RANDOM WAVE-INDUCED CURRENT IN SHALLOW WATER USING DEEP WATER WIND AND WAVE STATISTICS

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Abstract

This article addresses the random wave-induced current in shallow water based on deep water wind and wave statistics, where the wave-induced current is expressed in terms of the deep water seastate wave parameters significant wave height and mean zero-crossing wave period. The average statistical properties of the random wave-induced current in shallow water expressed in terms of the mean value and the standard deviation are presented. Results are exemplified by using long-term wind statistics from the Northern North Sea and long-term wave statistics from the same ocean area. Overall, it appears that there is agreement between the results based on these inputs from wind and wave statistics. The presented analytical method should be useful for making preliminary estimates of the random wave-induced drift in shallow water within seastates using either available deep water wind statistics or deep water wave statistics, which enhances the possibilities for assessing further the wave-induced current in, for example, near-coastal zones.

Keywords: Random surface gravity waves; Wave-induced current; Shallow water; Deep water wind statistics; Deep water wave statistics; Coastal zones.

1. Introduction

Near-coastal zones are in general characterized by their shallow water depths and flow conditions caused by ocean surface gravity waves and currents. Coastal flow circulation models are commonly used tools in near-coastal work, and these models usually include parameterizations of many flow mechanisms, for example, wave-induced current, also referred to as Stokes drift, as well as the Stokes transport, i.e. the result of integrating the Stokes drift over the water column. The Stokes drift is the mean Lagrangian velocity obtained from the water particle trajectory in the direction of wave propagation. In deep and finite water depths the Stokes drift has its maximum at the surface and decreases with the depth below the surface, while in shallow water it is independent of the elevation in the water column. Then, the Stokes transport in shallow water is obtained by multiplying the Stokes drift with the water depth. Further details of the wave-induced drift is given in e.g. Dean and Dalrymple (1984).

A recent comprehensive review of wave-induced drift is given by van den Bremer and Breivik (2018). They mainly identify three categories of applications of Stokes drift. First, in the coastal zone it contributes to wave-induced sediment transport and migration of sand bars as well as to drive an opposite return flow transport. Second, it it is important in exploring Langmuir turbulence, i.e. the formation of counter-rotating vortices in vicinity of the ocean surface characterized by bands of foam, floating seaweed and debris. Third, Stokes drift in combination with other current components such as those driven by wind, density gradients and tides, contributes to transport of e.g. heat, salt, plankton, fish eggs and larvae, as well as pollutants like oil spills, contaminated ballast water from ships, plastic and microplastic litter. It also contributes in air-sea mixing processes which occur across the ocean surface, as well as to environmental loading on structures and drift of sea ice. Thus, although the Stokes drift is the weakest of the current components it plays an important role (see van den Bremer and Breivik (2018) for more details).

Much attention has recently been given to random wave-induced drift. Among others the present author and co-authors have contributed, see e.g. Myrhaug and Ong (2015), Myrhaug et al. (2016, 2018, 2019), that also give brief reviews of the literature up to that dates. Myrhaug and Ong (2015) provided a simple analytical method of calculating the wave-induced drift due to individual long-crested random waves on mild slopes. Myrhaug et al. (2016) presented the statistical properties of Stokes drift for individual long-crested random waves in moderate intermediate water depth including spectral bandwidth effects. Myrhaug et al. (2018, 2019) addressed the Stokes transport in layers in the water column within seastates for deep water long-crested random waves based on wind and wave statistics, respectively. Furthermore, Paprota et al. (2016) presented results from an experimental study of wave-induced mass transport, while Paprota and Sulisz (2018) developed a theoretical model of the kinematics of water particles and mass transport beneath nonlinear waves generated in a closed flume and verified their results against the data from Paprota et al. (2016). Grue and Kolaas (2017) provided experimental results from wave tank measurements on particle paths and drift velocity in steep waves at finite water depth. Song et al. (2018) derived a theoretical statistical distribution of wave-induced drift for long-crested random waves in finite water depth.

