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Innovative Applications of O.R

Technology adoption in a declining market☆

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ABSTRACT

Rapid technological developments are inducing the shift in consumer demand from existing products towards new alternatives. When operating in a declining market, the profitability of incumbent firms is largely dependent on the ability to correctly time the introduction of product innovations. This paper contributes to the existing literature on technology adoption by determining the optimal time to innovate in the context of a declining market. We study the problem of a firm that has an option to undertake the innovation investment and thereby either to add a new product to its portfolio (add strategy) or to replace the established product by the new one (replace strategy). We find that it can be optimal for the firm to innovate not only because of the significant technological improvement, but also due to demand saturation. In the latter case profits of the established product may become so low that the firm will adopt a new technology even if the newest available innovation has not improved for some time. This way, our approach allows to explicitly account for the effect of a decline in the established market on technology adoption. Furthermore, we find that a substantial cannibalization effect occurring under the add strategy results in an inaction region. In this region the firm waits with innovation until the current technology level becomes either low enough to apply the add strategy, or the new technology becomes advanced enough to apply the replace strategy.

1. Introduction

The semiconductor industry has a profound impact on our daily lives. Computer chips (or integrated circuits) are crucial elements in electronic devices that most people use every day, as well as in more advanced industrial equipment. Broadly speaking, computer chips can be divided in two types[1], memory chips and logic chips. Memory chips are used for storage of data and logic chips are used to process data. In the memory segment companies like Samsung, SK Hynix, Micron, and Toshiba are active. The memory segment can be divided in DRAM and NAND segments. One of the main differences is that DRAM chips need power to store the data, whereas the NAND chips can retain data without power. TSMC, Samsung, and Intel are large suppliers of logic chips. Examples of logic chips are the processors in personal computers, and the computing chip in a smartphone or washing machine.

The developments in the semiconductor industry are driven by Moore's law: the number of transistors in a dense integrated circuit doubles about every two years. Producers of chips keep Moore's law alive by making use of new production processes, also called nodes in the semiconductor industry. In the logic segment these nodes have names like 14 nanometer, 10 nanometer, and 7 nanometer. The nodes lie at the basis of product improvements. Think of for example the main chip for the iPhone. In the iPhone XS, being the update of last year, Apple introduced the A12 chip, which is produced on the 7 nanometer node. In the previous

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[1] This division is a simplification of the division into four functionalities of chips: memory chips, microprocessors, standard chips, and complex systems-on-a chip (SoCs). For simplicity reasons we group the last three categories into one, namely logic chips.

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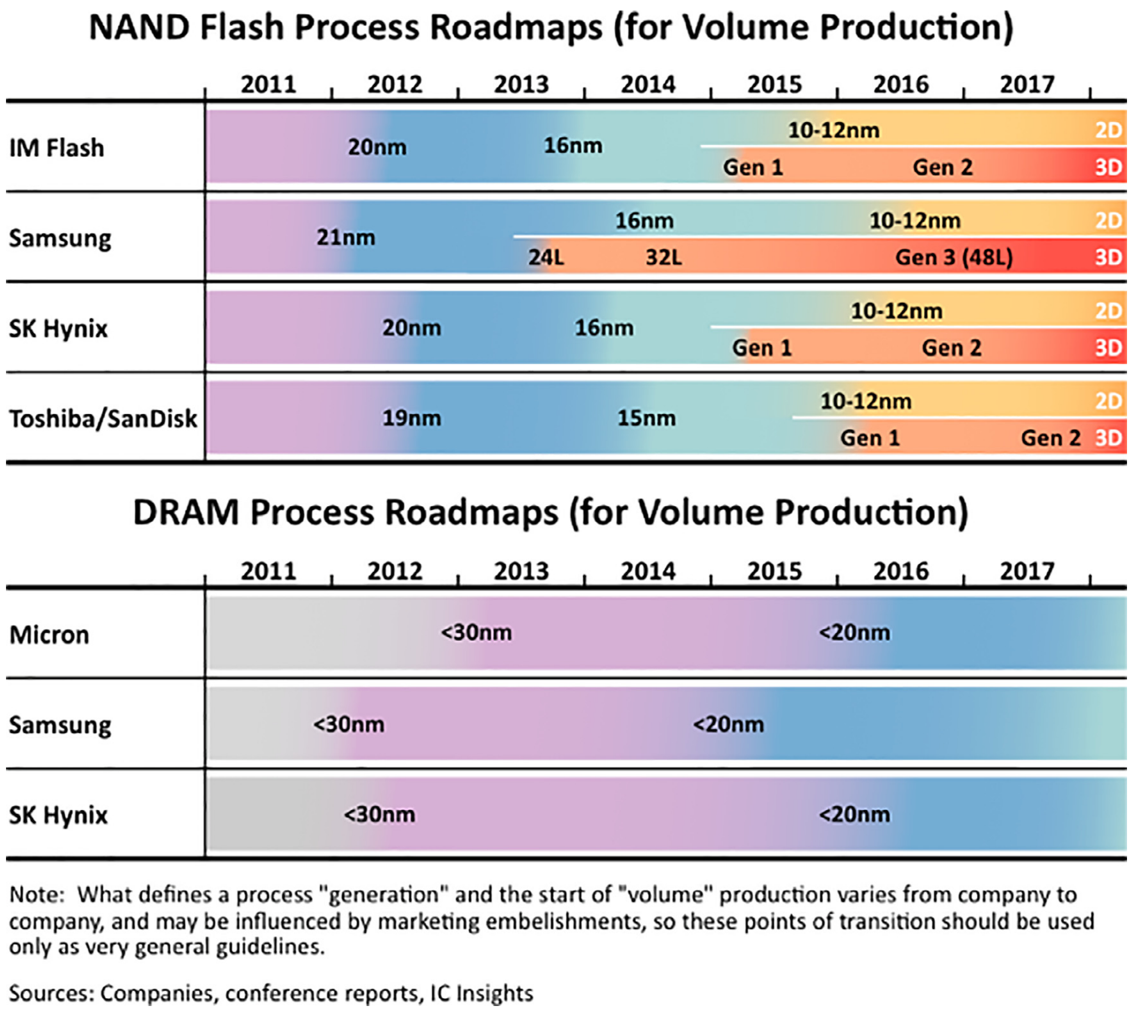


Fig. 1. High volume production nodes in NAND and DRAM segments, source: EE Times.

year the iPhone X used the A11 chip, which is produced on the 10 nanometer node. The 7 nanometer node was necessary to reach the specifications[2] for the A12 chip. It is important to note that A11 has 4.3 billion transistors and the A12 has 6.9 billion transistors, while at the same time the size of the A12 (83.27 millimetre^2) is smaller than the A11 (87.66 millimetre^2).[3]

Interestingly, there is a difference between logic nodes and memory nodes. Memory chip producers usually only run one node in high volume production, i.e. they *replace* one node completely over time by a new (smaller) node (see Fig. 1). However, in contrast logic chip producers *add* new nodes to their already existing product portfolio, mainly because there is demand left for chips from the older nodes, see Fig. 2. However, in doing so they are faced with the possibility that the market share of the existing product can be cannibalized by the launch of the new nodes.

Our paper explicitly pays attention to this issue of whether a firm should end a product innovation by adding a new product to its existing product portfolio, or to replace the existing product by the new one. The first strategy has the advantage that revenue can be collected from the established and the new product. However, the disadvantage is that introducing the new product to the market will cannibalize market share of the existing products. In our example the timing of a full-scale transition from 2D to 3D NAND memory depends on the point at which 3D becomes a cost-effective option to 2D. Even when the cost crossover point is reached, 2D and 3D NAND will likely coexist for several years, implying that the add strategy is employed.[4] The topic of cannibalization is central in the analysis of our model and is a main influencer of our results.

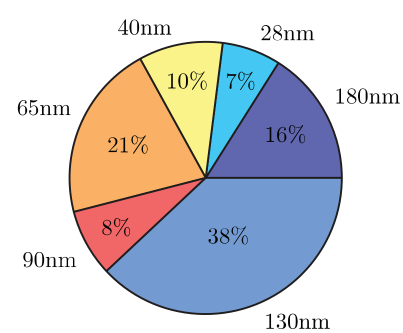


Fig. 2. Division of different production nodes in logic segment in 2016, source: anysilicon.

As documented in numerous studies (see, e.g., Rink & Swan, 1979, Klepper, 1996), demand for existing products decreases over time at the late stages of the product life cycle. One of the

[2] See, for example, https://www.faceofit.com/apple-a11-vs-a12-vs-a12x.

[3] See, for example, https://en.wikipedia.org/wiki/Apple_A12.

[4] https://www.eetimes.com/document.asp?doc_id=1323644.

important factors driving the decline is the arrival of more exciting alternatives[5]. This induces that firms need to change their product portfolio over time, and thus have to innovate in order to keep on making profits. This paper has the aim to study optimal firm behavior in such a setting. To do so, we study a problem of an existing incumbent producing an established product of which demand declines over time. The firm has an option to innovate, where, due to technological progress, a newer technology can produce better products. The resulting higher demand of the better product leads to higher profits. As time passes the best available new technology that can be adopted by the firm improves. So, the longer the firm waits with investing, the better the technology is that the firm can acquire and the better the products are the firm can produce.

In such a scenario the firm has the necessity to innovate, because otherwise the declining demand of the existing product diminishes its revenue over time[6]. In evaluating its innovation options the firm faces the following tradeoff. By adopting soon the firm is not affected too much by the reducing revenues from the existing technology, while it attracts a newer technology with higher profits. Adopting late means that, on the one hand, the firm suffers for a long time from declining profits due to the demand decrease of the established product. On the other hand, later adoption implies that, due to technological progress, the firm can attract a still better new technology with which the firm can obtain higher profits than when it adopted a new technology sooner.[7]

The existing analytical studies of technology adoption, like Balcer and Lippman (1984), Farzin, Huisman, and Kort (1998), and Hagspiel, Huisman, and Nunes (2015), consider similar innovation problems (see Huisman, 2001 and Hoppe, 1999 for an extensive survey about decision theoretic models of technology adoption), but they do not consider the important characteristic of declining demand for the existing product. As a result we obtain that the time to innovate can be governed by two different causes. First, like in Farzin et al. (1998), a firm innovates right at the moment of arrival of a far better technology, the use of which enables the firm to produce products with much higher demand, leading to a considerable profit increase. Second, the fact that demand for the existing product declines over time, implies that the firm's revenue gets lower and lower as long as it does not innovate. For this reason it could be optimal for the firm to adopt a new technology a time lag after its introduction.

