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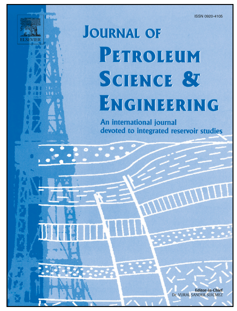

Interwell connectivity identification in immiscible gas-oil systems using statistical method and modified capacitance-resistance model: a comparative study

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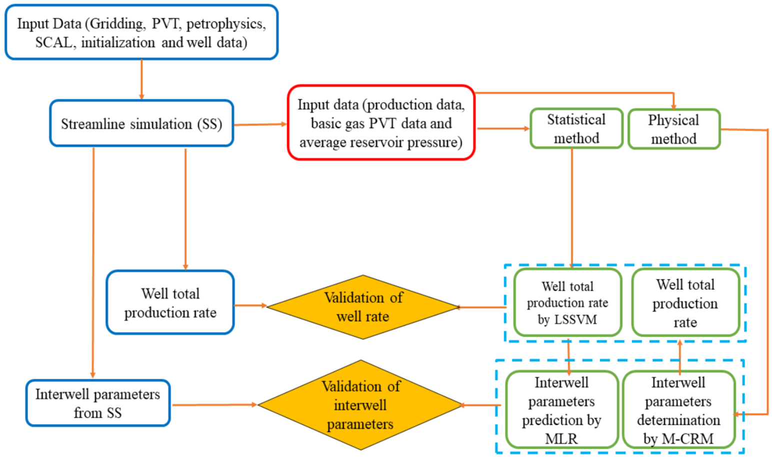
Input Data (Gridding, PVT, petrophysics, SCAL, initialization and well data)
Streamline simulation (SS)
Input data (production data, basic gas PVT data and average reservoir pressure)
Statistical method
Physical method
Well total production rate
Validation of well rate
Well total production rate by LSSVM
Well total production rate
Interwell parameters from SS
Validation of interwell parameters
Interwell parameters prediction by MLR
Interwell parameters determination by M-CRM

Interwell connectivity identification in immiscible gas-oil systems using statistical method and modified capacitance-resistance model: a comparative study

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Abstract

Interwell connectivity identification between injector-producer well pairs and hydrocarbon production estimation are essential parameters in reservoir management, which can determine unrecovered oil volume and reservoir continuity. Although there are several published methods for determination of interwell connectivity in water-oil systems, there is no such comprehensive study on gas flooded reservoirs. Due to the high mobility of gas, interwell connectivity is a critical criterion in channelized, faulted and heterogeneous reservoirs for reservoir characterization, production optimization, infill drilling and performance predication. There are physical and statistical techniques to determine interwell connectivity mathematically and identify reservoir flow dynamics without using any operational activities. All methods are working with limited production data and unlike the numerical simulators, they are simple and do not require detailed data. In this paper, modified capacitance-resistance model (or M-CRM as a physical approach) and combination of least square support vector machine and multiple linear regression (as a statistical approach) are applied to two immiscible gas injection cases with different assumptions, and the results are compared. The results show that both methods are reliable in terms of validity, speed and flexibility. The physical approach (M-CRM) is more

accurate for interwell connectivity prediction while the statistical method is more precise for producer total rate estimation.

Keywords: Capacitance-resistance model; Gas injection; Interwell connectivity; Least square support vector machine

1. Introduction

Interwell connectivity between producer and injector is defined as the injection rate effect on production rate of the surrounding producers. This parameter depends on well locations, reservoir heterogeneity and geometry.

Interwell connectivity characterization and estimation of reservoir performance in water/gas flooded reservoirs are important parameters in reservoir management and optimization. Accurate determination of interwell connectivity affects well placement optimization, infill drilling, reservoir sweep efficiency and identification of high oil saturation zones.

There are several methods for interwell connectivity calculation in the literature including direct and indirect methods. Direct methods are practical approaches which are utilized in a field such as 4D seismic [1-4], tracer test [1, 5-7], pulse test [8, 9] and interference well testing [10-12]. These methods monitor fluid flow in porous media and give reliable results using accurate interpretation. However, operational challenges, cost and time-consuming aspects are their disadvantages.

Indirect methods are categorized as physical and statistical methods. Physical methods consist of streamline simulation [13, 14], pressure-based methods [15], multi-well productivity index (MPI) [16, 17], network model [18, 19] and Capacitance-resistance model (CRM) [20-27]. Physical methods are based on mathematical derivation of reservoir flow models. However, statistical methods are data-driven models such as Spearman Rank Correlation (SRC) [28-30], Artificial Neural Network (ANN) [31-34], Extended Kalman Filter [35] and Wavelet Analysis [36, 37]. In these methods, signal processing technique is employed where injector and producer rates are the signals. Data availability, simplicity and high speed are the main advantages of the statistical methods.A brief description of all the methods studied in the literature is given in Table 1:

Table 1 Description of all the methods in the literature for interwell connectivity calculation

Category	Method	Description
Direct methods	4D seismic	Determining interaction between producer and injector pairs using water and gas front movement and pressure changes in field-scale by imaging the dynamic variation in the reservoir scale [2, 3]
	Tracer testing	Tracer injection into a reservoir and its production could be interpreted to evaluate well to well interaction in a reservoir, reservoir continuity, sub-layer communication and residual oil saturation determination [5-7]
	Pulse and interference well testing	dynamic methods in which a signal is generated from one active well and its effect is measured at an observation well. The analysis identifies reservoir heterogeneity and reservoir properties between two wells [9, 10]
Indirect - physical	Streamline simulation	numerical simulation method which is faster than 3D finite difference simulation. This technique uses implicit pressure explicit saturation (IMPES) formulation and calculates saturations explicitly along 1D streamlines. The weight allocation factor between injector and producer pairs is an output of this method [38, 39]
	pressure-based method	The basis o method is calculation of interwell connectivity from producers and injectors BHP fluctuations using nonlinear regression. Dinh and Tiab [15] developed an analytical answer in a closed system of water flooding projects
	MPI	A semi-analytical approach for interwell connectivity analysis which connectivity parameters from MPI method are not affected by changing the operational conditions, such as not using the existing wells or drilling new wells [16, 17]
	network model	In this method, two parameters (flow area and time-of-flight) are defined and by dividing the reservoir into some nodes, the diffusivity equation is solved [18, 19]
	CRM	A method based on the dynamic material balance, where reservoir is considered as a tank and rates of injectors and rates of producers are input and output signals. The method was introduced by Yousef et al [40, 41]
Indirect - statistical	SRC	Based on the conversion of rates to ranks and depends on summation of the square of the difference in the rankings [34] which was introduced by Heffer et al. [28] for injector and surrounding producers. Fedenczuk et al. [29] presented some plots to visualize injector and producer communication. Refunjol and Lake [30] utilized SRC with a specific time-lag to consider medium and distance effects.
	ANN	A rapid tool for determination of interaction between well pairs. The network includes different layers of input, hidden and output. The hidden layers convert the input into output using weights and transformation functions [31, 32].

	Extended Kalman Filter	Liu and Mendel [35] applied this method to multiple injectors and single producer system to determine connectivity between a producer and the surrounding injectors.
	Wavelet transformation	Employed by Jansen and Kelkar [36] to separate high frequency (details) from low frequency (smoothed) components. Then, the high-frequency section is used for characterizing connectivity. Lee et al. [37] formulated a production rate as a function of filtered injection rate and estimated interwell connectivity using Haar Wavelet.