The main purpose and the novelty of the present article is to demonstrate how deep water wind and wave statistics can be used to obtain statistical properties of random wave-induced drift in shallow water. Thus, these aspects are new compared with the other papers referred to.

The article contains an Introduction, followed by Section 2 giving the theoretical background and the general formulation of the wave-induced drift in shallow water used here given in terms of the deep water seastate wave parameters significant wave height and mean zero-crossing wave period. Section 3 gives the results for a Pierson-Moskowitz deep water wave amplitude spectrum with mean wind speed statistics from a deep water location in the Northern North Sea as input. Section 4 provides the results using a joint distribution of significant wave height and spectral wave steepness from the same ocean area as the wind statistics. Section 5 gives examples of results, by first providing the validity of results for shallow water (Section 5.1), and then presenting results based on wind and wave statistics (Section 5.2). Section 6 provides discussion of the results. Summary and conclusions are given in Section 6. Overall, the presented method provides an estimation of shallow water random wave-induced drift based on offshore wind statistics or wave statistics, that can be linked to this drift mechanism. Thus, it is demonstrated how the present analytical method can be used to make preliminary estimates of wave-induced drift in near-coastal zones.

2. Background

By following Myrhaug and Ong (2015) (see their Eq. (3)), the mean (time-averaged) Lagrangian mass transport for shallow water waves for a wave component with amplitude a_n is given as (by neglecting terms of higher order than O(a_n^2))

$$\bar{u}_{Ln} = \frac{a_n^2}{2h}\sqrt{\frac{g}{h}} \tag{1}$$

where h is the water depth and g is the acceleration due to gravity. One should notice that the Lagrangian mass transport for shallow water waves is independent of the elevation in the water column and the wave frequency. Often the Lagrangian mass transport is referred to as (surface) Stokes drift, which for shallow water waves represents a current constant over the depth. The magnitude of the wave-induced drift versus the other current components is discussed in Section 5.2.

The wave amplitude is related to the wave amplitude spectrum as $a_n^2 = 2S(\omega_n, h)\Delta\omega$ where $\Delta\omega$ is a constant frequency interval between the frequencies ω_n, and $S(\omega_n, h)$ is the wave amplitude spectrum for long-crested random waves in shallow water, which according to Eq. (18) in Myrhaug and Ong (2015) is given as $S(\omega_n, h) = (h/2g)\,\omega_n^2\, S(\omega_n)$ where $S(\omega_n)$ is the wave amplitude spectrum for long-crested random waves in deep water. Shallow water waves are valid for $\omega_n < (\pi/10)\sqrt{g/h}$, whilst deep water waves are valid for $\omega_n > \sqrt{\pi g/h}$ (Dean and Dalrymple, 1984). Further discussion on the validity of the results for shallow water is given in Section 5.1. It should be noted that no energy is lost in this transformation from deep to shallow water (see Section 6 for further discussion).

With an infinite number of wave components, the Stokes drift in shallow water within a seastate of random waves is

$$U_s = \frac{1}{2\sqrt{gh}} \int_0^\infty \omega^2\, S(\omega)\, d\omega = \frac{m_2}{2\sqrt{gh}} \tag{2}$$

where m_2 is the second deep water spectral moment. Here the nth order deep water spectral moments are defined as $m_n = \int_0^\infty \omega^n\, S(\omega) d\omega; n = 0,1,2,----$. Now $T_2 = 2\pi\sqrt{m_0/m_2}$ is the mean zero-crossing wave period in deep water, and $H_s = 4\sqrt{m_0}$ is the significant wave height in deep water, which can be combined to give

$$m_2 = \frac{\pi^2}{4}\left(\frac{H_s}{T_2}\right)^2 \tag{3}$$

Thus, Eqs. (2) and (3) give

$$U_s = \frac{\pi^2}{8\sqrt{gh}}\left(\frac{H_s}{T_2}\right)^2 \tag{4}$$

Similarly, the total mean (time- and depth-averaged) mass transport (often referred to as the Stokes transport) in shallow water for individual random waves with amplitude a_n is given as (see Eq. (5) in Myrhaug and Ong (2015))

