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STYLIZED DYNAMIC MODEL REPRESENTATIONS - A NOTE

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Stylized dynamic model representations—a note.

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There is marked difference between the complex dynamics found in empirical models — at least if they model the data—and the very simplified dynamics typically found in theoretical models. The purpose of this note is to motivate how the essence of an estimated dynamic model, say

$$\begin{aligned}\Delta y_t = 2 - 0.4\Delta y_{t-1} - 0.6\Delta y_{t-2} \\ + 0.2\Delta x_t - 0.5\Delta x_{t-1} + 3\Delta x_{t-2} - 1\Delta x_{t-3} \\ - 0.5\left(y_{t-3} - 4x_{t-4}\right) + v_t,\end{aligned}$$

can be approximated by a stylized model with simplified dynamics, in this example:

$$\Delta y_t = 1 + 0.85\Delta x_t - 0.25\left(y - 4x\right)_{t-1}.$$

This is achieved by using the mean of the dynamics of the variables.[1]

To ease exposition, let lowercase of the variables denote natural logarithms, so $\Delta z_t \approx \frac{Z_t - Z_{t-1}}{Z_{t-1}} = g_{z_t}$. If we assume that on average, the growth rates are constant—the variables could be "random walks with drift"—the expected values of the growth rates are constants:

$$\begin{aligned}E\Delta y_t &= g_y \forall t \\ E\Delta x_t &= g_x \forall t.\end{aligned}$$

If the variables also are cointegrated, the expectation of the linear combination in the "Equilibrium Correction" term is also constant, so

$$E\left(y_{t-3} - 4x_{t-4}\right) = E\left(y_{t-1} - 4x_{t-1}\right) = \mu \forall t.$$

Under these assumptions, the mean dynamics of the model becomes:

$$\begin{aligned}E\Delta y_t &= 2 - 0.4E\Delta y_{t-1} - 0.6E\Delta y_{t-2} \\ &\quad + 0.2E\Delta x_t - 0.5E\Delta x_{t-1} + 3E\Delta x_{t-2} - 1E\Delta x_{t-3} \\ &\quad - 0.5E\left(y_{t-3} - 4x_{t-4}\right) + Ev_t \\ g_y\left(1 + 0.4 + 0.6\right) &= 2 + \left(0.2 - 0.5 + 3 - 1\right) g_x - 0.5\mu \\ g_y &= \frac{2}{2} + \frac{1.7}{2} g_x - \frac{0.5}{2}\mu \\ g_y &= 1 + 0.85 g_x - 0.25\mu.\end{aligned}$$

[1] Although the derivations are presented for a single equation with exogenous regressors, for ease of exposition, the techniques are of course the same for systems.

We can therefore write the mean-approximated, or stylized, dynamic model as

$$\Delta y_t = 1 + 0.85\Delta x_t - 0.25\,(y - 4x)_{t-1}\,.$$

To illustrate, the dynamic behaviour of the model and its mean approximation are shown below. The upper panel shows the dynamic, or period, responses in y_t to a unit change in x_{t-i}. The latter panel shows the cumulative, or interim, response. The graphs illustrates how the cyclical behaviour—due to complex roots—is averaged out in the stylized representation.

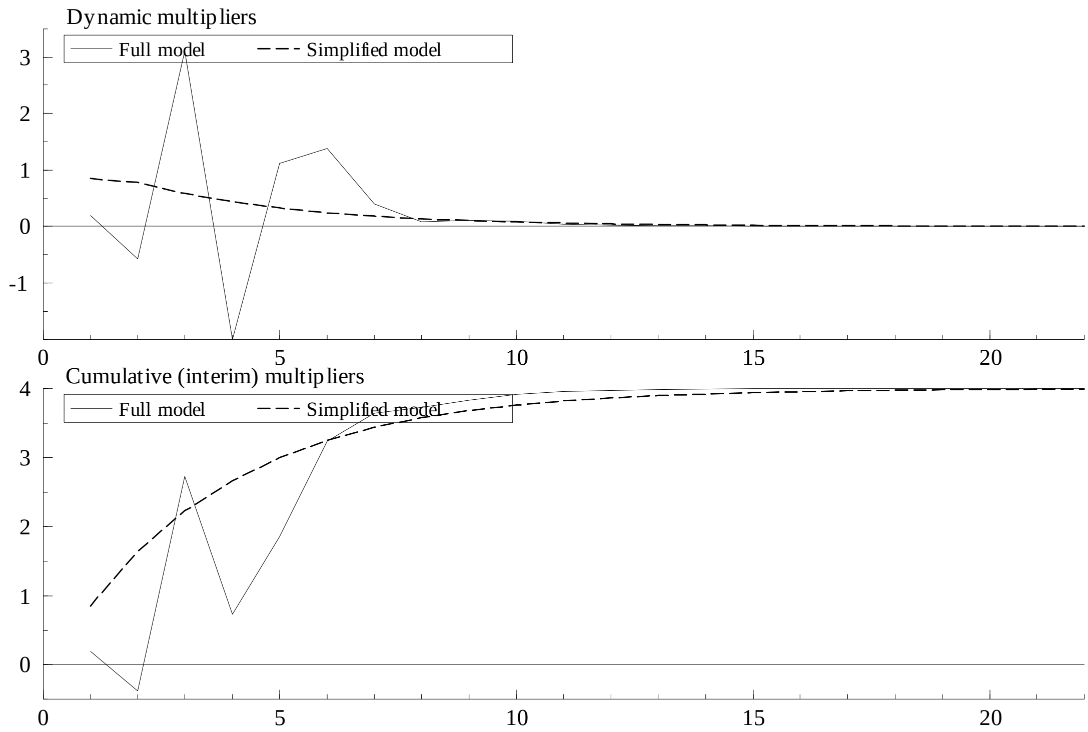

Figure 1: The dynamic responses of the example model and its mean approximation.

Note that all that is done is to exploit the first step in the derivation of the steady state—see (Harvey, 1990, p. 290). The steady-state growth

$$g_y = 4g_x$$

implies the steady-state mean μ as

$$\begin{aligned}(4 - 0.85)\,g_x &= 1 - 0.25\mu \\ \mu &= 4 - 12.6g_x,\end{aligned}$$

so the steady-state relationship between the variables is

$$y_t = (4 - 12.6g_x) + 4x_t,$$

which will hold both for the complete as well as the stylized dynamic representation of the model.

References

Harvey, A. C. (1990). *The Econometric Analysis of Time Series*. Phillip Allen, Oxford, 2nd edn.