Economical Secondary Control of DC Microgrids

Babak Abdolmaleki[1], Qobad Shafiee[2], Mahdieh S. Sadabadi[3], and Tomislav Dragičević[4]

1 *Department of Electric Power Engineering*, *Norwegian University of Science and Technology*, Trondheim, Norway

2 *Department of Electrical Engineering*, *University of Kurdistan*, Sanandaj, Iran

3 *Department of Automatic Control and Systems Engineering*, *University of Sheffield*, Sheffield, United Kingdom

4 *Department of Electrical Engineering*, *Technical University of Denmark*, Copenhagen, Denmark

babak.abdolmaleki@ntnu.no; q.shafiee@uok.ac.ir; m.sadabadi@sheffield.ac.uk; tomdr@elektro.dtu.dk

***Abstract*—This paper proposes a distributed secondary voltage and current control aiming at economic dispatch of DC microgrids. The dispatch problem is solved with the Lagrange method resulting in the equal incremental costs criterion. The proposed secondary controller realizes this criterion by using an average data consensus algorithm. A distributed dynamic data corrector is also proposed which ensures that the generators' output currents are always kept within the allowable ranges and that the total generation cost is as optimum as possible. Both physical and control systems considering constant impedance-current loads are formulated and stability of the system is briefly discussed. To verify the effectiveness of the proposed scheme, a test microgrid system is simulated in MATLAB/Simulink environment.**

***Index Terms*—Current sharing, DC microgrid (MG), economic dispatch (ED), secondary control, voltage control.**

I. Introduction

DC power systems are drawing more attention as time goes by and hence their control and optimization are of great importance [1]. Droop control is an effective, simple custom to integrate multiple converter-based distributed generators (DGs) into the DC MGs. However, this method is not successful when it comes to voltage formation and optimal current dispatch of the DGs which are both significant for effective operation of power systems [2]; these failures are due to droop-induced voltage deviations and the electric network asymmetry and in-optimal sizing and siting of the loads and generators [3]. To reach an appropriate voltage-current dispatch and to compensate for the voltage drifts, different centralized, distributed, and decentralized have been proposed. The centralized methods require a complex communication between the DGs and the central controller; moreover, the central control unit exposes a single point of failure to the system. The decentralized methods, on the other hand, do not provide an accurate current sharing for the DGs. Therefore, the distributed schemes seem to be the most promising way to control the secondary control of DC microgrids [3], [4].

Literature review: Distributed consensus-based secondary control techniques have shown acceptable performances [4]–[23]. Proportional current-sharing between the DGs within DC MGs has already been reported in numerous research works, e.g., [5]–[17]. None of the proposed controllers in the above works presents the DGs with economical current-sharing. In addition, they compensate for the voltage deviations by using an extra parallel controller. The research attempts in [4], [18]–[23], on the other hand, investigate the economic dispatch (ED) problem of DC MGs. Refs. [18], [19] consider economical and low loss operation of DC MGs, respectively. Moreover, the proposed schemes in these works are not of the secondary control class. None of the works in [4], [20] consider the generation limits. The controller in [4] introduces a trade-off between incremental cost consensus and voltage regulation. It should be noted that the proposed ED controllers in [21]–[23], similar to [20], are based on average voltage estimation and work as tertiary controllers.

Contributions: Motivated by the above mentioned works, a distributed consensus-based secondary controller is proposed for DC MGs which tries to minimize the total generation cost of the DGs. Unlike the previous methods, the droop-induced voltage deviations are compensated through the average of neighboring data; additionally, each DG requires only one integrator to compute the final control action. A smart mechanism, based on virtual dynamics, is also proposed which corrects the transmitted data (the DG's incremental cost) when the output current of a DG hits the limits. The proposed method requires local measurements and calculations and hence is distributed. In addition, the proposed method results in a *proportional-integral controller* and requires *only one data* to be communicated among the DGs.

Outline: The rest of the paper is structured as follows. System setup and economic dispatch problem are introduced in Section II. Section III is devoted to the proposed controller and the stability and equilibrium analyses of the controlled microgrid system. Simulation results and case studies are presented and discussed in Section IV. Finally, Section V concludes the paper.

