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Assessment of mixed mode fatigue crack growth under biaxial loading using an iterative technique

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Abstract

Crack growth is investigated numerically in a cruciform specimen with an inclined center crack subjected to biaxial fatigue loading. The mixed-mode fatigue crack growth was examined using a parametric study both on the biaxial stress ratio and the initial crack angle. A fatigue crack growth code for two-dimensional problems was developed to investigate the mixed-mode behavior of fatigue crack growth under biaxial loading. The effects of initial crack angle and the biaxial stress ratio on the specimen fatigue life are explored using finite element analysis and the results are discussed.

Keywords: Biaxial fatigue; Crack propagation; Fatigue crack growth; Finite element analysis.

1. Introduction

Cracks can be found in many structural parts during their service life. In structures such as pressure vessels and aerospace structures the loading is biaxial tension or compression and an inclined micro or macro crack present in the structure will grow under biaxial fatigue loading. In this case, the fatigue crack growth is also influenced by the presence of mixed mode loading conditions (mode I and II) at the crack tip. Under complex biaxial loading states the fatigue crack may grow in a mixed mode manner. There are several suggestions for the correlation of mixed mode

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fatigue crack growth under general uniaxial and biaxial loading conditions. Liu and Dittmer reported that biaxiality has no effect on the crack growth rates under constant amplitude loading (Liu and Dittmer, 1978). Yuuki et al. (1989) observed that only at high stresses the biaxiality has effect on the crack growth rates. Anderson and Garrett (1980) also observed a close relationship between crack growth rate and biaxial stress field.

The present paper is concerned with the problem of fatigue crack growth under the influence of biaxiality and mixed-mode loading conditions. The direction of crack propagation and cyclic life are discussed for a cruciform specimen made of aluminum alloy, and the results are compared with those when biaxiality is not present.

Nomenclature

a	crack length
a_0	structural crack length
C, n, p, q	empirical coefficients
C_{th}	curve control coefficient for different values of R
da/dN	crack growth rate
f	Newman's function
K_I	mode-I stress intensity factor
K_{II}	mode-II stress intensity factor
K_C	critical value of SIF
K_{max}	SIF for the maximum load in the cycle
K_{min}	SIF for the minimum load in the cycle
ΔK	*SIF range* = $K_{max} - K_{min}$
ΔK_{eff}	equivalent SIF range ΔK for mixed-mode I and II loading condition
ΔK_{th}	SIF threshold, i.e. minimum value of ΔK, from which the crack starts to propagate
N	number of load cycles
R	stress ratio
α, S_{max}/σ_0	Newman's empirical coefficients
Δa	crack growth incremental length
$\Delta\sigma_{\theta\ max}$	maximum tangential stress range at the crack tip
θ	angle of initial crack
θ_c	angle between the initial direction and the direction of new crack growth increment
APDL	ANSYS parametric design language
FCG	fatigue crack growth
LEFM	linear elastic fracture mechanics
MTS	maximum tangential stress criterion
SIF	stress intensity factor

2. Computational method for fatigue crack growth modeling

To investigate the fatigue crack growth behavior of cruciform specimens under mixed-mode biaxial loading, a numerical methodology is used to estimate the remaining fatigue life of components. The fatigue crack growth and the crack path prediction models required for the analyses are described below.

In the linear elastic fracture mechanic (LEFM), fatigue life is usually estimated for a cracked specimen by using an exponential function of SIF. An approach that describes all sections of the *da*/*dN* diagram is the so-called NASGRO equation, which is written as (AFGROW®, 1980).

$$\frac{da}{dN} = C\,(\frac{1-f}{1-R}\Delta K)^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_c}\right)^q} \tag{1}$$

The FCG life and FCG rate values determined by using the NASGRO model have been shown to be in good agreement with the experimental results. In Eq. 1, a is the crack length, N is the number of load cycles, C, n, p and q are the empirical coefficients, R is the stress ratio, ΔK_{th} is the SIF threshold (i.e. minimum value of ΔK from which the crack starts to propagate), K_C is the critical value of SIF and f is the Newman's function describing the crack closure. Moreover, the SIF range ΔK depends on the size of the specimen, the applied loads and the crack length ($\Delta K = K_{max} - K_{min}$), and K_{max} and K_{min} are the values of SIF corresponding to the maximum and minimum loads in the cycle. The coefficients of NASGRO equation have reported for some engineering materials in AFGROW database. For 6061-T651 aluminium alloy, these coefficients are given in Table 1.

