



Modeling Bond Spreads and Credit Default Risk in the Norwegian Financial Market Using Structural Credit Default Models

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Abstract

In this study, we examine the credit risk of banking bonds. We apply two option-based credit default models originally derived by Merton and Black and Cox, with the aim of producing objective credit ratings and credit spreads. A credit rating process can never be purely objective and typically credit rating assessments are highly dependent on subjective judgment on the part of credit analysts. We do believe, however, that the credit rating industry might benefit from employing objective methods to help foster consistency in the rating processes (which some CRAs already do, e.g., Moody's). Employing data from two Norwegian banks, our analysis is designed to capture the characteristics of the Nordic financial bond market. The results indicate that structural models are well suited to computing plausible credit default probabilities, as well as credit spreads and to performing credible credit ratings of Nordic banks, given that the input parameters are properly estimated.

Keywords

structural credit default models, credit spreads, credit ratings, reorganization boundaries, volatility measures

1 Introduction

Producing credible and consistent credit ratings is a vital responsibility of a credit rating agency (CRA). Credit ratings are an important tool for categorizing firms based on credit risk; the risk of loss due to default on contractual obligations, and risk management, are crucial to both financial and non-financial institutions. Credit ratings are outputs from a process employing both qualitative and quantitative factors combining objective data with subjective assessments on the part of experienced analysts. We believe however, that in general CRAs' credit rating processes would benefit from combining subjective assessments with output from credit default models relying on empirical input data. Some CRAs do

employ credit default models to support employees' analytical judgments; others do not. Local variations in financial markets and regional economies also affect credit risk.

The contribution of this study is: (i) Developing a structural credit default modeling framework for calculating probabilities of default, *reflecting Nordic parameters*. (ii) Constructing models that are able to evaluate and explain observed credit spreads in the Nordic region, as well as producing *credible bond ratings*. Our models are intended to serve as a quantitative tool supporting the credit rating processes of Nordic Credit Rating (NCR).

Broadly speaking, credit default models can be divided into two main types: structural and reduced-form models. Both methodologies have their own context-specific advantages and disadvantages, and should be considered as complements to each other (Timmins 2009). Structural models are based on an underlying financial theory of the evolution of asset prices, and utilize theoretical differential equations describing the evolution of the endogenous variables. However, many of the assumptions are difficult to assess. For the reduced-form models, on the other hand, the system of equations is already solved for the endogenous variables and as few structural assumptions as possible are introduced. Arora et al. (2005) assess both the reduced-form approach and the structural-modeling approach by investigating three acknowledged models on corporate default risk: two structural models known as the *Merton* and *Vasicek-Kealhofer* (VK) models, and one reduced-form model known as the *Hull-White* model. Based on cross-sectional variations on credit default swap spreads, the robustness of the models and their ability to predict default, they conclude that the structural models outperform the reduced-form model. In this analysis, we only consider structural models.

The conventional notion of the probability of default for a bond at a given point in time is equivalent to the probability that the value of the underlying firm's assets will fall below the face value of its debt at maturity. In theory, a firm is defined as bankrupt when the market value of the firm is less than the book value of its liabilities. In practice, however, firms often are capable of continuing operations as long as they are able to service their debt by providing periodic contractual payments. When modeling credit default risk, the definition of default is an essential part of the credit rating process, as are the related concepts of the distance to default or the remoteness of default. Obviously, the default probability of a company, or of any of its new and outstanding issues, heavily impact the final rating result. When acquiring bonds, investors face credit risk in terms of firms not fulfilling their contracted payments. Many investors may have neither the resources nor the skills necessary to perform detailed credit analyses of the bonds or the issuers. This is the task undertaken by credit rating agencies, and this work is of course even more important when issuers are non-listed, as are many Nordic companies. The banks we consider in this study are, however, listed.

In order to be attractive to investors, risky bonds should offer higher yields compared to similar risk-free bonds to compensate for the probability of losses. Consequently, a risky bond trades at a lower price than a risk-free bond. In the case of a failure, the bondholder receives a fraction of the predefined face value, often referred to as recovery rate (RR). Due to the presence of credit risk, the yield to maturity on the firm's debt, i.e., the bond, is greater than the risk-free interest rate, which introduces a credit spread compensating the bondholders for being exposed to the risk of default. A credit spread is thus the difference in yield between two bonds of similar maturity but of different credit quality.

In the next section we discuss relevant literature and specifically position our work among studies considering risk characteristics of Norwegian financial securities. We give an overview of data and statistics in section 3. The methodology and further details of the

calculations are provided in section 4, before the results are presented in section 5. We conclude and discuss further research in section 6. The appendices provide detailed derivations of mathematical results.

2 Literature Review

Modeling default risk has been an important topic in the financial literature for decades; however, there exists no general agreement on which modeling approach is superior. The two main approaches, reduced-form modeling and structural modeling are intensively debated by many authors and outperform one another in different settings (Arora et al. 2005). However, the structural models outperform the reduced-form models in their ability to connect a firm's capital structure to its credit risk, which is a major concern in real-life assessment of credit default risk. One of the best-known structural models is the Merton (1973) model. This model assumes that a company's equity can be regarded as a call option on its underlying assets with strike price equal to its liabilities, and utilizes the Black and Scholes (1973) option pricing formula.

The Merton model is often referred to as a benchmark among credit default models and is favored because of its simplicity. However, it is based on unrealistic assumptions and is therefore subject to limitations. In the academic literature there exist several modifications and extensions of the Merton model. Geske (1977) allows for re-financing in order to meet financial obligations, while Longstaff and Schwartz (1995) develop a two-factor model including a stochastic interest rate. The Merton model assumes zero tax and bankruptcy costs, factors which are included by Leland and Toft (1996). The most obvious drawback of the Merton model is that default can only happen at maturity. The more complex first-time-passage model by Black and Cox (1976) allows for default at any point in time by modeling the firm's equity as a knock-out barrier option. This study will exclusively focus on the Merton model and the Black & Cox model, because as far as we have been able to ascertain, more complex models generally do not perform better. See for instance (Zethraeus and Roos 2017).

Hao et al. (2010) provide a most useful literature review on credit risk modeling. Their paper describes and summarizes the development on credit rate modeling from January 1998 to April 2009. They identify the two broad categories we mentioned above being structural models and reduced-form models. They further divide structural models into exogenous default group models, to which the two models employed in this study belong, and endogenous default group models. The latter type of models allows obligors to choose the time of default strategically, which is often the case in practice. Further, reduced-form models are divided into *individual* level reduced-form models and *portfolio* reduced-form models. Interestingly the authors discover that individual level reduced-form models are only utilized in 9 percent of the reviewed papers. Finally, the paper also reviews some works studying performance tests of credit risk.

We note that score card models, which belong to the category of individual level reduced-form models, are now becoming popular amongst credit rating agencies, as a means to producing quantitative, *internal*, indicative credit assessments covering historic time periods. Also, back-testing the quality of credit rating processes is now compulsory for European CRAs and banks, and typically an integral part of the compliance and credit review processes of these entities. Finally, we note that only one of the 130 credit risk studies reviewed by Hao et al. focus on the Nordic markets, examining the effect of the Basel II rules for capital requirements in two of the largest Swedish banks.

Bank defaults are different from corporate defaults. In contrast to corporate defaults, a lot of regulatory action typically precedes bank defaults, and while pre-failure intervention often is voluntary for corporate institutions, for banks such measures are typically mandatory. In order to maintain control, policymakers and bank regulators rely on market-based measures to supplement accounting-based indicators. The distance-to-default is a commonly used measure which has earned prominence, partly due to its successful commercial implementation by Moody's KMV model ¹. When applied to banks, however, the concept of distance-to-default has limitations. The risk associated with leverage for a bank differs significantly from a non-financial corporation. The business model of a bank depends on leverage, as banks mainly fulfill an intermediary role between depositors and borrowers (Chan-Lau and Sy 2007). For a given credit rating or level of credit risk, banks are more leveraged than a non-financial corporation. The distance-to-default, however, would assign a higher risk score to the bank due to its higher leverage. We aim to overcome this limitation by introducing risk adjusted parameters. More details on this follow in section 4.

Different approaches for measuring credit default risk is valuable and might help catch early warning signs of distress applying to both individual firms and the market as whole. Structural models may of course be employed in performing credit ratings of issuers and bonds alike. A bond is a contractual agreement between the firm and the investor, by which the investor is guaranteed future payments in exchange for a payment at the date of issue (Zethraeus and Roos 2017). Many studies find that the Merton model generally underestimates credit spreads compared to real market spreads ². This study will determine if this is also the case for Norwegian financial bonds.

Altman and Saunders (1997) dispute the importance of credit ratings and explore how the rating processes of CRAs have changed over the last two decades. Their study emphasizes how the increase in the number of bankruptcies has resulted in the development of a new and more sophisticated method for credit ratings. One of the main objectives of credit rating agency regulations is to ensure that credit ratings are independent, objective, and of high credibility. In 2018 ESMA ³launched supervisory work-streams related to the quality of credit ratings, which in turn support the establishment of consistent approaches across the industry (ESMA 2019).

According to Becker and Millbourn (2011), it is important to stress the need for honest and structured rating processes, as the credibility of credit ratings is relevant for the proper functioning of the financial system. Many institutions, like commercial banks or insurance companies for instance, are forced to abide by regulations based on credit ratings. This might be due to investors who can only hold securities within a given rating category, or requirements to allocate a given fraction of their funds to different rating categories. A company's credit rating determines both funding costs and ability to issue debt, and is an essential factor for profitability or the survival of the firm. Gordy and Heiteld (2001) conclude that firms are more likely to be downgraded than upgraded, largely due to the fact that funding costs depend on their rating. Yet, the process of credit rating appears as a highly subjective judgment. Erdogan et al. (2018) identify lack of transparency and consistency in the process of credit rating and Schroeter (2013) expresses that the processes are a distinctive assessment based on individual judgment; *none can objectively predict the future*. Thus, although accepting the innate subjective nature of credit ratings, we and many

1. Originated by Kealhofer, McQuown, and Vasicek. The model is an integral part of Moody's Analytics credit rating processes.

2. Breccia (2012); Chan-Lau and Sy (2007); Vestbekken and Engebretsen (2016)

3. European Securities Markets Authority

authors believe that the rating agency industry would benefit from employing more objective credit default models supporting their credit rating processes.

2.1 Studies on Norwegian bond spreads and credit ratings

The Norwegian fixed income market is small compared to bond markets in Europe and the United States, and less data is available. There are relatively few studies examining Norwegian bonds.

Sæbø (2015*b*) studies risk premiums in Norwegian credit securities, examining all trades in fixed income securities (in NOK) registered on Oslo Stock Exchange in 2008–09. He discovers that on average only 21,5 percent of the spread is related to expected loss in case of default. The rest of the credit spread is due to other factors such as sector affiliation, firm size, and liquidity premium.

Further Sæbø (2015*a*) shows that the so-called credit spread puzzle is present in a sample from the Norwegian fixed income market from 2008–2013. He also finds that on average the structural model explains only 28 percent of spreads.

In their master thesis Knappskog and Ytterdal (2015) study credit spreads on Nordic high yield bonds, attempting to identify underlying explanatory factors. They use a Merton style structural modeling approach finding that on average about 65 percent of spreads can be explained by default risk. They further found that a liquidity premium variable proved statistically significant, explaining about 110 basis points of the spread for illiquid issuers.

Also Eskerud (2017), in his master thesis, examines a data set comprising 62 fixed coupon bonds issued by Norwegian corporates in the period 2000–2015. He employs a Merton style structural model in order to understand the pricing processes and spread formation of the bonds. Contrary to most authors employing structural models he finds that the model overestimates compensation for default risk for most bond price observations. He attributes this result to the manner in which he estimates the default point, i.e., the *reorganization boundary*.

P. E. de Lange et al. study spreads on Norwegian senior unsecured bank bonds and CoCo bonds in two papers; de Lange et al. (2018), de Lange et al. (2019). In the first paper, they analyze price formation processes, identifying underlying drivers of spreads on senior and subordinated bank bonds, using quantile regression. Being able to fully explain the entire distribution of credit spreads, they find that relations between the dependent and explanatory variables are stronger at the tails of the distributions of the dependent variables, than closer to the median.

In their second paper, de Lange et al. (2019), they model credit spreads on contingent convertible bonds (CoCos) in the Norwegian financial bond market, using a Merton style option model approach. They find that model spreads are narrower than market spreads in periods of market distress. They point out that this is precisely when liquidity vanishes from Norwegian bond markets, and liquidity premiums, which most structural models do not capture, soar. On the contrary, the model produces spreads that are fairly close to observed market spreads, when market conditions are benign and liquidity is less of a concern.

Our paper is related to de Lange et al. (2019) and Sæbø (2015*a*). It is however broader in scope and considers more securities than de Lange et al. (2019). We also attempt to capture some of the liquidity premium embedded in observable market spreads through our modelling of reorganization barriers/recovery ratios (ref. section 4.5.1). Not the least we make a comparison between the two prominent structural credit default models in the financial literature—the Merton model and the Black and Cox model—examining their ability to replicate Norwegian market spreads and produce reliable credit ratings of Norwegian financial

bonds. To our knowledge, the Black and Cox model has not previously been applied to a data set containing credit spreads on Norwegian financial bonds. We discover that calibrating the model parameters properly to underlying market characteristics, is equally important to the choice of model itself.

In section 3 below we briefly describe the different categories of quantitative data employed in this study.

3 Data

The core of our analysis is testing the ability of two structural models to replicate and predict credit spreads in the Norwegian financial market, and to contribute to the development of a structural credit default model intended to support the credit rating processes of Nordic Credit Rating. In order to build such a model, employing reliable and relevant data is a key factor, both for calibrating the model and for testing the results against real market spreads. In this analysis different types of data are used both for modeling and evaluating the two different approaches to spread calculations and credit ratings. The data sets collected apply for the two Norwegian banks, Den Norske Bank (DNB for short) and SpareBank1 SMN (SMN for short), which we briefly present below.

DNB is the biggest financial institution in Norway, and the second largest in the Nordic region, superseded only by Nordea. The company operates within the entire financial sector, including banking, finance, insurance, and brokerage. Table 1 gives an overview of credit ratings assigned to DNB by various rating agencies, collected from *Rating* (2019a). In addition, the AT1 securities (the CoCos) are rated BBB by *Fitch ratings*.

