ISSN 1503-299X

WORKING PAPER SERIES

No. 4/2003

GROWTH, DEVELOPMENT AND LAND-USE IN A SIMPLE AGRARIAN ECONOMY WITH ENDOGENEOUS POPULATION

Anders Skonhoft
Jan Tore Solstad

Department of Economics

Norwegian University of Science and Technology
N-7491 Trondheim, Norway
www.svt.ntnu.no/iso/wp/wp.htm

GROWTH, DEVELOPMENT AND LAND-USE IN A SIMPLE AGRARIAN ECONOMY WITH ENDOGENEOUS POPULATION

by

Anders Skonhoft
Department of Economics
Norwegian University of Science and Technology
N-7491 Trondheim, Norway
(E-mail: Anders.Skonhoft@sv.ntnu.no)

and

Jan Tore Solstad
Trondheim Business School
Jonsvannsveien 82
N-7004 Trondheim, Norway
(E-mail: Jan.Solstad@aoa.hist.no)

Abstract
The paper analyses the relation between demographic transformation, agricultural transformation and land-use pressure within a simple agrarian economy where population is treated both as a cause and consequence of economic changes. In this Malthusian-type of economy, population growth and food production are interrelated through two production activities. First, agricultural land and labour are tied up in production of agricultural products determining the current flow of consumption. Secondly, labour is used for converting the natural resource base in the form of wilderness, land into agricultural land. It is demonstrated that the economy can run into a poverty trap equilibrium which is the typical Malthusian situation. Alternatively, the equilibrium can be of a high income per capita type. Increasing returns to scale in food production together with an increasing amount of agricultural land, are the crucial factors behind this outcome. As increasing return to scale can be interpreted as if induced innovations take place, and can be seen in light of the presence of Boserup growth mechanisms.

Thanks to Amit Bhaduri for comments on an earlier draft. The paper has also benefited from discussions at seminars in Santiago, York, Oslo, Trondheim and Stocholm. The work has been partly funded by The Norwegian Research Council through the Biologial Diversity Programme.

1. Introduction

This paper analyses the connection between agricultural growth, demographic transformation and land-use in a simple agrarian economy. Studies of population growth in agricultural economies have to a large extent been related to the theory of Thomas Malthus(1798) and its counter thesis; the work by Esther Boserup (1965). Malthus' line of argument is that the living conditions of the people is the main determinant of the population growth. Emphasising the failure of humans to adapt their reproduction behaviour to a limited resource base (food availability), he argued that in good times a high income per capita generates rapid population growth through its positive effect on fertility and its negative effect on the mortality rate. Increased population, given decreasing marginal productivity of labour when producing food, reduces per capita consumption, and as the living conditions deteriorate the fertility rate declines and the mortality rate increases so that the population growth decreases until it finally settles down at a low level of per capita consumption. Hence, in the long run the economy will be trapped in an equilibrium of poor living conditions, the well known 'Poverty Trap'. This type of equilibrium was formally first modelled, under somewhat different assumptions, by Leibenstein (1954) and Nelson (1956).

Boserup (1965) is reversing the main argument of Malthus. Hence, rather than treating the capacity to produce food as given and the main determinant of population growth in poor rural communities, she maintains that population growth is independent of the living conditions of the people. Moreover, population growth is seen as the main factor determining labour productivity in agriculture. This argument basically hinges on the assumption that a higher population pressure induces a shift to more labour-intensive production techniques and opens new innovation possibilities. She even argued that increased population pressure was necessary for the realisation of new innovations. For a more fully discussion and interpretation of the population theory of Maltus and Boserup and extensions in various directions, see, e.g., Birdsall (1988), Cuffaro (1997), Kelley (1988), Nerlove and Raut (1997) and Robinson and Srinivasan (1997).

In light of the Western world economic history of the nineteenth and twentieth centuries, it has been widely argued that the predictions of Malthus have been wrong, and that Malthus was too pessimistic. He was wrong both about the assumption of diminishing returns in food production, and about mans reproductive behaviour and the various demographic transitions (Robinson and Srinivasan 1997, Birdsall 1988, Kelley 1988). However, when addressing Third World population problems today, optimism, typically based on strong beliefs in population-induced technological and institutional innovations, is clearly challenged (see, e.g., Platteau 1996 and Cuffaro 1997 for recent studies of sub-Saharan Africa) [1]. Criticism is typically based on the economic and demographic conditions for innovations. For example, the population growth rates in the Third World countries today are considerably higher than what the Western world ever experienced. While the yearly growth rate was 1.1 % at its highest in the Western world under the industrial revolution, the typical rates of todays

[1] Whether Boserup's theory gives rise to an optimistic view on the population-food chain is a matter of controversy. Some interpret Boserup as linking population-induced technological and institutional innovations with long term increases in labour productivity, while others confine her theory to explain the positive relation between population growth and land yields, leaving labour productivity basically unaffected by population growth in the long run.

developing countries are well above 2 % (e.g., 2.7 % for the Africa as a whole during the period 1960-1990). The difference stems mainly from the fact that the Third World countries, in spite of their considerably poorer living conditions, partly due to new and better medicines, experience far lower mortality rates than the earlier agrarian economies (Bairoch 1993).

Based on the arguments above and on the last decades' experience of economic stagnation and unprecedented high rates of population growth in many Third World countries, we therefore believe that Malthus' emphasis on the connection between population growth and the living conditions are of crucial importance in understanding agricultural and demographic transformations. Consequently, building on Malthus, we formulate a simple growth model within the framework of an agrarian economy. In addition to modelling the interlinkage between population growth and food production, a mechanism leading to land-use changes is introduced as well. Hence, we also emphasise the relationship between population growth, the living conditions and the natural resource base, the latter, after all, being the basis in which all economic activities depend (Arrow et al. 1995; Dasgupta and Mäler 1995).

The identification of the Poverty Trap is one a possible outcome of the model. The present model offers more, however, and moving out of the Poverty Trap can also take place. Indeed, the model substantiates two growth regimes; 'The Poverty Trap Regime' and 'The High Income Regime'. While the conditions leading to the Poverty Trap Regime are clearly Malthusian, the conditions leading to The High Income Regime will be associated with the reasoning of Boserup as the technological and institutional conditions here are favourable. More specific, under this growth regime, it is non-decreasing returns to scale in food production. In the spirit of Boserup, we interpret this as being due to population induced innovations. The present model represents therefore a synthesis of the population theory of Malthus and Boserup as the economy, based on a Malthusian model, can end up in a relatively prosperous state, in addition to the Malthusian Poverty Trap.

We start in section two by formulating the model. The model is to some extent complementary to the recent work by Brander and Taylor(1998), and as in this paper, the fertility behaviour is not modelled explicitly (see, e.g. Nerlove and Raut 1997 for an overview)[2]. Instead, a reduced form equation is applied to express a population growth pattern consistent with observed data for the demographic transition (Birdsall 1988; see also Dasgupta 1995). The amount of food production is related to the utilisation of the natural resource which is agricultural land, or wilderness land that can be converted into agricultural land. Undeveloped land is treated as a renewable resource because agricultural land regrow and wilderness land recover. This happens in a linear way in the present exercise.

The model is solved in section three. It is demonstrated that there generally will be two

[2] Brander and Taylor also analyse the interaction between human population growth and the utilisation of natural resources in a natural resource based economy. The natural resource in their model is a renewable one (interpreted as forest and soil) which grows in a logistic manner, while population growth is linearly related to consumption per capita of the natural resource. Contrary to this, we assume that the population grows in an unlinear manner depending on consumption of food per capita while the natural resource grows in a linear way (see below).

steady-states, and these equilibria can be approached in different ways. These different approaches constitute The Poverty Trap Regime and The High Income Regime and are studied in section four and five, respectively. The analysis of the steady-states and the dynamics in these sections are based on the assumption that there is no constraint on the amount of agricultural land. In section six we see what happens when there is no more agricultural land available.

