,t')) + K(p+F) - C(K). \end{aligned}$$

Since $-\lambda_0 t'(v_0(p) - v_1(p,t')) > 0$ and $-\lambda_0 t_1^*(p)(v_0(p) - v_1(p,t_1^*(p)) < 0$, we get

$$\begin{aligned} \partial_t v_1(p,t') &> rv_1(p,t_1^*(p)) - \lambda_0 t_1^*(p)(v_0(p) - v_1(p,t_1^*(p)) + K(p+F) - C(K) \\ &> 0 \text{ (as already been proved)} \end{aligned}$$

thet is a contradiction $(0 > 0)$.

- If $\exists t_1$ such that $v_1(p, t_1) \le v_0(p)$, then let t' be such that $t' = \inf\{t > t_1 : \frac{\partial v_1}{\partial t}(p,t) = 0\}$. Then,

$$\begin{aligned} 0 &= \partial_t v_1(p,t') = rv_1(p,t') - \lambda_0 t'(v_0(p) - v_1(p,t')) \\ &< rv_1(p,t_1) - \lambda_0 t'(v_0(p) - v_1(p,t')) + K(p+F) - C(K) \end{aligned}$$

Since $(v_0(p) - v_1(p,t')) > v_0(p) - v_1(p,t_1)$ and $t' > t_1$, then

$$\begin{aligned} 0 &< rv_1(p,t_1) - \lambda_0 t'(v_0(p) - v_1(p,t')) + K(p+F) - C(K) \\ &< rv_1(p,t_1) - \lambda_0 t_1(v_0(p) - v_1(p,t_1)) + K(p+F) - C(K) \\ &= \partial_t v_1(p,t_1) < 0. \end{aligned}$$

Thus, we have again the contradiction $0 < 0$.

Therefore, $v_1(p, t)$ is increasing in t, when $p \in (c_1, p_0^*)$, then

$$v_1(p,t) > -I(K)$$

(d) $p < c_1^*$

Let $A : \mathbb{R}^+ \to \mathbb{R}$ be a function such that

$$A(p) = v_1(p,0).$$

Then, we can write $v_1(p, t)$ as

$$\begin{aligned} v_1(p,t) &= e^{\int_0^t r+\lambda_0 \ sds}\left(A(p) + \int_0^t (-\lambda_0 \ sv_0(p) + K(p+F) - C(K))e^{-\int_0^s r+\lambda_0 udu}dt\right. \\ &= e^{rt+\lambda_0 \frac{t^2}{2}}\left(A(p) + \int_0^t (-\lambda_0 \ sv_0(p) + K(p+F) - C(K))e^{-rs-\frac{s^2}{2}}ds.\right. \end{aligned}$$

Analogously to the previous case, when $t \to \infty$, $v_1(p,\infty) = v_0(p)$. So,

$$\int_0^\infty (-\lambda_0 \ sv_0(p) + K(p+F) - C(K))e^{-\frac{\lambda_0}{2}s^2-rs}ds = -A(p).$$

Since $A(c_1^*) = -I(K)$, to prove that $v_1(p,t) > -I(K)$ for $p < c_1^*$, one just need to verify that $A(p)$ is a decreasing function. Let $x, y \in (0, c_1^*)$ be such that $x < y$. We will show that $A(x) > A(y)$, or equivalently, $-A(x) < -A(y)$.

$$\begin{aligned} -A(x) &= \int_0^\infty (-\lambda_0 \ sv_0(x) + Kx - C(K) + KF)e^{-\frac{\lambda_0}{2}s^2-rs}ds \\ &< \int_0^\infty (-\lambda_0 \ sv_0(y) + Kx - C(K) + KF)e^{-\frac{\lambda_0}{2}s^2-rs}ds \\ &< \int_0^\infty (-\lambda_0 \ sv_0(y) + Ky - C(K) + KF)e^{-\frac{\lambda_0}{2}s^2-rs}ds \\ &= -A(y). \end{aligned}$$

Combining the previous arguments, we have that $A(p) > -I(K)$ for all $p < c_1^*$. To get the inequality $v_1(p,t) > -I(K)$ for every $t > 0$, one may use similar arguments to the ones used in the previous point.