II. System Setup

A. Droop-Controlled DC Microgrids

Considering static network model and constant impedance-current load model, the dynamic model of a DC MG is

$$\tau_i^V \dot{V}_i = -V_i - R_i^d I_i^f + u_i + V_n,\ R_i^d = \Delta V_{\max}/I_i^{\text{rated}} \quad (1a)$$

$$\tau_i^f \dot{I}_i^f = -I_i^f + I_i, \quad (1b)$$

$$I_i = \sum_j^n Y_{ij}(V_i - V_j) + Y_i^c V_i + I_i^c, \quad (1c)$$

where V_i, I_i are ith DG's output voltage and current; R_i^d, I_i^{rated}, τ_i^V are droop coefficient, rated current, and equivalent time-constant of inner voltage controller of ith DG; $\Delta V_{\max}$ is the maximum allowable voltage deviation; I_i^f is the current filtered by the low-pass filter with the time constant τ_i^f; Y_{ij} is the admittance between the DGs i and j in the Kron-reduced network; Y_i^c and I_i^c are constant admittance and current values of the load at ith DG bus; u_i is the voltage correction term commanded by secondary controller.

B. Economic Dispatch and Equal Incremental Costs Principle

Let $C_i(x_i) = \alpha_i x_i^2 + \beta_i x_i + \gamma_i$ be ith DG's cost function. The ED can be formulated as the minimization problem

$$\min \left(\sum\nolimits_i^n C_i(I_i)\right), \begin{cases} \sum_i^n I_i = I_{\text{demand}}, \\ I_i^{\min} \leq I_i \leq I_i^{\max}. \end{cases} \tag{2}$$

The above inequality-constrained ED problem can be reformulated as

$$\min \left(\sum\nolimits_i^n s_i C_i(P_i)\right), \tag{3a}$$

$$0 = I_{\text{demand}} - \sum\nolimits_i^n (s_i I_i + s_i^{\max} I_i^{\max} + s_i^{\min} I_i^{\min}), \tag{3b}$$

where

$$s_i^{\max} = \begin{cases} 1, \ I_i > I_i^{\max} \\ 0, \ \text{otherwise} \end{cases}, s_i^{\min} = \begin{cases} 1, \ I_i < I_i^{\min} \\ 0, \ \text{otherwise} \end{cases},$$

$$s_i = \begin{cases} 1, \ s_i^{\min} = s_i^{\max} = 0 \\ 0, \ \text{otherwise} \end{cases}.$$

The ED optimization problem can be solved by Lagrangian method with the following Lagrangian function [2].

$$\begin{aligned} L(\mathbf{I}, \lambda) = &\sum\nolimits_i^n s_i C_i(I_i) + \lambda I_{\text{demand}} \\ &-\lambda \sum\nolimits_i^n (s_i I_i + s_i^{\max} I_i^{\max} + s_i^{\min} I_i^{\min}). \end{aligned} \tag{4}$$

The first order optimality criterion associated with (4), considering $\frac{\partial C_i(I_i)}{\partial I_i} = 2\alpha_i I_i + \beta_i$, are $\frac{\partial L}{\partial I_i} = \frac{\partial L}{\partial \lambda} = 0$ or

$$s_i\left(\frac{\partial C_i(I_i)}{\partial I_i} - \lambda_{\text{opt}}\right) = 0, \tag{5a}$$

$$I_{\text{demand}} = \sum\nolimits_i^n (s_i I_i + s_i^{\max} I_i^{\max} + s_i^{\min} I_i^{\min}). \tag{5b}$$

Hence, the DG's optimal currents are

$$I_i = \begin{cases} (\lambda_{\text{opt}} - \beta_i)/(2\alpha_i), & \forall i \mid s_i = 1 \\ I_i^{\min}, & \forall i \mid s_i^{\min} = 1 \\ I_i^{\max}, & \forall i \mid s_i^{\max} = 1 \end{cases}. \tag{6}$$

The criterion in first case of (6) is known as the Equal Incremental Costs (EIC) principle and is the fundamental principle for minimizing the total generation cost of the DGs with $s_i = 1$ which are operating in normal mode.

III. Economical Secondary Control Technique

A. Communication Network (CN) and Graph Theory

The CN among the DGs, can be regarded as a directed graph (digraph) with the DGs and communication links playing the roles of its nodes and edges, respectively. Consider the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N} = \{1, ..., n\}$, $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ are its node set, edge set, and adjacency matrix, respectively. If node i directly obtains data

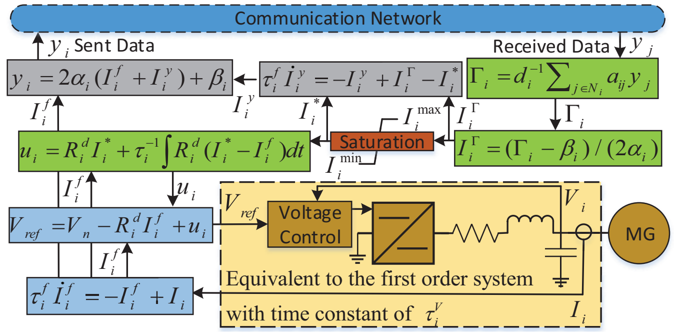

Fig. 1. Schematic of a converter-based microgrid engaged with the proposed secondary control.

