Collision Avoidance using Mixed H_2/H_∞ Control for an Articulated Intervention-AUV

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***Abstract*— In this paper we consider the problem of mixed H_2/H_∞ control to combine optimal and robust control for a double integrator system with nonlinear performance variables, and we apply this to control an articulated intervention autonomous underwater vehicle (AIAUV). The AIAUV has an articulated body like a snake robot, is equipped with thrusters, and can be used as a free-floating underwater manipulator. The objective is to control the joints of the AIAUV to desired setpoints without causing collisions between links or with obstacles in the environment. The mixed H_2/H_∞ problem is viewed as a differential game, and a set of matrix equations is solved in order to construct an approximate solution to the problem for a system described by double integrator dynamics and with nonlinear performance variables. A feedback linearising controller is derived to obtain the double integrator dynamics for the joints of the AIAUV, and the solution found for the mixed H_2/H_∞ control problem is applied to the resulting system. Simulations demonstrate that collisions between links of the manipulator are successfully avoided also in the presence of parameter uncertainties while regulating the joints to the desired setpoints, and the method can easily be extended to include collision avoidance with static and dynamic obstacles in the environment.**

I. Introduction

Robotic systems are increasingly taking over tasks in environments which are dangerous or inaccessible to humans, an example of which are deep seas. Exploration and intervention for research and industry purposes are more and more often not only performed by machines, but also done autonomously, removing the need for involving a human operator.

An articulated intervention autonomous underwater vehicle (AIAUV) combines the jointed body of an underwater snake robot with thrusters, which enable it to propel itself forward or hover in one place [1]. This, combined with its articulated body, allows it to be used as a free-floating manipulator arm to perform inspection, maintenance and light intervention tasks. The slender, articulated body allows it to access narrow, confined spaces, which makes it well-suited for operating at sites such as underwater constructions, caves or ship wrecks. However, in order to operate within confined spaces, the AIAUV must avoid colliding with obstacles in the environment, and also avoid collision with itself.

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Being an underwater vehicle, the AIAUV is subject to hydrodynamic effects, the parameters of which are difficult to identify [2]. This calls for robust control methods capable of handling parameter uncertainties, as well as other disturbances. Some robust control approaches have been considered for the AIAUV already in e.g. [3] for tracking of pre-planned trajectories for the joints and position of the vehicle. While collisions with static obstacles can be prevented by planning appropriate trajectories using methods such as those in e.g. [4], collisions with dynamic obstacles require a different approach.

Avoiding collisions with both static and dynamic obstacles was considered already in [5], where desired joint velocities of a manipulator arm are modified to avoid an obstacle, so long as it does not interfere with the main task. Multiple tasks in strict priority can be combined in a similar manner [6]. These approaches require the manipulator to be kinematically redundant, i.e. that it has more degrees of freedom (DOFs) than required to perform its primary tasks, which enables it to satisfy multiple objectives. This redundancy resolution is usually performed at the kinematic level, requiring a separate dynamic controller, but approaches including dynamics also exist [7]. Objectives such as collision avoidance, where the goal is to keep a task variable within a set of values, can be accomplished by considering them only when the variable is outside of or exiting the set of desired values, and deactivating them otherwise [8]. This results in discontinuous velocity or acceleration references. Methods for smooth transitions exist [7], [9], but in [7], where both kinematics and dynamics are considered, priority is lost during transitions. This allows lower-priority tasks to interfere with higher-priority ones, the consequences of which could be catastrophic in the case of safety-critical, high-priority tasks. Loss of priority is not an issue in [9], but here only the kinematic level is considered. Robustness of such approaches against modelling errors is examined in [10] for the case of redundancy resolution at the dynamic level, and errors are found to only affect the transient behaviour of the system. In the case of redundancy resolution at the kinematic level, robustness can be achieved by pairing it with a robust dynamic controller, such as in [11]. Neither of the works [10], [11] consider activation and deactivation of tasks, which is necessary for implementing tasks such as collision avoidance.

While the aforementioned methods have been developed primarily for redundant robotic manipulators, a method of collision avoidance which is applicable to a larger class of systems relies on the use of artificial potential fields, designed to push a robot or parts of it away from obstacles and towards a goal [12]. This is a local approach and vulnerable to the presence of local optima or equilibria. This issue has been addressed in [13], for scenarios in which the location of obstacles in the environment is known beforehand.

Local equilibria, such as e.g. deadlocks are avoided in [14], [15], where collision avoidance for multi-agent systems is achieved by formulating the problem as a differential game. A benefit of this game theoretic formulation is that it enables multiple objectives, which may or may not be conflicting, to be considered simultaneously. This can also include robustness with respect to disturbances. The problem of mixed H_2/H_∞ control, which concerns the combination of optimal control and disturbance attenuation, is formulated as a game played by the control input and an exogenous disturbance in [16] for linear systems, and in [17] for a more general class of nonlinear systems. Mixed H_2/H_∞ control for robotic manipulators was considered in [18] for tracking joint angle trajectories, but only with a quadratic optimisation objective with constant weights. For general nonlinear system dynamics and more complex objectives, approximate solutions to differential game problems can be found by using the approach in [19]. However, this requires the solution to a set of matrix equations which may be difficult to solve. The agents considered in [14], [15] satisfy or are made to satisfy single integrator dynamics by using feedback linearisation. An explicit solution for a manipulator not subject to any damping or gravitational forces was found in [20], for the case of either optimal control or H_∞-control.

In this paper, we consider the problem of regulating the joints of an AIAUV to desired setpoints, while simultaneously avoiding collisions, in the presence of parameter uncertainties. In order to obtain a tradeoff between optimality and robustness, we use the method of H_2/H_∞ control. We first develop a mixed H_2/H_∞ controller for the general class of systems described by double integrator dynamics, with nonlinear performance variables. Specifically, we find a solution to a set of equations required to apply the result from [17] to such systems. We then use a feedback linearising input to simplify the joint dynamics of the AIAUV to those of a double integrator, and combine it with the solution found for the mixed H_2/H_∞ control problem. Collision avoidance between the links of the AIAUV is demonstrated in simulations, and the method can easily be extended to consider also collisions with static and dynamic obstacles in the environment. Since the problem of mixed H_2/H_∞ control considers disturbance attenuation, the result can be used to compensate for possible partial cancellation of terms in the feedback linearisation due to parameter uncertainties. This is especially important for underwater systems, which are subject to hydrodynamic effects that are difficult to identify. The solution found in this paper is applicable to a wide class of systems, in particular mechanical systems for which feedback linearisation can be used to simplify the system to double integrator dynamics. It is also applicable to multi-agent systems in which all agents collaborate to accomplish a common objective.

