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# THE ECONOMICS OF A STAGESTRUCTURED WILDLIFE POPULATION MODEL 

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# The economics of a stage-structured wildife population model 

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#### Abstract

A simple three-stage model of the Scandinavian moose (Alces alces) (young, adult female and adult male) is formulated. Fecundity is density dependent while mortality is density independent. Two different harvesting regimes are explored: hunting for meat, and trophy hunting. The paper gives an economic explanation of the biological notion of females as 'valuable' and males as 'non-valuable'. The paper also demonstrates how this notion may change under shifting economic and ecological conditions.


Key words: Population model, wildlife, harvesting, trophy hunting

[^0]
## 1. Introduction

The aim of this paper is twofold: first, to demonstrate the economic content of a structured wildlife population model; and second, to show how this economic content may change under different management scenarios. Analysing structured wildlife harvesting models, i.e., models where the species are grouped in different classes according to age and sex, has a long tradition within biology. Caswell (2001) gives a recent in-depth overview; see also Getz and Haigh (1989). However, economic analysis plays only a minor role in these works. Economic analysis is introduced more explicitly in Cooper (1993), who formulates a simulation model that finds the economically optimal level of deer tags for hunting zones and where the deer population is structured in bucks and does. Skonhoft et al. (2002) analyses various management strategies for a mountain ungulate living in a protected area and a hunting area. Four stages are included: females and males within and outside the protected area. Because of the complexity of these models, however, it is difficult to understand the various economic mechanisms influencing harvesting and abundance.

The present paper analyses such economic mechanisms more explicitly. A simple three-stage model (young, adult females and adult males) is formulated. Our analysis is similar to that of Clark and Tait (1982), who studied the optimal harvest value in a sex-selective harvesting model where the population was grouped into two stages. See also the two-stage seal model in Conrad and Bjørndal (1991). As in Clark and Tait, we analyse biological equilibrium where natural growth is balanced by harvesting. However, in contrast to Clark and Tait, trophy hunting, in addition to meat-value maximization, is analysed. We also calculate the shadow values of the adult males and females. We are thus giving an economic explanation of the biological notion of females as 'valuable' and males as 'non-valuable'.

The model is applied for a moose population (Alces alces), and is studied within a Scandinavian ecological and institutional context. Moose is by far the most important game species in Scandinavia, and in Norway and Sweden about 40,000 and 100,000 animals, respectively, are shot every year. Moose hunting has traditionally been a local activity, and landowners receive the hunting value. The hunters have been the local people; the landowners and their families and friends, and the management goal has been to maximize the meat value to retain stable populations (more details are provided in Skonhoft and Olaussen 2005).

During the last few years, however, a more commercialized hunting and wildlife industry has emerged, and Scandinavian moose hunting is gradually shifting from a 'family and friend'
activity to a game-hunting market. The trophy value of old males plays an important role here. Both the traditional exploitation scheme and the new commercialized scheme are studied, and the consequences for harvesting and the population composition are analysed.

The paper is organized as follows. In the next section, the three-stage moose population model is formulated. Section three demonstrates what happens when the hunting is steered by the traditional landowner goal of maximizing meat value. In section four we study the sex and age composition under the new exploitation regime of trophy hunting. Section five illustrates the models by some numerical simulations, while section six summarizes our findings.

## 2. Population model

The Alces alces is a large ungulate with mean slaughter body weight (about $55 \%$ of live weight) for adult moose in Scandinavia of about 170 kg for males and 150 kg for females. The non-harvest mortality rates are generally low due to lack of predators, and there is no evidence of density-dependent mortality. On the other hand, fecundity has proven to be affected by the female density while the number of males, within the range of moose densities in Scandinavia, seems to be of negligible importance (see, e.g., Nilsen et al. 2005 for more details).

The population at time (year) $t$ is structured in three stages (Lande et al. 2003); calves $X^{0}{ }_{t}$, adult females ( $\geq 1$ year) $X^{f}{ }_{t}$ and adult males ( $\geq 1$ year) $X^{m}{ }_{t}$ so that the total population is $X_{t}=X^{0}{ }_{t}+X^{f}{ }_{t}+X^{m}{ }_{t}$. These three stages are henceforth called young, female and male. The population is measured in spring after calving. All stages are generally harvested, and the hunting takes place in September-October. All natural mortality is assumed to take place during the winter, after the hunting season, as the natural mortality throughout summer and fall is small and negligible. The same natural mortality rate is imposed for males and females. As indicated, natural mortality is fixed and density independent, while reproduction is density dependent. The same sex ratio is assumed for the young when they enter the old stages (again, see Nilsen et al. 2005).