from node j, then, node j is an in-neighbor (sender) of node i, node i is an out-neighbor (receiver) of node j, $(j, i) \in \mathcal{E}$, and $a_{ij} = 1$; otherwise, nodes i and j are not neighbors, $(j, i) \notin \mathcal{E}$, and $a_{ij} = 0$. Let $N_i = \{j \mid (j, i) \in \mathcal{E}\}$, $N_i^o = \{j \mid (i, j) \in \mathcal{E}\}$, $d_i = \sum_{j \in N_i} a_{ij}$, and $d_i^o = \sum_{j \in N_i^o} a_{ji}$ be the in-neighbor set, out-neighbor set, in-degree, and out-degree of node i, respectively. Laplacian matrix of $\mathcal{G}$ is $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_i\}$. A directed path from node j to node i is a sequence of pairs, belong to $\mathcal{E}$, expressed as $\{(j, n_1), ..., (n_m, i)\}$. A graph has a spanning tree, if there is a node r (called the root node), such that there is a directed path from the root node to every other node in the graph [2], [24]–[28].

B. Proposed Controller

To compensate for the voltage deviations and to establish the criterion (6), the correction term below is proposed.

$$u_i = R_i^d I_i^* + q_i, \tag{7a}$$

$$\tau_i \dot{q}_i = R_i^d (I_i^* - I_i^f), \tag{7b}$$

$$I_i^* = \begin{cases} I_i^{\min} & I_i^\Gamma < I_i^{\min} \\ I_i^\Gamma = (\Gamma_i - \beta_i)/(2\alpha_i) & I_i^{\min} \leq I_i^\Gamma \leq I_i^{\max} \\ I_i^{\max} & I_i^\Gamma > I_i^{\max} \end{cases}, \tag{7c}$$

$$\Gamma_i = \frac{1}{d_i} \sum\nolimits_{j \in N_i} a_{ij} y_j, \tag{7d}$$

$$y_i = 2\alpha_i (I_i^f + I_i^y) + \beta_i, \tag{7e}$$

$$\tau_i^f \dot{I}_i^y = -I_i^y + (I_i^\Gamma - I_i^*), \tag{7f}$$

where I_i^* is the optimal current reference; Γ_i is the average of neighboring data; y_i is the data which ith DG forwards to the other DGs; τ_i is the secondary control integral time constant; a_{ij}, d_i, and N_i are communication weighting between the DGs i, j, in-degree and neighbor set of ith DG, respectively; τ_i^f is time constant of the low-pass filter which, as a virtual dynamic, mimics the control path from I_i^* (as input) to I_i^f (as output). The general scheme of a converter-based MG under the proposed secondary controller is depicted in Fig. 1.

Remark 1: Please note that the current I_i^y, which reflects the error between I_i^Γ and I_i^*, is employed to correct the forwarded data. This is designed to ensure that the other DGs can still reach an agreement on a new optimum incremental cost, when ith DG reaches the capacity limits.

C. Stability and Equilibrium Analyses

The system (1), engaged with the controller (7), can be given in the following compact form.

$$\begin{aligned}
\boldsymbol{\tau}_V \dot{\mathbf{V}} &= -\mathbf{V} + \boldsymbol{\tau}\dot{\mathbf{q}} + \mathbf{q} + \mathbf{1}V_n, && \text{(8a)}\\
\boldsymbol{\tau}_f \dot{\mathbf{I}}_f &= -\mathbf{I}_f + (\mathbf{Y} + \mathbf{Y}_c)\mathbf{V} + \mathbf{I}_c, && \text{(8b)}\\
\boldsymbol{\tau}\dot{\mathbf{q}} &= \mathbf{R}_d[(\mathbf{I}_\Gamma - \mathbf{I}_f) + \boldsymbol{\varsigma}(\mathbf{I}_{\text{lim}} - \mathbf{I}_\Gamma)], && \text{(8c)}\\
\boldsymbol{\tau}_f \dot{\mathbf{I}}_y &= -\mathbf{I}_y + \boldsymbol{\varsigma}(\mathbf{I}_\Gamma - \mathbf{I}_{\text{lim}}), && \text{(8d)}\\
\mathbf{I}_\Gamma &= -0.5\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\mathbf{y} + \mathbf{I}_f + \mathbf{I}_y, && \text{(8e)}\\
\mathbf{y} &= 2\boldsymbol{\alpha}\mathbf{I}_f + 2\boldsymbol{\alpha}\mathbf{I}_y + \boldsymbol{\beta}. && \text{(8f)}
\end{aligned}$$