2. The model

As noted, we consider a situation where the human population growth depends on the living condition of the people in a Malthusian fashion, and where the population and land-use are interrelated through two production activities. First, land and labour are used in production of agricultural goods determining the current flow of consumption. Secondly, labour is used for converting wilderness land into arable land, i.e., investment in new land. Hence, in the model, as in reality, there is a trade off between using labour for consumption and investment. Equation (1) gives first the population growth at time t (the time subscript is omitted when no misunderstanding can occur) depending on the living conditions as given by consumption per capita (C/N) where N is the population and C is consumption,

(1) $dN/dt = F(C/N)$.

The function is specified to be in accordance with the empirical evidence of Birdsall (1988), and represents, in a stylised manner, the various phases of the demographic transition (see also the theoretical contribution by Haavelmo 1954, and Brown 1995). Accordingly, for 'low' levels of consumption per capita, population growth increases with increasing consumption per capita due to a decreasing mortality rate, while the fertility more or less is constant. For 'high' levels of consumption per capita, population growth decreases with increasing consumption per capita due to a decreasing fertility rate while the mortality is more or less constant. Hence, by assuming a non-linear negative relationship between consumption per capita and the mortality rate for the low level interval, and a non-linear negative relationship between the fertility rate and consumption per capita for the high level interval, together with the assumption of a fixed and similar intrinsic growth rate in both stages, the population growth can be approximated by the humped function

(1') $dN/dt = N\{r(C/N)[1 - (C/N)/k] - v\}$.

With this specification, the population growth rate is related to two terms; the consumption per capita growth term $r(C/N)[1 - (C/N)/k]$, and an autonomous growth term v. The consumption per capita related growth term, reflecting living conditions influencing fertility and mortality, is therefore assumed to be a logistic function with r and k as positive constants. r reflects the underlying (intrinsic) gross growth rate of the population, while k is the saturation level of the growth, i.e., the consumption per capita level where the growth depending on the living conditions is zero. $v > 0$ will typically depend on factors such as the education level of the people, availability of medicines, the quality of the health system, and so forth, all factors assumed to be exogeneous in the present analysis.

Equation (1') implies that there will be two equilibria where population growth is zero, both taking place when the per capita related growth term equals the autonomous growth term v. It will be one equilibrium for a consumption per capita level below that of k/2, while the other one takes place for a level above k/2. See Figure 1. The two consumption per capita ratios coexisting with zero population growth can be classified as a low income equilibrium, $(C/N)^{*I}$, and a high income equilibrium, $(C/N)^{*II}$, both depending only on the demographic parameters r, k and v. The low income equilibrium $(C/N)^{*I}$ will be associated with the first phase of the demographic transition with high mortality and fertility rates. The high income equilibrium $(C/N)^{*II}$ will be associated with the third phase of the demographic transition, that is, the state of low mortality and fertility rates. Finally, the interval where the population growth is positive corresponds with phase two of the transition.

Figure 1 about here

A crucial question will be which of these equilibria that can be approached in the long-term. For a given land-use so that the amount of agricultural land is fixed (see below), it can be shown that the low consumption per capita equilibrium will be a locally stable equilibrium, while the high consumption per capita level will be unstable. This points in the direction that there are strong forces that can drive this economy into the low-income equilibrium. When this happens, the equilibrium $(C/N)^{*I}$ is said to fall within The Poverty Trap Regime. Because the land-use changes so that the amount of agricultural land is not fixed, however, $(C/N)^{*II}$ can also be approached, and this equilibrium will be referred to fall within The High Income Regime.

We then look at the production side of the model. Equation (2) gives first the production of agricultural goods. There are two variable production factors, agricultural land A and labour. When N_1 persons out of N is allocated to this activity at time t, the production follows as

(2) $C = C(A,N_1)$.

The production function is also assumed to be twice differentiable, with positive and non-increasing marginal productivity of both production factors. Moreover, both production factors are essential for production; that is, $C(0,N_1) = C(A,0) = 0$. The Cobb-Douglas function

(2') $C = q_1 A^{\alpha} N_1^{\beta}$

with $0 < \alpha < 1$ and $0 < \beta < 1$, obeys these properties and will be used in the analysis.

The growth of agricultural land is given by

(3) $dA/dt = G(N_2) - H(A)$

where N_2 is labour used for converting wilderness land into agricultural land, and $G(N_2)$ expresses how this investment effort translates into land conversion. However, because cultivated land regrow and wilderness land recover so that agricultural land, depending on the amount of it, depreciates as given by the term $H(A)$, $G(N_2)$ represents gross investments into new land. Equation (3) reflects therefore two things. First, it is not costless to convert wilderness into arable land. Secondly, it is reversibility in the land-use so that wilderness land is just like a renewable resource. Consequently, $dA/dt > 0$ holds only as long as the effort use exceeds that of the natural decay, $G(N_2) > H(A)$.

$G(N_2)$ as well as $H(A)$ are non-negative with $G(0) = 0$ and $H(0) = 0$. In the analysis these functions are specified as $G(N_2) = q_2 N_2^a$ with $a > 0$, and $H(A) = bA$ with $b > 0$. It is therefore assumed to be a linear decay of agricultural land. Hence, (3) reads

(3') $dA/dt = q_2 N_2^a - bA$.

There are two constraints in this economy. The labour (population) constraint is first given by

(4) $N_1 + N_2 \leq N$.

All the time this constraint is assumed to hold as an equality meaning that it always is an opportunity cost of channelling labour to either of the two production activities.

We also have a constraint on the amount of land. The total available land is normalised to one. The land-use constraint therefore reads

(5) $A \leq 1$.

Hence, $(1 - A) \geq 0$ represents non-cultivated, or wilderness land.

When the land-use constraint is not binding and (5) holds as an inequality, the above system has one degree of freedom. To close the model, we therefore need one more equation. In what follows, the model is closed by assuming that a fixed fraction $0 < s < 1$ of the population is channelled to investment in new land,

(6) $N_2 = sN$.

This closure rule represents therefore a correspondence with the standard neoclassical (Solow-Swan) growth model.

3. Dynamics and equilibria

Combination of equations (1'), (2') and (6) yield the population growth, depending on the population level itself together with the size of agricultural land, as

(7) $dN/dt = N\{rq_1(1-s)^{\beta} A^{\alpha} N^{(\beta-1)}[1 - (q_1(1-s)^{\beta}/k)A^{\alpha}N^{(\beta-1)}] - v\}$.

There are two constellations of agricultural land and population that yield zero population growth. They are found when $rq_1(1-s)^{\beta} A^{\alpha} N^{(\beta-1)}[1 - (q_1(1-s)^{\beta}/k)A^{\alpha}N^{(\beta-1)}]$ intersects with v and will be characterised by a low and a high land-labour ratio, corresponding to $(C/N)^{*I}$ and $(C/N)^{*II}$, respectively (see Figure 1). Both N-isoclines will be positively sloped and run through the origin in the A-N plane (see the Appendix). Moreover, both isoclines will be linear if there is constant returns to scale and $(\alpha + \beta) = 1$ holds, concave functions of A when we have decreasing returns, and convex functions if there is increasing returns to scale (again, see the Appendix).

