

# Reservoir Characterization in Under-balanced Drilling with Nonlinear Moving Horizon Estimation **with manual and automatic control conditions**

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## Abstract

This paper discusses the state estimation of the Under-Balanced Drilling (UBD) system in the presence of parametric uncertainties. During this process, the production indices of oil and gas from the reservoir into the well **in the manual and automatic control conditions** are estimated by employing the nonlinear Moving Horizon Estimation (MHE) based on a low-order lumped (LOL) model. **The LOL model has a low computational load and is suitable for reservoir characterization during UBD operations.** The estimation algorithms are tested by using a difficult scenario which is created by the OLGA **multiphase** flow simulator. The simulation results indicate that the nonlinear MHE has a high performance and can identify the production indices of gas and oil. Moreover, the method has the capability to diagnose the rapid variation of the production constant in different conditions, such as working with a manual or automatic controller. The results of the scenario with the swift change in the production index of gas demonstrate that the nonlinear MHE has a higher performance than the **Unscented Kalman Filter (UKF)**. The effect of uncertainties and errors in the reservoir and well parameters on the nonlinear MHE is evaluated. It is revealed that the presence of parametric uncertainty in the reservoir pore pressure can

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significantly affect the estimators' performance.

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## 1. Introduction

Generally, for hydrocarbon production, conventional oil is more cost-effective than unconventional oil such as shale oil, tight oil, or oil sands. Thus, conventional oil continues to play a **significant** role in oil resources. In recent years, the demand for advanced technologies to increase hydrocarbon recovery and decrease the cost of oil and gas production is growing. Managed pressure drilling (MPD)[1, 2, 3], dual gradient drilling[4, 5], and Under-Balanced drilling (UBD) are some advanced drilling technologies that can handle reservoirs with challenging features such as deep water, high pressure, high-temperature, depleted reservoir, and reservoir with narrow pressure windows. Furthermore, these methods have the capability to solve drilling problems such as differential sticking, and lost circulation. The UBD process is an advanced drilling system that tries to maintain the bottom-hole circulating pressure less than the formation pressure. The positive difference between the hydrostatic pressure of the well and the reservoir pressure, which is known as the mud column, is the major barrier for safety in conventional drilling. Also, another essential barrier for safety is the Blow-Out Preventer (BOP). In the UBD operation, since there is no mud column barrier, the BOP is the main barrier. Thus, it is necessary to focus on the control and monitoring of this system.

In recent years, reservoir characterizations have been widely considered[6, 7, 8, 9, 10, 11, 12, 13, 14], and the main attentions are on the estimation of the reservoir properties under the assumption that the total flow rate from the reservoir is completely known[15]. In[9], a method is proposed for the estimation of both reservoir pore pressure and the reservoir permeability. However, there are notable uncertainties in the estimation of the reservoir pore pressure,

such as the fluid flow behavior variations in the bottom-hole and the annulus. In[12], the performance of the extended Kalman filter (EKF), the ensemble Kalman filter and the unscented Kalman filter (UKF) is assessed for estimation of the state and the production index based on a low order model in the UBD process. In[16], an ensemble Kalman filter is proposed based on a drift-flux model to estimate the uncertain parameters of a two-phase flow model. In[15, 13] for estimation of the reservoir pore pressure and the reservoir permeability during the bottom-hole pressure excitation, an ensemble Kalman filter and an off-line nonlinear least squares (LS) approach are applied by using the Levenberg-Marquardt optimization algorithm. In[17], an adaptive observer is designed for parameters and state estimation of the well based on a drift-flux model. This model consists of three partial differential equations (PDE's) that are developed based on the mass conservation in each phase and a combined momentum equation. Furthermore, it guarantees that the parameters and state estimates converge to their **actual** values. In[6], an EKF is used for the estimation of state and production index. The model uses an empirical slip law instead of the flow-regime predictions. Also, the paper proposes an algorithm which is combined with off-line calibration introduced in[15]. In[18], a UKF and an EKF are designed to estimate the state, production, and slip parameters by using the bottom-hole pressure and liquid and gas flow rates. The used model is a simplified drift-flux model. In[19], a Lyapunov-based adaptive observer, a recursive least squares (RLS) estimator, and a UKF are presented based on a low order lumped (LOL) model. The total mass of gas and liquid is used for the estimation of state and parameters in the UBD operation. Moreover, the performance of the adaptive estimators is assessed using a pipe connection scenario. In[20], the performance of the adaptive observer, which is proposed in[19], is evaluated for state and **parameter** estimation based on the choke and bottom-hole pressures. The method is evaluated using the drift-flux model for typical drilling. In[21], a Lyapunov-based adaptive observer is compared with a joint unscented Kalman filter (UKF) based on a low order lumped model. The real-time measurements of choke and bottom-hole pressures are obtained from

the OLGA simulator. The robustness of the adaptive observers with respect to the model parameters uncertainties such as the reservoir and well parameters is investigated.

60 This paper is an extended version of work published in[22], which presents the design of the Nonlinear MHE based on the LOL model. In[22], a Nonlinear Moving Horizon Estimator based on the LOL model is proposed to estimate the total mass of gas and liquid in the annulus and geological properties of the reservoir in the UBD operation. The considered scenario was a pipe connection  
65 procedure using a simple simulation model which is simulated by MATLAB, and the process and observer model are the same.

