2020 IEEE Industry Applications Society Annual Meeting | 978-1-7281-7192-0/20/$31.00 ©2020 IEEE | DOI: 10.1109/IAS44978.2020.9334744

A Recursive Prediction Error Method with Effective Use of Gradient-Functions to Adapt PMSM-Parameters Online

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***Abstract*— This paper proposes a method for online estimation of electrical parameters of interior permanent magnet synchronous machines (IPMSM) based on the recursive prediction error method (RPEM). The parameter-sensitivity functions (herein known as the gradient functions, $\mathbf{\Psi}^T$) both in dynamic and steady -states are exploited for this purpose. The RPEM has been computed using the stochastic gradient algorithm (SGA). The scalar Hessian matrix, r[k] appearing in the algorithm has been analyzed for both its steady and dynamic states. Different combinations of $\mathbf{\Psi}^T$ and r[k] -states have been simulated and compared with respect to performance when used for parameter adaptation.**

Keywords— Gain-sequence, gradient function, Hessian, magnet flux linkage, online identification, permanent magnet synchronous machine, recursive prediction error algorithm, sensitivity analysis, stochastic gradient

I. INTRODUCTION

Permanent magnet synchronous machines are used nowadays in a multitude of applications in a wide range of power ratings, from watts to several megawatts. Machines without magnetic saliency are typically used in pumps, fans, wind turbines and propulsion in the marine industry. Machines with saliency as such as IPMSM are used in the applications where a large field weakening range is required, in order to increase the torque in the highspeed region by utilizing the reluctance torque of the machine. Irrespective of the application, the preference has been an electric drive without a position or speed transducer to enhanced reliability and/or availability of the motor drive system.

Such sensorless control methods for electric drives have been an interest of research for more than three decades. The classical approach has been the use of estimators based on the mathematical model. Such estimators can be classified as either open-loop estimators/simulation models and closed-loop estimators/observers. This approach, however, fails to accurately estimate the rotor position for a longer operational time at and around zero rotational speed. As a solution, several methods utilizing high frequency signal injection (HFSI) [1][2][3] have been introduced. Such HFSI methods capitalize on the machine saliency, magnetic saturation or slot-effects, etc. Depending on the frequency of the injected signal, such methods can introduce torque harmonics and acoustic noise. Thus, HFSI methods are preferred in the lower speed range.

To limit the range where HFSI is required, it is necessary to improve the performance of the model-based estimators in the lower speed range. Accurate parameter estimation plays a vital role in this regard. Online identification of electrical parameters of the motor improves the performance of the drive and will enable condition monitoring of the machine without additional sensors. Stator winding resistance and permanent magnet flux linkage (ψ_m) are temperature-sensitive motor parameters that should be adapted on-line. Functions for inductances can be adapted in an off-line experiment.

This paper presents a parameter-estimation method which is based on the recursive identification theory explained in [4] and applied for an induction machine in [5]. The method utilizes the sensitivity of the prediction error against the varying parameter. The emphasis has been made on the effective use of gradient functions both in dynamic and steady state. In [5][6][7], the analyses of the prediction errors are used to select a gain matrix. Alternatively, in this paper, a rigorous analysis of the $\mathbf{\Psi}^T$ will be performed to choose the gain in the parameter estimation algorithm. Thus, different algorithms for computation of the gain are discussed. The systematic approach presented in [4] for selection of model set, experimental condition, criterion function, search direction and gain-sequence has been explicitly followed.

II. MOTOR MODEL & ESTIMATION

A. IPMSM Motor Model

The complete model of the electrical part of the machine is in stator and rotor co-ordinates:

$$\underline{u}_s^s = r_s \cdot \underline{i}_s^s + \frac{1}{\omega_n}\frac{d\underline{\psi}_s^s}{dt} \qquad \underline{u}_s^r = r_s \cdot \underline{i}_s^r + \frac{1}{\omega_n}\frac{d\underline{\psi}_s^r}{dt} + \mathbf{j}\cdot f_k \cdot \underline{\psi}_s^r \tag{1}$$
$$\underline{\psi}_s^s = \mathbf{x}_s^s(\vartheta)\cdot \underline{\mathrm{i}}_s^s + \underline{\psi}_m^s \qquad \underline{\psi}_s^r = \mathbf{x}_s^r \cdot \underline{\mathrm{i}}_s^r + \underline{\psi}_m^r$$