Note that the bold letters and symbols represent proper matrix or vector of their corresponding scalars; $\mathbf{1}$ and $\mathbf{0}$ are proper vectors/matrices of ones and zeros, respectively; $\mathbf{I}_{\text{lim}}$ is the vector of maximum/minimum currents of the DGs elements of which depend on the conditions in (7c); $\boldsymbol{\varsigma} = \text{diag}\{\varsigma_i\}$ is a matrix of switching signals where $\varsigma_i = 0$ if $I_i^* = I_i^\Gamma$, otherwise $\varsigma_i = 1$. With $\mathbf{x}^\top = [\mathbf{V}^\top \ \mathbf{I}_f^\top \ \mathbf{q}^\top \ \mathbf{I}_y^\top]$, the closed-loop system (8) can be written as

$$\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{A}_\varsigma)\mathbf{x} + (\mathbf{d} + \mathbf{d}_\varsigma); \quad \text{(9)}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_V & \mathbf{A}_{VI_f} & \mathbf{A}_{Vq} & \mathbf{A}_{VI_y} \\ \mathbf{A}_{I_fV} & \mathbf{A}_{I_f} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{qI_f} & \mathbf{0} & \mathbf{A}_{qI_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{I_y} \end{bmatrix}, \mathbf{d} = \begin{bmatrix} \mathbf{d}_V \\ \mathbf{d}_{I_f} \\ \mathbf{d}_q \\ \mathbf{0} \end{bmatrix};$$

$$\mathbf{A}_\varsigma = \begin{bmatrix} \mathbf{0} & \mathbf{A}^\varsigma_{VI_f} & \mathbf{0} & \mathbf{A}^\varsigma_{VI_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^\varsigma_{qI_f} & \mathbf{0} & \mathbf{A}^\varsigma_{qI_y} \\ \mathbf{0} & \mathbf{A}^\varsigma_{I_yI_f} & \mathbf{0} & \mathbf{A}^\varsigma_{I_y} \end{bmatrix}, \mathbf{d}_\varsigma = \begin{bmatrix} \mathbf{d}^\varsigma_V \\ \mathbf{0} \\ \mathbf{d}^\varsigma_q \\ \mathbf{d}^\varsigma_{I_y} \end{bmatrix}.$$

The components of the above matrices/vectors are as follows.

$$\begin{cases} \mathbf{A}_V = -\mathbf{A}_{Vq} = -\boldsymbol{\tau}_V^{-1} \\ \mathbf{A}_{VI_f} = -\boldsymbol{\tau}_V^{-1}\mathbf{R}_d\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\alpha} \\ \mathbf{A}_{VI_y} = \mathbf{A}_{VI_f} + \boldsymbol{\tau}_V^{-1}\mathbf{R}_d \\ \mathbf{A}_{I_fV} = \boldsymbol{\tau}_f^{-1}(\mathbf{Y} + \mathbf{Y}_c) \\ \mathbf{A}_{I_f} = \mathbf{A}_{I_y} = -\boldsymbol{\tau}_f^{-1} \\ \mathbf{A}_{qI_f} = -\boldsymbol{\tau}^{-1}\mathbf{R}_d\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\alpha} \\ \mathbf{A}_{qI_y} = \mathbf{A}_{qI_f} + \boldsymbol{\tau}^{-1}\mathbf{R}_d \end{cases}, \quad \text{(10a)}$$

$$\begin{cases} \mathbf{d}_V = -0.5\boldsymbol{\tau}_V^{-1}\mathbf{R}_d\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\beta} - \boldsymbol{\tau}_V^{-1}\mathbf{1}V_n \\ \mathbf{d}_{I_f} = \boldsymbol{\tau}_f^{-1}\mathbf{I}_c \\ \mathbf{d}_q = -0.5\boldsymbol{\tau}^{-1}\mathbf{R}_d\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\beta} \end{cases}, \quad \text{(10b)}$$

$$\begin{cases} \mathbf{A}^\varsigma_{VI_f} = \mathbf{A}^\varsigma_{VI_y} = \boldsymbol{\varsigma}\boldsymbol{\tau}_V^{-1}\mathbf{R}_d\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\alpha} - \boldsymbol{\varsigma}\boldsymbol{\tau}_V^{-1}\mathbf{R}_d \\ \mathbf{A}^\varsigma_{qI_f} = \mathbf{A}^\varsigma_{qI_y} = \boldsymbol{\varsigma}\boldsymbol{\tau}^{-1}\mathbf{R}_d\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\alpha} - \boldsymbol{\varsigma}\boldsymbol{\tau}^{-1}\mathbf{R}_d \\ \mathbf{A}^\varsigma_{I_yI_f} = \mathbf{A}^\varsigma_{I_y} = -\boldsymbol{\varsigma}\boldsymbol{\tau}_f^{-1}\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\alpha} + \boldsymbol{\varsigma}\boldsymbol{\tau}_f^{-1} \end{cases}, \quad \text{(10c)}$$