This paper is organised as follows: the method for constructing solutions of (general) mixed H_2/H_∞ control problems presented in [17] is recalled in Sec. II, where also the specific solution corresponding to systems described by double integrator dynamics is provided. In Sec. III the AIAUV model, the feedback linearising input for the joints and the objective used for collision avoidance are presented. Simulations illustrating the results from Secs. II and III are provided in Sec. IV, and conclusions are given in Sec. V.

II. Mixed H_2/H_∞ control

In this section we recall the method for constructing approximate solutions to the differential game formulation of the mixed H_2/H_∞ control problem provided in [17], and we find the solution required to apply the result to a system described by double integrator dynamics. While the dynamics themselves are linear, the performance variables considered in this paper will be nonlinear. Consequently, solutions for linear systems such as those in e.g. [16] cannot be applied.

A. Solution for a general nonlinear system

The systems considered in [17] are nonlinear systems of the form

$$\dot{x} = f(x) + g_1(x)w + g_2(x)u \tag{1}$$

with state $x \in \mathbb{R}^m$, $w \in \mathbb{R}^{m_1}$ a disturbance or exogenous input, and $u \in \mathbb{R}^{m_2}$ a control input. The functions $f(x)$, $g_i(x)$ with $i = 1, 2$ are continuous. The origin is assumed to be an equilibrium of the system (1) and $f(x)$ sufficiently smooth, such that there exists a mapping $F(x)$ with $f(x) = F(x)x$.

The system (1) has the associated performance variables

$$z_1 = h_1(x) + k_1(x)u, \quad z_2 = h_2(x) + k_2(x)u \tag{2}$$

with $z_1 \in \mathbb{R}^{p_1}$, $z_2 \in \mathbb{R}^{p_2}$ and where $h_i(x)$, $k_i(x)$ with $i = 1, 2$ are continuous in x. The performance variables z_1 and z_2 will be associated with the H_∞ and the H_2 criteria in the mixed H_2/H_∞ control problem, respectively.

The mappings in (2) are assumed to satisfy $h_i(0) = 0$, $h_i(x)^\top k_i(x) = 0$ and $k_i(x)^\top k_i(x) = I_{m_2}$ for $i = 1, 2$, where I_{m_2} denotes the $m_2 \times m_2$ identity matrix. From the first of these conditions it follows that we can write

$$h_i(x)^\top h_i(x) = x^\top Q_i(x)x \tag{3}$$

where $Q_i(x) \geq 0$ for $i = 1, 2$.

The mixed H_2/H_∞ control problem can be stated as follows.

Problem 1: Given the system (1), determine a feedback control law u^* such that

i) The origin is a (locally) asymptotically stable equilibrium of the system when $w(t) = w^*$, w^* being the worst-case disturbance, with region of attraction including a non-empty neighbourhood $\bar{\Omega}$ containing the origin.

ii) For every $w \in \mathscr{L}_2$ such that the trajectories of the system remain in $\bar{\Omega}$, the $\mathscr{L}_2$-gain from w to z_1 is less than some $\gamma > 0$, i.e.
$$\int_0^T \|z_1\|^2 \, \mathrm{d}t \leq \gamma^2 \int_0^T \|w\|^2 \, \mathrm{d}t \tag{4}$$

iii) The control input u^* regulates the state $x(t)$ in such a way as to minimise the output energy with respect to the output z_2 when the worst case disturbance w^* is applied to the system, i.e. u^* minimises
$$J_2(u, w) = \frac{1}{2}\int_0^T z_2^\top z_2 \, \mathrm{d}t = \frac{1}{2}\int_0^T x^\top Q_2(x)x + u^\top u \, \mathrm{d}t \tag{5}$$
when $w = w^*$.

We consider the infinite horizon case, i.e. when $T \to \infty$.

Problem 1 can be formulated as two-player differential game (see [16]). To this end, let

$$\begin{aligned} J_1(u,w) &= \frac{1}{2}\int_0^\infty -z_1^\top z_1 + \gamma^2 w^\top w \ \mathrm{d}t \\ &= \frac{1}{2}\int_0^\infty -x^\top Q_1(x)x - u^\top u + \gamma^2 w^\top w \ \mathrm{d}t \ . \end{aligned} \tag{6}$$

Problem 2: Consider the system (1). Determine a set of admissible feedback strategies $u^*,\ w^*$ which satisfy the Nash equilibrium inequalities

$$J_1(u^*,w^*) \le J_1(u^*,w), \quad J_2(u^*,w^*) \le J_2(u,w^*) \tag{7}$$

for all admissible pairs (u,w^*) and (u^*,w).

A pair of feedback strategies (u,w) is said to be admissible if the origin of the system (1) in closed loop with (u,w) is a (locally) asymptotically stable equilibrium point.

Note that

$$J_1(u,w)\ge 0 \quad \Rightarrow \quad \int_0^T \|z_1\|^2 \ \mathrm{d}t \le \gamma^2 \int_0^T \|w\|^2 \ \mathrm{d}t \ . \tag{8}$$

The factor $\gamma > 0$ in (6) is referred to as a disturbance attenuation level. When $J_1(u,w)\ge 0$, γ is the $\mathscr{L}_2$-gain from the disturbance w to z_1.

Solving this differential game requires the solution of a set of partial differential equations which are in general difficult to solve for nonlinear systems. Instead of finding the exact solution, a means of constructing approximate solutions was developed in [19], and applied to the mixed H_2/H_∞ control problem in [17]. A dynamic extension $\xi(t)\in\mathbb{R}^m$ is introduced, along with the extended value functions

$$\begin{aligned} V_1(x,\xi) &= \frac{1}{2}x^\top P_1(\xi)x - \frac{1}{2}(x-\xi)^\top R_1(x-\xi) \ , \\ V_2(x,\xi) &= \frac{1}{2}x^\top P_2(\xi)x + \frac{1}{2}(x-\xi)^\top R_2(x-\xi) \end{aligned} \tag{9}$$

where $R_i = R_i^\top > 0$, and $P_1(x)\le 0$, $P_2(x)\ge 0$ are so-called algebraic $\bar{P}$ solutions for the system (1), (2), to be introduced in Sec. II-C. Furthermore, the system (1) with output $y = x^\top Q_1(x)x + x^\top Q_2(x)x$ must satisfy the following assumption.

Assumption 1: The pair $\{f,y\}$ and the pair $\{f - \frac{1}{\gamma^2}g_1(x)g_1(x)^\top \frac{\partial V_1}{\partial x}^\top, y\}$ are both zero-state detectable.

An approximate solution of Problem 2 is provided in the following statement (see [17], [19] for details).