Neglecting any stochastic variations in biology and environment, and any dispersal in and out of the considered area, the number of young at time $(t+1)$ is first governed by:

$$
\begin{equation*}
X_{t+1}^{0}=r_{t} X^{f}{ }_{t} \tag{1}
\end{equation*}
$$

with $r_{t}>0$ as the fertility rate (number of young per female). The fertility rate is dependent on female density (number of females):

$$
\begin{equation*}
r_{t}=r\left(X^{f}{ }_{t}\right) \tag{2}
\end{equation*}
$$

with $d r / d X^{f}=r^{\prime}<0$ (when omitting the time subscript) and where $r(0)>0$ is fixed.
Combining (1) and (2) gives the recruitment function $X^{0}=r\left(X^{f}\right) X^{f}$ with $d X^{0} / d X^{f}=\left(r^{\prime} X^{f}+r\right)$. The recruitment function is assumed to be concave. For obvious reasons $d X^{0} / d X^{f} \geq 0$ should hold in an optimal harvesting programme.

The abundance of (old) females follows next as:

$$
\begin{equation*}
X^{f}{ }_{t+1}=0.5\left(1-m^{0}\right)\left(1-h_{t}^{0}\right) X^{0}{ }_{t}+(1-m)\left(1-h^{f}\right) X^{f}{ }_{t} \tag{3}
\end{equation*}
$$

where $m^{0}>0$ and $m>0$ are the density-independent mortality fractions of young and female (and male), respectively, while $h^{0}{ }_{t}$ and $h^{f}{ }_{t}$ are the harvesting fractions. Half of the young population is female, after harvesting and natural mortality. The number of (adult) males is finally given by:

$$
\begin{equation*}
X^{m}{ }_{t+1}=0.5\left(1-m^{0}\right)\left(1-{h^{0}}_{t}\right) X^{0}{ }_{t}+(1-m)\left(1-{h^{m}}_{t}\right) X^{m}{ }_{t} \tag{4}
\end{equation*}
$$

where $h^{m}{ }_{t}$ is the male harvesting fraction.

When combining equations (1)-(3), the female population dynamic reads $X^{f}{ }_{t+1}=0.5\left(1-m^{0}\right)\left(1-h_{t}^{0}\right) r\left(X^{f}{ }_{t}\right) X^{f}{ }_{t-1}+(1-m)\left(1-h_{t}^{f}\right) X^{f}{ }_{t}$. This is a second-order non-linear difference equation, and numerical analyses demonstrate that the equilibrium is stable for fixed harvesting fractions (see, e.g., Gandolfo 2001 for a theoretical exposition). Omitting the time subscript, the equilibrium reads:

$$
\begin{equation*}
X^{f}=0.5\left(1-m^{0}\right)\left(1-h^{0}\right) r\left(X^{f}\right) X^{f}+(1-m)\left(1-h^{f}\right) X^{f} \tag{5}
\end{equation*}
$$

There are two equilibria: the trivial one of $X^{f}=0$ and $X^{f}>0$ given by $1=0.5\left(1-m^{0}\right)\left(1-h^{0}\right) r\left(X^{f}\right)+(1-m)\left(1-h^{f}\right)$. Because $r^{\prime}<0$, the non-trivial equilibrium will be unique and may be written as:

$$
\begin{equation*}
X^{f}=F\left(h^{0}, h^{f}\right) \tag{5’}
\end{equation*}
$$

where $F(.$.$) represents a functional form. We find \partial F / \partial h^{0}=F_{0}<0$ and $F_{f}<0$. Therefore, the iso-population female lines slope downwards in the $\left(h^{0}, h^{f}\right)$ plane, and lines closer to the origin yield a higher stock.