In the Appendix it is also demonstrated that the population will decrease above the isocline with the lowest land-labour ratio, increase between the two isoclines and again decrease below the isocline with the highest land-labour ratio. Figure 2 illustrates the two isoclines together with the dynamics outside the isoclines when it is decreasing return to scale in food production, while Figure 3 illustrates the increasing returns to scale case. In both these cases we again stress that consumption per capita is constant and equal to $(C/N)^{*I}$ along the isocline with the lowest land-labour ratio and equal to $(C/N)^{*II}$ along the isocline with the highest land-labour ratio.

Combination of equations (3') and (6) yield the reduced form agricultural land-use growth as

(8) $dA/dt = q_2 s^a N^a - bA$.

It is only one A-isocline which will slope upwards and run through the origin. When $a < 1$ so that it is decreasing return to scale in the land-converting process, it will be a convex function of A. When $a = 1$ it will be a linear function (see Figure 2 and 3), and when there is increasing returns to scale the isocline will be a concave function. Above the A-isocline the investment effort exceeds that of the decay so that $dA/dt > 0$, and vice versa.

Because both the N-isoclines and the A-isocline slope upwards, the system is a symbiotic one; more agricultural land means a higher equilibrium human population size, while more people means more agricultural land in the long-term. When the land-use constraint is not effective and $A < 1$ holds all through the time, it will be two interior equilibria in this symbiotic system[3]. We will have A^{*I} and N^{*I}, and therefore C^{*I}, along the N-isocline with the low land-labour ratio, and N^{*II} and A^{*II} and C^{*II} along the N-isocline with the high land-labour ratio. Hence, A^{*I} and N^{*I} correspond to the low consumption per capita equilibrium $(C/N)^{*I}$ from Figure 1, while A^{*II} and N^{*II} correspond to the high consumption per capita equilibrium $(C/N)^{*II}$.

The stability of the two interior equilibria, and therefore the economic significance of the two equilibria, will be different. The crucial point is whether the A-isocline intersects with the N-isoclines from above or from below. Because the actual outcome hinges on the concavity properties of the isoclines, the economic significance of the two equilibria is related to the technology in the two production activities. In the Appendix it is demonstrated that the two equilibria can be approached under two different main conditions. The equilibrium along the low land-labour N-isocline can be approached when it is non-increasing returns to scale when producing agricultural products as well as in the land-converting process. Hence, A^{*I}, N^{*I} and $(C/N)^{*I}$ can be approached when $(\alpha + \beta) \leq 1$ together with $a \leq 1$ hold, and the inequality is

[3] It will be no interior equilibria when there are constant returns to scale in both activities as both the A-isocline and the N-isoclines then are straight lines running through the origin. However, because it is a constraint on the use of agricultural land, a meaningful solution of the model will take place even in this case. See section 6 below.

unbinding at least in one of the activities. We then have The Poverty Trap Regime. On the other hand, A^{*II}, N^{*II} and $(C/N)^{*II}$ can only be approached when $(\alpha + ß) \geq 1$ and a ≥ 1 hold, and the inequality is unbinding at least in one of the activities. We then have The High Income Regime.

It is therefore not 'history or luck', i.e., the initial conditions, that are the main determinants of the growth pattern and the long-run outcome of this economy. The main determinant is the technology, i.e., whether it is decreasing or increasing returns to scale, the last interpreted as if population induced technologies are taking place. From the above discussion it is also clear that the economy can not grow without bounds. Hence, even under the assumption of increasing returns to scale in both food production and the land-clearing activity and no effective land-use constraint, the economy will settle down to zero growth in the long term. This result hinges on the fact that as the economy moves through phase two of the demographic transformation, a small and declining population growth will finally choke economic growth even if there is more land to expand production on. The High Income Regime is analysed in details in section 5, but first The Poverty Trap Regime is examined.

4. The Poverty Trap Regime

The Poverty Trap Regime is analysed when we have $(\alpha+ß) <1$ together with a =1 so there is decreasing returns to scale when producing agricultural products while it is constant returns in the land-clearing activity[4]. This case is depicted in Figure 2. As indicated, the low land-labour equilibrium ratio with A^{*I} and N^{*I} is locally stable, while the equilibrium A^{*II} and N^{*II} is an unstable one. This is confirmed by the stability analysis in the Appendix, where it is demonstrated that point II is a saddle-point equilibrium. Under the present technology and scale conditions, the economy is therefore caught in a low-income trap because A^{*I} and N^{*I}, and therefore the low consumption per capita level $(C/N)^{*I}$, represent the only possible interior long-term equilibrium.

Figure 2 about here

Starting from a population level N_0 equipped with a small amount of agricultural land A_0, Figure 2 indicates a possible development trajectory. Initially, we have therefore a poor economy with high rates of both mortality and fertility, but where mortality dominates. Consequently, the population declines and Malthus' notion of 'positive checks' are in effect. Even though the population declines, however, it will be cultivated additional land for future food production. The economy makes therefore progress, both because the population declines and the amount of agricultural land increases. As living conditions improve, mortality decreases and turns the population decline into a positive growth. The increasing population will now, because of decreasing returns to scale, counterbalance the gain from new investments in land, and the living conditions will eventually start to deteriorate. From this stage on the economy expands with continued land conversion, a growing population and less food per capita until it settles at the long term steady state equilibrium (A^{*I}, N^{*I}).

---

[4]As already mentioned, having constant returns in food production together with decreasing returns to scale in the land clearing process ($\alpha+\beta=1$, a<1) as well as decreasing returns to scale in both activities ($\alpha+\beta<1$, a<1), yield qualitatively the same results.

Consequently, the economy do not approach the third and final phase of the demographic transformation although the population grows rapidly in periods, but is eventually and inevitably forced back to the first phase of high mortality and fertility rates.

During the entire course of development, the land-labour ratio in food production A/(1-s)N increases. Moreover, because of the fixed investment fraction, the labour productivity C/(1-s)N develops identically to consumption per capita. Recalling that the N-isoclines represent constant food per capita paths, it is therefore clear that labour productivity improves in the beginning. The labour productivity will also increase during the first part when the population grows, but eventually productivity growth ceases and starts to decline before it stabilises at the long-term equilibrium. It can also be shown that the land productivity C/A decreases all through the time. This will be so because the constant land productivity locus defines N as convex functions of A when we have decreasing returns to scale. By inspection of the labour productivity identity $C/(1-s)N \equiv (C/A)(A/(1-s)N)$, it is also clear that declining land productivity more than outweights the positive productivity effect from more land per agricultural worker during the last part of the development path.

Having seen a possible growth trajectory leading to the low income equilibrium, we proceed to analyse factors affecting the equilibrium. A^{*I} and N^{*I} will be influenced by all the economic and demographic parameters of the model, and the effects are found by taking the total differential of equations (7) and (8) when dN/dt =0 and dA/dt =0, respectively. First of all, an increased investment fraction s will generally have unclear effects as it influences the system from two different channels; it shifts the N-isoclines as well as the A-isocline down. Through its impact on the population relation (the N-isocline), the isolated effects will be triggered by the immediate reduction in per capita consumption as labour is moved away from food production. Hence, the population declines and implies less labour for land conversion, and the amount of agricultural land decreases accordingly. Through its impact on the land conversion relation (the A-isocline), the isolated effects will be an increased amount of agricultural land as a larger fraction of the labour force is moved to land conversion, and, triggered by the corresponding temporary increase in consumption per capita, the population will increase (see also the Appendix).

