Modelling creep in clay using the framework of hyper-viscoplasticity

GUSTAV GRIMSTAD*[1], DAVOOD DADRASAJIRLOU* and SEYED ALI GHOREISHIAN AMIRI*

*PoreLab, Department of Civil and Environmental Engineering, Norwegian University of Science and Technology (NTNU), Trondheim, Norway

[1] corresponding author, e-mail: gustav.grimstad@ntnu.no, address: Department of Civil and Environmental Engineering, Høgskoleringen 7A, 7491 Trondheim, Norway

Abstract: This paper addresses the derivation of creep models using the framework of hyper-viscoplasticity. It demonstrates that the formulations widely used already can easily be obtained using the hyper-viscoplastic formalism. This means that existing formulations (i.e. of the flow potential) are thermodynamically sound. The key assumptions are that the free energy is only a function of elastic strains and that there is no dissipation under pure volumetric swelling (tension). The presented derivations, using the framework of hyper-viscoplasticity, allows for further model development along the same lines, as presented here, with only minor modifications.

KEYWORDS: time dependence; creep; compressibility; clays; constitutive relations; plasticity

INTRODUCTION

Only a few attempts have been made to formulate a hyper-viscoplastic model for creep and rate dependence of soft clays that comply with the critical state soil mechanics concept. There is still a need for showing the derivations of such a model in a complete formalistic manner. Attempts has been made, e.g. Aung et al. (2019), but still some clarifications are necessary to demonstrate a thermodynamic sound formulation. Therefore, this paper demonstrates how to establish the well-known empirically based formulation for creep in clay from a thermodynamic perspective. The result is in strong resemblance to the creep formulation widely used in geotechnics already. As found in Šuklje (1963), Janbu (1969) and others, for 1D case, and extended to full stress space in e.g. Stolle et al. (1999), with the effect of fabric by Leoni et al. (2008). The presented derivation from the thermodynamic framework gives the same correction to the Stolle et al. (1999) formulation as suggested by Grimstad et al. (2008) to properly model the "dry side", as further discussed in Grimstad et al. (2010). The notation used, follows the book of Houlsby and Puzrin (2006), with concepts/terminology discussed in e.g. Collins and Kelly (2002), Collins and Houlsby (1997), Darabi et al. (2018) and Osman et al. (2020). Small strains are assumed, so additive decomposition of elastic and viscoplastic strains holds. Cauchy stresses are hence then also used. Triaxial stress (p-q) space is utilized to simplify the derivation, but extension to full stress space is straightforward. This note makes use of the normal geotechnical sign convention, i.e. compression positive. The principles of hyper-viscoplasticity are briefly presented in the appendix.

DERIVATION OF THE FLOW POTENTIAL

The starting point of developing models, using the thermodynamic framework, is to establish the free energy function and the force potential. In terms of Helmholtz free energy, f, it can take the form of eq. (1), where the free energy is implicitly a function of the elastic strain only.

$$f = \kappa \cdot p_{ref} \cdot \exp\left(\frac{\varepsilon_v - \varepsilon_v^{vp}}{\kappa}\right) + \frac{3G}{2} \cdot \left(\varepsilon_q - \varepsilon_q^{vp}\right)^2 \qquad (1)$$

Where G is the shear stiffness, κ is elastic compressibility parameter (the bulk stiffness increases linearly with mean effective stress), p_{ref} is an arbitrary reference pressure, ε_v is the volumetric strain and ε_q is the deviatoric strain, energy conjugates to the mean effective stress, p, and deviatoric stress, q, respectively. Elastic strains are defined as total strains minus viscoplastic strains, $\varepsilon_v - \varepsilon_v^{vp}$ and $\varepsilon_q - \varepsilon_q^{vp}$. Alternatively, the free energy, eq. (1), can be expressed in terms of Gibbs free energy, g:

$$g = -\kappa \cdot p \cdot \left(\ln\left(\frac{p}{p_{ref}}\right) - 1\right) - \frac{q^2}{6G} - \left(p \cdot \varepsilon_v^{vp} + q \cdot \varepsilon_q^{vp}\right) \qquad (2)$$

This form of free energy results in true stress equals to dissipative stress (see appendix), thus, one can further concentrate only on the force potential.