State and **parameter** estimation of dynamical systems is a critical challenge in control theory (e.g., see[23, 24, 25]). Nonlinear estimators such as unscented Kalman filter (UKF), extended Kalman filter (EKF), nonlinear adaptive ob-  
70 server, nonlinear MHE, etc. have been designed to handle this challenge. The nonlinear MHE that uses a window of the most recent measurements has the capability to deal with some issues such as weak persistent excitation, model uncertainty, and measurement noise[26, 27, 28, 29, 30].

Estimation of production indices of gas and liquid during UBD operations  
75 by using the nonlinear MHE is the **primary** purpose of this paper. The nonlinear MHE is implemented based on the LOL model. The bottom-hole and choke pressures are the measurements that are obtained by different simulated scenarios with the OLGA high-fidelity simulator. The OLGA dynamic multi-phase flow simulator is one of the well-known benchmarks for hydraulic models  
80 in drilling technologies. The main advantage of this paper is using the OLGA multiphase flow simulator that utilizes partial differential equations. However, in [22], the process and observer models are **precisely** the same. The ability of the algorithm to diagnose and track fast changes in the production index in different conditions is a criterion for performance evaluation. There are **vari-**  
85 **ous** conditions, such as working with a manual or automatic controller. The considered controller for bottom-hole pressure in the UBD operation is the PI controller. The performance of the nonlinear MHE and UKF is compared by

implementation on two challenging scenarios:

- 1) Changing the production index of gas with the manual controller
- 90 2) Changing the production index of gas with the automatic controller

The results show that the nonlinear MHE has a better performance than the UKF for the scenario with a rapid change in the production index. The robustness of estimator with respect to the reservoir and well parameters uncertainties is demonstrated.

95 The organization of this paper can be summarized as follows: Section 2 summarizes a LOL model which is based on mass and momentum balances in the UBD system and the model of the reservoir. Section 3 describes the nonlinear Moving Horizon Estimation and joint UKF for the estimation of state and parameters of the LOL model by employing the obtained measurements  
100 from the OLGA simulator. Simulation results are presented in Section 4. The simulations are carried out for different challenging conditions, such as working with a manual or automatic controller. Finally, Section 5 concludes the paper.

## 2. Modeling

The model has a crucial role in the success and speed of MHE because the  
105 prediction is performed based on this model. During drilling, since there are influx materials from the reservoir such as oil, gas, water, and rock cuttings, the UBD operation should be considered as a multiphase flow system. The modeling of UBD operation can be done through a distributed model or a simplified LOL model. Distributed models for multiphase flow in the UBD system are developed  
110 based on some nonlinear hyperbolic partial differential equations (PDE). **This model is difficult to solve both analytically or numerically because the source terms reflecting interphase drag are stiff and this can lead to significant problems in the numerical computation[31].** Generally, the distributed models are utilized for simulation purposes, but not for the model-based observer and controller  
115 design. **The LOL model is perhaps the simplest method for modeling multiphase flow in UBD.** This simplified model comprises the primary dynamics of UBD

systems in the presence of some simplifying assumptions. A LOL model is suitable for conventional model-based control design methods and can be used for prediction and estimation in an observer and controller algorithms. There are some critical assumptions for the modeling of the UBD system that are presented below:

- Ideal gas behavior
- Simplified choke valve model for gas, mud, and liquid
- No mass transfer between phases
- Isothermal condition and constant system temperature
- Constant mixture density with respect to pressure and temperature.

### 2.1. Low-Order Lumped (LOL) model

In this model, the mud, oil, and water are considered as a single-phase liquid. The mass equations relevant to gas and liquid in the annulus are written based on isothermal mass and momentum balances (see Fig. 1)[19].

$$\dot{m}_g = w_{g,d} + w_{g,res}(m_g, m_l) - \frac{m_g}{m_g + m_l} w_{out}(m_g, m_l) \quad (1)$$

$$\dot{m}_l = w_{l,d} + w_{l,res}(m_g, m_l) - \frac{m_l}{m_g + m_l} w_{out}(m_g, m_l) \quad (2)$$

where  $m_g$  and  $m_l$  are the total mass of gas and liquid, respectively. The liquid and gas phases are considered incompressible and compressible, respectively. The free volumes are occupied by the gas phase,  $w_{g,d}$  and  $w_{g,res}$  are the mass flow rates of gas from the drill string and the reservoir, and  $w_{l,d}$  and  $w_{l,res}$  are the mass flow rates of liquid from the drill string and the reservoir, respectively. The total mass outflow rate is

$$w_{out} = K_c Z \sqrt{\frac{m_g + m_l}{V_a}} \sqrt{p_c - p_{c0}}, \quad (3)$$

where  $Z$  is the control signal related to the choke valve opening that belongs to the interval  $(0, 1]$ .  $K_c$  is a constant that is determined by the choke valve