By combining equations (1), (2) and (4), the male population growth reads $X^{m}{ }_{t+1}=0.5\left(1-m^{0}\right)\left(1-h_{t}^{0}\right) r\left(X^{f}{ }_{t}\right) X^{f}{ }_{t-1}+(1-m)\left(1-h^{m}{ }_{t}\right) X^{m}{ }_{t}$. The dynamic of the males is therefore contingent upon the female growth (but not vice versa as only female abundance regulates fertility), and again numerical analyses demonstrate that the equilibrium is stable. The equilibrium is:

$$
\begin{equation*}
X^{m}=0.5\left(1-m^{0}\right)\left(1-h^{0}\right) r\left(X^{f}\right) X^{f}+(1-m)\left(1-h^{m}\right) X^{m} \tag{6}
\end{equation*}
$$

There are two equilibria for the male population as well: the trivial one, when $X^{f}=0$, and $X^{m}>0$ when $X^{f}>0$. Equation (6) may also be written as:

$$
\begin{equation*}
X^{m}=G\left(h^{0}, h^{m}\right) r\left(X^{f}\right) X^{f} \tag{6’}
\end{equation*}
$$

where $G\left(h^{0}, h^{m}\right)=0.5\left(1-m^{0}\right)\left(1-h^{0}\right) /\left[1-(1-m)\left(1-h^{m}\right)\right]$. Again, it is confirmed that higher harvesting rates mean fewer animals, $G_{0}<0$ and $G_{m}<0$. The male iso-population lines hence slope downwards in the $\left(h^{0}, h^{m}\right)$ plane, and lines closer to the origin yield a higher stock. On the other hand, a higher female sub-population shifts these iso-population lines away from the origin (suggesting that the slope of the recruitment function is positive, see
above), meaning that there is room for more male harvesting for a given young subpopulation harvest, and vice versa.

Equation (6') also indicates that the equilibrium male-female proportion decreases with more females. However, the male-female proportion may be more easily recognized when combining (5) and (6), which yields $X^{m} / X^{f}=\left[1-(1-m)\left(1-h^{f}\right)\right] /\left[1-(1-m)\left(1-h^{m}\right)\right]$. We therefore simply have $X^{m} / X^{f}=1$ if $h^{m}=h^{f}$, as the mortality of males and females is equal, and the same fraction of young enters the female and male sub-populations.

## 3. Exploitation. The traditional regime: hunting for meat

As indicated, the traditional exploitation of the Scandinavian moose has been directed by maximizing the meat value in ecological equilibrium. Because natural mortality takes place after the hunting season, the equilibrium number of animals removed is simply $H^{0}=h^{0} r X^{f}$, $H^{f}=h^{f} X^{f}$ and $H^{m}=h^{m} X^{m}$, so that the total harvest equals $H=H^{0}+H^{f}+H^{m}$. The management goal of the landowner(s) is accordingly to optimize the value:

$$
\begin{equation*}
\max _{X^{f}, X^{m}, h^{\prime}, h^{f}, h^{m}} U=p\left(w^{0} H^{0}+w^{f} H^{f}+w^{m} H^{m}\right)=p\left[w^{0} h^{0} r\left(X^{f}\right) X^{f}+w^{f} h^{f} X^{f}+w^{m} h^{m} X^{m}\right] \tag{7}
\end{equation*}
$$

subject to ecological constraints ( $5^{\prime}$ ) and ( $6^{\prime}$ ). $w^{0}<w^{f}<w^{m}$ are the (average) body slaughter weights (kilograms per animal) of the three stages while $p$ is the meat price (NOK per kilogram). However, for obvious reasons, the meat price will not affect the optimization except for scaling the shadow price values (see below).