In this paper, a new statistical method using integration of least square support vector machine (LSSVM) and multiple linear regression (MLR) is employed to identify the interwell connectivities for a gas flooded reservoir. Also, the Modified CRM which has been recently published [27] is used and the results of both models are compared. These models are applied to two cases. Signal processing workflow of this study is demonstrated in Figure 1 which shows the relationship between input, model and output.

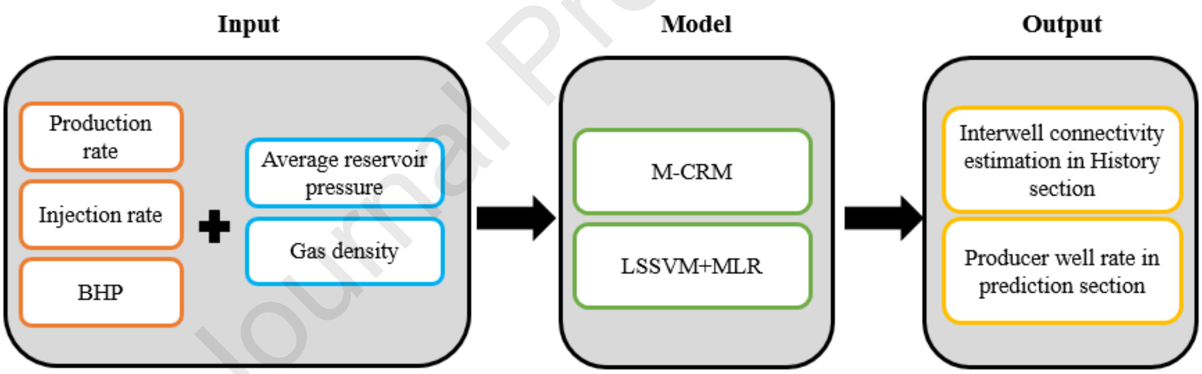


Figure 1 A schematic structure of signal processing workflow in this study

The main goals of this study are summarized as follows:

- Introducing two different methods for calculation of interwell connectivity in gas flooding projects.
- Combination of LSSVM and MLR for determination of well production rate and interwell connectivity as a statistical method.
- Comparison of the results of statistical and physical methods in gas flooded reservoirs, assessing their validity, complexity and run-time.

2. Model development

In this section, LSSVM-MLR as a statistical method and M-CRM as a physical method are thoroughly described. The basic equations and mathematical derivation of the models are presented and a literature review of previous studies and application of the models are discussed.

2.1.1. LSSVM model

Machine learning and intelligent systems have a wide application in engineering problems for optimization and prediction of parameters. In petroleum engineering, many optimization problems such as well placement, rate allocation and production optimization are solved by intelligent methods and data-driven models. In the prediction phase, development of empirical correlations for different parameters and proxy model design are some examples in the chemical and petroleum engineering field.

One of the robust intelligent tools in statistical methods is Support Vector Machine (SVM) developed by Vapnik [42, 43]. This method is used for classification, regression analysis and pattern recognition. Based on primary formulation of SVM, f(x) could be expressed as:

$$f(x) = w^T \varphi(x) + b \tag{1}$$

w^T, $\varphi(x)$ and b refer to transposed output layer vector, the Kernel function and the bias, respectively. x as an input, has a dimension of $N \times n$, where N and n stand for total data points and the input variables, respectively. To obtain w and b in this equation, Vapnik minimized the function below:

$$Cost\ function = \frac{1}{2} w^T + c \sum_{k=1}^{N} (\xi_k - \xi_k^*) \tag{2}$$

c is the tuning parameter in SVM, ξ_k and ξ_k^* are slack variables. The detailed derivation of equations and the constraints are discussed in Appendix A.

In this paper, the inputs are time, producer BHPs and injector rates and the output is producer rates. Finally, the objective function is:

$$\min z = \sum_{k=1}^{n_t}\sum_{j=1}^{n_{prod}}(q_{pred,j}{}^{k} - q_{obs,j}{}^{k})^2 \qquad (3)$$

where, $q_{pred,j}{}^{k}$ and $q_{obs,j}{}^{k}$ are predicted and observed rates.

There are different applications of LSSVM in the petroleum industry such as rock and fluid properties estimation [44], estimation of coning condition [45] and liquid rate at wellhead [46], etc. However, LSSVM has not been used for well production rate production in a reservoir scale. One of the goals of this study is to evaluate LSSVM applicability in rate prediction for two different cases.

2.2. MLR

MLR is a statistical method which uses several parameters as input and one parameter as a response or output. This simple method uses linear relationship between inputs to predict the output. In this study, inputs are time, well injection rate (time-dependent) and producer bottom hole pressure (time-dependent) and response is producer well rate. Using the same time interval in the model, we can eliminate the explicit term for time effect and remove it from input. Therefore, producer rate at each time is a function of constant term, injection rate and pressure difference and the following equation can be assumed:

$$q_j(t_k) = \sum_{i=1}^{N_{inj}} \alpha_{ij} i_i(t_k) + \beta_j \Delta P_{wf,j,k} + \gamma_j \qquad (4)$$

where, α_{ij} stands for interwell connectivity, γ_j is constant term and $\Delta P_{wf,j,k}$ is the difference between producer BHP and average reservoir pressure at time k. This equation is similar to CRM which is described in next section. Therefore, physics of problem could be preserved by the MLR method. Based on CRM formulation, the constant term (γ_j) is positive and less than q_0, β_j should be negative and greater than $-J_j\tau_j/\Delta t$ (these terms are introduced in Modified CRM section) and α_{ij} should be lower than 1. The number of unknowns is $N_{prod}\times(N_{inj}+2)$.

In this study, firstly producer well rate is predicted by LSSVM and then using Equation (4), MLR is employed to estimate interwell connectivity.

2.3. Modified CRM

In the CRM, reservoir is acting as a tank for which rates of injectors and rates of producers are considered as input and output, respectively. Common CRM requires injection/production rate, producer's BHP to train the model. Interwell connectivity (f_{ij}) and time constant (τ_j) are considered as two unknown parameters in the CRM which can be estimated using a nonlinear regression algorithm. The common CRM was developed for oil-water systems and is based on slightly compressible fluid flow and linear productivity index equation. Such assumptions are not reasonable in the systems with gas flow and should be modified to obtain reliable results for gas-oil systems. Table 2 summarizes different researches on the CRM, its developments and modifications.

Table 2 Previous studies on the CRM development

Researcher	Description
Yousef et al. [20, 40, 47]	Mathematical derivation of the CRM by combining the productivity index model and the mass balance equation to develop a tool for the determination of flow barriers in the reservoir
Sayarpour et al. [21, 48]	Development of three equations for different control volumes in the reservoir based on whole tank, one producer and injector-producer pair
Kaviani et al. [49]	Introducing the new models by considering BHPs variation during the period and adding or shutting in an active producer
Kaviani et al. [50]	Comprehensive sensitivity analysis and presenting a dimensionless number and the range of this number where the CRM can be applied
Mamghaderi and Pourafshary [22], Moreno [51]	CRM development for multi-layer reservoirs assuming crossflow between different layers or interwell connectivity changes with time
Moreno and Lake [52, 53]	Evaluation of signal noise on the CRM performance and investigation on uncertainty of interwell connectivity estimations
Soroush et al. [54]	Studying the effect of variable production well's skin factor, adding or shutting in-active wells
Cao et al. [23]	Development of two-phase CRM using solution of the equations for total fluid and oil simultaneously.
Tao and Bryant [24]	The CRM application in gas storage problems and determination of connectivity between extractor and injector
Eshraghi et al. [55]	The CRM application in CO_2 miscible injection and optimizing CO_2 injection rate
Mirzayev and Jensen [56]	Improvement of the CRM for a low permeable reservoir with high well densities and stimulation operations
Zhang et al. [25]	Applying ensemble Kalman filter for matching the parameters in the developed multilayer CRM
de Holanda et al. [57]	Derivation of matrix format of the CRM equations by state-space model
Naudomsup and Lake [58]	Extension of CRM to tracer flow for determining reservoir properties
Wang et al. [59]	Developing improved CRM considering the effect of existence of aquifer in the reservoir
Kim [60]	Developing Stochastic CRM to mitigate lack of data limitations by combining bootstrap with CRM.

