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Consensus-based Distributed Algorithm for Multisensor-Multitarget Tracking under Unknown–but–Bounded Disturbances

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Abstract: We consider a dynamic network of sensors that cooperate to estimate parameters of multiple targets. Each sensor can observe parameters of a few targets, reconstructing the trajectories of the remaining targets via interactions with "neighbouring" sensors. The multi-target tracking has to be provided in the face of uncertainties, which include unknown-but-bounded drift of parameters, noise in observations and distortions introduced by communication channels. To provide tracking in presence of these uncertainties, we employ a distributed algorithm, being an "offspring" of a consensus protocol and the stochastic gradient descent. The mathematical results on the algorithm's convergence are illustrated by numerical simulations.

Keywords: Sensor network, randomized algorithms, consensus, multitarget tracking

1. INTRODUCTION

Recently, multi-agent systems and multi-agent technologies have attracted enormous attention for the research community due to numerous applications in control theory (Olfati-Saber et al., 2007; Ren and Cao, 2011), distributed optimization (Boyd et al., 2011; D.Bertsekas and Tsitsiklis, 1989), mobile robotics (Bullo et al., 2009; Ren et al., 2007; Virágh et al., 2014) and modeling of complex natural and societal processes (Bar-Yam, 1997; Bhattacharya and Vicsek, 2010; Easley and Kleinberg, 2010; Proskurnikov and Tempo, 2018). It is known that teams of relatively simple and inter-replaceable agents applying a distributed algorithm can solve complex problems more efficiently than centralized systems, being more reliable and resilient. Unlike centralized solutions, multi-agent algorithms do not require collecting all data at a single node of the system; each agent communicates only with a few "neighbouring" (adjacent) agents.

One of the classical applications of multi-agent algorithms is multi-target tracking by networked sensors such as e.g. radars, sonars, infrared (IR) sensors or cameras (Blackman, 2004; Chen et al., 2015; He et al., 2018; Tugnait, 2004). Typically, a single agent has a limited range and can observe trajectories of few targets, so the information fusion requires cooperation between multiple agents. In large sensor networks, the quality of an individual sensor's measurements is often sacrificed for the low price and replaceability of the sensors. Besides this, the measurements reported by each sensor are typically distorted by clutter and noises. The models of targets (e.g. maneuvering objects) may be also partially uncertain due to e.g. drifting parameters.

Another application that has motivated the development of multi-target tracking theory is the air traffic control (ATC) and surveillance (Isaksson and Gustafsson, 1995; Li and Bar-Shalom, 1993; Oh et al., 2008). As the air traffic becomes denser, the centralized computation of its "live" radar map gets more complicated, so distributed algorithms can be used to increase the ATC system's performance. Tracking and classification of different objects which has to be done by vehicles with the usage of cameras, radars, and lidars are other problems at the crossroad of computer vision, sensor fusion and intelligent transportation (Battiato et al., 2015; Zou et al., 2019).

Traditionally, multi-target tracking problems have been handled by tools from filtering theory and advanced statistics such as e.g. multiple hypothesis tracking (MHT), interacting multiple model (IMM) methods, probabilistic data

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association filters (PDAF) etc. (Blackman, 2004; Li and Bar-Shalom, 1993; Tugnait, 2004). All of these methods, however, assume that some statistical characteristics of the uncertain parameters are known. In this paper, we are concerned with situation where the uncertain targets' parameters and sensor noises may be fully unknown yet supposed to be bounded (Granichin and Amelina, 2015), which makes many statistical methods inapplicable. Besides this, we take into account communication constraints, which always arise in practice and may be considered from several perspectives. On the one hand, communication constraints may be incorporated into the dynamics of the communication graph (the network's "topology"). This is convenient if the same communication channel has to be shared among multiple users, and each pair of sensors can communicate only during some interval allocated by an external scheduler. On the other hand, communication constraints may be interpreted as *costs* of data transmission. Such costs may e.g. penalize the power consumed by transmitters and receivers, processor time spent on processing the messages etc. The presence of *unknown-but-bounded disturbances, time-varying communication graph and communication costs* differs our problem from standard multi-target tracking problems addressed in the literature and leads to a problem of non-stationary *mean-risk* functional optimization which is to be solved in a distributed way.

The traditional approach to mean-risk functional optimization is based on the maximum likelihood estimator and stochastic approximation (SA) algorithms with slowly decaying step-size (Blum, 1954; Kiefer et al., 1952; Kushner and Yin, 2003; Robbins and Monro, 1951). SA algorithms use minimal information about random parameters and are very robust, although their convergence is rather slow. They have found numerous applications in adaptive signal processing, adaptive resource allocation, and artificial intelligence. In the case where the computation of the cost function's gradient during an operation is troublesome, it can be approximated by a "noisy gradient" computed using $2d$ random samples (where d is the dimension of the space). This idea naturally leads to the *simultaneous perturbation stochastic approximation* (SPSA) algorithms introduced by Spall (1992). SPSA may be considered as a special random search technique since the estimate of the optimum is updated by shifting in a randomly chosen direction rather than the direction of the steepest descent. At the same time, the gradient estimate is "almost" unbiased and on average the algorithm will nearly follow the steepest descent direction.

The study of distributed optimization has started long before the recent "boom" in multi-agent control (D.Bertsekas and Tsitsiklis, 1989; Tsitsiklis et al., 1986) and, in fact, has led to very general results on multi-agent coordination (Blondel et al., 2005). Most studied are methods for convex optimization, e.g. the alternating direction method of multipliers (ADMM) (Boyd et al., 2011) and subgradient methods (Nedic and Ozdaglar, 2009; Rabbat and Nowak, 2004). For non-convex optimization, methods of surrogate functions have been used (Di Lorenzo and Scutari, 2016; Zhu and Martínez, 2010).

