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INTERNATIONAL DIVERSIFICATION, GROWTH, AND WELFARE WITH NON-TRADED INCOME RISK AND INCOMPLETE MARKETS

Egil Matsen

Department of Economics

Norwegian University of Science and Technology
N-7491 Trondheim, Norway
www.svt.ntnu.no/iso/wp/wp.htm

International Diversification, Growth, and Welfare with Non-Traded Income Risk and Incomplete Markets

Egil Matsen*

egil.matsen@svt.ntnu.no

Abstract

We ask how the potential benefits from cross-border asset trade are affected by the presence of non-traded income risk in incomplete markets. We show that the mean consumption growth may be lower with full integration than in financial autarky. This can occur because: the hedging demand for risky high-return projects may fall as the investment opportunity set increases, and precautionary savings may fall as the unhedgeable non-traded income variance decreases upon financial integration. We also show that international asset trade increases welfare if it increases the risk-adjusted growth rate. This is always the case in our model, but the effect may be close to negligible. The welfare gain is smaller the higher the correlation between the domestic non-traded income process and foreign asset returns.

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1. Introduction

The turmoil in worldwide financial markets during 1997 and 1998 has lead to renewed discussions on the costs and benefits of free international capital mobility (see e.g. Obstfeld, 1998; Rogoff, 1999). Some economists point to the crisis as evidence on the risks of global financial trading, arguing for restraints on capital flows (Bhagwati, 1998; Krugman, 1998). On the other hand, economic theory predicts potential important advantages of international financial integration. The benefits are associated with consumption smoothing (across time and different states of nature) and the allocation of resources that the international financial markets facilitate. (See Obstfeld and Rogoff, 1996 for a comprehensive introduction.) The largest potential gains from international asset trade have been demonstrated in models where the ability to diversify risk increases the long-term growth rate by inducing producers to undertake riskier high-return projects (e.g. Obstfeld, 1994; Dumas and Uppal, 1999).[1]

This paper adds to the debate on capital mobility by asking how the growth and welfare effects of cross-border asset trade are affected by the presence of non-traded income risk. Earlier literature has ignored such income components despite its probable real-world importance. This would not be a problem if markets where complete, but some non-traded income risk (e.g. labor income) would be hard to diversify even under full capital mobility (Rodrik, 1997). Furthermore, income from non-traded assets will affect portfolio choice and savings decisions, exactly the channels through which financial integration may impact growth, growth-variability, and welfare.

Several recent papers have explored how free asset trade affects growth and welfare. Devereux and Smith (1994) show that increased ability to diversify risk can reduce the precautionary motive for saving, and this would lower growth with financial integration. While their analysis assume that agents have access to one technology only, Obstfeld (1994) analyses this issues in a model where investors can choose between a low-return risk-free and a high-return risky technology. Financial integration allows agents to lay off nation-specific risks in international markets, giving incentives to

[1] See also Acemoglu and Zilibotti (1997) on the link between the ability to diversify, risk-taking, and growth. Related research can be found in the financial development literature; see Pagano (1993) and Levine (1997) for useful surveys.

increase risk-taking. In Obstfeld's complete markets model, this leads to huge gains in terms of both growth and welfare. Dumas and Uppal (1999) introduce goods-market frictions in the same type of model. They show that goods-market imperfections reduce the gains from free asset trade, but not by much.[2]

This paper extends this research to the case where some income risk can not be traded and financial markets are incomplete. We study a two-country simple linear continuos-time stochastic growth model, along the lines of Cox *et al.* (1985) akin to several of the papers referred to above. Each country has a set of traded assets associated with constant-returns-to-scale no-adjustment-cost production technologies. In addition, we follow Svensson and Werner (1993) and include stochastic income that corresponds to income from non-traded assets. Such assets could be due to asset-market imperfections like transaction costs, moral hazard, capital controls, etc. We do not attempt to model the reason(s) for the (partially) missing insurance markets, but treat this absence as an exogenous constraint. In our context we may think of the market for claims on GDP, proposed by Shiller (1993), as an example of such missing markets.

Throughout, we compare our results with the all-assets-tradable, complete-markets models of Obstfeld (1994) and Dumas and Uppal (1999). Several interesting differences from these models emerge from our analysis: First, growth rates may be different across countries with full financial integration, even if preferences are equal. This happens because different countries choose different resource allocations depending on the covariance of their non-traded income process with the set of internationally traded assets.

Second, the equilibrium consumption growth rate may be lower with free asset trade than under financial autarky. This occurs when extending the set of available marketable assets leads to lower hedging demand for risky high-return projects, and/or when the increase in the hedgeable non-traded income variance that follows from integration gives lower precautionary savings.

Third, the welfare gain from international asset trade is always positive, but can approach zero in our model. The positive welfare effect is common with the Obstfeld and Dumas-Uppal models. However, we show by the means of numerical simulations

[2] Devereux and Saito (1997) study the effects of restricted asset trade, in the sense that agents can trade in non-contingent bonds only. In this case, some countries may experience both higher growth and welfare

that for certain parameter combinations, our model predicts a negligible welfare gain from cross-border asset trade while a model ignoring non-traded income risk would imply substantial gains. A key parameter in evaluating the size of the welfare gain is the correlation between the domestic non-traded income process and the return on foreign risky assets. The higher the correlation the lower is the welfare gain. This is because the risk-return benefits from diversifying into foreign assets are counteracted by reduced hedging ability of the risky asset portfolio when the correlation between domestic non-traded income and foreign assets is high.

The rest of this paper is organized in the following manner. In the next section we go through the basic model elements and assumptions that we build on. In section 3 we derive the equilibrium consumption growth rate, and its variance, for an economy in financial autarky. Section 4 contains the paper's central results on the link between asset trade, growth and welfare. Finally, in section 5 we present some numerical illustrations of the model, before concluding in section 6. The appendix contains the derivations of the optimal decision rules under autarky and integration.

2. Basic Model Elements

We will consider a world consisting of two countries, indexed by $i = H,F$. A large number of identical infinitely lived households populate each country. Time is continuous, and at time t the household in country i maximizes the intertemporal objective:

$$U_i(t) = E_t\left[(1-\gamma)^{-1}\int_t^{\infty} c_i(\tau)^{1-\gamma} e^{-\delta(\tau-t)} d\tau\right], \qquad (1)$$

where E_t is the conditional expectations operator, $c_i(\tau)$ the consumption level prevailing in country i at time τ, and γ and δ are the common constant coefficients of relative risk aversion and rate of time preference, respectively. Notice that E_t is independent of nationality, so that we assume homogenous expectations across countries.

In each country, there are two distinct constant-returns-to-scale production technologies for the production of the single consumption/investment good. One of the technologies is assumed to be risky while the other is risk-free. Adjustments in the

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under complete financial autarky.

allocation of capital to the different technologies are costless and instantaneous. Let K_i and B_i denote the quantity of the good invested in country i's risky and risk-free technology, respectively. Geometric Brownian motions drives all investment processes:

$$\frac{dK_i(t)}{K_i(t)} = \alpha_i dt + \sigma_i dz_i(t), \quad i = H, F \ , \tag{2}$$

$$\frac{dB_i(t)}{B_i(t)} = rdt, \quad i = H, F \ . \tag{3}$$

Here, α_i and r are the constant instantaneous expected rates of return (it is assumed that $\alpha_i > r$, $i = H,F$), σ_i the constant instantaneous standard deviation of returns, and $dz_i(t)$ a standard wiener process. As can be seen from equation (3), we assume that the returns on investment in risk-free technologies are equal across countries. The cross-country correlation of technology shocks are represented by the structure $dz_H dz_F = \kappa dt$, where κ is the correlation coefficient.