A modified CRM for gas-oil systems was developed in previous studies [27, 61]. The new model was developed considering density changes with pressure (Equation 5) and non-linear productivity equation (Equation 6).

$$V_p C_t \frac{d\overline{P}}{dt} = \frac{\rho_{g,inj}}{\overline{\overline{\rho_g}}} i(t) - \left(\frac{\rho_{o,prod}}{\overline{\overline{\rho_o}}} q_o(t) + \frac{\rho_{g,prod}}{\overline{\overline{\rho_g}}} q_g(t) \right) \tag{5}$$

$$q(t) = J(\overline{P}^2 - P_{wf}{}^2) \tag{6}$$

where, V_p: control volume pore volume, C_t: total (rock and fluid) compressibility, $\overline{P}$: reservoir pressure, t: time, $\rho_{g,ing}$: injected gas density, $\rho_{o,prod}$: produced oil density, $\rho_{g,\ prod}$: produced gas density, i: injection rate, q: total production rate, J: productivity index and P_{wf}: producer pressure.

Based on this formulation, simple ordinary differential equation (ODE) of total rate with respect to time can be shown as follows:

$$\frac{dq(t)}{dt} + \frac{1}{\tau} q(t) = \frac{1}{\tau} \frac{\rho_{g,inj}}{\overline{\overline{\rho_g}}} i(t) - 2JP_{wf} \frac{dP_{wf}}{dt} \tag{7}$$

where,

$$\tau = \frac{V_p C_t}{2J\overline{\overline{P}}} \tag{8}$$

The new model considers that τ is time-dependant and varies with pressure. Also, gas density variation effect in the equations is taken into account. The reliability of the model and impact of the two modifications were discussed in previous papers [27]. Although two variables (gas density and reservoir pressure) are added to the model as input in the M-CRM, the model accuracy is increased in comparison with the common CRM [27].

After deriving the analytical solution of Equation (7) and converting the integral form to time series, CRMP (CRM –Producer based) formulation assuming one producer with nearby injectors in a control volume of producing well j can be obtained as follows:

$$q_j(t_k) = q_j(t_{k-1})e^{\frac{-\Delta t_k}{\tau_j}} + \left(1 - e^{\frac{-\Delta t_k}{\tau_j}}\right)\left[\sum_{i=1}^{N_{inj}}\left[f_{ij}\frac{\rho_{g,inj}}{\overline{\rho_g}} i_i(t_k)\right] - 2J_j\tau_j P_{wf}\frac{\Delta P_{wf,j}}{\Delta t_k}\right] \tag{8}$$

where i , j and ij denote injectors, producers and well pairs indices respectively, while k stands for time index,. Also, f_{ij} is mathematically represented as follows:

$$f_{ij} = \frac{q_{ij}(t)}{i_i(t)} \quad f_{ij} \geq 0, \ \sum_{j=1}^{N_{prod}} f_{ij} \leq 1 \tag{9}$$

More details about the M-CRM equation and the effect of new parameters on the accuracy of the model and the number of unknowns have been discussed in [62, 63].

3. Workflow

In this research, a synthetic model and a sector model with immiscible gas injection have been selected and both LSSVM-MLR and M-CRM are applied to investigate their performance in interwell connectivity and well rate calculation. The detailed procedure for this workflow has been illustrated in Figure 2. First of all, input data are imported to the ECLIPSE FrontSim numerical simulator (streamline simulation) to calculate the outputs (producer well rate and interwell connectivities between injector and producer pairs). Validity and applicability of M-CRM and LSSVM-MLR outputs are then evaluated and compared with results of the streamline simulation. Next, required input data such as well rates, BHP, gas density and average reservoir pressure are used to build physical and statistical models. 80% and 20% of data were selected for training and testing sections, respectively. Both methods require history match (training) section. In M-CRM equations, using Genetic Algorithm (GA) optimizer, unknowns including f_{ij}, τ, J_j and q_0 are determined and then producer well rate is estimated. In statistical method, producer well rate is determined by LSSVM and then using MLR, interwell connectivities are optimized by GA. Finally, the results are compared with streamline simulation and the validity of these methods can be determined.

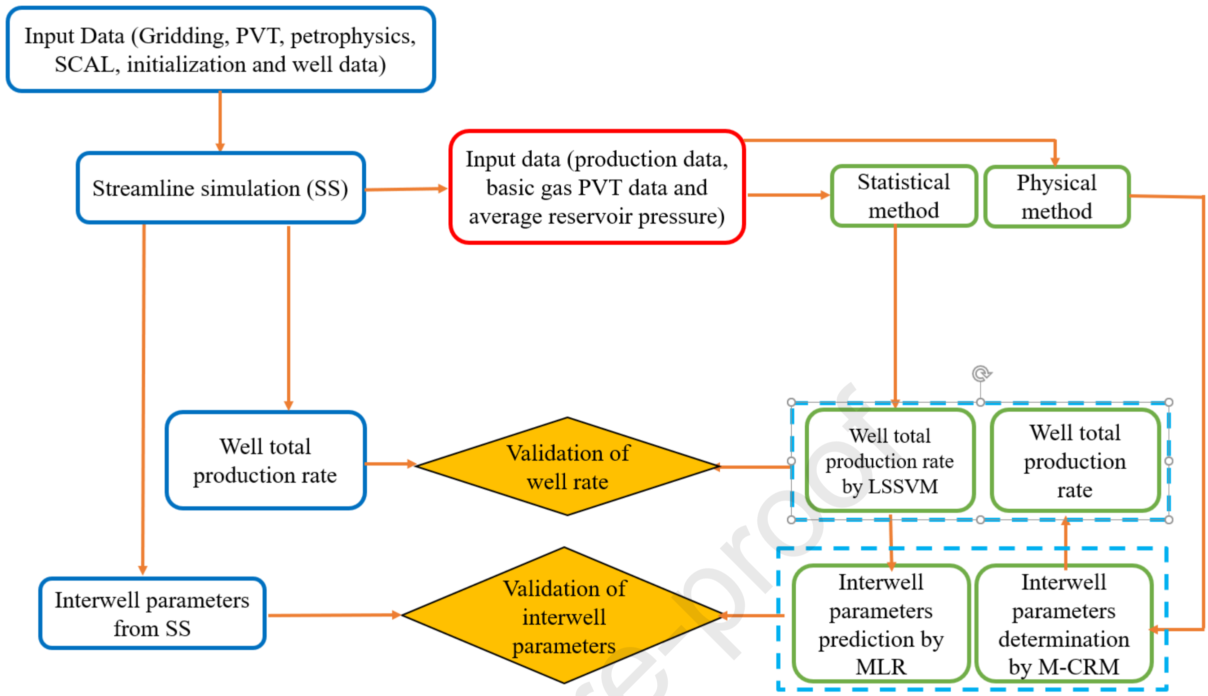


Figure 2 workflow procedure of this study

4. Results and Discussions

In this part, LSSVM-MLR and M-CRM are employed in two different cases. The first one is a simple case with one flow barrier and one high permeability streak, and the second case is a real sector model. Interwell connectivities and producers well rate for both cases are determined by both methods and a comparative study between the results is performed.