$$\begin{cases} \mathbf{d}^\varsigma_V = \boldsymbol{\varsigma}\boldsymbol{\tau}_V^{-1}\mathbf{R}_d[\mathbf{I}_{\text{lim}} + 0.5\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\beta}] \\ \mathbf{d}^\varsigma_q = \boldsymbol{\varsigma}\boldsymbol{\tau}^{-1}\mathbf{R}_d[\mathbf{I}_{\text{lim}} + 0.5\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\beta}] \\ \mathbf{d}^\varsigma_{I_y} = -\boldsymbol{\varsigma}\boldsymbol{\tau}_f^{-1}[0.5\boldsymbol{\alpha}^{-1}\mathcal{D}^{-1}\mathcal{L}\boldsymbol{\beta} + \mathbf{I}_{\text{lim}}] \end{cases}. \quad \text{(10d)}$$

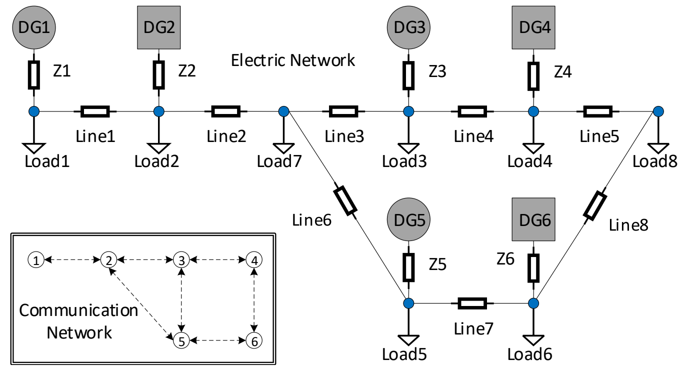

Fig. 2. Electrical and communication networks of the test microgrid system.

Theorem 1 (Stability): Suppose that there exists positive definite matrices $\mathbf{P}$ and $\mathbf{Q}$ such that for all possible forms of the switching signal matrix $\mathbf{A}_\varsigma$ one has $(\mathbf{A} + \mathbf{A}_\varsigma)^\top\mathbf{P} + \mathbf{P}(\mathbf{A} + \mathbf{A}_\varsigma) \leq -\mathbf{Q}, \forall \boldsymbol{\varsigma}$. Then, considering the fact that $(\mathbf{d} + \mathbf{d}_\varsigma)^\top(\mathbf{d} + \mathbf{d}_\varsigma) \leq \varphi^2$, i.e., $(\mathbf{d} + \mathbf{d}_\varsigma)$ is bounded, one can say that $\mathbf{x}$ is ultimately uniformly bounded.

Proof: Let $\mathcal{H}(\mathbf{x}) = \mathbf{x}^\top\mathbf{P}\mathbf{x}$ be a *common quadratic Lyapunov function* [29]. Then the proof follows the proof-lines of [2, Theorem 1]. ■

Theorem 2 (Equilibrium Analysis): Suppose that the communication network graph between the DGs is *strongly connected*, i.e., every node within it is a root node. Then, the proposed controller in (7) drives the microgrid to an steady state where (6) is satisfied.

Proof: Let $\bar{\mathbf{x}}$ stand for the steady-state value of $\mathbf{x}$ (same for the other vectors). Then, according to (8), in steady-state one has $\mathcal{L}\bar{\mathbf{y}} = \mathbf{0}$. From [27, Lemma 1 & Lemma 2], if the communication network graph is strongly connected, which posses at least one spanning tree, then this equation has a unique solution in the form of $\bar{\mathbf{y}} = \lambda_{\text{opt}}\mathbf{1}$. Therefore, according to (7) one has $\bar{\Gamma}_i = \lambda_{\text{opt}}, \forall i$ and $\bar{I}_i = \bar{I}_i^*$; hence one can write

$$\bar{I}_i = \begin{cases} (\lambda_{\text{opt}} - \beta_i)/(2\alpha_i), & \forall i \mid I_i^{\min} \leq \bar{I}_i^\Gamma \leq I_i^{\max} \\ I_i^{\min}, & \forall i \mid \bar{I}_i^\Gamma < I_i^{\min} \\ I_i^{\max}, & \forall i \mid \bar{I}_i^\Gamma > I_i^{\max} \end{cases} \quad \text{(11)}$$

which is equivalent to the optimal current defined in (6). ■

IV. Case Studies

To show the effectiveness of the proposed controller, a 48-Volt meshed DC MG, powered by six DGs, is simulated in MATLAB/Simulaink environment. It should be noted that the DGs with odd (resp. even) numbers are interfaced to the grid via Buck (resp. Boost) converters, which are depicted in Fig. 2 by circles and squares, respectively. The electrical and control specifications of the MG shown in Fig. 2 are given in Table I. The performance of the MG under the proposed controller is shown in Fig. 3.