The latter result is as such not new in the literature, but what is new is that it is caused by declining demand for the existing product. To exemplify, first consider Balcer and Lippman (1984) that also shows that as time passes without new technological improvements, it may become profitable to purchase an existing technology that is superior to the one in place even though it was not profitable to do so in the past. However, in that paper this is caused by the fact that the discovery time was not memoryless. Hagspiel et al. (2015) show that changing arrival rates over time of new technologies can result in firms adopting a new technology at a later point in time than when it was available for the first time. McCardle (1985) argues that such a time lag can be explained by the uncertainty regarding the profit potential of a new technology. Doraszelski (2004), who distinguishes between innovations and improvements, concludes that the possibility of further improvements gives the firm an incentive to delay the adoption of a new innovative technology until it is sufficiently advanced.

Unlike the just mentioned contributions, Kwon (2010) has in common with our paper that it also considers a firm with a declining profit stream over time. However, Kwon (2010), and also Hagspiel, Huisman, Kort, and Nunes (2016), that extends Kwon (2010) by considering capacity optimization, does not consider a sequence of new technologies arriving over time. Instead, it analyzes whether to exercise a single innovation opportunity. In addition, the firm also has an option to exit the industry, which exists before and after the investment. Matomaki (2013) generalizes the work of Kwon (2010) by considering different stochastic processes representing profit uncertainty. Strategic interactions in a declining industry are studied by Fine and Li (1986) and Murto (2004).

Similar to Kwon (2010), this paper focuses on a scenario where the firm has just one option to introduce a new product. This assumption is made for analytical tractability and allows us to obtain closed form solutions. The problem of multiple decisions to introduce successive generations of a product has been widely explored in the operations management literature. One stream of this literature, including, for example, Bayus (1997), Cohen, Eliashberg, and Ho (1997), Morgan, Morgan, and Moore (2001), Souza, Bayus, and Wagner (2004), focuses on the trade-offs between the early introduction of a new product and their quality where, unlike in our paper, there are no exogenous technology shocks.

Successive product launch policies with exogenous technology evolution was, among others, studied by Krankel, Duenyas, and Kapuscinski (2006), Lobel, Patel, Vulcano, and Zhang (2016) and Paulson Gjerde, Slotnick, and Sobel (2002). In these papers a firm chooses the time to replace a product from a previous generation by a new one, where later product introductions correspond to higher innovation levels. Compared to these contributions, our model considers that the firm has just one option to launch the new product. Our paper, however, provides additional insights on the choice between replacing the old product with the new one, or keeping them both in the firm's product portfolio, which induces a cannibalization effect.

The contributions that explicitly account for cannibalization between product of different quality typically use either logit or linear models of cannibalization. For example, in Bayus (1997), Cohen et al. (1997), Morgan et al. (2001) product market share is represented by a logit model, which is typical in marketing literature. In these contributions, however, the market share depends only on products' quality but not on their price. In the demand diffusion models, the cannibalization enters the demand function in a linear way. For example, Arslan, Kachani, and Shmatov (2009) assumes that sales for the old product drop by a constant factor as soon as the new generation is introduced. Savin and Terwiesch (2005) consider a game between two firms introducing a new product, where the innovation parameter, which represents the quality of a product, is linearly related to its market share. Klastorin and Tsai (2004) also model a competitive setting, where they focus on strategic consumers whose utility linearly depends on the innovation parameter that determines their preference for a particular product. In line with these contributions, we focus on the linear cannibalization model, where the market share of the old product cannibalized by the new one is linearly related to its quality (in our case technology level), which allows us to obtain a more stylized analytical solution.

[5] An example, among many others, is the introduction of solid state drives as an alternative for hard disk drives for data storage in computers. Before the current transition to solid state drives, the computer storage market has in the past decades gone through significant innovations from 14-inch, via 8-inch and 5.25-inch to 3.5-inch drives (see Kwon, 2010). Other examples include the arrival of LCD television sets that influenced demand of CRT television sets, the introduction of new iPhone models by Apple, and the replacement in the semiconductor industry of 200millimetre wafer plants by 300 millimetre wafer plants (see Cho & McCardle, 2009).

[6] In fact, in the computer data storage industry (see footnote 2), Western Digital (producer of hard disk drives) announced in October 2015 that it plans to acquire SanDisk (producer of solid state drives) in order to update their product portfolio (https://www.sandisk.com/about/media-center/press-releases/2015/western-digital-announces-acquisition-of-sandisk).

[7] In the computer storage industry of footnote 2, the 8-inch drives were eventually superseded by 5.25-inch drives, which are currently replaced by solid state drives (Kwon, 2010).

The described product innovation problem is attacked as follows in this paper. As in Farzin et al. (1998), Huisman (2001), Paulson Gjerde et al. (2002), technological progress is modeled as a Poisson process, where the level of the frontier technology jumps up at unknown points in time. Demand for the existing product decreases over time, resulting in a reduction of the associated profit with a fixed rate. At the moment the firm adopts the new technology it faces the choice as described in the example of the semiconductor industry: it either *adds* a new and technologically more advanced product to its product portfolio, meaning that it also keeps on producing the established product, or it *replaces* the old product by the new one. The revenue obtained from selling the new product is deterministic and increasing in the level of the adopted technology.

We start out by considering only the option to replace. Here, we obtain a threshold level for the technology that needs to be reached in order for the firm to invest optimally. The threshold level is increasing in the profit level of the established product, i.e. the firm delays the product innovation if the established product market is more profitable. We carry out a comparative statics analysis assuming a specific functional form for the profit flow in the new market. Among others, we find that the firm will innovate later in case of a slower decline of demand in the established product market.

We then proceed the analysis by also taking into account the option to add the innovative product to the product portfolio. The disadvantage of this strategy is that both products are competing in the sense that the new product cannibalizes demand of the old one and vice versa. Of course, the firm is still able to replace the old product by the new one, i.e. to stop production of the established product. Essentially, what we find is that the firm either innovates early and applies the add strategy or innovates late and applies the replace strategy. In the latter case, the firm waits for more technological improvements because its revenue solely depends on the new product upon adoption. Broadly speaking we found two different situations leading to qualitatively different solutions. In the first situation, it holds that the firm always innovates earlier if the current profit from selling the established product is lower, which is as expected. However, in the second situation an inaction region with respect to the technology level exists. In particular, in this inaction region the firm refrains from carrying out a product innovation, whereas for lower technology levels it would be optimal to innovate and add the new product to the product portfolio. If the technology level is sufficiently high the firm carries out the replace strategy. It turns out that such a situation occurs if the cannibalization effect is large enough, such that it dominates the increased revenue effect of a better technology.

The technical contribution of this paper lies in the fact that we are able to derive explicit expressions for the value function and the threshold boundary. Our optimal stopping problem has the special feature that the threshold boundary may either be reached in a continuous way (due to a gradual decrease in the profitability of the established product) or crossed in a discontinuous way, as a consequence of a technology arrival.

The literature offers several contributions that consider optimal stopping problems for diffusions combined with jump processes. In these studies, however, the solution is obtained either by making assumptions that simplify the problem, or by providing numerical approximations. An example of the former is Murto (2007). Although the paper starts with two stochastic processes, where one is a diffusion and the other one is a jump process, the author considers the following simplifications: either the volatility of the diffusion is zero (and, therefore, it becomes a deterministic process) or the jump process is purely deterministic. Therefore, instead of having a problem with two sources of uncertainty (that will lead to an exercise boundary and not to a point), the problem is transformed in a problem with just one source of uncertainty, where the classic tools (including verification theorems) may be used. Also Nunes and Pimentel (2017) provide analytical solution of the problem when the direction of the jumps is such that, contrary to the case that we analyze in the current paper, the stopping region is always attained through a continuous movement. This combined with the fact that the value function is homogeneous leads to an optimal stopping time problem where an analytical solution can be found.

In view of the difficulty to derive analytical solutions, we find some contributions on numerical solutions for jump-diffusion models; see, for instance, Cont and Tankov (2004), Cont and Voltchkova (2005), and d'Halluin, Forsyth, and Vetzal (2005) on numerical methods for solving partial integro-differential equations, and Feng and Linetsky (2008) on how to price path-dependent options numerically via variational methods and extrapolation.

The paper is organized as follows. Section 2 introduces the model. The replace strategy is analyzed in Section 3. Section 4 extends this analysis by also taking into account the add strategy. Section 5 concludes.

2. Model

We consider an incumbent firm currently producing an established product. As time passes, consumers get access to better alternatives in an evolving economy, shifting their demand away from the established product. Moreover, in case of durable goods the existing consumer base reduces as time passes, because more consumers already bought the product. For these reasons profits earned on the established product market decrease over time. The firm has been active in this market for some time, and we, therefore, assume that it has a perfect foresight about the future demand of the established product. Thus, the profit flow of the firm at time t is deterministic and equals $\pi_0(X_t) = z_0 X_t$, with $z_0 > 0$. The declining nature of the profit flow in the established market is captured by process $\boldsymbol{X} = \{X_t : t \geq 0\}$, where

$$dX_t = \alpha X_t dt,$$

with $X_0 = x_0$, where $x_0 > 0$ and $\alpha < 0$. This implies that the current product is already in the declining phase of the product life cycle (see, e.g., Bollen, 1999, Savin & Terwiesch, 2005, where a similar declining pattern was documented).

Facing a declining profit stream, the firm has an incentive to update its product portfolio. To do so it has to perform a product innovation by adopting a new, more advanced technology. Innovating requires an irreversible investment outlay of I. More significant technological improvements allow to produce products of higher quality. The adoption of the new technology, thus, boosts the firm's revenue, as it is able to attract more consumers.