This paper extends a number of results on SPSA algorithms published in the previous works of the authors. In (Granichin and Erofeeva, 2018; Granichin and Amelina, 2015), the SPSA algorithm is applied to an unconstrained problem of optimal target tracking. One of the main limitations is the property of strong convexity of the minimized mean-risk functional. In (Erofeeva et al., 2019; Granichin et al., 2019) this assumption was weaken by combining SPSA with the consensus algorithm from Amelina et al. (2015). In (Granichin et al., 2019), target tracking noisy measurements has been considered (where the noise does not need to satisfy standard statistical assumptions such as randomness, independence at different time instants or zero mean properties), and the cost constraints related to the network topology have been introduced.

Unlike the aforementioned works, in this paper we consider the situation where each sensor has a *limited number* of neighboring sensors with which it can communicate at each step. Due to this limitation, which is important in many practical problems, we suggest to use a *randomized* communication graph. Besides this, we apply the new SPSA algorithm to the tracking of multiple targets by a network of heterogeneous sensors, extending thus our previous results from (Erofeeva et al., 2019; Granichin et al., 2019).

The rest of this paper is organized as follows. Section 2 provides notations used in the paper. The formal problem is stated in Section 3. In Section 4 we suggest to use randomized topology to reduce a number of simultaneous connections between agents at each iteration. The modified SPSA-based consensus algorithm for tracking with different step-sizes is introduced in Section 5. The main result concerning stability properties of the proposed algorithm is shown in Section 6. In Section 7, we consider a simulation which illustrates the operability of the algorithm. Section 8 concludes the paper.

2. GRAPH THEORY

In subsequent sections we use the following notations.

Consider a dynamic network system of n intelligent collaborating sensors (agents). Without loss of generality, agents in the network system are numbered. Let $\mathcal{N} = \{1, \ldots, n\}$ be the set of agents, $i \in \mathcal{N}$ be the number of an agent, and E be the set of edges. $\forall i \in \mathcal{N}$ let $\mathcal{N}^i$ be a subset of all agents: $\mathcal{N}^i \subset \mathcal{N}$, which are able to send information to agent i. The corresponding adjacency matrix is denoted as $A = [a^{i,j}]$, where $a^{i,j} > 0$ if agent j is connected to agent i (i.e. if there is an arc from j to i) and $a^{i,j} = 0$ otherwise. Denote $\mathcal{G}_A$ the graph corresponding to adjacency matrix A. (Throughout the paper, the agent index i is used as a superscript and not as an exponent.)

Define *the weighted in-degree* of node i as the sum of i-th row of matrix A: $\deg_i^+(A) = \sum_{j=1}^n a^{i,j}$ and $\deg_{\max}^+(A)$ as the maximum in-degree of nodes contained in the graph $\mathcal{G}_A$. $\mathcal{D}(A) = \mathrm{diag}_n(\mathrm{col}\{\deg_1^+(A), \ldots, \deg_n^+(A)\})$ is the corresponding diagonal matrix. Henceforth, $\mathrm{col}\{\mathbf{x}^1, \ldots, \mathbf{x}^n\}$ denotes a vector obtained by stacking the specified vectors one on top of each other. $\mathrm{diag}_n(\mathbf{b})$ is a square diagonal matrix with elements of a vector $\mathbf{b}$ on the diagonal and other elements equal to zero. Let $\mathcal{L}(A) = \mathcal{D}(A) - A$ be the *Laplacian* of graph $\mathcal{G}_A$. $[\cdot]^{\mathrm{T}}$ is a vector or matrix transpose operation, $\langle \cdot, \cdot \rangle$ is a scalar product of two vectors. $\|A\|$ is the Frobenius norm: $\|A\| = \sqrt{\sum_i \sum_j (a^{i,j})^2}$.

$\mathrm{Re}(\lambda_2(A))$ is the real part of the second eigenvalue of matrix A ordered by the absolute magnitude; $\lambda_{\max}(A)$ is the eigenvalue of matrix A with maximum absolute magnitude; $\mathbf{1}_n = (1, \ldots, 1)^{\mathrm{T}}$ is the vector of n-times replication of ones; $\mathbf{e}_i = (\ldots, 0, 1, 0, \ldots)^{\mathrm{T}}$ is the unit orth-vector from $\mathbb{R}^n$ with all zeros except single one at i-th row; I_d is the identity matrix $d \times d$. $A \otimes B$ is the Kronecker product defined for any matrices A and B.

3. MULTISENSOR-MULTITARGET PROBLEM

Consider a distributed network of n intelligent sensors (agents) that have m targets in their zone of visibility whose state vectors are to be estimated.

Let $\mathcal{N} = \{1, 2, \ldots, n\}$ be the set of sensors, $\mathcal{M} = \{1, 2, \ldots, m\}$ be the set of targets. At time instant t $\mathbf{s}_t^i = [s_t^{i,1}, \cdots, s_t^{i,d}]^{\mathrm{T}} \in \mathbb{R}^d$ is the current state vector of sensor $i, i \in \mathcal{N}$, $\mathbf{r}_t^l = [r_t^{l,1}, \cdots, r_t^{l,d}]^{\mathrm{T}} \in \mathbb{R}^d$ is the state vector of target $l, l \in \mathcal{M}$, $\theta_t = \mathrm{col}\{\mathbf{r}_t^1, \ldots, \mathbf{r}_t^m\}$ is the common state vector of all targets. Two cases $d = 2$ and $d = 3$ are the most interesting from the practical point of view.