$$M_n = \frac{\rho}{2}\sqrt{\frac{g}{h}}\, a_n^2 \tag{5}$$

where ρ is the fluid density. From Eqs. (1) and (5) it is noticed that $M_n / \rho = h\bar{u}_{Ln}$. Thus, with an infinite number of wave components, the Stokes transport in shallow water within a seastate of random waves is

$$\frac{M}{\rho} = h\, U_s = \frac{1}{2}\sqrt{\frac{h}{g}}\, m_2 = \frac{\pi^2}{8}\sqrt{\frac{h}{g}}\left(\frac{H_s}{T_2}\right)^2 \tag{6}$$

As a result, U_s and M in shallow water are given for known wave conditions in deep water, i.e. which can be specified in terms of a deep water wave amplitude spectrum, or in terms of the seastate wave parameters H_s and T_2 in deep water. This will be further discussed in the next sections; Section 3 demonstrates how the Pierson-Moskowitz (*PM*) deep water wave amplitude spectrum can be used, while Section 4 uses H_s and the spectral wave steepness s_m as input. It should be noted that the spectral peak period T_p has been used in other papers on the topic, but this is not essential since T_2 and T_p are related. For the *PM* spectrum $T_2 = 0.71T_p$; for other model spectra other relationships exist (Tucker and Pitt, 2001), while for measured (or computed) wave data empirical relationships can be obtained.

The present model is a so-called point model, i.e. depending on the local wave parameters regardless of the history of the waves as they propagate from deep to shallow water. Further aspects of this model are discussed in Section 6.

3. Results for a *PM* spectrum with mean wind speed statistics as input

Here the *PM* spectrum with the mean wind speed at the 10 m elevation above the sea surface, U_{10}, as the parameter is chosen as the deep water wave amplitude spectrum. It should be noted that the *PM* spectrum is valid for fully developed wind waves, but as a compromise between simplicity and accuracy it is adopted here to demonstrate how wind statistics can be applied analytically. By following Tucker and Pitt (2001), the *PM* spectrum is

$$S(\omega) = \frac{A}{\omega^5} \exp\left(-\frac{B}{\omega^4}\right) \tag{7}$$

where the spectral moments for $n < 4$ are

$$m_n = \frac{1}{4} A\, B^{\frac{n}{4}-1}\, \Gamma(1-\frac{n}{4}) \tag{8}$$

Here Γ is the gamma function, $A = \alpha\, g^2$, $\alpha = 0.0081$, $B = 1.25\,\omega_p^4$, $\omega_p = 2\pi / T_p$, where ω_p and is the spectral peak frequency. Originally the *PM* spectrum for deep water waves was given with the mean wind speed at the 19.5 m elevation above the sea surface, $U_{19.5}$, as the parameter. From Eq. (8) it follows that

$$m_2 = \frac{1}{4} \alpha\, g^2 \sqrt{\pi}\; 1.25^{-0.5}\, \omega_p^{-2} \tag{9}$$

With $U_{10} = 0.93 U_{19.5}$, Tucker and Pitt (2001) give $T_p = 0.785\, U_{10}$, which substituted in Eq. (9) gives

$$m_2 = 0.00482\, U_{10}^2 \tag{19}$$

Thus, Eqs. (2) and (6) give, respectively,

$$U_s = \frac{0.00241}{\sqrt{gh}} U_{10}^2 \tag{11}$$

$$\frac{M}{\rho} = 0.00241\sqrt{\frac{h}{g}}\ U_{10}^2 \tag{12}$$

Now U_s and M/ρ in shallow water can be determined from known mean wind speed statistics at a deep water location. Here the Johannessen et al. (2001) cumulation distribution function (*cdf*) of U_{10} from a deep water location in the Northern North Sea is adopted in order to illustrate how long-term wind speed statistics can be used to assess U_s and M/ρ in shallow water. This *cdf* represents 1-hourly values of U_{10} covering the period 1973 – 1999, given by the two-parameter Weibull model

$$P(U_{10}) = 1 - \exp\left[-(\frac{U_{10}}{\theta})^{\beta}\right];\ U_{10} \geq 0 \tag{13}$$

with the Weibull parameters $\theta = 8.426\,m/s$ and $\beta = 1.708$.