Theorem 1 ([17]): Consider the system (1), the cost functionals (6), (5) and the resulting nonzero-sum differential game in Problem 2. Let R_1 and R_2 be such that $R_2 > R_1$ and $R_i(R_1+R_2)+(R_1+R_2)R_i > 0$, for $i=1,\,2$. There exists a neighbourhood Ω, containing the origin, and $\bar{\kappa}>0$ such that for all $\kappa\ge\bar{\kappa}$ the inequalities

$$\begin{aligned} \mathcal{HJI}_1 &= -\frac{1}{2}h_1(x)^\top h_1(x) - \frac{1}{2}\frac{\partial V_2}{\partial x}g_2(x)g_2(x)^\top\frac{\partial V_2}{\partial x}^\top \\ &- \frac{1}{2\gamma^2}\frac{\partial V_1}{\partial x}g_1(x)g_1(x)^\top\frac{\partial V_1}{\partial x}^\top + \frac{\partial V_1}{\partial x}f(x) \\ &- \frac{1}{2}\frac{\partial V_1}{\partial x}g_2(x)g_2(x)^\top\frac{\partial V_2}{\partial x}^\top + \frac{\partial V_1}{\partial \xi}\dot{\xi} \ge 0 \ , \end{aligned} \tag{10a}$$

$$\begin{aligned} \mathcal{HJI}_2 &= \frac{1}{2}h_2(x)^\top h_2(x) - \frac{1}{2}\frac{\partial V_2}{\partial x}g_2(x)g_2(x)^\top\frac{\partial V_2}{\partial x}^\top \\ &+ \frac{\partial V_2}{\partial x}f(x) - \frac{1}{2}\frac{\partial V_2}{\partial x}g_2(x)g_2(x)^\top\frac{\partial V_1}{\partial x}^\top + \frac{\partial V_2}{\partial \xi}\dot{\xi} \le 0 \end{aligned} \tag{10b}$$

with

$$\dot{\xi} = -\kappa\left(\frac{\partial V_2}{\partial \xi} - \frac{\partial V_1}{\partial \xi}\right)^\top \ , \tag{11}$$

are satisfied for all $(x,\xi)\in\Omega$. Suppose Assumption 1 is satisfied with V_1, V_2 given by (9). Then, the dynamic feedback strategy

$$u^* = -g_2(x)^\top\frac{\partial V_2}{\partial x}^\top, \quad w^* = -\frac{1}{\gamma^2}g_1(x)^\top\frac{\partial V_1}{\partial x}^\top \tag{12}$$

is such that the closed-loop system (1), (11), (12) is (locally) asymptotically stable.

The optimal input and worst case disturbance given by (12) are the Nash equilibrium strategies of a modified differential game with modified cost functionals $\tilde{J}_1(u,w)\le J_1(u,w)$ and $\tilde{J}_2(u,w)\ge J_2(u,w)$ (see [16], [17] for details).

B. Double integrator dynamics

Consider now a system described by double integrator dynamics, influenced by a disturbance at the acceleration level, i.e.

$$\dot{x} = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I_n \end{bmatrix} w + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u \tag{13}$$

with $x\in\mathbb{R}^{2n}$ and $w,\,u\in\mathbb{R}^n$.

The system (13) has associated performance variables (2) as defined previously. The matrices $Q_1(x), Q_2(x)$ are now assumed to satisfy the following.

Assumption 2: The matrices $Q_1(x)$ and $Q_2(x)$ can be partitioned into $n\times n$-blocks

$$Q_i(x) = \begin{bmatrix} Q_{i11}(x) & Q_{i12}(x) \\ Q_{i12}(x)^\top & Q_{i22}(x) \end{bmatrix} \tag{14}$$

where the blocks $Q_{i12}(x)$ satisfy

$$Q_{i12}(x)^\top Q_{i12}(x) < b_1 b_2 I_n \quad \forall x \tag{15}$$

for $i=1,\,2$ and some constants $b_1,\,b_2>0$.

C. Algebraic $\bar{P}$ matrix solution

An $\mathcal{X}$-algebraic $\bar{P}$ solution for the general, nonlinear system (1), (2) for the mixed H_2/H_∞ control problem is a set of matrix-valued $\mathcal{C}^1$ functions $P_1(x),\,P_2(x)\in\mathbb{R}^{m\times m}$, with $P_i(x)=P_i(x)^\top$, $i=1,\,2$, and such that for all $x\in\mathcal{X}\subseteq\mathbb{R}^m$ they satisfy

$$\begin{aligned} &-Q_1(x) - P_2(x)g_2(x)g_2(x)^\top P_2(x) \\ &-\tfrac{1}{\gamma^2}P_1(x)g_1(x)g_1(x)^\top P_1(x) + P_1(x)F(x) \\ &+F(x)^\top P_1(x) - P_1(x)g_2(x)g_2(x)^\top P_2(x) \\ &-P_2(x)g_1(x)g_1^\top P_1(x) - \Sigma_1(x) = 0 \ , \end{aligned} \tag{16a}$$

$$\begin{aligned} &Q_2(x) - P_2(x)g_2(x)g_2(x)^\top P_2(x) + P_2(x)F(x) \\ &+F(x)^\top P_2(x) - \tfrac{1}{\gamma^2}P_2(x)g_1(x)g_1(x)^\top P_1(x) \\ &-\tfrac{1}{\gamma^2}P_1(x)g_1(x)g_1^\top(x)P_2(x) + \Sigma_2(x) = 0 \ , \end{aligned} \tag{16b}$$

where $\Sigma_i(x)\in\mathbb{R}^{m\times m}$ are symmetric, positive semidefinite matrix-valued functions satisfying $\Sigma_i(0)>0$ for $i=1,\,2$, and such that the matrices $P_1(x), P_2(x)$ satisfy $\bar{P}_2 - \bar{P}_1 > 0$, where $\bar{P}_i = P_i(0)$, $i=1,2$, solve a set of coupled Riccati equations for the system (1), (2) linearised about the origin. Because the dynamics of the system (13) are linear, the linearised coupled Riccati equations coincide with the equations (16) evaluated at $x=0$. $P_1(x)$ and $P_2(x)$ are called algebraic $\bar{P}$ solutions if the equations (16) hold for all $x\in\mathbb{R}^{2n}$, i.e. if $\mathcal{X}=\mathbb{R}^{2n}$.