We assume that at time instant t sensor i is able to measure the squared distance

$$\rho(\mathbf{s}_t^i, \mathbf{r}_t^l) = \|\mathbf{r}_t^l - \mathbf{s}_t^i\|^2 = \sum_{d'=1}^{d} \left(r_t^{l,d'} - s_t^{i,d'}\right)^2$$

to moving target $\mathbf{r}_t^l$. It is well-known that sensor i can estimate state $\mathbf{r}_t^l$ if it gets similar data from d other sensors $j_1, \ldots, j_d \in \mathcal{N}^i$, which are its neighbours. For each such column $\mathbf{u} = \mathrm{col}\{i, j_1, \ldots, j_d, l\}$ of $(d+2)$ naturals denote $\bar{\rho}_t^q(\mathbf{u}) = \rho(\mathbf{s}_t^i, \mathbf{r}_t^{l(\mathbf{u})}) - \rho(\mathbf{s}_t^{j_q}, \mathbf{r}_t^{l(\mathbf{u})})$, $q = 1, \ldots, d$. Here and after, $l(\mathbf{u})$ is the map defining the last component of $\mathbf{u}$. In this case, we get d equations

$$\bar{\rho}_t^q(\mathbf{u}) = \sum_{d'=1}^{d} (s_t^{j_q,d'} - s_t^{i,d'})(2r_t^{l(\mathbf{u}),d'} - s_t^{j_q,d'} - s_t^{i,d'})$$

$q = 1, \ldots, d$, which allow us to derive

$$\mathbf{r}_t^l = [C_t^{\mathbf{u}}]^{-1} D_t^{\mathbf{u}} \tag{1}$$

when matrix $C_t^{\mathbf{u}} > 0$ is positive definite. Here we define

$$C_t^{\mathbf{u}} = 2 \begin{bmatrix} (\mathbf{s}_t^{j_1} - \mathbf{s}_t^i)^{\mathrm{T}} \\ \cdots \\ (\mathbf{s}_t^{j_d} - \mathbf{s}_t^i)^{\mathrm{T}} \end{bmatrix}, \; D_t^{\mathbf{u}} = \begin{bmatrix} \bar{\rho}_t^1(\mathbf{u}) + \|\mathbf{s}_t^{j_1}\|^2 - \|\mathbf{s}_t^i\|^2 \\ \cdots \\ \bar{\rho}_t^d(\mathbf{u}) + \|\mathbf{s}_t^{j_d}\|^2 - \|\mathbf{s}_t^i\|^2 \end{bmatrix}.$$

Denote U^i the set of all vectors $\mathbf{u}$ with the first element i and $|U^i|$ the amount of elements in U^i. So, for any $\mathbf{u} \in \cup_{i \in U^i}$ we would like to find estimates $\hat{\mathbf{r}}_t^{l(\mathbf{u})}$ of target $l(\mathbf{u})$ state vector $\mathbf{r}_t^{i,l(\mathbf{u})}$ that

$$f_t(\mathbf{u}, \hat{\mathbf{r}}_t^{l(\mathbf{u})}) = \|\hat{\mathbf{r}}_t^{l(\mathbf{u})} - [C_t^{\mathbf{u}}]^{-1} D_t^{\mathbf{u}}\|^2 \to \min_{\hat{\mathbf{r}}_t^{l(\mathbf{u})}}. \tag{2}$$

The overall multisensor–multitarget problem can be formulated as following minimization problem: at each time t, to find the overall estimate $\hat{\theta}_t = \mathrm{col}\{\hat{\mathbf{r}}_t^1, \ldots, \hat{\mathbf{r}}_t^m\}$ that minimizes the loss function

$$\bar{F}_t(\hat{\theta}_t) = \sum_{i \in \mathcal{N}} \bar{f}_t^i(\hat{\theta}_t) = \sum_{i \in \mathcal{N}} \frac{1}{|U^i|} \sum_{\mathbf{u} \in U_i} f_t(\mathbf{u}, \hat{\mathbf{r}}_t^{l(\mathbf{u})}) \to \min_{\hat{\theta}_t} \tag{3}$$

4. RANDOMIZED TOPOLOGY

Suppose that each sensor $i \in \mathcal{N}$ at every time instant t is able to measure with noise the squared distance to one target and to gather data only from two its neighbors. To satisfy these constraints at each time instant t we suggest to choose randomly independently and uniformly one $\mathbf{u}_t^i$ from U_t^i for each sensor (agent) $i \in \mathcal{N}$ (as in gossip algorithm Boyd et al. (2011)). In fact, we randomize topology graph $\mathcal{G}_A$ in a such manner as in Amelina et al. (2014). At each time instant t we use a randomly chosen subgraph $\mathcal{G}_{B_t} \subset \mathcal{G}_A$ with adjacency matrix B_t which rows contain not more than two nonzero entries: $b_t^{i,j} > 0$ if $j \in \mathbf{u}_t^i$.

Assume each sensor $i \in \mathcal{N}$ at time instant t for chosen estimates $\hat{\mathbf{r}}_t^{l(\mathbf{u}_t^i)}$ gets residuals observation

$$y_t^i = f_t(\mathbf{u}_t^i, \hat{\mathbf{r}}_t^{l(\mathbf{u}_t^i)}) + v_t^i \tag{4}$$

with *unknown–but-bounded* noise v_t^i.

Let $(\Omega, \mathcal{F}, P)$ be the underlying probability space corresponding to the sample space Ω with σ-algebra of all events $\mathcal{F}$ and the probability measure P, and $\mathbb{E}$ be a mathematical expectation symbol. Denote $\mathcal{F}_t$ the σ-algebra of all probabilistic events which happened up to time instant $t = 1, 2, \ldots$, $\mathbb{E}_{\mathcal{F}_t}$ is a symbol of the conditional mathematical expectation with respect to the σ-algebra $\mathcal{F}_t$.

It is not so hard to prove that according to definitions we have

$$\bar{f}_t(\theta_t) = \mathbb{E}_{\mathcal{F}_{t-1}} f_t(\mathbf{u}_t^i, \hat{\mathbf{r}}_t^{l(\mathbf{u}_t^i)}). \tag{5}$$

Hence, multisensors-multitargets estimation problem can be reformulated as distributed non-stationary mean-risk optimization problem (see Erofeeva et al. (2019)):

$$\bar{F}_t(\hat{\theta}_t) = \sum_{i \in \mathcal{N}} \mathbb{E}_{\mathcal{F}_{t-1}} f_t(\mathbf{u}_t^i, \hat{\mathbf{r}}_t^{l(\mathbf{u}_t^i)}) \to \min_{\hat{\theta}_t}. \tag{6}$$

5. SPSA-BASED CONSENSUS ALGORITHM

Considered problem (6) with residuals observation model (4) is similar to the distributed tracking problem studied in Erofeeva et al. (2019) where simultaneous perturbation stochastic approximation-based consensus algorithm was proposed. In this paper we generalize early proposed algorithm to the case when network topology randomly changes with time.