Remark 2: It should be noted that in the system modeling and theoretical analyses dealing with secondary control, the converters are modeled by an equivalent first order model as in (1a). However, in reality the converters use local inner

TABLE I
THE ELECTRICAL AND CONTROL SPECIFICATIONS OF THE TEST MG

DGs' Specifications with Base RL of $(0.5\Omega, 50\mu H)$						
	DG Number					
	1	2	3	4	5	6
$I_i^{\text{rated}} = I_i^{\max}(A)$	5	3	6	6	5	6
$\alpha_i(10^{-1}\$/A^2)$	0.8	1.9	1	1.4	1.2	1.6
$\beta_i(10^{-1}\$/A)$	1	2.5	1.2	1.8	1.5	2.1
$\gamma_i(10^{-1}\$)$	2	5	2	4	3	4
Zi: (p.u.)	0.5	0.4	0.55	0.6	0.45	0.5

DGs' Common Parameters						
$\forall i$	τ_i^V	τ_i^f	τ_i	$I_i^{\min}$	$\Delta V_{\max}$	V_n
	$\frac{1}{100\pi}$	$\frac{1}{4\pi}$	0.05	0	3	48

Line Specifications with Base RL of $(0.5\Omega, 50\mu H)$								
	Line Number							
	1	2	3	4	5	6	7	8
(p.u.)	1	2	2	1	1	3	1	2

Load Specifications								
	Load Number							
	1	2	3	4	5	6	7	8
$Y_c^{-1}(\Omega)$	30	20	20	20	30	20	10	10
$I^c(0.5A)$	1	1	1	1	1	1	1	1

voltage controllers to track the voltage reference given by droop control [1]. In this paper, Linear Quadratic Regulator (LQR) controller technique is used for the inner voltage control of converters. For this purpose, the linearized second order average model of converters augmented with a voltage-tracker integrator is utilized where the output current of the converter capacitor is considered as an external disturbance.

According to Fig. 3, prior to $t = 2s$ the MG is engaged with the droop control and hence the voltages are all deviated from the nominal voltage and neither current ratios nor the incremental costs have reached an agreement. At $t = 2s$, the proposed controller is activated; therefore, one can see that the voltages make a formation around the nominal voltage in a way that the DGs, which work within the current limits, reach a consensus on the incremental costs. However, one can see that the current of the DG 1 hits the upper limit, i.e., its current ratio becomes $\frac{I_1}{I_1^{\max}} = 1$. It is clear that even in this situation, the other DGs reach an agreement on incremental costs, while the current of the DG 1 remains at its maximum value. In other words, the steady-state point is as optimum as possible. On the other hand, one can see that the communication data of the first DG is corrected in a way that the data is different from its incremental cost and the consensus task for the other DGs is not interrupted.

To show the resiliency of the proposed controller, at $t = 10s$, a severe increase is planned for loads 7 and 8. The figures depict that after a rise in the amount of load current, all the DGs increase their production to reach a new optimal point and establish the EIC principle again. Nevertheless, the output current of 3rd DG, similar to that of 1st DG, reaches the limit; hence, it leaves the consensus-based optimization task and its data corrector is activated.