$$-Q_{111} - P_{212}P_{212}^\top - \frac{1}{\gamma^2}P_{112}P_{112}^\top - P_{112}P_{212}^\top - P_{212}P_{112}^\top - \Sigma_{111} = 0 \tag{18a}$$

$$-Q_{112} - P_{212}P_{222} - \frac{1}{\gamma^2}P_{112}P_{122} + P_{111} - P_{112}P_{222} - P_{212}P_{122} - \Sigma_{112} = 0 \tag{18b}$$

$$-Q_{122} - P_{222}^2 - \frac{1}{\gamma^2}P_{122}^2 + P_{112}^\top + P_{112} - P_{122}P_{222} - P_{222}P_{122} - \Sigma_{122} = 0 \tag{18c}$$

$$Q_{211} - P_{212}P_{212}^\top - \frac{1}{\gamma^2}P_{112}P_{212}^\top - \frac{1}{\gamma^2}P_{212}P_{112}^\top + \Sigma_{211} = 0 \tag{19a}$$

$$Q_{212} - P_{212}P_{222} + P_{211} - \frac{1}{\gamma^2}P_{112}P_{222} - \frac{1}{\gamma^2}P_{212}P_{122} + \Sigma_{212} = 0 \tag{19b}$$

$$Q_{222} - P_{222}^2 + P_{212}^\top + P_{212} - \frac{1}{\gamma^2}\left(P_{122}P_{222} + P_{222}P_{122}\right) + \Sigma_{222} = 0 \tag{19c}$$

The dependency of $P_i,\ Q_i,\ \Sigma_i$ on x will now be omitted for brevity. Let

$$P_i = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i12}^\top & P_{i22} \end{bmatrix}, \quad \Sigma_i = \begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} \\ \Sigma_{i12}^\top & \Sigma_{i22} \end{bmatrix}. \tag{17}$$

Substituting (13) for the double integrator dynamics and (2) for the performance variables into (16) yields a set of six equations, given in (18) and (19).

Algebraic $\bar{P}$ matrix solutions, i.e. matrices P_1 and P_2 satisfying (18) and (19), are not unique, and determining the block elements of P_i in (17) is nontrivial. In the following, we will show that choosing $P_{112} = -kP_{212}$, with $k > 0$, enables us to find an algebraic $\bar{P}$ solution for the game defined in Problem 2.

Inserting $P_{112} = -kP_{212}$ into (18a) and (19a) gives

$$\left(2k - 1 - \tfrac{k^2}{\gamma^2}\right)P_{212}{P_{212}}^\top = Q_{111} + \Sigma_{111} \tag{20a}$$

$$\left(1 - \tfrac{2k}{\gamma^2}\right)P_{212}{P_{212}}^\top = Q_{211} + \Sigma_{211} \tag{20b}$$

Since $Q_{i11} + \Sigma_{i11} \geq 0$, in order for a solution of (20) to exist γ and k should satisfy

$$2k - 1 - \tfrac{k^2}{\gamma^2} > 0\,, \quad 1 - \tfrac{2k}{\gamma^2} > 0 \tag{21}$$

which requires k to be chosen such that

$$\gamma^2 - \gamma\sqrt{\gamma^2 - 1} < k < \tfrac{\gamma^2}{2}\,. \tag{22}$$

Let k then be chosen as the solution to $2k - 1 - \frac{k^2}{\gamma^2} = 1 - \frac{2k}{\gamma^2}$ satisfying (22), which gives

$$k = 1 + \gamma^2 - \sqrt{1 + \gamma^4}\,. \tag{23}$$

Notice that in order for the interval in (22) from which k can be chosen to be non-empty, γ must satisfy

$$\gamma > \tfrac{2}{\sqrt{3}}. \tag{24}$$

Remark 1: A $\gamma > 1$ merely limits how much the disturbance w is amplified in the performance variable z_1. A solution admitting lower values for γ may be found by introducing lower weights on the input u in the cost functionals (5), (6), as was done in [18]. A similar effect can be achieved by instead increasing the elements of the matrix Q_1.

Now let $a = \frac{1}{1 - \frac{2k}{\gamma^2}}$. Choosing $\Sigma_{111} = Q_{211} + b_1 I_n$ and $\Sigma_{211} = Q_{111} + b_1 I_n$, with $b_1 > 0$, gives

$$P_{212}{P_{212}}^\top = a\left(Q_{111} + Q_{211} + b_1 I_n\right). \tag{25}$$

Since the right-hand side of (25) is positive definite, P_{212} can be chosen as the Cholesky decomposition of the right-hand side, and will also be positive definite [21].

Remark 2: The closer γ is chosen to its lower bound (24), the greater a becomes. In order for the optimal control input (12) to be feasible, the inequality (24) should be satisfied with some margin.

Now let $P_{122} = -cP_{222}$ with $c > 0$. Equations (18c) and (19c) give

$$\left(2c - 1 - \tfrac{c^2}{\gamma^2}\right)P_{222}^2 = \Sigma_{122} + Q_{122} + k\left({P_{212}}^\top + P_{212}\right), \tag{26a}$$

$$\left(1 - \tfrac{2c}{\gamma^2}\right)P_{222}^2 = Q_{222} + \Sigma_{222} + {P_{212}}^\top + P_{212}. \tag{26b}$$

Let $c = k$, and choose

$$\begin{aligned} \Sigma_{122} &= Q_{222} + (1-k)\left({P_{212}}^\top + P_{212}\right) + b_2 I_n \\ \Sigma_{222} &= Q_{122} + b_2 I_n \end{aligned} \tag{27}$$

with $b_2 > 0$. Since $k < 1$, Σ_{122} and Σ_{222} are both positive definite. We then have

$$P_{222}^2 = a\left(Q_{122} + Q_{222} + b_2 I_n + {P_{212}}^\top + P_{212}\right), \tag{28}$$

where the right-hand side is positive definite, and hence its root P_{222} exists and is likewise positive definite.

Finally, inserting $P_{112} = -kP_{212}$ and $P_{122} = -kP_{222}$ into equations (18b) and (19b) gives

$$P_{111} = \Sigma_{112} + Q_{112} - \frac{1}{a}P_{212}P_{222}, \tag{29a}$$

$$P_{211} = -\Sigma_{212} - Q_{212} + \frac{1}{a}P_{212}P_{222} \tag{29b}$$

where we choose $\Sigma_{112} = -Q_{112}$, $\Sigma_{212} = -Q_{212}$.

To be an algebraic $\bar{P}$ matrix solution, $P_1(x)$, $P_2(x)$ must satisfy $P_2(0) - P_1(0) > 0$, and the matrices $\Sigma_i(x)$ must satisfy $\Sigma_i(x) \geq 0$ and $\Sigma_i(0) > 0$ for $i = 1, 2$. The matrices $\Sigma_1(x)$ and $\Sigma_2(x)$ both have positive definite upper-left blocks $\Sigma_{i11}(x)$. They are then positive definite if they fulfil

$$\Sigma_{i22}(x) - \Sigma_{i12}(x)^\top \Sigma_{i11}(x)^{-1}\Sigma_{i12}(x) > 0\ . \tag{30}$$

Inserting for $\Sigma_1(x)$, and using that $P_{212}(x)$ and $Q_{211}(x)$ are positive (semi) definite gives

$$\begin{aligned} &Q_{222}(x) + (1-k)\left(P_{212}(x)^\top + P_{212}(x)\right) + b_2 I_n \\ &- Q_{112}(x)^\top\left(Q_{211}(x) + b_1 I_n\right)^{-1} Q_{112}(x) \\ &\geq Q_{222}(x) + b_2 I_n - \tfrac{1}{b_1}Q_{112}(x)^\top Q_{112}(x)\ . \end{aligned} \tag{31}$$

By Assumption 2 and since $Q_{222}(x)$ is positive semidefinite, this is positive definite and so $\Sigma_1(x)$ is positive definite. Likewise, $\Sigma_2(x)$ can also be shown to be positive definite.