Here the expected value and the variance of U_s and M/ρ are considered, which requires the calculation of the expected value and the variance of U_{10}^2, i.e. $E\left[U_{10}^2\right]$ and $Var\left[U_{10}^2\right]$, respectively. For a Weibull-distributed variable (Bury, 1975)

$$E\left[U_{10}^n\right] = \theta^n\,\Gamma(1+\frac{n}{\beta}) \tag{14}$$

$$\sigma^2\left[U_{10}^n\right] \equiv Var\left[U_{10}^n\right] = E\left[U_{10}^{2n}\right] - \left(E\left[U_{10}^n\right]\right)^2 \tag{15}$$

For the *cdf* in Eq. (13), $E\left[U_{10}^2\right] = 77.0\,\mathrm{m}^2/\mathrm{s}^2$ and $\sigma\left[U_{10}^2\right] = 90.8\,\mathrm{m}^2/\mathrm{s}^2$, i.e. the coefficient of variation is $\sigma\left[U_{10}^2\right]/E\left[U_{10}^2\right] = 1.18$. Substitution of this in Eqs. (11) and (12) gives, respectively,

$$E\left[U_s\right] = \frac{0.186}{\sqrt{gh}}\ \text{(m/s)} \tag{16}$$

$$E\left[\frac{M}{\rho}\right] = 0.186\ \frac{h}{\sqrt{gh}}\ (\mathrm{m^2/s}) \tag{17}$$

Thus, the mean value ± 1 standard deviation (*SD*) interval of the factor 0.186 is 0 to 0.405.

Furthermore, according to Tucker and Pitt (2001), $H_s = 0.0246\ U_{10}^2$ for a *PM* spectrum, which then gives $E[H_s] = 0.0246\, E\left[U_{10}^2\right] = 0.0246 \cdot 77.0\,\mathrm{m} = 1.89\,\mathrm{m}$.

4. Results for a joint *pdf* of H_s and s_m

Here the joint probability density functions (*pdfs)* of H_s and U_s as well as H_s and M/ρ are obtained from the joint *pdf* of H_s and the spectral wave steepness s_m provided by Myrhaug (2018). The spectral deep water wave steepness is defined as $s_m = H_s/((g/2\pi)T_2^2)$, and thus, $H_s/T_2^2 = (g/2\pi)s_m$. By defining $v = (H_s/T_2)^2$, v can be expressed in terms of s_m as $v = (g/2\pi)H_s s_m$.

The joint *pdf* of H_s and v is obtained from the joint *pdf* of H_s and s_m given in Appendix A by following the same procedure as in Myrhaug (2018), i.e. by a change of variables from H_s, s_m to H_s, v, which gives

$$p(H_s, v) = p(v\,|\,H_s)\,p(H_s) \tag{18}$$

where $p(H_s)$ is given in Eq. (A2). This change of variable from s_m to v only affects $p(s_m\,|\,H_s)$ since $s_m = (2\pi/g)H_s^{-1}\,v$, and by using the Jacobian $|ds_m/dv| = (2\pi/g)H_s^{-1}$, this gives the following conditional lognormal *pdf* of v given H_s

$$p(v\,|\,H_s) = \frac{1}{\sqrt{2\pi}\sigma_v v}\exp\left[-\frac{1}{2}\left(\frac{\ln v - \mu_v}{\sigma_v}\right)^2\right] \tag{19}$$

where μ_v and σ_v^2 are the mean value and the variance, respectively, of $ln\,v$, given by

$$\mu_v = \mu_{s_m} - ln\left(\frac{2\pi}{g} H_s^{-1}\right) \tag{20}$$

$$\sigma_v^2 = \sigma_{s_m}^2 \tag{21}$$

where μ_{s_m} and σ_{s_m} are given in Eqs. (A4) and (A5), respectively.