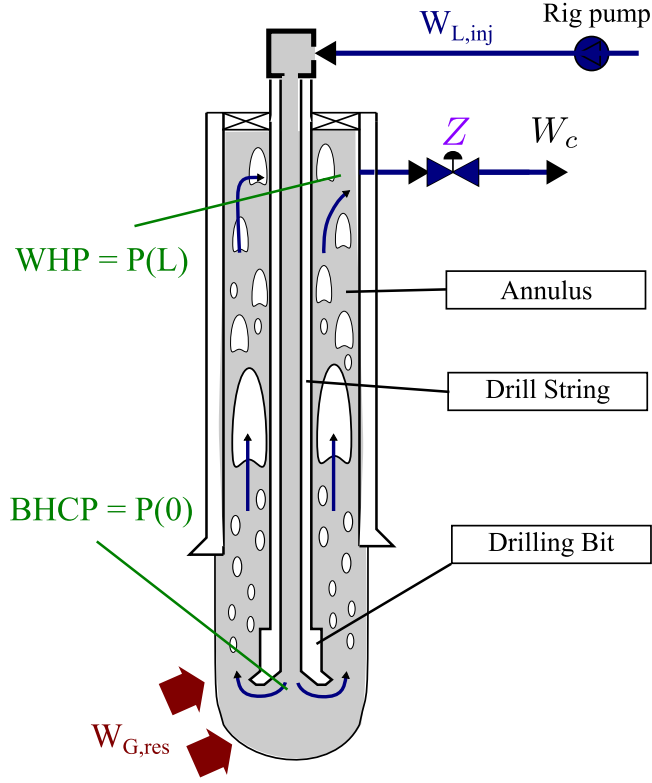


Figure 1: Schematic of a UBD system[32]

characteristics. The atmospheric pressure is denoted by  $p_{c0}$ , and  $V_a$  is the annulus volume. The choke valve pressure  $p_c$  is given by the ideal gas equation

$$p_c = \frac{RT}{M_{gas}} \frac{m_g}{V_a - \frac{m_l}{\rho_l}}, \quad (4)$$

where  $R$  is the gas constant,  $\rho_l$  is the density of the liquid,  $T$  and  $M_{gas}$  are the average temperature and the molecular weight of the gas, respectively. The bottom-hole pressure is obtained by the following equation

$$p_{bh} = p_c + \frac{(m_g + m_l)g \cos(\Delta\theta)}{A} + \Delta p_f, \quad (5)$$

where  $A$  is the cross-sectional area of the annulus, and  $g$  is the gravitational constant. The average angle between the positive direction of gravity and flow of the well is denoted by  $\Delta\theta$ , and the friction pressure loss in the well  $\Delta p_f$  is

computed by

$$\Delta p_f = K_f(w_{g,d} + w_{l,d})^2, \quad (6)$$

where  $K_f$  is the friction coefficient.

## 2.2. Reservoir flow

The mass flow from the reservoir into the well for both phases can be modeled by

$$w_{g,res} = \begin{cases} K_g(p_{res} - p_{bh}), & \text{if } p_{res} > p_{bh} \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

$$w_{l,res} = \begin{cases} K_l(p_{res} - p_{bh}), & \text{if } p_{res} > p_{bh} \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

130 where the production indices of the gas and liquid are indicated by  $K_g$  and  $K_l$ , and  $p_{res}$  is the reservoir pore pressure.

The measurements and information of formation properties such as porosity, gamma-ray, the thickness of bed, conductivity, acoustic velocity, etc. are collected by using logging while drilling (LWD)[33]. Generally, these measurements have some delays that can negatively affect automation systems. These measurements and geological properties of the reservoir are **essential** and are utilized for some purposes, such as the controllers' design, fault diagnosis systems, and safety applications. Thus, it is necessary to estimate these parameters and states on-line by employing appropriate Measurement While Drilling (MWD).  
 140 During UBD operations, reservoir engineers try to estimate the reservoir pressure by employing some flow rate tests. So long as the drilling is done through one reservoir, the reservoir pressure is considered stationary[34, 35]. The reservoir pore pressure is assumed to be known, and the estimation of the unknown production index of gas  $K_g$  and liquid  $K_l$  from the reservoir into the well is  
 145 necessary. Some parameters, such as the friction coefficient  $K_f$  and the choke constant  $K_c$  are supposed to be known because it is possible to estimate them



off-line[34]. Other parameters such as density, temperature, and well volume can be evaluated by well data.

### 3. Nonlinear Estimation

150 In this section, the nonlinear Moving Horizon Estimation is described for the simultaneous estimation of state and parameters using the LOL model in the UBD operation. Afterward, a joint unscented Kalman filter is explained for state and **parameter** estimation.

The equations (1)-(2) relevant to the LOL model can be formulated with the discrete state-space equation

$$x_k = f(x_{k-1}, u_{k-1}) + q_k, \quad (9)$$

where  $x_k$  is the state vector,  $u_k$  is the known input vector,  $q_k \sim N(0, Q_k)$  is assumed the zero-mean Gaussian process noise representing unknown disturbances and model uncertainties. The discrete measurement equation is given by

$$y_k = h(x_k) + r_k, \quad (10)$$

$$h(x_k) = [p_c, p_{bh}]^T, \quad (11)$$

155 where  $y_k$  is the output vector,  $r_k \sim N(0, R_k)$  is the zero-mean Gaussian measurement noise. The measurement and input variables are listed in Table 1.

#### 3.1. Nonlinear Moving Horizon Estimation

The nonlinear MHE calculates the unmeasured states and unknown parameters in each time step  $k$  by minimization of a constrained objective function 160 over the time horizon of the most recent measurements.