Consider now $P_2(x)$, with $P_{211}(x)$ given by (29b). Since $\frac{1}{a} > 0$ and both $P_{212}(x)$ and $P_{222}(x)$ are positive definite, so is their product $P_{211}(x)$ [21]. Then $P_2(x) > 0$ if the following holds:

$$P_{222}(x) - P_{212}(x)^\top P_{211}(x)^{-1}P_{212}(x) > 0 \tag{32}$$

Substituting (29b) into (32) gives

$$\left(P_{222}(x)^2 - aP_{212}(x)^\top\right)P_{222}(x)^{-1} > 0\ . \tag{33}$$

Since $P_{222}(x)$ is positive definite, so is its inverse. By rearranging (28) to find an expression for $P_{222}(x)^2 - aP_{212}(x)^\top$ we see that this term is also positive definite. Hence (33) holds, and $P_2(x)$ is positive definite for all x.

The matrix $P_1(x)$ is negative definite if $-P_1(x)$ is positive definite. Since $-P_{111}(x) > 0$, $-P_1(x)$ is positive definite if

$$\begin{aligned} -P_{122}(x) + P_{112}(x)^\top P_{111}^{-1} P_{112}(x) = \\ kP_{222}(x) - k^2 P_{212}(x)^\top P_{211}(x)^{-1} P_{212}(x) > 0 \ . \end{aligned} \tag{34}$$

Again inserting (29b) for $P_{211}(x)$, this can be rewritten as

$$\left(P_{222}(x)^2 - akP_{212}(x)^\top\right) P_{222}(x)^{-1} > 0 \tag{35}$$

Since $k < 1$, $P_{212}(x)$ is positive definite and the inequality (33) holds, the inequality (35) also holds. Since $P_2(x)$ and $-P_1(x)$ are positive definite for all x, it follows that $P_2(0) - P_1(0) > 0$.

The result of this section is summarised in the following

Lemma 1: Consider the the system (13) with output variables (2), where $Q_1(x)$, $Q_2(x)$ are subject to Assumption 2 and with $\gamma > \frac{2}{\sqrt{3}}$. The matrices $P_1(x)$, $P_2(x)$ with P_2 given by (25), (28), (29b), and $P_1(x)$ given by (29a) and

$$P_{112}(x) = -kP_{212}(x), \ P_{122}(x) = -kP_{222}(x) \tag{36}$$

constitute an algebraic $\bar{P}$ solution for Problem 2.

III. Application to AIAUVs

In this section we give the dynamical model of the AIAUV. We then derive the feedback linearising controller for the joints, and define the performance variables z_1 and z_2 for achieving collision avoidance between links.

A. Vehicle model

An AIAUV can be modelled as an underwater vehicle-manipulator system (UVMS) using the model from [2], where the backmost link can be considered to be the vehicle base. The dynamics of an AIAUV with n 1-DOF joints and $n+1$ links are given by

$$M(q)\dot{\zeta} + C(q,\zeta)\zeta + D(q,\zeta)\zeta + g(q,\eta_2) = \tau(q) \tag{37}$$

where $q \in \mathbb{R}^n$ is a vector of joint angles, and $\zeta = \begin{bmatrix} v^\top & \omega^\top & \dot{q}^\top \end{bmatrix}^\top$ is the generalised velocity vector consisting of the linear and angular velocities $v, \omega \in \mathbb{R}^3$ of the base. The matrix $M(q)$ is the inertia matrix, including added mass effects, $C(q,\zeta)$ is the Coriolis and centripetal matrix and $D(q,\zeta)$ is the damping matrix, as given in [22]. The gravity and buoyancy forces acting on the AIAUV are given by the term $g(q,\eta_2)$, where η_2 is an appropriate choice of representation for the orientation of the base.

The generalised input $\tau(q) \in \mathbb{R}^{6+n}$ consists of the total forces and moments on the base, and total joint torques resulting from the control inputs. Based on [22], $\tau(q)$ can be expressed as

$$\tau(q) = \begin{bmatrix} B(q) & \begin{matrix} 0_{6\times n} \\ I_n \end{matrix} \end{bmatrix} \begin{bmatrix} \tau_{\text{thr}} \\ \tau_q \end{bmatrix}. \tag{38}$$

For an AIAUV equipped with m thrusters, the thrust configuration matrix $B(q)$ is an $(6+n) \times m$-matrix. The control inputs $\tau_{\text{thr}} \in \mathbb{R}^m$, $\tau_q \in \mathbb{R}^n$ are thruster forces and joint torques, respectively.

B. Joint dynamics

In this paper we consider only the joint motion of the AIAUV will be considered. The position and orientation of the base are assumed to be controlled using the thrusters independently of the joint motion, using control methods such as those in e.g. [3].

The dynamics of the joints alone are described by the lower n rows of the dynamics (37), which can be written as

$$\begin{aligned} \ddot{q} = \begin{bmatrix} 0_{n\times 6} & I_n \end{bmatrix} M(q)^{-1} \Bigg(B(q)\tau_{\text{thr}} + \begin{bmatrix} 0_{6\times n} \\ I_n \end{bmatrix} \tau_q \\ - C(q,\zeta)\zeta - D(q,\zeta)\zeta - g(q,\eta_2) \Bigg) \end{aligned} \tag{39}$$

Assume now that only an estimate of the parameters of the manipulator inertia matrix is available. In particular hydrodynamic parameters due to added mass effects are difficult to identify. The inertia matrix estimate $\hat{M}(q)$ must be positive definite. Let $\tilde{M}(q) = M(q) - \hat{M}(q)$ be the error between the inertia matrix and its estimate. Further let the lower-right $n \times n$-block of $\hat{M}^{-1}(q)$ be denoted $\hat{M}_{\text{inv},q}(q)$, such that (39) can be written as

$$\begin{aligned} \ddot{q} = \hat{M}_{\text{inv},q}(q)\tau_q + \begin{bmatrix} 0_{n\times 6} & I_n \end{bmatrix} \hat{M}(q)^{-1} \Big(B(q)\tau_{\text{thr}} \\ - C(q,\zeta)\zeta - D(q,\zeta)\zeta - g(q,\eta_2) - \tilde{M}(q)\dot{\zeta} \Big) \end{aligned} \tag{40}$$

Since the mass matrix estimate $\hat{M}(q)$ is positive definite, the inverse and its lower-right block $\hat{M}_{\text{inv},q}(q)$ are likewise positive definite, and hence invertible.