To finish the proof we need to obtain the function V_1. Indeed, it is a matter of computations to see that

$$\begin{aligned} E_{1,p,t}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] &= E_{p,1}\left[\int_0^\nu e^{-rs}(K(p+F) - C(K))ds\right] + E_{p,0}\left[\int_\nu^\infty e^{-rs}(KP_s - C(K))ds\right] \\ &= \frac{K(p+F)}{r} - \frac{C(K)}{r} + \left(\frac{Kp}{r-\mu} - \frac{K(p+F)}{r}\right)E_{1,t}[e^{-r\nu}] \\ E_{1,t}[e^{-r\nu}] &= 1 - re^{\frac{(r+\lambda_0 t)^2}{2\lambda_0}}\sqrt{\frac{2\pi}{\lambda_0}}\left(1 - \Phi\left(\frac{r+\lambda_0 t}{\sqrt{\lambda_0}}\right)\right) \end{aligned}$$

$$\int_a^b e^{-\frac{\lambda_0}{2}s^2-rs}ds = e^{\frac{r^2}{2\lambda_0}}\sqrt{\frac{2\pi}{\lambda_0}}\int_a^b \frac{1}{\sqrt{2\pi/\lambda_0}}e^{-\frac{(s+(r/\lambda_0))^2}{2/\lambda_0}}ds = e^{\frac{r^2}{2\lambda_0}}\sqrt{\frac{2\pi}{\lambda_0}}\left(\Phi\left(\frac{r+\lambda_0 b}{\sqrt{\lambda_0}}\right)-\Phi\left(\frac{r+\lambda_0 a}{\sqrt{\lambda_0}}\right)\right),$$

which implies that

$$\int_{t_1^*(p)}^{\infty}(-\lambda_0\, sv_0(p)+K(p+F)-C(K))e^{-\int_{t_1^*(p)}^s r+\lambda_0 u du}ds = v_0(p)$$
$$+\ (rv_0(p)+K(p+F)-C(K))e^{\frac{(r+\lambda_0 b)^2}{2\lambda_0}}\sqrt{\frac{2\pi}{\lambda_0}}\left(1-\Phi\left(\frac{r+\lambda_0 t_1^*(p)}{\sqrt{\lambda_0}}\right)\right).$$

Equation (14) follows immediately from the previous calculations. Additionally, in light of the previous comments one may check that $v_1(p,t) = Ap^{d_1}E_{1,t}[e^{-r\nu}]$, for $(p,t)\in \big(]0,c_1^*[\times[0,\infty[\cup[c_1^*,p_0^*[\times[t_1^*(p),\infty[\big)$.

A3. Proposition 3

The result follows in view of the expected hitting time of a geometric Brownian motion; see, for instance, Willmott et al. (1995).

A4. Proposition 4

To prove Proposition 4 we notice that the condition for $t_1^*(p)$ is equivalently given by (30). Therefore,

$$\frac{\partial t_1^*(p)}{\partial\gamma} = \frac{\frac{\partial v_0(p)}{\partial\gamma}e^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)}\int_{t_1^*(p)}^{\infty}\lambda s e^{-\frac{\lambda}{2}s^2-rs}ds}{I(K)(\lambda t_1^*(p)+r)+\lambda t_1^*(p)v_0(p)-K(p+F)+C(K)},\ \text{for } \gamma=\mu,\sigma.$$

From the literature, it is known that $\frac{\partial v_0(p)}{\partial\mu}<0$ and $\frac{\partial v_0(p)}{\partial\sigma}>0$ (see for instance (Guerra et al., 2017)). Additionally,

$$I(K)(\lambda t_1^*(p)+r)+\lambda t_1^*(p)v_0(p)-K(p+F)+C(K)$$
$$< \frac{K(p+F)-C(K)-rI(K)}{v_0(p)+I(K)}(I(K)+v_0(p))+rI(K)-K(p+F)+C(K)=0.$$

Combining the arguments above with the fact that $e^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)}>0$ and $\int_{t_1^*(p)}^{\infty}\lambda s e^{-\frac{\lambda}{2}s^2-rs}ds>0$, we get the intended result for μ and σ.