In addition to the return on their portfolio of traded assets, households in both countries receive an exogenous stochastic income from a non-traded asset. We can interpret this as income from some production factor in fixed supply (e.g. labor or land). Non-traded income are driven by the processes:

$$\frac{dy_i(t)}{W_i(t)} = \mu_i dt + \sigma_{y,i} d\zeta_i(t), \quad i = H, F \ . \tag{4}$$

In (4) $W_i(t)$ is aggregate marketable wealth in country i at time t, μ_i a constant drift coefficient, $\sigma_{y,i}$ the constant instantaneous standard deviation and $d\zeta_{i,}(t)$ another wiener process. (We will often refer to (4) as simply the non-traded income process. It implicitly understood that this refers to the process of non-traded income to marketable wealth.)

The assumption made in (4) implies that non-traded income is proportional to marketable wealth. It follows Losq (1978), whom adopts it to study consumption behavior in this setting. He does not, however, study the growth and welfare properties of the model. These properties are the focus of our paper. The analysis is greatly simplified by imposing this assumption and does not loose its illustrative power. Indeed, the process in (4) allows us to solve the model in closed form, given the assumed CRRA-preferences. Svensson and Werner (1993) demonstrate that the consumption/ asset-allocation problem considered below can be analytically solved in more general cases with CARA-preferences, but such preferences would prevent us from deriving closed-form solutions of the growth rates that are very central to our analysis.[3]

3. Financial Autarky

Let us first imagine that the two economies can not trade its (marketable) assets with each other. This experiment provides a benchmark upon which we evaluate the gains from cross-border asset trade in the next section.

3.1 Individual Behavior

In the absence of international asset trade, the wealth of a representative household will evolve according to (country subscripts are ignored in this section):

$$dW(t) = \left[\omega(t)\alpha + (1-\omega(t))r\right]W(t)dt + \omega(t)\sigma W(t)dz(t) + dy(t) - c(t)dt, \tag{5}$$

where $\omega(t)$ denotes the fraction of wealth invested in the domestic risky technology at time t. We will assume that negative allocations are non-feasible; $0 \le \omega(t) \le 1$.[4] The change in wealth is determined by (i) the return on the investment in the risky asset, (ii) the return on the (composite) risk-free asset, (iii) exogenous income growth and (iv) the instantaneous consumption rate.

The representative household in the closed economy chooses a consumption path $\{c(\tau)\}_{\tau=t}^{\infty}$ and a portfolio path $\{\omega(\tau)\}_{\tau=t}^{\infty}$, to maximize (1) subject to (4), (5) and the current

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[3] Losq (1978) interprets the non-traded income process as dividends on human wealth. He argues that it is plausible to assume that the ratio of human wealth income to financial wealth stays relatively constant through time, in which case (4) is a reasonable representation of the labor income process.

[4] This restriction is imposed to ensure consistency with the assumption of a constant risk-free interest rate. As such, it is an innocuous restriction in the closed economy of this section. Since domestic agents are homogenous, we could easily have introduced a market for an instantaneous risk-free bond and showed that the endogenously determined interest rate on this bond would be constant in the closed economy equilibrium. In the open version of the model studied in the next section agents are heterogeneous across countries. Then, allowing non-positive allocations would imply that the equilibrium risk-free interest rate would be a time varying process, as would be the optimal consumption policy and the asset demand function. It is in general very difficult to calculate the equilibrium path with heterogeneous agents (Den Haan, 1994). We wish to make the model tractable and hence impose the above restrictions on the portfolio weights.

level of wealth. This problem is solved in the appendix.[5] Here, we summarize the solution as follows:

Lifetime utility evaluated at time t is given by (time indexes are ignored from now on when they are unnecessary):

$$J(W) = (1-\gamma)^{-1} A^{-\gamma} W^{1-\gamma}, \tag{6}$$

where

$$A \equiv \frac{1}{\gamma}\Big[\delta - (1-\gamma)\big(r + \mu + \omega(\alpha - r) - \tfrac{1}{2}\gamma(\omega^2\sigma^2 + 2\omega\sigma_{Ky} + \sigma_y^2)\big)\Big] \tag{6a}$$

is a constant assumed to be positive. In (6a), σ_{Ky} is the instantaneous covariance between the ratio of non-traded income to marketable wealth and the traded risky asset. The consumption policy and asset demand of the household in the closed economy is

$$\frac{c}{W} = A, \tag{7}$$

and

$$\omega = \begin{cases} 0 & \text{if } \varpi < 0 \\ \varpi & \text{if } 0 \le \varpi \le 1 \\ 1 & \text{if } \varpi > 1, \end{cases} \tag{8}$$

respectively, where

$$\varpi = \frac{\alpha - r}{\gamma\sigma^2} - \frac{\sigma_{Ky}}{\sigma^2}\,. \tag{8a}$$

Equation (7) shows that the optimal consumption-wealth ratio is constant. For interior solutions, equation (8a) tells that the optimal portfolio is a combination of the tangency portfolio, corresponding to the first term on the right-hand-side of the equality, and a hedge portfolio given by the second term. Without non-traded income risk, the tangency portfolio would have been the only part of the household's asset demand. In our model, the household wishes to hedge against fluctuations in non-traded income and thus adjusts their portfolio holdings. The hedge portfolio is the portfolio that has the maximum negative correlation with non-traded income (see Ingersoll, 1987 for a general discussion of the tangency and hedge portfolios).

The result that both optimal consumption and asset-allocation are constant fractions of wealth replicates the standard model where all assets are marketable (Merton,

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[5] Losq (1978) also consider this problem, deriving equations (6) and (7) below.

1969). We have obtained this replication by using the special process in equation (4). Still non-traded income risk gives rise to important differences from the standard model. The following two observations underline this:

First, the existence of a non-traded income component affects the equilibrium, even if this income is uncorrelated with the return on the traded assets (i.e., $\sigma_{Ky} = 0$). The asset demand would be identical to the standard model, but both the consumption function and welfare are affected by the non-traded income component.

Second, non-traded income affects consumption and asset allocation decisions even if the domestic financial market is complete. Complete markets in this closed economy mean that the non-traded income risk is spanned by the risky technology. Formally, it implies that the unhedgeable variance of the ratio of non-traded income to marketable wealth conditional upon the set of traded assets, denoted $\sigma_{y|K}$, is 0. In the closed economy:

$$\sigma_{y|K} = \sigma_y^2 - \frac{\sigma_{Ky}^2}{\sigma^2} = \sigma_y^2(1-\rho_{Ky}^2) = 0, \tag{9}$$

where ρ_{Ky} is the instantaneous correlation coefficient between returns on the risky technology and the growth rate of the ratio non-traded income to marketable wealth. Obviously, spanning in the closed economy requires $|\rho_{Ky}| = 1$. Still, equations (6)-(8) would be different from the corresponding ones without non-traded income. It may thus be misleading to ignore non-traded income components even if one believe that financial markets are complete.

3.2 Equilibrium

We have three possible equilibria in the closed economy: one where both types of technology are demanded, and one for each of the corner solutions with zero investment in either risk-free or risky capital.