At time  $t$ , the information vector  $I_t$  consists of  $N + 1$  last measurements and  $N$  last inputs

$$I_t = \text{col}(y_{t-N}, \dots, y_t, u_{t-N}, \dots, u_{t-1}), \quad (12)$$

Table 1: Measurements and inputs

Variables	Type
Choke pressure ( $p_c$ )	Measurement
Bottom-hole pressure ( $p_{bh}$ )	Measurement
Drill string mass flow rate of gas ( $w_{g,d}$ )	Input
Drill string mass flow rate of liquid ( $w_{l,d}$ )	Input
Choke opening ( $Z$ )	Input

where  $N + 1$  is the finite horizon. This information can be separated into two vectors

$$Y_t = \begin{bmatrix} y_{t-N} \\ y_{t-N+1} \\ \vdots \\ y_t \end{bmatrix}, \quad U_t = \begin{bmatrix} u_{t-N} \\ u_{t-N+1} \\ \vdots \\ u_{t-1} \end{bmatrix}. \quad (13)$$

The nonlinear MHE is formulated as an objective function

$$J(\hat{X}_{t-N,t}, \bar{X}_{t-N,t}, I_t) = \|W(Y_t - H_t(\hat{X}_{t-N,t}))\|^2 + \|V(\hat{X}_{t-N,t} - \bar{X}_{t-N,t})\|^2, \quad (14)$$

where  $V \in \mathbb{R}^{n_x \times (N+1)} \times \mathbb{R}^{n_x \times (N+1)}$  and  $W \in \mathbb{R}^{n_y \times (N+1)} \times \mathbb{R}^{n_y \times (N+1)}$  are the positive definite weight matrices[36, 37]. These weight matrices and the finite horizon are tuning parameters that should be adjusted to achieve **an excellent** performance.

165 The objective function (14) comprises two main parts. The first part tries to minimize the output estimation error. It is reasonable that the weight matrix  $W$  is chosen as a small value in the presence of an uncertain measurement model and noisy measurements. The second part of the objective function minimizes the deviation between the estimated state at the start of the horizon and its  
170 prediction. Although choosing small tuning matrices  $V$  and large  $W$  will result in rapid convergence in the estimation, there will be more uncertainties. Choosing larger tuning matrices  $V$  and small  $W$  will result in slow convergence, but a

smoother estimation. The nonlinear MHE minimizes the objective function (14) using the current and historical measurements during a time window, subject to nonlinear model equations. The solution to this optimization problem is  $X_{t-N,t}^o$ . The state estimation  $\hat{X}_{i,t}$ , ( $i = t-N, \dots, t$ ) can be computed using the nonlinear observer model, which is Eq. 9 without the process noise. Consequently, the output estimation vector  $\hat{Y}_t$  can be written as follows

$$\hat{Y}_t = H(\hat{X}_{t-N,t}, U_t) = \begin{bmatrix} h(\hat{X}_{t-N,t}) \\ h(f^{u_{t-N}}(\hat{X}_{t-N,t})) \\ \vdots \\ h(f^{u_{t-1}}(\dots(f^{u_{t-N}}(\hat{X}_{t-N,t})))) \end{bmatrix}. \quad (15)$$

By employing  $X_{t-N-1,t-1}^o$  and the observer model, a one-step prediction  $\bar{X}_{t-N,t}$  is obtained as follows

$$\bar{X}_{t-N,t} = f(X_{t-N-1,t-1}^o, u_{t-N-1}) \quad t = N+1, N+2, \dots \quad (16)$$

Because of the simultaneous estimation of unmeasured states  $x$  and unknown parameters  $\theta$ , they are augmented. The production indices of the gas  $K_g$  and liquid  $K_l$  are denoted by  $\theta_1$  and  $\theta_2$ , respectively. The discrete state-space equation of the augmented system is given by

$$\begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \theta_{1,k} \\ \theta_{2,k} \end{bmatrix} = \begin{bmatrix} f_1(X_{k-1}, \theta_{1,k-1}) \\ f_2(X_{k-1}, \theta_{2,k-1}) \\ \theta_{1,k-1} \\ \theta_{2,k-1} \end{bmatrix} = f^a(X_{k-1}, \theta_{k-1}, u_{k-1}) + q_k, \quad (17)$$

where  $m_g$  and  $m_l$  in (1)-(2) are labeled respectively by  $x_1$  and  $x_2$ .

### 180 3.2. Joint Unscented Kalman Filter

The unscented Kalman filter (UKF) was presented by Julier and Uhlman[38]. The fundamental concept behind this approach is that it is easier to approximate a Gaussian distribution than an arbitrary nonlinear function. Then, the

unscented transform, a deterministic sampling approach, was introduced to generate a minimal set of sample points (sigma points) around the mean for the estimation of the mean and covariance matrix of estimation error[39, 40]. Since UKF does not need to use an explicit linearization of the model, the step relevant to the computation of the explicit Jacobian or Hessian matrix is omitted.

There are two **standard** methods for parameter estimation by using the UKF. The first method is a dual UKF that computes the state and parameters separately by employing two UKFs. **The state is calculated by state estimator, and then it is utilized for parameter estimation**[41, 42]. The second and more common method is a joint UKF, which uses the augmentation of original state variables and parameters. In this way, a single UKF estimates the augmented state vector. The implementation of the second method is easier and more efficient than dual filtering[43, 40]. **Details of UKF are provided in Appendix.**

## 4. Simulation results

### 4.1. *Simulation With Perfect Model Data*

The results have been produced by using the OLGA simulator. The OLGA dynamic multiphase flow simulator is a precise high-fidelity simulation tool which has the capability to simulate the oil and gas processes meticulously[44]. The parameter values related to the well and reservoir are presented in Table 2. During simulations, the measurement sampling interval was chosen as 10 secs, and the time step of the model was 10 secs.