Table 1. The coefficients of NASGRO equation for Al 6061-T651 and stress ratio of R = 0.1, from AFGROW®.

α	S_{max}/σ_0	a_0 (mm)	ΔK_{th} (MPa.m$^{0.5}$)	K_C (MPa.m$^{0.5}$)	C	n	p	q
1.5	0.3	0.0381	3.846	59.338	2.733e-9	2.248	0.5	1

Kitagawa et al. (1981) extended the maximum tangential stress criterion to the fatigue crack propagation. They assumed in this modified criterion that the direction θ_c corresponds to that of the maximum tangential stress range $\Delta\sigma_{\theta\ max}$ at the crack tip, as

$$\theta_C = 2\tan^{-1}\left\{0.25\left[\frac{\Delta K_I}{\Delta K_{II}} \pm \sqrt{\left(\frac{\Delta K_I}{\Delta K_{II}}\right)^2 + 8}\right]\right\} \tag{2}$$

where θ_c is the angle between the initial direction and the direction of new crack growth increment.

According to the mixed mode condition of the crack growth, an equivalent SIF must be used, so the equivalent SIF range ΔK of the mixed-mode I and II crack (Tanaka et al., 2005) is assumed as

$$\Delta K_{eff} = \Delta K_I \cos^3\left(\frac{\theta_C}{2}\right) - 3\Delta K_{II}\cos^2\left(\frac{\theta_C}{2}\right)\sin\left(\frac{\theta_C}{2}\right) \tag{3}$$

The value of θ_c in the above equation is obtained from Eq. 2.

In the present study, the fatigue crack initiation and growth are simulated by an iterative procedure that is based on the fatigue models described earlier. For this purpose, the finite element software ANSYS is linked to the fatigue code to simulate the initiation and extension of crack. The SIF values required for the fatigue models are calculated automatically by ANSYS and are used as input data for the FCG code. A constant prespecified incremental length of crack growth is considered in every computation step (Fig. 1). If the crack growth incremental length (Δa) and the numerical results of the effective SIF range ΔK_{eff} before and after the crack extension in each step are substituted into Eq. 1, the number of load cycles for each step of crack propagation can be determined. The summation of the values of incremental load cycles gives the total FCG life at the end of each iteration (N_i). The crack geometry is redefined by the extension of incremental crack segment in every iterative computation step. The FE mesh is modified, and the previous computational steps are repeated until the crack length reaches its critical length for which $K = K_C$. The main objective of fatigue analysis was to investigate the effect of mode mixity on the fatigue life of cracked components under biaxial loading. This numerical technique has been previously used for more simple cases to evaluate fatigue crack initiation and propagation under various loading condition. Further validation of the methodology can be found in (Ayatollahi et al., 2014a,b, 2015, 2016; Razavi et al. 2017).

3. Numerical model

A cruciform specimen with the initial crack length of a = 10 mm was considered for fatigue analyses with linear elastic properties assumption. The geometry of specimen and its finite element model is illustrated in Fig. 2. The specimen was assumed to be made from a 6061-T651 aluminum alloy with the Young's modulus of 68.9 GPa and the

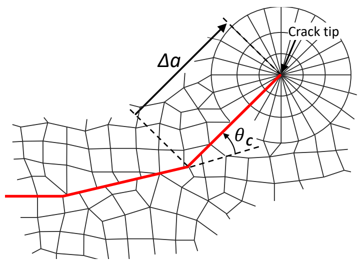

Fig. 1. Schematic illustration of incremental fatigue crack growth.

Poisson's ratio of 0.33.The fatigue analyses were conducted under constant amplitude fatigue loading at the load ratio of $R = 0.1$. The initial angle of crack (θ, theta) varied as 0, 15°, 30° and 45°. Also, the analyses were conducted under loading conditions with biaxiality ratios of $L = -0.5$, 0, 0.5 and 1 ($L = F_x / F_y$). The maximum level of vertical applied cyclic loading of 2.5kN were considered in the analyses. The 6-node plane strain elements were used in the finite element models. Higher mesh density was used near the crack tip to improve the accuracy of the results. Besides, the singular elements were used for the first ring of elements around the crack tip. A mesh convergence study was also undertaken to ensure that a proper number of elements was used in fatigue loading modeling. The appropriate values of the crack growth incremental length (Δa) and the crack tip element size were found to be equal to 1mm and 0.1mm, respectively.