It is a matter of computations to see that the derivative of $t_1^*(p)$ in order to λ is given by

$$\frac{\partial t_1^*(p)}{\partial\lambda} = \frac{\int_{t_1^*(p)}^{\infty}\left[v_0(p)s-(-\lambda s v_0(p)+K(p+F)-C(K))\left(\frac{t_1^*(p)^2}{2}-\frac{s^2}{2}\right)\right]e^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)-\frac{\lambda}{2}s^2-rs}ds}{I(K)(\lambda t_1^*(p)+r)+\lambda t_1^*(p)v_0(p)-K(p+F)+C(K)}.$$

We already know that the denominator is negative. Then, we just need to check that the numerator is positive. Taking into account that $p>c_1*>c^*=\frac{C(K)-KF+rI(K)}{K}$, which implies that $K(p+F)-C(K)>rI(K)$, we get

$$\int_{t_1^*(p)}^{\infty}\left[v_0(p)s-(-\lambda s v_0(p)+K(p+F)-C(K))\left(\frac{t_1^*(p)^2}{2}-\frac{s^2}{2}\right)\right]e^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)-\frac{\lambda}{2}s^2-rs}ds$$
$$> \int_{t_1^*(p)}^{\infty}\left[v_0(p)s+(-\lambda s v_0(p)+rI(K))\left(\frac{s^2}{2}-\frac{t_1^*(p)^2}{2}\right)\right]e^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)-\frac{\lambda}{2}s^2-rs}ds.$$

In light of the fact that $v_0(p)+I(K)>0$, we have

$$v_0(p)s+(-\lambda s v_0(p)-rv_0(p))\left(\frac{s^2}{2}-\frac{t_1^*(p)^2}{2}\right)>v_0\left[s+(-\lambda s-r)\left(\frac{s^2}{2}-\frac{t_1^*(p)^2}{2}\right)\right].$$

The result follows noticing that

$$\int_{t_1^*(p)}^{\infty}\left[s+(-\lambda s-r)\left(\frac{s^2}{2}-\frac{t_1^*(p)^2}{2}\right)\right]e^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)-\frac{\lambda}{2}s^2-rs}ds=0.$$

Lastly, we want to study the effect of F on the curve $t_1^*(p)$. Since

$$\frac{\partial t_1^*(p)}{\partial F} = \frac{-Ke^{\frac{\lambda}{2}t_1^*(p)^2+rt_1^*(p)}\int_{t_1^*(p)}^{\infty}e^{-\frac{\lambda}{2}s^2-rs}ds}{I(K)(\lambda t_1^*(p)+r)+\lambda t_1^*(p)v_0(p)-K(p+F)+C(K)},$$

the result follows immediately from the fact that the denominator is negative.

A5. Proposition 5

In this case, the value function V_1 (when the subsidy is still active) does not depend on time, as the process is time-homogeneous. Following the same line of arguments as for the benchmark case, we find that

$$V_1(p) = E_{1,p}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] + \sup_\tau E_{1,p}\left[\int_0^\tau e^{-rs}(-\Pi_\theta(P_s))ds - e^{-r\tau}I(K)\right] =$$
$$= E_{1,p}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] + v_1(p)$$

where

$$E_{1,p}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] = \frac{K(p+F)}{r} - \frac{C(K)}{r} + \left(\frac{Kp}{r-\mu} - \frac{K(p+F)}{r}\right)\frac{\hat{\lambda}}{r+\hat{\lambda}}$$

and v_1 in the continuation region is solution of the following equation:

$$rv_1(p) - \hat{\lambda}(v_0(p) - v_1(p)) + K(p+F) - C(K) = 0.$$

Therefore

$$V_1(p) = \frac{K(p+F)}{r} - \frac{C(K)}{r} + \left(\frac{Kp}{r-\mu} - \frac{K(p+F)}{r}\right)\frac{\hat{\lambda}}{r+\hat{\lambda}} + \begin{cases} \frac{C(K)+\hat{\lambda}v_0(p)-K(p+F)}{r+\hat{\lambda}} & p < p^* \\ -I(K) & p \geq p^* \end{cases},$$

which allows us to get the value function. Moreover, by using the continuity of the value function, we get that p^* satisfies the equation

$$f(p) := \frac{A(p^*)^{d_1}\hat{\lambda}}{r+\hat{\lambda}} - \frac{K(p^*+F)}{r+\hat{\lambda}} + \frac{C(K)}{r} - \frac{Kp^*}{r-\mu}\frac{\hat{\lambda}}{r+\hat{\lambda}} + I(K) = 0.$$