4.1. Case 1

An inverted five-spot model is considered for immiscible gas injection analysis. The reservoir is heterogeneous with respect to porosity and permeability and one sealing fault and one high permeability streak with permeability of 350 md are located in the reservoir. All layers for one injector and four producers are perforated and open to flow. The reservoir geometry with permeability distribution is shown in Figure 3. General data and rock/fluid properties are presented in Table 3.

In case 1, BHP is not constant; oil rate control for production wells and gas rate control for injection wells are selected. Minimum well pressure is controlled above bubble point pressure.

Therefore, gas production is only originated from the injected gas. The number of data points is 210 and the time window for the analysis is the period of 222-640 days after the simulation start. In this period (168 initial points for train and 42 next points for test), all of the input data are imported to the models. The well total (oil and gas) production rate in reservoir volume (bbl/day) is illustrated in Figure 4.

Figure 5 shows producer total rate from LSSVM and M-CRM vs. streamline simulation in the test section (42 points) in the format of cross plot. Based on the results, LSSVM and M-CRM predict the total rate accurately with a relative error of 3.05% and 3.13%, respectively.

Table 4 presents the results of interwell connectivities estimated by the M-CRM and the statistical method (LSSVM for rate prediction and MLR for interwell connectivity estimation). There are 16 unknowns (including f_{ij}, τ_j, J_j and q_{0j}) in M-CRM and 12 unknowns (α_i, β_j, γ_j) in MLR for Case 1. Optimization of unknowns in the M-CRM and MLR with respect to objective function and all constraints is accomplished by GA. Overall, both M-CRM and MLR can accurately predict interwell parameters. However, based on lower relative error and higher correlation coefficient, M-CRM results are more reliable than MLR (2.47% vs. 13.11% relative error). Moreover, last row in Table 4 shows that well total rate has been predicted by LSSVM and M-CRM with a similar range of accuracy. Results of well total production rate for both methods are similar and have an acceptable range of accuracy (within 3% relative error).

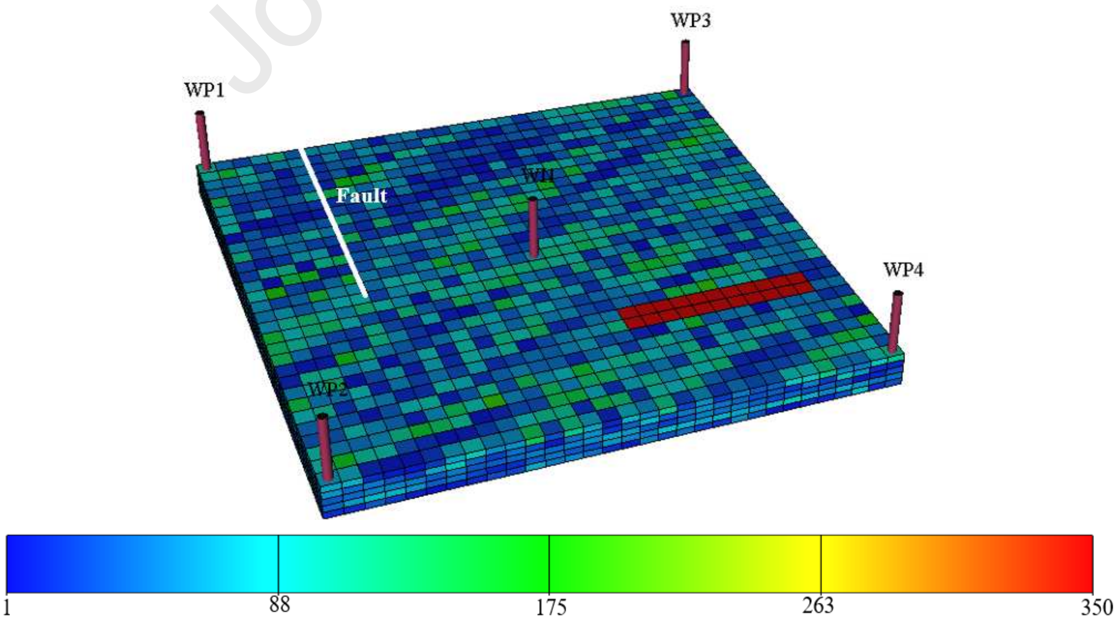


Figure 3 3D geometry and horizontal permeability of Case 1

Table 3 Rock and fluid properties of Case 1

Model dimension	31×31×5	Reservoir pressure at D.D	5200
Reservoir area, Acre	108.1	Initial condition	Oil and irreducible water
Datum depth (D.D), ft	11250	STOIIP, MMSTB	1.88
Mean porosity (%)	15	Saturation pressure, psi	4014
Mean horizontal permeability (K_h), md	57	Initial GOR, scf/STB	1270
Thickness, ft	35	Mechanism	Compressibility and gas flooding

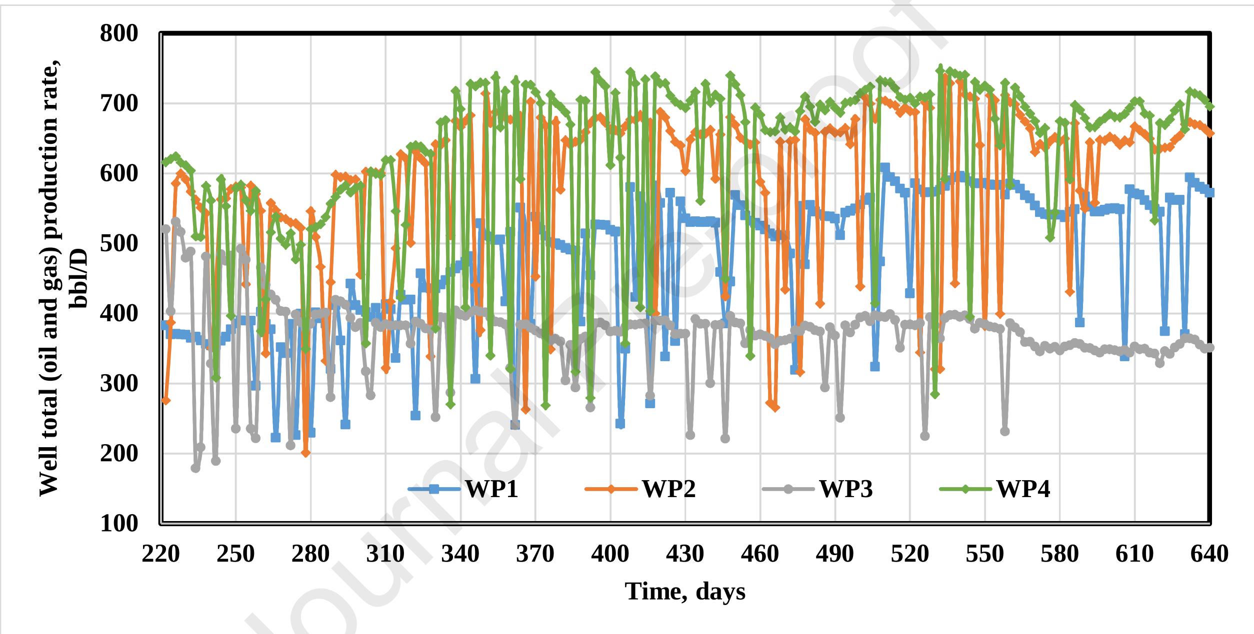


Figure 4 well total production rates (bbl/d) for producers of Case 1

Table 4 Interwell connectivity parameters of Case 1; Streamline vs. M-CRM and LSSVM