The Lagrangian of this problem writes
$\left.L=p\left[w^{0} h^{0} r\left(X^{f}\right) X^{f}+w^{f} h^{f} X^{f}+w^{m} h^{m} X^{m}\right)\right]-\lambda\left[X^{f}-F\left(h^{0}, h^{f}\right)\right]-\mu\left[X^{m}-G\left(h^{0}, h^{m}\right) r\left(X^{f}\right) X^{f}\right]$ with $\lambda \geq 0$ and $\mu \geq 0$ as the shadow prices of the female and male population, respectively. ${ }^{1}$ The first-order conditions of this maximizing problem are (the second-order conditions are fulfilled due to the concavity of the recruitment function):

[^1]\[

$$
\begin{equation*}
\partial L / \partial X^{f}=p\left[w^{0} h^{0}\left(r^{\prime} X^{f}+r\right)+w^{f} h^{f}\right]-\lambda+\mu G\left(r^{\prime} X^{f}+r\right)=0, \tag{8}
\end{equation*}
$$

\]

$$
\begin{equation*}
\partial L / \partial X^{m}=p w^{m} h^{m}-\mu=0, \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
\partial L / \partial h^{0}=p w^{0} r X^{f}+\lambda F_{0}+\mu G_{0} r X^{f} \leq 0 ; 0 \leq h^{0}<1, \tag{10}
\end{equation*}
$$

$$
\partial L / \partial h^{f}=p w^{f} X^{f}+\lambda F_{f} \leq 0 ; 0 \leq h^{f}<1,
$$

and

$$
\begin{equation*}
\partial L / \partial h^{m}=p w^{m} X^{m}+\mu G_{m} r X^{f} \geq 0 ; 0<h^{m} \leq 1 . \tag{12}
\end{equation*}
$$

Conditions (8) and (9) steer the shadow price values, and (9) suggests that the male shadow price should be equal to its marginal harvested value. Equation (8) is somewhat more complex, but indicates that the female shadow price should be equal to the sum of the marginal harvested value of the female and the young sub-populations, plus the indirect male marginal harvested value, evaluated at its shadow price. Rewriting equation (8) when using condition (9) yields $\lambda=p\left(w^{0} h^{0}+w^{m} h^{m} G\right)\left(r^{\prime} X^{f}+r\right)+p w^{f} h^{f}$. As the slope of the recruitment function is non-negative, $\left(r^{\prime} X^{f}+r\right) \geq 0$ (see above), $\lambda \geq p w^{f} h^{f}$ holds. Hence, while the shadow value of the male population is exactly equal to its marginal harvested value, the shadow value of the female population is above its marginal harvested value. In this sense, females may be considered as more 'valuable' than males in line with the biological notion of females as valuable and males as non-valuable.

Conditions (10)-(12) are the control conditions with the actual complementary slackness conditions stated. From the male control condition (12), harvesting the whole population could be considered as a possibility as this is the biological 'end' product. On the other hand, keeping the female and young sub-populations unexploited are also options as these stages represent the reproductive and potentially reproductive biological capital. Condition (10) indicates that the harvesting of young should take place up to the point where the harvesting benefit is equal to, or below, the cost in terms of reduced population of males and females evaluated at their respective shadow prices. When (10) holds as an inequality, the marginal
harvesting benefit is below its marginal cost and harvesting is thus not profitable, $h^{0}=0$. The interpretation of the female harvesting condition (11) is somewhat simpler. Because of the fecundity density effect, meaning that one more female on the margin yields a smaller recruitment when the female population is 'high' than when 'low', $h^{f}=0$ seems less likely.

The male harvesting condition (12) is analogous to the female harvesting condition (11), but the cost-benefit ratio generally works in the opposite direction. This condition always holds as an inequality. This is revealed when first combining conditions (9), (12) and ( $6^{\prime}$ ), which yield $\left(G+h^{m} G_{m}\right) \geq 0$. When next substituting for $G$ (and $G_{m}$ ) from equation (6), we find that $\left\{0.5\left(1-m^{0}\right)\left(1-h^{0}\right) /\left[1-(1-m)\left(1-h^{m}\right)\right]\right\}\left\{1-(1-m) h^{m} /\left[1-(1-m)\left(1-h^{m}\right)\right]\right\} \geq 0$. After some small rearrangements, it reduces to $m \geq 0$. Accordingly, because $m>0, h^{m}=1$ and the whole male population should be harvested. Notice that this result holds irrespective of the meat value of males and females (as given by the body weights).