In addition, the equation for flux linkages can be used to remove the current vector from the model. The other option is to select the currents as state variables:

$$\underline{u}_s^s = r_s \cdot \underline{i}_s^s + \frac{\mathbf{x}_s^s(\vartheta)}{\omega_n}\cdot\frac{d\underline{\mathrm{i}}_s^s}{dt} + \mathbf{j}\cdot 2\cdot n\cdot \mathbf{x}_g^s(\vartheta)\cdot \underline{\mathrm{i}}_s^s + \mathbf{j}\cdot n\cdot \underline{\psi}_m^s$$
$$\underline{u}_s^r = r_s \cdot \underline{i}_s^r + \frac{\mathbf{x}_s^r}{\omega_n}\cdot\frac{d\underline{\mathrm{i}}_s^r}{dt} + \mathbf{j}\cdot n\cdot \mathbf{x}_s^r\cdot \underline{\mathrm{i}}_s^r + \mathbf{j}\cdot n\cdot \underline{\psi}_m^r \tag{2}$$

Here ϑ is the electrical angle of the mechanical position $p^*\vartheta_{mech}$, where p is the number of pole pairs. The inductance matrix can be expressed as:

$$\mathbf{x}_s^s(\vartheta) = x_{s\sigma}\cdot \mathbf{I}_2 + \mathbf{x}_{sm}^s + \mathbf{x}_g^s(\vartheta)$$
$$\mathbf{x}_{sm}^s = x_{sm}\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad \mathbf{x}_g^s(\theta) = x_g\cdot\begin{bmatrix}\cos 2\vartheta & \sin 2\vartheta\\ \sin 2\vartheta & -\cos 2\vartheta\end{bmatrix} \tag{3}$$

The rotor-oriented inductance matrix becomes:

$$\mathbf{x}_s^r(\theta)=\begin{bmatrix} x_d & 0 \\ 0 & x_q \end{bmatrix} \qquad \mathbf{j}=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \mathrm{x}_g=\frac{x_d-x_q}{2} \tag{4}$$

B. Open-loop/Simulation Model

The open-loop or simulation -models are the models based on the equations presented above. Unlike the observers, open-loop models do not have feedback from measurements to correct the model. Thus, these models become more sensitive to parameter-errors and measurement noise.

In a full-order open-loop/simulation model $\mathcal{M}_{u\vartheta}$, either stator flux linkages or stator currents can be chosen as state variables. When selecting the currents as state variables, the rotor- oriented model is preferred:

$$\underline{u}_s^r=\hat{r}_s\cdot\hat{\underline{i}}_s^r+\frac{\mathbf{x}_s^r}{\omega_n}\cdot\frac{d\hat{\underline{i}}_s^r}{dt}+\mathbf{j}\cdot n\cdot\hat{\mathbf{x}}_s^r\cdot\hat{\underline{i}}_s^r+\mathbf{j}\cdot n\cdot\underline{\hat{\psi}}_m^r$$

$$\underline{\hat{\theta}}=\begin{bmatrix}\hat{r}_s & \hat{x}_d & \hat{x}_q & \hat{\psi}_m\end{bmatrix}^T \qquad \underline{\hat{\psi}}_m^r=\begin{bmatrix}\hat{\psi}_m & 0\end{bmatrix}^T \tag{5}$$

$$\underline{\mathrm{i}}_s^r=\mathbf{T}_{ss}^r(\vartheta)\cdot\underline{\mathrm{i}}_s^s \qquad \underline{u}_s^r=\mathbf{T}_{ss}^r(\vartheta)\cdot\underline{u}_s^s$$

As shown in (5), position and speed must be either measured or estimated. This model will be used as an example to show how to develop a recursive parameter estimation routine and models for the prediction error sensitivity function. In this paper the full-order model $\mathcal{M}_{u\vartheta}$ and stator currents as state variables has been chosen. To focus only the proposed parameter-estimation method, it is assumed that the rotor position is measured.

III. Parameter Sensitivity of the Control Strategy

Sensitivity of the max Torque/Amp Control algorithm with respect to parameter variations will be investigated for the case with measured rotor position. The temperature dependent parameters ψ_m and r_s should be adapted on-line. The value of r_s is mainly influencing the control in the field weakening range due to limited available voltage from the inverter. The value of ψ_m , however, is influencing the torque control in the complete torque-speed plane. A 10% under-estimated ψ_m will give an error in the torque control as shown in Figure 1.

IV. Parameter Estimation by Use of the $\mathcal{M}_{u\vartheta}$ Model

When developing an estimation algorithm, one needs to choose a Model Set and a criterion function. The model set chosen is the $\mathcal{M}_{u\vartheta}$ model. This is a second order system and the eigenvalues of this model are speed dependent. The system matrix **A** of the linearized system can be expressed as:

$$\lambda\cdot I_2-\mathbf{A}=\begin{bmatrix} \lambda+\dfrac{\omega_n\cdot\hat{r}_s}{\hat{x}_d} & -\dfrac{\omega_n\cdot n\cdot\hat{x}_q}{\hat{x}_d} \\ \dfrac{\omega_n\cdot n\cdot\hat{x}_d}{\hat{x}_q} & \lambda+\dfrac{\omega_n\cdot\hat{r}_s}{\hat{x}_q}\end{bmatrix} \tag{6}$$

The eigenvalues become:

$$\lambda_{1,2}=-\frac{1}{2}\cdot\left(\frac{1}{\hat{T}_d}+\frac{1}{\hat{T}_q}\right)\pm\sqrt{\left(\frac{1}{2}\cdot\left(\frac{1}{\hat{T}_d}+\frac{1}{\hat{T}_q}\right)\right)^2-\left(\frac{1}{\hat{T}_d\cdot\hat{T}_q}+(\omega_n\cdot n)^2\right)} \tag{7}$$

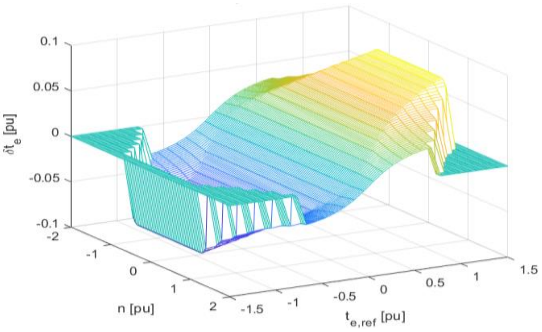


Fig 1: Torque error due to 10% under-estimated ψ_m

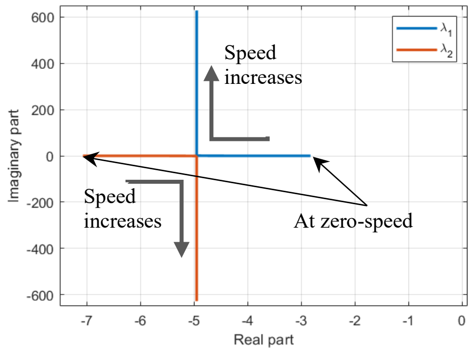


Fig 2: Eigenvalues, $\lambda_{1,2}$ of the full-order model

The eigenvalues are plotted as function of speed in Figure 2 and shows that the model is stable for all values of n. The imaginary part of the eigenvalues increases with the speed.

A quadratic criterion with a prediction error based on stator currents are chosen. The continuous version of the estimation algorithm then becomes:

$$\underline{\varepsilon}_s^r(t,\underline{\hat{\theta}})=\delta\underline{i}_s^r=\underline{i}_s^r-\hat{\underline{i}}_s^r \qquad \underline{\mathrm{i}}_s^r=\mathbf{T}_{ss}^r(\vartheta)\cdot\underline{\mathrm{i}}_s^s$$

$$\underline{\hat{\theta}}=\begin{bmatrix}\hat{r}_s & \hat{x}_d & \hat{x}_q & \hat{\psi}_m\end{bmatrix}^T \tag{8}$$

$$\frac{d\underline{\hat{\theta}}}{dt}=\mathbf{L}_c\cdot\underline{\varepsilon}_s^r(t,\underline{\hat{\theta}}) \qquad \underline{\varepsilon}_s^r=\begin{bmatrix}\varepsilon_d(t,\underline{\hat{\theta}}) & \varepsilon_q(t,\underline{\hat{\theta}})\end{bmatrix}^T$$

The values of the prediction errors for 10% under-estimated ψ_m is shown in Figure 3. It is evident that the d-axis component of the prediction error is more sensitive to incorrect ψ_m and should be used for estimation of this parameter. This can be shown by inspection of the steady-state prediction errors:

$$\varepsilon_d=-\frac{n^2\cdot\hat{x}_q}{\hat{r}_s^2+n^2\cdot\hat{x}_q\cdot\hat{x}_d}\cdot(\psi_m-\hat{\psi}_m)$$

$$\varepsilon_q=-\frac{n\cdot\hat{r}_s}{\hat{r}_s^2+n^2\cdot\hat{x}_q\cdot\hat{x}_d}\cdot(\psi_m-\hat{\psi}_m) \tag{9}$$

C. Gradient $\mathbf{\Psi}$ of the prediction errors

When selecting the search direction in the parameter space, the negative gradients of the prediction errors with respect to the parameters is a very important matrix. While the measured rotor position is used in the transformation, the measured currents are not dependent on the model parameters such that the gradient function becomes:

$$\boldsymbol{\Psi}^T = -\frac{d\underline{\varepsilon}_s^r\left(t,\hat{\underline{\theta}}\right)}{d\hat{\underline{\theta}}} = \left[\frac{d\hat{\underline{i}}_s^r\left(t,\hat{\underline{\theta}}\right)}{d\hat{\theta}_1} \quad \frac{d\hat{\underline{i}}_s^r\left(t,\hat{\underline{\theta}}\right)}{d\hat{\theta}_2} \quad \cdots\cdots\right] \tag{10}$$

The differential equations for this gradient function are given by the equation below. The order of the model set $\mathcal{M}_{u\vartheta}$ is n=2. Please observe that the dynamic model for $\boldsymbol{\Psi}$ is of order nxd, where d is the dimension of the parameter vector $\underline{\theta}$. This means that the order of this model increases with n for each parameter that is estimated. The dynamic model of $\boldsymbol{\Psi}$ can be expressed as:

$$\frac{d\left(\frac{d\hat{\underline{i}}_s^r}{d\hat{\underline{\theta}}}\right)}{dt} = \frac{d}{d\hat{\underline{\theta}}}\left(\omega_n \cdot \hat{\mathbf{x}}_s^{-r} \cdot \left(-\left(\hat{r}_s + \mathbf{j}\cdot n \cdot \hat{\mathbf{x}}_s^r\right)\cdot \hat{\underline{i}}_s^r - \mathbf{j}\cdot n\cdot \hat{\underline{\psi}}_m^r + \underline{u}_s^r\right)\right) \tag{11}$$

When only ψ_m is estimated, the dynamic model becomes on component form:

$$\begin{aligned} \frac{d\left(\frac{d\hat{i}_d}{d\hat{\psi}_m}\right)}{dt} &= -\frac{\omega_n\cdot \hat{r}_s}{\hat{x}_d}\cdot\frac{d\hat{i}_d}{d\hat{\psi}_m} + \frac{\omega_n\cdot n\cdot\hat{x}_q}{\hat{x}_d}\cdot\frac{d\hat{i}_q}{d\hat{\psi}_m} \\ \frac{d\left(\frac{d\hat{i}_q}{d\hat{\psi}_m}\right)}{dt} &= -\frac{\omega_n\cdot \hat{r}_s}{\hat{x}_q}\cdot\frac{d\hat{i}_q}{d\hat{\psi}_m} - \frac{\omega_n\cdot n\cdot\hat{x}_d}{\hat{x}_q}\cdot\frac{d\hat{i}_d}{d\hat{\psi}_m} - \frac{\omega_n\cdot n}{\hat{x}_q} \end{aligned} \tag{12}$$

This model has the same eigenvalues as the model $\mathcal{M}_{u\vartheta}$ and is thus stable. The dynamics of the gradients are given by d- and q-axis time constants T_d , T_q and speed n. The speed n is also the input or excitation for this dynamic system. The steady state solution of these equations are:

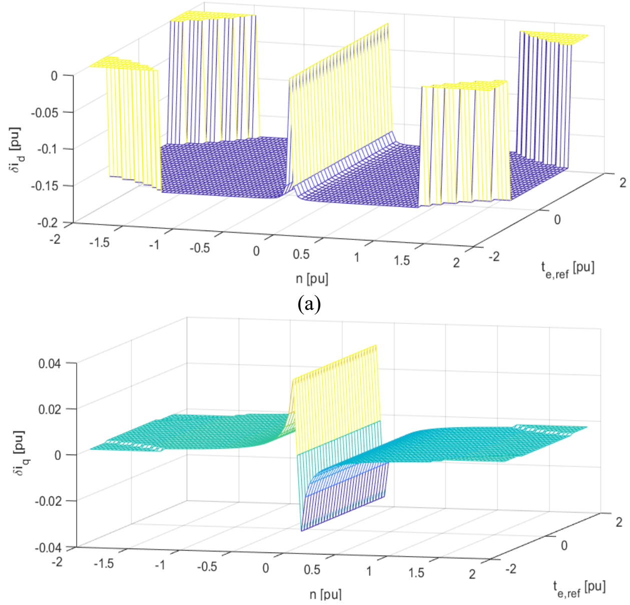
$$\begin{aligned} \frac{d\hat{i}_d}{d\hat{\psi}_m} &= -\frac{n^2\cdot\hat{x}_q}{\hat{r}_s^2 + n^2\cdot\hat{x}_q\cdot\hat{x}_d} \\ \frac{d\hat{i}_q}{d\hat{\psi}_m} &= -\frac{n\cdot\hat{r}_s}{\hat{r}_s^2 + n^2\cdot\hat{x}_q\cdot\hat{x}_d} \end{aligned} \tag{13}$$

The gradient of the d-axis prediction error becomes $-1/x_d$ in most of the speed range and is independent of torque. The q-axis component becomes quite small due to r_s -dependency. Both functions are equal 0 at zero speed. These functions are plotted in the torque-speed plane in Figure 4. From these plots it can be concluded that the d-component of the prediction error should be used for estimation of ψ_m. When implementing the model in a digital controller, the method of discretization has to be considered as well. Usually the Forward Euler Method is sufficient. It is then important to investigate the stability of the discrete model for both $\mathcal{M}_{u\vartheta}$ and $\boldsymbol{\Psi}^T$.

D. Search direction and gain-sequence

The parameter estimation algorithm on discrete form based on this Forward Euler Method becomes:

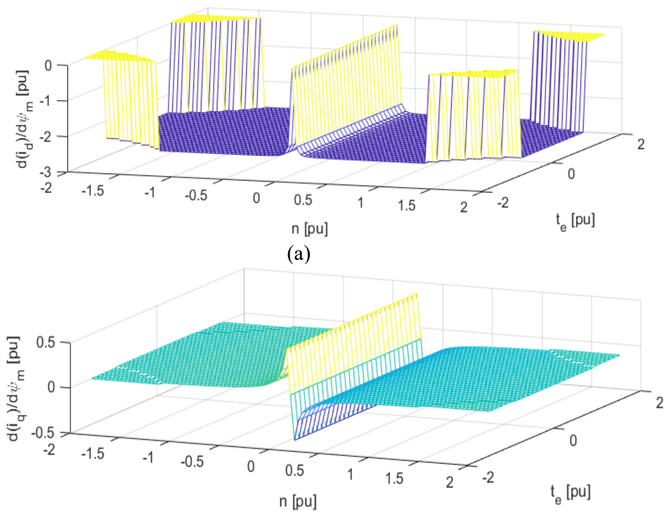
$$\hat{\underline{\theta}}[k] = \left[\hat{\underline{\theta}}[k-1] + \mathbf{L}\cdot\underline{\varepsilon}_s^r[k]\right]_{D_M} \tag{14}$$



(a)

(b)

Fig 3: Sensitivity plot of prediction errors (a) d-axis prediction error (b)q-axis prediction error



(a)

(b)

Fig 4: Gradients of prediction errors for (a) d-axis gradient (b) q-axis gradient

where $D_{\mathcal{M}}$ is in the stable region of the model $D_{\mathcal{M}} \subseteq D_s$. This means that all model parameters and the sampling time T_{samp} must be chosen such that the discrete model is stable.

General *stochastic gradient algorithm* can be expressed as:

$$\begin{aligned} \mathbf{L} &= \gamma[k]\cdot\frac{\boldsymbol{\Psi}[k]}{r[k]}\cdot\underline{\varepsilon}[k] \\ r[k] &= r[k-1] + \gamma[k]\cdot\left\{tr\left[\boldsymbol{\Psi}[k]\cdot\boldsymbol{\Psi}^T[k]\right] - r[k-1]\right\} \end{aligned} \tag{15}$$

Here r[k] is the scalar version of the Hessian matrix used in stochastic gradient algorithm and the trace of a matrix is the sum of the diagonal elements. The gain sequence γ could be time dependent, but a constant value γ_0 is chosen. This memory coefficient γ_0 of the algorithm should be chosen such that the parameter is "almost constant" within the time period $T_0 = T_{samp}/\gamma_0$ [4]. The initial value of r[k] must be different from zero to prevent division by zero. It is possible to choose different γ[k] in the filter for r[k] and the gain $\mathbf{L}$.

Four versions of this algorithm will be investigated:

- Dynamic solution of $\mathbf{\Psi}^T$ and dynamic r[k]
- Dynamic solution of $\mathbf{\Psi}^T$ and steady state r[k]
- Steady state solution of both $\mathbf{\Psi}^T$ and r[k]
- Steady state solution of $\mathbf{\Psi}^T$ and dynamic r[k]

For *dynamic solution of both* $\mathbf{\Psi}^T$ *and r[k]* the algorithm becomes:

$$\begin{aligned}
\hat{\psi}_m[k] &= \left[\hat{\psi}_m[k-1] + \frac{\gamma_0}{r[k]}\cdot\frac{d\hat{i}_d[k]}{d\hat{\psi}_m}\cdot\varepsilon_d[k]\right]_{D_M} \\
r[k] &= r[k-1] + \gamma_0\cdot\left\{\left(\frac{d\hat{i}_d[k]}{d\hat{\psi}_m}\right)^2 - r[k-1]\right\} \qquad (16) \\
D_M &= \left\{\psi_{m,\min} \le \hat{\psi}_m \le \psi_{m,\max}\right\}
\end{aligned}$$

The value of r[k] is always positive and limited to a minimum value of $r_{\min}$. The initial value r[0] has to be chosen as well. Here r[0] is chosen to be $r_{\min}$=0.01.

For *dynamic solution of* $\mathbf{\Psi}^T$ *and steady state r[k]* one obtains the gain **L** as follows:

$$L = \frac{\gamma_0}{r[k]}\cdot\frac{d\hat{i}_d[k]}{d\hat{\psi}_m} \qquad r[k] = \left(\frac{d\hat{i}_d[k]}{d\hat{\psi}_m}\right)^2_{\lim} \qquad (17)$$

For *steady state solution of both* $\mathbf{\Psi}^T$ *and r[k]* one obtains the gain L as follows:

$$L = -\gamma_0\cdot\left(\frac{\hat{r}_s^2}{n^2[k]\cdot\hat{x}_q} + \hat{x}_d\right) \qquad |n| > n_{o,\lim it} \qquad (18)$$

It is important to limit the gain L at low speeds to avoid amplification of the noise in the current measurements.

For *steady state solution of* $\mathbf{\Psi}^T$ *and dynamic r[k]* one obtains the gain L as follows:

$$\begin{aligned}
L &= \frac{\gamma_0}{r[k]}\cdot\frac{d\hat{i}_d[k]}{d\hat{\psi}_m} \qquad \frac{d\hat{i}_d[k]}{d\hat{\psi}_m} = -\frac{n^2[k]\cdot\hat{x}_q}{\hat{r}_s^2 + n^2[k]\cdot\hat{x}_q\cdot\hat{x}_d} \qquad (19) \\
r[k] &= r[k-1] + \gamma_0\cdot\left\{\left(\frac{d\hat{i}_d[k]}{d\hat{\psi}_m}\right)^2 - r[k-1]\right\}
\end{aligned}$$

In general, the *Gauss-Newton* search direction is preferred. This method usually gives faster convergence than the stochastic gradient algorithm. Due to the relatively slow variation of the temperature, the convergence rate versus complexity of the algorithm should be evaluated. Different implementations of such Gauss-Newton algorithms exist [4]. The intention is to reduce the number of inversion of matrices to reduce the computation burden. There are also some solutions which give better individual gains for the different parameters. In this work, however, only one parameter and one prediction error are used. Thus, the algorithm becomes much simpler and identical to the stochastic gradient algorithm.

V. Simulation Results

An IPMSM drive with a 2-level inverter and quadratic load has been simulated in MATLAB Simulink/Simscape toolbox. Asymmetrical modulation with 3rd harmonic injection has been used. The switching frequency is 3 kHz and the sampling frequency of the controller is 6 kHz. The motor data are:

U_N= 690 [V] I_N= 478 [A] f_N= 50 Hz   polepairs =1

$$\underline{\hat{\theta}} = \begin{bmatrix}\hat{r}_s & \hat{x}_d & \hat{x}_q & \hat{\psi}_m\end{bmatrix}^T = \begin{bmatrix}0.009 & 0.4 & 1 & 0.66\end{bmatrix}^T$$

In these simulations, the parameter estimation algorithm is started immediately at start-up of the drive. The d- and q-current references are calculated by help of a 3rd order approximation of the of the 4th order equation of i_d as function of the torque:

$$i_{dref} = \frac{\dfrac{\hat{\psi}_m}{3} - \sqrt[3]{\left(\dfrac{\hat{\psi}_m}{3}\right)^3 + \dfrac{\left(\hat{x}_q - \hat{x}_d\right)^2\cdot\tau_{eref}^2}{3\cdot\hat{\psi}_m}}}{x_q - x_d} \qquad (20)$$

$$i_{qref} = \frac{\tau_{eref}}{\hat{\psi}_m - \left(\hat{x}_q - \hat{x}_d\right)\cdot i_d} = \frac{\tau_{eref}}{\hat{\psi}_T}$$

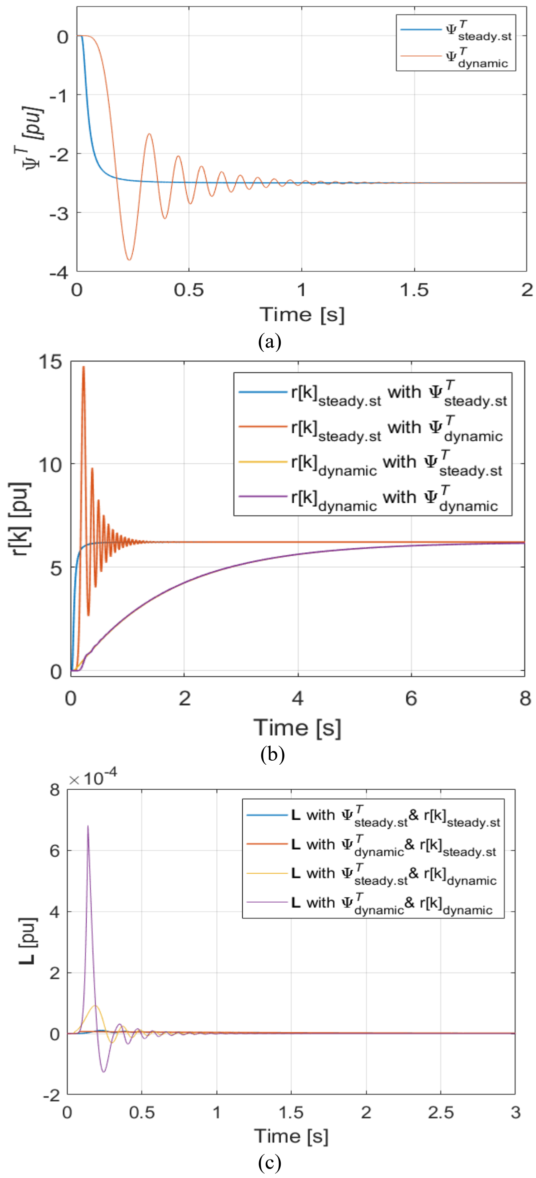
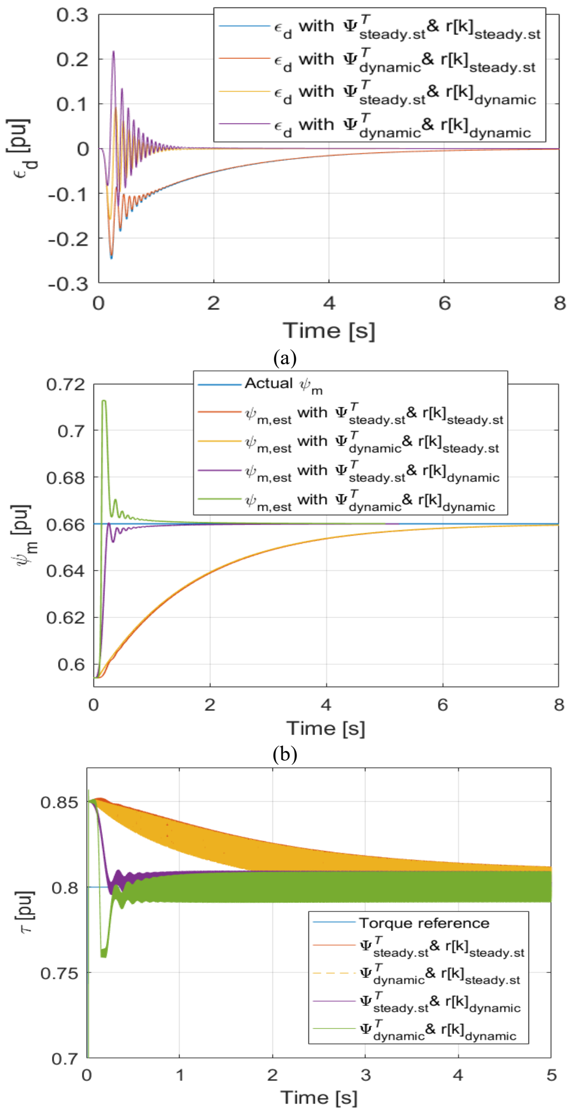


