Real time optimization of systems with fast and slow dynamics using a lookahead strategy

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Abstract— **Systems with fast and slow dynamics give rise to objectives in different time scales which may not be aligned. The existing dynamic optimal control methods might become computationally infeasible due to the fine discretization required to capture the fast dynamics. On the other hand, a real time optimization (RTO) method based on steady-state models, which is computationally efficient, can greedily drive the plant towards optimal operation. The drawback of the RTO approach is that it may yield actions that only focus on near future goals and the objectives involving the slower dynamics are neglected. In this paper, we propose to extend RTO with a lookahead strategy by introducing a predictor to capture the effect of changing the current controls on the long-term objective. In this way, we introduce the long-term objectives in RTO while maintaining its computational efficiency and not losing focus of short-term objectives. The proposed approach is demonstrated in a simulation study from offshore petroleum production, that compares the proposed method with both an "industry-standard" RTO method, and a full fledged dynamic optimization method that takes both slow and fast dynamics into account. The proposed methodology performs almost as well as the dynamic optimization method while maintaining a low computational effort.**

I. INTRODUCTION

Complex industrial processes have goals in different time scales ranging from the long-term planning and scheduling to automatic actions for stable and safe operation. Such goals can be of different nature, and it can be challenging to achieve an overall good performance treating the problem as a whole. Thus, a proper manner to deal with such problems is to decompose the decision making process into several layers [1]. This hierarchical methodology is accepted as a standard industrial practice [2] and also well-known in academic literature as plant-wide control [3]–[6].

The decisions in a hierarchical control system are divided in layers depending on their time frame [3] as shown in Fig. 1. The top layer, commonly known as Asset Management, focuses on long-term decisions which typically involve risk handling, investments and infrastructure planning. Next are the decisions taken in a time frame of a couple of days or weeks such as operations scheduling and plant-wide operations. Next follows the Real-time Optimization (RTO) layer, which is responsible for decisions that have to be taken in a time scale of hours or days. Shorter-term operational decisions regarding the control and automation of the plant aim to keep operations stable and mitigate disturbances in a range of seconds to hours, being performed automatically without human interference.

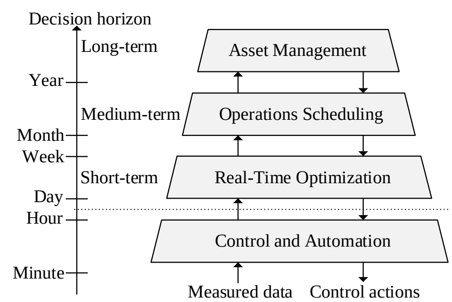

Fig. 1. Multi-level control hierarchy, adapted from [7].

Traditionally the interplay between the layers functions with the upper layers providing setpoints to the layers below such that near-optimal economic operation of the process is achieved. This hierarchical scheme is suitable in many industrial applications, especially when the objectives in the layers are aligned. However, in some applications, these goals might be conflicting in different time scales such that the overall objective of the control system becomes less obvious.

An example of such problems appears in resource-constrained production optimization. In the short-term, the top-ranked objective is the maximization of economic revenues obtained with the current production, while in a long-term perspective this might lead to suboptimal utilization of resources. In other words, the decision variables might be shared between different layers, but the objectives are not necessarily aligned, which means that prioritizing short-term economic performance comes at the cost of long-term economic performance.

In some cases, problems of this type are approached by constructing objectives that weigh control performance against usage of inputs, yielding thereby a multi-objective optimization problem for which some tuning effort is required [8], [9].

Depending on the research field, this approach may be known as Dynamic Real Time Optimization (DRTO) or Economic Model Predictive Control (EMPC). These frameworks are capable of handling complex dynamical systems with conflicting objectives and constraints. The main limitations of this approach are the requisite of computational power,

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and the potential unexpected behaviour due to an improper choice of weights, which is typically problem-dependent. The former limitation becomes evident as the size of the decision variable vector grows due to the increasing amount of control steps needed to capture the dynamics of the faster system.

The challenge addressed in this paper is inspired by production optimization problems involving multiple time scales within the petroleum industry. The proposed solution is generalized into a systematic approach which can possibly be extended to other application areas. It consists of a novel control method that reconciles problems with conflicting goals and heterogeneous time scales by incorporating long-term objectives in the RTO layer using a lookahead strategy. This involves some level of integration between the models from different layers, but in such a way that the integrated problem remains computationally tractable.

The paper is organized as follows: Section II presents a motivating example from the petroleum industry. In Section III we provide a general formulation for control problems of systems containing both fast and slow dynamics. Section IV contains the RTO with a lookahead strategy and the solution method. In Section V we demonstrate the feasibility of the proposed methodology by applying it to the motivating example. Section VI contains a discussion. We conclude with Section VII.

II. MOTIVATING EXAMPLE

In offshore petroleum production, there is typically a set of wells producing from a hydrocarbon reservoir to the topside facilities, see Fig. 2. While the reservoir consists of considerable amounts of hydrocarbons trapped in subsurface porous media, the network is a set of pipelines connecting the wells to the platform.

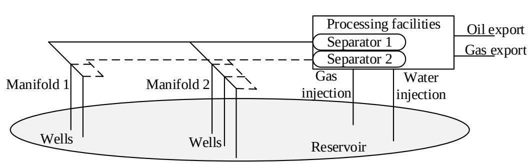

Fig. 2. Integrated production system with a reservoir, a production gathering network, and the processing facilities [7].