The reason for harvesting the whole biological 'end' product as the best option is the lack of any trade-offs when the meat value is maximized; there is neither any biological feed-back effects from the other stages nor any price demand response. Accordingly, the male-female proportion becomes $X^{m} / X^{f}=\left[1-(1-m)\left(1-h^{f}\right)\right]$ (section two above) in the optimal programme while one more male (cf. also footnote 1) yields a benefit of $\mu=p w^{m}$ (NOK per animal). If the optimal policy at the same time gives $h^{0}=0$, the female shadow price reads $\lambda=p w^{m} G\left(r^{\prime} X^{f}+r\right)+p w^{f} h^{f}$. As $G=0.5\left(1-m^{0}\right)$ when $h^{m}=1$ and $h^{0}=0$ (equation $6^{\prime}$ ) and $w^{f}<w^{m}$, the female shadow price may be lower than the male shadow price in contrast to the above notion of females as more 'valuable' than males. In addition, from condition (8), it may also be shown that if $1-G\left(r^{\prime} X^{f}+r\right)>w^{f} h^{f}$, then $\mu>\lambda$. This shows more directly that a low female slaughter weight may pull in the same direction.

## 4. Exploitation. Present time: trophy hunting

The moose-harvesting regime in Scandinavia (like wildlife hunting in other places, see, e.g., Anderson and Hill (1995)) is gradually changing, and a hunting and wildlife industry is emerging. 'Present times' are modelled by introducing a market for trophy hunting of males while still having meat-value hunting of the other two stages. The market for trophy hunting is probably something between a competitive market and a monopoly. One of these extremes
is chosen, and we assume that trophy-hunting licences are supplied under monopolistic conditions. Following the practice in Scandinavia, one licence allows the buyer to kill one animal, which is paid only if the animal is killed. In addition to price, the demand for trophyhunting licences may also be contingent upon 'quality', expressed by the abundance of males. The inverse market demand for male hunting licences is hence given as:

$$
\begin{equation*}
q=q\left(h^{m} X^{m}, X^{m}\right) \tag{13}
\end{equation*}
$$

The licence price $q$ (NOK per animal) decreases with a higher harvest, $q_{H}=\partial q / \partial\left(h^{m} X^{m}\right)<0$, while it increases with more animals available, $q_{X}>0$. Supplying trophy-hunting licences is also costly and depends on the number of animals shot:

$$
\begin{equation*}
C=C\left(h^{m} X^{m}\right) \tag{14}
\end{equation*}
$$

where fixed cost $C(0)>0$, and variable cost $C^{\prime}>0$ and $C^{\prime \prime} \geq 0$. The fixed component includes the cost of preparing and marketing the hunting, whereas the variable component includes the cost of organizing the permit sale, the costs of guiding and various transportation services.

The landowner management goal is now to find a harvesting policy that maximizes the sum of the meat value and trophy-hunting profits:

$$
\begin{equation*}
\max _{X^{f}, X^{m}, h^{p}, h^{f}, h^{m}} \pi=p\left[w^{0} h^{0} r\left(X^{f}\right) X^{f}+w^{f} h^{f} X^{f}\right]+\left[q\left(h^{m} X^{m}, X^{m}\right) h^{m} X^{m}-C\left(h^{m} X^{m}\right)\right] \tag{15}
\end{equation*}
$$

again subject to the constraints ( $5^{\prime}$ ) and ( $6^{\prime}$ ). The first-order conditions of this problem are (where $L$ again refers to the Lagrange function):

$$
\begin{equation*}
\partial L / \partial X^{m}=q_{H}\left(h^{m}\right)^{2} X^{m}+q h^{m}-C^{\prime} h^{m}+q_{X} h^{m} X^{m}-\mu=0 \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
\partial L / \partial h^{m}=q_{H}\left(X^{m}\right)^{2} h^{m}+q X^{m}-C^{\prime} X^{m}+\mu G_{m} r X^{f} \geq 0 ; 0<h^{m} \leq 1, \tag{17}
\end{equation*}
$$

in addition to conditions (8), (10) and (11).

