Estimation of Oscillatory Mode Activity from PMU Measurements

Hallvar Haugdal, Kjetil Uhlen
Department of Electric Power Engineering
Norwegian University of Science and Technology
Trondheim, Norway
{hallvar.haugdal, kjetil.uhlen}@ntnu.no

Daniel Müller, Hjörtur Jóhannsson
Department of Electrical Engineering
Technical University of Denmark
Kgs. Lyngby, Denmark
{danmul, hjjo}@elektro.dtu.dk

***Abstract*—We propose a method for estimating the activity of oscillatory modes in power systems. The frequencies and mode shapes of the modes of interest are assumed to be known beforehand, either from linear modal analysis or from empirical mode estimation methods, and are used in combination with measurements from Phasor Measurement Units to estimate the instantaneous mode excitation in terms of amplitude and phase. The estimation is carried out using non-linear least squares to fit a set of curves to the measured data. Combining mode shapes with measured data allows the activity to be estimated from only a low number of consecutive measurement snapshots, resulting in a problem of low computational complexity that can be solved fast enough for the method to run online.**

The purpose of estimating the mode activity is, firstly, to contribute to increased situational awareness and facilitate methods that build further upon this information, and secondly, to be able to synthesize signals that can serve as input to controllers for power oscillation damping. It is expected that using this excitation measure will result in a more robust controller that is less prone to disturbances and noise.

***Index Terms*—Empirical modal analysis, non-linear least squares, power oscillations, wide area monitoring and control**

I. Introduction

Inter-area oscillations often appear in power systems consisting of areas of generation that are connected by weak interties. Increasing the intertie transfer generally reduces the damping of the oscillations [1], potentially leaving the full thermal capacity of the lines unexploited. Low-damped or negatively damped oscillations might also appear when the system is operated in a critical state, e.g. following severe faults or disturbances [2]. This motivates the development of monitoring applications that increase our knowledge about oscillatory modes of the power system. More accurate information allows the system to be operated closer to the stability limits, and enables us to take proper action in case of critical, growing oscillations.

Further measures can be implemented to ensure secure operation of the grid; A Power Oscillation Damper (POD) can be introduced, whose basic functioning is to measure the activity of oscillatory modes, and to apply control action attempting to damp the oscillations. The control action could for instance be applied through generator excitation systems, SVCs or HVDC links.

To measure the activity of the oscillatory mode, serving as the input to the controller, the voltage angle can be measured using a Phasor Measurement Unit (PMU) at a location where the oscillatory mode to be damped has a high observability. The observability of a mode at a specific location can be obtained by performing an offline model based modal analysis. The right eigenvectors resulting from the modal analysis determine the relative amplitude and phase of the oscillatory mode within a measurement, often referred to as the observability mode shape. Choosing the location with the highest amplitude therefore ensures a good observability of the mode.

In [3], a Wide Area POD is described, which uses the angle difference between two widely separated locations as input to the POD, which controls the voltage setpoint of a Static VAr Compensator. In [4] a Phasor POD is described, which controls a Thyristor Controlled Series Capacitor in order to mitigate oscillations. The frequency of the particular mode to be damped is assumed to be known on beforehand in this case, and a phasor which represents the amplitude and phase of the mode is estimated and used to modulate the reactance reference of the device.

The method presented in this paper can be viewed as the first development steps towards an extended version of the Phasor POD described above. Given recent development of empirical modal analysis [5]–[9], from which estimates of mode shapes can be obtained in near real-time, we assume that not only the frequency of the mode to be damped is known, but also that the corresponding observability mode shape is prior knowledge. We further use this in combination with a high number of input measurements to estimate the mode phasors.

The scope of this paper is solely demonstrating the proposed phasor estimation method. The obvious next step, which will constitute the continuation of this work, is to use this information to synthesize control signals to be injected with a suitable phase at suitable locations to damp oscillations. Ultimately, this is expected to result in a power oscillation damper that applies control action only when all the generators involved in the observability mode shape oscillate with the particular frequency of the mode, and also with the relative phase and amplitude as determined by the mode shape.