3.2.1 Investments in Both Types of Technology

Consider first the case when the representative household wishes to incur positive investments in both assets. Notice that, since there are no adjustment costs, asset supply always accommodate the equilibrium asset demand, given by equation (8a). By substituting equations (4) and (7) into (5), wealth accumulation in the closed economy can be written as

$$dW(t) = \left[\omega\alpha + (1-\omega)r + \mu - A\right]W(t)dt + \left(\omega\sigma dz(t) + \sigma_y d\zeta(t)\right)W(t)\,. \qquad (10)$$

Using this expression in (7), we obtain the stochastic process for per capita consumption:

$$dc(t) = \left[\omega\alpha + (1-\omega)r + \mu - A\right]c(t)dt + \left(\omega\sigma dz(t) + \sigma_y d\zeta(t)\right)c(t)\,. \qquad (11)$$

By defining g as the instantaneous expected per capita consumption growth rate, we find from equation (11):

$$\tfrac{E_t[dc(t)/dt]}{c(t)} \equiv g = \omega\alpha + (1-\omega)r + \mu - A\,. \qquad (12)$$

That is, g is endogenously determined as the expected return on the traded assets, plus the instantaneous expected non-traded income growth, minus the consumption-wealth ratio.

The growth rate can be expressed in closed-form by substituting for ω from (8a) in (6a) and (12):

$$g = m + n, \qquad (13)$$

where

$$m \equiv \frac{r-\delta}{\gamma} + \frac{(1+\gamma)(\alpha-r)^2}{2\gamma^2\sigma^2}, \qquad (13a)$$

and

$$n \equiv \frac{1}{\gamma}\left[\mu - (\alpha - r)\frac{\sigma_{Ky}}{\sigma^2} - \tfrac{1}{2}\gamma(1-\gamma)\sigma_{y|K}\right]. \qquad (13b)$$

We have split the components of (13) in two, because m is the consumption growth rate that would prevail if only the traded assets were present. With the non-traded income process (4) the growth rate needs to be adjusted by the term n. In (13b), we see that the growth adjustment is expected non-traded income plus the expected excess return on the hedge portfolio less the (risk-aversion weighted) unhedgeable variance $\sigma_{y|K}$. All this is multiplied by the elasticity of intertemporal substitution (1/γ). We notice that non-traded income affects the growth rate of the economy also in the cases where there is spanning ($\sigma_{y|K} = 0$) and when the return on the traded assets is uncorrelated with non-traded income ($\sigma_{Ky} = 0$).

The instantaneous variance of the mean growth rate can be derived from equation (11):

$$\frac{\operatorname{var}[dc/c]}{dt} \equiv s^2 = \omega^2\sigma^2 + 2\omega\sigma_{Ky} + \sigma_y^2 = \frac{(\alpha - r)^2}{\gamma^2\sigma^2} + \sigma_{y|K} , \tag{14}$$

where the last equality follows upon substitution from (8a). The consumption growth variance is simply the sum of the instantaneous variances of the return on the traded assets and the non-traded income. Notice that the first term after the last equality is the consumption variance that would prevail without the non-traded income component. Unless markets are complete, consumption growth becomes more volatile when we add non-traded income.

We can obtain insight into how international asset trade may affect the equilibrium growth rates of this model, by considering the growth impacts of a fall in σ. Imagine that the households hold both types of assets also after the parameter shift. In an economy without non-traded income risk, we would then have $\partial g/\partial\sigma = \partial n/\partial\sigma < 0$; lower rate-of-return risk stimulates consumption growth. This clear prediction arises because a lower σ unambiguously shift investments towards the high-productive, risky technology, dominating a possible fall in saving (Obstfeld 1994).

The direction of the portfolio shift is ambiguous when there is non-traded income risk, as can be seen from equation (8a). Resembling the economy where all income risks are traded, the fraction of wealth invested in the tangency portfolio will increase. However, lower rate-of-return risk has an ambiguous effect on the fraction of wealth invested in the hedge portfolio. This fraction can be written as $-\frac{\rho_{Ky}\sigma_y}{\sigma}$. Holding ρ_{Ky} fixed, a fall in σ will decrease the optimal investment in the hedge portfolio if $\rho_{Ky} > 0$, contributing to lower overall risk-taking. More precisely, it will increase further the absolute value of the negative amount invested in the hedge portfolio.[6] Whether the increase in the amount invested in the tangency portfolio will dominate this effect is theoretically undetermined. A similar ambiguity is present in the relationship between σ and the consumption-wealth ratio (equation (7)). Ultimately, this implies a theoretically

[6] The effect is similar if we hold the covariance fixed. Higher ρ_{Ky} and lower σ both contribute to lower hedging demand (given that $\rho_{Ky} < 0$). I thank Diderik Lund for this point.

undetermined sign on the term $\partial m/\partial\sigma$ in equation (13), so there is an uncertain impact on the growth rate from lower rate-of-return risk.[7]

3.2.2 Investment in One Type Only

In the equilibrium where all investments are in the risky technology the instantaneous expected growth rate would be $g = \alpha + \mu - A$. By (6a), the consumption-wealth ratio is

$$A = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(\alpha + \mu - \tfrac{1}{2}\gamma(\sigma_K^2 + 2\sigma_{Ky} + \sigma_y^2)\right)\right]$$

when ω is forced to 1. Then the consumption growth rate can be written as

$$g = m' + n', \tag{15}$$

where

$$m' \equiv \frac{\alpha - \delta}{\gamma} - \tfrac{1}{2}(1-\gamma)\sigma_K^2, \tag{15a}$$

and

$$n' \equiv \frac{\mu}{\gamma} - \tfrac{1}{2}(1-\gamma)\left(2\sigma_{Ky} + \sigma_y^2\right). \tag{15b}$$

Here, m' is the growth rate that would prevail without non-traded income risk. Again, the equilibrium per capita consumption growth rate could be higher or lower than in an economy with only traded income risk, depending on the covariance between the traded asset return and non-traded income growth. The instantaneous variance of the growth rate is simply $\sigma + 2\sigma_{Ky} + \sigma_y$ when all marketable assets held are of the risky form.

The growth impact of lower technological uncertainty is ambiguous in this case as well, depending on the effect on σ_{Ky} and on whether γ is smaller than or larger than 1. The latter point is valid also if we ignore non-marketable assets, as can be seen from (15a) and as shown earlier by Devereux and Smith (1994) and Obstfeld (1994). Since the portfolio allocation is fixed in this case, the ambiguous effect on the growth rate from a fall in σ is due to a undetermined impact on the consumption-wealth ratio (confer equation (12)).

[7] A similar argument for the parameter α gives parallel conclusions. Without non-traded income risk, the equilibrium growth rate increases in α, while the link is theoretically ambiguous in the model with non-traded income.

The last possible equilibrium in the closed economy is one where there is investment in risk-free technology only. By (12), the consumption growth rate in this case is simply $g = r + \mu - A$. This can be expressed in closed form by observing that the definition of A simplifies to

$$A = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \mu - \tfrac{1}{2}\gamma\sigma_y^2\right)\right],$$

whenever $\omega = 0$. Hence, the consumption growth rate is

$$g = \tfrac{1}{\gamma}(r + \mu - \delta) - \tfrac{1}{2}(1-\gamma)\sigma_y^2 . \tag{16}$$

It is noteworthy that higher non-traded income variance increases saving (lowers the consumption-wealth ratio), and thus growth, only when the elasticity of intertemporal substitution ($1/\gamma$) is smaller than 1. This resembles the classic analysis of uncertainty and saving in Sandmo (1970).