Table 1: DNB's credit ratings given by various credit rating agencies.

DNB			
Rating Agency	Long-term Rating	Outlook	Year of Rating
Moody's	Aa2	Stable	2019
S&P	AA-	Stable	2019
Scope	AA-	Stable	2019
DBRS	AA(low)	Stable	2019

SpareBank1 SMN is a Norwegian savings and loans bank with headquarter located in Trondheim. The credit ratings assigned to SMN can be found in table 2 (Rating 2019b). Hybrid capital, i.e., the CoCos, are rated Baa2, subordinated debt is rated one notch down from the viability rating of a- and senior unsecured debt is rated A1 (Moody's Investors Services 2019).

Table 2: SMN's credit ratings given by various credit rating agencies.

SMN			
Rating Agency	Long-term Rating	Outlook	Year of Rating
Moody's	A1	Stable	2019
Fitch	AA	Stable	2019

In order to test whether the two structural models we employ in section 4 are able to replicate observed market spreads, and thus credibly predict future credit spreads, we collected data on credit spreads for senior bonds, subordinate bonds and contingent convertible bonds (CoCos) issued by SMN and DNB. The data sets are collected from *Nordic Bond Pricing*⁴ and contain quotes from the time period of 2014–2019. These quoted market spreads are compared to spreads computed by the models. For the purpose of modeling equity volatility, we collected stock prices for DNB and SMN over the time span 2000–2019. We retrieved daily closing prices of SMN equity (MING) and DNB equity from *Bloomberg*. The stock prices were used for calculating daily returns and as input to the equity volatility models. Further, the equity volatility estimates were converted into asset volatility estimates, by the models outlined in section 4.4.1. In order to compute the default trigger, given as an asset-to-debt ratio, asset-values, debt-values, and other accounting key numbers are required. We collected complete balance sheets from both SMN and DNB from *Proff Forvalt*⁵. The data set includes accounting entries from the time period of 2014–2019. In addition, we obtained the number of issued stocks in order to calculate the market value of the equity of each company. Interest rates are important parameters in both of the structural models used in this analysis. We have collected two sets of interest rates, which are relevant for calibrating credit default models on data in the Nordic region. The first set contains the three-month NIBOR⁶ and the second set includes the five years treasury rate. Both series include daily observations from the year 2014 until the beginning of 2019, and are collected from Bloomberg.

In table 3 we exhibit long-term expected default rates for different credit ratings. This table indicate that NCR expects, when looking back at their ratings population, that the representative rating category will be assigned the listed default probabilities.

Table 3: Matrix of expected average default rate within each classification retrieved from www.nordiccreditrating.com. The values are given in percentage.

	AAA	AA	A	BBB	BB	B	CCC and below
1Y	0.00	0.00	0.02	0.20	1.80	4.00	25.00
3Y	0.00	0.03	0.25	1.00	5.00	13.00	40.00
5Y	0.00	0.08	0.50	1.90	7.50	18.00	45.00
10Y	0.01	0.20	1.40	4.00	13.50	25.00	50.00

4 Methodology

In this section we describe the models and methods we apply. Fundamentally, our objective is to develop a framework for assigning credit ratings and calculating credit spreads of financial bonds. To this end we implement two different structural models; the Merton model (Merton 1973) and the Black and Cox model (Black and Cox 1976). Our approach aims at developing a framework, which is able to rate all kinds of debt. In this analysis three different classes of capital are tested: Additional Tier 1 capital (CoCos), Tier 2 capital (subordinated bonds) and senior bonds.

4. www.nordicbondpricing.no

5. www.forvalt.no

6. Norwegian Interbank Offered Rate

4.1 Assumptions

In order to simplify the complex workings of financial markets, both the Merton model and the Black & Cox model rest on several assumptions: 1. Markets are perfect and all assets are tradeable. 2. Trading takes place in continuous time and there are no taxes or transaction costs. 3. Short sales are possible without constraints. 4. There is one risk-free interest rate, and every asset can be discounted at this rate. 5. The debt structure of individual firms is also assumed to be constant through the time interval $[t, T]$ consisting of one zero-coupon bond only with maturity at time T . 6. Each company's asset value is assumed to follow a Geometric Brownian Motion (GBM) process, given by the stochastic differential equation:

$$\frac{\partial A}{A_t} = (\mu_t - \delta_t)dt + \sigma dW_t \quad (1)$$

where A is the firm's asset value, μ is the expected rate of return, δ is the dividend, σ is the volatility of the firm's assets and dW is a Wiener Process. For simplicity and to avoid prior beliefs δ is naively set to zero and μ is replaced with r in order to obtain a risk-neutral probability measure. The reason behind this simplification is the fact that the Black & Scholes option pricing formula, underlying the Merton model and the Black & Cox model, does not include any parameter affected by investors' risk preferences. We may therefore assume that investors are risk-neutral using the risk-free interest rate when discounting securities prices.

When the above assumptions are satisfied, the Black-Scholes-Merton formulas (Eq. 2 and 3) below apply. As pointed out earlier, both these models are option based models representing the firms' equity as a call option on the firms' underlying value, with strike price equal to the face value of its debt (Bharath and Shumway 2008). The analogy to option pricing originates from the fact that debt holders are ranked higher than equity holders in the default hierarchy. If the company survives until liquidation, the debt holders are repaid the nominal amount of debt, while the equity holders receive the remaining company value. However, in the case of a default, the debt holders are only repaid what is left after bankruptcy costs, and the equity holders receive nothing. This payment structure makes owning equity equivalent to holding a European call option on the firm's assets. Similarly owning debt is equal to owning a default free bond and writing a put option on the assets of the firm. In general, this is a simplification of reality, but nevertheless, an intuitive interpretation of corporate debt and equity and the essence of structural models (Zethraeus and Roos 2017).

$$\text{Call} = E = AN(d_1) - e^{-rT}DN(d_2) \quad (2)$$

where

$$d_1 = \frac{\ln\left(\frac{A}{D}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

where

$$d_2 = d_1 - \sigma\sqrt{T}$$

Equation 2 expresses the value of the firm's equity as the firm's asset value in excess of the properly discounted face value of debt. A and E denote the firm's asset and equity value, respectively, D is the book value of its liabilities, r denotes the risk-free interest rate and N denotes the normal cumulative distribution. The d_1 term expresses the distance between the value of the assets and the value of the debt, measured in units of standard deviations.

Similarly, the credit risk arising from buying the company's debt, is equal to the value of a put option on the value of the assets of the firm, with strike price equal to its liabilities. This can be explained by realizing that a portfolio combining the bond and the put will be risk-free. In other words, the portfolio will yield a payout of D irrespective of the value of the firm's assets when the bond matures at time T . The value of the put option is simply the cost of eliminating the credit risk associated with the bond (de Lange et al. 2019) and it is given by

$$\text{Put} = e^{-rT}DN(-d_2) - AN(d_1) \quad (3)$$

4.2 Credit Spreads

The probability that a bond will default at a given point in time is equivalent to the probability that the value of the firm's assets will fall below the face value of debt at maturity (Zethraeus and Roos 2017), i.e. the probability that $A_T < D$. In the case of a failure the bondholders receive a fraction of the predefined face value, often referred to as the recovery rate (RR). Due to the presence of credit risk, the yield to maturity on the firm's debt, i.e., the bond, is greater than the risk-free interest rate. This implies that $D_t < De^{-r(T-t)}$, which introduces a credit spread compensating the bondholders for being exposed to the risk of default (de Lange et al. 2019). A credit spread is a difference in yield between two bonds of similar maturity but of different credit quality. Yield to maturity is the total yield resulting from all coupon payments and any gains from a "built-in" price appreciation. The current yield is the portion generated by coupon payments (Harper 2018).

Different securities have various trigger levels for default. As a fixed income security, a bond is classified according to collateral, convertibility, maturity, and price. A secure bond is secured by an underlying asset that can be sold by the bondholder to satisfy a claim. A convertible bond can be exchanged for other securities of the issuing company at a future date. Contingent convertible securities (CoCos) are hybrid capital instruments that absorb losses in accordance with their contractual terms when the capital of the issuing bank falls below a certain threshold. Owing to their capacity to absorb losses, CoCos have the potential to satisfy regulatory capital requirements (Avdjiev et al. 2013).

Equation 4 gives the implied credit spread, S , which is calculated with reference to the bond's yield to maturity. See Appendix A.1 for more details.

$$S = -\frac{1}{(T-t)} \ln(\text{PD}) \quad (4)$$

Further, Chen et al. (2009) show that the credit spread can be expressed by equation (5), below, where T is time to maturity, RR is the recovery rate, PD_{DC} is the probability of default given by the distance-to-capital approach (section 4.2.1) and $\theta = \frac{\mu - r}{\sigma}$ is the Sharpe ratio.

We employ this formula when calculating credit spreads by the Merton approach.

$$S_{DC} = -\frac{1}{(T-t)} \ln\left(1 - (1 - RR)N\left(N^{-1}\left(PD_{DC} + \theta\sqrt{T}\right)\right)\right) \quad (5)$$

4.2.1 The Merton Approach

In order to obtain the credit spread, it is necessary to derive the probability of default. Calculating the probability of default can be done in different ways. One of the most well-known methods, which is based on the Merton model, is Moody's KMV model. This is an approach utilizing the concept of distance-to-default (DD) (Eq. 6), as a way of categorizing

firms' credit risks by their respective distances to default. The KMV model further combines this measure with a substantial amount of empirical data to obtain an empirically inferred default frequency (Lu 2008). In this study, μ is replaced by the risk-free interest rate r . K is the preassigned asset value *at* default, comprised by the current value of liabilities and the default trigger.

$$DD = \frac{\ln\left(\frac{A}{K}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad (6)$$

When applied to banks, however, the concept of distance-to-default has limitations. In contrast to corporate defaults, regulatory actions often anticipate bank defaults, and while pre-failure intervention often is voluntary for corporate institutions, banks are typically forced by statutory regulations (Chan-Lau and Sy 2007). The business model of a bank depends on leverage, as banks mainly fulfill an intermediary role between depositors and borrowers. For a given credit rating or level of credit risk, banks are more leveraged than non-financial corporations and the distance-to-default measure would assign a higher risk score to the bank due to its higher leverage. In order to overcome this limitation the distance-to-default measure is replaced by distance-to-capital (DC).

$$DC = \frac{\ln\left(\frac{A}{\lambda K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad (7)$$

were

$$\lambda = \frac{1}{1 - PCAR}$$

and PCAR is the capital adequacy threshold. The capital adequacy requirement (also known as regulatory capital) is the amount of capital required by the financial regulator. These requirements are put into place to ensure that financial institutions do not take on excessive leverage and become insolvent, and is usually expressed as a capital adequacy ratio of equity that must be held as a percentage of risk-weighted assets (*Capital requirement 2019a*). Capital requirements govern the ratio of equity to debt, recorded on the liabilities' side of a firm's balance sheet. These requirements should not be confused with reserve requirements, which govern the assets side of a bank's balance sheet, i.e., the proportion of its assets it must hold in cash or highly liquid securities (*Capital requirement 2019b*).

The basic capital requirement is 8%, but in modeling credit spreads each security class operates within different levels of capital requirement before default. Figure 1 illustrates the ranking of capital within a firm and regulatory capital requirements. The lowest ranking capital will absorb losses first.

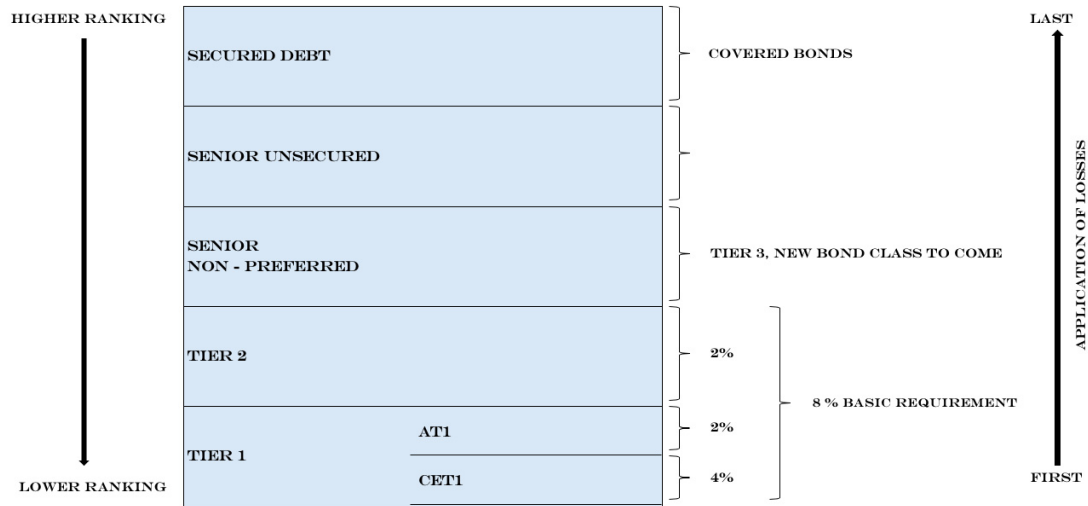


Figure 1: Illustration of the capital ranking and required capital buffer in financial institutions.

In order to compare the distance-to-default approach with the Black & Cox modeling approach, and instead of relying on non-existing empirical data, we compute a Merton style theoretical probability of default. The model-assumption that the asset value follows a Geometric Brownian Motion process implies that the incremental changes of asset values are normally distributed. Based on this assumption, the probability of default is given by the normal cumulative distribution

$$PD_{DC} = N(-DC) \tag{8}$$

4.2.2 The Black & Cox Approach

Due to its simplicity, Merton’s model is a very appealing specification of credit risk. Yet, the model implies that firms can only default *at maturity* and actual credit spreads tend to be underestimated. An extension of Merton’s model, and an attempt to overcome this limitation, is the Black & Cox model (1976), built on the foundation of a knock-out barrier option model. We shall refer to this model as the BC model. To our knowledge, this model has not previously been applied to a data set containing credit spreads on Norwegian financial bonds. This is a first-time-passage model. In addition to calculating the probability that the company will default at maturity, it also accounts for the risk that the firm might default prior to maturity of its debt, i.e., over the time interval $[t, T]$ by introducing a time-dependent safety covenant given by $K(\tau) = RR \cdot De^{-\gamma(T-t)}$.