Let $\mathbf{u}_k^i$ and $\boldsymbol{\Delta}_k^i$, $k = 1, 2, \ldots$, $i \in \mathcal{N}$, be observed sequences of independent random vectors from $\mathbb{N}^{d+2}$ and from $\mathbb{R}^d$. The sequence of appointments $\mathbf{u}_k^i$ has uniform distribution on pre-defined sets of indices which are determined by matrix B_{2k} and availabilities for server i to observe target l. $\boldsymbol{\Delta}_k^i$ has Bernoulli distribution with each component independently taking values $\pm\frac{1}{\sqrt{d}}$ with probabilities $\frac{1}{2}$. This sequence is usually called the *simultaneous test perturbation*. Let us take fixed nonrandom initial vectors $\widehat{\theta}_0^i \in \mathbb{R}^{md}$, $i \in \mathcal{N}$, positive step-size α, gain coefficient γ, and choose the scale of perturbation $\beta > 0$.

We consider the algorithm with two observations of distributed sub-functions $\bar{f}_t^i(\theta)$ for each agent $i \in \mathcal{N}$ for constructing sequences of points of observations $\{\mathbf{x}_t^i\}$ and estimates $\{\widehat{\theta}_t^i\}$ of overall state vectors of all targets:

$$\begin{cases} \mathbf{x}^i_{2k} = \widehat{\theta}^i_{2k-2} + \beta \mathbf{e}_{l(\mathbf{u^i_k})} \otimes \mathbf{\Delta}^i_k, \\ \mathbf{x}^i_{2k-1} = \widehat{\theta}^i_{2k-2} - \beta \mathbf{e}_{l(\mathbf{u^i_k})} \otimes \mathbf{\Delta}^i_k, \\ \widehat{\theta}^i_{2k-1} = \widehat{\theta}^i_{2k-2}, \\ \widehat{\theta}^i_{2k} = \widehat{\theta}^i_{2k-1} - \alpha \Bigg[\mathbf{e}_{l(\mathbf{u^i_k})} \otimes \mathbf{\Delta}^i_k \dfrac{y^i_{2k} - y^i_{2k-1}}{2\beta} + \\ \qquad \gamma \sum_{j \in \mathcal{N}^i_t} b^{i,j}_t (\widehat{\theta}^i_{2k-1} - \widehat{\theta}^j_{2k-1}) \Bigg]. \end{cases} \quad (7)$$

The formulation of problem (6) says about one minimized common vector $\widehat{\theta}_t$. We consider n parallel sequences of estimates. In the next section we give the main theoretical result of this paper that all these n sequences converge to the neighbour of the true overall vector θ_t of all targets.

Algorithm (7) is similar to corresponding one in Erofeeva et al. (2019): but in the latter we use brackets above defined random coefficients $b^{i,j}_t$ instead of $a^{i,j}$.

Consider the last equation of the algorithm (7): the first part is similar to SPSA-like algorithm from Granichin and Amelina (2015) and the second one coincides with a local voting protocol (LVP) from Amelina et al. (2015), where it was studied for stochastic networks in the context of load balancing problem. The SPSA part represents a stochastic gradient descent of sub-functions $\bar{f}^i_{\xi_t}(\theta)$, and LVP part is determined for each agent i by the weighted sum of differences between the information about the current estimate $\widehat{\theta}^i_{2k-1}$ of agent i and available information about the estimates of its neighbors.

Further, we use notation $\bar{\theta}_t = \text{col}\{\widehat{\theta}^1_t, \ldots, \widehat{\theta}^n_t\}$ for the common vector of estimates of all agents at time instant t. Also we introduce the following notations:

$$\bar{\mathbf{y}}_t = \text{diag}_n(\text{col}\{y^1_t, \ldots, y^n_t\}),$$
$$\bar{\Delta}_{t\div 2} = \text{col}\{\mathbf{e}_{l(\mathbf{u^i_{t\div 2}})} \otimes \Delta^1_{t\div 2}, \ldots, \mathbf{e}_{l(\mathbf{u^i_{t\div 2}})} \otimes \Delta^n_{t\div 2}\}.$$

Using the notations introduced above, algorithm (7) can be rewritten in the following form

$$\bar{\theta}_{2k} = \bar{\theta}_{2k-1} - \alpha \left[\left(\frac{\bar{\mathbf{y}}_{2k} - \bar{\mathbf{y}}_{2k-1}}{2\beta} \otimes I_{md} \right) \bar{\Delta}_k + \gamma(\mathcal{L}(B_{2k-1}) \otimes I_{md}) \bar{\theta}_{2k-1} \right]. \quad (8)$$

6. MAIN RESULT

This section presents Theorem 1 for the convergence of estimates generated by algorithm (7).

First, let us formulate assumptions about the movement of targets, noise, disturbances, and network topology.

Assumption 1: Denote the differences $\xi^l_t = \mathbf{r}^l_t - \mathbf{r}^l_{t-1}$, $l \in \mathcal{M}$. a) Norms of changing of targets' positions are uniformly bounded: $\forall l \in \mathcal{M}\ \|\xi^l_t\| \le \delta < \infty$, or $\mathbb{E}\|\xi^l_t\|^2 \le \delta^2$ and $\mathbb{E}\|\xi^l_t\|\|\xi^l_{t-2}\| \le \delta^2$ if a sequence $\{\xi^l_t\}$ is random;

b) all matrices $C^{\mathbf{u}^i_k}_{2k}, C^{\mathbf{u}^i_k}_{2k-1}$ are invertible and $\forall i, k\ \mathbb{E}\|Q^i_k\|^2 \le \bar{q}\delta^2$, where $Q^i_k = [C^{\mathbf{u}^i_k}_{2k}]^{-1} D^{\mathbf{u}^i_k}_{2k} - [C^{\mathbf{u}^i_k}_{2k-1}]^{-1} D^{\mathbf{u}^i_k}_{2k-1}$.