4. Integrated Capital Markets

4.1 Trade in Marketable Assets

Assume now that the marketable assets can be traded internationally. Given the setup in section 2, wealth dynamics in the two countries are:

$$dW_i = \left[\left(\sum_{j=H}^{F}\omega_i^j(\alpha_j - r) + r + \mu_i\right)W_i - c_i\right]dt + \left(\sum_{j=H}^{F}\omega_i^j\sigma_j dz_j + \sigma_{y,i}d\zeta_i\right)W_i, \quad i = H, F \tag{17}$$

where ω_i^j is the fraction of country i's wealth invested in the risky asset of country j, i,j = H,F. To ensure consistency with the assumption of a constant risk-free interest rate we need to impose the short sale constraints: $0 \le \omega_i^j \le 1$ $i,j = H,F$, and $\sum_j \omega_i^j \le 1$, $i = H,F$. The problem solved by the representative households is as in the closed economy, with the budget constraint (17) replacing (5). We show how to proceed in the appendix.

Maximal utility is given by $J_i(W_i) = (1-\gamma)^{-1}(A_i^*)^{-\gamma}W_i$, where

$$A_i^* = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \mu_i + \mathbf{w}'_i(\mathbf{a} - r\mathbf{1}) - \tfrac{1}{2}\gamma(\mathbf{w}'_i\,\boldsymbol{\Omega}\mathbf{w}_i + 2\mathbf{w}'_i\,\mathbf{V}_i + \sigma_{y,i})\right)\right], \quad i = H, F . \tag{18}$$

In this expression $\mathbf{w}_i \equiv [\omega_i^H\ \omega_i^F]'$ is the portfolio weight vector, $\mathbf{a} \equiv [\alpha_H\ \alpha_F]'$, $\mathbf{1} \equiv [1\ 1]'$, $\boldsymbol{\Omega} \equiv [\sigma_H\ \sigma_F\ \kappa]$ is an invertible 2 x 2 variance-covariance matrix, and $\mathbf{V}_i \equiv [\sigma_{Hy,i}\ \sigma_{Fy,i}]'$ is the vector of the covariance of each of the traded risky assets with the ratio of non-traded income. The optimal consumption policies are

$$c_i = A_i^* W_i, \quad i = H, F \,, \tag{19}$$

Due to the constraints on the portfolio weights, the asset allocation policy is somewhat complicated:

$$\mathbf{w}_i' = \left[\omega_i^H \omega_i^F\right] \equiv \begin{cases} [0,0] & \text{if } \hat{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F < 0 \\ \left[0, \hat{\omega}_i^F\right] & \text{if } \breve{\omega}_i^H < 0 \text{ and } 0 \le \hat{\omega}_i^F \le 1 \\ [0,1] & \text{if } \tilde{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F > 1 \\ \left[\hat{\omega}_i^H, 0\right] & \text{if } 0 \le \hat{\omega}_i^H \le 1 \text{ and } \breve{\omega}_i^F < 0 \\ \left[\varpi_i^H, \varpi_i^F\right] & \text{if } 0 \le \varpi_i^H \le 1,\ 0 \le \varpi_i^H \le 1 \text{ and } \varpi_i^H + \varpi_i^F \le 1 \\ \left[\frac{\varpi_i^H}{\varpi_i^H + \varpi_i^F}, \frac{\varpi_i^F}{\varpi_i^H + \varpi_i^F}\right] & \text{if } \varpi_i^H > 0,\ \varpi_i^F > 0 \text{ and } \varpi_H^H + \varpi_H^F > 1 \\ [1,0] & \text{if } \hat{\omega}_i^H > 1 \text{ and } \tilde{\omega}_i^F < 0 \end{cases} \tag{20}$$

$i = H,F$, where

$$\hat{\omega}_i^j \equiv \frac{\alpha_j - r}{\gamma \sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2}, \quad i, j = H, F \,,$$

$$\tilde{\omega}_i^j = \frac{\alpha_j - r}{\gamma \sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2} - \frac{\sigma_{HF}}{\sigma_j^2}, \quad i, j = H, F \,,$$

$$\breve{\omega}_i^H \equiv \frac{\alpha_H - r}{\gamma \sigma_j^2} - \frac{(\alpha_F - r)\sigma_{HF}}{\gamma \sigma_H^2 \sigma_F^2} - \frac{\sigma_{Hy,i}}{\sigma_H^2} + \frac{\sigma_{Fy,i}\sigma_{FH}}{\sigma_H^2 \sigma_F^2}, \quad i = H, F \,,$$

$$\breve{\omega}_i^F \equiv \frac{\alpha_F - r}{\gamma \sigma_F^2} - \frac{(\alpha_H - r)\sigma_{HF}}{\gamma \sigma_H^2 \sigma_F^2} - \frac{\sigma_{Fy,i}}{\sigma_F^2} + \frac{\sigma_{Hy,i}\sigma_{FH}}{\sigma_H^2 \sigma_F^2}, \quad i = H, F \,,$$

and

$$\varpi_i^j = \gamma^{-1} \sum_{k=H}^{F} \nu_{jk} (\alpha_k - r) - \sum_{k=H}^{F} \nu_{jk} \sigma_{jy,i}, \quad i, j = H, F \,, \tag{21}$$

In (21), ν_{jk} are the elements of $\mathbf{\Omega}^{-1}$.

In comparing the asset allocation policy above to a world with only tradable assets, we restrict attention to the case where none of the short-sale constraints bind; that is, case 5 in equation (20). Equation (21) gives the asset demand functions in this case. It is instructive to rewrite it in matrix form:

$$\bar{\mathbf{w}}_i = \gamma^{-1} \Omega^{-1} (\mathbf{a} - r\mathbf{1}) - \Omega^{-1} \mathbf{V}_i, \quad i = H, F \,. \tag{22}$$

Define the scalars $D \equiv \gamma[\mathbf{1}'\mathbf{\Omega}^{-1}(\mathbf{a} - r\mathbf{1})]$ and $H_i \equiv -\mathbf{1}'\mathbf{\Omega}^{-1}\mathbf{V}_i$, so that (22) can be written as

$$\bar{\mathbf{w}}_i = D_i\mathbf{t} + H_i\mathbf{h}_i, \qquad \mathbf{t} \equiv \frac{\Omega^{-1}(\mathbf{a}-r\mathbf{1})}{\mathbf{1}'\Omega^{-1}(\mathbf{a}-r\mathbf{1})}, \qquad \mathbf{h}_i = \frac{\Omega^{-1}\mathbf{V}_i}{\mathbf{1}'\Omega^{-1}\mathbf{V}_i}, \quad i = H, F\ . \tag{23}$$

The two portfolios **t** and $\mathbf{h}_i$ are the tangency and hedge portfolio respectively (Ingersoll, 1987), and these are independent of preferences. A household from country *i* form a portfolio of risky assets by buying shares in the two mutual funds **t** and $\mathbf{h}_i$. The construction of the tangency portfolio is identical across nations, while the composition of the hedge portfolio depends on the covariance between non-traded income in country *i* and the (global) set of traded risky assets. Hence, the portfolio of risky assets will be different across nations. This contrasts the case where all assets are marketable, in which all households would construct an identical mutual fund regardless of nationality, consisting of the tangency portfolio only (Obstfeld, 1994).

The optimal fraction of wealth invested in risky assets is given by the scalar $\mathbf{1}'\mathbf{w}_i$, while the composition of the risky asset portfolio can be found from the 2 x 1 vector $\mathbf{q}_i \equiv \mathbf{w}_i/\mathbf{1}'\mathbf{w}_i$, $i = H,F$.[8] Because of the short-sale constraints, the optimal asset allocation is time-invariant. This enables us to derive closed-form solutions of the mean growth rates, as will shown below.