The male harvesting benefit is now expressed by a marginal profit term plus a marginal stock effect through the demand quality effect. The interpretation of (16) and (17) is straightforward (see also above). Combining these conditions and ( $6^{\prime}$ ) yields $\left(q_{H} X^{m} h^{m}+q-C^{\prime}\right)\left(G+h^{m} G_{m}\right)+q_{X} h^{m} X^{m} G_{m} \geq 0$, where the term $\left(G+h^{m} G_{m}\right)$ is still strictly positive because $m>0$ (section three above). When first disregarding the quality effect, $q_{X}=0$, not harvesting all males, $h^{m}<1$, will hence represent the optimal solution if the marginal harvesting profit is equal to zero, $\left(q_{H} X^{m} h^{m}+q-C^{\prime}\right)=0$. From condition (16) (as well as from equation 17), it is seen that this implies a zero value male shadow price. While not harvesting down the whole stage, a zero value shadow price is a counterintuitive result, but hinges on the biological 'end' product nature of the adult males; the number of males does not affect fertility. The zero marginal harvesting profit condition may be met if the marginal cost is high and/or the inverse demand schedule is steep (inelastic). On the other hand, if the marginal revenue exceeds the marginal cost for $h^{m}=1$, the same solution type as above is obtained and where $\mu$ is positive.

When taking the demand quality effect into account, $q_{X}>0, h^{m}<1$ may still hold as an optimal solution when the marginal revenue exceeds the marginal cost $\left(q_{H} X^{m} h^{m}+q-C^{\prime}\right)>0$, as $G_{m}^{\prime}<0$ and $\left(G+h^{m} G_{m}^{\prime}\right)>0$ (see above). The economic reason for this result is simple, as constraining the harvest and keeping a high stock size works in the direction of a higher trophy-hunting licence price through the quality shift in demand. From equation (16) it is seen that this situation implies that $\mu>0$. The corner solution of $h^{m}=1$ is also now a possibility, but the marginal harvesting profit must then exceed a certain minimum, equal to the shadow price.

While the first-order conditions for harvesting female and young are the same as in the traditional harvesting regime, the new conditions for male harvesting will obviously spill over to these stages. With $h^{m}<1$, we may typically find that the male-female proportion $X^{m} / X^{f}$ increases compared with the traditional regime, which may be reinforced if $h^{f}$ shifts up at the same time. Moreover, while the meat price $p$ had no effect on the optimal harvesting policy
in the traditional regime, it may now influence the optimal harvesting policy of all three stages. This will generally occur when the quality effect is included and we have $\mu>0$. In line with standard harvesting theory, a larger harvest and fewer male and young animals is accompanied by a higher price. On the other hand, with no quality effect and with a zero shadow price value of the males, $p$ will have no effect as the conditions (5), (8), (10) and (11) then alone determine $h^{0}, h^{f}, X^{f}$ and $\lambda$.

## 5. Numerical illustration

## Data and specific functional forms

The two exploitation schemes will now be illustrated numerically. The fecundity rate, decreasing in the number of females, is specified as a sigmoidal function with an increasing degree of density dependence at high densities (Nilsen et al. 2005):

$$
\begin{equation*}
r=r\left(X^{f}\right)=\frac{\tilde{r}}{1+\left(X^{f} / K\right)^{b}} \tag{2’}
\end{equation*}
$$

with $\tilde{r}>0$ as the intrinsic growth rate (maximum number of young per female) and $K>0$ as the female stock level for which density-dependent fertility is equal to density-independent fertility. Thus, for a stock level above $K$, density-dependent factors dominate. The compensation parameter $b>0$ indicates to what extent density-independent effects compensate for changes in the stock size. (2’) implies a recruitment function $X^{0}=r\left(X^{f}\right) X^{f}=\tilde{r} X^{f} /\left[1+\left(X^{f} / K\right)^{b}\right]$, which is of the so-called Sheperd type.

The trophy demand function is specified linear. In addition, it is assumed that the quality effect as given by the number of males, through the parameter $\gamma \geq 0$, shifts the demand uniformly up:

$$
\begin{equation*}
q=\alpha e^{\gamma X^{m}}-\beta h^{m} X^{m} \tag{13'}
\end{equation*}
$$

Accordingly, the choke price $\alpha>0$ gives the maximum willingness to pay with a zero quality effect, $\gamma=0$, whereas $\beta>0$ reflects the market price response in a standard manner. The trophy cost function is given linearly as well:

$$
\begin{equation*}
C=\bar{c}+c h^{m} X^{m} \tag{14’}
\end{equation*}
$$

so that $\bar{c} \geq 0$ is the fixed cost and $c>0$ is the constant marginal cost. Table 1 gives the baseline parameter values. For these demand and cost functions we find that the optimal number of hunted males will be $h^{m} X^{m}=(\alpha-c) / 2 \beta$ without the demand quality effect and when $\mu=0$ holds at the same time (equation 16).