Assumption 2: For $k = 1, 2, \ldots$, the successive differences $\tilde{v}^i_k = v^i_{2k} - v^i_{2k-1}$ of observation noise are bounded: $|\tilde{v}^i_k| \le c_v < \infty$, or $\mathbb{E}(\tilde{v}^i_k)^2 \le c^2_v$ if a sequence $\{\tilde{v}^i_t\}$ is random.

Assumption 3: For any $i, j \in \mathcal{N}$ a) vectors $\mathbf{u}^i_k$, $\mathbf{\Delta}^i_k$, $k = 1, 2, \ldots$, $i \in \mathcal{N}$, are mutually independent; b) $\mathbf{u}^i_k$, $\mathbf{\Delta}^i_k$, ξ^l_{2k-1}, ξ^l_{2k}, and $\mathbf{s}^i_{2k-1}$, $\mathbf{s}^i_{2k}$, (if they are random) do not depend on the σ-algebra $\mathcal{F}_{2k-2}$; c) if $\xi^l_{2k-1}, \xi^l_{2k}, \bar{v}^i_n$ are random, then random vectors $\mathbf{u}^i_k$, $\mathbf{\Delta}^i_k$, and elements $\mathbf{s}^i_{2k-1}$, $\mathbf{s}^i_{2k}$, $\xi^l_{2k-1}, \xi^l_{2k}, \bar{v}^i_n$ are independent.

Assumption 4: a) For all $i \in \mathcal{N}$, $j \in \mathcal{N}^i_t$ weights $b^{i,j}_t$ are independent random variables with mean values (mathematical expectations): $\mathbb{E}b^{i,j}_t = b^{i,j}_{av}$, and bounded variances: $\mathbb{E}\|\mathcal{L}(B_t) - \mathcal{L}(B_{av})\|^2 \le \sigma^2_B$ where $B_{av} = [b^{i,j}_{av}]$.
b) Graph $\mathcal{G}_{B_{av}}$ is strongly connected.

To analyze the quality of estimates we apply the following definition for the problem of minimum tracking for mean-risk functional (6).

Definition. A sequence of estimates $\{\bar{\theta}_{2k}\}$ has *an asymptotically efficient upper bound* $\bar{L} > 0$ *of residuals of estimation* if $\forall \varepsilon > 0\ \exists \bar{k}$ such that $\forall k > \bar{k}$

$$\sqrt{E\|\bar{\theta}_{2k} - \mathbf{1}_n \otimes \theta_{2k}\|^2} \le \bar{L} + \varepsilon.$$

Denote $\bar{\mathcal{L}} = \mathcal{L}(B_{av})$, $\bar{\lambda}_2 = \text{Re}(\lambda_2(\bar{\mathcal{L}}))$, $\bar{\lambda}_{\max} = \lambda^{\frac{1}{2}}_{\max}(\bar{\mathcal{L}}^{\mathrm{T}}\bar{\mathcal{L}})$, $\mu = 2\gamma\bar{\lambda}_2 - \alpha(\gamma^2\bar{\lambda}^2_{max} + 4\gamma\bar{\lambda}_{max} + \frac{2\bar{q}\delta^2}{\beta^2} + 4 + \gamma^2\sigma^2_B)$.

The following theorem shows the asymptotically efficient upper bound of estimation residuals provided by algorithm (7).

Theorem 1: If Assumptions *1–4* hold, and positive constant α is sufficiently small: $\alpha < \min(2\gamma\bar{\lambda}_2/\gamma^2(\bar{\lambda}^2_{\max}+\sigma^2_B)+\frac{\bar{q}\delta^2}{\beta^2}+2; 1/\mu)$
then the sequence of estimates provided by algorithm (7) has an asymptotically efficient upper bound which equals

$$\bar{L} = \frac{1}{\mu}\left(h + \sqrt{h^2 + l\mu}\right), \quad (9)$$

where $h = \delta(2\gamma\sqrt{nm}\bar{\lambda}_{\max} + 6\sqrt{m} + \sqrt{\bar{q}} + \alpha\gamma\bar{\lambda}_{max}(\sqrt{\bar{q}} + 2\sqrt{m}) + \frac{\alpha}{\beta^2}(\bar{q}\delta^2 + 2\beta^2)(2\sqrt{m} + \sqrt{\bar{q}}))$, $l = n(\frac{\alpha}{2\beta^2}c^2_v + \delta^2(\frac{4m}{\alpha} + 8m + 4\sqrt{m}\sqrt{\bar{q}} + \frac{\alpha}{2\beta^2}(\bar{q}\delta^2 + 2\beta^2)(2\sqrt{m} + \bar{q})^2))$.

See the proof of Theorem 1 in Appendix.

Remarks. 1. The observation noise v_t in Theorem 1 can be said to be almost arbitrary since it may either be nonrandom but bounded or it may also be a realization of some stochastic process with arbitrary internal dependencies. In particular, to prove the results of Theorem 1, there is no need to assume that v_t and $\mathcal{F}_{t-1}$ are independent.

2. The result of the Theorem 1 shows that for the case $\delta = 0$ (all targets do not change the position with time) we have $\bar{L} = \frac{c_v\sqrt{\alpha}}{\beta\sqrt{2\mu}}$. Under any noise level c_v this bound can be made infinitely small by choosing sufficiently small α. At the same time, in the case of moving targets, the bigger norm of changes δ can be compensated by choosing a bigger step-size α. This leads to a tradeoff between making α smaller because of noisy observations and making α bigger due to the drift of optimal points.

7. SIMULATION

In this section, we consider the numerical experiment, which illustrates the performance of the suggested algorithm (7).

Consider an example of distributed network of 3 planar intellectual sensors (agents) that have in their zone of visibility 6 planar targets whose state vectors are to be estimated. Algorithm (7) working on each node has the following parameters: $\alpha = 0.05$, $\beta = 0.1$, $\gamma = 0.1$. We consider three types of noise: uniformly distributed random variable falling within the interval $[-1; 1]$, an unknown constant, and periodic oscillation near $+1$ and -1. For the last one we switch from $+1$ to -1 every 50th iteration and add $0.1\sin(k)$.