Estimating the mode activity is carried out by generating a set of functions that represent curves one would expect to observe given the frequency and observability mode shape of the mode. Further, non-linear least squares is used to find the amplitude and phase of the mode that best fits with the measurements.

The work presented is a continuation of previous research on power oscillation monitoring. In [8], [9], it is shown that the frequency and observability mode shape of oscillatory modes can be estimated within seconds after standing oscillations appear in measurements. This motivates the development of monitoring and control applications that make use of this information.

The current stage of development of the method is described in the succeeding sections, specifically: In Section II relevant modal analysis theory is presented, Section III presents the proposed method, Section IV presents the results obtained when testing the method on simulated data and on PMU measurements, and finally Section V and VI contains discussion and conclusions.

II. Modal Analysis

Analysing electromechanical oscillations in power systems is often approached using modal analysis on a linearized system model. The linearized state space equations, describing the dynamics of the system perturbed by small disturbances at a given operating point, can be written as [10]:

$$\Delta\dot{\mathbf{x}}(t) = \mathbf{A}\Delta\mathbf{x}(t) + \mathbf{B}\Delta\mathbf{u}(t) \tag{1}$$

$$\Delta\mathbf{y}(t) = \mathbf{C}\Delta\mathbf{x}(t) + \mathbf{D}\Delta\mathbf{u}(t) \tag{2}$$

Performing an eigendecomposition of the system matrix $\mathbf{A}$, we get a set of eigenvalues and left- and right eigenvectors. The eigenvalues characterize the modes of the system, of which the ones with a non-zero imaginary part are related to oscillatory modes. Further, the left and right eigenvectors describe controllability- and observability mode shapes for each of the modes. The time evolution of the system states can be written as a function of the modes of the system:

$$\Delta\mathbf{x}(t) = \sum_{j=1}^{n_s} \mathbf{\Phi}_j z_j(t) \tag{3}$$

Here, $\mathbf{\Phi}_j$ is the observability mode shape (right eigenvectors) of mode j, z_j is the time variation of mode j and n_s is the order of the system. For oscillatory modes, the right eigenvectors determine the amplitude and phase with which mode j is observed in the states. We can also define the mode shape describing how the modes are observed in each of the measurements:

$$\Delta\mathbf{y}(t) = \mathbf{C}\Delta\mathbf{x}(t) = \sum_{j=1}^{n_s} \mathbf{C}\mathbf{\Phi}_j z_j(t) \tag{4}$$

Assuming that the system is excited by an arbitrary input $\Delta\mathbf{u}$, the time variation of mode j can be written as [11]

$$z_j(t) = \int_0^t e^{\lambda_j(t-\tau)} \mathbf{\Psi}_j \mathbf{B}\Delta\mathbf{u}(\tau) d\tau \tag{5}$$

where λ_j and $\mathbf{\Psi}_j$ are the eigenvalue and left eigenvector corresponding to mode j. The product of left eigenvector and input matrix $\mathbf{B}$ describes how the mode j is excited by the disturbance $\Delta\mathbf{u}$ and therefore indicates the controllability of mode j by the inputs related to the disturbance.

Since the eigenvalues corresponding to oscillatory modes are complex, the series $z_j(t)$ is also complex for these modes. One can therefore describe the mode activity in terms of amplitude $|z_j(t)|$ and phase $\arg z_j(t)$. The proposed method described in the following section allows to extract this information by estimating the amplitude and phase of a given mode from PMU measurements.

III. Amplitude and Phase Estimation using Non-Linear Least Squares

Applying to intuition, one could consider the following simple example: We assume a power system consisting of a number of generators divided into two main areas, where the dominant mode is the inter area mode related to oscillations between the areas. We assume that the frequency and observability mode shape of the dominant mode is available. Following a disturbance, one would from studying only a few consecutive PMU snapshots be able to say that the inter area mode was excited if the generators in the first area accelerated while those in the other area decelerated. An even clearer indicator would be that the magnitude of the acceleration of each individual generator was according to the corresponding magnitudes in the observability mode shape of the mode.

This idea forms the basis of the proposed method where we use the mode shape and a set of measurements to formulate a fitting problem that is solved using non-linear least squares minimization [12].