Here results will be exemplified by considering $E\left[H_s\right]$, $E\left[v \mid H_s\right]$ and the coefficient of variation $R = \sigma\left[v \mid H_s\right] / E\left[v \mid H_s\right]$ given by (Bury, 1975)

$$E\left[H_s\right] = \varepsilon_h + \zeta_h \, \Gamma\left(1 + \frac{1}{\theta_h}\right) \tag{22}$$

$$E\left[v \mid H_s\right] = \exp\left(\mu_v + \frac{1}{2}\sigma_v^2\right) \tag{23}$$

$$R = \left(e^{\sigma_v^2} - 1\right)^{1/2} \tag{24}$$

Now the results are exemplified by using the wave data from the deep water location at Utsira in the Northern North Sea as given in Appendix A. First, by substituting the Weibull parameters in Eq. (A6) in Eq. (22), the result is $E\left[H_s\right] = 2.11\,\text{m}$ (which compared with $E\left[H_s\right] = 1.89\,\text{m}$ based on the *PM* spectrum in Section 3, is about 10 percent larger). Second, by using this value of H_s in Eqs. (A4) and (A5) together with the coefficients in Eqs. (A7) and (A8), substitution in Eqs. (20), (21), (23) and (24) gives, respectively,

$$\mu_v = -1.925\,\text{m}^2/\text{s}^2 \tag{25}$$

$$\sigma_v^2 = 0.0936\ \text{m}^4/\text{s}^4 \tag{26}$$

$$E\left[v \mid E\left[H_s\right] = 2.11\,\text{m}\right] = 0.153\,\text{m}^2/\text{s}^2 \tag{27}$$

$$R = 0.313 \tag{28}$$

Thus, it follows from Eq. (27) and Eqs. (4) and (6) that

$$E\left[U_s \mid E[H_s] = 2.11\,\text{m}\right] = \frac{1}{\sqrt{gh}} \cdot 0.189\,(\text{m/s}) \tag{29}$$

$$E\left[\frac{M}{\rho} \mid E[H_s] = 2.11\,\text{m}\right] = \frac{h}{\sqrt{gh}} \cdot 0.189\,(\text{m}^2/\text{s}) \tag{30}$$

where the mean value $\pm 1\,SD$ interval of the factor 0.189 is 0.130 to 0.248.

It appears that the results from the wind statistics (with $E[H_s] = 2.11\,\text{m}$ and Eqs. (16), (17)) and the wave statistics (with $E[H_s] = 1.89\,\text{m}$ and Eqs. (29), (30)) are consistent, i.e. that for a given water depth in shallow water there is agreement between the results obtained from the present deep water wind and wave statistics. This is addressed further in Section 5.2. However, it should be noted that the coefficient of variation is smaller based on wave statistics (R = 0.313) than that based on wind statistics (R = 1.18), which is a result of inherent features of the distributions.

5. Examples of results

To our knowledge no data exist in the open literature for random wave-induced drift in shallow water to compare with, and therefore examples of estimating the wave-induced drift based on the results in Sections 3 and 4 are provided in Section 5.2. However, first the validity of the results due to the shallow water approximation is given.

5.1 Validity of results for shallow water

By following Hedges (1995), the validity is given in terms of the wave steepness and the Ursell number. First, the upper limit of the wave steepness for linear regular waves in deep water is 0.04. By taking this upper limit to be valid for the spectral wave steepness s_m defined in Section 4, then

$$s_m = \frac{H_s}{\frac{g}{2\pi} T_2^2} \leq 0.04 \tag{31}$$

Second, the Ursell number $U_R = ka / (kh)^3 \leq 0.5$ for linear regular waves, where k is the wave number, and a is the wave amplitude. For linear harmonic waves propagating over a gently sloping flat bottom towards a straight coastline at normal incidence, the wave amplitude in shallow water is derived assuming that the energy flux is constant, i.e. $a = a_\infty / (2kh)^{1/2}$ (Dean and Dalrymple, 1984), where $a_\infty = H_\infty / 2$ is the wave amplitude and H_∞ is the wave height in deep water. By using the dispersion relationship in shallow water, $k = 2\pi / (T\sqrt{gh})$ where T is the wave period, and replacing H_∞ and T with H_s and T_2, respectively, the Ursell number criterion in shallow water becomes