Consider the following force potential, z:

$$z = \frac{p_0}{2^n} \cdot \frac{r^{1-n}}{n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp}\right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp}\right)^2} + \dot{\varepsilon}_v^{vp}\right)^n \qquad (3)$$

Where p_0 is a state variable equivalent to the isotropic "pre-consolidation" stress, r is a reference rate, n is a number slightly larger than 1 but significantly less than 2. $n = 2$ would mean linear increase with strain rate. From experiments it is well documented that this has a logarithmic nature, see e.g. the early work of Buisman (1936), Šuklje (1963) or Bjerrum (1967): this implies a n between 1 and 2. M is the critical state line in p-q space. This force potential gives expectation of a behavior that scales the behavior of the Modified Cam-Clay model (MCCM) (Roscoe and Burland, 1968). In fact, with $n = 1$, the dissipation function of the MCCM is retrieved from this force potential in a form as discussed in e.g. Collins and Houlsby (1997), with the "shift" included in the dissipation function.

The dissipation, d, equals, after differentiation:

$$d = \frac{\partial z}{\partial \dot{\varepsilon}_v^{vp}} \cdot \dot{\varepsilon}_v^{vp} + \frac{\partial z}{\partial \dot{\varepsilon}_q^{vp}} \cdot \dot{\varepsilon}_q^{vp} = \frac{p_0}{2^n} \cdot r^{1-n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp}\right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp}\right)^2} + \dot{\varepsilon}_v^{vp} \right)^n \geq 0 \tag{4}$$

See that $d \geq 0$ holds for any strain rate. For pure volumetric unloading, it results in $d = 0$. It means under pure volumetric swelling, there is no dissipation, as all applied energy is spent in volume increase. One will see later that this happens only in the Origin of stress space, which is a realistic behavior in many cases (i.e. liquefaction). However, when using a free energy function as the one in eq. (1), such a state is actually impossible, as it will require infinite negative elastic volumetric strain to reach zero mean effective stress.

The flow potential, w, is found from the difference between dissipation and force potential:

$$w = d - z = \frac{p_0}{2^n} \cdot \frac{n-1}{n} \cdot r^{1-n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp}\right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp}\right)^2} + \dot{\varepsilon}_v^{vp} \right)^n \tag{5}$$

(note again that for $n = 1$ the dissipation is linear in strain rate, and w will then define the yield surface (y) in dissipative stress space (χ); generally $y(\chi_{ij}, \sigma_{ij}, p_0)$ as $w = y = 0$). Which, in true stress space, will give the plastic potential function and yield surface. In this case, it will result in an associated flow rule as $y = y(\chi_{ij}, p_0)$. However, if e.g. $M = M(\sigma_{ij})$, a non-associated flow rule will be predicted.

The dissipative stresses are derived from the force potential as:

$$\begin{bmatrix} \chi_p \\ \chi_q \end{bmatrix} = \begin{bmatrix} \dfrac{\partial z}{\partial \dot{\varepsilon}_v^{vp}} \\ \dfrac{\partial z}{\partial \dot{\varepsilon}_q^{vp}} \end{bmatrix} = \frac{\dfrac{p_0}{2^n} \cdot r^{1-n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp}\right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp}\right)^2} + \dot{\varepsilon}_v^{vp} \right)^n}{\sqrt{\left(\dot{\varepsilon}_v^{vp}\right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp}\right)^2}} \cdot \begin{bmatrix} 1 \\ \dfrac{M^2 \cdot \dot{\varepsilon}_q^{vp}}{\sqrt{\left(\dot{\varepsilon}_v^{vp}\right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp}\right)^2} + \dot{\varepsilon}_v^{vp}} \end{bmatrix} \tag{6}$$

Rearranging the equation results in:

$$\dot{\varepsilon}_q^{vp} = \frac{2 \cdot \dfrac{\chi_q}{\chi_p}}{M^2 - \left(\dfrac{\chi_q}{\chi_p} \right)^2} \cdot \dot{\varepsilon}_v^{vp} \tag{7}$$

Zero shift stress (i.e. $\bar{\chi}_{ij} = \sigma_{ij}$) and assumption of maximum dissipation rate (i.e. $\chi_{ij} = \bar{\chi}_{ij}$ the Ziegler's orthogonality assumption (Ziegler, 1983), see the appendix), results in the well-known MCCM flow rule in the true stress space:

$$\dot{\varepsilon}_q^{vp} = \frac{2 \cdot \dfrac{q}{p}}{M^2 - \left(\dfrac{q}{p} \right)^2} \cdot \dot{\varepsilon}_v^{vp} \tag{8}$$

Rearranging and eliminating the viscoplastic strain rates give the flow potential as a function of dissipative stresses:

$$w = p_0 \cdot r \cdot \frac{n-1}{n} \cdot \left(\frac{p_{eq}}{p_0} \right)^{\frac{n}{n-1}} \tag{9}$$

Where, one may identify the known equivalent stress measure, p_{eq} as:

$$p_{eq} = p + \frac{1}{M^2} \cdot \frac{q^2}{p} \tag{10}$$

Note that eq. (9) results in a family of similar ellipses with a "similarity center" at the origin of the true stress space.