4.2 Equilibrium

Let us now characterize the equilibrium in which the two economies above can trade marketable assets. The absence of adjustment costs has the convenient implication that the price of marketable assets relative to each other will be unchanged upon integration. We fix these relative prices at 1. Accordingly, given **a**, **Ω**, *r*, $\mathbf{V}_H$ and $\mathbf{V}_F$, it is quantities that adjust to balance the demands given by (20). The equilibrium conditions are thus simply

$$K_i = \sum_{j=H}^{F} \omega_j^i W_j, \quad i = H, F\ .$$

To derive the mean consumption growth rates we can proceed as in subsection 3.2, obtaining

$$g_i^* = \mathbf{w}'_i(\mathbf{a}-r\mathbf{1}) + r + \mu_i - A_i^*, \quad i = H, F\ . \tag{24}$$

[8] The composition is undetermined in the first case of equation (20) when the households hold no risky assets.

We thus have seven possible mean growth rates for each country, depending on the short-sale constraints (see equation (20)). Again we concentrate on the case where no constraint is binding (case 5 in equation (20)).[9] Using (22) in (18), we find that the consumption-wealth ratios are

$$A_i^* = \frac{1}{\gamma}\left[\delta - (1-\gamma)\left(r + \mu + \frac{(\mathbf{a}-r\mathbf{1})'\Omega^{-1}(\mathbf{a}-r\mathbf{1})}{2\gamma} - (\mathbf{a}-r\mathbf{1})'\Omega^{-1}\mathbf{V}_i\right)\right] + \frac{1}{2}(1-\gamma)V_i\,,$$

$i = H,F$, where $V_i = \sigma_{y,i}^2 - \mathbf{V}'_i\,\Omega^{-1}\mathbf{V}_i$ is the unhedgeable variance of the process (4) conditional upon the set of internationally traded assets. Using this in (24), the expected growth rate can be written as

$$g_i^* = m^* + n_i^*, \tag{25}$$

where

$$m^* = \frac{r-\delta}{\gamma} + \frac{(1+\gamma)}{2\gamma^2}(\mathbf{a}-r\mathbf{1})'\Omega^{-1}(\mathbf{a}-r\mathbf{1})\,, \tag{25a}$$

and

$$n_i^* = \frac{1}{\gamma}\left[\mu_i - (\mathbf{a}-r\mathbf{1})'\Omega^{-1}\mathbf{V}_i - \tfrac{1}{2}\gamma(1-\gamma)V_i\right]. \tag{25b}$$

A model without non-traded income would predict a common world growth rate, given by m^*, in a financially integrated equilibrium. This may no longer be the case when we include non-traded income, since different countries choose different resource allocations depending on the covariance of their non-traded income process with the set of traded assets. This is reflected in the growth adjustment term (25b) above.

Equation (25) corresponds to (13) in autarky. From Obstfeld's (1994) work we know that $m^* > m$, whenever there is investment in both risk-free and risky technologies both prior to and after trade has been opened. Such an unambiguous ranking is not present for n_i and n_i^*. The second term in the square brackets of (25b) is the expected excess return on the hedge portfolio under financial integration. This may be higher or lower than the corresponding return under financial autarky. As for last term in the square brackets, V_i is a decreasing function of available marketable assets and will accordingly decrease upon integration. Hence, when $\gamma > 1$ this term contributes to lower growth under integration, while the opposite is true for $\gamma < 1$. This reflects precautionary

[9] We can derive the mean consumption growth rates for the other asset demand policies in equation (20) in the same manner as below.

savings behavior: With CRRA-utility lower unhedgeable income risk reduce (increase) savings when $\gamma > 1$ ($\gamma < 1$), slowing (spurring) growth in this model.

Now since n_i^* can be higher or lower than its autarky counterpart, we have an ambiguous effect on growth from financial integration. Underlying this indeterminacy is the uncertain effect that the increase in the investment opportunity set has on the hedging demand for the high-return risky technologies, and the ambiguous savings response to lower unhedgeable income risk.

The instantaneous variance of the consumption growth rates with financial integration are given by

$$(s_i^*)^2 = \mathbf{w}'_i \Omega \mathbf{w}_i + 2\mathbf{w}'_i \mathbf{V}_i + \sigma_{y,i}^2, \quad i = H, F \; . \tag{26}$$

This can be expressed in terms of the parameters of the model simply by plugging in the relevant optimal portfolio allocation from equation (20).

4.3 The Effect of Integration on Welfare

A convenient measure for evaluating the welfare effects of financial integration in this type of models is equivalent variation. This gives the percentage change in wealth under autarky necessary to make the households as well off as with integrated markets. That is, we wish to compute EV_i, which is implicitly defined as

$$A_i^{-\gamma}(1-\gamma)^{-1}\left[W_i(1+EV_i)\right]^{1-\gamma} = (A_i^*)^{-\gamma}(1-\gamma)^{-1}W_i^{1-\gamma} \, . \tag{27}$$

The left hand-side of (27) corresponds to (6), while the right hand-side is maximal utility with financial integration (equation (18)). Solving for EV_i, we obtain

$$EV_i = \left(\frac{A_i^*}{A_i}\right)^{\frac{\gamma}{\gamma-1}} - 1 \, . \tag{28}$$

To interpret (28), it is instructive to notice that A_i can be written as

$$A_i = \delta - (1-\gamma)\left(g_i - \tfrac{1}{2}\gamma s_i^2\right), \quad i = H, F \; , \tag{29}$$

by substituting from (12) and (14) into (6a). The term in the last parenthesis of (29) is the *risk-adjusted* (or certainty equivalent) growth rate in country *i*. There is a similar expression for the consumption-wealth ratio under financial integration. Then, it follows from (28) that financial integration has a positive welfare effect for country *i* if, and only if, its risk-adjusted expected consumption growth rate is higher under financial integration than under financial autarky.

By choosing the same resource allocations under both autarky and integration, the households can always obtain the same expected risk-adjusted growth rate. Utility-maximizing agents will never choose an allocation that implies lower welfare, so we can conclude that the risk-adjusted growth rate is non-decreasing upon integration and that financial integration improves welfare.

This qualitative result is common with the all-assets-tradable, complete-markets models of Obstfeld (1994) and Dumas and Uppal (1999). To investigate whether there may be significant quantitative differences between the welfare gains in those models and ours, we rewrite the autarky risk-adjusted growth rates in full:[10]

$$
\begin{aligned}
& g_i - \tfrac{1}{2}\gamma s_i^2 = \tilde{m}_i + \tilde{n}_i, \\
& \tilde{m}_i \equiv \frac{r-\delta}{\gamma} + \frac{(\alpha_i - r)^2}{2\gamma\sigma_i^2}, \\
& \tilde{n}_i \equiv \frac{\mu_i}{\gamma} - \frac{(\alpha_i - r)\sigma_{Ky}}{\gamma\sigma_i^2} - \tfrac{1}{2}\sigma_{y|K},
\end{aligned}
\tag{30}
$$

for $i = H,F$. The risk-adjusted growth rate that would prevail if we ignored non-traded income risk is given by $\tilde{m}_i$. Since welfare would be increasing upon financial integration in that case, this term must increase. This comes through as an increase in the expected excess return on the tangency portfolio (the last term in the definition of $\tilde{m}_i$). As explained earlier, the unhedgeable non-traded income variance is decreasing in available assets so that the last term in the definition of $\tilde{n}_i$ reinforces the welfare gain. What may counteract this, is the expected excess return on the hedge portfolio. That is, the second term in the definition of $\tilde{n}_i$ may be lower with asset trade than in autarky, contributing to lower welfare. This happens when the expected excess return on the optimal portfolio of risky assets is lower with full integration, and/or when the covariance between the non-traded income process and the portfolio of risky assets is higher with free asset trade. Although this can never dominate the combined effect of increased expected excess return on the tangency portfolio and lower unhedgeable income risk, the numerical examples constructed in the next section show that it could be important.