The reservoir fluids are produced by the wells and collected by a manifold before reaching the surface for separation, processing and further exportation. The oil is stored in tanks, while the water is treated before disposal or reinjection, and the gas is either flared, reinjected or exported. There are also some wells injecting either water or gas into the reservoir to provide pressure support and enhance the overall recovery factor of the asset.

The control hierarchy presented in Fig. 1 is also suitable to describe the decisions in petroleum applications. In asset management the production infrastructure and topside facilities are designed. This provides a constraint in terms of number of wells and processing capacity for the reservoir management layer. This layer, on the other hand, determines well locations and drainage strategies that improve the long-term gains. Then, the RTO layer determines the well controls and the platform settings to maximize the production such that the operational constraints are honored. Finally, the control and automation layer guides the system towards the optimum, and autonomously stabilizes the rates and pressures of the wells. Notice that rate control is performed in both the reservoir management and the RTO layers, often with fairly different operational strategies. While in reservoir management the focus is to maintain the reservoir pressure and improving the overall recovery factor, the RTO layer acts greedily, prioritizing short-term gains often ignoring effects on the long-term objective.

A. *Reservoir management: long-term optimization*

The oil-water fluid flow in the reservoir can be described with a set of differential equations based on the mass-conservation principle, the Darcy law, and the capillary pressure principle [10]. After the discretization in time and space, the pressure of oil $p_{\mathrm{oil},j,k}$ and the water saturation $S_{\mathrm{water},j,k}$ typically suffice to describe the conditions in each grid block $j \in \mathcal{G}$ at time step $k \in \mathcal{K}$. The flows through the wells are a boundary condition for two-phase fluid flow in porous media, with $q_{\mathrm{p},i,k}$ denoting the flow rate of phase p in well $i \in \mathcal{N}_{\mathrm{w}}$, and $p_{\mathrm{bh},i,k}$ being its Bottom-Hole Pressure (BHP) at time step k. A state space model can be utilized to formulate the aforementioned set of equations:

$$x_{j,k} = (p_{\mathrm{oil},j,k}, S_{\mathrm{water},j,k}), \forall j \in \mathcal{G}, \tag{1a}$$
$$v_{i,k} = (q_{\mathrm{p},i,k}, p_{\mathrm{bh},i,k}), \qquad \forall i \in \mathcal{N}_{\mathrm{w}}, \tag{1b}$$

where the states x_k are the pressure of oil and water saturation in the grid blocks, and the algebraic variables v_k are the flows and pressures in the wells. The control vector u_k contains one variable of v_k per well which are used as a setpoint whereas the remaining variables in v_k are calculated (cf. (2c)). Notice that, while the algebraic vector v_k has the length of the same magnitude of the number of wells, the state vector x_k have the same magnitude of the number of grid blocks in $\mathcal{G}$, which might range from thousands to a few million. The implicit equations to be solved at each simulation step are:

$$0 = R(x_{k-1}, x_k, v_k), \tag{2a}$$
$$0 = V(x_k, v_k), \tag{2b}$$
$$0 = B(v_k, u_k), \tag{2c}$$

where $R(\cdot)$ represents the fluid flow in the reservoir, $V(\cdot)$ maps the reservoir states and the well algebraic variables, and $B(\cdot)$ are the relationship between the setpoints and the algebraic variables, see [10] for more details.

The long-term rate control problem can be formulated as the maximization of the undiscounted Net Present Value (NPV):

$$\sum_{k=1}^{N} t_k \left[\sum_{i \in \mathcal{N}_{\mathrm{w,prod}}} (q_{o,i,k} r_o - q_{w,i,k} r_w) - \sum_{i \in \mathcal{N}_{\mathrm{w,inj}}} (q_{w,i,k} r_i) \right] \tag{3}$$

subject to the implicit system of equations (2) and the well control bounds:

$$u_{\mathrm{lb},i,k} \leq u_{i,k} \leq u_{\mathrm{ub},i,k}, \quad \forall i \in \mathcal{N}_{\mathrm{w}}, k \in \mathcal{K}, \tag{4}$$

where $\mathcal{N}_{\mathrm{w,prod}}$ and $\mathcal{N}_{\mathrm{w,inj}}$ are the sets of production and injection wells, respectively, with $\mathcal{N}_{\mathrm{w}} := \mathcal{N}_{\mathrm{w,prod}} \cup \mathcal{N}_{\mathrm{w,inj}}$. The flow rates $q_{o,i,k}$ and $q_{w,i,k}$ are part of the algebraic variables $v_{i,k}$ of well $i \in \mathcal{N}_{\mathrm{w}}$. Further, the oil price, and the water production and injection costs are denoted by r_o, r_w and r_i, respectively. Finally, the t_k is the step length.