RESULTS AND DISCUSSION

The derived flow potential, eq. (9), can be used directly for modelling creep in clay. However, the material parameters *r* and *n* does not give direct physical meaning. Therefore, it is more convenient to change them to engineering ones. By comparing eq. (9) with the formulation for the plastic multiplier found in e.g. Grimstad et al. (2010) (after integration), one gets:

$$w = p_0 \cdot \frac{\dot{\lambda}_{ref}}{\frac{\lambda-\kappa}{\mu}+1} \cdot \left(\frac{p_{eq}}{p_0}\right)^{\frac{\lambda-\kappa}{\mu}+1} = p_0 \cdot r \cdot \frac{n-1}{n} \cdot \left(\frac{p_{eq}}{p_0}\right)^{\frac{n}{n-1}} \qquad (11)$$

As a result, *n* and *r* relates to "classical" parameters through:

$$n = \frac{\lambda-\kappa+\mu}{\lambda-\kappa} \text{ and } \dot{\lambda}_{ref} = r \qquad (12)$$

For typical values of $\lambda - \kappa$ (= 0.09) and μ (= 0.0036) for soft clays (i.e. a creep number $[(\lambda-\kappa)/\mu]$ of 25), one observes that the number *n* is 1.04. Note that all these parameters are found from conventional laboratory tests.

Further, see that the over consolidation ratio, *OCR*, is identified as:

$$OCR = \frac{p_0}{p_{eq}} \qquad (13)$$

Also, in practice, it is not convenient to use a reference rate as input parameter. Therefore, Grimstad et al. (2010), already defined this in terms of more conventional parameters through:

$$\dot{\lambda}_{ref} = \frac{\mu}{\tau} \cdot \frac{M^2}{M^2 - \eta_{K0NC}{}^2} \qquad (14)$$

Where, τ is the reference time for which p_0 (or OCR) is determined, typically 1 day for incrementally loaded oedometer tests. The last term is there to generalize the oedometer condition to general condition, where η_{K0NC} is the stress ratio, q/p, under 1D normal compression. This derivation shows that the creep formulation used in e.g. Grimstad et al. (2017) (with $OCR_{\max} \rightarrow \infty$) can be exactly derived in the framework of hyper-viscoplasticity. With a modified force potential, e.g. including the effect of fabric and Lode angle dependency, a more advance model could be retrieved following the same steps as above. Also, if a linear term is added to the force potential, it is quite straightforward to include a type of $OCR_{\max}$ parameter, ending up with a formulation with some similar characteristics as the one suggested in Grimstad and Degago (2010) and Grimstad et al. (2017), with $OCR_{\max}$ representing an inner limit surface corresponding to zero viscoplastic strain rate.

To complete the scheme, the viscoplastic strain rates are calculated from the flow potential, giving the following viscoplastic strain rates:

$$\begin{bmatrix} \dot{\varepsilon}_v^{vp} \\ \dot{\varepsilon}_q^{vp} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial w}{\partial \chi_p} \\ \dfrac{\partial w}{\partial \chi_q} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial w}{\partial p} \\ \dfrac{\partial w}{\partial q} \end{bmatrix} = \dot{\lambda}_{ref} \cdot \left(\frac{p_{eq}}{p_0} \right)^{\frac{\lambda-\kappa}{\mu}} \cdot \begin{bmatrix} \dfrac{\partial p_{eq}}{\partial p} \\ \dfrac{\partial p_{eq}}{\partial q} \end{bmatrix} \tag{15}$$

The hardening rule of p_0, eq. (16), takes the same form as for MCCM, where strains are considered rather than specific volume.

$$\dot{p}_0 = \frac{p_0}{\lambda - \kappa} \cdot \dot{\varepsilon}_v^{vp} \tag{16}$$