### 4.2. *Case 1: Tracking production index in the manual control condition*

Firstly, in the UBD operation, the reservoir pressure is estimated by reservoir engineers. Then during process, this value is updated by performing flow rates. During drilling through the same reservoir, the reservoir pressure is assumed reasonably homogeneous. Also, the PI increases as progressively larger parts of the production matrix is opened up, and the drilling bit potentially encounters faults[34, 35]. The considered scenario is the UBD operation of a vertical well

Table 2: Parameter values for well and reservoir measurements

Name	LOL	Unit
Reservoir pressure ( $p_{res}$ )	279	bar
Collapse pressure ( $p_{coll}$ )	155	bar
Well total length ( $L_{tot}$ )	2530	m
Drill string outer diameter ( $D_d$ )	0.1206	m
Well inner diameter ( $D_a$ )	0.1524	m
Liquid pump flow rate ( $w_{l,d}$ )	13.33	kg/s
Gas pump flow rate ( $w_{g,d}$ )	0	kg/s
Liquid density ( $\rho_L$ )	1000	kg/m <sup>3</sup>
Gas average temperature ( $T$ )	285.15	K
Average angle ( $\Delta\theta$ )	0	rad
Choke constant ( $K_c$ )	0.0053	m <sup>2</sup>

drilled into an oil and gas reservoir, and the reservoir pressure is presumed known. In the beginning, the drilling process is in the steady-state condition with the choke opening of 8 %. After 2 hours, the production index of gas increases rapidly from 0.07 to 0.13. The choke valve is opened to 9 % after 3 hours for manual control of influx.

For performance assessment of the nonlinear MHE on the UBD operation, UKF is utilized as a nonlinear estimator. In[32], it is shown that UKF has a higher performance than other Kalman filters for the UBD system. More details of UKF can be found in[40]. The parameter values of these nonlinear estimators are summarized in Table 3.

The tuning parameters of the nonlinear MHE and UKF are chosen based on trial and error. Trade-offs in the UKF is determined by the process noise covariance matrix  $Q$ . The covariance matrices of state  $Q_s$  and parameter  $Q_p$

Table 3: Parameter values for estimators

Parameter	Value
$W$	$diag(0.1, 0.05)$
$V$	$diag(0.1, 0.1, 100, 200)$
$N + 1$	15
$\beta$	2
$\kappa$	0
$\alpha$	0.1

are chosen based on the normal variations of states and parameters as

$$Q = diag(Q_s, Q_p), \quad (18)$$

$$Q_s = \begin{bmatrix} 9 \times 10^{-4} & 0 \\ 0 & 10^{-2} \end{bmatrix}, \quad (19)$$

$$Q_p = \begin{bmatrix} 4 \times 10^{-9} & 0 \\ 0 & 8 \times 10^{-9} \end{bmatrix}. \quad (20)$$

The measurements (the choke and the bottom-hole pressures) are corrupted by additive white Gaussian noise. The measurement noise covariance matrix is

$$R = \begin{bmatrix} 0.2^2 & 0 \\ 0 & 0.4^2 \end{bmatrix}.$$

The **actual** values and the initial conditions of estimation relevant to the unknown parameters are presented below.

$$K_g = 0.07, \quad K_l = 0.1$$

$$\hat{K}_g = 0.098, \quad \hat{K}_l = 0.13$$

Fig.2 and Fig.3 show the estimation of production indices of gas and liquid from the reservoir into the well for both the nonlinear MHE and the UKF. The results illustrate that the rapid change in the production index of gas is detected.

225 However, there is a small difference between the estimation of the production index of gas and its **real** value, which is resulting from the model uncertainties such as the friction coefficient variations.