Interwell connectivity	Well pair		Streamline well factor	M-CRM well factor	Statistical well factor	M-CRM MAPE* (%)	M-CRM CC**	Statistical MAPE (%)	Statistical CC
	I1	P1	0.226	0.238	0.177	2.47	0.99	13.11	0.95
		P2	0.302	0.291	0.319				
		P3	0.169	0.168	0.154				
		P4	0.302	0.303	0.351				
Total rate production (average for all producers)						3.13	0.98	3.05	0.95

* Mean Absolute Percentage Error

** Correlation coefficient

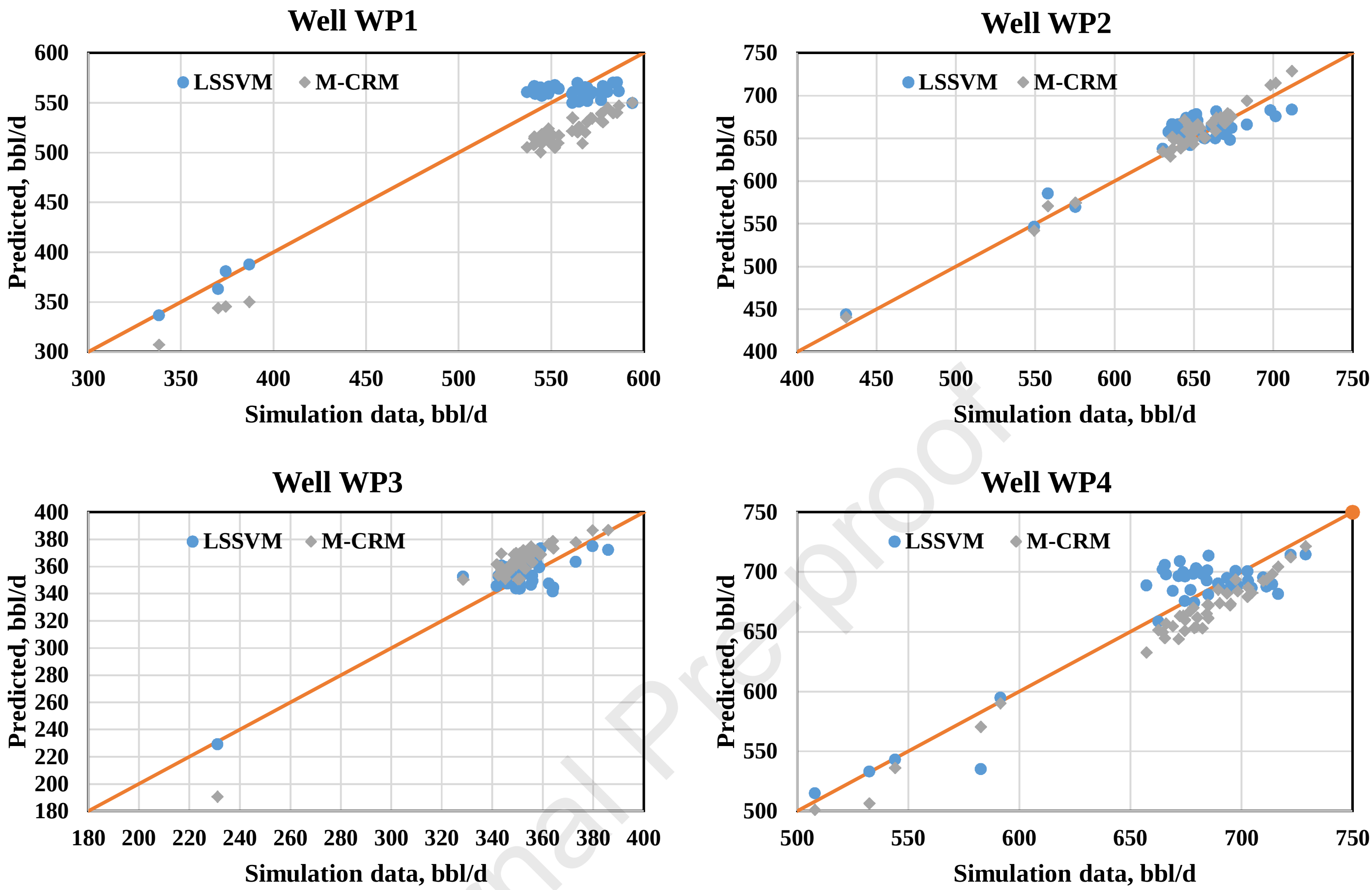


Figure 5 cross plot of well total production rates for LLSVM and M-CRM vs. streamline in test section – Case 1

4.2. Case 2 – sector model

This is a real model of an oil reservoir which is located in Iran. The lithology of reservoir is carbonate rock; dolomite with local fractures and vugs. There is no evidence for gas cap, sealing fault and aquifer in the reservoir. This sector contains 6 producers and 2 injectors. 3D configuration of the model and horizontal permeability distribution are illustrated in Figure 6 and reservoir properties of the sector are reported in Table 5. Immiscible gas injection by 2 injectors has been simulated in the model. The reservoir only contains oil and irreducible water at initial condition. Dynamic simulation data for producers and injectors are imported using the oil rate control for producers and gas rate control for injectors.

This analysis is performed for data points between the period 3600-9000 days after the simulation start (181 data points) in which the initial 145 points for training and the remaining 36 points for the test have been used. All of the input data in this period are imported to the models. Figure 7 shows the well total (oil and gas) production rate for six producers in Case 2.

The number of unknowns in this case in M-CRM and MLR methods are 30 and 24, respectively, and GA is employed for both optimization problems. Figure 8 illustrates the cross plot of total rate of six production wells for LSSVM and M-CRM vs. streamline simulation points in the test section (36 points). LSSVM results are in good agreement with streamline simulation (only 1% relative error). Therefore, LSSVM is reliable for the prediction of well rate. The relative error of 5.20% for M-CRM shows the acceptable results, However lower accuracy in comparison with LSSVM.