B. RTO: short-term optimization

In a shorter time-scale, rate control consists in determining well controls and platform settings that optimize the daily or weekly production. In this case, since the dynamics in the reservoir are considerably slower than in the gathering network, it is common to assume the reservoir at steady-state, and disregard the long-term effects of current actions [11]–[14]. Thus, the system of difference equations (2a) is replaced with a static curve known as the Inflow Performance Relationship (IPR), which describes the relationship between the well BHP and its flow rates, and needs to be adjusted routinely to match field measurements [15]. The short-term rate control problem in a given time step k can be formulated as follows:

$$\begin{aligned}
\min_{u_k} \quad & -\sum_{i \in \mathcal{N}_{\mathrm{w,prod}}} (\hat{q}_{o,i,k+1} r_o - \hat{q}_{w,i,k+1} r_w) && \text{(5a)}\\
\text{s.t.} \quad & \hat{q}_{l,i,k+1} = a_{i,k} u_{i,k} + b_{i,k}, \quad \forall i \in \mathcal{N}_{\mathrm{w,prod}}, && \text{(5b)}\\
& \hat{q}_{w,i,k+1} = \hat{q}_{l,i,k+1} \gamma_{i,k}, \quad \forall i \in \mathcal{N}_{\mathrm{w,prod}}, && \text{(5c)}\\
& \hat{q}_{o,i,k+1} = \hat{q}_{l,i,k+1}(1 - \gamma_{i,k}), \quad \forall i \in \mathcal{N}_{\mathrm{w,prod}}, && \text{(5d)}\\
& \Delta \hat{P}_{k+1} = h(u_k, \hat{q}_{l,k+1}) \geq 0, && \text{(5e)}\\
& u_{\mathrm{lb},i,k} \leq u_{i,k} \leq u_{\mathrm{ub},i,k}, \quad \forall i \in \mathcal{N}_{\mathrm{w,prod}}, && \text{(5f)}
\end{aligned}$$

where the control $u_{i,k}$ is the BHP and $\hat{q}_{l,i,k+1}$, $\hat{q}_{w,i,k+1}$, and $\hat{q}_{o,i,k+1}$ are the flow rates of well i obtained with a linear IPR with coefficients $a_{i,k}$ and $b_{i,k}$. The IPR can be assumed linear because there is no gas being produced at reservoir conditions. The parameter $\gamma_{i,k}$ is the proportion of water in the liquid flow rate, also known as water cut. Further, the lower bounds $u_{\mathrm{lb},i,k}$ are typically defined as the pressure at the inlet of the topside facilities, and the upper bounds $u_{\mathrm{ub},i,k}$ are the minimum bottom-hole pressure given as reservoir boundary conditions. In practice, this is a safe margin to avoid the injection of oil into the reservoir. Each well has a valve at the top, called choke, which can adjust the rates to ensure feasibility with respect to the network infrastructure. The pressure drops over the chokes $\Delta \hat{P}_{i,k+1}$, $\forall i \in \mathcal{N}_{\mathrm{w,prod}}$ are calculated by a network simulation procedure, here denoted by $h(u_k, \hat{q}_{l,k+1})$, and must be positive such that the chokes do not become virtual pumps.

C. Long-term vs. short-term optimization

The rate control of reservoirs in the long-term is often performed in a resolution of a couple of months to a year, meaning that the same settings should be kept at a constant value for a long time. A review of methods for long-term production optimization may be found in [16]. In practice, such controls are typically changed every week, daily or even several times a day on a platform. Further, because many constraints are ignored while performing long-term rate control planning, these controls may not be feasible with respect to the gathering facilities. Among the attempts to overcome this issue is a framework to handle output constraints using a multiple shooting paradigm [17], and a further extension to handle network constraints [18].

While it is common to treat RTO and reservoir management separately, the limitations of standalone approaches are evident: in the long-term, a coarse solution is obtained at a high computational cost, while the short-term solutions are greedy and disregard long-term effects of current actions. We propose an optimization procedure that considers both short- and long-term gains while keeping the computational effort low, thereby enabling the use of the proposed method in real-world decision-making workflows.

III. OPTIMAL CONTROL PROBLEMS WITH FAST AND SLOW DYNAMICS

In this section, a mathematical formulation of the problem is presented followed by two fairly standard approaches to tackle it. A conceptual formulation of an optimal control problem with fast and slow dynamics is given below:

$$\begin{aligned}
\min_{u(\cdot)} \quad & \int_{t_0}^{t_f} \alpha L_c^s(x(t), z(t), u(t)) \\
& \qquad + (1-\alpha) L_c^l(x(t), z(t), u(t)) \, dt && \text{(6a)}\\
\text{s.t.} \quad & \dot{x}(t) = f_c(x(t), z(t), u(t)), \quad \forall t \in [t_0, t_f], && \text{(6b)}\\
& \dot{z}(t) = g_c(x(t), z(t), u(t)), \quad \forall t \in [t_0, t_f], && \text{(6c)}\\
& 0 \leq h_c(x(t), z(t), u(t)), \quad \forall t \in [t_0, t_f], && \text{(6d)}
\end{aligned}$$

where L_c^s and L_c^l are the possibly conflicting economical objectives for the short-term and long-term perspective, respectively, $f_c(\cdot)$ and x represent the dynamics and states of the system with slow dynamics, respectively, while $g_c(\cdot)$ and z are the equivalent for the system with fast dynamics. Equation (6d) are the path constraints. The decision variables are the control inputs $u(t)$ $\forall t \in [t_0, t_f]$, and the objective function is given as the integral in (6a) with $\alpha \in [0, 1]$.