We start by collecting the last n voltage angle snapshots from m PMU locations in the matrix $\mathbf{Y}$ as follows:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & & \ddots & \\ y_{m1} & y_{m2} & & y_{mn} \end{bmatrix} \tag{6}$$

The indexing is such that the element y_{ik} corresponds to the measurement obtained at location i at time t_k. We assume that each measurement closely resembles the state corresponding to the rotor angle of one generator. (This assumes deployment of PMUs in major substations. If necessary, the rotor angle could also be estimated based on measured voltage and current.)

We further assume that the frequency f_j of the mode is known. For the observability mode shape, we need to know the elements which correspond to the states we are measuring, i.e. the states corresponding to rotor angles. The elements of the mode shape, which are complex for oscillatory modes, can be written in terms of amplitude and angle:

$$\mathbf{C}\mathbf{\Phi}_j = \begin{bmatrix} a_{1j}\angle\delta_{1j} \\ a_{2j}\angle\delta_{2j} \\ \vdots \\ a_{mj}\angle\delta_{mj} \end{bmatrix} \tag{7}$$

In the following, we focus on only one mode and skip the mode index j. From the frequency and mode shape of the mode, we construct the following set of functions, which represent signals one would expect to observe given activity of the mode:

$$\mathbf{g}(A,\theta,t) = \begin{bmatrix} g_1(A,\theta,t) \\ g_2(A,\theta,t) \\ \vdots \\ g_m(A,\theta,t) \end{bmatrix} \tag{8}$$

The individual functions are defined as follows:

$$g_i(A,\theta,t) = Aa_i \cos\left(\omega t + \delta_i + \theta\right) \tag{9}$$

Here, $\omega = 2\pi f$ is the angular frequency of the mode of interest.

Further, we evaluate the functions at the same time instants as the measurements in $\mathbf{Y}$ are obtained:

$$\mathbf{S}(A,\theta) = \left[\mathbf{g}(A,\theta,t_1) \quad \mathbf{g}(A,\theta,t_2)\ldots\mathbf{g}(A,\theta,t_n)\right] \tag{10}$$

The matrix $\mathbf{S}$ now has the same dimension as $\mathbf{Y}$, where each element is a function that can be fitted to the corresponding measured data in $\mathbf{Y}$. The residuals are given by

$$\mathbf{R}(A,\theta) = \mathbf{Y} - \mathbf{S}(A,\theta) \tag{11}$$

Finally, we perform a non-linear least squares minimization to find the amplitude A and angle θ that minimizes the error, given by

$$e = \sum_{i=1}^{m}\sum_{k=1}^{n} r_{ik}(A,\theta)^2 \tag{12}$$

where $r_{ik}(A,\theta)$ denotes the elements of $\mathbf{R}(A,\theta)$.

The minimization is performed for each new PMU snapshot received at t_k, resulting in one estimate of amplitude and phase of $z(t_k)$ per time stamp.

IV. Results

The above described method is tested on simulated data from the Kundur Two-Area System and on measured PMU-data from the nordic power grid.

A. Simulated data from the Kundur Two-Area System

The Kundur Two-Area system, described in [1] is often used as a benchmark system for small signal stability related methods. The system consists of four generators distributed among two areas which are connected by a relatively weak intertie. The system has three electromechanical oscillatory modes; one negatively damped interarea mode whith a frequency of $0.61\,\text{Hz}$, and two local modes with a frequency around $1\,\text{Hz}$ and a damping of about $9\,\%$.

In order to demonstrate the performance of the proposed method, a time series based on the linearized system model is generated where the modes are excited by two separate disturbances: The first disturbance occurs at $t = 1\,\text{s}$, where a step change of $0.01\,p.u.$ is applied to the voltage reference of generator 1. Additionally the voltage reference of generator 3 is increased by $0.02\,p.u.$ at $t = 6\,\text{s}$.