[10] We discuss only the case when all short-sale constraints are slack both under autarky and integration. Equation (30) is derived by using (13) and (14).

5. Numerical Illustrations

The preceding subsection has demonstrated that the growth-stimulus of international asset trade, implied by a complete market model, might be overturned when one introduces non-traded income components, while the positive welfare effect is retained. In this section we construct a few simple examples to demonstrate that non-traded income may significantly amplify the gains from cross-country asset trade in some cases, while it practically removes the gains in other instances.

Example 1: Consider a situation where $r = 0.02$, $\alpha_H = \alpha_F = 0.08$, $\sigma_H = \sigma_F = 0.20$, and $\kappa = 0.554$. These numbers are used by Dumas and Uppal (1999) in calibrating a friction-free version of their model, and are (roughly) based on stock-market data from the US and Germany presented by Obstfeld (1994). Let us also adapt Dumas and Uppal's preference parameters, setting $\delta = 0.02$ and $\gamma = 4$. To this, we add some imaginary parameters for the non-traded income processes. We assume that $\mu_H = \mu_F = 0.02$, $\sigma_{y,H} = \sigma_{y,F} = 0.05$, $\rho_{Hy,H} = \rho_{Fy,F} = 0.5$, and $\rho_{Hy,F} = \rho_{Fy,H} = 0$. That is, we start by using a fairly high domestic correlation between risky (marketable) asset return and non-traded income growth, while the domestic risky assets and foreign non-traded income growth are uncorrelated.

Under autarky, equation (8a) implies that both countries invest ω = 25 % of their marketable wealth in the risky asset. This is the sum of investing long 37.5 % of wealth in the tangency portfolio and shorting 12.5 % of wealth in a hedge portfolio. Since there are investments in both types of technologies, we use equation (13) to find that the mean consumption growth rate is $g = m + n$ = 1.41 % + 0.59 % = 2.00 %. By equation (14) the instantaneous standard deviation of the growth rate is s = 8.66 %, giving a risk-adjusted mean growth rate of 0.5 %.

In the integrated equilibrium we use equations (20)-(22) to calculate that $\mathbf{1}'\mathbf{w}_H = \mathbf{1}'\mathbf{w}_F = 0.402$. Risk-taking increases upon integration. This is the result of increasing the fraction of wealth invested in the tangency portfolio to 48.2 % and reducing the short hedge position to 8 % of wealth. In both countries, the portfolio of risky assets consists of 15 % invested in the domestic risky technology and 85 % in the foreign. This symmetric investment behavior occurs because we assume that the two countries are identical. From (25) we find that the growth rate increases to $g^* = m^* + n^* = 1.81\ \% + 0.62\ \% = 2.43\ \%$ in both countries upon integration. The standard deviation of the consumption growth rates is also higher however. Equation (26) gives $s_i^* = 9.40\ \%$, $i = H,F$. Still, the risk-adjusted mean growth rates increase to 0.66 %.

By (28) we can then calculate that households in both nations requires an increase in marketable wealth of 18.9 % in autarky to obtain the same level of life-time utility as with financial integration. This is a large welfare gain; it is more than 40 % higher than Dumas and Uppal (1999) find in their frictions-free calibration. Hence, the covariance structure between marketable assets and non-traded income assumed above amplify the gains from international asset trade.

Example 2: Consider a second example where $\rho_{Hy,F} = \rho_{Fy,H} = 0.7$, while the other parameters are left unchanged. The foreign risky technology is less attractive in this example, leading to an increase in the fraction of wealth invested in risky assets to 29 % only. This allocation to tangency portfolio is still 48.2 %, but the non-traded income/risky assets covariance structure now imply a short hedge position of 19.3 % of wealth. The risky asset portfolio composition is 69.4 % in the domestic asset and 30.6 % in the foreign.

Since risk-taking increases less than in example 1 the impact on the mean growth rate is also smaller. It is still substantial though, increasing to $g^* = m^* + n^* = 1.81\ \% + 0.39\ \% = 2.20\ \%$ in both countries. We notice that the growth-adjustment term n contributes to lowering growth upon integration in this situation, but this is dominated by the increase in m. The instantaneous standard deviation of the growth rate increases to $s = 9.20\ \%$, giving a slightly higher expected risk-adjusted growth rate of 0.51 %. The implied welfare gain from integration is accordingly quite small, with $EV = 1.25\ \%$ in both nations. This demonstrates that the gain from international asset trade need not be very large, even though the positive impact on the growth rate is significant. An equivalent-variation gain of 1.25 % is less than a tenth of what this example would yield if we ignored non-traded income risk.

Examples 1 and 2 illustrate that the gains from asset trade are sensitive to the hedging ability of foreign marketable assets relative to the domestic ones. In figure 1 we represent the gains from trade for country H, varying $\rho_{Hy,F}$ between –0.3 and 0.9 (the other parameters are fixed at the values of ex. 1). The lower the correlation between domestic non-traded income shocks and foreign risky assets return, the higher are hedging benefits from including foreign assets in the portfolio of risky assets, and the higher are the gains from trade. For high values of $\rho_{Hy,F}$, the risk-return benefits from diversifying into foreign assets are counteracted by the fact that this diversification reduces the hedging ability of the risky-assets portfolio. As shown in figure 1, the lower hedging potential can wipe out practically all gains from financial integration if $\rho_{Hy,F}$ is sufficiently high.

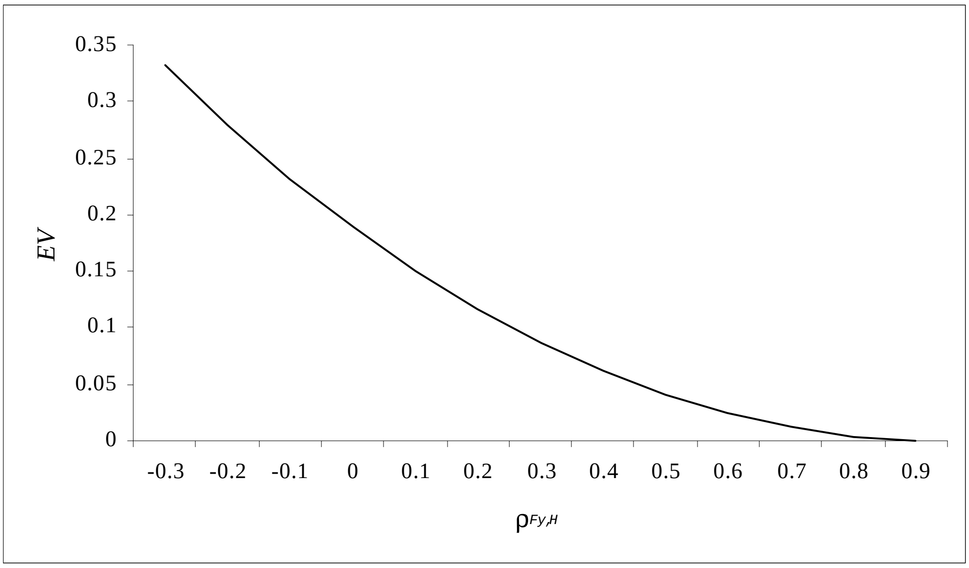

Figure 3.1: The welfare gain as a function of the correlation between the non-traded income process and foreign risky assets return.