In this paper we focus on a modified version of (6) in which the fast dynamics are seen at steady-state conditions, i.e., $\dot{z}(t) = 0$. This assumption is made because we assume the slow dynamics to be much slower than the fast dynamics such that the transients of the fast dynamics system do not affect the integrated system. Thus, the fast dynamics are replaced with algebraic equations (cf. (7c) below). The resulting formulation can be written in discrete time domain as follows:

$$\begin{aligned}
\min_{u} \quad & \sum_{\forall k \in \mathcal{K}} t_k \left[\alpha L^s(x_{k+1}, u_k) + (1-\alpha) L^l(x_{k+1}, u_k) \right] && \text{(7a)}\\
\text{s.t.} \quad & x_{k+1} = f(x_k, u_k), \quad \forall k \in \mathcal{K}, && \text{(7b)}\\
& 0 = g(x_k, u_k), \quad \forall k \in \mathcal{K}, && \text{(7c)}\\
& 0 \leq h(x_k, u_k), \quad \forall k \in \mathcal{K}, && \text{(7d)}
\end{aligned}$$

where $\mathcal{K} := [0, 1, \ldots, N-1]$ is the set of time steps. The functions in (7) are the discrete versions of the same functions in (6). The long-term rate control problem, given in the motivating example, in combination with the network gathering constraints in (5e) would be an example of (7) if we weigh the two objective functions (3) and (5a). Note that variables with subscript k denote (in general) vector-valued quantities corresponding to the time instant k. The same variable without the subscript will be a matrix, e.g., $u = [u_0, u_1, \ldots, u_{N-1}]$ and $x = [x_1, x_2, \ldots, x_N]$. The control i at the time step k is denoted $u_{i,k}$ whereas u_k indicates a vector of all these controls.

A. Solving the optimal control problem

A common way to solve the optimal control problem (6) is by a direct transcription method such as Multiple Shooting (MS) or Single Shooting (SS) [19]. However, these may run into computational limitations as the complexity is typically cubic in state- and input dimension, and time horizon.

B. Solving a sequence of quasi steady-state optimization problems

To circumvent the computational issues of solving the problem as a large dynamic optimization problem, one alternative is to decompose it into smaller and separate sub-problems, one for each time step k, such that the integrated state from step k becomes the initial state for step $k+1$. The formulation of an individual subproblem at time step k resembles an RTO problem, and is given by:

$$\min_{u_k} \quad L^s(x_{k+1}, u_k) \tag{8a}$$
$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k), \tag{8b}$$
$$0 = g(x_{k+1}, u_k), \tag{8c}$$
$$0 \leq h(x_{k+1}, u_k). \tag{8d}$$

Each subproblem (8) is a quasi steady-state optimization problem since the fast dynamics are assumed to be steady-state whereas the slow dynamics is integrated one step ahead. Notice that, if possible, it is only necessary to integrate the states that are needed to calculate the constraints and the objective function while solving the RTO. In the subproblem (5) in the motivating example, we use a linear approximation of the integration.

This approach greedily solves a sequence of quasi steady-state problems such that the optimized variables are purely based upon what is the best action in the current step. In other words, it is expected that the final value of (7a) will be worse than the one from a direct optimal control approach. On the other hand, the greedy strategy is computationally tractable since each subproblem is solved individually. As a result of each step being treated separately, this strategy avoids running into dimensionality issues due to an increase in the number of control steps. Thus, the computational bottleneck for this method can be taken as the time it takes to solve a single instance of (8).

IV. LOOKAHEAD REAL TIME OPTIMIZATION

The greedy strategy is efficient but might yield considerable losses in the long-term objective in comparison to the direct optimal control solution. The latter, on the other hand, might become computationally infeasible for a fine resolution in the discretization scheme. We propose a methodology that extends the greedy approach with a lookahead strategy in order to improve the cumulative objective (7a). The elements of the methodology and an algorithm with the solution method are provided in what follows.

A. The Predictor

A key ingredient of the methodology is the predictor. It is used to predict the impact of changing the current controls, u_k, on the final value of the long-term objective, $\sum L^l(\cdot)$, in (7a):

$$P(u_k; k, N, x_k, u^{\text{init}}), \tag{9}$$

where N is the total amount of time steps. The last parameter, u^{init}, is a matrix containing an initial guess for the optimal controls over the whole time period $k \in \mathcal{K}$. This is needed since it is required to simulate the slow dynamics from step k to N to find the impact of changing u_k. For simplicity, the size of u^{init} is fixed. The predictor is a scalar function.

As an example of a predictor, which will be used in this work, consider a linear predictor which can be used to estimate the impact of changing u_k on the long-term objective, $L^l(\cdot)$, in (7a). The linear predictor can be obtained by perturbing u_k, as described in Algorithm 1.