The conclusion is therefore that more effort channelled to investments not necessarily will be associated with more capital in the form of agricultural land in the long-term. However, it can be shown that both the amount of agricultural land and the population size, reach maximum values during the course of a changing s. In the Appendix it is demonstrated that the investment fraction maximising the amount of agricultural land is given as $(1-\beta)$. This investment fraction will therefore maximise current welfare if welfare is made up of consumption per capita together with wealth in the form of agricultural land. Consumption per capita is namely determined by demographic factors alone, and hence, maximising welfare is the same as maximising agricultural land. The optimal investment rule $(1-\beta)$ has a close connection to the so-called Hartwick's rule in the theory of exhaustible resources (see, e.g., Dasgupta and Heal 1978). In fact, if we have constant returns to scale in food production, it is exactly Hartwick's rule.

A positive efficiency shift in the production of agricultural products through an increased q_1, shifts up the N-isocline while the A-isocline stays unchanged. As a consequence, there will be a higher population size as well as more agricultural land, both increasing in the same proportion when there is constant returns to scale in the land-clearing process as we have in Figure 2. The food production increases accordingly, but will also now be exactly counterbalanced by an increased population size so that the long-term effect on consumption per capita is unaffected. The mechanisms are obviously Maltusian; as food production becomes more efficient, more food is produced for the given inputs of labour and land and the living conditions improve accordingly. Thus, in a next step, the population increases and the availability of labour for land clearing increases. Under the condition of decreasing returns to scale, the economy therefore expands with deteriorating living conditions until it settles down at a long-term equilibrium where the initial (lower) consumption per capita level is reached.

Table 1
Comparative statics of the long-run (locally)
stable equilibrium when $(\alpha + ß) <1$, and $a =1$.
The Poverty Trap Regime

	s	q_1	q_2	b	v	r	k
A^{*I}	?	+	+	-	-	+	+
N^{*I}	?	+	+	-	-	+	+
C^{*I}	?	+	+	-	-	+	+
$(C/N)^{*I}$	0	0	0	0	+	-	-

A more efficient land-clearing process through an increased q_2, shifts down the A-isocline. The outcome will therefore be more agricultural land and a higher population size in the long-term. Once again the Malthusian mechanism is apparent; as the land-clearing process becomes more efficient, the given amount of labour is capable of converting land more rapidly than that of the natural decay. More agricultural land drives food production up in a next step and improves the living conditions. But also now the improved living conditions will be only temporary as a higher food production is counterbalanced by a subsequent increase in the population size in the long-term.

The long-term effects of changes in the demographic parameters are also clear, and a positive shift in the growth parameter v works unambiguously in the direction of improved living conditions of the people as $(C/N)^{*I}$ increases, cf. equation (1') and Figure 1. At the same time, the long-term use of agricultural land as well as the population size decrease as the N-isocline shifts down. Hence, vaccination programmes aiming at reducing mortality through v will increase the population size and worsen the long-term living conditions under The Poverty Trap Regime. These results fit intuitive reasoning as a higher population size, coexisting with no technological progress and decreasing returns to scale, not will be offset by the same amount of food production. Fertility control will obviously work in the opposite direction.

5. The High Income Regime

The long-term equilibrium represented by A^{*II}, N^{*II}, and $(C/N)^{*II}$ can be approached when it is non-decreasing returns to scale so that we have $(\alpha + ß) \geq 1$ and a ≥ 1. As mentioned, these scale properties can be associated with the reasoning of Boserup (1965) as they represent favourable technological and institutional growth conditions. Under these scale properties, and an additional condition related to the magnitude of the decay parameter b (for details, see the Appendix), the equilibrium point II will be locally stable, and can be reached either by cyclical or monotonic convergence (again, see the Appendix), while point I is a saddle point. Hence, because a small and declining population growth finally chokes economic growth, the economy also settles down to zero growth when there is non-decreasing returns to scale. In what follows, The High Income Regime is analysed when we have $(\alpha+ß) > 1$ together with a =1 so that there is increasing return to scale in food production while there is constant returns in the land-clearing process[5]. The difference compared to The Poverty Trap Regime is therefore only due to different scale conditions when producing consumption goods; increasing returns replaces decreasing returns.

Figure 3 about here

A possible development trajectory is shown where the economy initially is equipped with a small amount of agricultural land and where the land-labour ratio is low, A_0 and N_0. See Figure 3. Initially the situation is just as under the The Poverty Trap Regime; the 'positive checks' are in effect and the population shrinks. However, the low land-labour ratio means that it is room for additional cultivation, and the amount of agricultural land grows. Both these factors work in the direction of improved living conditions. Hence, mortality decreases and the population decline slows down and eventually it starts to increase. From this stage on, being in phase two of the demographic transition, the economy expands with continued land conversion, more people and a corresponding improvement in the living conditions because of the prevailing increasing returns to scale. When the High Income Equilibrium (A^{*II}, N^{*II}) is reached, the economy has run through all the phases of the demographic transition. As already noted, the economy may adjust cyclically or monotonically towards the steady-state.

The labour productivity in food production C/(1-s)N again goes hand in hand with the amount of food per capita and, consequently, it increases steadily during this growth trajectory. From Figure 3 it is also evident that the land-labour ratio A/(1-s)N, just as under The Poverty Trap Regime, increases as well. The fixed land productivity locus C/A are now concave functions running through the origin. The land productivity will therefore decrease during the first part of the growth trajectory. However, after population has started to increase, the land productivity will eventually start to increase as well. When again inspecting the labour productivity identity $C/(1-s)N \equiv (C/A)(A/(1-s)N)$, we must now have that the increased land labour ratio more than outweights the reduced land productivity during the first part of the growth path.

[5] As mentioned, constant returns in food production together with increasing returns to scale in the land clearing process ($\alpha+\beta=1$, a>1) as well as increasing returns to scale in both activities ($\alpha+\beta>1$, a>1), yield qualitatively the same results.

The comparative static of this long-run steady-state is given in Table 2. As above, a positive shift in the investment fraction s also now gives unclear results because of its two-sided effects. Through its impact on the population growth function, the isolated effects will also now be triggered by the immediate reduction in food per capita and, since this economy has gone through the demographic transition and the population response to more food per capita is of the opposite of The Poverty Trap case, a corresponding increase in the population as well as an increase in the amount of agricultural land will be the result as labour is moved away from food production. Through its impact on the land conversion relation, the isolated effects will be a reduction in the amount of agricultural land as well as a decrease in the population as labour is moved to the land conversion activity. The fact that agricultural land is reduced when a higher proportion of the labour force is channelled to land conversion contrasts intuition. However, it hinges on the fact that as the amount of agricultural land initially starts to increase due to a larger investment fraction, the living conditions of the people improve and, being in the third phase of the demographic transition, the population will start to decline. Hence, the total availability of labour for land conversion shrinks and affects land conversion negatively. Eventually, this negative impact will dominate the initial increase so that the amount of agricultural land starts to shrink. It will do so until the economy settles at the new equilibrium where the amount of agricultural land is less than initially.

Contrary to what takes place under The Poverty Trap Regime, an increased s from a small initial value now works in the direction of less agricultural land, while an increased s from a large initial value means more agricultural land in the long-term (see the Appendix). Hence, under The High Income Regime, it exists a value of s that minimises the amount of agricultural land, or what is the same, that maximises the amount of non-cultivated land. This investment fraction is again given as $(1-\beta)$, and is therefore still in accordance with Hartwick's rule if welfare is maximised and now made up of consumption per capita together with wealth in the form of wilderness land.