From the HJB equation, it is known that $v_1(p) \geq -I(K)$ for every $p > 0$, which allows us to conclude that p^* is the smallest root of the HJB equation, when f has roots.

In fact, taking into account that $f(0) = 0,\ \lim_{p\to\infty} f(p) = \infty$ and

$$f'(p) = \frac{Ad_1(p^*)^{d_1-1}\hat{\lambda}}{r+\hat{\lambda}} - \frac{K}{r+\hat{\lambda}} - \frac{K}{r-\mu}\frac{\hat{\lambda}}{r+\hat{\lambda}},$$

we can conclude that f has a unique minimiser, $\tilde{p}$ in the interval $\tilde{p} \in (0, +\infty)$, and f is decreasing for $p < \tilde{p}$ and increasing for $p > \tilde{p}$. Existence of p^* is guaranteed if and only if $f(\tilde{p}) < 0$, and, moreover, p^* is such that $f(p^*) = 0$ and $f'(p^*) < 0$. Additionally, it is a matter of calculations to check that $\frac{\partial f(\tilde{p})}{\partial F} < 0$. The result follows from the fact that

$$f(0) = 0 \Leftrightarrow F = \tilde{F} \quad \text{and} \quad f'(0) < 0$$

$$f(p_0^*) = 0 \Leftrightarrow F = \underset{\sim}{F} \quad \text{and} \quad f'(p_0^*) < 0.$$

A6. Propositions 6 and 9

The proofs of Propositions 6 and 9 are based on the same argument: the tower property. Indeed,

$$E[P_{\nu_t} \mid \nu_t] = pe^{\mu\nu}.$$

Therefore, the results will be obtained noticing that

$$E[P_{\nu_t}] = p\int_0^\infty e^{\mu t}\left(1 - e^{-\int_t^{t+s}\lambda(s)ds}\right)' dt,$$

where $\lambda(t) = \lambda_0 t$ in Proposition 6 and $\lambda(t) = \hat{\lambda}$ in Proposition 9.

A7. Proposition 7

The value function V_1^A of problem (23) admits a decomposition like (6), where in this case v_θ has to be replaced by v_1^A and

$$E_{1,p,t}\left[\int_0^\infty e^{-rs}\Pi_\theta(P_s)ds\right] = \frac{K(p+F)}{r} - \frac{C(K)}{r} + E_{1,p,t}\left[e^{-r\nu_t}\right]\left(\frac{Kp_1}{r-\mu} - \frac{K(p+F)}{r}\right).$$

Furthermore, v_1^A satisfies the HJB equation

$$\min\left(rv_1^A(p, p_1, t) - \tilde{\mathcal{L}}_1 v_1^A(p, p_1, t) + \Pi_1(p_1), v_1^A(p, p_1, t) + I(K)\right) = 0,$$

where $\mathcal{L}_1^A$ is the operator defined by

$$\mathcal{L}_1^A v_1^A(p, p_1, t) = \frac{\partial v_1^A(p, p_1, t)}{\partial t} + \lambda(t)\big(v_0(p_1) - v_1^A(p, p_1, t)\big).$$

Therefore, proposition 7 can be proved along the same lines of Proposition 2.