Example 3: Consider finally an example where the two nations are asymmetric. We impose this asymmetry in the simplest possible manner, assuming that expected return on the risky technology in *F* is lower than in *H*. Specifically, we assume that α_F = 0.045, while the rest of the parameters have the same values as in example 1.

We start by evaluating country *H*, which has autarky equilibrium as in example 1. The risky asset in *F* is less attractive than in that example since it now has a lower expected return. With integrated financial market, country *H* will bear only slightly more risk than in autarky, investing 26.1 % of wealth in the risky assets. The long position in the tangency portfolio is lower than in example 1 (now 34.1 % of wealth), but so is the short hedge position (which falls to 8.0 % of wealth) and the latter effect dominates. The portfolio of risky assets is heavily concentrated in the domestic technology; country *H* households invest 90.2 % of their risky-assets portfolio at home.

Even though there is a small increase in fraction of wealth invested in risky assets, the mean consumption growth rate falls to 1.97 % in this case. This arises because the expected return on the portfolio held with integrated markets gives a lower expected return than under autarky. Households choose a portfolio with lower expected return because it provides a better hedge against non-traded income risk.

The instantaneous standard deviation of consumption growth would decrease to s_H = 8.59 % in country *H* upon integration. This ensures a tiny increase in the risk-adjusted growth rate to 0.501 %. The welfare gain from financial integration is accordingly very small; the equivalent-variation gain is only 0.1 % of wealth. Even moderate trading costs would thus swamp the gains from trade in country *H*.

Turning to country *F*, we have an autarky equilibrium characterized by $\omega_F = 0.03$, $g_F = m_F + n_F = 0.24\ \% + 0.70\ \% = 0.94\ \%$, $s_F = 5.34\ \%$, and a risk-adjusted growth rate of 0.38 %. The low expected return on the risky asset in *F* implies little investment in this asset, and low consumption growth. With free asset trade between the two countries the short-sale constraints binds for country *F*. It wish to short its own risky technology and invest the proceedings in *H*'s risky asset, since the latter provides both a better risk-return tradeoff and a better hedge against non-traded income risk. The short-sale constraints prohibit such allocations, however. By inspection it turns out that case 4 in equation (20) gives the optimal constrained asset allocation for country *F*, implying an investment of 37.5 % of wealth in the foreign risky asset while nothing is invested in the domestic counterpart. Using equations (24) and (26), we find that this allocation implies $g_F^* = 2.84$ % and $s_F^* = 10.89$ %, giving a risk-adjusted growth rate equal to 0.47 %. The equivalent variation measure of the welfare gain from financial integration is 11.9 % of initial wealth. Hence, country *F* derives large benefits from cross-country asset trade even with a binding short-sale constraint.

6. Conclusions

Our starting point was the debate on the costs and benefits of free capital mobility. This paper has dealt with the possible benefits, an asked how these might be affected by non-traded income risk when financial markets are incomplete.

We have shown that non-traded income risk can substantially alter the conclusions found in earlier research on the gains from cross-border asset trade. In our model, extending the set of available marketable assets may imply lower hedging demand for risky high-return projects, and lower precautionary savings as the unhedgeable non-traded income variance is reduced. These effects counteract the growth stimulus that all-assets-traded complete-markets models predict from international asset trade. It also implies that the welfare gains from financial integration may be negligible.

These results will typically occur if the non-traded income process has a higher correlation with foreign risky asset returns than the domestic equivalent. If, on the other hand, the non-traded income process has a lower correlation with the returns on foreign assets than with the domestic ones, the gains from asset trade would typically be amplified compared to a complete markets model.

Labor income fluctuations are probably the most important non-traded income risk in reality. Baxter and Jermann (1997) presents evidence from Germany, Japan, UK and US, where the pattern is that both (a synthetically computed) return on human capital and labor income growth is more highly correlated with domestic capital returns than foreign. This points to the conclusion that labor income risk strengthens the case for trade in financial assets. Bottazzi *et al.* (1996) concludes different, however. Using data for a large set of OECD-countries and taking into consideration redistributive shocks between capital and labor, they find that foreign assets generally are less attractive than domestic assets for hedging labor income uncertainty. Whether the presence of non-traded income risk strengthens or weakens the arguments in favor of international asset trade is therefore an empirical question that seems open.

Appendix

A.1 Individual Choice in the Closed Economy

We want to derive lifetime utility, asset demand, and the consumption policy of the households in the closed economy of section 2. The consumption path $\{c_\tau\}_{\tau=t}^{\infty}$ and portfolio path $\{\omega_\tau\}_{\tau=t}^{\infty}$ are chosen to maximize (1) subject to (4), (5), the current level of wealth $W(t)$. Let $J(W)$ denote the implied indirect utility function. The Hamilton-Jacobi-Bellman equation for this problem is

$$0=\max_{\{c,\omega\}}\left\{\tfrac{c^{1-\gamma}}{1-\gamma}-J\delta+J_W\left[W\left(\omega\alpha+(1-\omega)r+\mu\right)-c\right]+\tfrac{1}{2}J_{WW}W^2\left[\omega^2\sigma^2+2\omega\sigma_{Ky}+\sigma_y^2\right]\right\} \quad (A.1)$$

where subscripts on J denotes partial derivatives. The implied first order conditions with respect to the instantaneous consumption rate can be written as:

$$c^{-\gamma}=J_W\,. \quad (A.2)$$

Substituting this back into (A.1), we find that the differential equation for the value function J is

$$0=\tfrac{\gamma}{1-\gamma}J_W^{\gamma-1/\gamma}-J\delta+J_W W\left[\omega\alpha+(1-\omega)r+\mu\right]+\tfrac{1}{2}J_{WW}W^2\left[\omega^2\sigma^2+2\omega\sigma_{Ky}+\sigma_y^2\right], \quad (A.3)$$

where the portfolio weights are optimized under the constraint that they must lie between 0 and 1:

$$\omega=\begin{cases}0 & \text{if } \varpi<0\\ \varpi & \text{if } 0\le\varpi\le 1\\ 1 & \text{if } \varpi>1,\end{cases} \quad (A.4)$$

where

$$\varpi=-\frac{J_W}{J_{WW}W}\frac{\alpha-r}{\sigma^2}-\frac{\sigma_{Ky}}{\sigma^2}\,. \quad (A.5)$$

Whenever there is an interior solution, (A.5) tells us that the optimal portfolio is a combination of the tangency portfolio, corresponding to the first term on the right-hand-side, and a hedge portfolio given by the second term.

Preferences of the CRRA-form leads to the conjecture that the indirect utility function is of the form $J(W)=A^{-\gamma}(1-\gamma)^{-1}W^{1-\gamma}$ for some constant A. Plugging the conjectured function into (A.3) confirms that it is indeed correct, and that A is given by equation (6a) in the main text. Finally, using (6) in (A.2) and (A.5) gives the consumption policy (equation (7)) and asset demand (equation (8)) for the representative household in the closed economy.