A more efficient food production through an increased q_1 is also now associated with a upward shift in the N-isocline. However, because the N-isoclines intersect with the A-isocline from below, a reduction in the amount of agricultural land as well as a lower population size will be the result in the long-term. Consequently, the long term food production will also shrink. These effects contrast intuition, but the reason is that the economy now has gone through the demographic transitions and has passed the stage where improved living conditions trigger a rapid population growth. Rather, fertility is reduced and population declines and subsequently leads to a decline in the amount of agricultural land. Because of increasing returns to scale, the economy therefore contracts until it settles down at a long-term equilibrium where the initial (lower) level of consumption per capita again is reached.

The effect of a more efficient land-clearing process through a positive shift in q_2 will also be of the opposite of The Poverty Trap Regime as it leads to a decrease both in the amount of agricultural land and in the population. The effects of shifts in the demographic parameters are, however, the same except for the effects on per capita consumption. Reduced v and hence, more people, works therefore now in the direction of improved living conditions. The reason is that increased population accompanied by more land means an even higher consumption growth because of the presence of increasing returns to scale in the food production.

Table 2
Comparative statics of the long-run (locally) stable equilibrium when $(\alpha + ß) > 1$, and $a = 1$.
The High Income Regime

	s	q_1	q_2	b	v	r	k
A^{*II}	?	-	-	+	-	+	+
N^{*II}	?	-	-	+	-	+	+
C^{*II}	?	-	-	+	-	+	+
$(C/N)^{*II}$	0	0	0	0	-	+	+

6. <u>Constraints on the use of agricultural land</u>
The above analysis of dynamics and equilibria rest on the assumption that it is no constraint on the amount of agricultural land; that is, relation (5) holds as an inequality all through the time. However, many agricultural economies, past or present, have very limited land resources so that all, or almost all, usable land have been converted into agricultural land. In addition, physical property rights, government land-use policies, and so forth, imply that there in many instances will be little land left over for further agricultural expansion. See Timmer(1988) for a general discussion.

When the land-use constraint becomes effective and $A = 1$ holds from relation (5), the growth pattern and the properties of the long-term equilibrium change. This is illustrated in Figure 4 where the equilibrium at the high land-labour N-isocline is locally stable. Consequently, without any land-use constraints, we are in The High Income Regime. Starting from the same initial situation as above (cf. Figure 3), the population will therefore first decrease and then increase while the amount of agricultural land grows. At the same time the labour productivity in food production increases and land productivity decreases. When the land-use constraint becomes effective, however, the population growth and the development pattern change compared to the above case. We will then have that $A = 1$ replaces A in equation (7) and the population dynamics reads

$$(9)\ dN/dt = N\{rq_1(1-s)^ß N^{(ß-1)}[1 - (q_1(1-s)^ß/k)N^{(ß-1)}] - v\}.$$

This equation together with (8) as

$$(10)\ q_2 s^a N^a = b$$

represent therefore the dynamics of the system. Hence, when the land-use constraint is binding there are two equations in just one variable and the model is over determined, i.e., a fixed size of agricultural land is inconsistent with a relatively large and growing population. In other words, for the given technology and labour allocation, there are too many hands in land conversion and parts of the labour force are redundant. To overcome this inconsistency one or more of the parameters in equation (10), reflecting changes in behaviour, have to adjust. There are two possibilities within the present setting; labour productivity in the land conversion activity can be reduced, or redundant labour can be removed from food production through a change in the fraction of people channelled to land conversion. In what follows, adjustments in q_2 and s are analysed separately[6].

We start to analyse what happens when the efficiency parameter in the land conversion changes through q_2. Because q_2 is included only in the land equilibrium condition (10), equation (9) determines the population dynamics alone while the degree of efficiency and organisational slack adjust to the population growth through equation (10). Under this scenario, it can easily be demonstrated that the low consumption per capita equilibrium $(C/N)^{*I}$ will be the stable long-term equilibrium (cf. also the discussion related to Figure 1), and the equilibrium will be approached as indicated in Figure 4.

Figure 4 about here

Consequently, being in a situation where per capita consumption is lower than the level required for equating fertility with mortality when the land constraint becomes effective, population will continue to grow. Since there is no more land available, however, the productivity in the land conversion activity q_2 decreases and the population increases even though the amount of agricultural land is constant. The food production can therefore not keep pace with the growing number of people and the living conditions will deteriorate. This process continues until the consumption per capita level has fallen to $(C/N)^{*I}$. An equilibrium within The Poverty Trap Regime will therefore replace an equilibrium within The High Income Regime.

As mentioned, the adjustment can also take place by changing the fraction of people channelled to the land-clearing activity. Because s influences the population growth as well as the land-clearing activity, this adjustment process is more complex. By studying equations (9) and (10) in the N-s plane and under the assumption that s adjusts instantly, it can be demonstrated (see the Appendix) that, if any, there will either be one or two stable and attainable long-term equilibria. One is associated with the low consumption per capita equilibrium $(C/N)^{*I}$ while the other one is associated with $(C/N)^{*II}$. If the fixed investment fraction s before running into the land-use constraint is under a certain level, it can be shown that $(C/N)^{*I}$ will represent the only attainable long-term equilibrium. Hence, also under this adjustment process an equilibrium within the Poverty Trap Regime can replace an equilibrium within The High Income Regime, and in such a case the outcome will be quite similar to that of the adjustment in q_2. Now, however, the economy will be able to feed a higher population as redundant labour is made productive by channelling it to food production because s decreases as N increases through the adjustment process, cf. equation (10). More interesting, however, is the fact that $(C/N)^{*II}$ also is attainable, and this may

[6] The productivity in the land-clearing activity can also be adjusted through the scale elasticity a, but this works exactly as adjusting q_2.

happen if s is above a certain level. In such a case, the living conditions will therefore still improve while there is no more land to cultivate and labour is the only variable production factor. Decreasing returns to scale in food production means therefore more food per capita and the reason is the successive adjustments in the fraction of people channelled to food production (again, see the Appendix).

The above reasoning demonstrates that a decisive factor determining the performance of our simple agrarian economy is whether the land shortage occurs before the third phase of the demographic transition is reached. Technically, the location of the intersection between the high land-labour ratio N-isocline and the A-isocline versus the location of the constraint A =1 is the crucial factor. The values of the parameters determining the A-isocline and the N-isoclines, in addition to the amount of available land, will therefore together determine whether the land-use constraint becomes effective or not. The land-use constraint A =1 is therefore a relative concept. This point can be illustrated by indicating changes taking place in a comparative dynamic setting.

Let the initial situation, the efficiency and the investment fraction, be just as under The High Income Regime studied above. However, now the growth and development process is assumed to take place for a *permanent* lower value of the autonomous population growth term v. The direct consequences of a lower v, say, due to a general improvement of the health system, means that the high land-labour N-isocline will shift down while the low land-labour N-isocline shifts up. Consequently, the intersection between the high land-labour N-isocline and the A-isocline, which is unaffected, shift to the right and takes place for a higher value of A (as well as a higher value for N). As a result, a previous situation of no constraint on A can change to a situation with an effective land-use constraint. If this happen, not only an equilibrium taking place within The Poverty Trap Regime can replace an equilibrium within The High Income Regime, the whole growth path will change as well. A permanent lower value of the autonomous population growth term can therefore change the qualitative structure of the whole growth process, as well as the nature of the equilibrium. The same can obviously also happen for permanent changes of the efficiency parameters or the investment fraction.