Assume further that the parameters of the thrust configuration matrix $B(q)$ and hydrostatic forces $g(q,\eta_2)$ are known exactly, whereas for the parameters of $C(q,\zeta)$ and $D(q,\zeta)$ only estimates are available, due to uncertain hydrodynamic parameters. In particular, the estimated matrix $\hat{C}(q,\zeta)$ should be computed from the estimate $\hat{M}(q)$. Denote the estimated damping matrix $\hat{D}(q,\zeta)$, and let

$$\begin{aligned} \tau_q = \hat{M}_{\text{inv},q}(q)^{-1} \Big(u - \begin{bmatrix} 0_{n\times 6} & I_n \end{bmatrix} \hat{M}(q)^{-1} \Big(B(q)\tau_{\text{thr}} \\ + \hat{C}(q,\zeta)\zeta + \hat{D}(q,\zeta)\zeta - g(q,\eta_2) \Big) \Big) \end{aligned} \tag{41}$$

where u is a new, virtual input to be assigned later.

Remark 3: The parameters of the thrust configuration matrix and hydrostatic forces should be known exactly in order for the disturbance due to parameter uncertainties to have bounded energy, as considered in Problem 1. In some cases also the parameters of $g(q,\eta_2)$ may be uncertain, e.g. in the presence of tunnel thrusters through links, making it difficult to know their volume exactly. The simulation results in [18], where an error in the gravitational term is included, suggests some robustness to this type of errors as well.

Inserting the input (41) into the dynamics (40), and introducing error terms $\tilde{C}(q,\zeta) = C(q,\zeta) - \hat{C}(q,\zeta)$ and $\tilde{D}(q,\zeta) = D(q,\zeta) - \hat{D}(q,\zeta)$, yields

$$\ddot{q} = u + \underbrace{\begin{bmatrix} 0_{n\times 6} & I_n \end{bmatrix} \hat{M}(q)^{-1} \Big(-\tilde{C}(q,\zeta)\zeta - \tilde{D}(q,\zeta)\zeta - \tilde{M}(q)\dot{\zeta} \Big)}_{=w} \tag{42}$$

The dynamics of the joint angles q can then be written as

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ I_n \end{bmatrix} w + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u \tag{43}$$

Finally, in order to achieve setpoint regulation, let $\tilde{q} = q - q_d$, where q_d is a vector of constant, desired joint setpoints. Then

$$\dot{\tilde{q}} = \dot{q}, \quad \ddot{\tilde{q}} = \ddot{q} \tag{44}$$

which gives

$$\begin{bmatrix} \dot{\tilde{q}} \\ \ddot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} + \begin{bmatrix} 0 \\ I_n \end{bmatrix} w + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u \tag{45}$$

since the linearised system dynamics (43) do not depend on the original state q. The control input (41) still requires the original state q, which can be found as $q = \tilde{q} + q_d$. We will now let $x = \begin{bmatrix} \tilde{q}^\top & \dot{\tilde{q}}^\top \end{bmatrix}^\top$ denote the error state of the system, and note that it satisfies double integrator dynamics (13).

The virtual input u will be selected as the optimal control input in (12).

C. Performance variables

We wish to design the cost functional (5) to be minimised by the virtual control input u such that it leads to a trajectory along which collisions between links of the AIAUV are avoided, similarly to what has been done in [14] for collision avoidance between multiple agents. In addition, joint limits should be obeyed.

The matrix $Q_1(x)$ will be chosen simply as a constant, diagonal matrix. A natural choice is the identity matrix I_{2n}.

For the matrix $Q_2(x)$, the lower right block $Q_{222}(x)$ will be chosen simply as a diagonal matrix. The upper-left block $Q_{211}(x)$ will be a diagonal matrix with elements $Q_{211_i}(x)$, $i = 1, ..., n$ chosen as

$$Q_{211_i}(x) = \alpha_i + \beta_i^l g_i^l(q_i) + \beta_i^c g^c(q) \tag{46}$$

where α_i, β_i^l, β_i^c are positive weights, $g_i^l(q_i)$ is a joint limit avoidance function for joint i, and $g^c(q)$ is a collision avoidance function penalising collisions between links of the AIAUV.

The joint limit avoidance function $g_i^l(q_i)$ is chosen as

$$g_i^l(q_i) = \frac{1}{(q_i^2 - q_{\text{lim},i}^2)^2} \tag{47}$$

where $q_{\text{lim},i}$ is the absolute value of the maximum joint angle allowed for joint i, typically due to physical limitations.

Collision avoidance between links will be achieved similarly as in the case of separate agents in [14], by using inverse barrier functions as weights in the matrix $Q_2(x)$. Each joint may cause collisions between not only the links which it connects, but any two links on opposite sides of the joint. Therefore, creating separate functions including only the collisions a given joint may cause would give a large overlap between the individual functions. To simplify this, all joint errors will be weighted by the same function which takes into account all possible collisions.

Denote by l_i half of the length of link i. A sphere of radius l_i can then almost encompass the link in the case of cylindrical links. To simplify the collision avoidance function, this sphere will be used as an approximation of the shape of the link. A collision between two links i and j can then be avoided by ensuring that the distance between their centers is greater than $l_i + l_j$. A less conservative behaviour can be achieved by approximating the link shapes as ellipsoids instead. In the case of neighbouring links, collisions can easily be avoided simply by respecting appropriate joint angle limits, and will therefore not be included in the collision avoidance function.

The distance between the centers of two links can be found using homogeneous transformations. Let each link have a reference frame attached to its back, as in [1]. The homogeneous transformation from frame i to $i + 1$, for $i = 1, ..., n$ is

$$T_{i+1}^i = \begin{bmatrix} R_{q_i} & \begin{matrix} 2l_i \\ 0 \\ 0 \end{matrix} \\ 0_{1\times 3} & 1 \end{bmatrix} \tag{48}$$

where R_{q_i} is the rotation matrix describing the rotation caused by joint q_i. Consequently, the position p_j^i of the center of link j relative to the center of link i in homogeneous coordinates, with $j > i$, can be found as

$$p_j^i = \begin{bmatrix} R_{q_i} & \begin{matrix} l_i \\ 0 \\ 0 \end{matrix} \\ 0_{1\times 3} & 1 \end{bmatrix} T_{i+2}^{i+1} \ ... \ T_j^{j-1} \begin{bmatrix} l_j \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{49}$$

Then the distance between the links i and j, squared, is $d_{ij}(q) = p_j^{i\top} p_j^i - 1$. We now define the collision avoidance function as

$$g^c(q) = \sum_{i=1}^{n+1} \sum_{j=i+2}^{n+1} \frac{1}{(d_{ij}(q) - (l_i + l_j)^2)^2}. \tag{50}$$

As in [14], neither the initial state $q(0)$ nor the desired state q_d can be states in which a collision occurs, and there must exist a collision-free path from $q(0)$ to q_d in the state space.