Finally, it is interesting to note that when there is a difference between the dissipative stress and the true stress (coming from a different choice of free energy function), it will result in a creep formulation where the "similarity center" is not in the origin of p-q space. As an example, eq. (17) gives the Gibbs free energy for a case where the “similarity center” will be in the center of the Cam-Clay ellipse.

$$g = -\kappa \cdot p \cdot \left(\ln\left(\frac{p}{p_{ref}} \right) - 1 \right) - \frac{q^2}{6G} - \left(p \cdot \varepsilon_v^{vp} + q \cdot \varepsilon_q^{vp} \right) + \left(\lambda - \kappa \right) \cdot \frac{p_{0,ref}}{2} \cdot e^{\frac{\varepsilon_v^{vp}}{\lambda-\kappa}} \tag{17}$$

The generalized stresses are found by differentiation. And when combined with the integral of eq. (16) from $p_{0,ref}$ to p_0 , one obtains:

$$\begin{aligned} \overline{\chi}_p &= -\frac{\partial g}{\partial \varepsilon_v^{vp}} = p - \frac{p_{0,ref}}{2} \cdot e^{\frac{\varepsilon_v^{vp}}{\lambda-\kappa}} = p - \frac{p_0}{2} \\ \overline{\chi}_q &= -\frac{\partial g}{\partial \varepsilon_q^{vp}} = q \end{aligned} \tag{18}$$

Here one identifies the “shift stress” $p - \overline{\chi_p}$ as $p_0/2$. The force potential is now:

$$z = \frac{p_0}{2^n} \cdot \frac{r^{1-n}}{n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp} \right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp} \right)^2} \right)^n \tag{19}$$

From this, the calculation of differentials and the transformation to a flow potential as a function of an equivalent stress measure is done once more, eq. (20) to (25):

$$d = \frac{\partial z}{\partial \dot{\varepsilon}_v^{vp}} \cdot \dot{\varepsilon}_v^{vp} + \frac{\partial z}{\partial \dot{\varepsilon}_q^{vp}} \cdot \dot{\varepsilon}_q^{vp} = \frac{p_0}{2^n} \cdot r^{1-n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp} \right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp} \right)^2} \right)^n \geq 0 \tag{20}$$

$$w = d - z = \frac{p_0}{2^n} \cdot \frac{n-1}{n} \cdot r^{1-n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp} \right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp} \right)^2} \right)^n \tag{21}$$

$$\begin{bmatrix} \chi_p \\ \chi_q \end{bmatrix} = \begin{bmatrix} \dfrac{\partial z}{\partial \dot{\varepsilon}_v^{vp}} \\ \dfrac{\partial z}{\partial \dot{\varepsilon}_q^{vp}} \end{bmatrix} = \frac{p_0}{2^n} \cdot r^{1-n} \cdot \left(\sqrt{\left(\dot{\varepsilon}_v^{vp} \right)^2 + M^2 \cdot \left(\dot{\varepsilon}_q^{vp} \right)^2} \right)^{n-2} \cdot \begin{bmatrix} \dot{\varepsilon}_v^{vp} \\ M^2 \cdot \dot{\varepsilon}_q^{vp} \end{bmatrix} \tag{22}$$

Flow rule:

$$\dot{\varepsilon}_q^{vp} = \frac{1}{M^2} \cdot \frac{\chi_q}{\chi_p} \cdot \dot{\varepsilon}_v^{vp} \tag{23}$$

Using eq. (22) to transform eq. (21) gives:

$$w = p_0 \cdot r \cdot \frac{n-1}{n} \cdot \left(\frac{2 \cdot p_{eq}^*}{p_0} \right)^{\frac{n}{n-1}} \tag{24}$$

Where:

$$p_{eq}^* = \sqrt{\chi_p{}^2 + \frac{\chi_q{}^2}{M^2}} \tag{25}$$

Which defines a family of ellipses with a similarity center at $p_0/2$ in p-q stress space, as $\chi_{ij} = \bar{\chi}_{ij}$ (see Figure 1, Alt. 2). Note that this type has similar characteristics as the flow potential of Apriadi et al. (2013). The consequences of such a choice are prediction of rate dependent (creep) swelling under isotropic unloading for positive (compressive) mean stress (but negative χ_p), and a critical state line in p-q space (M) corresponding only to the reference rate. I.e. interpreted negative “cohesion” (interception of the M line with the q axis) for undrained test ran slower than the reference and a positive interpreted “cohesion” for tests ran faster than the reference. The differences between the two alternative options are summarized in Figure 1.