$$U_R = 0.062 \frac{H_s T_2^{5/2}}{h^{9/4}} \leq 0.5 \tag{32}$$

From Eq. (31) it follows that $T_2 \geq 4 H_s^{1/2}$. By substituting the lower value $T_2 = 4 H_s^{1/2}$ in Eq. (32), it follows that the results in shallow water is valid for

$$h \geq 1.85 H_s \tag{33}$$

However, it should be noted that if $T_2 > 4 H_s^{1/2}$, the results will be valid for a larger water depth.

5.2 Results based on wind and wave statistics

First, for wind statistics from Section 3, $E[H_s]=1.89\,\text{m}$, which substituted in Eq. (33) gives $h \geq 3.5\,\text{m}$. Substitution of $h=3.5\,\text{m}$ in Eqs. (16) and (17) gives, respectively,

$$E[U_s]=0.032\,\text{m/s} \tag{34}$$

$$E\left[\frac{M}{\rho}\right]=0.112\ \text{m}^2/\text{s} \tag{35}$$

Furthermore, the expected value $\pm 1SD$ interval of the factor 0.186 is 0 to 0.405, which for $E[U_s]$ gives the interval 0 to 0.070 m/s, and for $E[M/\rho]$ the interval is 0 to 0.244 m^2/s (these results are summarized in Table 1).

Second, for wave statistics from Section 4, $E[H_s]=2.11\,\text{m}$, which substituted in Eq. (33) gives $h \geq 3.9\,\text{m}$. Substitution of $h=3.9\,\text{m}$ in Eqs. (29) and (30) gives, respectively,

$$E\left[U_s \mid E[H_s]=2.11\,\text{m}\right]=0.031\,\text{m/s} \tag{36}$$

$$E\left[\frac{M}{\rho} \mid E[H_s]=2.11\,\text{m}\right]=0.121\,\text{m}^2/\text{s} \tag{37}$$

Similarly, the expected value $\pm 1SD$ interval of the factor 0.189 is 0.130 to 0.248, which for Eq. (36) gives the interval 0.021 m^2/s to 0.041 m^2/s, and for Eq. (37) the interval is 0.083 m^2/s to 0.159 m^2/s (as summarized in Table 1). Estimation obtained from wind statistics using this water depth, i.e. by substituting $h=3.9\,\text{m}$ instead of $h=3.5\,\text{m}$ in Eqs. (16) and (17) are also summarized in Table 1, showing that the results are slightly changed compared with those obtained for $h=3.5\,\text{m}$. If the standard deviations are taken into account, the range of values associated with U_s and M/ρ based on wave statistics are within the range of values associated with these quantities based on wind statistics. Thus, this demonstrates that there is agreement between the results based on wind statistics and wave statistics from the same deep water ocean area as discussed in Section 4. One should notice that the presented statistical results for the wave-induced drift are valid for this ocean area.

As mentioned previously, the wave-induced drift is weak compared to other current components. The wind-driven component is often approximated by 2 % of U_{10}, where the wave-induced current will be included in this estimate (Faltinsen, 1990). Now, according to Eqs. (13) and (14), $E\left[U_{10}\right] = 7.52\,\mathrm{m}/s$, and with this wind speed the wind-induced current is estimated to be 0.15 m/s where the wave-induced current of 0.03 m/s is included. Typical current speeds along the Norwegian coast are in the range 0.2 – 0.5 m/s, with maximum current speeds exceeding 1 m/s (Sætre, 2007).