Collision avoidance with obstacles in the environment can be similarly achieved by taking into account the position of the base relative to the obstacle.

The matrices $Q_1(x)$, $Q_2(x)$ have now been chosen such that $Q_1(x) + Q_2(x)$ is positive definite and hence has full rank. Then if the output $y = x^\top Q_1(x)x + x^\top Q_2(x)x$ is identically equal to zero, so is the state x, and Assumption 1 is satisfied for the system for any choice of $Q_1(x)$, $Q_2(x)$ such that their sum is positive definite. Then using the algebraic $\bar{P}$ matrix solutions for the double integrator dynamics from Sec. II-C, Theorem 1 states that the system (45) is asymptotically stable if u, w are the optimal input and worst-case disturbance, respectively, as given by (12) with dynamic extension ξ as given by (11) and V_1, V_2 as in (9).

TABLE I

LINK PROPERTIES

Link no.	Length [m]	Mass [kg]	Thrusters
1	0.62	14.3	None
2, 4, 6, 8	0.104	6.0	None
3	0.584	12.7	2: Z, Y
5	0.726	9.8	3: X, X, Z
7	0.584	12.7	2: Y, Z
9	0.37	7.8	None

IV. SIMULATION RESULTS

A. Implementation

The simulation model is implemented in MATLAB/Simulink based on [22]. The model has 9 cylindrical links and 8 joints, out of which joints 1, 3, 5 and 7 rotate about the z-axis, and joints 2, 4, 6 and 8 about the y-axis. The links have radius 0.085 m, and further properties given in Table I. The notation "2: Z, Y" means that the link has two thrusters, one acting in the z-direction and one in the y-direction. The vehicle has 7 thrusters in total. The even-numbered links are short links separating two joints, simulating a physical vehicle with 2DOF-joints and 5 actual links. The collision function (50) is therefore implemented such that it only considers the odd-numbered links.

B. Simulations

The initial joint configuration is $q(0) = [\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{6}]^\top$, and all initial joint velocities are 0. The desired setpoint is $q_d = [\frac{\pi}{4}, -\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{4}, -\frac{\pi}{6}]^\top$, and the initial and

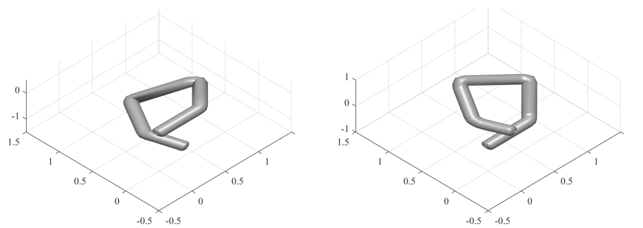

Fig. 1. Initial (left) and desired final (right) joint configuration.

desired configurations are shown in Fig. 1. Motions like this may for instance arise if the vehicle has attached one end to a structure, while the other end has to interact with multiple points on the structure.

The initial value $\xi(0)$ is chosen by selecting a start value ξ_0, then finding the nearest value for $\xi(0)$ such that the inequalities (10) are satisfied at $t = 0$, and $\xi(0) + q_d$ satisfy joint limits and there are no collisions between links. The search is performed using the MATLAB function `fmincon` to minimise the norm of the error $\xi(0) - \xi_0$ with $\xi(0)$ subject to the aforementioned constraints. The search is initiated at $\xi_0 = [\text{-}\frac{\pi}{2}, \frac{\pi}{3}, \text{-}\pi, \frac{\pi}{3}, \text{-}\pi, \frac{\pi}{3}, \text{-}\frac{\pi}{2}, \frac{\pi}{3}, \text{-}10, 0, \text{-}10, 0, \text{-}10, 0, \text{-}10, 0]^\top$, and the final choice of $\xi(0)$ is such that $\|\xi(0) - \xi_0\| = 0.07$.

The physical joint limits of all joints are $\pm\frac{\pi}{2}$, and the limits in (47) are set to $q_{\text{lim},i} = \frac{\pi}{2} + 0.1$ to allow the full range of the joints to be used. The parameters of the cost functionals (6), (5) are chosen as $\gamma = 1.2$, $Q_1(x) = I_{2n}$, $Q_{222}(x) = \frac{1}{2} I_n$, and $Q_{211}(x)$ with elements as given by (46) with $\alpha_i = \frac{1}{2}$, $\beta_i^l = 0.1$ and $\beta_i^c = 0.01$ for all i. The parameters of the algebraic $\bar{P}$ solution are chosen to be $b_1 = 4.5$, $b_2 = 1.5$, and the parameters of $\dot{\xi}$ and the value functions (9) are selected as $\kappa = 1$, $R_1 = \left[\begin{smallmatrix} 1.1 & -1 \\ -1 & 1.1 \end{smallmatrix}\right]$ and $R_2 = \left[\begin{smallmatrix} 1.2 & 1 \\ 1 & 1.2 \end{smallmatrix}\right]$. No thrust is used in the simulations, causing the AIAUV to float freely and rotate due to the joint motion.

Simulations are performed for three cases:

1) With $w = w^*$,
2) With $\hat{M}(q) = 0.5M(q)$, $\hat{C}(q,\zeta) = 0.5C(q,\zeta)$ and $\hat{D}(q,\zeta) = 0.5D(q,\zeta)$ in the controller (41),
3) With estimates as above, but with $Q_1(x) = 10I_{2n}$.

The simulation results are shown in Figs. 2, 3, 4 for cases 1, 2 and 3, respectively. In Fig. 2, the behaviour of joints 1 and 7, 3 and 5, and 2, 4, 6 and 8 coincides. The inequalities (10) are fulfilled in the first simulation case, but not cases 2 and 3. In case 3, only $\mathcal{HJI}_1$ does not fulfil the inequality.

C. Discussion

The error converges to zero while successfully avoiding collision between links in all the simulated cases, as can be seen from Figs. 2, 3 and 4, despite Theorem 1 only stating that the system is asymptotically stable when $w = w^*$. While γ is the same in all cases, comparing Figs. 3a and 4a show that better disturbance attenuation and faster convergence can be achieved by increasing $Q_1(x)$, which corresponds to reducing both γ and the weight on the input u in the cost functional J_1 (6). This way, disturbance attenuation can be improved despite there being a lower bound on γ, but at the expense of greater control effort, as can be seen in Figs. 3b and 4b.