7. Discussion and concluding remarks

The model analysed above is Malthusian since the population growth depends on the living conditions while the living conditions again depends on the population growth. However, there are two crucial extensions compared to the standard Malthusian theory. First of all, by introducing an investment activity so that uncultivated land can be converted into agricultural land, increasing labour productivity accompanied by growing population represent a possible development although the typical Malthus assumption of diminishing returns to labour is present in food production. Secondly, because the population growth is assumed to be a humped function depending on consumption per capita, it will be an additional long-term equilibrium with zero population growth coexisting with no land-use changes and a fixed consumption per capita. When present, this equilibrium yields a higher consumption per capita than what follows from the Malthusian poverty trap. So while Malthus argued for the absence of 'preventive checks' to keep population from outgrowing food production when the economy is in a prosperous state, the present model puts preventive checks into effect by allowing net fertility to decrease with improved living conditions when consumption per capita is above a certain level. This is the demographic foundation for The High Income Regime within the model.

The above model represents therefore a generalisation of the standard Malthusian theory where the Malthusian poverty trap is one of two possible equilibria. Our approach gives two explanations for being trapped in the Malthusian low-income equilibrium level; either by having decreasing returns to scale in the production activities, or by running into an effective land-use constraint. These two factors also explain low productivity in food production. Non-decreasing returns to scale in food production and when converting uncultivated land into agricultural land, together with a sufficient amount of arable land, are therefore the two conditions to be fulfilled for escaping the poverty trap and increase the labour productivity in the long term. Non-decreasing returns to scale can be interpreted as if population induced innovations are taking place. Hence, The High Income Regime can be associated with the reasoning of Boserup as technological and institutional conditions here are favourable.

The development trajectories leading to the equilibria in The Poverty Trap Regime and The High Income Regime will be quite different, and changes in demographic and economic factors influence the trajectories in different manners. It has also been seen that when the land-use constraint becomes effective during the course of development, growth within The High Income Regime can shift direction; instead of leading to the high consumption per capita equilibrium, it may end up in the traditional Malthusian low income equilibrium. However, depending on the size of the investment fraction, an equilibrium within The High Income Regime can still be attainable when there is no more land to cultivate. As mentioned, an effective constraint on the amount of agricultural land is probably a fairly realistic situation for many agricultural communities in sub-Saharan Africa as well as in other Third World countries.

The various transformations discussed in the model can be seen in light of the empirical overview given by Timmer(1988) and Hayami and Ruttan(1985). Timmer suggests that there have been various stylised transformation paths for the agricultural sector. One characterises the newly opened countries in the last part of the 19th century (USA and Canada and Australia) with lot of surplus land. These countries moved almost uniformly in the direction of higher labour productivity and lower land productivity. The growth in the land productivity was quite slow until quite recently. Another path can be seen in light of the developing countries in Asia. Among these, high population growth and land scarcity are common characteristics together with rapidly increasing land productivity and slowly increasing, or even falling (e.g., Bangladesh), labour productivity. Timmer also sees a particular African path over the last decades. Between 1965 and 1973, Africa's productivity performance in the agricultural sector was much like that of the newly opened countries in the last century; slow growth in land productivity and more rapid growth in labour productivity. However, something new happened after 1973 as the productivity in both land and labour declined for the entire continent between Sahara and South Africa. At the same time the land-labour ratio also declined. So while the development path under The High Income Regime to some extent can be related to what happened when the new countries were opened during the last century, the paths within The Poverty Trap Regime share some similarities with the development in Africa and to some extent, Asia.

The present model exercise can also be related to the conditions for agricultural and economic 'take off' as discussed by Bairoch (1975). Bairoch sees the agricultural sector as the key sector for economic and social development in most ’backward’ Third World economies, and pinpoints that an essential condition for increased labour productivity in the agricultural sector is that the area cultivated per agricultural worker shifts up. This point is well illustrated by the path discussed under The High Income Regime where the productivity gain, leading the economy to the high equilibrium level consumption per capita, is partly the result of a rapidly growing land-labour ratio. However, as demonstrated, the land-labour ratio increases as well in the scenario discussed under The Poverty Trap Regime. The model therefore clearly illustrates that more land per labour is not a sufficient condition for improved labour productivity and better living conditions for the rural people as it can be offsett by a rapidly declining land productivity.

Literature

Arrow, K. et al. (1995): Economic growth, carrying capacity, and the environment. *Science* 268, 520-521

Bairoch, P. (1975): *The Economic Development in Third World since 1900*. University of California Press, Berkeley

Bairoch, P.(1993): *Economics and World History*. Harvester-Wheatsheaf, New York

Birdsdall, N. (1988): Economic approaches to population growth. In H. Chenery and T. N. Srinivasan (eds.): *Handbook of Development Economics* Vol I. Elsevier, Amsterdam

Boserup, E. (1965): *The Conditions of Agricultural Growth*. Chicago University Press, Chicago

Brander, J. and M. S. Taylor (1998): The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. *American Economic Review* 88, 119-138

Brown, G. (1995): Crowding, endogeneous population growth and economic development. *Paper*, presented at the Environment and Development Conference Beijer Institute, Stockholm

Cuffaro, N. (1997): Population growth and Agriculture in poor countries: A review of theoretical issues and empirical evidence. *World Development* 25, 1151-1163

Dasgupta, P. (1995): The population problem: Theory and evidence. *Beijer Institute Discussion Paper*, Stockholm

Dasgupta, P. and G. Heal(1978): *Economic Theory and Exhaustible Resources*. Cambridge University Press, Cambridge

Dasgupta, P. and K.G. Mäler (1995): Poverty, institutions and environmental resource base. In J. Behrman and T.N. Srinivasan (eds.): *Handbook of Development Economics* Vol III. Elsevier, Amsterdam

Hayami, Y. and V. Ruttan (1985): *Agricultural Development*. John Hopkins University Press, Baltimore

Haavelmo, T. (1954): *A Study in the Theory of Economic Evolution*. North-Holland, Amsterdam

Kelley, A. (1988): Economic consequences of population changes in the third world. *Journal of Economic Literature* 26, 1685-1728

Leibenstein, H.(1954): *A Theory of Economic-Demographic Development*. Princeton

University Press, Princeton

Malthus, T.(1798): *An Essay on the Principle of Population*. Oxford University Press, Oxford

Nelson, R.(1956): A theory of the low-level equilibrium trap in underdeveloped economies. *American Economic Review* 56, 894-908

Nerlove, M. and L. Raut (1997): Growth models with endogeneous population: A general framework. In M. Rosenzweig and O. Stark (eds.): *Handbook of Population and Family Economics* Vol Ib. Elsevier, Amsterdam

Platteau, J.-P. (1996): The evolutionary theory of land rights as applied to sub-Saharan Africa: acritical assesment. *Development and Change* 27, 29-86

Robinson, J.A. and T.N. Srinivasan (1997): Long-term consequences of population growth: Technological change, natural resources and the environment. In M. Rosenzweig and O. Stark (eds.): *Handbook of Population and Family Economics* Vol Ib. Elsevier, Amsterdam

Timmer, P. (1988): The agricultural transformation. In H. Chenery and T. N. Srinivasan (eds.): *Handbook of Development Economics* Vol I. Elsevier, Amsterdam.

Appendix

The N-isoclines

Solving equation (7) when $\frac{dN}{dt} = 0$, we obtain the N-isoclines explicitely as

$$\text{(A1)} \quad N = \left(\frac{k \pm \sqrt{k(k - 4v/r)}}{2q_1(1-s)^{\beta}} \right)^{\frac{1}{(\beta-1)}} A^{\frac{\alpha}{(1-\beta)}}.$$

The isocline with the highest population for a given amount of agricultural land corresponds to the low consumption per capita equilibrium $(C/N)^{*\mathrm{I}}$ in Figure 1 while the other isocline corresponds to the high per capita consumption equilibrium $(C/N)^{*\mathrm{II}}$. After some manipulations, the slope reads $\frac{dN}{dA} = \frac{\alpha}{(1-\beta)} \frac{N}{A} > 0$.