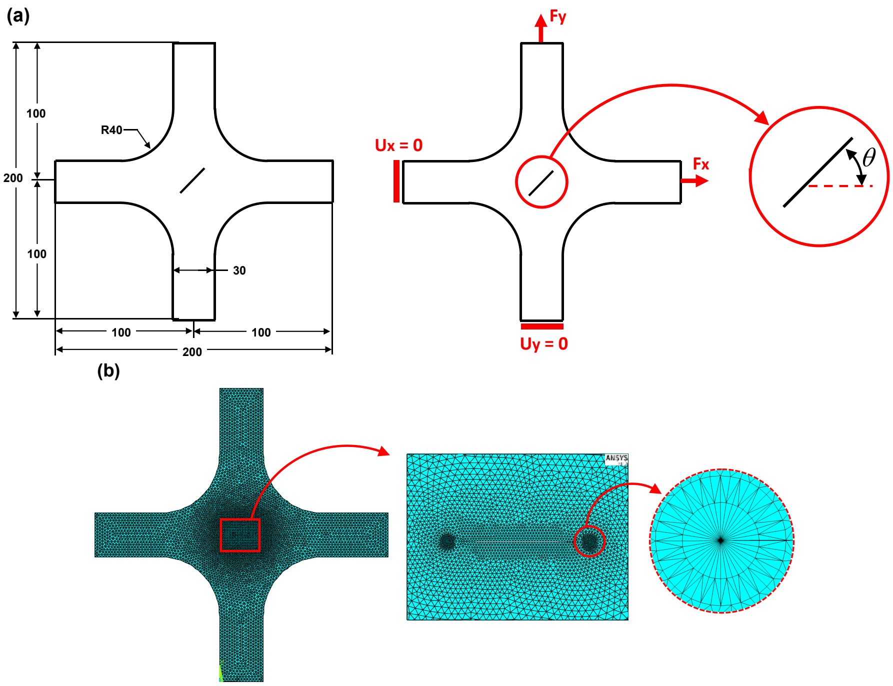

Fig. 2. Geometry, boundary conditions and finite element model of the cruciform specimen.

4. Results and Discussion

The FCG code validated previously is now developed to study the effect of mixed mode biaxial loading on the FCG path and FCG life in a cruciform specimen. The angle of initial crack was varied within a specific range as described in section 3. Fig. 3 illustrates the FCG path for different mode mixites and biaxiality ratios. For $\theta = 0$, the biaxiality ratio didn't affect the FCG path and for all biaxiality ratios the FCG trajectory was straight. But, by increasing the θ, the biaxiality ratio shows its role in the FCG tracking. According to the Fig. 3, it is obvious that the most effect of biaxiality ratio on the path is resulted for $\theta = 45^o$. For all cases, the higher biaxial loading ratios causes more crack inclination in the FCG process.

The ratio of the mode-II SIF range to mode-I SIF can show the mode mixity condition of the fatigue crack. The values of mode mixity ratios are given in Table 2. It can be seen that for initial crack angle of zero, in different biaxial ratios, the value of mode mixity ratio has a small value showing that the FCG is mostly under mode-I loading condition and it grows on its initial direction. For the specimens with initial crack angles of 15°, 30° and 45°, the mode-I condition is occurred for the biaxial ratio of L = 1. For the non-zero initial crack angles (θ), increasing the biaxial ratio causes the loading condition to accede to mode-I loading condition.

Fig. 4 illustrates the fatigue crack growth curves for different mode mixities under uniaxial and biaxial loading conditions. In FCG problems, the mode-I loading is more critical than the mode-II and the FCG life of cracked specimens under mode-I is less than the FCG life of mode-II and mixed mode loading conditions. According to Fig. 4(b) for biaxial loading condition (L = 1), higher initial crack angles resulted in shorter FCG lives. This observation is due to higher levels of SIFs in the case of initial crack with non-zero initial angles. The FCG life behavior for the specimen under biaxial loading of L = 1 is unlike the behavior of the specimen under uniaxial. For uniaxial loading condition, increasing the initial crack angle increases the FCG life.