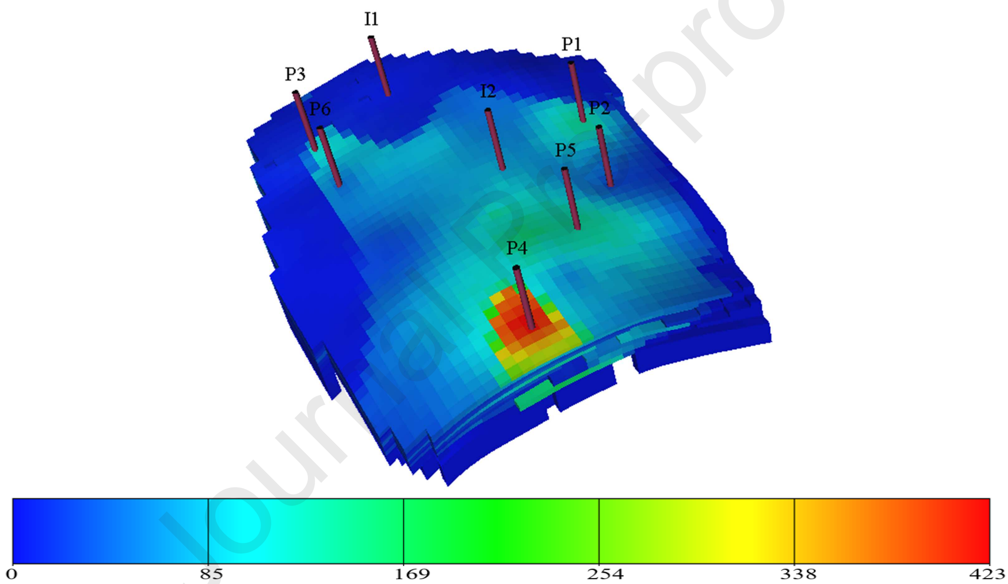


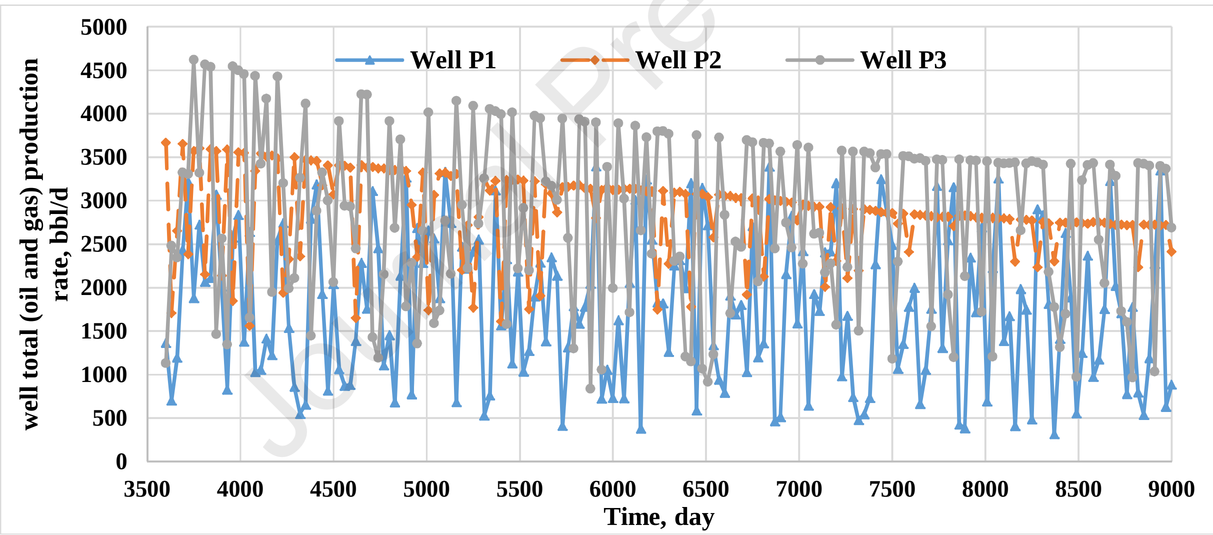
Figure 6 3D geometry and horizontal permeability distribution of the sector model

Table 5 Rock and fluid properties of the sector model

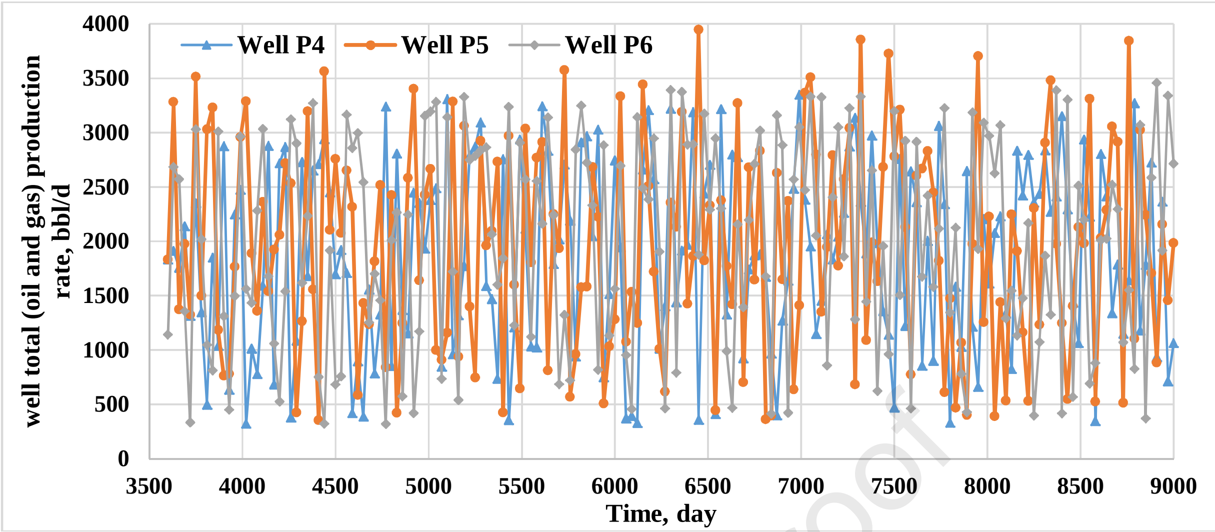
Model dimension	35×44×11	Initial condition	Oil and irreducible water
D.D, ft	8645	Total wells	6 (production wells) and 2 (injection wells)
Mean porosity (%)	5.35	S_{wir}	0.12
Mean horizontal permeability (K_h), md	39.97	S_{or}	0.35
Average NTG*	0.87	Active cells	11929
Initial pressure @ D.D, psi	4900	STOIIP, MMSTB	679.93
Rock compressibility, psi^{-1}	3×10^{-6}	Bubble point pressure, psi	3157
Reservoir temperature, deg F	140	Mechanism	Compressibility and gas flooding

* Net to Gross ratio

Table 6 summarizes interwell connectivities and producer total rates which were calculated by statistical and physical methods. Based on these results of MAPE and correlation coeffieicent in the last row in Table 6, LSSVM is more accurate than M-CRM in well total rate prediction (1.19% vs. 5.20% relative error); however, M-CRM is a more reliable tool for prediction of interwell connectivities compared to MLR (11.95% vs. 46.2% relative error). Therefore, quantitative analysis of interwell connectivity should be performed by physical model and only qualitative connectivity evaluation is recommended by statistical method i.e. well connectivities for I1 injector show that P3 and P6 have strong connectivity, However, P4 and P5 have a weak connection with the injector. Again for I2, P2 and P4 connectivities are strong and P3 is weak. Hence, it is possible to determine which well pairs are connected together qualitatively based on the results. However, the exact determination of interwell connectivity by LSSVM is not possible due to its high relative error.