Table 1 about here

## Results

Table 2 reports the results. As a benchmark, a no-hunting scenario is also included (first row). Since the young enters the (adult) male and female stages at the same sex ratio, the number of males and females are here the same. In the traditional regime with meat-value maximization, the female harvest rate becomes 0.26 while no harvest of young represents an optimal policy. The marginal harvesting benefit of young is hence below the marginal cost in term of losses from reduced harvesting of males and females. Notice that the number of young is lower in the no-hunting scenario than in the traditional regime. The reason is that the number of females is above the value representing the peak value of the recruitment function and $d X^{0} / d X^{f}=\left(r^{\prime} X^{f}+r\right)<0$ without harvesting. The male shadow value is about four times above that of the female shadow value. As demonstrated in section three, the male shadow value is exactly equal to its marginal harvesting value of $\mu=p w^{h}$, while the female shadow value is above its marginal harvesting value. However, because of the low female harvesting fraction and an optimal harvesting policy close to the peak of the recruitment function (that is, $\left(r^{\prime} X^{f}+r\right)$ is small and positive (see above)), the female shadow value becomes low.

Table 2 about here

The current exploitation scheme is first studied when the quality effect is disregarded; that is $\gamma=0$ and the inverse demand function (13') reads $q=\alpha-\beta h^{m} X^{m}$. Harvesting down all the males is no longer the optimal policy, and the harvesting fraction is substantially reduced, $h^{m}=0.24$. As expected, the male-female proportion increases, and at the same time the
female harvesting fraction shifts up slightly compared with the traditional regime. It is still beneficial, however, to keep the young population unexploited.

When the quality effect is included, the male harvesting rate, as expected, is further reduced and is accompanied by a positive shadow value indicating that the marginal harvesting income exceeds the marginal cost in optimum. The difference between the male and female shadow values is now quite small. The female harvesting rate decreases somewhat as well. As a consequence, the total stock size is higher when the quality effect is included and substantially higher than that of the traditional harvesting scheme of meat-value maximization. The table also demonstrates that the profit increases compared with the traditional regime, and that it further increases when the demand quality effect is added. However, for obvious reasons, the specification of the demand function and parameterization play a critical role here.

Shifting up the meat price $p$ simply scales up the shadow price values in the traditional regime. In the trophy-hunting regime with no quality effect and with a zero shadow price value of the males, the harvesting activity and stock sizes will, as explained above, be unaffected as well. On the other hand, with the quality included and $\mu>0$, the male harvesting activity interacts with the other stages and hence $p$ has an allocation effect as well. However, sensitivity analyses show that the female harvest rate increases only modestly even for a quite dramatic price increase. The reason is that female stock is close to the peak of the recruitment function.

## 6. Concluding remarks

The paper has analysed a three-stage model of the Scandinavian moose with densitydependent fertility. Two exploitation schemes have been studied and it is demonstrated that harvesting down the whole biological 'end' product, i.e., the (adult) male population in this model, always represents the best option when meat-value maximization is the goal. In the numerical examples, this option is accompanied by zero harvesting of the young and modest female harvesting. Within this regime, the biological notion of females as 'valuable' and males as 'non-valuable' is easily recognized, even if the shadow value of the males might be higher than that of the females. The current exploitation scheme with a market for trophy hunting changes the optimal harvesting condition of males. Hunting down the whole
population will no longer be the best option if a well developed market for trophy hunting is present. In addition, the trophy-hunting market allocation also spills over to the conditions for meat-value maximization of young and females. The male-female population ratio will increase, and more female harvesting may take place.