The development of technologies over time is governed by a stochastic process, which is exogenous to the firm. Similar to Farzin et al. (1998), Huisman (2001), Paulson Gjerde et al. (2002), the state of technological progress is modeled by a compound Poisson process, $\boldsymbol{\theta} = \{\theta_t : t \geq 0\}$. We may express $\theta_t = \theta_0 + uN_t$, where $\theta_0 > 0$ denotes the state of technology at the initial point in time, $u > 0$ is the jump size and $\{N_t, t \geq 0\}$ follows a homogeneous Poisson process with rate $\lambda > 0$. This formulation implies that new technologies arrive at rate λ, and each arrival increases the technology level by u. This is typical, for example, for the semiconductor industry where technological progress is driven by Moore's law, which describes that the technology level jumps upwards at discrete points in time.

Similar to Klastorin and Tsai (2004), we can interpret θ as a level of attractiveness of the new technology, which could include elements like performance level, robustness and reliability, which sums up to the overall quality of the product.

It follows that initially the firm is producing with a technology ξ_0, for which it holds that $\xi_0 \leq \theta_0$. Without loss of generality we impose that $\xi_0 < \theta_0$, implying that initially the firm is not producing with the best available technology. The reason for this could be that the firm exists for some time and adopted its current technology at some time in the past.

Essentially, the firm has two reasons to innovate. The first reason is that over time alternative technologies have been invented with which the firm could enter markets that are more profitable than the established product market. The second reason is that the established product market profit has reduced too much so that to keep on producing this established product is not economically viable for the firm. In practice, firms often innovate not solely because of technological progress, but rather because of demand saturation. This is reflected in the fact that some technologies are not adopted by firms immediately after they emerge, but rather after the demand on established technology declines enough. This, for example, happened in the case of Fujifilm that entered the digital camera market rather late. This was driven by a dramatic decline in their revenues from film, where "film went from 60% of its profits in 2000 to basically nothing" (The Economist, January 14th, 2012).[8] Translated to our model, the first reason is equivalent to a high value of θ, whereas the second reason implies a low value of X. We conclude that innovating is optimal for low values of X, and high values of θ, while the firm should keep on being active on the established product market when X is high and θ is low.

The objective of the firm is, thus, to determine the optimal time to adopt the new technology. At that time the firm has to decide whether to simply replace the old product by the new one, or to add the new product to the existing product portfolio, so that the firm will produce both products at the same time. The next section fully concentrates on the replace case. This is an interesting case by itself as, for instance, in the two-period model of Levinthal and Purohit (1989) it is established that replacing the existing version of the product with an upgrade gives higher profits than joint production. The option to add is taken into account in Section 4.

3. Option to replace

In case the firm replaces the old product by the new one, it has to decide on the timing. Therefore, the firm solves the following optimal stopping problem:

$$F(\theta, x) = \sup_{\tau} \mathbb{E}\left[\int_0^{\tau} \pi_0(X_s)e^{-rs}ds + \left\{\int_{\tau}^{+\infty} \pi_1(\theta_\tau)e^{-rs}ds - Ie^{-r\tau}\right\} \times \chi_{\{\tau<+\infty\}} \middle| \theta_0 = \theta, X_0 = x\right], \quad (1)$$

in which F is the value of the firm, τ denotes the investment timing, $r > 0$ represents the discount rate, π_1 is the profit flow in the new market, and χ_A represents the indicator function of set A.

In this setting the firm faces a trade-off between early adoption and the significance of the technological improvement. In particular, waiting for a better technology comes at a cost of operating longer with lower profits.

If the firm decides to innovate at the current level of θ, it earns a profit flow of $\pi_1(\theta)$. Adding it up and discounting gives a total discounted profit stream $\frac{\pi_1(\theta)}{r}$. Since innovating requires an investment outlay of I, this results in the following value of instantaneous investment,

$$V(\theta) = \frac{\pi_1(\theta)}{r} - I. \quad (2)$$

In this section we do not propose any particular instance of π_1; instead we simply assume that it is an increasing and concave function of θ, with $\pi_1(0) = 0$ and $\lim_{\theta\to+\infty} \pi_1(\theta) = +\infty$. This entails that V is also increasing and concave, guaranteeing the existence of a unique solution of the optimal stopping problem (Alvarez, 2003).

The corresponding Hamilton–Jacobi–Bellman (HJB) equation for the optimization problem (1) is given by

$$\min\{rF(\theta, x) - [\pi_0(x) + \mathcal{L}F(\theta, x)], F(\theta, x) - V(\theta)\} = 0, \quad (3)$$

where the infinitesimal generator is defined by

$$\mathcal{L}f(\theta, x) = \alpha x \frac{\partial f(\theta, x)}{\partial x} + \lambda[f(\theta + u, x) - f(\theta, x)], \quad (4)$$

with f being continuous in θ and continuous with derivative absolute continuous in x.

Let the set $\mathcal{C} := \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : F(\theta, x) > V(\theta)\}$ denote the *continuation region*, and $\mathcal{S} := \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : F(\theta, x) = V(\theta)\}$ denote the *stopping region*. The firm adopts the new technology at the moment that the boundary between stopping and continuation region is passed. This happens at the optimal investment timing, denoted by τ^*, which is given by

$$\tau^* = \inf\{t > 0 : (\theta_t, X_t) \notin \mathcal{C}\}.$$

The expression for the boundary, or threshold curve, is derived in the following proposition.

Proposition 1. *The boundary (threshold curve) that separates the continuation and stopping region is defined as follows:*

$$\partial\mathcal{S} = \left\{(\theta, x) : \theta \geq \bar{\theta} \wedge x = b(\theta)\right\},$$

where

$$b(\theta) = \frac{(r + \lambda)V(\theta) - \lambda V(\theta + u)}{z_0}, \quad (5)$$

and $\bar{\theta}$ is implicitly defined by $(r + \lambda)V(\bar{\theta}) - \lambda V(\bar{\theta} + u) = 0$.[9] Moreover, b is an increasing function of θ.

Proof of Proposition 1. See Appendix A.1 for the proof.

From Proposition 1 we conclude that b is an upward sloping curve in the (θ, x)–plane. This implies that adoption of the new technology does not happen only due to a technology arrival, which corresponds to a horizontal jump in the (θ, x)–plane. It can also happen that the existing revenue for the established product becomes so low that innovating is optimal. This is reflected by the decrease in x over time, which corresponds to a vertical movement in the (θ, x)–plane, such that innovating takes place at the moment the b–curve is hit from above. These two possibilities are graphically illustrated in Fig. 3.

In this figure the current level of (θ, x) is marked by a star ($\star$). The solid lines correspond to the profit decline in the established market, whereas the dashed lines illustrate the technology arrivals. As said before, the threshold curve can be crossed in two ways. In one case, an additional decline of the profit flow in the current market is necessary for the investment to be optimal after two technology arrivals, and b is hit from above. In the other case, innovating is optimal immediately after two technology arrivals and b is crossed from the left.

Passing the boundary in these different ways has to be taken into account in the derivation of the value function of the firm in the continuation region, which we present in Proposition 2.

[8] https://www.economist.com/business/2012/01/14/the-last-kodak-moment.

[9] Here, $\bar{\theta}$ represents the level of the technology that triggers investment when there is no market left for the old product.

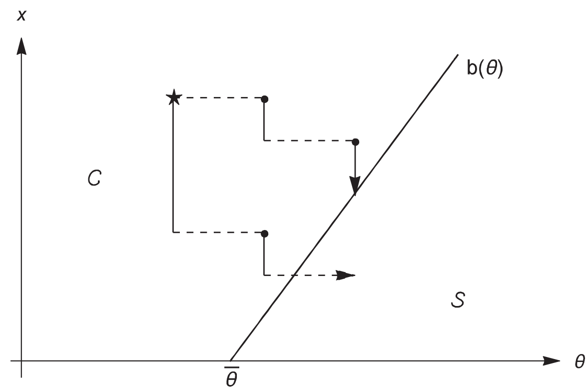


Fig. 3. Illustration of the two possible ways of adopting: at the arrival of a new technology (left-right horizontal crossing of threshold curve) or after a sufficient decrease of the profitability of the current market (downward vertical crossing of threshold curve).

Proposition 2. *Let the number of arrivals of new technologies until it is optimal to innovate be given by*

$$n(\theta, x) = \left\lceil \frac{b^{-1}(x) - \theta}{u} \right\rceil, \tag{6}$$

where, for $k \geq 0$, $\lceil k \rceil = \min\{n \in \mathbb{N} : n \geq k\}$. *Then the value of the firm in the continuation region is equal to*

$$\begin{aligned} F(\theta, x) = &\left(\frac{\lambda}{r+\lambda}\right)^{n(\theta,x)} V(\theta + n(\theta, x)u) \\ &+ \frac{z_0 x}{r-\alpha}\left[1 - \left(\frac{\lambda}{r+\lambda-\alpha}\right)^{n(\theta,x)}\right] \\ &+ \sum_{k=0}^{n(\theta,x)-1} \left\{\left[\frac{x}{b(\theta+ku)}\right]^{\frac{r+\lambda}{\alpha}} z_0 b(\theta+ku)\lambda^k \right. \\ &\times \sum_{m=0}^{k} \frac{1}{m!\,(-\alpha)^m}\left[\frac{1}{(r+\lambda)^{k-m+1}} - \frac{1}{(r+\lambda-\alpha)^{k-m+1}}\right] \\ &\left.\times \left[\ln\left[\frac{x}{b(\theta+ku)}\right]\right]^m\right\} \chi_{\{\theta > \bar{\theta} - ku\}}, \end{aligned} \tag{7}$$

where χ_A *represents the indicator function of set A.*

Proof of Proposition 2. See Appendix A.2 for the proof.

The value function in the continuation region consists of three parts. The first term in (7) can be interpreted as the expected discounted value of adopting the new technology upon its arrival. Here the fraction $\frac{\lambda}{r+\lambda}$ accounts for the stochastic discount factor under a Poisson process (Huisman, 2001, p.46).

The second term in (7) represents what the firm earns on sales of the established product until it innovates. Here $\frac{z_0 x}{r-\alpha}$ stands for the discounted revenue stream if the firm were active on the established product market forever. However, after the firm innovates, it discontinues this activity. Therefore, we need to subtract the amount $\left(\frac{\lambda}{r+\lambda-\alpha}\right)^{n(\theta,x)} \frac{z_0 x}{r-\alpha}$. The denominator $r+\lambda-\alpha$ makes sure that the resulting expected revenue stream is discounted (r), it is corrected for the fact that the revenue stream lasts up until the innovation time (λ), and that the revenue decreases over time with rate $-\alpha$ due to the declining demand of the established product.