Safety covenants provide the bond holders with a right to force the company into bankruptcy if the firm is doing poorly, compared to a predefined financial standard. As soon as the value of the company’s assets crosses this lower threshold, the bondholders take over the firm. The safety covenant is simply a properly discounted value of the liabilities, where γ is the discount rate reflecting the company’s weighted average cost of capital (WACC). In our experiments we fix this rate at 8%, a choice also confirmed by Koziol (2013). The parameter τ denotes the continuous time $\tau \in [t, T]$, D is the debt-value and RR is the recovery rate. The probability that the asset value has *not* reached its reorganization boundary, i.e., the probability of survival over the time interval, is further given by

$$PS_{BC} = N(u_1) - \left(\frac{A}{K(\tau)} \right)^{1 - \frac{2(\mu - \delta - \gamma)}{\sigma^2}} N[u_2] \quad (9)$$

Where

$$u_1 = \frac{\frac{\ln(A)}{\ln(D)} + \left(\mu - \delta - \frac{1}{2}\sigma^2 \right) (\tau - t)}{\sqrt{\sigma^2 (\tau - t)}}$$

and

$$u_2 = \frac{2 \ln(K(\tau)) - \frac{\ln(A)}{\ln(D)} + \left(\mu - \delta - \frac{1}{2}\sigma^2 \right) (\tau - t)}{\sqrt{\sigma^2 (\tau - t)}}$$

The BC model comprises two main components. $N(u_1)$ denotes the probability of the asset value, A , not falling below some preassigned threshold at time T . This term is equivalent to the *complement*⁷ of the default probability of the Merton model and can be interpreted as the probability of *not* defaulting *at* maturity. The second part includes the probability of the asset value not reaching the default barrier over the time interval $[t, T]$ i.e. the probability of not defaulting *prior* to maturity. The probability of default is thus given as $PD_{BC} = 1 - PS_{BC}$. By using the distribution of the first-time passage model to the valuation of a bond we obtain the formula given by Black and Cox (1976), which can be found in A.2. Further the credit spread can be constructed by calculating the yield on the debt and subtracting the interest rate, given in equation 10, where D_{BC} is the debt value obtained by subtracting the equity value produced by the Black & Cox formula from the asset value, and D is the accounting debt value obtained from the firm's balance sheet.

$$S_{BC} = y - r = -\frac{1}{(T-t)} \left(\ln \left(\frac{D_{BC}}{D} \right) \right) - r \quad (10)$$

4.3 Bond Rating

4.3.1 Debt Ratings in relation to Issuer Rating

Credit ratings on long-term debt securities are based on both the issuer's credit rating and the specific terms, conditions, and characteristics of the debt instrument itself. Consequently, long-term debt ratings could be higher, lower, or equal to the issuer's rating. In general, senior bonds obtain more or less the same rating, while subordinated debt is usually rated lower than the issuer rating (Scope 2018).

4.3.2 Senior Bonds

Senior bonds are high quality bonds with first priority of repayment in case of a default, and thus rarely experience losses. In one way senior capital can be viewed as equal to the issuing firm itself. When rating senior bonds, the result is often identical to the issuer rating.

7. $PD_{DC}^c = 1 - PD_{DC}$

4.3.3 Tier 3 Senior non-preferred

Tier 3 (T3) securities rank lower than senior securities and investors are exposed to principal-loss absorption risk. These securities are convertible into equity in a bail in event. This is a new class of securities issued by banks and will be used to recapitalize banks in a bail-in event. Norwegian banks that are supposed to be recapitalized in a bail-in event will need to fulfill new capital requirements by end of 2022 by issuing T3 bonds. Also, T3 securities rank above CET1, Tier 1 and Tier2 debt in insolvency and bail-in situations, and do not require a trigger for write-down. T3 bonds will not be further analyzed in this paper.

4.3.4 Tier 2

Tier 2 (T2) securities rank lower than senior securities and investors are exposed to principal-loss-absorption risks. Consequently, when rating T2 securities, the assigned rating is typically two notches down from the rating of the bank's senior unsecured debt. However, T2 securities rank above Tier 1 debt in insolvency and bail-in situations, and do not require a trigger for write-down or conversion.

4.3.5 Tier 1

Tier 1 (T1) are the lowest ranked securities and when rating T1 capital, rating agencies often assign four notches down from the rating of the issuing bank's senior unsecured debt. This notching is supposed to reflect the twin risks of both coupon cancellation and principal-loss absorption. Further, there are instances of additional down-notching, due to security- or issuer-specific applications, such as trigger level, capital structure and regulatory requirements.

4.4 Variable Estimation

Parameter estimation is an important part of credit default modeling. A central parameter is volatility. In the Nordic region asset values are generally more volatile compared to the US market, partly due to the fact that fewer firms are publicly listed. Another important parameter is the default trigger. Within the framework of structural models, default occurs when the firm is unable to meet its financial obligations. In the case of a default, the bondholders receive a fraction of the preassigned par value. Also, the Sharpe ratio measuring asset dependent risk premium per unit of risk, is sometimes used as a component of the credit spread.

4.4.1 Volatility

A vital parameter in structural models, which has a substantial impact on companies' theoretical probability of default, is the volatility of firms' assets. However, estimating asset volatility is challenging. The value of a firm depends on both equity and debt, and thus asset volatility should be represented by a combination of the volatility of these balance sheet items. One possible approach to volatility estimation is based on the asset variance equation given by Schaefer and Strebulaev (2008)

$$\sigma_A^2 = (1-L)^2 \sigma_E^2 + L^2 \sigma_D^2 + 2L(1-L)\sigma_{ED} \quad (11)$$

In equation 11 σ_E is the equity volatility, σ_D is the debt volatility, σ_{ED} is the co-variance between the returns on debt and equity and L is the leverage ratio. Two possible simplifications given by Feldhutter and Schaefer (2015) are computing the upper or lower bound of the asset variance. The upper bound assumes zero leverage and is simply the equity volatility, while the lower bound is determined as $\sigma_E^2 (1-L)^2$ where L is the market leverage, i.e., assu-

ming that the debt carries no asset risk and a theoretical hedge ratio of zero. Even though equation 11 is a more realistic estimate of asset variance, as debt in fact carries some asset risk and since equity and debt co-vary, it is reasonable to assume that the lower bound estimate produces close to identical estimates for firms within the higher credit rating categories (Schaefer and Strebulaev 2008). However, in an attempt to make yet another adjustment for differing levels of leverage ratios, the lower bound variance estimate is multiplied by a scalar, x . In the banking sector, which includes both SMN and DNB, debt is frequently refinanced and interest payments are stochastic. This results in volatile liabilities, and to remedy the simplification of deterministic debt, our final estimate for asset variance is given by

$$\sigma_A^2 = x(1-L)^2 \sigma_E^2 \quad (12)$$

In order to calculate equity volatility (σ_E) we collect time series data of equity returns. An intuitive first approach might be using long run volatility which is known to be relatively stable over time (Christoffersen 2012). However, in terms of credit risk modeling, the long-run volatility has severe limitations in that it tends to underestimate the overall probability of default. Two of the most commonly used methods for estimating future volatility are the EWMA⁸ model (Morgan 1996) (Eq. 13), and the GARCH(1,1)⁹ model (Bollerslev 1996) (Eq. 14). Both models combine historical volatility and daily returns in order to estimate future volatility. In addition, the GARCH(1,1) assumes that volatility is mean-reverting, with a tendency to revert towards its long-run mean.

$$\sigma_{E,t+1}^2 = (1-\lambda)^2 R_t^2 + \lambda \sigma_{E,t}^2 \quad (13)$$

$$\sigma_{E,t+1}^2 = \omega + \alpha (R_t - \psi \sigma_{E,t})^2 + \beta \sigma_{E,t}^2 \quad (14)$$

However, none of these models entirely capture the fact that a negative return yields a substantially higher credit risk, compared to a positive return of the same magnitude. In order to fully capture the increased credit risk in case of a downturn, often referred to as the leverage effect, we also test the nonlinear GARCH (NGARCH) model

$$\sigma_{E,t+1}^2 = \omega + \alpha (R_t - \psi \sigma_{E,t})^2 + \beta \sigma_{E,t}^2 \quad (15)$$

where R_t is the daily log return, and the persistence parameters ($\alpha, \beta, \psi, \omega$) are estimated by MLE¹⁰. Having estimated daily equity variances, we next seek to find the annual equity volatility. This is obtained by multiplying the square-root of the daily variance by the square-root of 250 trailing days, resulting in the following equation

$$\sigma_E = \sqrt{250 \sigma_{E,t}^2} \quad (16)$$

In structural credit default models, asset volatility is estimated from observable equity volatility. In our modeling approach we let each capital tranche on the liability side of the balance sheet be affected by the leverage ratio of that particular debt class, i.e. we expose Tier 1, Tier 2 and senior debt to different levels of asset volatility. This results in different Sharpe ratios for the bonds, which will be further discussed in section 4.5.2

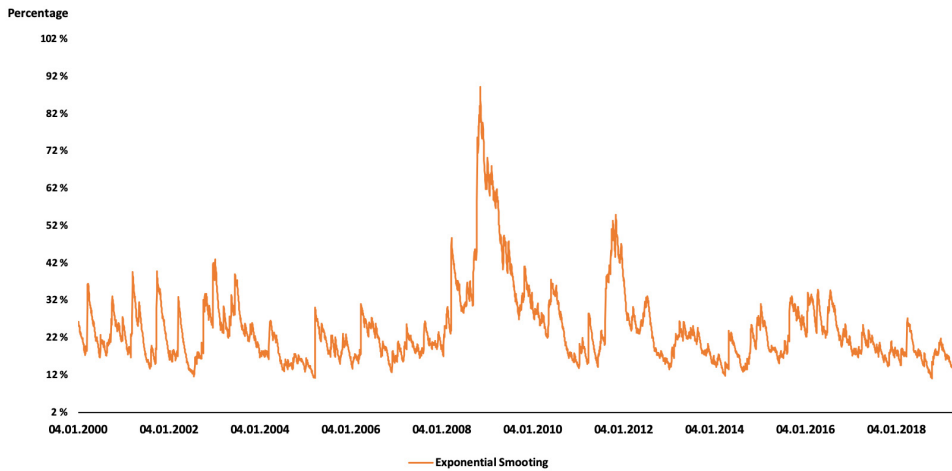
8. Exponentially Weighted Moving Average

9. Generalized Autoregressive Conditional Heteroskedasticity

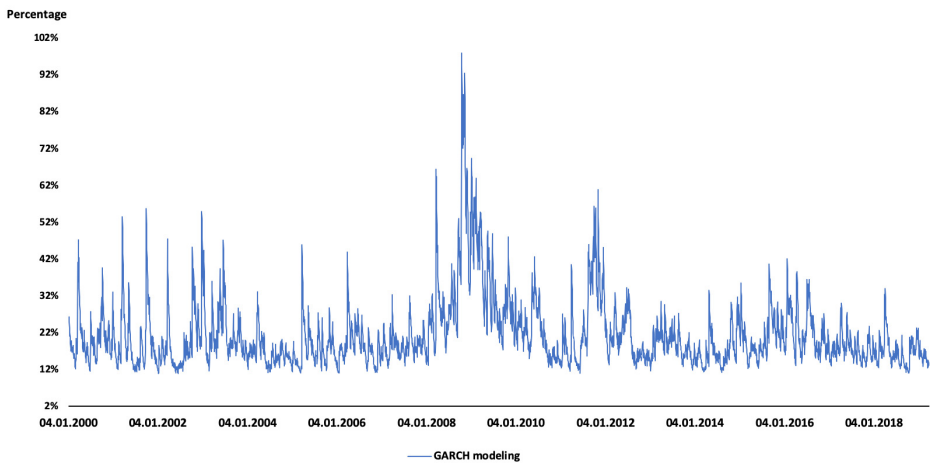
10. Maximum Likelihood Estimation

4.4.2 Equity Volatility Estimates

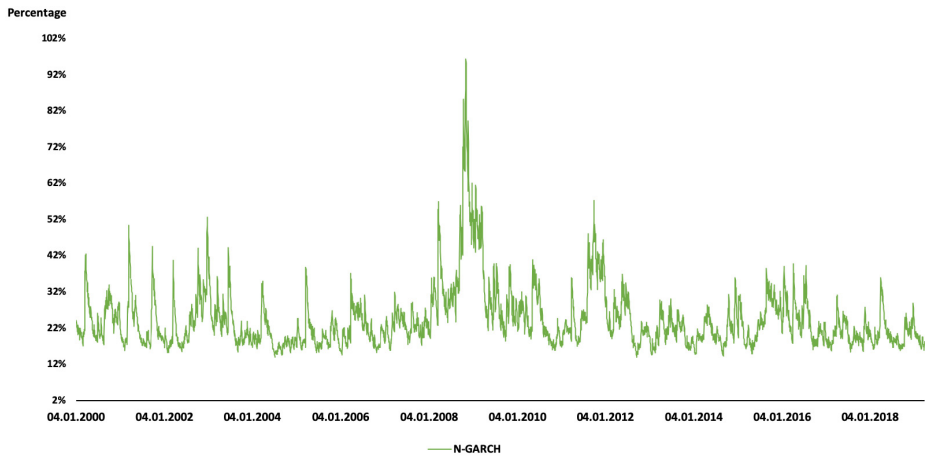
Volatility is conceivably the most important input parameter to structural credit default models since model outputs are highly sensitive to volatility changes.