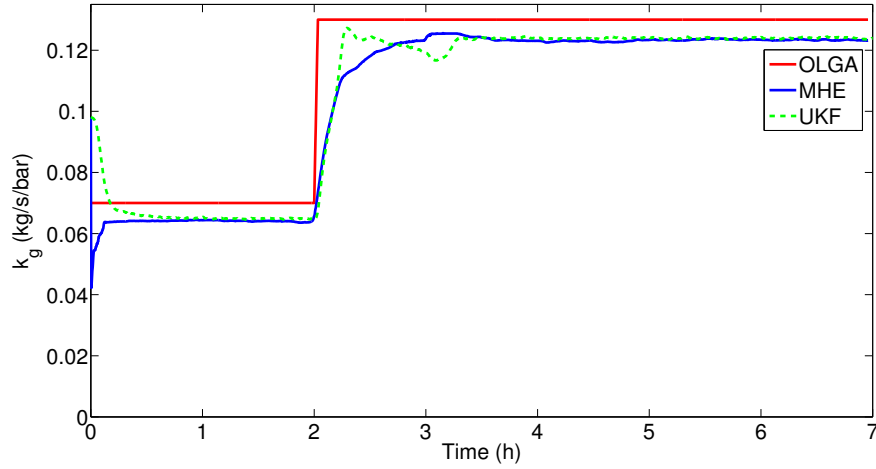


Figure 2: Estimation of production index of gas

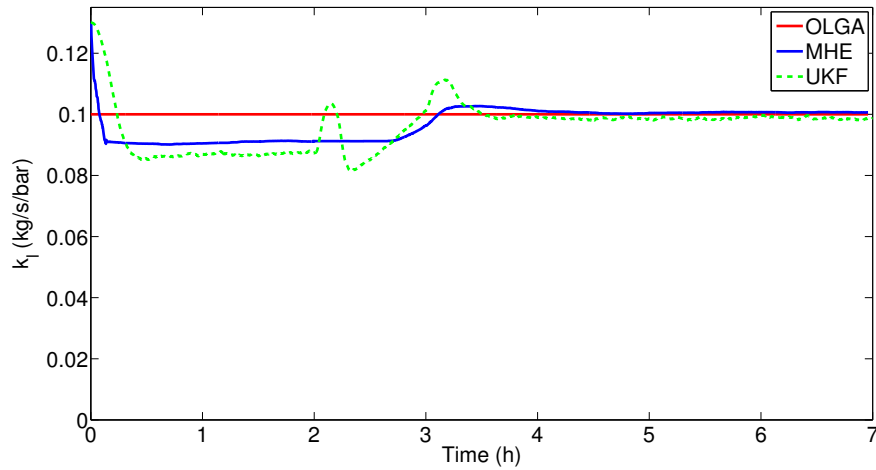


Figure 3: Estimation of production index of liquid

The root mean square error (RMSE) index is used for the performance assessment of utilized nonlinear estimators. The RMSE metrics for the unknown  
 230 parameter estimation are summarized in Table 4. In this paper, it is found that

Table 4: RMSE metric

Method	$K_g$	$K_l$
UKF with manual control	0.0099	0.0089
MHE with manual control	0.0098	0.0060

this choice of the parameters gives the sensible performance of the estimator algorithms. Still, we emphasize that the tunable parameters of the estimator algorithms are not tuned to optimize the performances. Based on the RMSE indices, the simulation results show that the nonlinear MHE has a higher performance than the UKF in the estimation of the production constants of gas and liquid. The results illustrate that the proposed method can diagnose and track the rapid change in the production index of gas.

#### 4.3. Case 2: Tracking production index in the automatic control condition

A Proportional-Integral (PI) controller is used for bottom-hole pressure control in the UBD operation since it is a standard and useful industrial controller, and it can be tuned easily. The proportional and integral gains of the controller are chosen as 0.005 and 0.001, respectively. More information about the design and tuning of the PI controller can be founded in [45, 46, 47]. Set-point is selected as 245 bar for desired bottom-hole pressure.

The bottom-hole pressure  $p_{bh}$  and the choke pressure  $p_c$  with the manual controller and PI controller are shown in Fig. 4. It is illustrated that the PI controller has a better performance than the manual controller for the bottom-hole pressure. The PI controller has the capability to regulate the set-point efficiently and mitigate the negative impacts of production index change. The choke valve opening related to the two scenarios is presented in Fig. 5.

The estimations of production indices of gas and oil with the manual controller and PI controller by using the nonlinear MHE are shown in Fig. 6 and Fig. 7. The nonlinear MHE has a good performance and high convergence rate in the presence of the manual or PI controller. The RMSE metrics of the parameters  $K_g$  and  $K_l$  for the nonlinear MHE by using the manual and PI controller



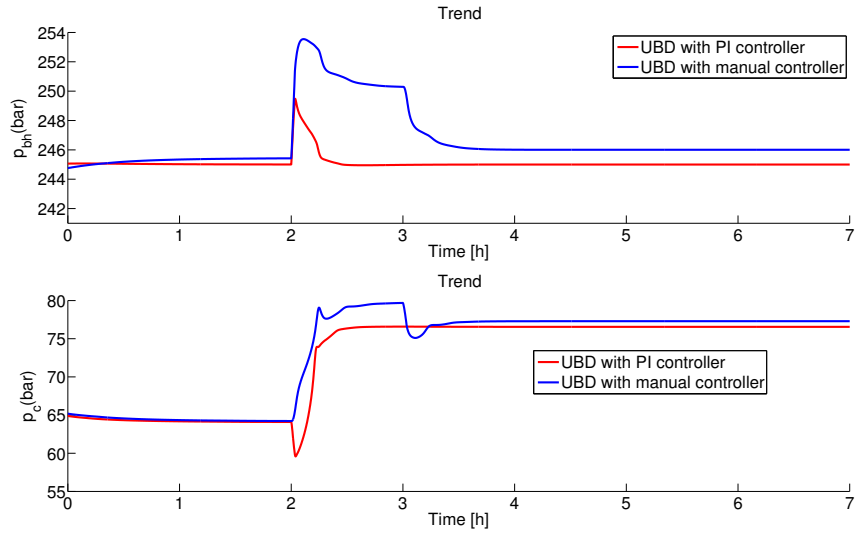


Figure 4: Measured bottom-hole pressure and choke pressure for changing production index scenario.