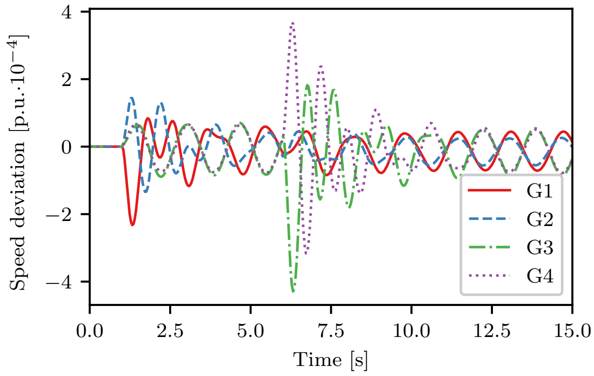

Fig. 1. The speed response obtained from the linear simulation of a model of the Kundur Two-Area System is shown.

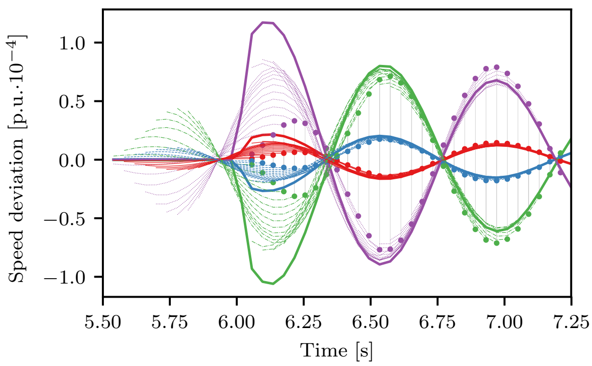

Fig. 2. The oscillations caused by the second local mode, involving G_3 & G_4, are shown (where the same colors as those in Fig. 1 are used for each generator). The four thick lines shows the oscillations for each generator computed using the linear model. The thin lines show the fitted curves for each time instant, and the markers correspond to the instantaneous estimate at that time.

The simulated time series is shown in Fig. 1, where the sampling frequency is $25\,\text{Hz}$. The method is applied, using 15 consecutive PMU snapshots in the curve fitting problem, resulting in matrices $\mathbf{Y}$ and $\mathbf{S}$ both having dimensions 4×15. An impression of the curve fitting is presented in Fig. 2 where the estimate of local mode 2 during the second disturbance is compared with the response of the linear system. The estimates of amplitude and phase are shown in Fig. 3, along with the mode shapes corresponding to each of the modes.

The estimation converges to the value obtained from the linearized system showing that it is possible to accurately estimate the phase and magnitude of the mode activity. As might be expected, the amplitude and phase are estimated more accurately with an increasing level of excitation. From both the linear system and the estimate it is apparent that the two disturbances affect both the amplitude and phase of all the modes.