CONCLUSIONS AND RECOMMENDATIONS

The classical creep model formulation, widely used for clay, was successfully derived using the framework of hyper-viscoplasticity. The derived flow potential was compared to existing formulations and classical model parameters were identified. The consequences of the choice of free energy function and force potential to include the so-called shift stress are highlighted and compared to the alternative form. Further developments in constitutive modelling of clays using the framework of hyper-viscoplasticity can be done along the same lines, as presented here. This means inclusion of e.g. fabric, structure and Lode angle dependency is quite straightforward. For modelling of cyclic behavior of clay, it should be noted that including a plastic part of the free energy (with some similarities as in alt.2) is essential to define a kinematic hardening mechanism in a multisurface formulation as like e.g. Likitlersuang and Houlsby (2007).

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NOTATION

d	dissipation function
f	Helmholtz free energy function
G	Shear stiffness
g	Gibbs free energy function
n	power number
OCR	Over Consolidation Ratio
p	mean effective stress
p_0	isotropic pre-consolidation stress

p_{0ref}	reference isotropic pre-consolidation stress
p_{eq}	equivalent effective stress
p_{ref}	reference pressure
q	deviatoric stress
r	reference rate
w	flow potential function
y	yield surface
z	force potential function
ε_v	volumetric strain
ε_q	deviatoric strain
ε_{ij}^{vp}	viscoplastic strain tensor
ε_v^{vp}	volumetric viscoplastic strain
ε_q^{vp}	deviatoric viscoplastic strain
η_{K0NC}	stress ratio, q/p, given under 1D normal compression
κ	elastic compressibility parameter (strain based)
$(\lambda - \kappa)$	plastic compressibility parameter (strain based)
$\dot{\lambda}_{ref}$	reference rate for viscoplastic multiplier
M	critical state line in p-q space
μ	creep number (strain based)

σ_{ij} true stress tensor

τ reference time

χ_{ij} dissipative generalized stress tensor

χ_p dissipative generalized mean stress

χ_q dissipative generalized deviatoric stress

$\overline{\chi_{ij}}$ generalized stress tensor

$\overline{\chi_p}$ generalized mean stress

$\overline{\chi_q}$ generalized deviatoric stress

APPENDIX: THE PRINCIPLES OF HYPER-VISCOPLASTICITY [ADAPTED FROM THE BOOK OF Houlsby and Puzrin (2006)]

First law of thermodynamics says that the change (with time) in internal energy, $\dot{U}$, is equal to the sum of applied heat $\dot{Q}$ and applied work $\dot{W}$.

$$\dot{U} = \dot{Q} + \dot{W} \quad (26)$$

For the case of mechanical behavior of materials, it is more natural to speak of specific internal energy, $\dot{u}$ (energy per volume and time, kW/m^3 = kPa/s). In this case the change in work, with time, (i.e. the power) is simply the stress times the strain rate:

$$\dot{W} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} \quad (27)$$

Where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\dot{\boldsymbol{\varepsilon}}$ is the increment in the work conjugative strain tensor (i.e. the Almansi strain). However, in practice (for small strains) the increment of

Almansi strain matches the engineering strain increment, calculated on the updated geometry. The specific change in heat is:

$$\dot{Q} = -\nabla \cdot \mathbf{q} \tag{28}$$

Where **q** is the gradient of heat (heat flux). The second law of thermodynamics in terms of specific entropy, $\dot{s}$, can be written as:

$$\dot{s} \geq -\nabla \cdot \frac{\mathbf{q}}{\theta} \tag{29}$$

Also called the Clausius-Duhem inequality (entropy increase). Where θ is the temperature. This can be rewritten as:

$$\dot{s} \geq -\frac{\nabla \cdot \mathbf{q}}{\theta} + \frac{\mathbf{q} \cdot \nabla \theta}{\theta^2} \tag{30}$$

Or:

$$d = \theta \dot{s} + \nabla \cdot \mathbf{q} \geq \frac{\mathbf{q} \cdot \nabla \theta}{\theta} \tag{31}$$

Where d is the dissipation. Since one can assume small temperature gradients in most geotechnical problems, eq. (31) can be rewritten to a more strict condition:

$$d \geq 0 \tag{32}$$

The specific internal energy can now be written as:

$$\dot{u} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \nabla \cdot \mathbf{q} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + \theta \dot{s} - d \tag{33}$$