The points $l = 1 \dots m$ start their motion at the position consisting of randomly chosen components from the interval $[0; 100]$. Dynamics of the targets motion is as follows: $\mathbf{r}_t^l = \mathbf{r}_{t-1}^l + \chi_{t-1}^l$. Let χ_{t-1}^l be a random vector uniformly distributed on the ball of radius equal to 0.2. Observers don't move and their coordinates are random values uniformly distributed in interval $[100; 120]$.

Figures 1, 2 show that there exists the time instant t starting with which the estimations converge to the actual value and move next to it.

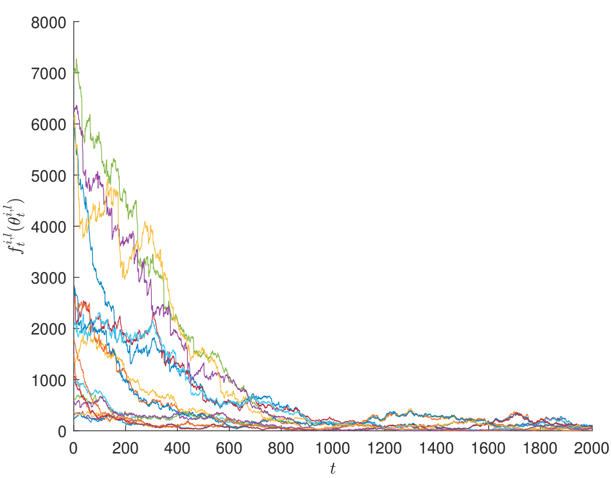

Fig. 1. Residuals $f_t^{i,l}(\mathbf{u}_t^i, \mathbf{r}_t^{i,l})$ obtained by nodes.

Figure 3 shows that we provide the estimations that are typical of every noise, due to similar behaviour of residuals observed with different types of it.

8. CONCLUSION

In this paper we propose the new state estimation method for networked systems with randomized topology combining Simultaneous Perturbation Stochastic Approximation and the consensus algorithm. The SPSA algorithm itself is well-studied and may be used in various applications. However, the new approach makes it possible to relax the assumption regarding the strong convexity of the minimized mean-risk functional. This assumption may not be fulfilled in the distributed optimization problems. We have obtained a finite bound of residual between estimates and time-varying unknown parameters. We have also applied the new algorithm on the multisensor-multitarget problem and validated it through simulation.

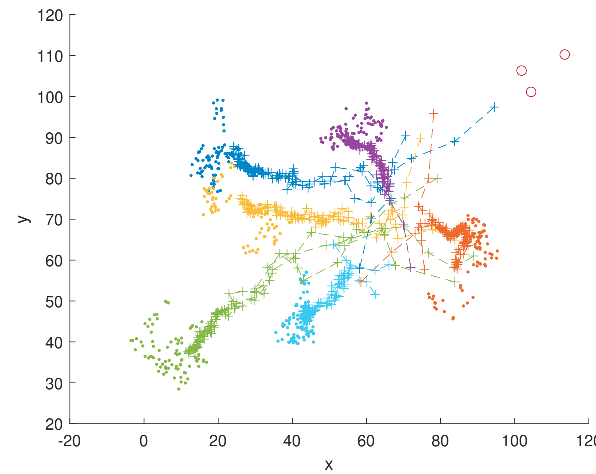

Fig. 2. The estimates $\hat{\mathbf{r}}_t^{i,l}$ obtained by nodes and actual targets positions $\mathbf{r}_t^{i,l}$. (Empty circles denote sensor positions, targets movement is depicted as a series of shaded circles and plus signs show the estimated target positions.) The figure shows sparse data for clarity: each 50th position of targets and the estimates.

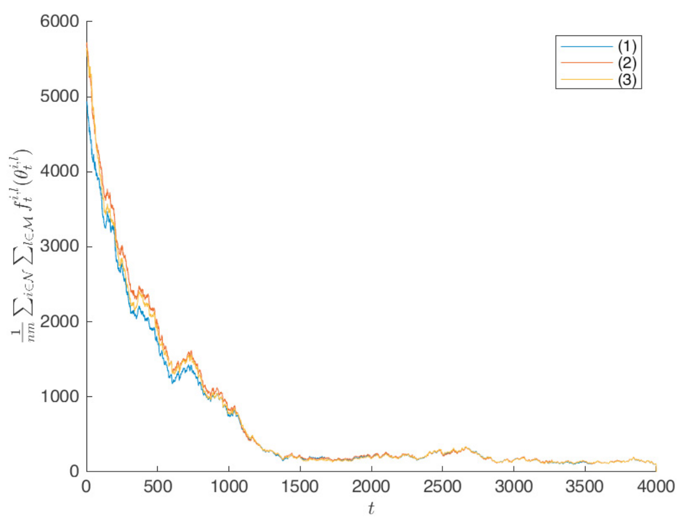

Fig. 3. Average residuals $f_t^{i,l}(\mathbf{u}_t^i, \mathbf{r}_t^{i,l})$ observed with different types of noise: (1) — uniformly distributed random variable falling within the interval $[-1; 1]$, (2) — an unknown constant, (3) — periodic oscillation near $+1$ and -1.

APPENDIX

The following Lemma 1 in Granichin et al. (2009) is instrumental to the proof of Theorem 1.

Lemma 1 Granichin et al. (2009): *If* $e_k > 0$, $\mu, \alpha > 0$, $0 < \mu\alpha < 1$, $h, l \geq 0$,

$$e_k^2 \leq (1 - \mu\alpha)e_{k-1}^2 + 2\alpha h e_{k-1} + \alpha l, \; k = 1, 2, \dots$$

then $\forall \varepsilon > 0 \; \exists K$ such that $\forall k > K$ the following inequality holds: $e_k \leq \frac{1}{\mu}(h + \sqrt{h^2 + l\mu}) + \varepsilon$.

The proof of Theorem 1: Denote $\mathbf{d}_t^i = \widehat{\theta}^i_{2\lceil\frac{t-1}{2}\rceil} - \theta_t$, $\bar{\mathbf{d}}_t = \mathrm{col}\{\mathbf{d}_t^1, \dots, \mathbf{d}_t^n\}$, where $\lceil\cdot\rceil$ is a ceiling function,

$\nu_k = \|\bar{\mathbf{d}}_{2k}\|$, $\bar{\mathbf{s}}_k = \frac{\alpha}{2\beta}((\bar{\mathbf{y}}_{2k} - \bar{\mathbf{y}}_{2k-1}) \otimes I_{md})\bar{\Delta}_k$, $\bar{\mathbf{v}}_t = \mathrm{col}\{\tilde{v}_t^1, \ldots, \tilde{v}_t^n\}$.

Let $\bar{\mathcal{F}}_{k-1} = \sigma\{\mathcal{F}_{k-1}, \bar{\mathbf{v}}_{2k-1}, \bar{\mathbf{v}}_{2k}, \xi_{2k-1}, \xi_{2k}, \bar{\Delta}_k\}$ be the σ-algebra of probabilistic events generated by $\mathcal{F}_{k-1}$, $\bar{\mathbf{v}}_{2k-1}$, $\bar{\mathbf{v}}_{2k}, \xi_{2k-1}, \xi_{2k}, \bar{\Delta}_k$, and $\tilde{\mathcal{F}}_{k-1} = \sigma\{\mathcal{F}_{k-1}, \bar{\mathbf{v}}_{2k-1}, \bar{\mathbf{v}}_{2k}, \xi_{2k-1}, \xi_{2k}\}$,

$$\mathcal{F}_{k-1} \subset \tilde{\mathcal{F}}_{k-1} \subset \bar{\mathcal{F}}_{k-1} \subset \mathcal{F}_k.$$

According to the algorithm (8), we have
$\nu_k = \|\bar{\theta}_{2k-2} - \mathbf{1}_n \otimes \theta_{2k} - \bar{\mathbf{s}}_k - \alpha\gamma(\mathcal{L}(B_{2k-2}) \otimes I_{md})\bar{\mathbf{d}}_{2k-2}\|$ since it is not so hard to prove that $(\mathcal{L}(B_{2k-2}) \otimes I_{md})\mathbf{1}_n \otimes \theta_{2k-2} = 0$ based on the properties of Laplasian matrix $\mathcal{L}(B_{2k-2})$. By virtue of Assumption *4* we get $\mathbb{E}_{\bar{\mathcal{F}}_{k-1}}(\mathcal{L}(B_{2k-2}) - \bar{\mathcal{L}})\bar{\mathbf{d}}_{2k-2} = 0$, where $\bar{\mathcal{L}} = \mathcal{L}(B_{av})$, and taking the conditional expectation over σ-algebra $\bar{\mathcal{F}}_{k-1}$ we derive

$$\mathbb{E}_{\bar{\mathcal{F}}_{k-1}}\nu_k^2 = \|\bar{\mathbf{g}}_k\|^2 + \alpha^2\gamma^2\sigma_B^2\nu_{k-1}^2 - 2\langle\bar{\mathbf{g}}_k, \mathbb{E}_{\bar{\mathcal{F}}_{k-1}}\bar{\mathbf{s}}_k\rangle + \mathbb{E}_{\bar{\mathcal{F}}_{k-1}}\|\bar{\mathbf{s}}_k\|^2, \quad (10)$$

where $\bar{\mathbf{g}}_k = (I_{mnd} - \alpha\gamma\bar{\mathcal{L}} \otimes I_{md})\bar{\mathbf{d}}_{2k-2} + \mathbf{1}_n \otimes (\theta_{2k-2} - \theta_{2k})$.

Denote $\Delta_k^{i,q}$ the q-th component of vector $\mathbf{\Delta}_k^i$, $q = 1, 2, \ldots, d$, and $\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)}$ the components of $\mathbf{d}_{2k-1}^i$ corresponded to target $l \in \mathcal{M}$. Non-zero components of $\bar{\mathbf{s}}_k$ equal to

$$(y_{2k}^i - y_{2k-1}^i)\Delta_k^{i,q} = (\tilde{f}_k^i + \tilde{v}_k^i)\Delta_k^{i,q},$$

where

$$\tilde{f}_k^i = f_{2k}(\mathbf{u}_k^i, \hat{\mathbf{r}}_{2k-2}^{l(\mathbf{u}_k^i)} + \beta\mathbf{\Delta}_k^i) - f_{2k-1}(\mathbf{u}_k^i, \hat{\mathbf{r}}_{2k-2}^{l(\mathbf{u}_k^i)} - \beta\mathbf{\Delta}_k^i) = (2\beta\mathbf{\Delta}_k^i - Q_k^i)^{\mathrm{T}}(2\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)} - Q_k^i).$$

By virtue of Assumption *3* we have $\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}\tilde{v}_k\mathbf{\Delta}_k^i = 0$, $\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}(\mathbf{\Delta}_k^i)^{\mathrm{T}}(2\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)} - Q_k^i) = 0$. Hence, taking the conditional expectation over σ-algebra $\tilde{\mathcal{F}}_{k-1}$ of both sides of the (10) and using observation model (4), we can assert for $\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}\nu_k^2$ as follows

$$\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}\nu_k^2 = \|\bar{\mathbf{g}}_k\|^2 + \alpha^2\gamma^2\sigma_B^2\nu_{k-1}^2 + 2\alpha\sum_{i\in\mathcal{N}}\langle -\mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)} + \alpha\gamma\bar{\mathcal{L}}\mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)} - (\theta_{2k-2} - \theta_{2k}), 2\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)} - Q_k^i\rangle + \frac{\alpha^2}{4\beta^2}\sum_{i\in\mathcal{N}}\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}\left(\tilde{v}_k^i + \tilde{f}_k^i\right)^2\|\Delta_k^i\|^2. \quad (11)$$