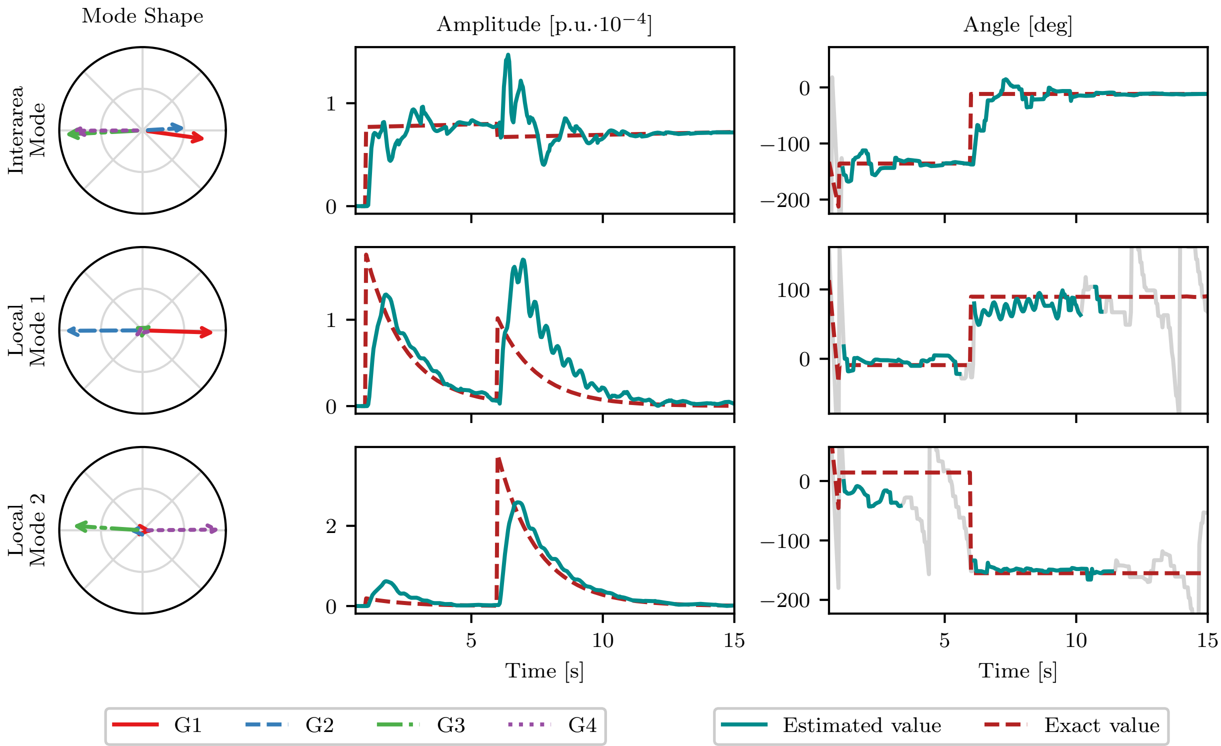

Fig. 3. The result from applying the method to simulated time series from the Kundur Two-Area System is shown. Each row corresponds to one mode, where the mode shapes (which are prior knowledge) are shown to the left. The estimated amplitude $|z_j|$ and phase $\arg z_j$ are shown in cyan for each mode in the two plots to the right, while the values from the linear model are shown in dashed red lines. The estimated phase is colored in grey in regions where the estimated amplitude of the mode is approximately zero.

B. Measured PMU-data from the Nordic Power Grid

The measured PMU-data is shown in the first plot of Fig. 4, captured with a sampling frequency of 10 Hz. The voltage angle is measured at seven locations distributed across Norway. The mode being analyzed describes the severe oscillations starting around $t = 280\,\text{s}$, lasting until around $t = 400\,\text{s}$. The frequency and mode shape of the oscillations are estimated beforehand using the empirical method described in [9]. The frequency of the oscillation is 1.04 Hz in average, and the observability mode shape is shown in Fig. 5.

Applying the proposed method using five consecutive snapshots in the fitting problem (i.e. the matrices $\mathbf{Y}$ and $\mathbf{S}$ both have dimensions 7×5) yields the result shown in the two lower plots of Fig. 4. The second plot shows the estimated amplitude of the oscillations, while the last plot shows the estimated angle (where the regions where the amplitude is approximately zero is colored in grey).

V. Discussion

The results indicate that the method provides reasonable estimates of amplitude and phase of oscillatory modes. The accuracy generally increases with the level of excitation.

From the result when testing the method on measured PMU data, two main conclusions can be drawn: Firstly, the estimated amplitude and phase appear to fit well with what one would expect by observing the measured time series. Both the amplitude and angle estimates are relatively smooth during the course of the oscillations. This indicates that a reliable control signal can be synthesized based on these quantities. Given knowledge of the controllability mode shape, synthesized signals could be injected with a suitable phase at suitable locations to contribute to the damping of the oscillations.

Secondly, the estimated angle and amplitude contributes to situational awareness. Observing the third plot, we see that the oscillations are increasing in amplitude until about $t = 310\,\text{s}$, where they start decreasing. There is also a distinct turn in the trajectory of the angle at about the same time. The continually decreasing angle before $t = 310\,\text{s}$ indicates that the frequency of oscillations is slightly lower than the mean frequency used as input to the method, while the opposite is true for the period after $t = 310\,\text{s}$. Observing that the trajectories of both the amplitude and angle make distinct turns at the same time could indicate that some remedial action was applied that changed the operating point. The same observation is also made in [8].