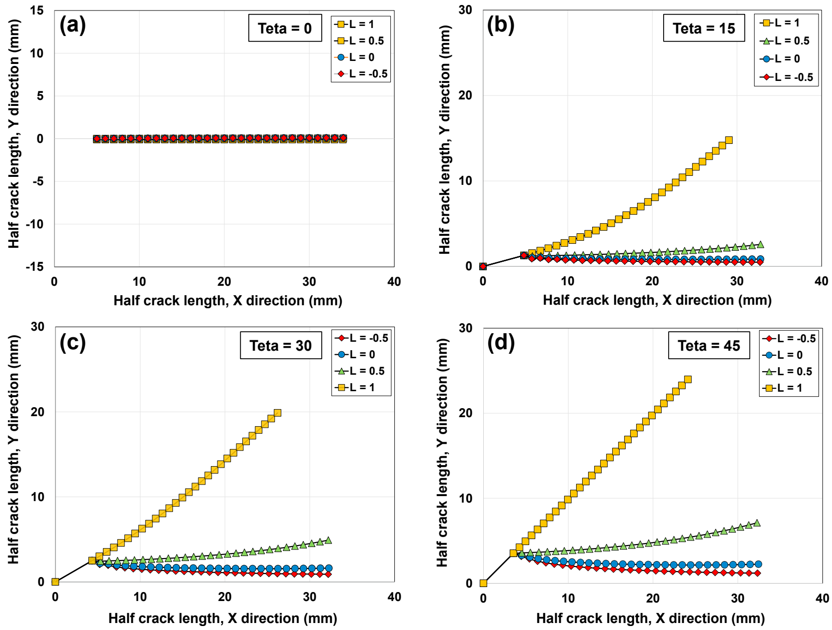

Fig. 3. Fatigue crack growth path for different loading conditions.

Table. 2. The ratio of the SIF mixed mode range for different loading conditions.

θ (degree)	L	$\Delta K_{II} / \Delta K_I$	θ (degree)	L	$\Delta K_{II} / \Delta K_I$
0	-0.5	-0.010	30	-0.5	1.209
0	0	-0.006	30	0	0.770
0	0.5	-0.001	30	0.5	0.367
0	1	0.005	30	1	-0.004
15	-0.5	0.454	45	-0.5	4.813
15	0	0.330	45	0	1.636
15	0.5	0.180	45	0.5	0.552
15	1	-0.004	45	1	0.005

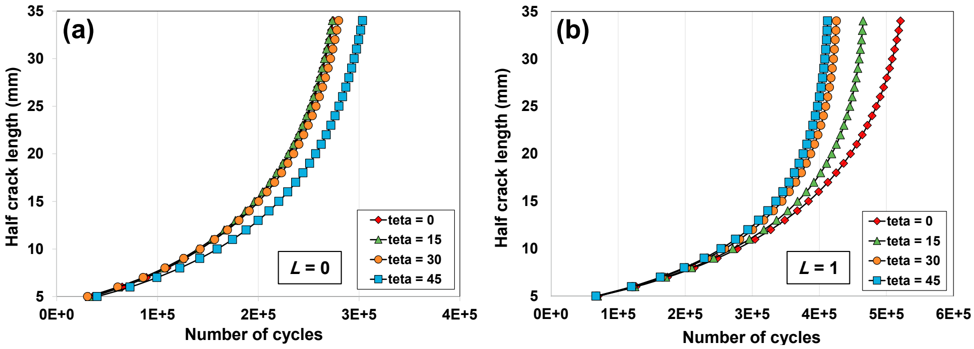

Fig. 4. Fatigue crack growth life for different loading conditions

5. Conclusion

The effects of mode mixity on the biaxial loading condition was studied numerically in center cracked cruciform specimens made of 6061-T651 aluminum alloy. All specimens with initial crack angle experienced the mixed mode loading conditions. In the specimens with zero initial angle, the biaxiality ratio of loading didn't affect the FCG path. For the biaxiality ratio of $L = 1$ the fatigue crack propagated on its initial angle and the FCG trajectory didn't changed. The FCG life behavior for the specimen under biaxial loading of $L = 1$ is unlike the behavior of the specimen under uniaxial. For this loading condition, increasing the initial crack angle decreased the FCG life. The related results are limited to specific biaxiality ratios investigated in this paper; the same approach can be developed to estimate the FCG behavior of cracked specimens under more loading.

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