Fig 5: Steady-state and dynamic behaviours of (a) gradient functions (b) Hessian function (c) Gain-sequence



(a) (b) (c)

Fig 6: Convergence of (a) prediction error to zero (b) estimated parameter to actual parameter (c) Actual torque to reference torque

VI. DISCUSSION

As shown in Figure 5(a), the dynamic gradient function will have oscillations due to the complex eigenvalues at higher speeds. However, the average value of the dynamic gradient function does not differ significantly from the steady-state gradient function. In addition, the speed, which is the input to the gradient function model, does not change very rapidly due to the inertia of the system. This means that the filtering effect of the dynamic gradient function is not required in this context, thus, the steady state solution of the gradient is good enough and it's relatively less demanding to compute. Use of dynamic r[k], however, is a preferred solution while this will boost the gain **L** at start-up by diving on a small number. How large this boosting will be, and for how long time it will last, is given by the filter time constant in γ_0 and the initial value r[0].

As seen in figure 6(c), the fastest parameter convergence is achieved with dynamic solution of $\mathbf{\Psi}^T$ and dynamic r[k]. However, this method gives large overshoot in the estimate of ψ_m. This is due to the slow increase of r[k] as a result of slower rise in the gradient function. For dynamic solution of $\mathbf{\Psi}^T$ and steady state r[k] the convergence rate is rather low, while the gain **L** becomes inversely proportional with the dynamic gradient function. The gain obtained by steady state solution of both $\mathbf{\Psi}^T$ and r[k] is the same gain obtained by inspection of the prediction errors. This method gives marginally slower convergence rate compared to the previous method, but has the least computational burden. The method with steady state solution of $\mathbf{\Psi}^T$ and dynamic r[k] gave the fastest and most well damped convergence of the parameter estimate. This is thus the preferred solution.

Oscillations in the predicted current is not so critical, while we use this predicted current for parameter estimation and not for control. Unlike observers, the open-loop model is more sensitive for parameter errors thus, optimal for parameter estimation. Observer, on the other hand, is less sensitive and thus preferred for control. In addition, with observers, the poles can be located to achieve a more well-damped system.

VII. CONCLUSION

Permanent magnet flux linkage of IPMSM has been estimated with the use of recursive prediction error method by using stochastic gradient algorithms. The gradient functions and scalar versions of the Hessian matrix have been investigated to calculate the gain in the parameter estimation algorithm. The investigation resulted in 4 different algorithms in combination of steady and dynamic states of gradient functions and the scalar Hessian matrix. The dynamic gradient function does not provide faster convergence than its steady-state counterpart. Considering the relative simplicity in computation, it can be concluded that steady-state gradient function is a sensible choice. On the contrary, the dynamic Hessian results in a larger initial gain over its steady-state counterpart. Such initial boost in the gain leads to a faster parameter convergence, thus torque convergence. Therefore, it can be concluded that for the slow-changing parameter ψ_m, RPEM with steady-state gradient function and dynamic Hessian matrix gives the most optimal performance.

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