Under fulfilment of Assumption *4b*, we have $\bar{\lambda}_2 > 0$ (see Olfati-Saber and Murray (2004)). Hence, for the first term in (11) we derive

$$\|\bar{\mathbf{g}}_k\|^2 \leq \bar{\mathbf{d}}_{2k-2}^{\mathrm{T}}(I_{mnd} - \alpha\gamma(\bar{\mathcal{L}} \otimes I_{md}))^{\mathrm{T}} \times (I_{mnd} - \alpha\gamma(\bar{\mathcal{L}} \otimes I_{md}))\bar{\mathbf{d}}_{2k-2} + 2\alpha\gamma \times \bar{\mathbf{d}}_{2k-2}^{\mathrm{T}}(I_{mnd} - \alpha\gamma(\bar{\mathcal{L}} \otimes I_{md}))^{\mathrm{T}}\mathbf{1}_n \otimes (\theta_{2k-2} - \theta_{2k}) + \|\mathbf{1}_n \otimes (\theta_{2k-2} - \theta_{2k})\|^2 \leq (1 - 2\alpha\gamma\bar{\lambda}_2 + \alpha^2\gamma^2\bar{\lambda}_{\max}^2)\nu_{k-1}^2 + 4\alpha\gamma\sqrt{nm}\bar{\lambda}_{\max}\delta\nu_{k-1} + 4nm\delta^2.$$

Note that $0 \leq (1 - 2\alpha\gamma\bar{\lambda}_2 + \alpha^2\gamma^2\bar{\lambda}_{\max}^2) \leq 1$ according to condition of Theorem 1.

Considering Assumption *1*, and $\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)} = \mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)} + (\theta_{2k-2} - \theta_{2k-1})$ we can evaluate the third term in (11) as following

$$\cdots \leq 2\alpha(2\sqrt{m}\delta + \sqrt{\bar{q}}\delta)\nu_{k-1} + 4\alpha^2\gamma\bar{\lambda}_{max}\nu_{k-1}^2 + 2\alpha^2\gamma\bar{\lambda}_{max}(\sqrt{\bar{q}}\delta + 2\sqrt{m}\delta)\nu_{k-1} + 8\alpha\sqrt{m}\delta\nu_{k-1} + 4\alpha n\sqrt{m}\delta(2\sqrt{m}\delta + \sqrt{\bar{q}}\delta).$$

Consider the squared difference $(\tilde{v}_k^i + \tilde{f}_k^i)^2$ which can be represented as sum of three terms

$$\tilde{v}_k^i + \tilde{f}_k^i = a_1 + a_2 + a_3,$$

where $a_1 = \tilde{v}_k^i$, $a_2 = -(Q_k^i)^{\mathrm{T}}(2\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)} - Q_k^i)$, $a_3 = (2\beta\mathbf{\Delta}_k^i)^{\mathrm{T}}(2\mathbf{d}_{2k-1}^{i,l(\mathbf{u}_k^i)} - Q_k^i)$.

The first two terms do not depend on $\mathbf{\Delta}_k^i$ and $\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}a_q\mathbf{\Delta}_k^i\|\Delta_k^i\|^2 = 0$, $q = 1, 2$, by virtue the Assumption *3*. Hence, we derive $\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}(\tilde{v}_k^i + \tilde{f}_k^i)^2\|\Delta_k^i\|^2 \leq$

$$\mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}(a_1 + a_2)^2 + a_3^2 \leq \mathbb{E}_{\tilde{\mathcal{F}}_{k-1}}2a_1^2 + 2a_2^2 + a_3^2.$$

We need to estimate $\mathbb{E}_{\mathcal{F}_{k-1}}a_q^2$, $q = 1, \ldots, 3$. Taking the conditional expectation over σ-algebra $\mathcal{F}_{k-1}$, by virtue Assumptions *1–3* we evaluate

$$\mathbb{E}_{\mathcal{F}_{k-1}}a_1^2 \leq c_v^2,\ \mathbb{E}_{\mathcal{F}_{k-1}}a_2^2 \leq \bar{q}\delta^2(4\|\mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)}\|^2 + 4\delta(2\sqrt{m} + \sqrt{\bar{q}})\|\mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)}\| + \delta^2(2\sqrt{m} + \sqrt{\bar{q}})^2),\ \mathbb{E}_{\mathcal{F}_{k-1}}a_3^3 \leq 4\beta^2 (4\|\mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)}\|^2 + 4\delta(2\sqrt{m} + \sqrt{\bar{q}})\|\mathbf{d}_{2k-2}^{i,l(\mathbf{u}_k^i)}\| + \delta^2(2\sqrt{m} + \sqrt{\bar{q}})^2).$$

Taking the conditional expectation over σ-algebra $\mathcal{F}_{k-1}$ for the last term in (11) we get

$$\frac{\alpha^2}{4\beta^2}\mathbb{E}_{\mathcal{F}_{k-1}}\sum_{i\in\mathcal{N}}(\tilde{v}_k^i + \tilde{f}_k^i)^2\|\Delta_k^i\|^2 \leq \frac{\alpha^2}{2\beta^2}(nc_v^2 + 4(\bar{q}\delta^2 + 2\beta^2)\nu_{k-1}^2 + 4\delta(\bar{q}\delta^2 + 2\beta^2)(2\sqrt{m} + \sqrt{\bar{q}})\nu_{k-1} + n\delta^2(\bar{q}\delta^2 + 2\beta^2)(2\sqrt{m} + \sqrt{\bar{q}})^2).$$

Summing up the bounds and taking the conditional expectation over σ-algebra $\mathcal{F}_{k-1}$, we derive the following from (11)

$$\mathbb{E}_{\mathcal{F}_{k-1}}\nu_k^2 \leq (1 - \mu\alpha)\nu_{k-1}^2 + 2\alpha h\nu_{k-1} + \alpha l. \quad (12)$$

By virtue condition of Theorem 1, we have $\alpha\mu < 1$. Taking the unconditional expectation of both sides of (12), we see that all conditions of Lemma 1 hold for $e_k = \sqrt{\mathbb{E}\nu_k^2}$.

This completes the proof of Theorem 1.

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