The dynamics outside the isoclines can be found by differentiation of $\dot{N}$ evaluated at the equlibrium. After some rearrangements this yields

$$\text{(A2)} \quad \left. \frac{d\dot{N}}{dN} \right|_{\dot{N}=0} = (\beta - 1) r q_1 A^{\alpha} (1-s)^{\beta} N^{(\beta-1)} \left(1 - \frac{2q_1 A^{\alpha} (1-s)^{\beta} N^{(\beta-1)}}{k} \right).$$

Hence, we have

$$\left. \frac{d\dot{N}}{dN} \right|_{\dot{N}=0} \begin{cases} < 0 & \text{if} \quad N > \left(\dfrac{k}{2q_1(1-s)^{\beta}} \right)^{\frac{1}{(\beta-1)}} A^{\frac{\alpha}{(1-\beta)}} \\ = 0 & \text{if} \quad N = \left(\dfrac{k}{2q_1(1-s)^{\beta}} \right)^{\frac{1}{(\beta-1)}} A^{\frac{\alpha}{(1-\beta)}} \\ > 0 & \text{if} \quad N < \left(\dfrac{k}{2q_1(1-s)^{\beta}} \right)^{\frac{1}{(\beta-1)}} A^{\frac{\alpha}{(1-\beta)}} \end{cases}$$

$N = \left(\frac{k}{2q_1(1-s)^{\beta}} \right)^{\frac{1}{(\beta-1)}} A^{\frac{\alpha}{(1-\beta)}}$ will be a line between the two isoclines. Above this line $\left. \frac{d\dot{N}}{dN} \right|_{\dot{N}=0}$ is negative, meaning that N decreases when being above the upper isocline having the lowest land-labour ratio, and increases when being below it. On the other hand, below this line $\left. \frac{d\dot{N}}{dN} \right|_{\dot{N}=0}$ is positive, meaning that N increases when being above, and decreases when being below the isocline with the highest land-labour ratio.

Stability conditions

The Jakobi-matrix of the reduced form system (7) and (8) is given as $J = \begin{bmatrix} \frac{\partial \dot{A}}{\partial A} & \frac{\partial \dot{A}}{\partial N} \\ \frac{\partial \dot{N}}{\partial A} & \frac{\partial \dot{N}}{\partial N} \end{bmatrix}$.

Evaluated at equilibrium we obtain after some manipulations

$\frac{\partial \dot{A}}{\partial A} = -b$, $\frac{\partial \dot{A}}{\partial N} = aq_2 s^a N^{(a-1)}$, $\frac{\partial \dot{N}}{\partial A} = \alpha r q_1 A^{(\alpha-1)}(1-s)^\beta N^\beta \left(1 - \frac{2q_1 A^\alpha (1-s)^\beta N^{(\beta-1)}}{k}\right)$

and $\frac{\partial \dot{N}}{\partial N} = (\beta - 1) r q_1 A^\alpha (1-s)^\beta N^{(\beta-1)} \left(1 - \frac{2q_1 A^\alpha (1-s)^\beta N^{(\beta-1)}}{k}\right)$.

The trace is then

(A3) $$Trace(J) = (\beta - 1) r q_1 A^\alpha (1-s)^\beta N^{(\beta-1)} \left(1 - \frac{2q_1 A^\alpha (1-s)^\beta N^{(\beta-1)}}{k}\right) - b$$

$$= (\beta - 1) r \frac{C}{N}\left(1 - \frac{2C/N}{k}\right) - b,$$

and the determinant is

(A4) $$Det(J) = brq_1 A^\alpha (1-s)^\beta N^{(\beta-1)} \left(1 - \frac{2q_1 A^\alpha (1-s)^\beta N^{(\beta-1)}}{k}\right)(1 - \beta - a\alpha)$$

$$= br\frac{C}{N}\left(1 - \frac{2C/N}{k}\right)(1 - \beta - a\alpha).$$

At the equilibrium (A^{*I}, N^{*I}) we know that $C/N < k/2$, i.e., $\left(1 - \frac{2C/N}{k}\right) > 0$ while $\left(1 - \frac{2C/N}{k}\right) < 0$ holds at (A^{*II}, N^{*II}). The stability properties are then as follows:

Non-increasing returns to scale ($1 - \beta - a\alpha > 0$):

Equilibrium (A^{*I}, N^{*I}):	*Det*(*J*) > 0, *Trace*(*J*) < 0	Locally stable node
Equilibrium: (A^{*II}, N^{*II}):	*Det*(*J*) < 0	Unstable saddle point

Non-decreasing returns to scale ($1 - \beta - a\alpha < 0$):

Equilibrium (A^{*I}, N^{*I}):	*Det*(*J*) < 0	Unstable saddle point
Equilibrium (A^{*II}, N^{*II}):		

$$Det(J)>0, Trace(J) \begin{cases} <0 & \text{if } b > (\beta-1)\dfrac{C}{N}\left(1-\dfrac{2C/N}{k}\right) & \text{Locally stable} \\ >0 & \text{if } b < (\beta-1)\dfrac{C}{N}\left(1-\dfrac{2C/N}{k}\right) & \text{Locally unstable} \end{cases}$$

The locally stable equilibrium when non-decreasing returns to scale is approached by monotonic convergence if

$$Trace(J)^2 - 4Det(J) = \left((\beta-1)r\frac{C}{N}\left(1-\frac{2C/N}{k}\right)-b\right)^2 - 4br\frac{C}{N}\left(1-\frac{2C/N}{k}\right)\left(1-\beta-a\alpha\right) > 0$$

, and by cyclical convergence if

$$Trace(J)^2 - 4Det(J) = \left((\beta-1)r\frac{C}{N}\left(1-\frac{2C/N}{k}\right)-b\right)^2 - 4br\frac{C}{N}\left(1-\frac{2C/N}{k}\right)\left(1-\beta-a\alpha\right) < 0$$

Comparative statics

The comparative static results are found by taking the total differential of equations (7) and (8) when $\dot{N} = \dot{A} = 0$.

The investment fraction:

$$\frac{dA^*}{ds} = \frac{a(1-\beta-s)}{s(1-s)(1-\beta-\alpha a)}A = ?\,,\ \frac{dN^*}{ds} = \frac{((1-s)\alpha a - s\beta)}{s(1-s)(1-\beta-\alpha a)}N = ?\text{, and}$$

$$\frac{dC}{ds} = \frac{q_1 A^\alpha (1-s)^\beta N^\beta ((1-s)\alpha a - s\beta)}{s(1-s)(1-\beta-\alpha a)} = \frac{((1-s)\alpha a - s\beta)}{s(1-s)(1-\beta-\alpha a)}C = ?\,.$$

Productivity in food production:

$$\frac{dA^*}{dq_1} = \frac{a}{q_1(1-\beta-\alpha a)}A = ?\,,\ \frac{dN^*}{dq_1} = \frac{1}{q_1(1-\beta-\alpha a)}N = ?\text{, and}$$

$$\frac{dC^*}{dq_1} = \frac{A^\alpha (1-s)^\beta N^\beta}{(1-\beta-\alpha a)} = \frac{1}{q_1(1-\beta-\alpha a)}C = ?\,.$$