Algorithm 1 Linear Predictor

Require: $x_k, u^{\text{init}}, e_k$.
Ensure: x_k, u^{init} are feasible w.r.t. (7b)-(7d).
for $i \leftarrow 1, 2, \ldots, n_u$ **do**
  $u^{\text{copy}} \leftarrow u^{\text{init}}$
  $u^{\text{copy}}_{i,k} \leftarrow u^{\text{copy}}_{i,k} + e_{i,k}$
  $\hat{\Delta}_{i,k} \leftarrow \dfrac{I(x_k, u^{\text{copy}}, k, N) - I(x_k, u^{\text{init}}, k, N)}{e_{i,k}}$
end for
$P(u_k; k, N, x_k, u^{\text{init}}) \leftarrow (u_k - u^{\text{init}}_k)^T \hat{\Delta}_k$

The perturbation vector e_k has the same size of the control vector u_k. To calculate the impact of changing one element in u_k, the slow dynamics are simulated from x_k to x_N. This requires the use of an initial guess containing the controls for the whole horizon, u^{init}, for the matrix u. We assume here that this guess is feasible with respect to all the constraints in (7). The notation $u^{\text{init}}_{i,k}$ denotes the element i of the controls at step k of the initial guess u^{init}. Further, the function $I(x_k, u, k, N) = \sum_{i=k}^{N-1} L^l(x_{i+1}, u_i)$ where the future states of the slow dynamics are obtained by integration.

B. Conflicting objectives

The predictor consists of an estimate of how the value of the long-term objective changes with respect to the current control actions, while the short-term objective prioritizes immediate profit. To deal with these possibly conflicting objectives, we propose a two-stage optimization approach. The first stage consists of optimizing the short-term objectives by solving the quasi steady-state RTO (8). We denote the integrated state and optimized control actions by x'_{k+1} and u'_k. The second stage is performed with the lookahead, i.e., using the predictor $P(u_k; k, N, x_k, u^{\text{init}})$, but allowing only a certain deviation from the objective function value obtained with the RTO. The second optimization stage can be formulated as:

$$\min_{u_k} \quad P(u_k; k, N, x_k, u^{\text{init}}) \tag{10a}$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k), \tag{10b}$$

$$0 = g(x_{k+1}, u_k), \tag{10c}$$

$$0 \leq h(x_k, x_{k+1}, u_k), \tag{10d}$$

$$L^s(x_{k+1}, u_k) \leq \beta L^s(x'_{k+1}, u'_k), \tag{10e}$$

where the additional constraint (10e) imposes a limit on the deviation from the first-stage RTO objective $L^s(x'_{k+1}u'_k)$. Note that $L^s(x'_{k+1}u'_k)$ is assumed to be negative. The parameter $\beta \in [0, 1]$ determines how much the short-term objective function $L^s(\cdot)$ may worsen, e.g., $\beta = 0.9$ implies an acceptable worsening of 10%. The objective function in (10a) is the predictor replacing the first stage RTO objective.

In a given time step k, the solution obtained at the first stage RTO (8) is optimal with respect to the objective (8a). Thus, the solution obtained with the Lookahead RTO (LRTO) is expected to worsen the instantaneous objective in exchange for a potential improvement in the long-term objective $\sum L^l(\cdot)$ in (7a). An alternative to solve the two-stage optimization problem is to combine the two, possibly conflicting, objective functions into one by introducing weights. However, by doing so, the guarantee given by the parameter β would be lost. This guarantee is of interest because it limits the risk taken by reducing the near future objectives. This risk reflects that it is likely that the models used in optimization of $L^s(x'_{k+1}u'_k)$ are more certain than those used for calculation of the predictor (9).

C. The algorithm

The building blocks of the proposed methodology are put together in a two-stage optimization method described in Algorithm 2. For a given initial state x_0 and nominal path u^{init} that are feasible with respect to the slow dynamics system (8b), the algorithm first solves the RTO to obtain the greedy solution u'_k. Then a predictor is calculated and used in the second stage optimization to obtain the current optimized control u^*_k. The calculated control is applied to the plant (or simulator) to obtain the next state. The process is repeated until the end of the control interval. Notice that Lookahead RTO is defined as the procedure performed at lines 2 to 4 of Algorithm 2.

Algorithm 2 The workflow of Lookahead RTO

Require: x_0, u^0.
Ensure: x_0, u^0 are feasible w.r.t. (8b).
1: **for** $k \leftarrow 0, 1, \ldots, N-1$ **do**
2: $x'_{k+1}, u'_k \leftarrow$ Solve RTO in (8).
3: Obtain the predictor, (9).
4: $u^*_k \leftarrow$ Solve modified RTO in (10) with x'_{k+1}, u'_k.
5: $x_{k+1} \leftarrow$ Apply u^*_k to the plant.
6: **end for**

V. SIMULATION ANALYSIS

The Lookahead RTO method is assessed in the rate control optimization problem described in Section II. We consider an oil production system consisting of an oil-water reservoir, a production gathering network, and the topside facilities.

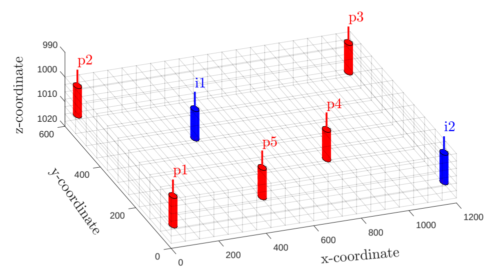

Fig. 3. Illustration of the reservoir model.