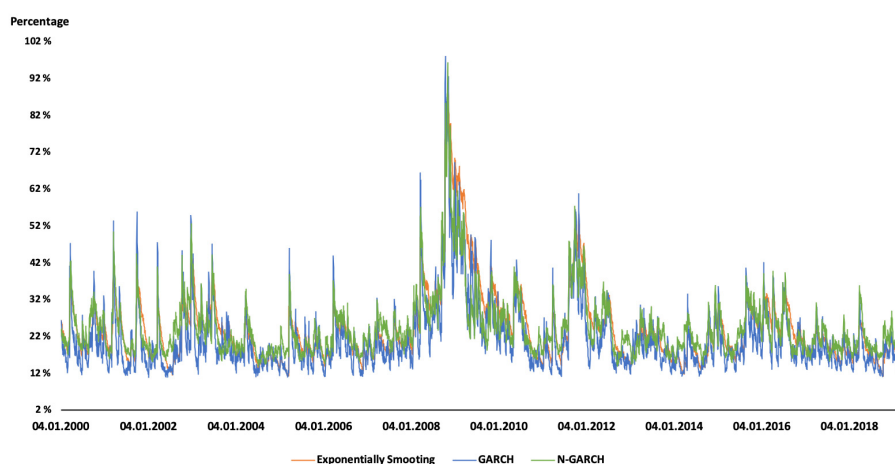
2a



2b



2c



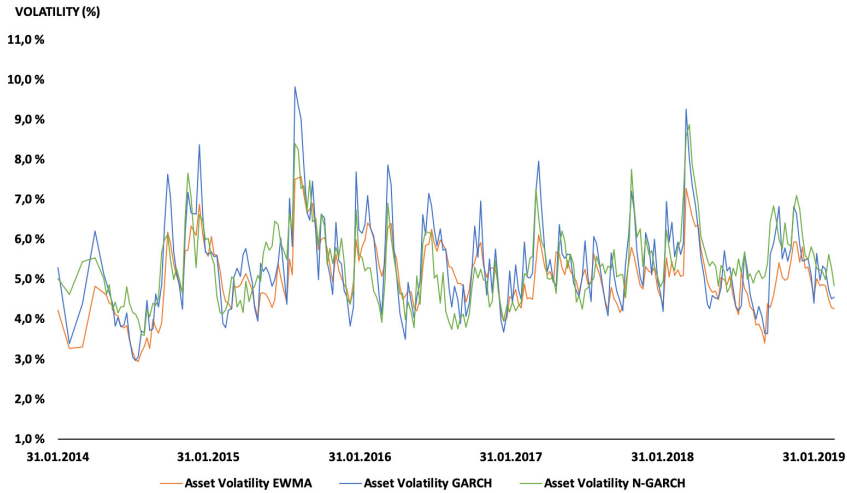
2d

Figure 2: The plots illustrate estimated equity volatilities for SMN over a time-period of 19 years from 2000 to 2019, based on the bank's quoted equity price (MING). All estimates indicate an annual equity volatility oscillating around 25%, except during the financial crisis where equity volatility made a jump. (a) Volatility estimates based on Exponentially Weighted Moving Average (EWMA). (b) Volatility estimates based on Generalized Autoregressive Conditional Heteroskedasticity (GARCH). (c) Volatility estimates based on Non-linear Autoregressive Conditional Heteroskedasticity (N-GRACH). (d) Comparison of volatility estimates indicating that all models obtain similar results.

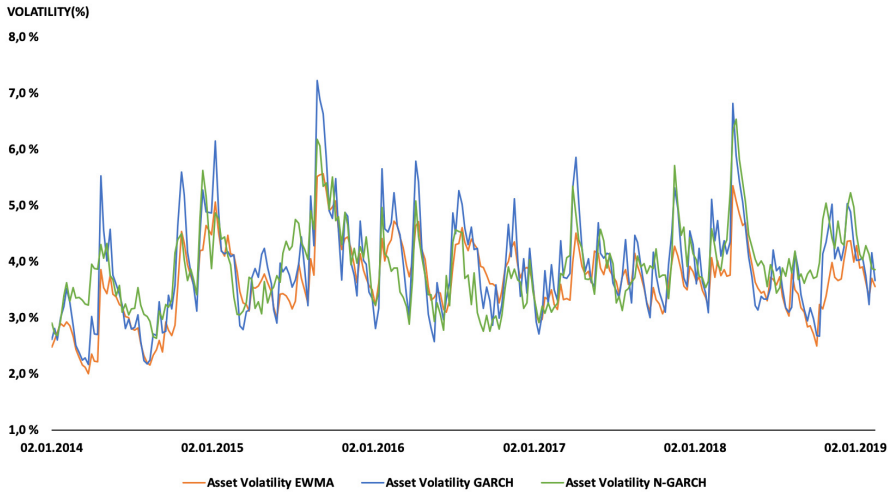
Thus volatility estimation is key to default modeling. Figure 2 illustrates the periodic changes in equity volatility over a time span of 19 years (2000–2019), generated by three different volatility modeling approaches. We recognize two significant peaks: the financial crisis in 2009 and the sovereign debt crisis in 2012. We further observe a high degree of correlation between the three different volatility models. The EWMA approach is the simplest model, but still it does perform well compared to the more complicated GARCH models. We also refer readers to the figures in Appendix D. However, the N-GARCH model is the most realistic approach and is viewed as a more prudent way of capturing credit risks.

4.4.3 Asset Volatility Estimates

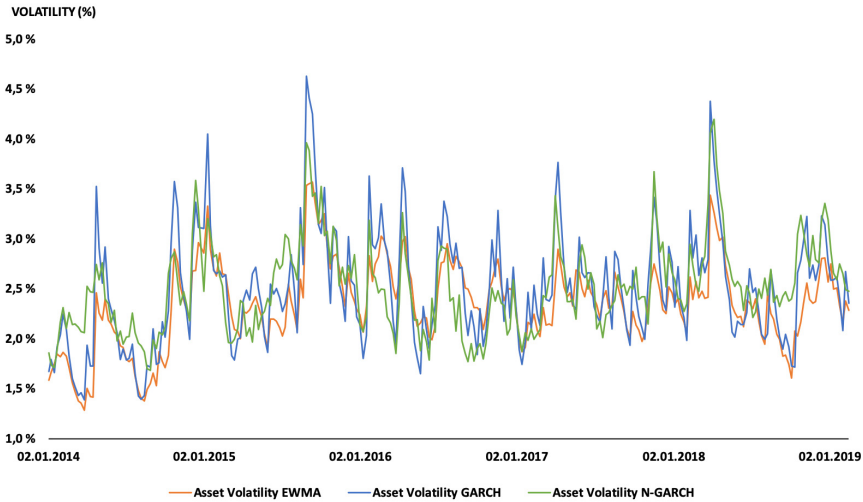
In figure 3 we focus on the five-year default modeling period (2014–19) and convert equity volatility into asset volatility. Compared to the financial crisis in 2009 and the sovereign debt crisis in 2012, other peaks become negligible. However, we notice two smaller peaks in asset volatility in 2015 and 2018. One possible explanation might be the decline in oil prices, which started in 2014 and had a negative impact on the Norwegian economy. Also economic uncertainty in Europe about the outcome of the complicated Brexit negotiations may have contributed to increased financial volatility. In 2018, international trading conflicts between the US and China caused turbulence in the entire financial market. The shapes of the estimated volatilities roughly follow the same variations across debt classes. Note, however, that Tier 1 volatility oscillates around 5 %, Tier 2 has a volatility of almost 3.5 % and senior bonds have the lowest volatility of approximately 2.5 %.



3a



3b



3c

Figure 3: The plots illustrate estimated asset volatilities for SMN over the 5 year time period 2014–2019, based on the three different volatility models (EWMA, GARCH, N-GARCH). (a) Asset volatility Tier 1. (b) Asset volatility Tier 2. (c) Asset volatility senior.

In an attempt to account for differing levels of leverage ratios, the asset volatility estimates are multiplied by a scalar, x . Feldhutter and Schaefer (2015) operates with a scalar of 1.8 for leverage ratios greater than 75%. In this study, we aim at producing suitable credit spreads within three different classes of seniority. Since the degree of leverage varies between the distinct securities, we also introduce unique scalars for each security class. In table 4 we report the scalars assigned to SMN bonds, observing that senior debt has the lowest numerical scalar value resulting in less volatility compared to lower quality bonds. We note however, that the *optimized* scalar values differ between the two banks.

Table 4: Overview of the relevant scalars multiplying volatility to adjust for level of leverage ratio, based on seniority.

Seniority	Scalar
Senior	2.00
Tier 2	3.00
Tier 1	4.00

4.5 Parameter Calibration

Many scientific studies (ref. section 2) conclude that the Merton model tends to underestimate credit spreads. In this analysis, however, we have tried to overcome this limitation by introducing more realistic *recovery ratios* and *Sharpe ratios*. Examining the Merton model, it seems like the results are rather sensitive to changes in the recovery rate (RR) and the Sharpe ratio (Θ). A higher Sharpe ratio produces higher credit spreads, while a lower Sharpe ratio reduces the spread. Changes in the recovery ratio produce the opposite effect: a higher recovery ratio results in lower spreads and vice versa. Note that these recovery ratios and Sharpe ratios are only employed in the Merton modeling approach. In the Black & Cox model, on the other hand, changes in the discount value of the safety covenant, γ , changes the barrier and thus influence the results. A higher γ increases the peaks in the model and amplify the oscillation of model spreads. It is reasonable to believe that the value of γ should be somewhat lower for senior bonds than for Tier 1 and Tier 2 capital.

4.5.1 Recovery Rate

When modeling credit spreads on contingent convertible bonds (CoCos) issued by two Norwegian banks, de Lange et al. (2019) emphasize the fact that liquidity premiums are not captured by the Merton model even though liquidity is an important issue, especially in the Norwegian market. However, Arora et al. (2005) finds empirical evidence showing that the liquidity premium is implicitly reflected in the *recovery ratio* (RR). When evaluating bonds, the relevant recovery rate is the trigger level where the bonds start absorbing losses. This trigger level is associated with the default point of the firm. In the Merton model, the recovery rate is interpreted as the partial expectation of the firm's assets given default, divided by the probability of default. It is a model *output*. In practice, recovery rates are affected by corporate issues, instrument type and macroeconomic conditions. The rate is directly proportional to the instrument's seniority, which indicates that an instrument that is more senior in the capital structure will usually have a higher recovery rate than one which is lower in the capital structure (Kenton 2019). Macroeconomic conditions affect financial markets. If the percentage of companies defaulting is high, as it is during a financial crisis, recovery rates decrease. For example, Standard & Poor's estimated that for all issuers that emerged from default during the chal-

lenging 2008–2010 period, the average recovery rate across all instruments was 49.5%, which is somewhat lower than the 51.1 % average recovery rate over the 1987–2007 period (Kenton 2019). Also, the lower a company’s debt-to-asset ratio, the higher is the expected recovery rate.

Historical market estimates of recovery rates (RR) vary a lot. Nevertheless, seniority does affect the recovery rate and it is well known that RRs for high quality bonds are superior to RRs on lower quality bonds ¹¹. Table 5 gives an overview of the calibrated (theoretical) recovery ratios for this analysis. We do obtain similar results for SMN and DNB, but there are minor differences. The main difference between DNB and SMN as such, is the size of the firms. Despite statement by Altman and Kishore (1996), who express that neither the size of the issue nor the time to default from its original date of issuance has any association with the recovery rate, it might be reasonable to believe that there is a size-effect which should be included in the analysis. Our estimated parameters are supported by Moody’s average RR of 38 % and the average RR of the financial sector of 49.20% (Chen et al. 2009).

Table 5: Recovery rates for the three different securities classes in the two banks under study.

Security class	Recovery Ratio SMN	Recovery Ratio DNB
Senior	0.48	0.50
Tier 2	0.34	0.38
Tier 1	0.31	0.00

4.5.2 Sharpe ratio

The Sharpe ratio is a means of measuring the performance of investments by adjusting for risk dividing excess return (alpha) by the standard deviation, i.e., a risk reward relationship (Martin 2017). We employ Sharpe ratios in our formula for credit spreads in the Merton model (eq. 5). The Sharpe ratio measures the risk premium per unit of standard deviation in an investment asset. It is therefore reasonable to believe that Tier 1 capital will achieve the highest Sharpe ratio, since lower seniority is riskier and therefore requires higher return. When calibrating our model we have tested different levels of Sharpe ratios. We display the results in table 6. In view of the risk of over-fitting the model and due to lack of sample data, further investigation may be necessary before concluding on the correct size of the Sharpe ratio. Still, we notice that there are differences between Sharpe ratios for all security classes, which we expected. Similar results can be found by de Lange et al. (2019), who use a Sharpe ratio of 0.23 when modeling the credit spread of Norwegian CoCo bonds, and Chen et al. (2009) who estimate the Sharpe ratio to be 0.22. We further found that SMN’s Sharpe ratios clearly differ from DNB’s Sharpe ratios, which equal zero for all securities, posing the question of whether some kind of size-effect should be included.

Table 6: Overview of Sharpe ratios for the different securities classes of SMN included in the Merton approach for modeling credit spreads.

Security Class	Sharpe ratio SMN
Senior	0.05
Tier 2	0.10
Tier 1	0.20

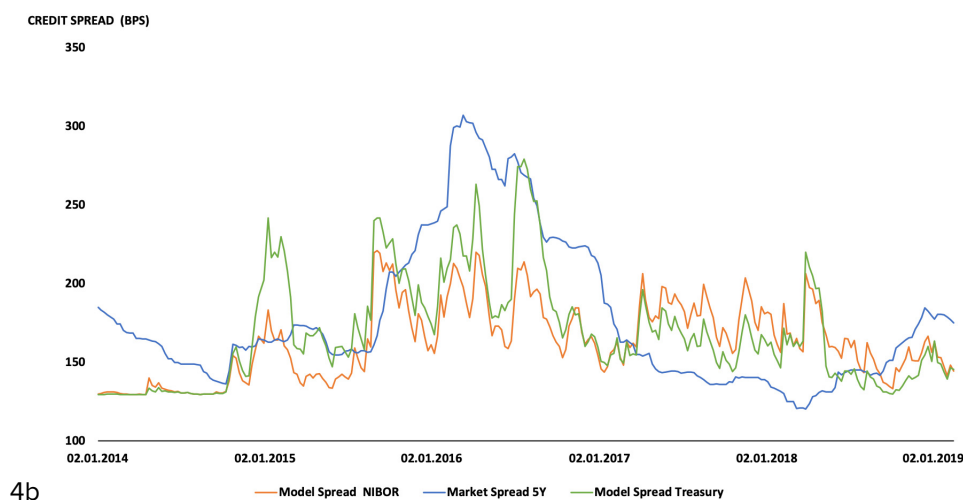
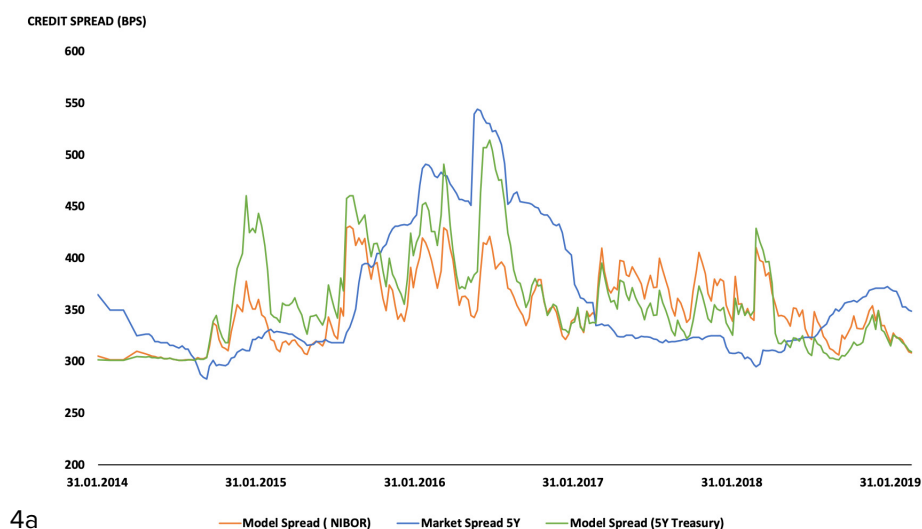
11. Jark (2019), Altman and Suggit (1999), Asarnow and Edwards (1995), (Grossman et al. 1997), Carty and Lieberman (1996), Altman and Kishore (1996)

5 Results, Analysis and Discussions

In this section we present the most relevant results from our study. We have chosen to illustrate each step in the analysis by showing the outcome from applying the models to Spare-Bank 1 SMN. A similar approach is applied to DNB. More details on this can be found in Appendix B. Comprehensive discussions and analysis will be presented successively.