The third term in (7) accounts for the fact that the innovation can occur not only due to the technology jump, but also by the decline in the established market. In order to illustrate this, consider a scenario in which the current demand in the established market and the technology level are such that the innovation will always be optimal after two jumps. Let $\mathcal{C}_n$ denote the subset of the continuation region where stopping is optimal after n jumps in θ, i.e. if $(\theta, x) \in \mathcal{C}_n$ then $(\theta + nu, x) \in \mathcal{S}$. Thus, in the region $\mathcal{C}_2$ we can simplify the value function in (7) – considering $n(\theta, x) = 2$ – as follows

$$\begin{aligned} &\left(\frac{\lambda}{r+\lambda}\right)^2 V(\theta+2u) + \frac{z_0 x}{r-\alpha}\left[1 - \left(\frac{\lambda}{r+\lambda-\alpha}\right)^2\right] \\ &+ \left\{\left[\frac{x}{b(\theta)}\right]^{\frac{r+\lambda}{\alpha}} z_0 b(\theta)\left[\frac{1}{r+\lambda} - \frac{1}{r+\lambda-\alpha}\right]\right\}\chi_{\{\theta>\bar{\theta}\}} \\ &+ \left\{\left[\frac{x}{b(\theta+u)}\right]^{\frac{r+\lambda}{\alpha}} z_0 b(\theta+u)\left[\frac{\lambda}{(r+\lambda)^2} - \frac{\lambda}{(r+\lambda-\alpha)^2}\right.\right. \\ &\left.\left. - \frac{\lambda}{\alpha}\left[\frac{1}{r+\lambda} - \frac{1}{r+\lambda-\alpha}\right]\ln\left[\frac{x}{b(\theta+u)}\right]\right]\right\}\chi_{\{\theta>\bar{\theta}-u\}}. \end{aligned} \tag{8}$$

Fig. 4 shows the four alternative ways the stopping region can be reached from an initial level of $(\theta, x) \in \mathcal{C}_2$.

The first two terms in (8) capture the case when the technology level $\theta + 2u$ is reached after two jumps, as depicted in Fig. 4a. The last three terms in (8) correct for the fact that in certain scenarios the demand in the established market may decline enough for the firm to be willing to adopt a lower technology level than $\theta + 2u$. In particular, the firm might end up adopting a technology level, $\theta + u$, if the established market declines enough before the second jump takes place to trigger the investment. In this case the stopping region can be reached in two different ways. The first is illustrated in Fig. 4b, where the first technology arrival happens relatively early. This brings the firm in the region one jump away from adopting, $\mathcal{C}_1$, where a further decline in the established market triggers the investment. This situation is captured by the last correction term in (8). The second possibility, when the jump occurs relatively late, is shown in Fig. 4c. In this case the decline in the established market brings the firm to the region $\mathcal{C}_1$, after which the first technology arrival triggers the investment. This scenario is accounted for by the second correction term in (8). Finally, as shown in Fig. 4d the firm may eventually adopt the current level of technology, θ, if the market declines even further before any jump occurs. The first correction term in (8) corrects for that.

The following remark highlights important properties of function F defined in (7).

Remark 1. F is continuous in both arguments, θ and x, and has derivative absolute continuous in x.

This result follows directly from the proof of Proposition 2 presented in Appendix A.2.

3.1. Example

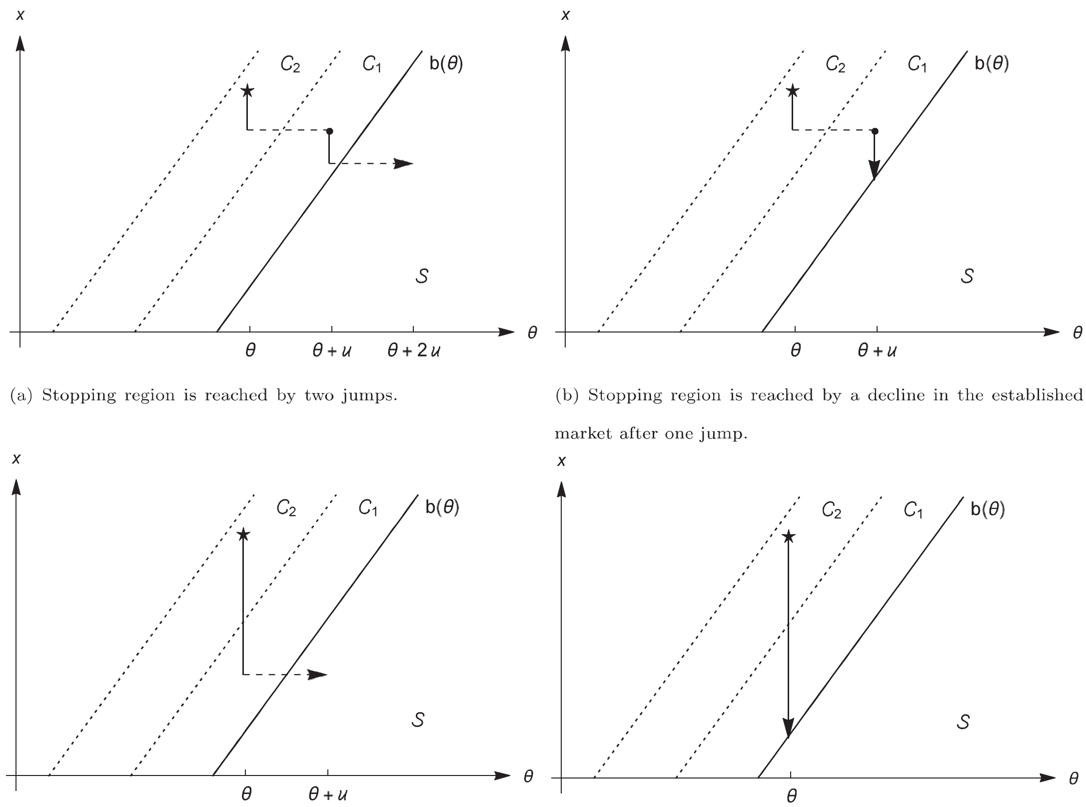
In Section 3 we derived the analytical solution of the optimal stopping problem for a general expression of the profit flow in the new market. In this section we analyze a specific example for a functional form of π_1. In particular we consider a profit flow expressed by $\pi_1(\theta) = z_1\theta^\beta$ with $0 < \beta \leq 1$. After plugging this expression into (2) and (5), we obtain that the threshold curve b is given by the following equation

$$b(\theta) = \frac{1}{z_0}\left[z_1 \frac{(r+\lambda)\theta^\beta - \lambda(\theta+u)^\beta}{r} - rI\right]. \tag{9}$$

From the expression of the threshold curve (9) the following comparative statics results are derived.

Proposition 3. *The threshold curve b is increasing in* z_1, *and decreasing in* λ, z_0, *I, and u, for a given value* θ.

Proof of Proposition 3. See Appendix A.3 for the proof.



(a) Stopping region is reached by two jumps.

(b) Stopping region is reached by a decline in the established market after one jump.

(c) Stopping region is reached by one jump after a decline in the established market.

(d) Stopping region is reached by a decline in the established market.

Fig. 4. Four different ways of reaching the stopping region from an initial level of $(\theta, x) \in \mathcal{C}_2$.

Proposition 3 implies that the firm will innovate later if profits on the established product market are higher. On the other hand, the firm will innovate later if the revenue from innovating is lower.[10] Intuitively, waiting for the next technology arrival is more appealing if it is expected to occur sooner or when the technology arrival results in a higher increase of the technology level. If innovating is more expensive it will happen later. The location of the threshold curve is not affected by the rate of decline α, implying that a larger decline rate of the revenue in the established product market will result in reaching the threshold curve sooner. The conclusion is that the firm will innovate sooner for a larger absolute value of α, which makes sense from an intuitive point of view.

Concerning the discount rate r there are opposing effects. On the one hand, when r increases the firm is less inclined to wait for future technological breakthroughs and therefore wants to innovate sooner. This effect dominates for small r. On the other hand, the firm innovates later, because the net present value of the investment decreases with r. Fig. 5 illustrates the relationship between a point in the threshold curves for a fixed level of θ and different values of r.

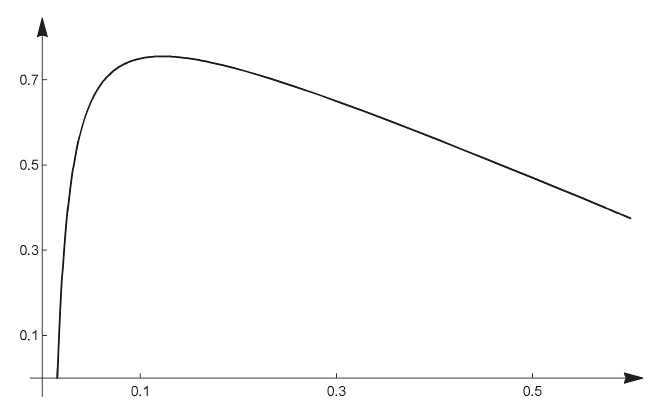


Fig. 5. Example showing that the threshold curve $b(\theta)$ for a given $\theta = 5$ first increases in the discount rate r and then decreases. Parameter values used: $\lambda = 0.05$, $u = 0.5$, $I = 50$, $z_0 = 50$, $z_1 = 10$ and $\beta = 1$.

4. Option to add or replace

In this section we give the firm the option to keep producing the established product after investing in the innovative product. The firm can still replace the established product right away, as we analyzed before. We denote by π_1^A (respectively, π_1^R) the profit that results from adding the new product to the product portfolio (respectively, replacing the old product by the new one).

[10] A real world illustration of these two results is the transition from film to digital photography. At that time firms realized that "digital photography itself would not be very profitable" and therefore, concluded that "it was best not to hurry to switch from making 70 cents on the dollar on film to maybe five cents at most in digital" (The Economist, January 14th, 2012). Source: https://www.economist.com/business/2012/01/14/the-last-kodak-moment.