A8. Proposition 8

To prove Proposition 8, one may notice that in light of the strong Markov property we get that

$$\overline{V}_1(p,t) = \sup_{\tau>0} E_{1,p,t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau}I(K)\right]$$

where Π_θ is defined as in (4), $\tilde{P}$ satisfies the stochastic differential equation and

$$d\tilde{P}_s = \mu(\theta_s)\tilde{P}_s ds + \sigma(\theta_s)\tilde{P}_s dW_s, \quad \tilde{P}_0 = P_{\nu_t}$$

where

$$\mu(\theta) = \begin{cases} 0, & \theta = 1 \\ \mu, & \theta = 0 \end{cases}, \quad \sigma(\theta) = \begin{cases} 0, & \theta = 1 \\ \sigma, & \theta = 0 \end{cases}$$

and

$$dP_s = \mu P_s ds + \sigma P_s dW_s, \quad P_0 = p.$$

Additionally, for any $\tau > 0$ we have that

$$E_{1,p,t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau}I(K)\right] = E_{1,p,t}\left[E_{1,P_{\nu_t},t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau}I(K)\right]\right].$$

Moreover,

$$V^A(p, P_{\nu_t}, t) = \sup_{\tau>0} E_{1,P_{\nu_t},t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau}I(K)\right] = E_{1,P_{\nu_t},t}\left[\int_{\tau^*_{P_{\nu_t}}}^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau}I(K)\right],$$

where $V^A(p, P_{\nu_t}, t)$ is defined by (23) and $\tau^*_{P_{\nu_t}}$ is the optimal time when p_1 is replaced by P_{ν_t}. This implies that

$$E_{1,p,t}\left[\int_\tau^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau}I(K)\right] \le E_{1,p,t}\left[V^A(p, P_{\nu_t}, t)\right] = E_{1,p,t}\left[\int_{\tau^*_p}^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau^*_p}I(K)\right].$$

On the other hand, we have

$$\overline{V}_1(p,t) \ge E_{1,p,t}\left[V^A(p, P_{\nu_t}, t)\right] = E_{1,p,t}\left[\int_{\tau^*_p}^\infty e^{-rs}\Pi_\theta(\tilde{P}_s)ds - e^{-r\tau^*_p}I(K)\right],$$

that proves the result.

A9. Proposition 10

The proof of Proposition 10 can be written along the same lines of proof of Proposition 1. Therefore, we will focus on some particularities of the proof not discussed yet.

The value function in the waiting region is the solution of the following equation

$$(r+\hat{\lambda})v_1(p) - \mu p\frac{\partial v_1(p)}{\partial p} - \frac{1}{2}\sigma^2 p^2\frac{\partial^2 v_1(p)}{\partial p^2} = \hat{\lambda}Ap^{d_1}, \tag{26}$$

where the parameter A is defined in Proposition 1. The main difficulty in solving Equation (42) is the fact that d_1 can be a non-integer. A general solution to this equation, which we denote by $y(p)$, is derived in Nunes et al. (2019) and verifies

$$y(p) = Bp^{\beta_1} + Cp^{\beta_2} - \frac{2\hat{\lambda}A}{\sigma^2}\frac{1}{Q(d_1)}p^{d_1},$$

with

$$\beta_1(\beta_2) = \frac{1}{2}\left[1 - \frac{2\mu}{\sigma^2} \pm \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + \frac{8(r+\hat{\lambda})}{\sigma^2}}\right],$$

$$Q(d_1) = d_1(d_1 - 1) + \frac{2\mu}{\sigma^2}d_1 - \frac{2(r+\hat{\lambda})}{\sigma^2} = -2\frac{\hat{\lambda}}{\sigma^2}.$$

Taking into account the definition of d_1 we get $Q(d_1) = -2\frac{\hat{\lambda}}{\sigma^2}$. Additionally, given the finiteness of the function v_1 we get that $C = 0$. Therefore,

$$v_1(p) = Bp^{\beta_1} + Ap^{d_1}.$$

Combining the smooth pasting conditions with the fact that

$$E_{1,p,t}\left[\int_0^{\infty} e^{-rs}\Pi_{\theta}(P_s)ds - I(K)\right] = \frac{K(p+F)}{r+\hat{\lambda}} - \frac{C(K)}{r} + \frac{Kp}{r-\mu}\frac{\hat{\lambda}}{r-\mu+\hat{\lambda}} - I(K),$$

we get the result stated in Proposition 10.

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