5.1 Credit Spreads Merton

In order to be attractive to investors, risky bonds should offer higher yields than comparable risk-free bonds, compensating bond holders for the probability of losses. Consequently, a risky bond trades at a lower price than a risk-free bond. Based on our results in figure 4, we can observe that the Merton model captures the characteristics of all three security classes quite well, compared to observed market spreads. The shapes of all curves are similar, but credit spread levels differ. Roughly speaking, Tier 1 capital has a credit spread in the range of 300–500 basis points (bps), Tier 2 capital obtains credit spreads in the range 150–300 bps and finally the senior capital spread varies between 50–150 bps. Further, we observe that the orange line, which is spread to 3M NIBOR, and the green line, which is spread to the 5Y treasury rate, provide the highest spread at different times. This can be explained by variations in the market. Credit spreads of DNB securities produced by the Merton approach exhibit a similar pattern and can be found in appendix B.1.



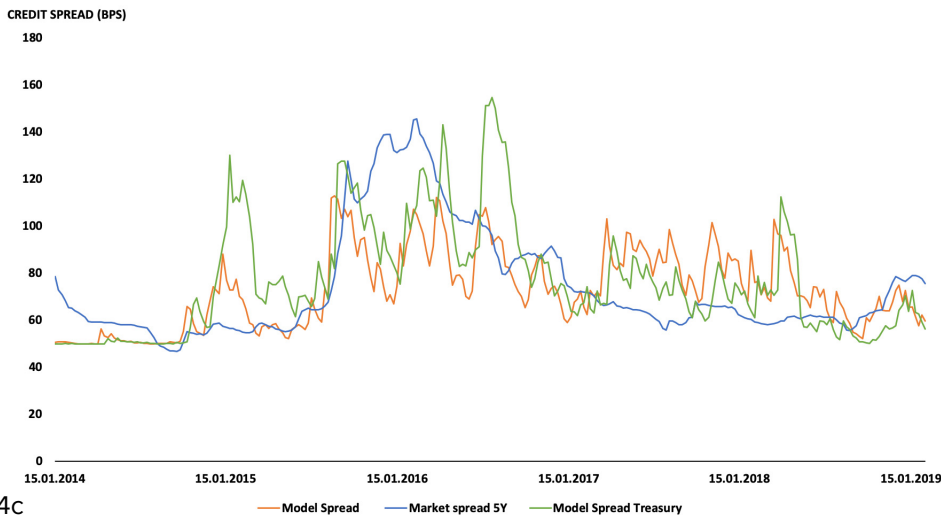
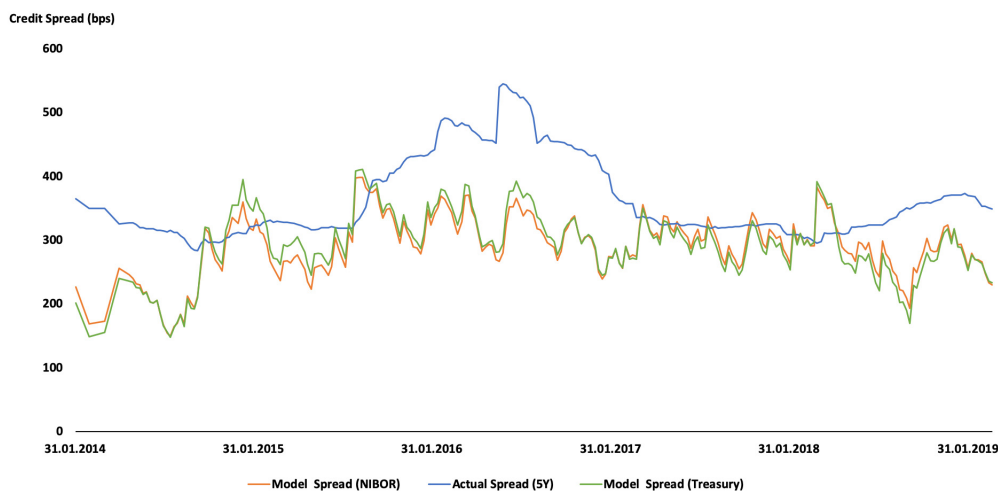


Figure 4: Overview of credit spreads of SMN, produced by the Merton model in all types of securities classes. Asset volatility estimates by EWMA and time to maturity equals 5 years. The orange line utilizes 3M NIBOR, while the green line uses 5Y treasury rate. The blue line is the market credit spread for 5Y maturity. Sharpe ratios and recovery rates (RR): (a) Tier 1: $\Theta = 0.19$ and $RR = 0.31$. (b) Tier 2: $\Theta = 0.10$ and $RR = 0.38$. (c) Senior: $\Theta = 0.05$ and $RR = 0.50$

5.2 Credit Spreads Black & Cox

Similar to the Merton model, the Black & Cox approach manages to capture the level of credit spreads quite well, but has a minor tendency to under-predict the spreads. As can be seen in figure 5, credit spreads for Tier 1 range between 200–400 bps, for Tier 2 credit spreads lie between 100–200 bps and for senior bonds the credit spreads fluctuate in the interval of 40–100 bps. Further, the results indicate that the model spreads, for both models, tend to oscillate more than observed market spreads. This oscillation can also be observed for the DNB model spreads illustrated in Appendix B.2. These oscillations are highly related to movements in volatility.



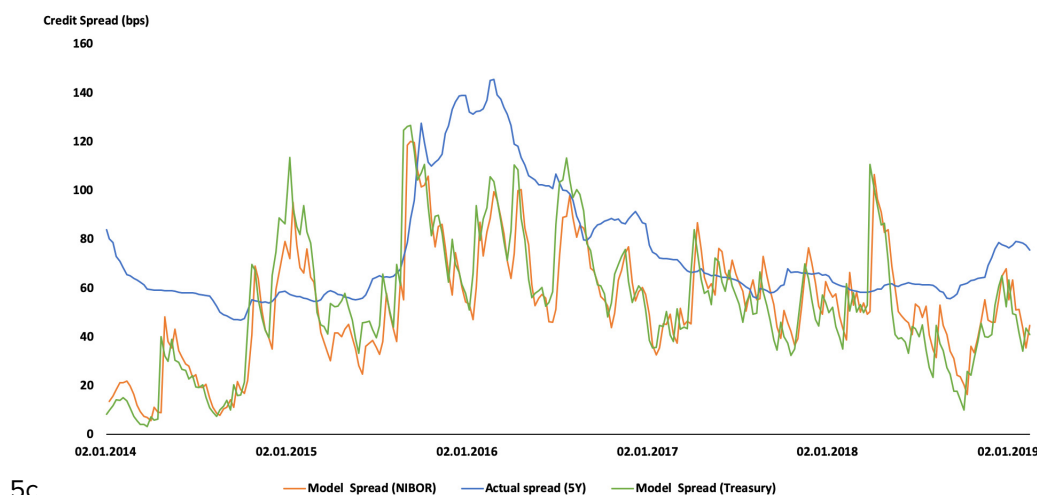
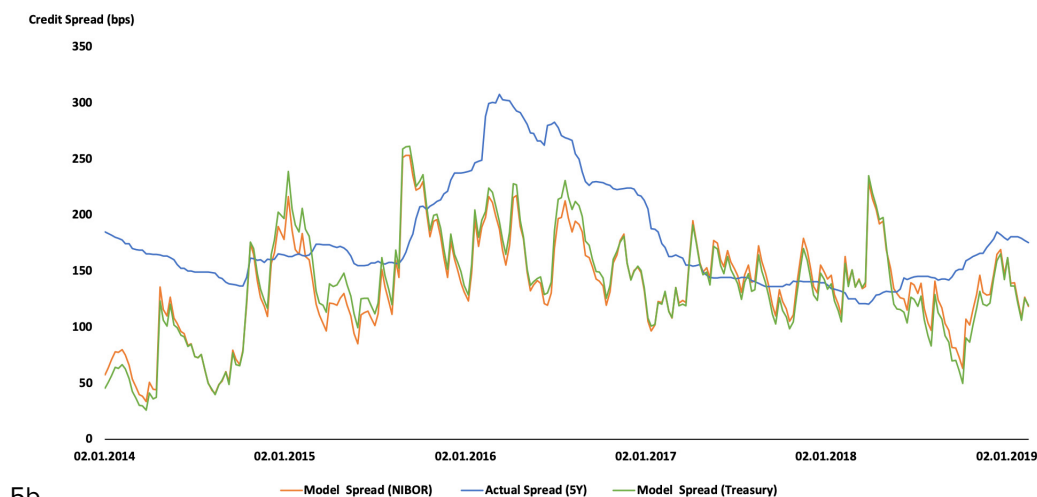


Figure 5: Overview of credit spreads of SMN, produced by the Black & Cox model in all securities classes. In this comparison the credit spreads are based on EWMA estimated volatility and time to maturity is equal 5 years. The orange line uses 3M NIBOR and the green line uses the 5Y treasury rate, while the blue line is the actual credit spread. (a) Tier 1 (b) Tier 2. (c) Senior.

5.3 Bond Rating

Credit ratings on long-term debt securities are affected by both the issuer's credit rating and specific characteristics of the debt instrument itself. In order to obtain accurate credit ratings for bonds, it is necessary to introduce subjective assessments of each individual bond and notch the rating up or down according to seniority, maturity, and macroeconomic conditions. However, in order to complete the framework for bond rating, we include a "pure" Merton approach, where recovery ratios and Sharpe ratios are set to zero in order to derive comparable probabilities of default. Our aim is to develop a model which can produce consistent and adequate probabilities of default, i.e. a senior bond has smaller probability of default than Tier 1 capital.

In table 7 we provide an overview of the average default probabilities calculated by the "pure" Merton approach. The model default probabilities are intuitively appealing. Comparing these estimated default probabilities with Nordic Credit Rating's matrix of expected default rates presented in section 3, we discover that estimated values correspond to ratings ranging between triple A (AAA) and triple B (BBB). In practice, one should be careful to compare default frequencies directly across seniority because default practices may be dif-

ferent across asset classes. If the recovery rate is expected to be high, a default typically will occur immediately at the technical default point, while investors in lower quality bonds may accept the asset value to drop below the technical default point before a default kicks in, since the recovery rate on actual default is low.

Table 7: Calculated average probability of default produced by the "plain" Merton approach and the BC model. The values are given in percentage.

Probability of Default						
	Merton			Black & Cox		
	Tier 1	Tier 2	Senior	Tier 1	Tier 2	Senior
DNB	1.5722	0.8621	0.2084	1.3967	0.7811	0.0000
SMN	1.4937	0.1952	0.0896	1.3874	0.4196	0.0003

Based on the calculated default probabilities, table 8 shows an overview of the corresponding bond ratings. Both the Merton model and the Black & Cox model obtain BBB ratings for Tier 1 capital, irrespective of which bank it belongs to. For Tier 2 capital we expect a smaller probability of default, and hence a higher credit rating. In practice, this is exactly what we observe. Yet, for DNB the Tier 2 rating is still in the range of a BBB, while SMN's Tier 2 capital is assigned a single A. Both models produce close to zero probability of default for senior debt. This is also the case in reality, as senior bonds rarely default. High quality bonds thus receive the lowest probability of default, hence also the highest credit ratings.

Table 8: Derived bond ratings for all classes of securities in both banks, based on the calculated probabilities of default from the two structural models.

Bond Rating						
	Merton			Black & Cox		
	Tier 1	Tier 2	Senior	Tier 1	Tier 2	Senior
DNB	BBB	BBB	A	BBB	BBB	AAA
SMN	BBB	A	AA	BBB	A	AAA

6 Conclusion and Further Research

The purpose of this analysis is to examine the ability of structural models to *predict credit spreads and determine credit ratings* on financial bonds issued by Norwegian banks. We have implemented two structural option-based models, originally derived by *Merton* and *Black & Cox*. In order to test the models' accuracy, we collected data of observed market spreads over a time span of five years (2014–2019). Our results show that both models are capable of capturing the differing risk levels of different securities classes well, illustrated by the models clearly distinguishing between credit spreads in Tier 1, Tier 2, and senior bonds respectively. The fact that our models yield plausible results gives us reason to believe that the chosen parameters and estimated variables are well calibrated to the Norwegian financial market. As for choosing between structural models, it appears that well-estimated parameters are as important as the choice of the model itself.

In terms of employing structural models as an integrate part of CRAs' credit rating processes, we believe that our research has demonstrated that the two models we have imple-

mented are capable of producing reliable default probabilities and plausible credit spreads. However, it appears that the two purposes, of calculating credit spreads and determining bond ratings, rely on different calibrations of the input parameters. We therefore recommend performing these operations separately. Although CRAs might be searching for a more objective assessment process, a credit rating process will always be a composite and complex analysis, in which a formal model at best might supplement subjective judgments on the part of credit analysts.