The inequalities (10) are not always satisfied in the cases when the disturbance w is not the worst-case w^*. When the inequalities are not satisfied, the $\mathscr{L}_2$-stability with respect to the disturbance w is no longer guaranteed, and neither is boundedness of $h_2(x)^\top h_2(x)$, which is what ensures collision avoidance. While Theorem 1 states that there exists a region Ω in which the inequalities (10) are satisfied, it gives no guarantees that the solution will remain in Ω. Whether or not the solution remains in Ω depends, at least in part, on the choice of $\xi(0)$. Finding such a $\xi(0)$ is not easy and often relies on much trial and error. In addition, a new $\xi(0)$ is required for every new motion to be performed. This difficulty can be overcome by following an approach similar to that presented in [23]. Namely, global asymptotic stability can be achieved by enabling ξ to evolve according to hybrid dynamics. This may come at the cost of discontinuities in the input, which is difficult to realise and causes strain on physical actuators.

The task considered here is setpoint regulation of joints. If the true goal is to e.g. control the end-effector position, there may be multiple joint configurations which give the same end-effector position. In the case of obstacles in the environment, some of them might be impossible to reach from a given initial configuration, as pointed out in [4]. The selection of desired setpoints therefore requires careful planning.

V. Conclusions and Future Work

In this paper, an algebraic $\bar{P}$ matrix solution was found for a system satisfying double integrator dynamics and nonlinear cost functionals in order to construct an approximate solution to a mixed H_2/H_∞ control problem, formulated as a differential game. Then, using feedback linearisation, the joint dynamics of an AIAUV were simplified into those of a double integrator, and the solution for mixed H_2/H_∞ control was applied to the task of joint setpoint regulation while avoiding joint limits and collisions between the links of the AIAUV. Simulation results show that collisions are successfully avoided while regulating the joints to desired setpoints, also in the case of parameter uncertainties in the feedback linearising controller. The solution found in this paper is applicable to any system which can be feedback linearised into double integrator dynamics.

Future work to be considered includes developing a trajectory tracking version, and using the results from [23] to achieve a global result. In addition, the properties of the solution in the case of disturbances other than the worst-case disturbance should be examined closer in order to be able to give guarantees for the collision avoidance.

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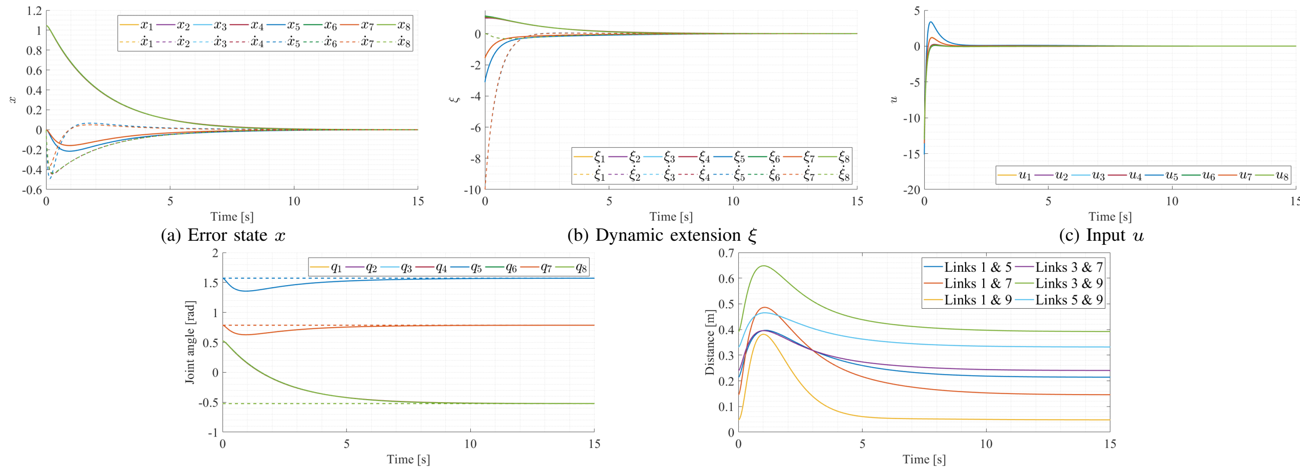
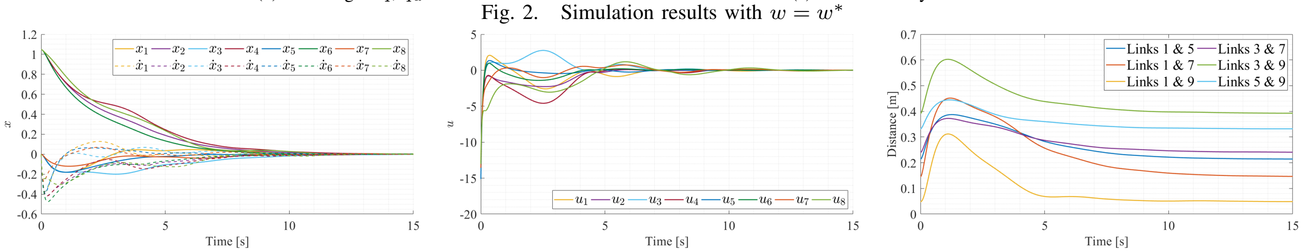


(a) Error state x (b) Dynamic extension ξ (c) Input u

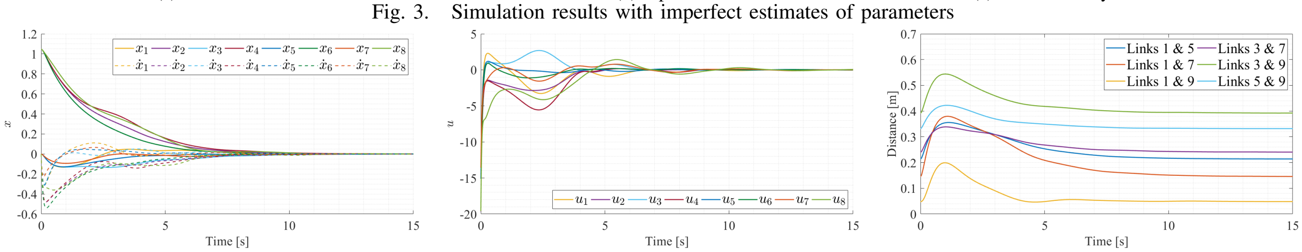
(d) Joint angles q, q_d in dashed lines (e) Distance away from collision

Fig. 2. Simulation results with $w = w^*$



(a) Error state x (b) Input u (c) Distance away from collision

Fig. 3. Simulation results with imperfect estimates of parameters



(a) Error state x (b) Input u (c) Distance away from collision

Fig. 4. Simulation results with imperfect estimates of parameters and $Q_1(x) = 10I_{2n}$

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