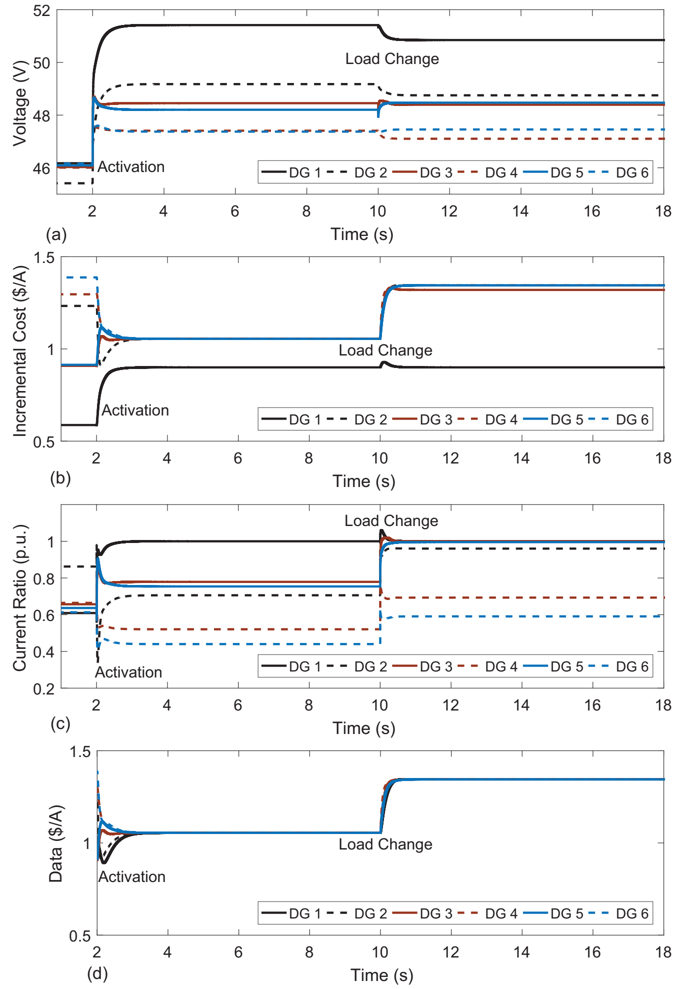

Fig. 3. Performance of the controller; the DGs' (a) voltages, (b) incremental costs, (c) actual per rated currents, and (d) communication data.

V. CONCLUSION AND FUTURE WORKS

In this article, a distributed consensus-based secondary control is proposed for DC microgrids which tries to minimize the total generation cost of the DGs by establishing the equal incremental costs criterion. This principle, which is the first order optimality criterion for the minimization problem, is realized by using a data consensus algorithm. In this algorithm, the transmitted data of each DG is defined by using a virtual dynamic control system. This virtual dynamic ensures that the DG's current remains within the allowable limits and that the steady-state point is as optimum as possible. Finally, the simulation results validated that, under the proposed controller, the DGs can establish the EIC principle and the voltages are close to the nominal voltage.

Proposing a secondary controller considering both voltage and current limits of the DGs is one of the future works based on this work. Moreover, studying the resiliency of the controller against the cybernetic attacks and/or in the presence of non-ideal realistic communications is an interesting, challenging research topic of future.

References

[1] M. S. Sadabadi, Q. Shafiee, and A. Karimi, “Plug-and-play robust voltage control of dc microgrids,” *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 6886–6896, Nov. 2018.

[2] B. Abdolmaleki and Q. Shafiee, “Online kron reduction for economical frequency control of microgrids,” *IEEE Trans. Ind. Electron.*, vol. 67, no. 10, pp. 8461–8471, Oct. 2020.

[3] Y. Han, X. Ning, P. Yang, and L. Xu, “Review of power sharing, voltage restoration and stabilization techniques in hierarchical controlled dc microgrids,” *IEEE Access*, vol. 7, pp. 149 202–149 223, 2019.

[4] J. Zhao and F. Dörfler, “Distributed control and optimization in dc microgrids,” *Automatica*, vol. 61, pp. 18–26, Nov. 2015.

[5] V. Nasirian, S. Moayedi, A. Davoudi, and F. L. Lewis, “Distributed cooperative control of dc microgrids,” *IEEE Trans. Power Electron.*, vol. 30, no. 4, pp. 2288–2303, Apr. 2015.

[6] S. Sahoo and S. Mishra, “A distributed finite-time secondary average voltage regulation and current sharing controller for dc microgrids,” *IEEE Trans. Smart Grid*, vol. 10, no. 1, pp. 282–292, Jan. 2019.

[7] J. Lai, X. Lu, W. Yao, J. Wen, and S. Cheng, “Robust distributed cooperative control for dc mircogrids with time delays, noise disturbances, and switching topologies,” *Journal of the Franklin Institute*, vol. 354, no. 18, pp. 8312–8332, Dec. 2017.

[8] S. Sahoo, D. Pullaguram, S. Mishra, J. Wu, and N. Senroy, “A containment based distributed finite-time controller for bounded voltage regulation & proportionate current sharing in dc microgrids,” *Applied Energy*, vol. 228, no. 15, pp. 2526–2538, Oct. 2018.

[9] R. Han, H. Wang, Z. Jin, L. Meng, and J. M. Guerrero, “Compromised controller design for current sharing and voltage regulation in dc microgrid,” *IEEE Trans. Power Electron.*, vol. 34, no. 8, pp. 8045–8061, Aug. 2019.

[10] X. Liu, H. He, Y. Wang, Q. Xu, and F. Guo, “Distributed hybrid secondary control for a dc microgrid via discrete-time interaction,” *IEEE Trans. Energy Convers.*, vol. 33, no. 4, pp. 1865–1875, Dec. 2018.

[11] D. Pullaguram, S. Mishra, and N. Senroy, “Event-triggered communication based distributed control scheme for dc microgrid,” *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 5583–5593, Sep. 2018.