6. Discussion

This section provides further aspects of the present point model as well as some comments on this approach versus a procedure which commonly is used. For calculating the random wave-induced current in shallow water, common practice would be to start with available data on joint statistics of H_s and T_2 (or other characteristic wave periods); preferably within directional sectors at a nearby location offshore (in deep water). The next step would be to apply an appropriate physics-based wave transformation model (e.g. SWAN (Holthuijsen, 2007)) containing nonlinear wave transformation processes involving air-sea interaction and resulting waves, flow and transport processes between deep water and nearshore locations, to obtain the joint statistics of H_s and T_2 at the shallow water site; then finally to use this as input for calculating the wave-induced drift. In general this practice would also include shallow water regions exposed to sea states with combined wind waves and swell waves from different directions. Here an alternative is presented providing a simple analytical method which can be used to make first-order estimates of random wave-induced drift based on given values of H_s and T_2 in deep water from observed wind and wave statistics. The wind and wave statistics are based on in-situ data obtained from the same ocean area. The transition from deep water to the shallow water site is assumed to be smooth, neglecting wave energy dissipation effects over changing bed conditions with varying intermediate and shallow water depths. The feature of a point model also implies that the dependence on the spatial coordinates are discarded; it only depends on the local water depth and the local wave conditions via the transformed deep water wave amplitude spectrum for long-crested waves in terms of the sea state parameters H_s and T_2. Consequently, several effects affecting the assessment of the wave-induced drift are neglected, i.e. dissipation due to bottom friction and wave breaking; that the wave field is inhomogeneous; from where the waves are coming and the location of the assessment point; return flows from dissipation effects which in turn will affect the local wave amplitude spectrum. It should also be noted that the wave-induced longshore current is not included. However, a simple analytical approach as the one provided here should be appropriate for making quick estimates for some preliminary engineering studies. However, the estimates so obtained need be validated by other models and field data before used in any practical application design cases. Under field conditions such an easily accessible and simple tool might also be useful as there is usually limited time and access to computational resources. Although the presented results are based on a specific wave amplitude spectrum, wind speed distribution and joint (H_s, s_m) distribution, the method can also be applied for other wave amplitude spectra, wind speed distributions and joint distributions of sea state wave parameters, or for a given deep water wave amplitude spectrum including directional spreading effects. However, in such cases numerical calculations are most probably required. It is important, however, to assess the accuracy of this approach versus common practice, which is only possible to quantify by comparing with such methods over a wide parameter range, also including a sensibility analysis of the results regarding the assumptions considered, but this is beyond the scope of this short article.

7. Summary and conclusions

A simple analytical method for estimating random wave-induced current in shallow water using deep water wind and wave statistics is provided. It is based on the wave-induced drift for regular waves and is applied for long-crested random waves by transformation of deep water waves to shallow water. The deep water wave conditions are given in terms of the seastate wave parameters significant wave height and mean zero-crossing wave period. Consequently, the wave-induced drift in shallow water is expressed in terms of these deep water seastate wave parameters. Average statistical properties of the random wave-induced drift expressed in terms of the mean value and the standard deviation are presented. First, results are provided by applying a Pierson-Moskowitz model wave spectrum for deep water wind waves with the mean wind speed at the 10 m elevation above the sea surface as the parameter, and using long-term mean wind speed statistics from a deep water location in the Northern North Sea as input. Second, results are given for seastates described by a joint distribution of significant wave height and spectral wave steepness representing swell, wind waves and combined swell and wind waves from the same ocean area as the wind statistics. Examples of results using these inputs from long-term wind statistics and wave statistics typical for field conditions are provided.

The main conclusions are:

1. Overall, it appears that the present assessment of random wave-induced current in shallow water based on long-term wind statistics from a deep water location in the Northern North Sea is in agreement with that using long-term wave statistics from the same deep water ocean area.

2. The strength and novelty of this work is that it demonstrates how the presented analytical method can be applied to make preliminary estimates of the random wave-induced current in shallow water within seastates using either available deep water wind statistics or deep water wave statistics, which enhances the possibilities of assessing further the wave-induced drift in e.g. near-coastal zones. The present formulation may also serve as a useful parameterization of random wave-induced drift. However, the method should be validated by comparing with other models and field data before a firm conclusion regarding its validity can be made.