A. The simulator setup

The reservoir model is an adapted case instance from Roysem [20], which is based on the SPE1 benchmark, and implemented in the Matlab Reservoir Simulation Toolbox 2019a [21]. As illustrated in Fig. 3, it consists of a Cartesian three-dimensional box, split in 1200 regular grid blocks being $20 \times 10 \times 6$ in x, y, and z coordinates, respectively. Each block is 60 meters long, 60 meters wide and 3 meters high. The fluid utilized is the incompressible black-oil model, and the initial oil pressure $p_{\text{oil},i,0}$ is set to 200 bar, while the water saturation $S_{\text{water},i,0}$ is set to 0.15 in all grid blocks $i \in \mathcal{G}$. The field has 5 production wells and 2 water injectors, while its life time is taken as 10 years of 360 days, with time steps of 30 days, resulting in 120 time steps. We chose such a coarse resolution to maintain the problem computationally feasible for the aforementioned dynamic optimization approach.

The gathering network has 5 wells producing to a common manifold at the inlet of the topside facilities. Each producer is equipped with a choke at the top of the well, i.e. the well-head, which allows the regulation of the well flow. The set of pipelines transporting the fluids are divided into the production tubings and the flowlines. The tubings are pipes in the well bore through which the reservoir fluids are produced. The tubings of all wells have the same features, a length of 250 meters, diameter of 76 millimeters, inclination of 90 degrees, i.e. they are vertical. The flowlines are the pipes that take the fluids from wells to the platform. Each well has a separate flowline with the same features, a total length of

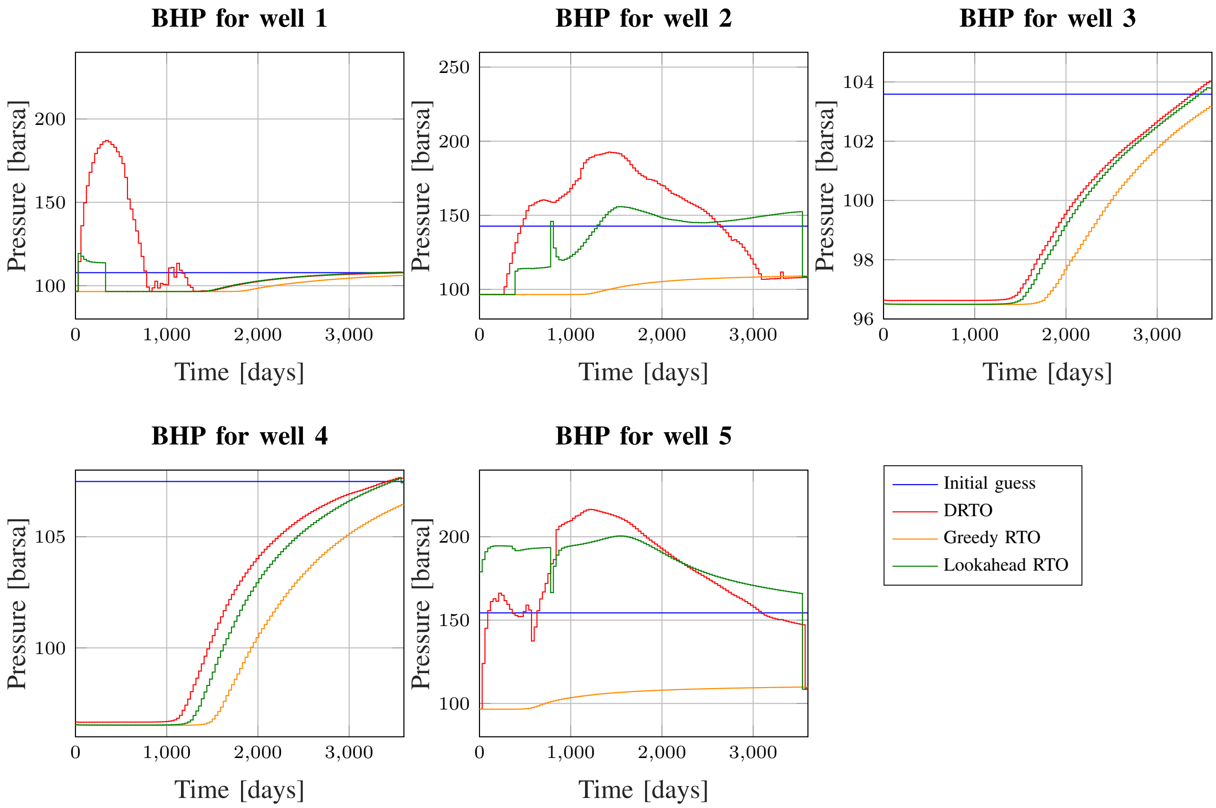

Fig. 4. Different control strategies.

1000 meters, diameter of 240 millimeters, and an inclination angle of 55 degrees. The downstream boundary condition of the network is a constant pressure of 5 bar at the inlet of the processing facilities.

The fluid flow in the gathering network is modeled and simulated with a network solver developed in [18]. The pressure drops are calculated using the framework for nodal analysis in [22], which consists of several correlations to compute fluid properties and pressure-drop derivatives. The calculations are performed by an integrator that utilizes an interface to the ODE solver CVODES, which is part of Suite of Nonlinear and Differential/Algebraic Equation Solver (Sundials), version 2.6.2 [23].

The long-term objective given in (3) has the parameters r_o, r_w and r_i set to 60, 10 and 10 USD/stb, respectively. We chose $\alpha = 1$ and $L^s(\cdot) = L^l(\cdot)$ in (7a) such that the stage costs of LRTO, RTO and the dynamic optimization method remain the same. Note that the objectives are still conflicting, due to the different horizons. This selection implies that the injectors are not a part of the decision variables. The injection rates of wells "i1" and "i2" are set to 302.0778 m^3/d and 88.7446 m^3/d, respectively. The IPRs for the wells which are needed for the production optimization, as explained in Section II, are identified by perturbation of the control inputs by -10 bar. This way we can find a linear approximation of what the flows will be at step $k+1$ given a control action u_k. The IPRs are reconstructed at each iteration of Algorithm 2. The separator pressure is used as the lower bound for the BHPs of the wells in all time steps. The upper bound for the BHP for each well is set to 99% of the minimum value of the pressures in the surrounding grid blocks. Just like the IPRs, these values will be updated at each iteration. Ideally, the upper bounds should be made for the state at $k+1$, but to avoid having to add a nonlinear constraint, the upper bound is created by first integrating along the nominal path for one step and then use this state to determine the upper bound. The reservoir pressure is a part of the slow dynamics, and thus this approximation is considered justified.

The second stage optimization problem for this experiment, solved at step 4 of Algorithm 2, will be similar to the one solved at the first stage, which is given the motivating example (5), but with the two following changes. The first is the additional constraint

$$-\sum_{i \in \mathcal{N}_w} (\hat{q}_{o,i,k+1} r_o - \hat{q}_{w,i,k+1} r_w) \leq \beta L(\hat{q}'_{o,k+1}, \hat{q}'_{w,k+1})$$

with $\beta = 0.75$. The second change is that the objective function is replaced by the linear predictor given in Algorithm 1:

$$P(u_k; k, N, x_k, u^{\text{init}}) = (u_k - u_k^{\text{init}})^T \hat{\Delta}_k.$$

A small modification was done to Algorithm 2. In addition to applying u_k^* to the plant, it was also copied into u_{k+1}^{init}. This change was done due to the assumption that the controls should not aggressively change from one step to another. This would provide a better linearization point for the linear predictor and the IPRs.

B. Results

Four different strategies are compared in the simulation analysis. In addition to the methods Greedy RTO and Lookahead RTO addressed in the paper, two different strategies obtained by a multiple shooting framework [18] are presented. The first, named DRTO, is configured to have a 30-day step size. In the second trajectory each well will keep the same control for 10 years. This trajectory is used as an initial guess for the other strategies and is named Initial guess.

The quality of the prediction obtained with the linear predictor described in Algorithm 1, with a perturbation size of 1.5 bar, is measured as the ratio of the actual change to the predicted change, which means that a value of 1 indicates a perfect prediction. Some characteristics of the quality of the linear predictor are given in Tab. I. The predictor performed well on average, which justifies its choice as the predictor for the experiments.

TABLE I

MEASUREMENTS OF THE QUALITY OF THE LINEAR PREDICTOR.

Min	Max	Mean	RMSE
-2.34	3.70	1.01	0.47

The different trajectories for the controls for the four strategies are shown in Fig. 4. The controls in the Greedy RTO case are all changing smoothly. This is because the only constraint is that the pressure loss over the choke has to be greater than zero and the wells still produce a good amount of oil at the end, and thus $q_{o,i,k+1}r_o \geq q_{w,i,k+1}r_w$ throughout the 10 year period. This implies that it is most valuable to simply produce as much as possible as the production network allows at all times. If $\Delta P = 0$, then maximum flow rate is achieved. At the very end of the simulation we can see that there is a large jump in the control for both well 2 and well 5 for the Lookahead RTO strategy, this is due to the fact that the sign of the linear predictor changes and the greedy strategy aligns with the predictor. Phrased differently, the short-term and long-term objectives are aligned.

The values of the long-term goal for the different strategies may be seen in Fig. 5. We can see, not unexpectedly, that the Greedy RTO strategy gives a higher income in the beginning. Around day 1000, the Lookahead RTO takes the lead and remains most profitable until around day 2600. From this point forward, the DRTO strategy is the best. The two most interesting comparisons are the two RTO approaches, and the Lookahead RTO vs. DRTO. The solution obtained with the Lookahead RTO method is 7.1 % (or 16.3 million USD) better NPV than the greedy RTO. When compared to DRTO, the Lookahead RTO method achieved a solution that is only 0.4% (or 1.1 million USD) lower in terms of NPV. Although the original objective of Lookahead RTO was to improve the greedy strategy, which is similar to daily routines in real world platforms, it also performed almost as well as DRTO.

To illustrate the differences in terms of production achieved by the different methods, the accumulated flows of oil and water are plotted in Fig. 6. The Greedy RTO approach

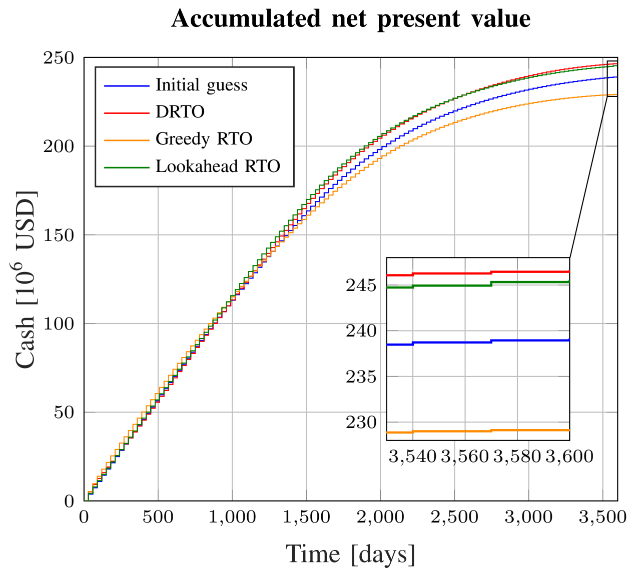

Fig. 5. Accumulated NPV for the different control strategies.