Even though our analysis yields promising results, it is important to recognize the limitations of the models. Evaluation of the input parameters suggests that model accuracy is sensitive to changes in volatility, recovery rates, Sharpe ratios and the discount value of the safety covenant. All of these parameters are to be estimated, with some degree of error. Inaccuracy in modeling volatility is potentially serious since volatility is probably the most important input variable to structural models. In order to increase the robustness of the conclusions, it would be useful to examine an extended data sample including several different banks and other securities classes. Likewise, improved estimates of model-parameters might strengthen the results. In this study, the volatility estimates are based on stock prices of two banks and thus reflect firm specific volatility over the last years. Empirical analysis by Ray and Tsay (2000) shows that a company's sector affiliation significantly affects the number of common long-range dependent components in volatility. Developing sector-specific volatilities in order to capture the systematic risk within the financial sector, is a possible enhancement of our analysis. In addition, different ways of calculating the default barrier might prove a valuable extension of the framework.

There are several potential and interesting approaches to further developing this analysis. An obvious extension of the research is to include covered bonds. Covered bonds rank even higher in the capital hierarchy of the banks than senior bonds. However, because these bonds are supported by a pool of collateral assets, their risk characteristics cannot be deduced from the capital structure of the underlying firm, especially when it comes to volatility estimates. Obviously, this debt class poses a challenge for structural modelling.

Further, there exists a relation between the credit spread and the maturity of the bond, often referred to as the credit spread curve¹², which might prove beneficial to investigate further. This relationship could be upward sloping, downward sloping or even humped-shaped. According to Annaert and De Ceuster (1999), the intuition behind this is that high-quality bonds can hardly become better, however, it is perfectly possible that their quality deteriorates. Similarly, the reverse applies to low-quality bonds, where longer maturity increases the probability of an upgrade.

Credit spreads are correlated with overall market conditions and by capturing changes in the stock price of the issuer the models indirectly capture some of the market features. However, the models do not precisely capture fluctuations such as changes in oil prices, liquidity premiums, political conflicts, etc. Larsson and Magne (2010) investigate the ability to predict defaults during a financial crisis in the US and the EU. An interesting extension of our research might be examining how a financial crisis affects the ability to predict default probabilities in the Nordic region. Another aspect of the credit rating business is identified by Becker and Millbourn (2011), who states that increased competition between agencies lowers the credibility of ratings and that strong competition is not always good. This is a contradicting proposition to the common notion that new local rating agencies are a positive contribution to the market and should be further examined.

12. Merton (1973), Longstaff and Schwartz (1995), Jarrow et al. (1997)

Appendixes

A Bond Valuation

A.1 The Merton Approach

In the Merton model, default can only happen at maturity and the default boundary is equal to the original debt value. This results in the following pay-off:

$$\phi(A_t) = \begin{cases} K, & A_t \geq K \\ A, & A_t < K \end{cases} \quad (17)$$

Further, the price of the bond at time t in a risk-neutral world, where T is maturity, K is the face value of debt and σ is the asset volatility, is given by

$$P_t = e^{-(T-t)r} K (1 - N(-d_2)) + e^{-(T-t)\delta} N(-d_1) \quad (18)$$

The value of the bond should equal the risk-free yield to maturity

$$e^{-(r+S)(T-t)} K = P_t \quad (19)$$

Solving for the relation between price and yield to maturity, the credit spread can be derived as

$$S = -\frac{1}{(T-t)} \ln\left(\frac{P_t}{K}\right) - r \quad (20)$$

which can be simplified to

$$S = -\frac{1}{(T-t)} \ln(PD) \quad (21)$$

A.2 The Black & Cox Approach

Valuation of bonds by the Black and Cox (1976) model relies on an underlying stochastic differential equation.

$$\frac{1}{2} \sigma^2 V^2 B_{VV} + (r-a) V B_V - rB + B_t = 0 \quad (22)$$

with boundaries condition

$$B(V, T) = \min(V, P)$$

$$B\left(Ce^{-\gamma(T-t)}, t\right) = Ce^{-\gamma(T-t)}$$

and were the stock price, X , must satisfy

$$\frac{1}{2} \sigma^2 V^2 X_{VV} + (r-a) V X_V - rX + X_t + aV = 0 \quad (23)$$

$$X(V, T) = \min(V - P, 0)$$

$$X\left(Ce^{-\gamma(T-t)}, t\right) = 0$$

Based on the first-passage-time distribution, the valuation formula for bond, B , becomes equation 24 below. This serves as the calculated equity value.

$$\begin{aligned}
B(A, t) = & De^{-r(T-t)} \left[N(z_1) - y^{2\theta-2} N(z_2) \right] + Ae^{-\delta(T-t)} N(z_3) \\
& + y^{2\theta} N(z_4) + y^{\theta+\xi} e^{-\delta(T-t)} N(z_5) \\
& + y^{\theta-\xi} e^{-\delta(T-t)} N(z_6) - y^{\theta-\eta}(z_7) - y^{\theta-\eta}(z_8)
\end{aligned} \tag{24}$$

where

$$y = \frac{Ce^{-\gamma(T-t)}}{A}$$

$$\theta = \frac{\left(r - \delta - \gamma + \frac{1}{2}\sigma^2 \right)}{\sigma^2}$$

$$d = \left(r - \delta - \gamma - \frac{1}{2}\sigma^2 \right)^2 + 2\sigma^2(r - \gamma)$$

$$\eta = \frac{\sqrt{d}}{\sigma^2}$$

$$z_1 = \frac{\ln(A) - \ln(D) + \left(r - \frac{1}{2}\sigma^2 \right)(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_2 = \frac{\ln(A) - \ln(D) + 2\ln(y) \left(r - \frac{1}{2}\sigma^2 \right)(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_3 = \frac{\ln(D) - \ln(A) + \left(r - \frac{1}{2}\sigma^2 \right)(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_4 = \frac{\ln(A) - \ln(D) + 2\ln(\theta) \left(r - \frac{1}{2}\sigma^2 \right)(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_5 = \frac{\ln(y) + \xi\sigma^2(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_6 = \frac{\ln(y) - \xi\sigma^2(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_7 = \frac{\ln(y) + \eta\sigma^2(T-t)}{\sqrt{\sigma^2(T-t)}}$$

$$z_8 = \frac{\ln(y) - \xi\sigma^2(T-t)}{\sqrt{\sigma^2(T-t)}}$$

Further, this calculated equity value is used to obtain a representative debt value by subtracting it from the asset value of the firm. The yield, y , on the debt can then be calculated by the following formula:

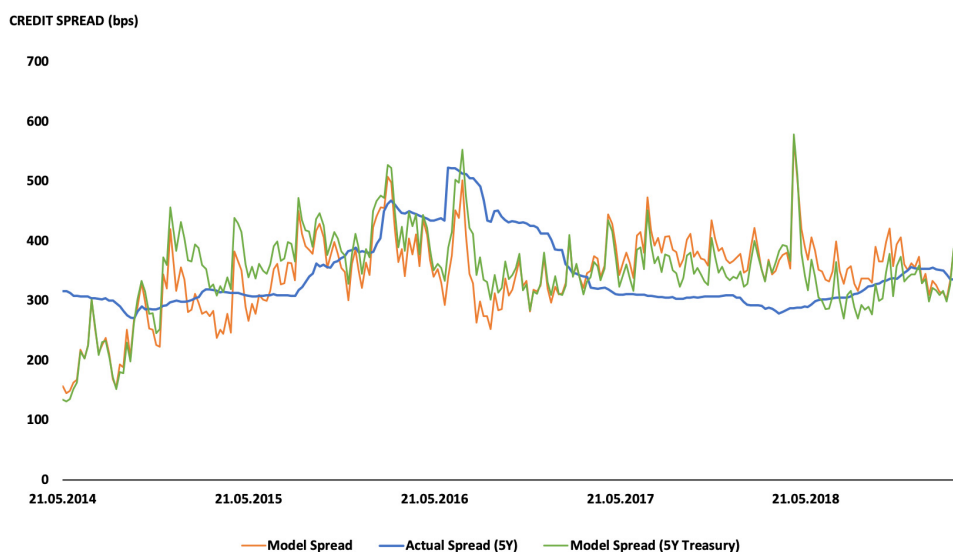
$$y = -\frac{1}{T-t} \ln\left(\frac{D_{BC}}{D}\right) \tag{25}$$

where D_{BC} is the debt value obtained through the Black & Cox formula and D is the debt value from the balance sheet. Finally, the credit spread is the difference between the yield and the interest rate, converted into basis points.

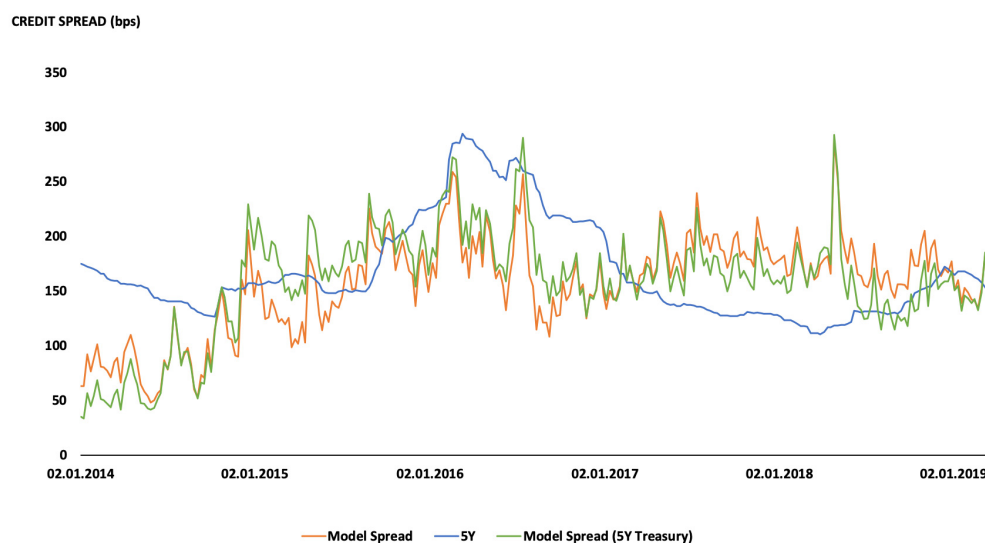
B Model Spreads DNB

We start out by looking at the model spreads from the Merton model.

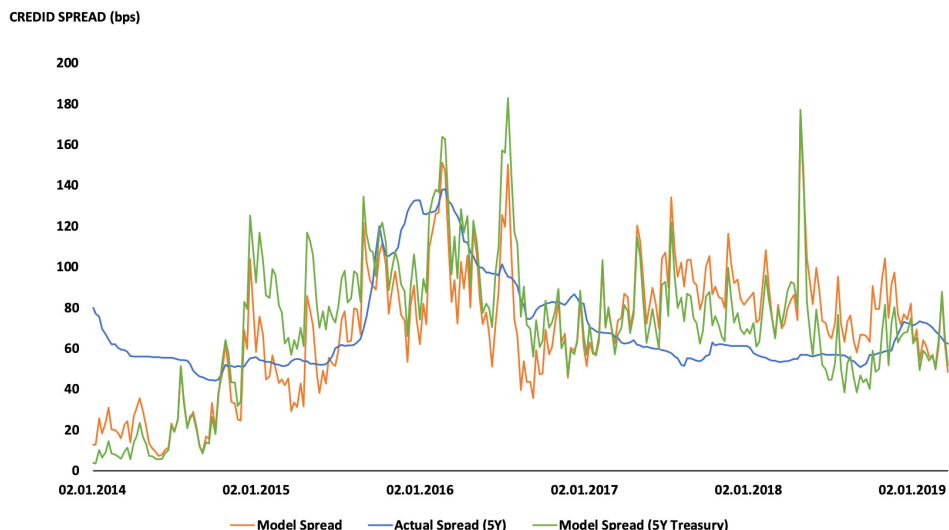
B.1 The Merton Approach



6a



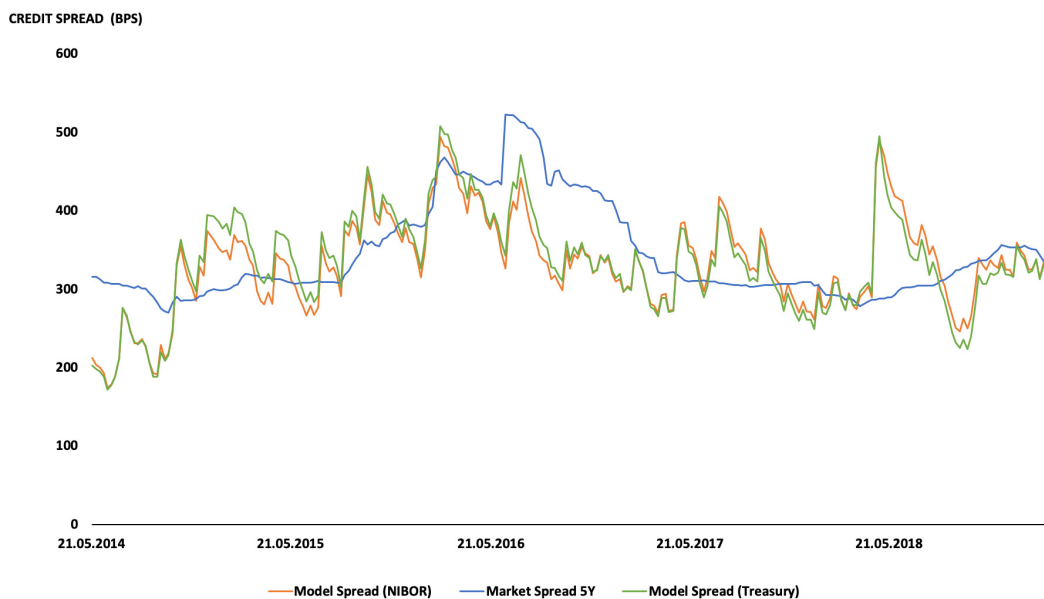
6b



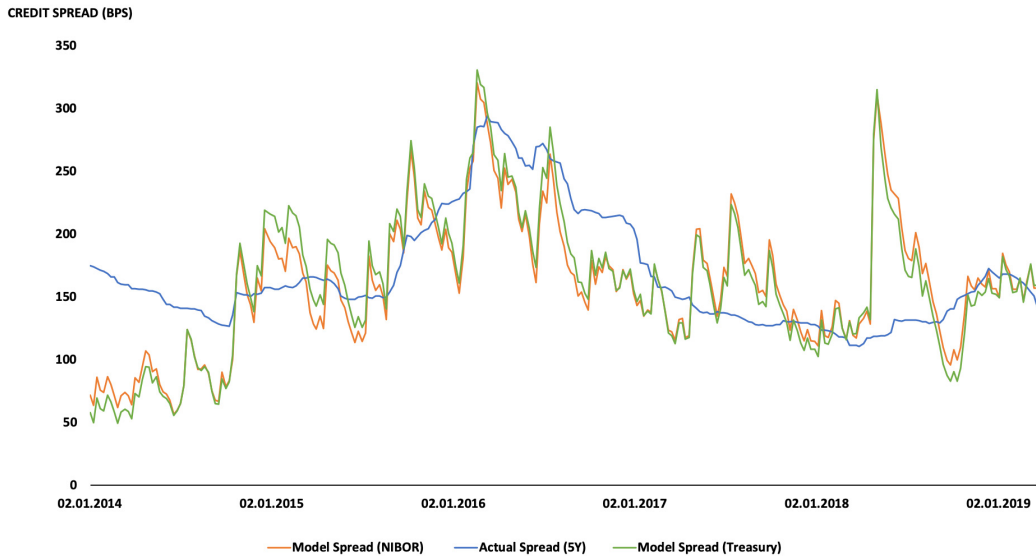
6c

Figure 6: Overview of DNB’s credit spreads produced by the Merton model for all types of capital. Volatility estimates are based on N-GARCH. For all securities classes we have maturity of 5Y, Sharpe ratio equal zero. (a) Tier 1 capital, $RR = 0$, $x = 4$. (b) Tier 2 capital, $RR = 0.38$, $x = 3$. (c) Senior capital, $RR = 0.50$, $x = 2$.