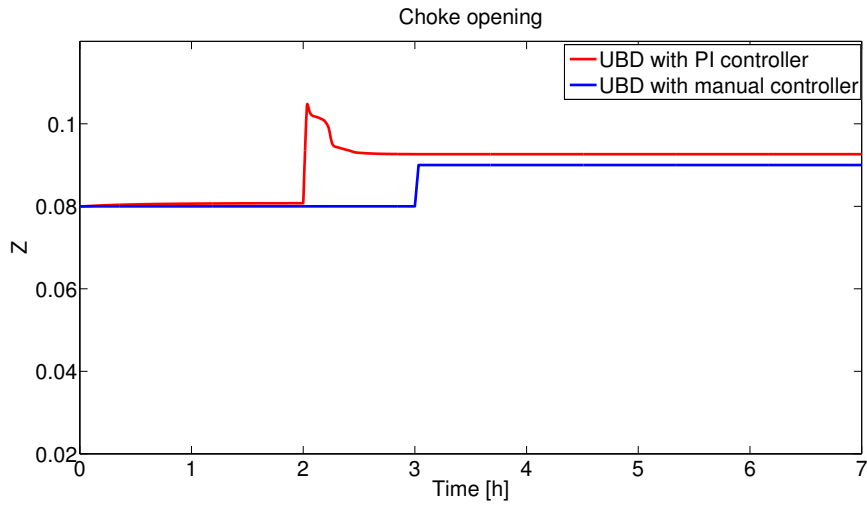


Figure 5: Choke opening for changing production index scenario.

are summarized in Table 5.

Although generally, the type of controller does not affect the estimation accuracy, the simulation results illustrate that the estimation accuracy of pro-

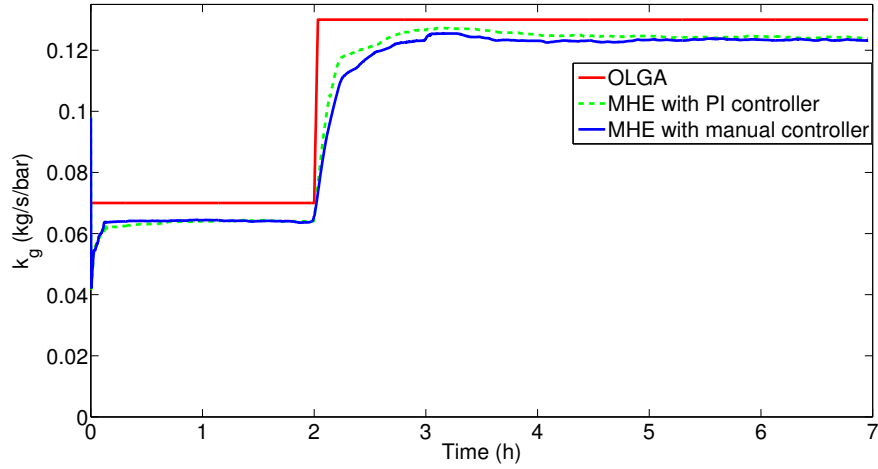


Figure 6: Estimation of production index of gas.

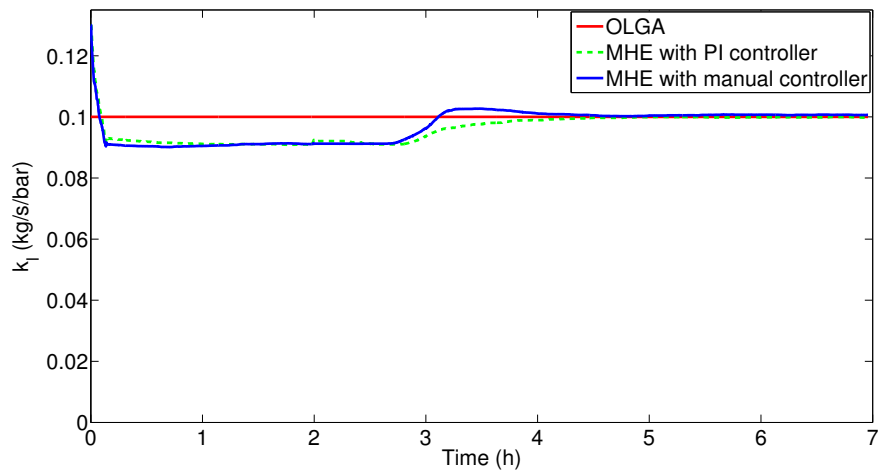


Figure 7: Estimation of production index of liquid.

duction indices can be significantly improved by the PI controller since this  
 260 controller can considerably reduce the reservoir influx. However, the nonlinear  
 MHE evaluates the unknown parameters with reasonable accuracy with either

Table 5: RMSE metric for estimate of  $K_g$  and  $K_l$  for changing production index scenario

Method	$K_g$	$K_l$
MHE with manual control	0.0098	0.0060
MHE with PI control	0.0084	0.0059

Table 6: RMSE metric in case of error in the reservoir pressure value

Method	$K_g$	$K_l$	$p_{res}$ model
UKF with manual control	0.0175	0.0144	282
UKF with PI control	0.0179	0.0143	282
MHE with manual control	0.0182	0.0122	282
MHE with PI control	0.0163	0.0122	282

manual or automatic controller.

#### 4.4. Case 3: Robustness analysis of MHE in case of uncertainties and errors in the reservoir and well parameters of the model

265 Two simulations are carried out to investigate the effects of uncertainties and errors in the model parameters. It is assumed that there are 1 % error on the reservoir pore pressure  $p_{res}$  and 5 % errors on liquid density  $\rho_L$ . The RMSE metrics for the nonlinear estimators related to these two scenarios are presented in Table 6 and Table 7, respectively. Because there is a direct relation  
 270 between the mass flow rates from the reservoir into the well and the reservoir pore pressure, the production constants estimation can be significantly affected by small uncertainties in the reservoir pore pressure. Thus, the sensitivity of the proposed method to the reservoir pore pressure uncertainties is extremely high.