Appendix A. Myrhaug (2018) joint *pdf* H_s and s_m

The joint *pdf* of H_s and s_m provided by Myrhaug (2018) is given as

$$p(H_s, s_m) = p(s_m \mid H_s)\, p(H_s) \tag{A1}$$

where $p(H_s)$ is the marginal *pdf* of H_s given by the following three-parameter Weibull *pdf*

$$p(H_s) = \frac{\theta_h}{\zeta_h}\left(\frac{H_s - \varepsilon_h}{\zeta_h}\right)^{\theta_h - 1} \exp\left[-\left(\frac{H_s - \varepsilon_h}{\zeta_h}\right)^{\theta_h}\right];\ H_s \ge \varepsilon_h \tag{A2}$$

where θ_h, ζ_h, ε_h are the Weibull parameters. Furthermore, $p(s_m \mid H_s)$ is the conditional *pdf* of s_m given H_s, given by the following lognormal *pdf*

$$p(s_m \mid H_s) = \frac{1}{\sqrt{2\pi}\sigma_{s_m} s_m} \exp\left[-\frac{1}{2}\left(\frac{ln\ s_m - \mu_{s_m}}{\sigma_{s_m}}\right)^2\right] \tag{A3}$$

where μ_{s_m} and $\sigma_{s_m}^2$ are the mean value and the variance, respectively, of $ln\, s_m$, given as

$$\mu_{s_m} = ln\left(\frac{H_s}{g/2\pi}\right) - 2(a_1 + a_2\ H_s^{a_3}) \tag{A4}$$

$$\sigma_{s_m}^2 = 4(b_1 + b_2\, e^{b_3 H_s})^2 \tag{A5}$$

where a_1, a_2, a_3, b_1, b_2 and b_3 are coefficients. In this study the joint *pdf* of H_s and s_m representing deep water swell, wind waves and combined swell and wind waves conditions at the Utsira location (in the Northern North Sea) on the Norwegian continental shelf is chosen to exemplify the results. For these data the Weibull parameters in Eq. (A1) and the coefficients in Eqs. (A4) and (A5) are given as (see Myrhaug (2018) for more details)

$$\zeta_h = 1.50 \text{ m}, \theta_h = 1.15, \ \varepsilon_h = 0.679 \text{ m} \tag{A6}$$

$$a_1 = 0.933, \ a_2 = 0.578, \ a_3 = 0.395 \tag{A7}$$

$$b_1 = 0.0550, \ b_2 = 0.336, \ b_3 = -0.585 \tag{A8}$$

Figure A1 shows the isocontours of $p(H_s, s_m)$, and it appears that the shape of the *pdf* is nearly symmetric with respect to $s_m \approx 0.05$ for $H_s > 5$ m. Furthermore, the peak value $p_{\max} = 15.7$ is located at $H_s = 0.85$ m and $s_m = 0.022$.

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Table 1. Test case results of estimates obtained from deep water wind parameters versus those obtained from deep water wave parameters. The results for waves are given for the shallow water depth h = 3.9 m, while the results for wind are given for h = 3.5 m/ h = 3.9 m. Here SD = standard variation.

Wind		Waves	
$E[H_s]$(m)	1.89	$E[H_s]$(m)	2.11
$E[U_s]$(m/s)	0.032/0.030	$E[U_s \mid E[H_s]=2.11\mathrm{m}]$(m/s)	0.031
$E[U_s]\pm 1SD$(m/s)	0, 0.070/0.065	$E[U_s \mid E[H_s]=2.11\mathrm{m}]\pm 1SD$(m/s)	0.021, 0.041
$E\left[\frac{M}{\rho}\right](\mathrm{m}^2/\mathrm{s})$	0.112/0.117	$E\left[\frac{M}{\rho} \mid E[H_s]=2.11\mathrm{m}\right](\mathrm{m}^2/\mathrm{s})$	0.121
$E\left[\frac{M}{\rho}\right]\pm 1SD(\mathrm{m}^2/\mathrm{s})$	0, 0.244/0.255	$E\left[\frac{M}{\rho} \mid E[H_s]=2.11\mathrm{m}\right]\pm 1SD(\mathrm{m}^2/\mathrm{s})$	0.083, 0.159

Figure caption

Figure A1. Isocontours of $p(H_s, s_m)$.