Which by using the assumption of small increments must be equal to:

$$\dot{u} = \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}} + \frac{\partial u}{\partial s} : \dot{s} + \frac{\partial u}{\partial \boldsymbol{\varepsilon}^{vp}} : \dot{\boldsymbol{\varepsilon}}^{vp} \tag{34}$$

Each term is identified as:

$$
\begin{aligned}
\boldsymbol{\sigma} &= \frac{\partial u}{\partial \boldsymbol{\varepsilon}} \\
\theta &= \frac{\partial u}{\partial s} \\
d &= -\frac{\partial u}{\partial \boldsymbol{\varepsilon}^{vp}} : \boldsymbol{\varepsilon}^{vp} = \overline{\boldsymbol{\chi}} : \boldsymbol{\varepsilon}^{vp}
\end{aligned}
\qquad (35)
$$

Selecting d as a homogenous function of degree n and using the Euler's theorem for homogeneous functions, one can write:

$$
d = \frac{1}{n} \cdot \frac{\partial d}{\partial \dot{\boldsymbol{\varepsilon}}^{vp}} : \dot{\boldsymbol{\varepsilon}}^{vp} = \boldsymbol{\chi} : \dot{\boldsymbol{\varepsilon}}^{vp} = \frac{\partial z}{\partial \dot{\boldsymbol{\varepsilon}}^{vp}} : \dot{\boldsymbol{\varepsilon}}^{vp} \qquad (36)
$$

Where z is defined as the force potential.

Eliminating d and $\dot{u}$ between the equations (33), (34) and (36) results in the following requirement:

$$
\left(\overline{\boldsymbol{\chi}} - \boldsymbol{\chi}\right) : \dot{\boldsymbol{\varepsilon}}^{vp} = 0 \qquad (37)
$$

The simplest choice is that:

$$
\overline{\boldsymbol{\chi}} = \boldsymbol{\chi} \qquad (38)
$$

Called Ziegler's orthogonality assumption (alternatively, orthogonality between $\left(\overline{\boldsymbol{\chi}} - \boldsymbol{\chi}\right)$ and $\dot{\boldsymbol{\varepsilon}}^{vp}$ must be satisfied). The ingredients in hyper-viscoplasticity are hence the free energy function, the force potential and any potential hardening rules. The remaining formulations in more convenient variables follows through derivations/transformations.

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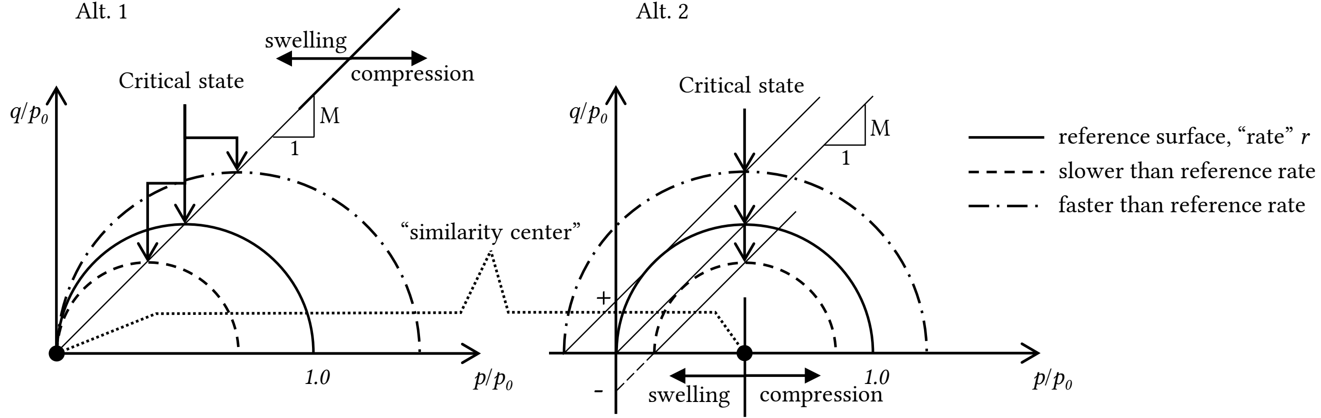
Figures:

Figure 1 Graphical representation of the two different presented options, alt. 1 without shift stress, alt. 2 with shift stress.



Alt. 1
swelling
compression
Critical state
M
1
q/p0
1.0
p/p0
"similarity center"
Alt. 2
Critical state
M
1
q/p0
+
-
swelling
compression
1.0
p/p0
reference surface, "rate" r
slower than reference rate
faster than reference rate