In this problem the firm not only needs to decide on when to invest in the new product but also when to stop producing the old product. This means that the firm solves the following optimal stopping problem:

$$F(\theta,x)=\sup_{\tau_1}\mathbb{E}\Bigg[\int_0^{\tau_1}\pi_0(X_s)e^{-rs}ds + \Bigg\{\sup_{\tau_2:\tau_2\geq\tau_1}\mathbb{E}\Bigg[\int_{\tau_1}^{\tau_2}\pi_1^A(\theta_{\tau_1},X_s)e^{-rs}ds - Ie^{-r\tau_1} + \Bigg\{\int_{\tau_2}^{+\infty}\pi_1^R(\theta_{\tau_1})e^{-rs}ds\Bigg\}\chi_{\{\tau_2<+\infty\}}\Bigg]\Bigg\} \times \chi_{\{\tau_1<+\infty\}}\Bigg|\theta_0=\theta, X_0=x\Bigg], \quad (10)$$

where τ_1 denotes the time the firm adopts the new technology, i.e. adds the innovative product to its product portfolio; and τ_2 denotes the time the firm stops producing with the old technology, i.e. it replaces the established product by the new one that was added to the portfolio at τ_1.

If the firm has an option to keep the old product alive after investing in the innovative product, it has to take into account that some market share of the old product will be cannibalized by the upgrade. This is because a fraction of the consumers will switch to the new version once it becomes available. To take this effect into account, we specify the inverse demand functions for the old and new products, p_0^A and p_1^A, as follows[11]

$$p_0^A(\theta,x)=(1-\gamma q_0-\eta q_1\theta)x,$$
$$p_1^A(\theta,x)=(1-\gamma q_1-\eta q_0 x)\theta,$$

where q_0 denotes the quantity of the established product, q_1 is the quantity of the new product, and γ represents the demand sensitivity of the product to its quantity. The parameter η reflects how much the products are competing with each other, which is mainly affected by the extent to which both products have appeal to the same consumers. Intuitively, a larger θ corresponds to a higher quality of the new product, which becomes therefore more attractive to consumers. As a result, the new product will cannibalize a larger market share of the established product. Similarly, x represents the attractiveness of the established product, which negatively affects the demand for the new product, cannibalizing its market share. If the firm produces only one product, either solely the old one or the new one, the cannibalization effect is not present. Therefore, before the firm introduces the upgrade, the price for the existing product is given by $p_0(x)=(1-\gamma q_0)x$. If the firm decides to replace the old product by the new one, the price for the latter is $p_1^R(\theta)=(1-\gamma q_1)\theta$. In the notation of the model presented in Section 2, we have $z_0=(1-\gamma q_0)q_0$. We further introduce $z_1=(1-\gamma q_1)q_1$ and $\kappa=2\eta q_0 q_1$, where the latter represents the strength of the cannibalization effect. This leads to the following profit functions

$$\pi_0(x)=p_0(x)q_0=z_0x,$$
$$\pi_1^R(\theta)=p_1^R(\theta)q_1=z_1\theta,$$
$$\pi_1^A(\theta,x)=p_0^A(\theta,x)q_0+p_1^A(\theta,x)q_1=\pi_0(x)+\pi_1^R(\theta)-\kappa\theta x, \quad (11)$$

where π_0 denotes the profit before innovation, π_1^R is the profit of the firm producing only the innovative product, and π_1^A is the profit of the firm producing both products. Note that π_1^R corresponds to the linear profit function example in Section 3. The expression for $\pi_1^A(\theta,x)$ reflects that when producing both products the firm benefits from collecting profits on both markets. On the other hand, the drawback is that the products are competing with each other. In particular, selling the new product cannibalizes demand for the old product, whereas keep on selling the old product will decrease the profit of the new one. Both these effects are contained in the term $-\kappa\theta x$ that affects the profit π_1^A negatively, as confirmed by expression (11).

The following proposition states that in fact, given the chosen demand functions, the decision of the firm is either never replace the old product (and instead produce both products forever) or replace upon investing.[12]

Proposition 4. *The firm will keep producing the old product upon adoption of a new technology with level* θ *if* $0<\theta<\widehat{\theta}$, *and will replace the old product if the new technology level is such that* $\theta\geq\widehat{\theta}$, *with* $\widehat{\theta}=\frac{z_0}{\kappa}$.

Proof of Proposition 4. See Appendix A.5 for the proof.

As a result, the space (θ,x) is split in two regions: for $\theta<\widehat{\theta}$, upon investment the firm produces both products, whereas for $\theta\geq\widehat{\theta}$ the firm produces just the innovative one upon investment.

This condition implies that *replace* is a strictly dominating alternative when $z_0<\kappa\theta$, i.e. the profitability per unit of demand of the old product is lower than the marginal cannibalization effect. Intuitively, this means that the marginal benefit from keeping the old product alive is smaller than the marginal cost represented by the cannibalization. In other words, the replace option is preferable when the new technology is mature enough, the profitability of the old product is low enough and (or) the cannibalization effect of the new technology is large enough. This is, for example, typical for production nodes for memory chips. There the cannibalization effect is large because chips in the memory segment are characterized by little differentiation among them, implying a larger demand sensitivity to changes in technology. In this segment, new generation chips significantly outperform the old generations in speed and energy consumption, which in turn causes a severe drop in demand for the old technologies when innovations are adopted. This is consistent with Fig. 1 where we see that memory chip producers use only one high volume production node at a time. Contrarily, the logic chips producers operate in an environment where it is profitable to use several nodes in parallel (Fig. 2). One potential reason for this is that they often produce a set of heterogeneous chips for different customers, resulting in a lower cannibalization effect due to residual demand for the old technologies.

Taking into account Proposition 4, simple manipulations allow us to rewrite the problem (10) as

$$F(\theta,x)=\sup_{\tau_1}\mathbb{E}\Bigg[\int_0^{\tau_1}\pi_0(X_s)e^{-rs}ds+e^{-r\tau_1}V(\theta_{\tau_1},X_{\tau_1}) \times \chi_{\{\tau_1<+\infty\}}\Bigg|\theta_0=\theta,X_0=x\Bigg], \quad (12)$$

[11] The demand system can be derived from the following utility function

$$U=xq_0-\frac{1}{2}\gamma q_0^2x-\eta q_0q_1\theta x+\theta q_1-\frac{1}{2}\gamma q_1^2\theta+\lambda(\mathbb{Y}-p_0^Aq_0-p_1^Aq_1),$$

in which λ is a Lagrange parameter and $\mathbb{Y}$ is the income of the representative consumer.

[12] This in fact says that the policy of holding both products in the portfolio for a finite time and discarding the old product at some point is never the optimal policy. The reason is that in our current model all uncertainty has disappeared after the innovation has taken place. An interesting extension could be that innovative product demand is uncertain where the uncertainty is for instance generated through a geometric Brownian motion (GBM) process, which would also affect the cannibalization term for the established product. In other words, if demand is booming, i.e. the GBM variable is large, the new product takes away a considerable part of the demand of the old product. At such a point it could be optimal to discard the old product and keep on producing just the new one.

with

$$V(\theta, x) = \frac{\pi_1^A(\theta, x) - \pi_1^R(\theta)}{r - \alpha}\chi_{\{0<\theta<\widehat{\theta}\}} + \frac{\pi_1^R(\theta)}{r} - I,$$

which can be re-written as

$$V(\theta, x) = \begin{cases} V^A(\theta, x) = \dfrac{[z_0 - \kappa\theta]x}{r - \alpha} + \dfrac{z_1\theta}{r} - I & \text{if} \quad 0 < \theta < \hat{\theta}, \\ V^R(\theta, x) = \dfrac{z_1\theta}{r} - I & \text{if} \quad \theta \geq \hat{\theta}. \end{cases}$$

The HJB equation corresponding to the optimization problem (12) is given by

$$\min\{rF(\theta, x) - [\mathcal{L}F(\theta, x) + \pi_0(x)], F(\theta, x) - V(\theta, x)\} = 0,$$

where the infinitesimal generator is the same as defined in (4). As in the previous section, let the set $\mathcal{C} := \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : F(\theta, x) > V(\theta)\}$ denote the *continuation region*, and $\mathcal{S} := \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : F(\theta, x) = V(\theta)\}$ denote the *stopping region*. Then the optimal investment timing, denoted by τ^*, is given by $\tau^* = \inf\{t > 0 : (\theta_t, X_t) \notin \mathcal{C}\}$.

Unlike in Section 3, we have to distinguish two different cases concerning the shape of the optimal exercise boundary, and, consequently of the continuation region $\mathcal{C}$. In the first case, the boundary is monotonically increasing in θ. In the second case, the boundary exhibits non-monotonic behavior. The next proposition states a condition that needs to be satisfied for the threshold boundary to be monotonically increasing in the (θ, x)-plane.

Proposition 5. *The investment threshold is monotonically increasing in θ if the following condition is satisfied:*

$$\kappa\left(\frac{rI}{z_1} + \frac{u\lambda}{r}\right) > \frac{\lambda}{r + \lambda - \alpha}z_0. \tag{13}$$

Proof of Proposition 5. See Appendix A.6 for the proof.

The threshold boundary being monotonically increasing in the (θ, x)-plane is the result to be expected. It implies that, given the level of established product demand parameter x, it is optimal for the firm to innovate when the technology level θ is large enough. This is intuitive because for a larger θ the revenue of the innovative product increases in the case of the replace strategy where the revenue equals $\pi_1^R(\theta) = z_1\theta$. This also has a positive effect on the revenue in the add strategy, because we see in expression (11) that $\pi_1^A(\theta, x)$ increases with $\pi_1^R(\theta)$.