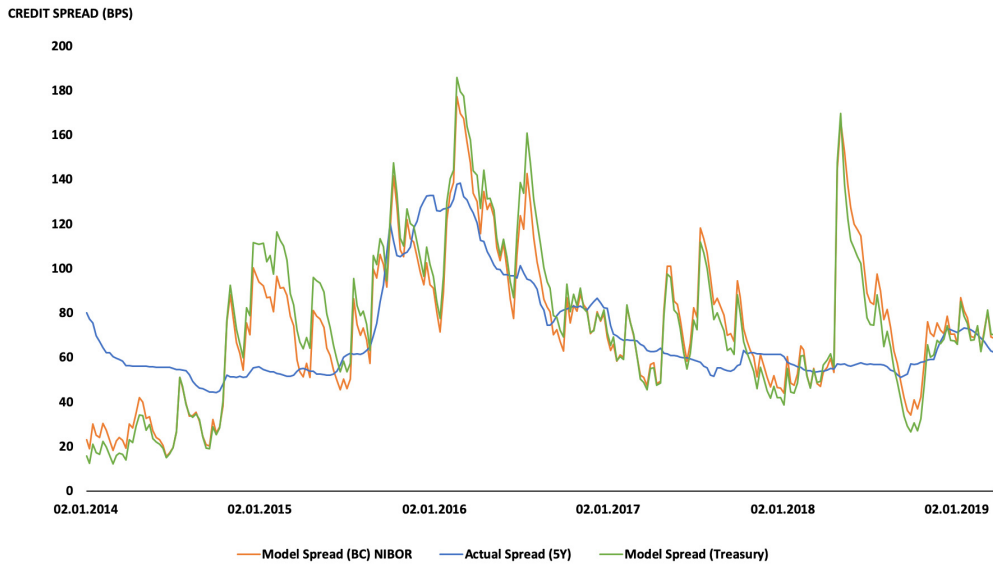
B.2 The Black & Cox Approach



7a



7b



7c

Figure 7: Overview of DNB's credit spreads based on the Black & Cox model for all securities classes. (a) Tier 1 capital, $\gamma = 0.08$ and $x = 20$. (b) Tier 2 capital, $\gamma = 0.08$, $x = 2$. (c) Senior capital, $\gamma = 0.04$, $x = 2$.

C Volatility Estimates DNB

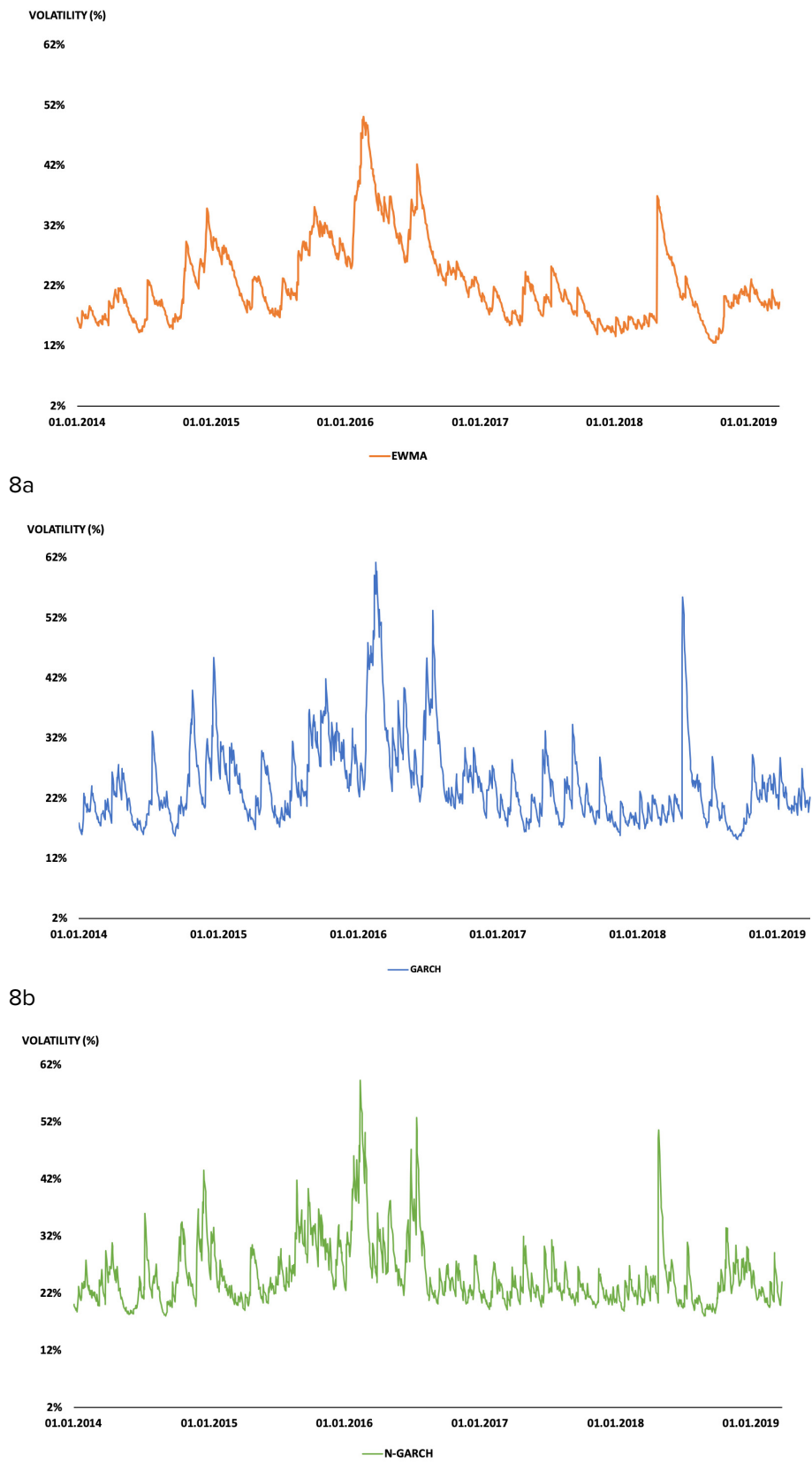


Figure 8: The plots illustrate the estimated equity volatility for DNB employing three different models over the relevant time-period of 5 years, from 2014 to the beginning of 2019, based on listed DNB stock prices.

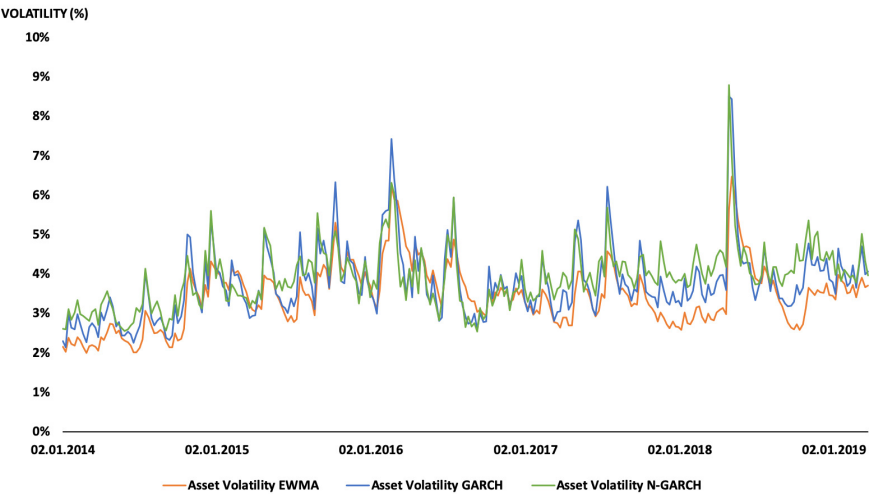
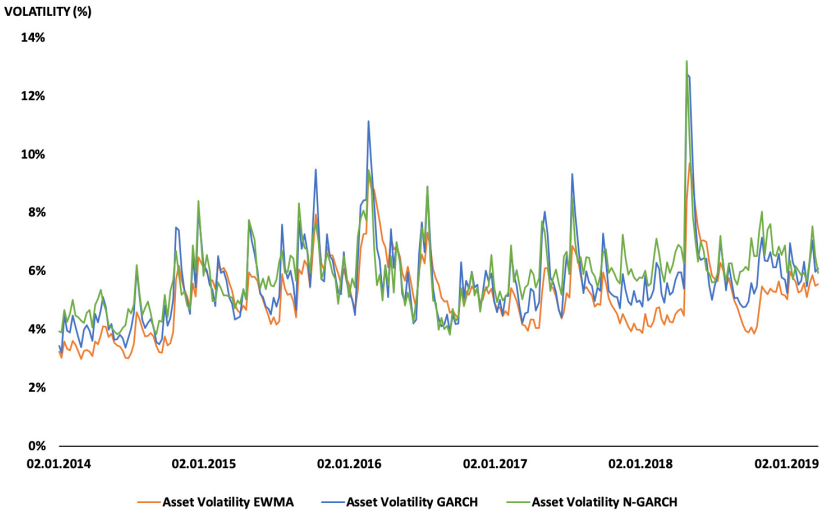
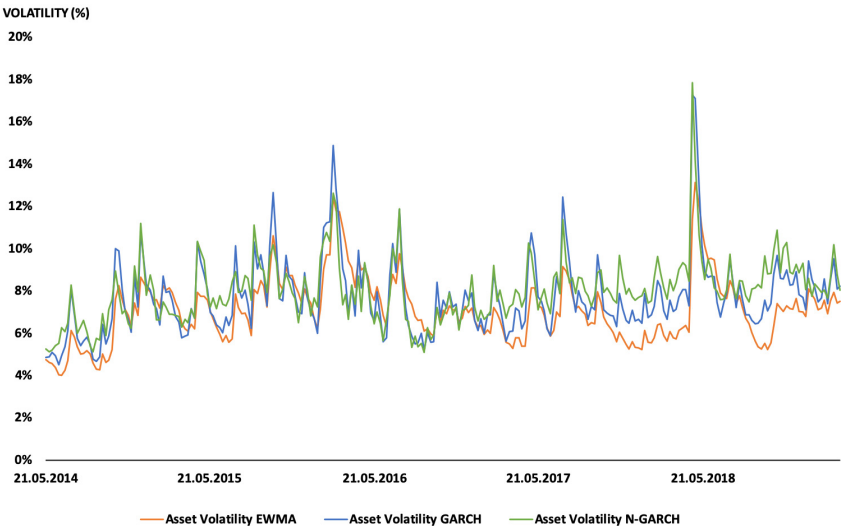
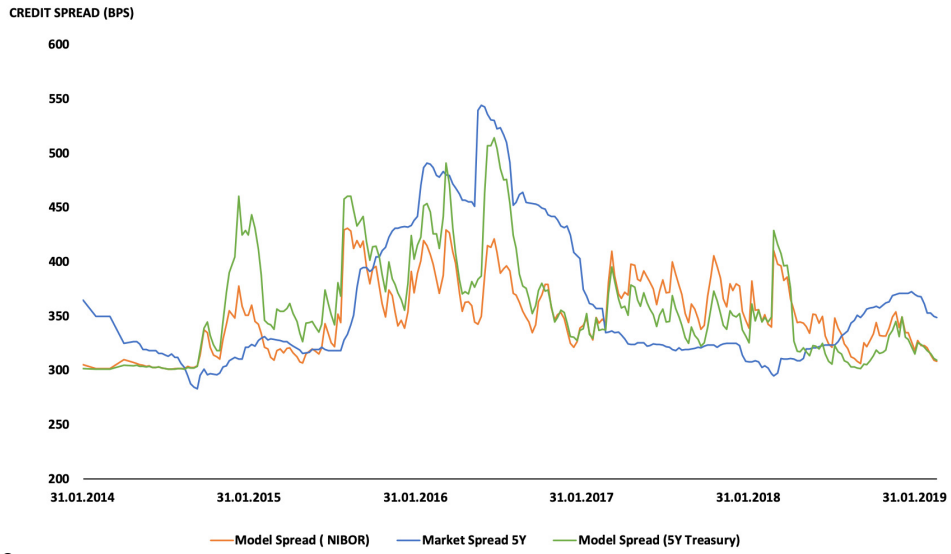
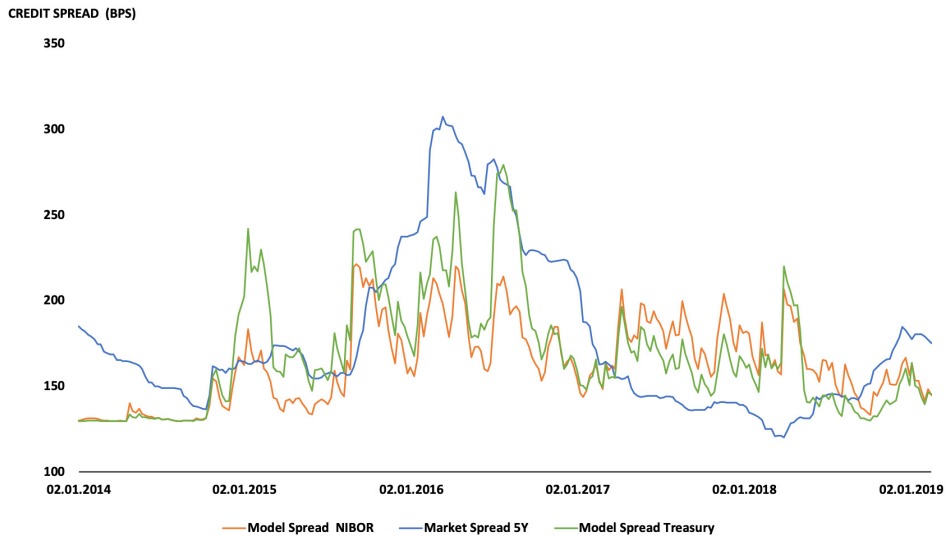


Figure 9: The plots illustrate the estimated asset volatility of DNB over the relevant time period of 5 years (2014–2019), based on all the different volatility estimates (EWMA, GARCH, N-GARCH). (a) Asset volatility Tier 1 capital. (b) Asset volatility Tier 2 capital. (c) Asset volatility senior debt.