Although the model is simple, it encompasses some general results that will survive in more complex stage-structured models. Most importantly, we have highlighted the economic forces influencing harvest in three different stages that, in various degrees, are present in many structured population models. Our model has two recruiting stages that recruit in different ways. The young represents a value through recruitment to the (adult) male and female stages. As long as density-dependent growth factors are weak, or non-existent (as here), harvesting young does not pay off. For the females, on the other hand, a traditional trade-off between recruitment and harvest is present through the density-dependent fertility mechanism. This mechanism will also be present in more complex models. Finally, the (adult) male stage is considered as the biological 'end' product, and thus does not influence recruitment. It is therefore tacitly assumed that there are always enough males for reproduction. However, irrespective of this, our model demonstrates that the male optimal harvest policy depends critically on economic conditions.

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SSB, Statistics Norway, Hunting Statistics, Oslo (2004)

Table 1: Biological and economic parameter values

| Parameters | Description | Baseline value | Reference/source |
| :---: | :---: | :---: | :---: |
| $\tilde{r}$ | max. specific growth rate | 1.15 | Nielsen et al. (2005) |
| K | female stock level where density dependent factors dominates density independent factors | 1,000 animal | Nielsen et al. (2005) |
| b | density compensation parameter | 2 | Nielsen et al. (2005) |
| $w^{0}$ | average weight young | 60 kg | SSB (2004) |
| $w^{\text {f }}$ | average weight females | 150 kg | SSB (2004) |
| $w^{\text {m }}$ | average weight male | 170 kg | SSB (2004) |
| $\mathrm{m}^{0}$ | natural mortality young | 0.05 | Nielsen et al. (2005) |
| m | natural mortality female and male | 0.05 | Nielsen et al. (2005) |
| p | meat price | $50 \mathrm{NOK} / \mathrm{kg}$ | Storaas et al. (2001) |
| $\alpha$ | choke price | 30,000 NOK/animal | Calibrated |
| $\gamma$ | quality parameter demand | 0.0001 | Calibrated |
| $\beta$ | slope parameter demand | 60 NOK/animal ${ }^{2}$ | Calibrated |
| $\bar{c}$ | fixed harvest cost | 500,000 NOK | Calibrated |
| c | marginal harvest cost | 2,000 NOK/animal | Calibrated |

Table 2: Ecological and economic equilibrium, different management regimes. $h^{0}$ harvest fraction young, $h^{f}$ harvest fraction female, $h^{m}$ harvest fraction male , $h$ total harvest fraction, $X^{0}$ number of young (in 1,000 animals), $X^{f}$ number of females (in 1,000 animals), $X^{m}$ number of males (in 1,000 animals), $X$ total stock (in 1,000 animals), $\lambda$ female shadow price (in 1,000 NOK per animal), $\mu$ male shadow price (in 1,000 NOK per animal) and, $\pi$ profit (in 1,000 NOK)

| Hunting regimes | $h^{0}$ | $h^{f}$ | $h^{m}$ | $h$ | $X^{0}$ | $X^{f}$ | $X^{m}$ | X | $\lambda$ | $\mu$ | $\pi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No harvest | 0 | 0 | 0 | 0 | 0.33 | 3.15 | 3.15 | 6.63 | -- | -- | -- |
| Traditional regime, hunting for meat | 0 | 0.26 | 1 | 0.29 | 0.57 | 0.92 | 0.27 | 1.76 | 2.2 | 8.5 | 4,099 |
| Modern times; trophy hunting. No quality effect | 0 | 0.28 | 0.24 | 0.20 | 0.57 | 0.85 | 0.97 | 2.39 | 2.1 | 0 | 4,599 |
| Modern <br> times; <br> trophy <br> hunting. <br> With quality effect | 0 | 0.25 | 0.09 | 0.12 | 0.57 | 0.95 | 2.01 | 3.53 | 2.0 | 1.9 | 5,594 |

(-- indicates value not calculated)


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[^1]:    ${ }^{1}$ The interpretation of $\lambda$ and $\mu$ as shadow prices is not obvious as the population sizes are determined within the model. However, when adding $\bar{X}^{f}$, interpreted as an exogenous number of introduced females, to the stock constraint ( $5^{\prime}$ ), it can be shown that $\partial U^{*} / \partial \bar{X}^{f}=\lambda$, and where $U^{*}$ denotes the maximum value of $U$. In the same manner, adding $\bar{X}^{m}$ as an exogenous number of introduced males to ( $6^{\prime}$ ) gives $\partial U^{*} / \partial \bar{X}^{m}=\mu$.