One potential application building further upon the information provided by this method could be to estimate information about controllability mode shapes. From (5), we see that the activity of a mode $z_j(t)$ can be determined from convolutions of the mode impulse response and input signals. Assuming that we also measure the inputs, the result from the proposed method could be used to estimate the elements of the matrix $\mathbf{\Psi}_j\mathbf{B}$. In practice, this could be used to indicate at what locations and with which phase control signals could be injected to mitigate the oscillations, or point out locations

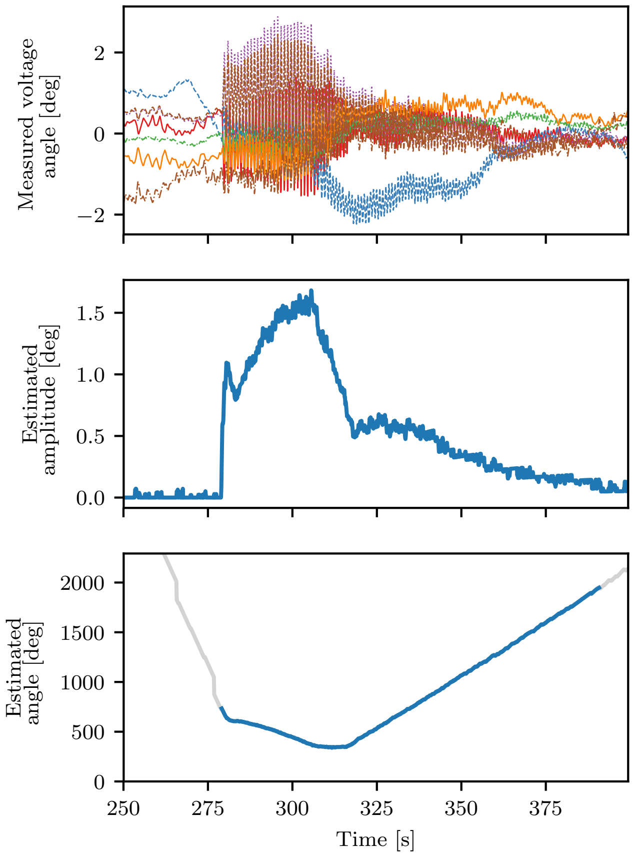

Fig. 4. The first plot shows the measured voltage angle time series. The oscillations last from about $t = 280\,\mathrm{s}$ to $t = 380\,\mathrm{s}$. The second and third plots show the estimated amplitude and phase, respectively. The phase is colored in grey in time regions where the amplitude is approximately zero.

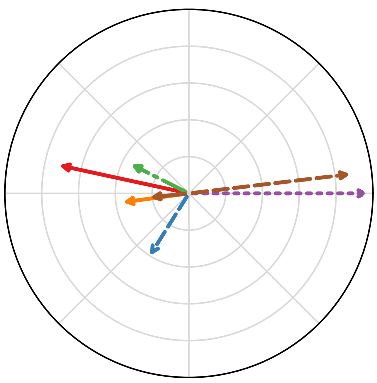

Fig. 5. The observability mode shape describing the phase and amplitude of the oscillations captured in the PMU measurements analysed in Fig. 4 is shown.

where disturbances should be minimized.

The computation time for each instantaneous estimate in the results above is in the range of milliseconds to tens of milliseconds. The current implementation uses a general non-linear least squares minimization. It is expected that significant improvement can be achieved regarding the speed by customizing the minimization algorithm, such that there should be no issue with running the method online.

It should be mentioned that accurate frequency and observability mode shapes might be difficult to obtain in real life. Assessing the performance of the method subject to inaccurate frequency and mode shape estimates will constitute an important part of the continuation of this research.

VI. Conclusion

The results from applying the method to simulated and measured PMU data are promising. This indicates that the fundamental idea, i.e. combining measurements with prior knowledge of oscillatory modes in the form of frequency and mode shapes, is feasible. At the current stage of development, the activity estimates appear suitable for generating control signals for damping of power oscillations, and also arguably contributes to situational awareness. Further development will reveal the potential benefit of using the output of the proposed method as input in power oscillation dampers, and the potential for estimating controllability from the mode activity.

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