The remarkable conclusion from Proposition 5, however, is that a condition, namely (13), needs to hold for this completely intuitive shape of the threshold boundary to be true. The point is that, as can be obtained from (11), in case of the add strategy the innovation level θ also influences the profit $\pi_1^A(\theta, x)$ in a negative way, namely via the cannibalization effect $-\kappa\theta x$. This is because the negative effect on the demand of the established product is stronger if the new product is of higher quality. Then more consumers are attracted to this new product, which goes at the expense of old product demand. It follows that the innovation payoff decreases in the innovation level θ as soon as the cannibalization effect $-\kappa\theta x$ dominates the "direct" revenue effect $z_1\theta$. In this case, it may happen that, given some level of x, for a certain level of θ it is optimal for the firm to innovate while applying the add strategy, whereas for a higher level of θ it is optimal for the firm to refrain from innovating. This implies that, as also demonstrated in Fig. 9, the threshold boundary $b(\theta, x)$ is not overall increasing.

From Proposition 5 we obtain that the threshold boundary is not monotonically increasing, implying that for some levels of θ, given x, the cannibalization effect dominates the direct revenue effect, if the complement of (13) holds, i.e.

$$\kappa\left(\frac{rI}{z_1} + \frac{u\lambda}{r}\right) < \frac{\lambda}{r + \lambda - \alpha}z_0. \tag{14}$$

Surprisingly, the non-monotonicity in $b(\theta, x)$ can occur if the cannibalization parameter κ is small. The explanation is that for a small κ the add strategy is more attractive. Then the firm innovates relatively early thus when established product demand represented by x is still large. And this boosts the cannibalization effect $-\kappa\theta x$.

As obtained from (14), a similar effect is observed when we consider the marginal increase in profits due to innovation, z_1, and the decline in investment cost, I. A large z_1 and a small I increase the attractiveness of the innovation. A more profitable innovation opportunity, in turn, makes the firm more eager to invest, i.e. it does so for larger values of x. This drives the cannibalization effect $-\kappa\theta x$ up, such that it dominates the direct increase in revenue from investment, $z_1\theta$. Then it is more likely that the threshold boundary has a decreasing part in the (θ, x)-plane.

In addition, expression (14) reflects that the cannibalization effect is likely to dominate for a large profitability of the old product, z_0. Since under the add strategy the firm will keep on producing the old product after having carried out the product innovation, in such a situation it will innovate soon and thus invest when x is relatively large. This implies that the cannibalization effect, $-\kappa\theta x$, is large in absolute terms and could dominate the direct revenue effect $z_1\theta$ at the threshold boundary, so that this boundary could decrease for some levels of θ.

Also when the technology jump size, u, is low the firm will innovate for large values of x, and the threshold boundary need not be monotonic in the (θ, x)-plane. The early innovation is triggered by the fact that, due to the small increase in the technology level associated with each arrival of a new generation, the firm is less inclined to wait for new generations to arrive before investing.

In the next two sections, we give a more formal illustration of these results and analyze the optimal solutions in the two different cases, i.e. the monotonic and the non-monotonic threshold boundaries.

4.1. Monotonic threshold boundary

Proposition 6 gives the expression for the optimal exercise boundary in case the boundary is monotonically increasing in θ.

Proposition 6. *Let us assume that the condition in Proposition 5 is satisfied. For $\theta > \overline{\theta}$, where $\overline{\theta} = \frac{rI}{z_1} + \frac{u\lambda}{r}$, the boundary between the stopping and the continuation region is given by*

$$b(\theta) = b^A(\theta)\chi_{\{\overline{\theta} \leq \theta < \widehat{\theta} - u\}} + b^{AR}(\theta)\chi_{\{\widehat{\theta} - u \leq \theta < \widehat{\theta}\}} + b^R(\theta)\chi_{\{\theta \geq \widehat{\theta}\}} \tag{15}$$

where

$$b^A(\theta) = \frac{z_1\left(\theta - \frac{u\lambda}{r}\right) - rI}{\kappa\left(\theta - \frac{u\lambda}{r-\alpha}\right)}$$

$$b^{AR}(\theta) = \frac{z_1\left(\theta - \frac{u\lambda}{r}\right) - rI}{\frac{(r+\lambda-\alpha)\kappa\theta - \lambda z_0}{r-\alpha}}$$

$$b^R(\theta) = \frac{z_1\left(\theta - \frac{u\lambda}{r}\right) - rI}{z_0}.$$

Proof of Proposition 6. See Appendix A.7 for the proof.

Note that depending on the model parameters, some of the regions defined in (15) could vanish.[13] In this study, we are always considering the more comprehensive case where all the regions are non-empty.

[13] The boundary defined by (15) is obtained when $\overline{\theta} < \widehat{\theta} - u < \widehat{\theta}$. The boundary can also be defined as $b(\theta) = b^{AR}(\theta)\chi_{\{\overline{\theta} \leq \theta < \widehat{\theta}\}} + b^R(\theta)\chi_{\{\theta \geq \widehat{\theta}\}}$ in case $\widehat{\theta} - u < \overline{\theta} < \widehat{\theta}$ or $b(\theta) = b^R(\theta)\chi_{\{\theta \geq \overline{\theta}\}}$ when $\widehat{\theta} < \overline{\theta}$.

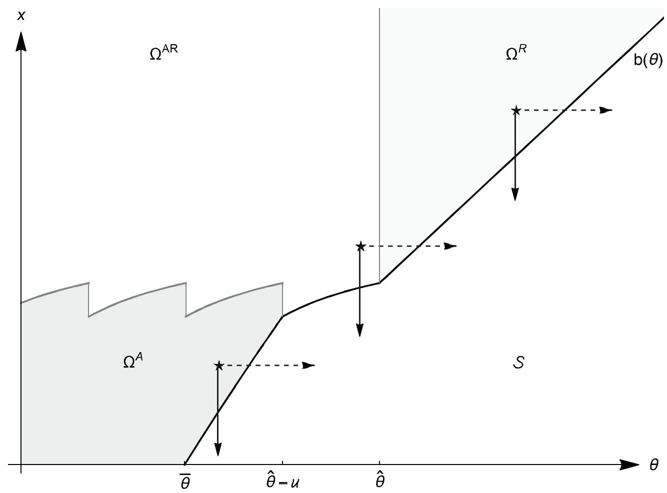


Fig. 6. Illustration of optimal theshold boundary and the regions Ω^R, Ω^A and Ω^{AR}.

Fig. 6 provides an illustration of this exercise boundary. We also show the possible ways for the bivariate process (θ, x) to enter the stopping region. As before, this may happen either due to the decline in the profitability of the existing market, i.e. when x decreases (solid vertical arrow), or due to the arrival of a sufficiently better technology, i.e. a jump in θ (dashed horizontal arrow). Fig. 6 has in common with Fig. 3 that both threshold curves are monotonically increasing in the (θ, x) – plane. However, whereas in Fig. 3 the threshold curve has the same slope everywhere, in Fig. 6 the threshold curve consists of three different pieces. To explain, we distinguish three subsets in the continuation region, which will be formally defined later in this section: Ω^R, Ω^A and Ω^{AR}. In the first two the stopping region is either entered in the replace region or in the add region, respectively. In the latter both parts of the stopping region can still be reached, depending on the trajectory of the bivariate process (θ, x).

On the part of the threshold curve, where $\theta \in \left[\widehat{\theta}, +\infty\right)$, the firm is indifferent between waiting with investing and replacing production of the established product by producing the new one. For these values of θ, the firm solves exactly the same optimal stopping problem as in Section 3. Therefore, in that part the position of the curves of Figs. 3 and 6 coincide. The subset of the continuation region bounded to this part of the curve is Ω^R. If the starting values of (θ, x) belong to Ω^R, then upon investment the firm implements the strategy of replacing the old product by the new both in case of a decline in x and in case of jumps in θ.

On the part of the threshold curve, where $\theta \in \left[\bar{\theta}, \widehat{\theta} - u\right)$, the firm is indifferent between waiting and investing in the new technology after which the firm will jointly produce the established and the new product. The subset of the continuation region, denoted by Ω^A, contains the starting values of (θ, x) such that upon investment the firm implements the strategy of adding the new product to its product portfolio both in case of a decline in x and in case of a jump in θ.

On the part of the threshold curve, where $\theta \in \left[\widehat{\theta} - u, \widehat{\theta}\right)$, the firm is indifferent between waiting with investing and adding the innovative product to its product portfolio. The subset of the continuation region, denoted by Ω^{AR}, contains the starting values of (θ, x) for which it is not established beforehand, whether the firm will apply the "add" or "replace" strategy upon investment. In particular, for starting values of (θ, x) only one jump away from the stopping region, it is optimal for the firm to add the new product to the existing one if it enters the stopping region by a decline in x. However, in case it enters the stopping region by a jump in θ, it is optimal to replace the old product by the new one.

In what follows we present the formal definition of the subsets Ω^R, Ω^A and Ω^{AR}, as well as the optimal value functions in each subset.

- We can define $\Omega^R = \bigcup_{n=1}^{\infty} \Omega_n^R$, where

$$\Omega_n^R = \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : \theta \geq \widehat{\theta} \wedge b^R(\theta + (n-1)u) < x \leq b^R(\theta + nu)\},\ n \in \mathbb{N}.$$

In this region, the firm replaces the old product by the innovative one, once it decides to undertake an investment. The value function for the replace region is already derived in Section 3. Incorporating the new notation, we let $F(\theta, x) \equiv F^R(\theta, x)$ for $(\theta, x) \in \Omega^R$, and get the following expression

$$\begin{aligned} F^R(\theta, x) = &\left(\frac{\lambda}{r+\lambda}\right)^{n(\theta,x)} V(\theta + n(\theta, x)u) \\ &+ \frac{z_0 x}{r-\alpha}\left[1 - \left(\frac{\lambda}{r+\lambda-\alpha}\right)^{n(\theta,x)}\right] \\ &+ \sum_{k=0}^{n(\theta,x)-1}\left[\frac{x}{b^R(\theta+ku)}\right]^{\frac{r+\lambda}{\alpha}}\left(z_1\left(\theta + ku - \frac{u\lambda}{r}\right) - rI\right)\lambda^k \\ &\times \sum_{m=0}^{k}\frac{(-\alpha)^{-m}}{m!}\left[\frac{1}{(r+\lambda)^{k-m+1}} - \frac{1}{(r+\lambda-\alpha)^{k-m+1}}\right] \\ &\times \left[\ln\left[\frac{x}{b^R(\theta+ku)}\right]\right]^m, \end{aligned} \tag{16}$$

where $n(\theta, x) = \left\lceil \frac{b^{-1}(x) - \theta}{u} \right\rceil$.

- We can define $\Omega^A = \bigcup_{n=1}^{\bar{n}-1} \Omega_n^A$, with $\bar{n} = \left\lceil \frac{\widehat{\theta}}{u} \right\rceil$, which represents the number of jumps needed to exceed $\widehat{\theta}$ starting from zero[14], and, for $n \in \mathbb{N}$,

$$\Omega_n^A = \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : 0 < \theta < \widehat{\theta} - nu \wedge b^A(\theta + (n-1)u) < x \leq b^A(\theta + nu) \wedge 0 < x \leq b^{AR}(\theta + nu)\}.$$

In this region, the firm adds the innovative product to its product portfolio, i.e. keeps the old product alive, upon investment. Using a similar reasoning as the one for the replace case, we derive that the value function in the this region, i.e. $F(\theta, x) \equiv F^A(\theta, x)$ for $(\theta, x) \in \Omega^A$, is given by

$$\begin{aligned} F^A(\theta, x) = &\left(\frac{\lambda}{r+\lambda}\right)^{n(\theta,x)} V(\theta + n(\theta, x)u) + \frac{z_0 x}{r-\alpha} \\ &\times\left[1 - \frac{\kappa(\theta + n(\theta,x)u)}{z_0}\left(\frac{\lambda}{r+\lambda-\alpha}\right)^{n(\theta,x)}\right] \\ &+ \sum_{k=0}^{n(\theta,x)-1}\left\{\left[\frac{x}{b^A(\theta+ku)}\right]^{\frac{r+\lambda}{\alpha}}\left(z_1\left(\theta + ku - \frac{u\lambda}{r}\right) - rI\right)\lambda^k\right. \\ &\times \sum_{m=0}^{k}\frac{(-\alpha)^{-m}}{m!}\left[\frac{1}{(r+\lambda)^{k-m+1}} - \frac{1}{(r+\lambda-\alpha)^{k-m+1}}\right] \\ &\left.\times\left[\ln\left[\frac{x}{b^A(\theta+ku)}\right]\right]^m\right\}\chi_{\{\theta > \bar{\theta} - ku\}}. \end{aligned} \tag{17}$$

- Finally, we can define $\Omega^{AR} = \bigcup_{n=1}^{\infty} \bigcup_{p=1}^{\min\{n, \bar{n}\}} \Omega_n^p$, where, for $n, p \in \mathbb{N}$, if $n \neq p$ we have

$$\Omega_n^p = \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : \widehat{\theta} - pu \leq \theta < \widehat{\theta} - (p-1)u \wedge b^R(\theta + (n-1)u) < x \leq b^R(\theta + nu)\}$$

[14] The number of jumps in this case is limited, as θ and x are bounded by 0.

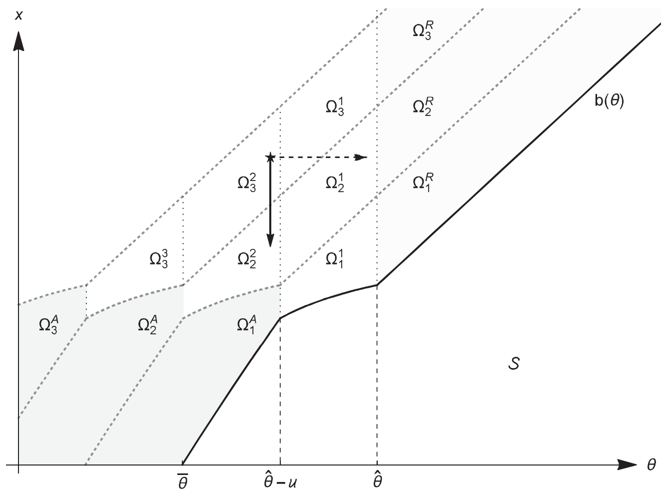


Fig. 7. Illustration of the increasing threshold curve $b(\theta)$.

otherwise

$$\Omega_n^p = \{(\theta, x) \in \mathbb{R}^+ \times \mathbb{R}^+ : \widehat{\theta} - pu \le \theta < \widehat{\theta} - (p-1)u \wedge b^{AR}(\theta + (n-1)u) < x \le b^R(\theta + nu)\}.$$

In this region, upon investment the firm may either produce both products or replace the old one by the new one. In fact Ω_n^p represents the sets of values for θ and x such that either x decreases continuously and reaches region Ω_{n-1}^p, or a jump occurs, leading to the region Ω_{n-1}^{p-1}. These possible transitions for the case of Ω_3^2 are illustrated in Fig. 7, where the vertical solid arrow represents a continuous decrease in the value of x, whereas the horizontal dashed arrow represents a jump in the technology level.

We note that for the values of θ and x in the previous two regions, the decision regarding the type of investment is clear: when one is in Ω^R, we do not know for how many jumps in the technology we will need to wait and when the jumps will take place, but we do know for sure that once the firm invests, it will replace the old product by the new one. Similarly, in Ω^A the same holds for the investment timing but the firm knows that upon investment, the two products will be produced forever. However, Ω^{AR} is a region where not only there is uncertainty regarding the timing of the investment, but also regarding the strategy upon investment (either add or replace). In particular, for $(\theta, x) \in \Omega^{AR}$, the value function is given by $F(\theta, x) \equiv F^{AR}(\theta, x)$, where

$$\begin{aligned} F^{AR}(\theta,x) = &\left(\frac{\lambda}{r+\lambda}\right)^{n(\theta,x)} V(\theta + n(\theta,x)u) \\ &+ \frac{z_0 x}{r-\alpha}\left[1 - \left(\frac{\lambda}{r+\lambda-\alpha}\right)^{n(\theta,x)}\right] \\ &+ \sum_{k=0}^{n(\theta,x)-1}\left\{\left[\frac{x}{b(\theta+ku)}\right]^{\frac{r+\lambda}{\alpha}} i(\theta+ku)\lambda^k \right.\\ &\times \sum_{m=0}^{k}\frac{(-\alpha)^{-m}}{m!}\left[\frac{1}{(r+\lambda)^{k-m+1}} - \frac{1}{(r+\lambda-\alpha)^{k-m+1}}\right] \\ &\left.\times\left[\ln\left[\frac{x}{b(\theta+ku)}\right]\right]^m\right\}\chi_{\{\theta>\bar{\theta}-ku\}}. \end{aligned} \tag{18}$$

The function (18) reflects that, given the state of (θ, x), different parts of the boundary may be reached. In order to illustrate this, we consider a specific example of the value function, when $(\theta, x) \in \Omega_3^2$. In this case (18) becomes

$$\begin{aligned} &F^{AR}(\theta,x) \\ &= \left(\frac{\lambda}{r+\lambda}\right)^3 V(\theta+3u) + \frac{z_0 x}{r-\alpha}\left[1-\left(\frac{\lambda}{r+\lambda-\alpha}\right)^3\right] \\ &+ \left[\frac{x}{b^A(\theta)}\right]^{\frac{r+\lambda}{\alpha}} i(\theta)\left[\frac{1}{(r+\lambda)} - \frac{1}{(r+\lambda-\alpha)}\right] \\ &+ \left[\frac{x}{b^{AR}(\theta+u)}\right]^{\frac{r+\lambda}{\alpha}} i(\theta+u)\sum_{m=0}^{1}\frac{\lambda(-\alpha)^{-m}}{m!} \\ &\times\left[\frac{1}{(r+\lambda)^{2-m}} - \frac{1}{(r+\lambda-\alpha)^{2-m}}\right]\left[\ln\left[\frac{x}{b^{AR}(\theta+u)}\right]\right]^m \\ &+ \left[\frac{x}{b^R(\theta+2u)}\right]^{\frac{r+\lambda}{\alpha}} i(\theta+2u)\sum_{m=0}^{2}\frac{\lambda^2(-\alpha)^{-m}}{m!} \\ &\times\left[\frac{1}{(r+\lambda)^{3-m}} - \frac{1}{(r+\lambda-\alpha)^{3-m}}\right]\left[\ln\left[\frac{x}{b^R(\theta+2u)}\right]\right]^m, \end{aligned} \tag{19}$$

where $i(\theta) = z_1\left(\theta - \frac{u\lambda}{r}\right) - rI$.

The value function in (19) consists of five terms. The first two terms are similar to the value function in Section 3. They reflect the possibility to reach the stopping region after three jumps, and the revenues that the firm earns until that point. The last three terms account for the fact that different boundaries may be hit. If for $(\theta, x) \in \Omega_3^2$ no jump in technology occurs and x declines sufficiently then the Add boundary is hit, which is reflected by the third term. If before investment there occurs only one jump in technology, which brings it to the level $\theta + u$, and x declines sufficiently, the firm will enter the stopping region through the Add/Replace boundary from Ω_1^1. This is captured by the fourth term. Finally, if after two jumps, i.e. when the firm will achieve the level of technology $\theta + 2u$, it will reach the Replace boundary by decline in x and enter the stopping region from Ω_1^R. This is lastly reflected in the fifth term.

4.2. Non-monotonic threshold boundary

We start out with presenting the threshold boundary in Proposition 7 in case the boundary is non-monotonic in θ. In addition, in the proof of this proposition, we demonstrate the formal intuition behind the non-monotonic behavior.

Proposition 7. *Let us assume that the condition of the Proposition 5 is not satisfied. For $\theta > \bar{\theta}$, the optimal exercise boundary is given by*

$$\begin{aligned} b(\theta) = &b^A(\theta)\chi_{\{\bar{\theta}\le\theta<\tilde{\theta}_1\}} + b^{AC_A}(\theta)\chi_{\{\tilde{\theta}_1\le\theta<\hat{\theta}-u\}} + b^{AC_R}(\theta)\chi_{\{\hat{\theta}-u\le\theta<\tilde{\theta}_2\}} \\ &+ b^{AR}(\theta)\chi_{\{\tilde{\theta}_2\le\theta<\hat{\theta}\}} + b^R(\theta)\chi_{\{\theta\ge\hat{\theta}\}}, \end{aligned} \tag{20}$$

where b^{AC_A} and b^{AC_R} are implicitly defined in Appendix A.8 by (42) and (43), respectively. Moreover, $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are defined as $b^A(\tilde{\theta}_1) = b^{AR}(\tilde{\theta}_1 + u)$ and $b^{AR}(\tilde{\theta}_2) = b^R(\tilde{\theta}_2 + u)$, respectively, which are explicitly given in Appendix A.8 by (40) and (41).

Proof of Proposition 7. *See Appendix A.8 for the proof.*

In Fig. 8 we provide an illustration for this case. Here the sets Ω^R, Ω^A and Ω^{AR} have a similar interpretation as in the previous case. For values (θ, x) in Ω^R or Ω^A we know beforehand that upon investing the firm will replace or add, respectively. Contrarily, for values (θ, x) in Ω^{AR} the decision whether to add or replace will depend on the realization of the process (θ, x).

Whereas Figs. 3 and 6 have in common that the threshold curve $b(\theta)$ is monotonically increasing, it is evident from Fig. 8 that the threshold curve has a decreasing part when the condition of

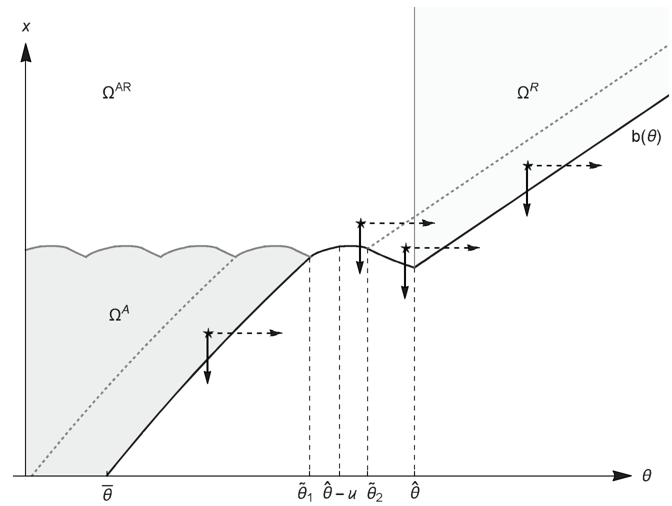


Fig. 8. Illustration of the optimal threshold boundary and the regions Ω^R, Ω^A and Ω^{AR}.

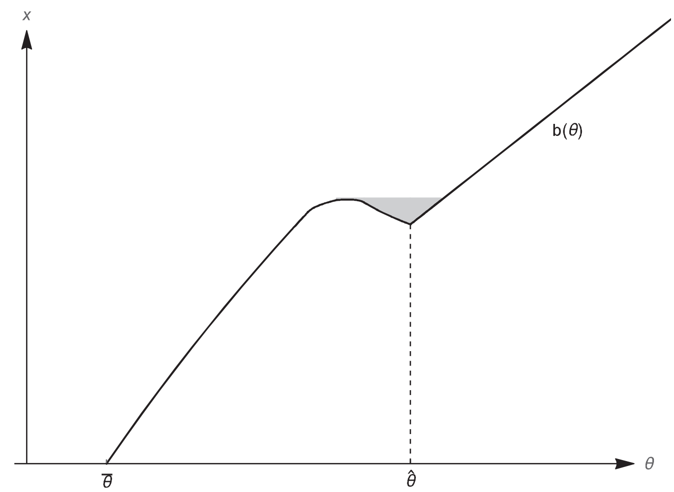


Fig. 9. Illustration of the hysteresis region (shaded area).

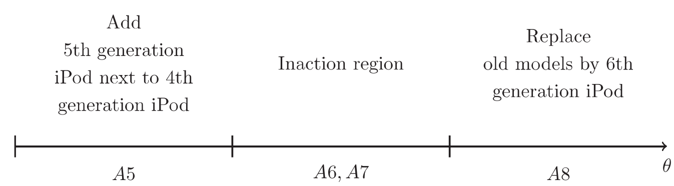


Fig. 10. iPod technology timeline.

Proposition 5 is not satisfied. In this case, the optimal threshold curve is a piece-wise function consisting of five regions, as defined in Proposition 7 and illustrated in Fig. 8. The derivation of the value function in this case can be done analogously to the previous case, i.e. we need to track the transitions of the bivariate process (θ, x) and take into account all possibilities for it to enter the stopping region. The difference is that now we need to take into account that instead of three potential boundary functions, in the non-monotonic case we have five. The value function, thus, has to reflect that either of the five different parts of the boundary can be reached from above.

In the monotonic case, the regions adjacent to the investment boundary in the continuation region have one common feature. More specifically, they all represent the situation when the stopping region is one jump away. This does not need to hold when the boundary is non-monotonic. In this case a continuation region two or more jumps away from the stopping region, can also be connected with the investment boundary. For example, in Fig. 8 the starred value of (θ, x) positioned between $\hat{\theta} - u$ and $\tilde{\theta}_2$ in Ω^{AR} illustrates such a case. One jump brings the firm to the region Ω^R, which is still part of the continuation region. However, at the same time only a relatively small vertical movement is needed to reach the investment region. In the former case the optimal strategy upon investment is to replace the old product. In the latter case the optimal strategy is to add the new product to the product portfolio, which still contains the old one.

In addition, Fig. 8 shows three more possible transitions of the bivariate process (θ, x) from the continuation region to the stopping region. Like with the monotonic case the starred values all have in common that they are one jump away from the stopping region. In particular, for $\theta \in \left[\widehat{\theta}, +\infty\right)$ the threshold can be reached by a decline in x or a jump in θ, leading to the strategy of replacing the old product by the new one (represented by the rightmost arrows). The same applies to $\theta \in \left[\bar{\theta}, \tilde{\theta}_1\right)$, but in this case the optimal strategy is to add the new product to the firm's portfolio (represented by the leftmost arrows). If $\theta \in \left[\tilde{\theta}_2, \widehat{\theta}\right)$ the threshold is reached by a decline in x then the firm will add the new product to its portfolio, whereas if the threshold is reached by a jump in θ then the firm will replace the old product by the new one (represented by the arrows starting between $\tilde{\theta}_2$ and $\widehat{\theta}$). Note that in this case the subset of the continuation region containing the starting points which are one jump away from the stopping region is a union of two disjoint sets.

As can be seen in Fig. 9, due to the decreasing behavior of the threshold curve, a hysteresis region arises. This region is illustrated by the shaded area. In the hysteresis region the firm in fact refrains from investing, while at the same time for a smaller level of the technology parameter adopting the new technology is optimal applying the add strategy. This at first sight counter-intuitive innovation strategy makes sense, because in the hysteresis region the firm wants to keep the option open to apply the replace strategy with a higher technology level instead of jointly producing the established and the new product once the decrease of x has resulted in reaching the threshold boundary. This is beneficial here, because θ is relatively large, implying that the cannibalization effect, given by $2\eta q_0 q_1 \theta x$, will make the add strategy unattractive compared to replace.

Furthermore, there exist real world examples of firms refraining from innovating in a way predicted by our inaction region. Consider, for example, one of the key technology components of Apple iPod Touch – the processor. In 2010, Apple introduced the 4th generation iPod based on the Apple A4 chip. In 2012, 5th generation iPods based on A5 chip was added to Apple's product portfolio. In the following years, Apple developed more advanced processors A6 and A7, which, however, were never brought to iPod Touch products. Instead, after 3 years they introduced the 6th generation iPod based on A8 in 2015, which completely replaced the old versions. This behavior can be explained by the presence of the inaction region predicted by our model, as illustrated in Fig. 10. In particular, in the years 2013 and 2014, Apple already had access to a better technology, i.e. A6 and A7 chips, however, it decided not to introduce a new version of iPod. At the same time, for A5 it used the *add* strategy, whereas for A8 chip they used the *replace* strategy.

5. Conclusion

This paper studies the product innovation option of an incumbent. Initially the firm is active in selling its established product. However, the firm's profit associated with the established product decreases over time due to the facts that, in case of durable goods, over time the consumer base declines because more consumers have already bought the product, and other firms introduce products that compete with the established product of the focal firm. For this reason the firm wants to change its product portfolio by innovating. Due to technological progress the firm is able to introduce a better product if it innovates later. Therefore, the firm faces the following trade-off. If it innovates early, it stops the profit decline associated with its established product early, but the adopted new product only incrementally improves the established one. If the firm innovates late, it is able to launch a product of much better quality, but at the same time it has to deal with a long period of declining demand of its established product. Depending on the realizations of the technological breakthroughs, the paper determines the firm's optimal product innovation timing. We show that such an innovation can occur either right at the moment of a technological breakthrough, or some time after such an event. In the latter case the firm adopts the new product, because demand of the established product has reduced too much. We further obtain an explicit expression for the value of the firm, reflecting a weighted average of all possible innovation patterns.

A product innovation implicitly creates another problem: what to do with the old product? To analyze this problem we explicitly distinguish between two strategies: introducing the new product while keeping the old product alive (add strategy), or abolishing the old product when introducing the new one (replace strategy). Producing both products at the same time generates a cannibalization effect. We find that when the cannibalization effect is substantial, a hysteresis effect arises. That particular case provides the following managerial insight. Waiting for new technology arrivals could be beneficial in case the currently best available new technology is not advanced enough to provide a complete replacement to the one in place, yet it has developed enough to cannibalize a considerable market share of the existing product, which also makes the add strategy not profitable.

This model is a first step considering this innovation problem and taking into account *add* and *replace* decisions. With this model we can already explain various real world phenomena as we illustrated in the main text. However, extending the model to multiple technology adoption decisions would bring our model even more closer to reality.

Another interesting idea for future research is to add a fixed operational cost to the model. Then, if the current instantaneous profit keeps on decreasing and the innovative product is not profitable enough to adopt it while incurring a high innovative cost, at some point the option to exit will be "in the money". The exit option is explicitly taken into account in Hagspiel et al. (2016), but their model does not consider ongoing technological innovations and the possibility to produce the established and the innovative product at the same time as we have in the present paper.

This paper also provides a solid basis for further interesting extensions. Here we think about determining the optimal production capacity associated with launching the new product, including the innovation strategy of competitors and how to optimally react to that, and to incorporate learning effects regarding the production processes of the different products.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2020.01.056.

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