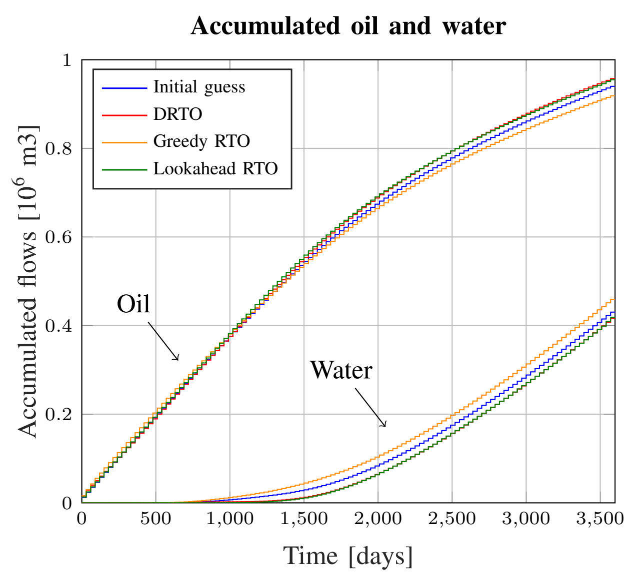

Fig. 6. Accumulated flows for the different control strategies.

produces the least oil and most water. The Lookahead RTO and the DRTO approaches produces almost identical amounts of water throughout the entire horizon, implying that the shape of the NPV curves and accumulated oil flow curves will be similar due to (3). This means that, with the lookahead strategy, it is possible to avoid the excessive water production that the greedy method causes, while maintaining a low computational effort.

The solving time for the most computationally heavy iteration of the lookahead method was less than 1 minute, while the DRTO took around 3.5 hours. Keep in mind that we are using a toy reservoir with a long step size. The computational efficiency of the lookahead method allows its use in a RTO fashion in daily operations, while, in a similar manner, using the multiple shooting approach in a Model

Predictive Control fashion would probably not be reasonable. Further, the entire trajectory for the Lookahead RTO in the figures took less than 1 hour to compute.

VI. DISCUSSION

The suggested methodology is computationally inexpensive because it only uses the simulator of the slow dynamics to create the predictors and to evolve in time. Since the simulator is not embedded into the smaller optimization problems, the method does not inherit its computational complexity and the problem dimensionality is kept low. This strength enables the use of Lookahead RTO in similar fashion to the standard RTO.

These days it is becoming more common to have easily available aggregated plant data. The LRTO may incorporate such information in the same manner as RTO. For example, if we gather data and new knowledge about the system, we may update the models in between iterations of Algorithm 2. Further, the u^{init} trajectory may also be updated if we obtain a better model for the slow dynamics.

The β parameter indicates how much of a worsening of the first stage objective function is accepted. It could also be tuned based upon how much we trust the model for the slow dynamics, or the projected oil price. If the slow dynamics are highly uncertain, then this parameter could be increased. This parameter can also be used in another way. Assume we have a good model for the slow dynamics. It would make sense to allow for a larger deviation in the beginning, because there is less future to consider when the end is getting closer. In this sense, the β could be gradually increased as time passes. This would result in a more greedy approach towards the end.

One issue one might run into when using Algorithm 2 is infeasibility in the slow dynamics. The initial guess, u^{init}, is feasible. However, after solving (10) we change u_k^* ($\neq u_k^{\text{init}}$). The fact that $u^{\text{init}} = [u_0^{\text{init}}, u_1^{\text{init}}, \dots, u_{N-1}^{\text{init}}]$ is feasible does not imply feasibility of $[u_0^*, u_1^{\text{init}}, \dots, u_{N-1}^{\text{init}}]$. This could perhaps be solved by, e.g., by running the multiple shooting method again with a coarser resolution such that the infeasibility is avoided. Note that this optimization should happen after one has applied u_k^* to the plant, but before a new iteration of Algorithm 2 is started.

A consequence of solving the optimal control problem in (7) as a sequence of smaller optimization problems, instead of one larger problem, is that the predictor, which is based upon a guess of the future control actions, will not be perfect unless this guess is followed.

VII. CONCLUSIONS

In the simulation study, it was shown that including the long-term effects into the short-term decision making process was valuable. The existing methods in literature, to optimize systems with both fast and slow dynamics, may suffer from the computational requirements due to the dimensionality. Disregarding the long-term effects gives rise to a method that is fast but non-optimal in the long run. The suggested methodology combines the advantages of the traditional real time optimization with a lookahead strategy to improve the long-term goal. The simulation study shows promising results for this novel methodology.

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