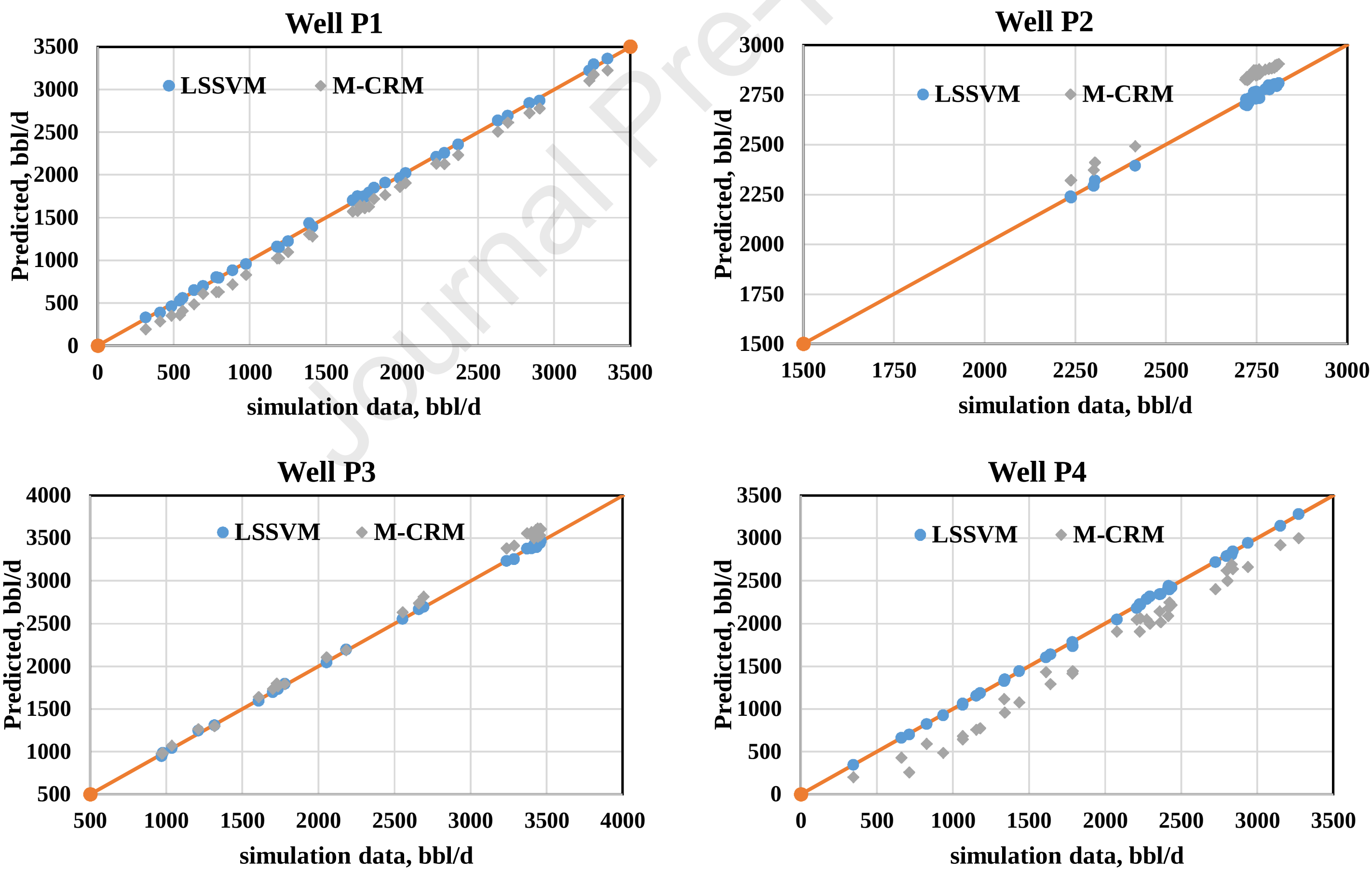


A - total production rates of wells P1, P2 and P3



B - total production rates of wells P4, P5 and P6

Figure 7 well total production rates (bbl/d) for producers of Case 2



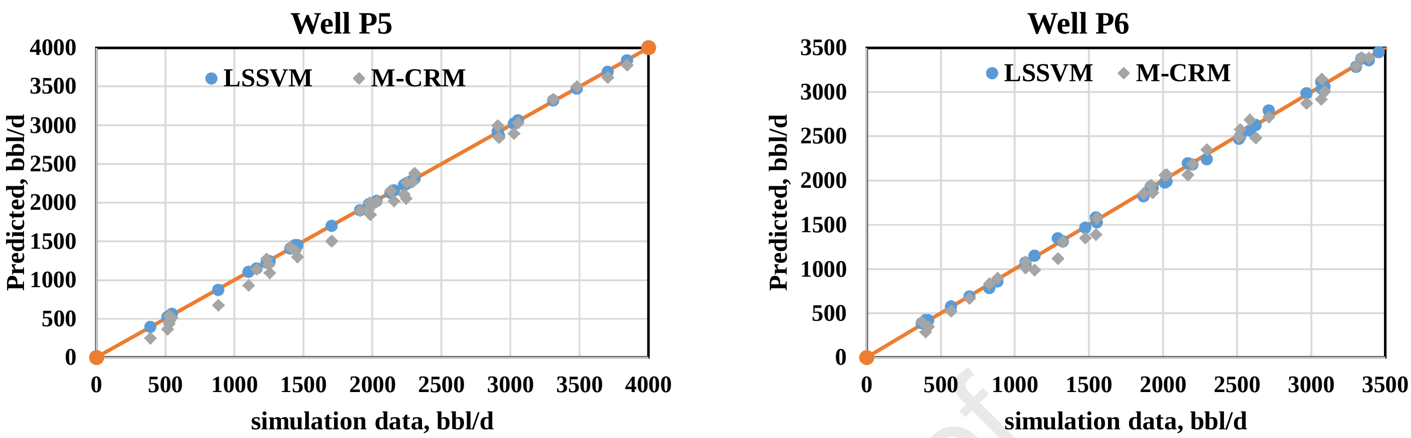


Figure 8 cross plot of well total production rates for LLSVM and M-CRM vs. streamline in test section – sector model

Table 6 Interwell connectivity calculation of Case 2; Streamline vs. M- CRM and LSSVM

	Well pair		Streamline well factors	M-CRM well factor	Statistical well factor	M-CRM MAPE (%)	M-CRM CC	Statistical MAPE (%)	Statistical CC
Interwell connectivity	I1	P1	0.141	0.120	0.086	11.95	0.99	46.20	0.81
		P2	0.025	0.023	0.012				
		P3	0.548	0.554	0.407				
		P4	0.011	0.009	0.007				
		P5	0.004	0.000	0				
		P6	0.271	0.278	0.488				
	I2	P1	0.115	0.106	0.084				
		P2	0.362	0.353	0.215				
		P3	0.005	0.007	0.006				
		P4	0.201	0.177	0.161				
		P5	0.252	0.242	0.399				
		P6	0.065	0.070	0.136				
Total rate production (average for all producers)						5.20	0.99	1.19	1.00

4.3. Analysis of the results

Overall, M-CRM and statistical methods have some advantages and both of them are applicable to different types of reservoirs. In this section, M-CRM and LSSVM-MLR methods are compared in terms of input data, accuracy and speed.

- Results of two different cases (see Table 4 and Table 6) show that for the calculation of producer total rate, statistical method (LSSVM) gives more accurate results than M-CRM; however, the range of accuracy for M-CRM is acceptable. Interwell connectivities determined by M-CRM are more reliable than results of statistical method (MLR), and it is recommended to use the MLR method just as a qualitative method.