A.2 Behavior with International Asset Trade

We can follow the same methodology, as above to derive the optimal policies when there is financial integration. Let us start by considering the problem without restrictions on the portfolio weights. The Hamilton-Jacobi-Bellman equations are

$$0=\max_{\{c_i,\omega_i^H,\omega_i^F\}}\left\{\begin{aligned}&\frac{c_i^{1-\gamma}}{1-\gamma}+J\delta+J_{W_i}\left[W_i\left(\sum_{j=H}^{F}\varpi_i^j(\alpha_j-r)+r+\mu_i-c_i\right)\right]\\&+\tfrac{1}{2}J_{W_iW_i}W_i^2\left[\sum_{j=H}^{F}\sum_{k=H}^{F}\varpi_i^j\varpi_i^k\sigma_{jk}+\sigma_{y,i}^2+2\sum_{j=H}^{F}\varpi_i^j\sigma_{jy,i}\right]\end{aligned}\right\},\ i=H,F \qquad \text{(A.6)}$$

In (A.6) the ϖ_i^j's are the unconstrained portfolio weights, σ_{jk} is the instantaneous variance/covariance of risky asset returns, and $\sigma_{jy,i}$ is the instantaneous covariance between non-traded income in i and the return on the risky asset in country j, $j = H,F$. From this, we find that the optimal consumption policy is as in the closed economy: $c_i^{-\gamma}=J_{W_i},\ i=H,F$.

The optimal unconstrained portfolio weights satisfy

$$0=J_{W_i}W_i(\alpha_j-r)+J_{W_iW_i}W_i^2\left(\sum_{k=H}^{F}\varpi_i^k\sigma_{jk}+\sigma_{jy,i}\right),\quad i,j=H,F. \qquad \text{(A.7)}$$

This condition can conveniently be rewritten in matrix form as

$$\mathbf{0}=J_{W_i}(\mathbf{a}-r\mathbf{1})+J_{W_iW_i}W_i\left(\Omega\bar{\mathbf{w}}_i-\mathbf{V}_i\right),\quad i=H,F\ , \qquad \text{(A.8)}$$

where $\bar{\mathbf{w}}_i = [\varpi_i^H\ \varpi_i^F]'$. (The other notation is explained in the main text.) Solving for $\bar{\mathbf{w}}_i$, we obtain:

$$\bar{\mathbf{w}}_i=-\frac{J_{W_i}}{J_{W_iW_i}W_i}\Omega^{-1}(\mathbf{a}-r\mathbf{1})-\Omega^{-1}\mathbf{V}_i,\quad i=H,F\ , \qquad \text{(A.9)}$$

which implies that

$$\varpi_i^j=-\frac{J_{W_i}}{J_{W_iW_i}W_i}\sum_{k=H}^{F}\nu_{jk}(\alpha_k-r)-\sum_{k=H}^{F}\nu_{jk}\sigma_{jy,i},\quad i,j=H,F\ . \qquad \text{(A.10)}$$

In (A.10), ν_{jk} are the elements of $\mathbf{\Omega}^{-1}$. We notice that the tangency and hedge portfolio is given by the first and second term, respectively, on the right hand side of (A.9).

Taking into account the short sale constraints $0 \le \omega_i^j \le 1$, $i,j = H,F$, and $\sum_j \omega_i^j \le 1$, $i = H,F$, the asset allocation policies are considerably more complicated. The Hamilton-Jacobi-Bellman equation is as (A.6), with the unconstrained portfolio weights ϖ_i^j replaced by the constrained ones ω_i^j. The first order conditions with respect to ω_i^H and ω_i^F are:

$$\omega_i^H=-\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_H-r}{\sigma_H^2}-\frac{\sigma_{Hy,i}}{\sigma_H^2}-\omega_i^F\frac{\sigma_{HF}}{\sigma_H^2},\quad i=H,F \qquad \text{(A.11)}$$

$$\omega_i^F = -\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_F - r}{\sigma_F^2} - \frac{\sigma_{Fy,i}}{\sigma_F^2} - \omega_i^H \frac{\sigma_{HF}}{\sigma_F^2}, \quad i = H, F\ . \tag{A.12}$$

Next, define the following subsets of (A.11) and (A.12):

$$\hat{\omega}_i^j \equiv -\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_j - r}{\sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2}, \quad i, j = H, F\ , \tag{A.13}$$

$$\tilde{\omega}_i^j = -\frac{J_{W_i}}{J_{W_iW_i}W_i}\frac{\alpha_j - r}{\sigma_j^2} - \frac{\sigma_{jy,i}}{\sigma_j^2} - \frac{\sigma_{HF}}{\sigma_j^2}, \quad i, j = H, F\ , \tag{A.14}$$

and

$$\breve{\omega}_i^H \equiv -\frac{J_{W_i}}{J_{W_iW_i}W_i}\left(\frac{\alpha_H - r}{\sigma_j^2} - \frac{(\alpha_F - r)\sigma_{HF}}{\sigma_H^2\sigma_F^2}\right) - \frac{\sigma_{Hy,i}}{\sigma_H^2} + \frac{\sigma_{Fy,i}\sigma_{FH}}{\sigma_H^2\sigma_F^2}, \quad i = H, F\ , \tag{A.15}$$

where there is an analogous expression for $\breve{\omega}_i^F$. Together with (A.10) and (A.13)-(A.15), equations (A.11) and (A.12) gives us the following constrained asset allocation policies:

$$\mathbf{w}_i' = \left[\omega_i^H, \omega_i^F\right] \equiv \begin{cases} [0,0] & \text{if } \hat{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F < 0 \\ \left[0, \hat{\omega}_i^F\right] & \text{if } \breve{\omega}_i^H < 0 \text{ and } 0 \le \hat{\omega}_i^F \le 1 \\ [0,1] & \text{if } \tilde{\omega}_i^H < 0 \text{ and } \hat{\omega}_i^F > 1 \\ \left[\hat{\omega}_i^H, 0\right] & \text{if } 0 \le \hat{\omega}_i^H \le 1 \text{ and } \breve{\omega}_i^F < 0 \\ \left[\varpi_i^H, \varpi_i^F\right] & \text{if } 0 \le \varpi_i^H \le 1,\ 0 \le \varpi_i^H \le 1 \text{ and } \varpi_i^H + \varpi_i^F \le 1 \\ \left[\frac{\varpi_i^H}{\varpi_i^H + \varpi_i^F}, \frac{\varpi_i^F}{\varpi_i^H + \varpi_i^F}\right] & \text{if } \varpi_i^H > 0,\ \varpi_i^H > 0 \text{ and } \varpi_H^H + \varpi_H^F > 1 \\ [1,0] & \text{if } \hat{\omega}_i^H > 1 \text{ and } \tilde{\omega}_i^F < 0 \end{cases} \tag{A.16}$$

for $i = H,F$.

Substituting the optimal consumption policy into the constrained Hamilton-Jacobi-Bellman equation gives the differential equations for the value functions $J_i(W_i)$:

$$0 = \tfrac{\gamma}{1-\gamma} J_{W_i}^{\gamma-1/\gamma} + J_i\delta + J_{W_i}W_i\left[\mathbf{w}'_i(\mathbf{a} - r\mathbf{1}) + r + \mu_i\right] + \tfrac{1}{2} J_{W_iW_i}W_i^2\left[\mathbf{w}'_i\,\mathbf{\Omega}\mathbf{w}_i + 2\mathbf{w}'_i\,\mathbf{V}_{y,i} + \sigma_{y,i}^2\right],$$

where $\mathbf{w'}_i$ are given by (A.16), and $i = H, F$. The functional form of the intertemporal indirect utility function does not change upon financial integration. For country i it is still $J_i(W_i) = (1-\gamma_i)^{-1}(A_i^*)^{-\gamma_i}W^{1-\gamma_i}$, for some constant A_i^*. Using this in (A.8) we find that A_i^* is given by (18) in the main text and that the optimal asset allocation is given by (20) and (21).

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