[12] R. Han, M. Tucci, A. Martinelli, J. M. Guerrero, and G. Ferrari-Trecate, “Stability analysis of primary plug-and-play and secondary leader-based controllers for dc microgrid clusters,” *IEEE Trans. Power Syst.*, vol. 34, no. 3, pp. 1780–1800, May 2019.

[13] J. Zhou, Y. Xu, H. Sun, L. Wang, and M. Chow, “Distributed event-triggered h_∞ consensus based current sharing control of dc microgrids considering uncertainties,” *IEEE Trans. Ind. Informat.*, 10.1109/TII.2019.2961151.

[14] M. Shi, X. Chen, J. Zhou, Y. Chen, J. Wen, and H. He, “Distributed optimal control of energy storages in a dc microgrid with communication delay,” *IEEE Trans. Smart Grid*, vol. 11, no. 3, pp. 2033–2042, May 2020.

[15] S. Sahoo, J. C. Peng, S. Mishra, and T. Dragičević, “Distributed screening of hijacking attacks in dc microgrids,” *IEEE Trans. Power Electron.*, vol. 35, no. 7, pp. 7574–7582, Jul. 2020.

[16] L. Xing, Y. Mishra, F. Guo, P. Lin, Y. Yang, G. Ledwich, and Y. Tian, “Distributed secondary control for current sharing and voltage restoration in dc microgrid,” *IEEE Trans. Smart Grid*, vol. 11, no. 3, pp. 2487–2497, May 2020.

[17] M. Dong, L. Li, Y. Nie, D. Song, and J. Yang, “Stability analysis of a novel distributed secondary control considering communication delay in dc microgrids,” *IEEE Trans. Smart Grid*, vol. 10, no. 6, pp. 6690–6700, Nov. 2019.

[18] C. Li, F. de Bosio, F. Chen, S. K. Chaudhary, J. C. Vasquez, and J. M. Guerrero, “Economic dispatch for operating cost minimization under real-time pricing in droop-controlled dc microgrid,” *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 5, no. 1, pp. 587–595, Mar. 2017.

[19] J. Yang, W. Feng, X. Hou, Q. Xia, X. Zhang, and P. Wang, “A distributed cooperative control algorithm for optimal power flow and voltage regulation in dc power system,” *IEEE Trans. Power Del.*, vol. 35, no. 2, pp. 892–903, Apr. 2020.

[20] V. Nasirian, F. L. Lewis, and A. Davoudi, “Distributed optimal dispatch for dc distribution networks,” in *Proc. 2015 IEEE First International Conference on DC Microgrids (ICDCM)*, Atlanta, GA, USA, Jun. 2015, pp. 97–101.

[21] M. Zaery, E. M. Ahmed, and M. Orabi, “Low operational cost distributed prioritised coordinated control for dc microgrids,” *IET Smart Grid*, vol. 2, no. 2, pp. 233–241, 2019.

[22] Z. Cheng, Z. Li, J. Liang, J. Gao, J. Si, and S. Li, “Distributed economic power dispatch and bus voltage control for droop-controlled dc microgrids,” *Energies*, vol. 12, no. 7, p. 1400, Apr. 2019.

[23] Z. Lv, Z. Wu, X. Dou, M. Zhou, and W. Hu, “Distributed economic dispatch scheme for droop-based autonomous dc microgrid,” *Energies*, vol. 13, no. 2, p. 404, Jan. 2020.

[24] D. P. Spanos, R. Olfati-Saber, and R. M. Murray, “Dynamic consensus on mobile networks,” in *Proc. 16th IFAC World Congress*, Prague, Czech Republic, 2005.

[25] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus and cooperation in networked multi-agent systems,” *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[26] W. Ren, R. W. Beard, and E. M. Atkins, “Information consensus in multivehicle cooperative control,” *IEEE Control Syst. Mag.*, vol. 27, no. 2, pp. 71–82, Apr. 2007.

[27] B. Abdolmaleki, Q. Shafiee, and H. Bevrani, “Kron reduction and l2-stability for plug-and-play frequency control of microgrids,” in *Proc. 2018 Smart Grid Conference (SGC)*, Sanandaj, Iran, Nov. 2018, pp. 1–6.

[28] K. Fitch, “Effective resistance preserving directed graph symmetrization,” *SIAM Journal on Matrix Analysis and Applications*, vol. 40, no. 1, pp. 49–65, 2019.

[29] H. Lin and P. J. Antsaklis, “Stability and stabilizability of switched linear systems: A survey of recent results,” *IEEE Trans. Autom. Control*, vol. 54, no. 2, pp. 308–322, Feb. 2009.