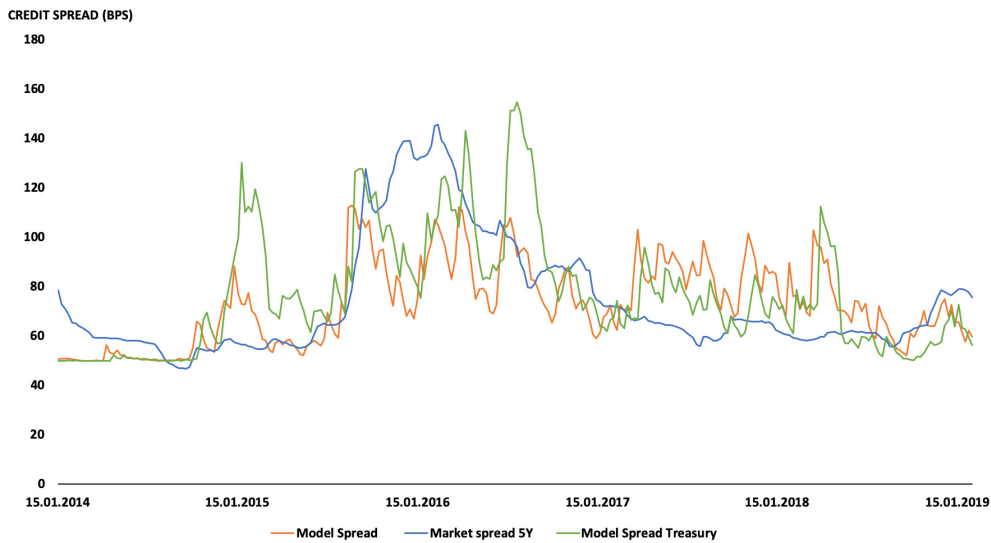
D Model Spreads by Different Volatilities



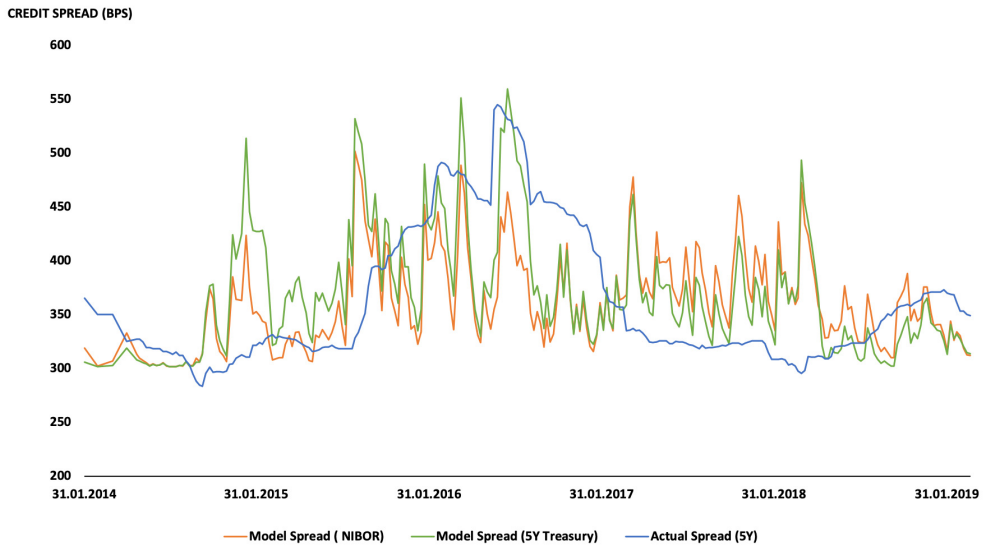
10a



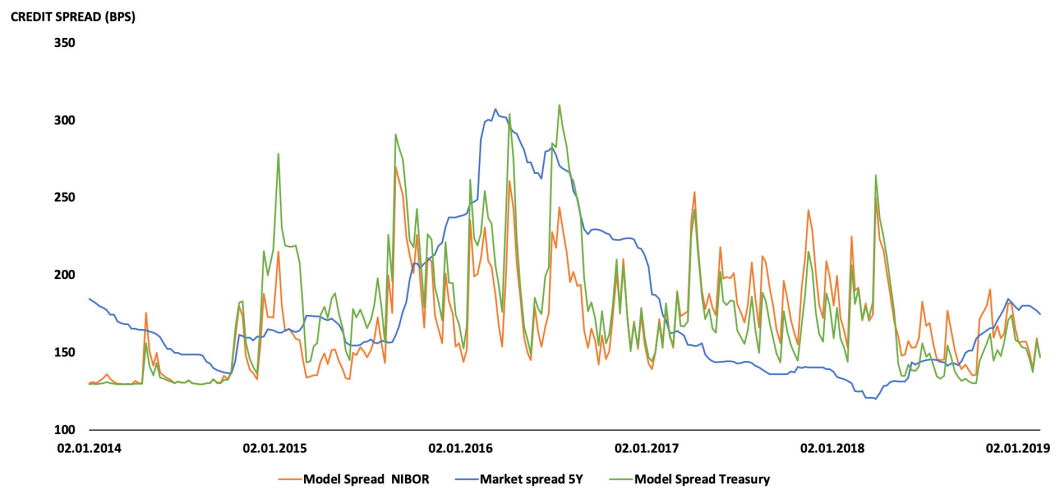
10b



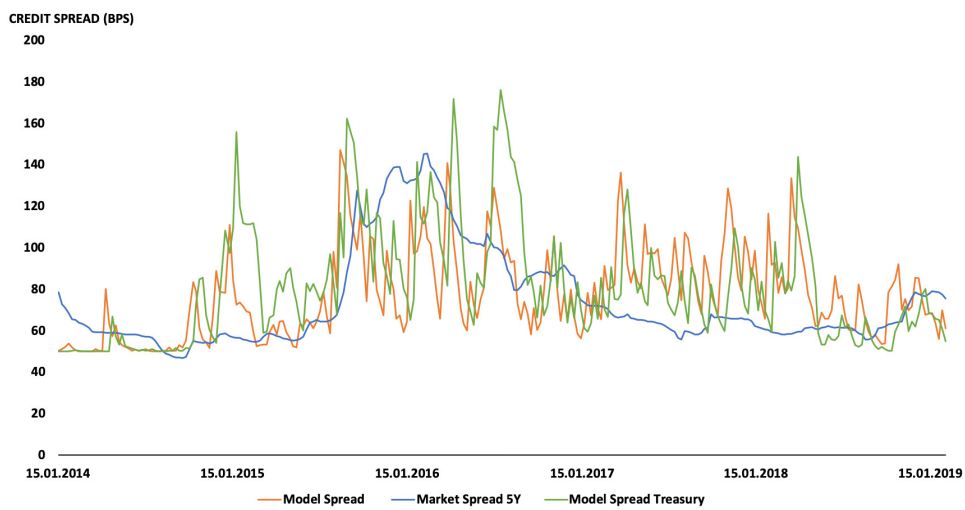
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10d



10e



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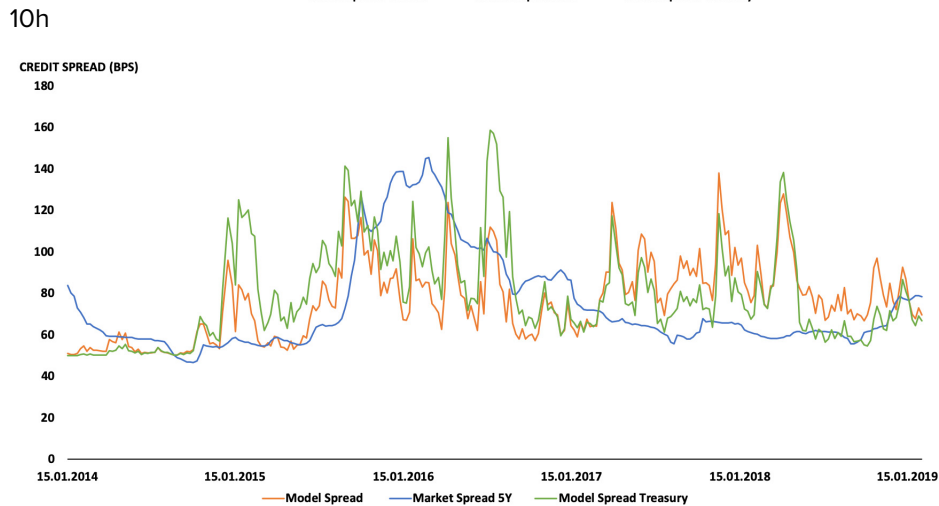
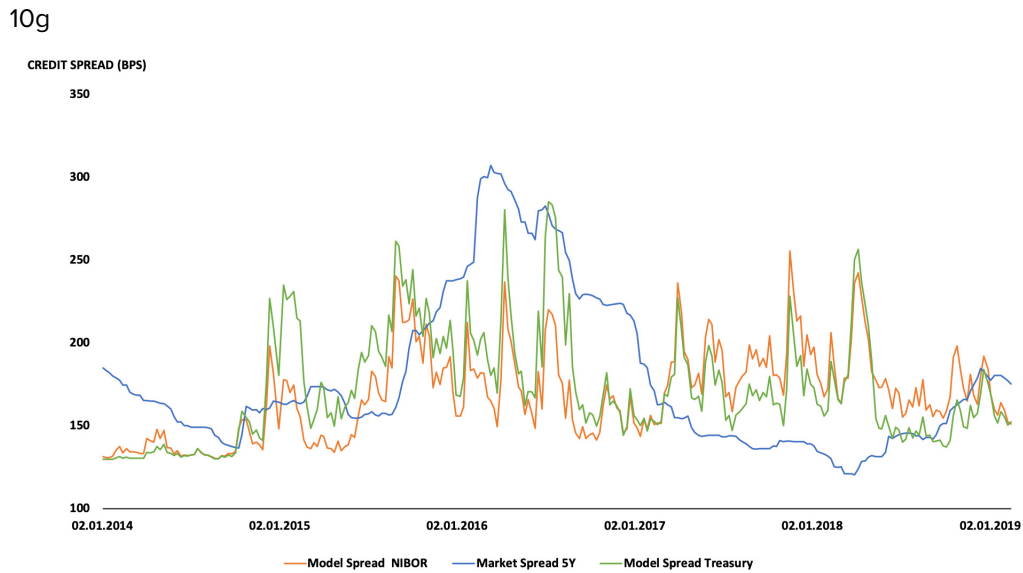
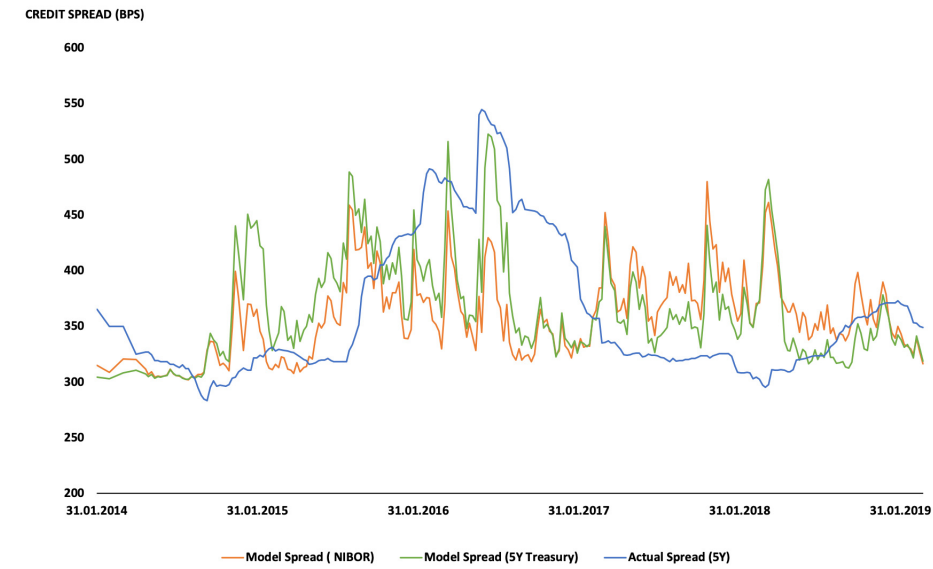


Figure 10: Credit spreads for all SMN security classes with different volatility estimates, compared to 5Y maturity market spreads. (a) – (c) Model spreads when volatility is estimated by EWMA. (d) – (f) Model spreads when volatility is estimated by GARCH. (g) – (i) Model spreads when volatility is estimated by N-GARCH.

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