## 275 5. Conclusions

The simultaneous estimation of state and geological properties of the reservoir (production parameters) based on the LOL model during the UBD process using the nonlinear MHE is addressed in this paper. Using the OLGA simulator, the real-time measurements of the choke and bottom-hole pressures are

Table 7: RMSE metric in case of error in the liquid density value

Method	$K_g$	$K_l$	$\rho_l$ model
UKF with manual control	0.0119	0.0080	950
UKF with PI control	0.0121	0.0078	950
MHE with manual control	0.0127	0.0048	950
MHE with PI control	0.0110	0.0044	950

280 obtained. The nonlinear MHE can be used temporarily instead of the mud-pulse telemetry, which has a lot of difficulties such as transmission delay and slow sampling rate. Moreover, during drill pipe connection operations, the nonlinear MHE can estimate the unmeasured state and reservoir characterizations in the UBD operation. The geological properties of the reservoir are incredibly  
 285 crucial in this process and must be estimated. The results illustrate that the nonlinear MHE can estimate the production indices of gas and liquid and track a change in the production index of gas either in manual or automatic control mode. Moreover, the simulation results demonstrate the nonlinear MHE has a higher performance than UKF in the estimation of production indices. The  
 290 nonlinear estimators have the capability to identify a rapid change in the production index in order of minutes. It is observed that the type of controller does not affect estimation accuracy. Finally, the robustness of the estimators against the errors in the reservoir pore pressure and liquid density is investigated. It is shown that the methods are highly sensitive to the reservoir pore pressure  
 295 uncertainties.

The nonlinear Moving Horizon Estimation (MHE) uses a history of measurements and has this capability to assign different weights to different measurements. Using adaptive weights in this method can be considered as future work. During drill pipe connection operations, when the measurements are  
 300 noisy or unreliable, the proposed method can be used as a temporary approach and reduce impacts of these kinds of measurements by choosing a small weight.

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## Appendix

The UKF consists of two separate steps: 1) prediction, 2) correction. The state mean  $\hat{x}_k^-$  and the error covariance  $P_k^-$  are predicted in the first step as follows

$$(\chi_k)_i = f^a((\chi_{k-1})_i), \quad i = 0, \dots, 2L \quad (21)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} (\chi_k)_i, \quad (22)$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} [(\chi_k)_i - \hat{x}_k^-][(\chi_k)_i - \hat{x}_k^-]^T + Q_k, \quad (23)$$

where  $L$  and  $Q_k$  are the dimension of the augmented state vector and process covariance matrix, respectively.  $(\chi_{k-1})_i$  is the  $i^{th}$  column of the sigma point matrix  $\chi_{k-1}$ .  $W_i^{(m)}$  and  $W_i^{(c)}$  are the weighting matrices that affect the computation of state mean and covariance, respectively.

$$W_0^{(m)} = \frac{\lambda}{(L + \lambda)},$$

$$W_i^{(m)} = \frac{1}{2(L + \lambda)}, \quad i = 1, \dots, 2L \quad (24)$$

$$W_0^{(c)} = \frac{\lambda}{(L + \lambda)} + (1 - \alpha^2 + \beta),$$

$$W_i^{(c)} = \frac{1}{2(L + \lambda)}, \quad i = 1, \dots, 2L \quad (25)$$

The tuning parameter  $\lambda$  is given by

$$\lambda = \alpha^2(L + \kappa) - L. \quad (26)$$

The constant  $\alpha$ , which is chosen between  $10^{-4}$  and 1, indicates the distribution of sigma points around the state estimation. The scaling parameter  $\kappa$  is normally selected as 0[43]. The prior knowledge of state vector distribution is incorporated by using the parameter  $\beta$ . The optimal value of  $\beta$  for Gaussian distribution is 2[38]. In the correction step of the UKF, the predicted weighted mean measurement is calculated by using

$$(Y_k)_i = h((\chi_k)_i), \quad i = 0, \dots, 2L \quad (27)$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} (Y_k)_i. \quad (28)$$

The Kalman gain of the UKF is calculated by

$$K_k = P_{\hat{x}_k \hat{y}_k} P_{\hat{y}_k \hat{y}_k}^{-1}, \quad (29)$$

$$P_{\hat{x}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [(\chi_k)_i - \hat{x}_k^-] [(Y_k)_i - \hat{y}_k^-]^T, \quad (30)$$

$$P_{\hat{y}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [(Y_k)_i - \hat{y}_k^-] [(Y_k)_i - \hat{y}_k^-]^T + R_k, \quad (31)$$

where  $P_{\hat{y}_k \hat{y}_k}$  is the covariance of the measurement, and  $P_{\hat{x}_k \hat{y}_k}$  is cross-covariance of the state and measurement. The measurement noise covariance matrix is denoted by  $R_k$ . Finally, the updated state mean  $\hat{x}_k$  and the updated error covariance  $P_k$  are calculated by

$$\hat{x}_k = \hat{x}_k^- + K_k (y_k - \hat{y}_k^-), \quad (32)$$

$$P_k = P_k^- - K_k P_{\hat{y}_k \hat{y}_k} K_k^T. \quad (33)$$

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