Productivity in land-clearing:

$$\frac{dA^*}{dq_2} = -\frac{(\beta-1)}{q_2(1-\beta-\alpha a)}A = ?\,,\ \frac{dN^*}{dq_2} = \frac{\alpha}{q_2(1-\beta-\alpha a)}N = ?\text{, and}$$

$$\frac{dC^*}{dq_2} = \frac{\alpha q_1 A^\alpha (1-s)^\beta N^\beta}{q_2(1-\beta-\alpha a)} = \frac{\alpha}{\mathrm{q}_2(1-\beta-\alpha a)}C = ?\,.$$

Decay of agricultural land:

$$\frac{dA^*}{db} = \frac{(\beta-1)}{b(1-\beta-\alpha a)}A = ?\,,\ \frac{dN^*}{db} = -\frac{\alpha}{b(1-\beta-\alpha a)}N = ?\text{, and}$$

$$\frac{dC^*}{db} = -\frac{\alpha q_1 A^{\alpha}(1-s)^{\beta} N^{\beta}}{b(1-\beta-\alpha a)} = -\frac{\alpha}{b(1-\beta-\alpha a)} C = ?\,.$$

The exogeneous population growth rate:

$$\frac{dA^*}{dv} = -\frac{ba}{Det(J)} A < 0\,,\ \frac{dN^*}{dv} = -\frac{b}{Det(J)} N < 0\,, \text{ and}$$

$$\frac{d(C/N)^*}{dv} = \frac{bq_1 A^{\alpha}(1-s)^{\beta} N^{(\beta-1)}(1-\beta-\alpha a)}{Det(J)} = \frac{b(1-\beta-\alpha a)}{Det(J)} \frac{C}{N} = ?\,.$$

The intrinsic growth rate:

$$\frac{dA^*}{dr} = \frac{abq_1 A^{(\alpha+1)}(1-s)^{\beta} N^{(\beta-1)}\left(1-\frac{q_1 A^{\alpha}(1-s)^{\beta} N^{(\beta-1)}}{k}\right)}{Det(J)} > 0\,,$$

$$\frac{dN^*}{dr} = \frac{bq_1 A^{\alpha}(1-s)^{\beta} N^{\beta}\left(1-\frac{q_1 A^{\alpha}(1-s)^{\beta} N^{(\beta-1)}}{k}\right)}{Det(J)} > 0\,,$$

$$\frac{dC^*}{dr} = \frac{bq_1^2 A^{2\alpha}(1-s)^{2\beta} N^{(2\beta-1)}\left(1-\frac{q_1 A^{\alpha}(1-s)^{\beta} N^{(\beta-1)}}{k}\right)}{Det(J)}(\alpha a+\beta) > 0\,, \text{ and}$$

$$\frac{d(C/N)^*}{dr} = -\frac{bq_1^2 A^{2\alpha}(1-s)^{2\beta} N^{(2\beta-2)}\left(1-\frac{q_1 A^{\alpha}(1-s)^{\beta} N^{(\beta-1)}}{k}\right)}{Det(J)}(1-\beta-\alpha a) = ?\,.$$

The saturation coefficient:

$$\frac{dA^*}{dk} = \frac{abrq_1^2 A^{(2\alpha+1)}(1-s)^{2\beta} N^{(2\beta-2)}}{k^2 Det(J)} = \frac{abr}{k^2 Det(J)}\left(\frac{C}{N}\right)^2 A > 0\,,$$

$$\frac{dN^*}{dk} = \frac{brq_1^2 A^{2\alpha}(1-s)^{2\beta} N^{(2\beta-1)}}{k^2 Det(J)} = \frac{br}{k^2 Det(J)} \frac{C}{N} C > 0\,,$$

$$\frac{dC^*}{dk} = \frac{brq_1^3 A^{3\alpha}(1-s)^{3\beta} N^{(3\beta-2)}}{k^2 Det(J)}(\alpha a+\beta) = \frac{br(\alpha a+\beta)}{k^2 \alpha Det(J)}\left(\frac{C}{N}\right)^2 C > 0\,, \text{ and}$$

$$\frac{d(C/N)^*}{dk} = -\frac{brq_1^3 A^{3\alpha}(1-s)^{3\beta} N^{(3\beta-3)}}{k^2 Det(J)}(1-\beta-\alpha a) = -\frac{br(1-\beta-\alpha a)}{k^2 Det(J)}\left(\frac{C}{N}\right)^3 = ?\,.$$

The optimal investment fraction

In section 4 and 5, the existence of Hartwick's rule was observed. By combination of equations (A1) and (8) when *dA/dt =0* and solving for A, we obtain

(A5) $A = A(s) = ps^{m_1}(1-s)^{m_2}$,

where $p > 0$ always holds, and $m_1 = \frac{a(1-\beta)}{1-\beta-a\alpha}$ and $m_2 = \frac{a\beta}{1-\beta-a\alpha}$.

When non-increasing returns to scale, m_1 and m_2 are positive. $A(s)$ is then a logistic function with $A(0)=0$ and $A(1)=0$ and has a peak value for s as given by $(1-\beta)$, i.e., the Hartwick rule.

Under non-decreasing returns to scale, m_1 and m_2 are negative. $A(s)$ is then an U-shaped curve having a minimum value as determined by $(1-\beta)$, i.e., the Hartwick rule again.

Adjustment of the investment fraction under an effective land-use constraint

The most convenient way to study what happens when *s* adjusts under an effective land constraint, seems to be to analyse equations (9) and (10) in the *N-s* plane. We first look at equation (9). It can be demonstrated that the $N-$isoclines of (9), corresponding to the constant consumption per capita levels $(C/N)^{*I}$ and $(C/N)^{*II}$, will slope downwards. For both isoclines we will have that $N=0$ for $s=1$ and they will intersect with the N-axis. However, as indicated by Figure A1, the isocline corresponding to $(C/N)^{*I}$ will be steeper than that of $(C/N)^{*II}$. It also follows that N decreases when being above the upper isocline and when being below the lower isocline, while N will increase when being between them, cf. the arrows in the figure. When expressing equation (10) as $N = \left(\frac{b}{q_2}\right)^{\frac{1}{a}}(1/s)$, it is directly seen at it is a downward sloping convex curve and where $N \underset{s\to 0}{\to} \infty$ and $N \underset{s\to\infty}{\to} 0$.

Depending on the position of equation (10) in relation to the two $N-$isoclines, there will either be two, four or none equilibria. Figure 1A demonstrates the case of four equilibria. Because (10) always holds and because *N* increases between the two N-isoclines and declines above and below, we will have that point I and II are stable equilibria while I' and II' are unstable ones. Hence, one of the stable equilibria is represented by the low consumption per capita equilibrium $(C/N)^{*I}$ while the other one is represented by $(C/N)^{*II}$.

Figure A1 about here

If the investment fraction is fixed as s_0 and the population size is 'high' when the economy runs into the land-use constraint (point B_1), it is therefore seen that I will represent the long-term equilibrium if *s* adjusts instantly to secure that (10) holds. On the other hand, if the population size is 'low' when A =1 holds, but we also now are in the second phase of the demographic transition at the time the when the land constraint is met (point B_2), II will represent the long-term equilibrium. Consequently, depending on the size of the population for a given investment fraction, the long-term attained equilibrium can either be represented by $(C/N)^{*I}$ or by $(C/N)^{*II}$. Figure 1A also clearly indicates that if the fixed investment fraction before running into the land-use constraint is under a certail level, $(C/N)^{*I}$ will be the only attainable long-term equilibrium. On the other hand, $(C/N)^{*II}$ can also, depending on the size of the initial *s*, be the only attainable equilibrium.