- M-CRM uses average reservoir pressure and gas density besides well rates and producer pressure which leads to more reliable results in immiscible gas flooding projects compared with common CRM. Therefore, in physical approaches such as M-CRM, input data are very important. However, the flexibility of input data in statistical approaches is high i.e. the type and the number of input data can be changed case by case based on the learning algorithm results. It leads to find the most important input data for each case and make it more flexible. However, it cannot be generalized for all cases. In this paper, for LSSVM, injector rates and producer BHPs are the input which give accurate results. However, it may be needed to consider other parameters for another case.
- The order of speed in both M-CRM and statistical methods is a couple of minutes. However, in the streamline, it takes hours for simulation. Training section which needs optimization algorithm for determination of unknowns, does not take more than a few minutes. Statistical method which is a combination of LSSVM and MLR requires a two-step procedure and may therefore require more time. Also, using GA in M-CRM may need some time. Summing up, both methods run at a higher speed in comparison with conventional numerical simulators

5. Conclusions

In this research, physical method (M-CRM) and statistical method (LSSVM/MLR) were employed to obtain interwell connectivities and producer well rate in immiscible gas flooding projects. Both methods were examined in two different cases and the results were compared. Streamline simulation was used as a reference tool for validation of the results. The main advantages of these two methods over numerical simulators are their simplicity, speed and the limited number of input data. The following results were concluded from this study:

- M-CRM applies to immiscible gas flooding projects by importing average reservoir pressure and gas density in the common CRM equations. Interwell connectivities and producer total rate as outputs of M-CRM were obtained with an acceptable range of accuracy.

- LSSVM was employed to predict producer total rate in two cases using injection/production data and BHP as inputs. The results show that this method gives reliable estimations.
- To obtain interwell connectivities, MLR was used. MLR predictions are less precise compared with M-CRM results; however, this method could be used for qualitative analysis.
- A comparison of physical (M-CRM) and statistical (LSSVN-MLR) models in terms of validity, precision, data requirements and speed revealed their advantages and limitations. M-CRM is more precise than statistical model. However, statistical model is more flexible than M-CRM. Calculation speed for both methods is similar.

6. Appendix A

LSSVM equations

The following constraints are applied to the cost function:

$$
\begin{aligned}
& y_k - w^T\varphi(x_k) - \mathrm{b} \le \varepsilon + \xi_k, && k = 1,2,3,....,N \\
& w^T\varphi(x_k) + \mathrm{b} - y_k \le \varepsilon + \xi_k^*, && k = 1,2,3,....,N \\
& \xi_k, \xi_k^* \ge 0, && k = 1,2,3,....,N
\end{aligned}
\tag{A1}
$$

where x_k, y_k and ε stand for k[th] input, k[th] output and precision of the approximation, respectively. For minimization purpose, the Lagrangian of the problem is applied as:

$$L\ (a\ \ , a^*) = -\frac{1}{2}\sum_{k,l=1}^{N}(a_k - a_k^*)(a_l - a_l^*)K(x_k - x_l) - \varepsilon\sum_{k=1}^{N}(a_k - a_k^*) + \sum_{k=1}^{N} y_k(a_k - a_k^*) \tag{A2}$$

$$\sum_{k=1}^{N}(a_k - a_k^*) = 0,\ a_k,\ a_k^* \in [0, c] \tag{A3}$$

$$K\ (x_k - x_l) = \varphi(x_k)^T\varphi(x_l)\ \ ,\ \ k = 1,2,....,N \tag{A4}$$

a_k and a_k^* are Lagrangian multipliers. Therefore, SVM becomes:

$$f(x) = \sum_{k,l=1}^{N}(a_k - a_k^*)K(x\ - x_k) + \mathrm{b} \tag{A5}$$

Unknown parameters in Equation (A5) should be obtained by quadratic programming. This method may result in computational problems. Therefore, Suykens and Vandewalle [42, 43] introduced Least Square SVM (LSSVM) to overcome the problems. The following equation shows the cost function of the proposed method:

$$Cost\ function = \frac{1}{2}w^T w + \frac{1}{2}\gamma\sum_{k=1}^{N} e_k^2 \tag{A6}$$

where, γ and e_k are the tuning and error parameters in the LSSVM, respectively. The constraint for this function is as follows:

$$y_k = w^T \varphi(x_k) + b + e_k \tag{A7}$$

Again, for minimization of the function, Lagrangian function is used:

$$L(w,b,e,a) = \frac{1}{2} w^T w + \frac{1}{2} \gamma \sum_{k=1}^{N} e_k^2 - \sum_{k=1}^{N} a_k \left(w^T \varphi(x_k) + b + e_k - y_k \right) \tag{A8}$$

Therefore, to optimize the problem, the following expressions should be established:

$$\begin{aligned}
&\frac{\partial L}{\partial w} = 0 \;\Rightarrow w = \sum_{k=1}^{N} a_k \varphi(x_k) \\
&\frac{\partial L}{\partial b} = 0 \;\Rightarrow \sum_{k=1}^{N} a_k = 0 \\
&\frac{\partial L}{\partial e_k} = 0 \;\Rightarrow a_k = \gamma e_k, \;\; k = 1,2,....,N \\
&\frac{\partial L}{\partial a_k} = 0 \;\Rightarrow w^T \varphi(x_k) + b + e_k - y_k = 0 \quad k = 1,2,....,N
\end{aligned} \tag{A9}$$

According to Equation (A9), $2N$+2 unknowns and $2N$+2 equations exist. Hence, all unknowns in LSSVM could be obtained.

7. Nomenclature

- ***Variables***

ANN	Artificial Neural Network
a_k	Lagrangian multiplier
BHP	Well bottom hole pressure
b	bias
CRM	Capacitance-Resistance Model
CRMP	Capacitance-Resistance Model (Injector/Producer based)
CC	Correlation coefficient
C_t	total compressibility
c	tuning parameter of the SVM
D.D	Datum depth
e_k	error parameter in the LSSVM
f	interwell connectivity between injector and producer
GA	Genetic Algorithm
GOR	gas-oil ration
IMPES	implicit pressure explicit saturation
i	injection rate
J	productivity index
LSSVM	least square support vector machine
M-CRM	Modified Capacitance-Resistance Model
MAPE	mean absolute percentage error
MLR	multiple linear regression
MPI	multi-well productivity index
N_{inj}	total number of injection wells
N_{prod}	total number of production wells
NTG	Net to Gross
ODE	Ordinary Differential Equation
$\bar{P}$	average reservoir pressure
P_{wf}	flowing wellbore pressure
q	Total (oil and gas) production rate
q_g	rate of gas production
q_o	rate of oil production
S	saturation
SRC	Spearman Rank Correlation
STOIIP	Standard oil initially in-place
SVM	Support Vector Machine
t	time
V_p	Pore Volume
w^T	transposed output layer vector

- ***Greek symbols***

τ	time constant
ρ	density

$\bar{\rho}$	average density
$\varphi(x)$	Kernel function
ξ_k	slack variables
γ	tuning parameter in the LSSVM
ε	fixed precision of the function approximation

- ***Subscripts and superscripts***

g	gas
i	injection well index
ij	well pair (injector-producer) index
j	production well index
k	time index
o	oil
obs	observed output
or	residual oil
pred	predicted output
wir	irreducible water

8. Unit conversion

Pressure	1 psi	=	6.895 E+3	Pa
Volume	1 bbl	=	1.590 E-1	m^3
Volume	1 scf	=	2.831 E-2	sm^3
length	1 ft	=	3.048 E-1	m
permeability	1 md	=	9.869 E-16	m^2
area	1 Acre	=	4.047 E+3	m^2

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Highlights

- Interwell connectivity in gas flooded reservoir was calculated by two methods.
- Statistical method of support vector machine was used for well rate prediction.
- The accuracy of support vector machine and linear regression was discussed.
- Statistical method was compared with modified capacitance resistance model.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: