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Optimal fleet renewal plans for a liner shipping company

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NTNU
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- uttak av masteroppgave

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Oppgavetekst/Problembeskrivelse

The fleet renewal problem (FRP) is a recurring strategic problem for shipping companies, with decisions that may involve major capital investments. The purpose of the paper is to develop a model to optimize investment returns for the fleet renewal problem for a major liner shipping company. The purpose of the model is to compare decisions made from a model that maximizes profit and a model that maximizes investement return.

The following topics will be studied

- Develop, implement, and test stochastic models for maximizing profit and maximizing investment returns for the MFRP.
- Investigate the value of the models by comparing the models to each other as well as using measures such as value of stochastic solution (VSS).
- Investigate the results by using different parameters as input.

Hovedveileder ved institutt	Medveileder(e) ved institutt
Professor Kjetil Fagerholt	

Merknader
1 uke ekstra p.g.a påske.

4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

Trondheim, 22.05.14

Sted og dato

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Originalen lagres i NTNUs elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.

Preface

This master's thesis is the result of the final work for my MSc. in Industrial Economics and Technology Management, with specialization in Managerial Economics and Operations Research. The thesis is a continuation of my work for the specialization project, done in the fall semester 2013.

The thesis examines optimal fleet renewal plans for a major liner shipping company. This master's thesis is a part of the MARFLIX (Maritime Fleet Size and Mix) project partly funded by the Norwegian Research Council.

I would like to express my gratitude to my supervisor Professor Kjetil Fagerholt (NTNU), who has been an invaluable resource and guidance for me in my work. Also, I would like to thank my co-supervisors, Giovanni Pantusso (DTU) and Jørgen Glomvik Rakke (MARINTEK) for their support and contribution to my work. At last, I would like to acknowledgde the contribution from Michal Kaut (Sintef), through allowing the use of his scenario generation code, which has been a valued contribution to the work done in this thesis.

Trondheim, June 5th

Ove Mørch

Abstract

This thesis considers the maritime fleet renewal problem (MFRP) for a major liner shipping company. The MFRP is a strategic problem with a planning horizon of several years. The literature study shows that there is a general lack of research about these types of problems. One hypothesis that can explain this lack, is that the current profit maximizing or cost minimizing models presented in research does not coincide with the objectives of the decision makers at the strategic level.

A new model that maximizes investment returns, in terms of Return on Capital Employed (ROCE), is developed, as well as a profit maximizing model. The model maximizing ROCE is believed to better describe the objectives of decision makers at the strategic level than the profit maximizing or cost minimizing models found in current literature. The profit maximizing model will be used as a comparison to the new ROCE maximizing model, to evaluate the solutions suggested.

To be able to solve the ROCE maximizing model, which has a fractional objective formula with decision variables both in the numerator and the denominator, a transformation method has to be applied. The inclusion of both binary and integer variables complicates this picture further, and the transformations of these variables have to be handled explicitly.

The computational study compares the solutions from the two models, including testing for the impact of varying input parameter values, as well as testing expansions of the model including charters and a second hand market, as well as using both a

two-stage and three-stage stochastic model.

The results from the computational study shows that there are major structural differences between the solutions from the two models, with the ROCE maximizing model being far more conservative in investments than the profit maximizing model. The findings from using the ROCE maximizing model give new insight for operations research considering strategic, maritime problems. This will hopefully contribute to the development of better decision support from operations research for these types of problems.

Sammendrag

Denne avhandlingen vurderer det maritime flåtefornyelseproblemer (MFFP) for et stort shippingselskap. Det MFFP er et strategisk problem med en planleggingshorisont som strekker seg over flere år. Litteraturstudien viser at det er generell mangel av forskning på denne typen problemer. En hypotese som kan forklare denne mangelen, er at objektfunksjonen i dagens profittmaksimerende og kostnadsminimerende modeller ikke sammenfaller med objektivene for beslutningstakere på strategisk nivå.

En ny modell som maksimerer avkastningen på investeringene, i form av avkastning på sysselsatt kapital (ROCE), er utviklet, samt en profittmaksimerende modell. Den ROCE-maksimerende modellen antas å bedre beskrive objektivene for beslutningstakere på strategisk nivå, enn de profittmaksimerende og kostnadsminimerende modellene som er presentert i litteraturstudiet. For å kunne sammenligne resultatene fra den nye modellen, er en profittmaksimerende modell også utviklet.

For å være i stand til å løse den ROCE-maksimerende modellen, som har en fraksjonell objektivformel med beslutningsvariabler både i teller og nevner, blir en transformasjonsmetode tatt i bruk. Binære og heltallsvariable kompliserer dette bildet ytterligere, transformeringen av disse håndteres eksplisitt.

Et resultatstudie sammenligner løsningene fra de to modellene. Det inkluderer også analyser for justering av parameterverdier, samt utvidelser av modellen, inkludert charter-muligheter og et bruktmarked for skip, samt en to-trinns og tre-trinns stokastisk modell.

Resultatene viser at det er store strukturelle forskjeller mellom løsningene fra de to modellene, der den ROCE-maksimerende modellen er langt mer konservativ i investeringsbeslutningene enn den profittmaksimerende modellen. Funnene fra bruken av den ROCE-maksimerende modellen gir ny innsikt til optimeringsfeltet innen strategiske, maritime problemer. Dette vil forhåpentligvis bidra til utvikling av bedre beslutningsstøtte fra optimeringsfeltet for slike problemer.

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Chapter 1

Introduction

The shipping industry is part of the seaborne trade, and consists of transportation of cargo by sea. The distance covered can range from local (e.g. ferries) to global (e.g. goods from Asia to Europe) transportation. Over the last decades, seaborne trade has been steadily increasing, from a volume of 2 605 million tons in 1970 to 8 748 million tons in 2012 (Asariotis et al., 2012). Figure 1.1 shows the development of world trade, seaborne trade, world gross domestic product (GDP) and the growth in OECD countries. It can be seen that world trade has been growing faster than the world GDP. This can be explained by the increasing globalization seen over the last decades, demanding more trade between the different geographic areas of the world. Looking towards the future, as unpredictable as it may be, there seems no reason to expect any major setbacks in globalization. Thus it seems reasonable to assume that the world seaborne trade will continue to develop in line with the economic development in the world in the future as well.

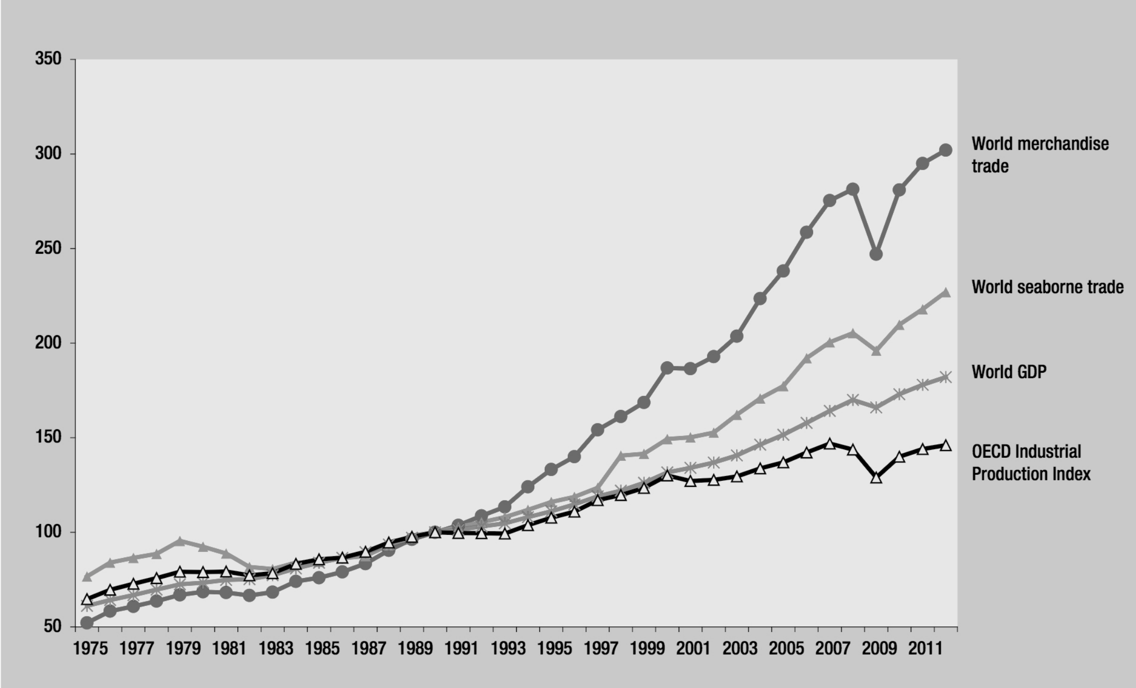


Figure 1.1: The OECD Industrial Production Index and indices for world GDP, world merchandise trade and world seaborne trade (1975-2012) (1990 = 100) Asariotis et al. (2012).

Even though the seaborne trade has been increasing steadily over the last decades, the shipping industry is considered cyclical of nature, with varying peaks and troughs (Stopford, 2008). Peaks are often recognized by increasing demand which is not met to full extent through increase in supply, while troughs can be recognized for instance by increasing supply with demand which cannot keep up, or even decreases. Stopford (2008) indicates an average length of about 7 years for a cycle of a peak and a trough, but the variance is large, and changes in market conditions can happen rapidly from year to year. This was last shown in the downturn after the financial crisis in 2008, leading for example to a decrease in global container trade of almost 10 % from 2008 to 2009 (Asariotis et al., 2012).

The shipping industry in general have long planning horizons when it comes to acquisition and disposal of ships. Lead times for new ships are normally over a year, sometimes as much as four years, from order until the ship is delivered. The lifetime of a ship is often considered around thirty years. Taking this into account, it is difficult to plan years ahead for investments that will last several decades, when one knows that the market changes in the same period can be large and unpredictable. It has been shown that shipowners tend to adapt to the current economic conditions of the market by extending the lifetime of ships in their fleet in good times and sometimes expedite scrapping of ships in bad economic times (Stopford, 2008).

In the last four years, when the world economy has seen little growth following the financial crisis in 2008, the total supply in the shipping market has increased by 37 %, far more than the worldwide economic growth in the same period. The oversupply in the freight market leads to lower freight rates, pushing the margins of the shipping companies. Figure 1.2 shows the development of container rates from 2007 to 2012. At the same time as rates are decreasing, stable or increasing oil prices have lead to increased fuel prices, making the bunker fuel costs a larger share of the total cost for the shipping companies (Asariotis et al., 2012).

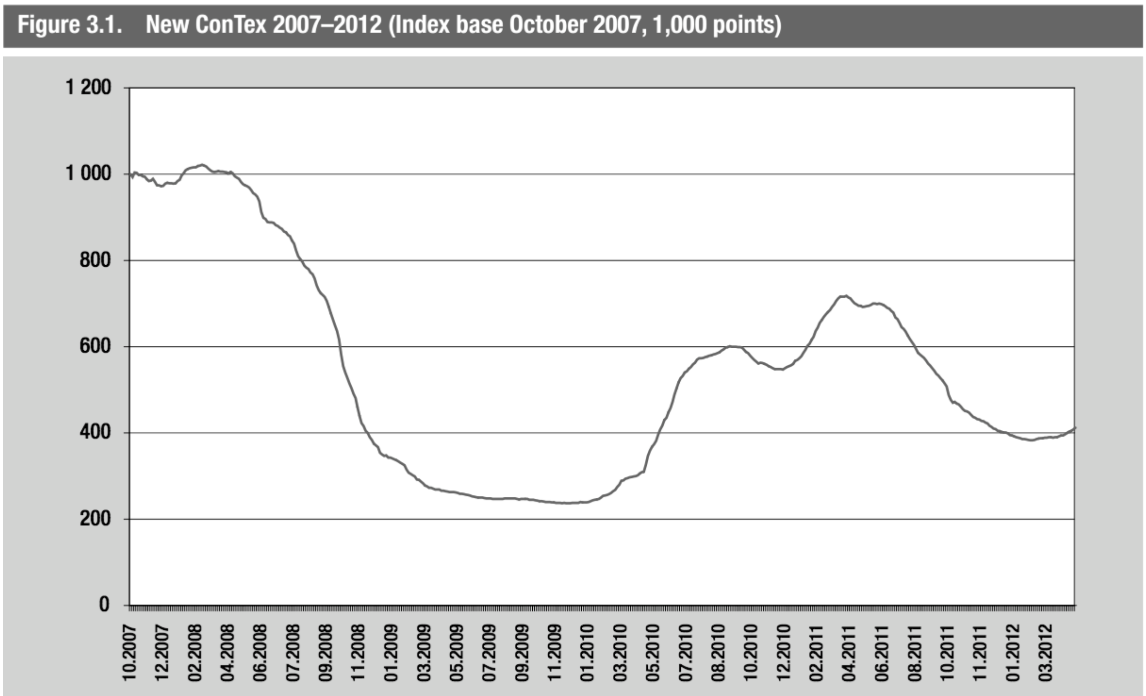


Figure 1.2: New ConTex Index 2007 - 2012 (Index base 2007 = 1000)(Asariotis et al., 2012)

Shipping operations were classified into three modes by Lawrence (1972); industrial, tramp, and liner shipping. In industrial shipping the cargo owner also controls the fleet of ships, and tries to transport its cargo at the lowest possible cost. In tramp shipping the ships operate where there is cargo, as a taxi operation. The companies will usually have a large amount of the capacity tied to long term contracts, but will also try to maximize profit from optional spot cargoes. Liner shipping companies operate with published routes and schedules, much like a bus company. There are not necessarily clearly defined boundaries between the three shipping modes presented, and a company can operate with multiple modes for different parts of the fleet.

For shipping companies, we often categorize the decisions they have to make into operational, tactical and strategic decisions. The difference between the categories are in the timeline of planning horizon for the decisions that need to be taken. Operational decisions look into the "here and now" level of planning, for example which speed should be used for a ship going from A to B. The tactical level looks into problems with a longer planning horizon, e.g. which ships should be used to service a trade, and fleet size and mix problems (FSMP) for shorter planning horizons. At the strategic level one looks at decisions which spans up to several years, such as fleet size and mix problems for planning horizons spanning over several years. With longer planning horizons, uncertainty tends to increase, meaning that strategic problem usually have more uncertainty involved than tactical and operational problems.

To handle the nature of the shipping industry, shipping companies must rely on optimal decisions at all decision levels. Increased performance of the fleet in terms of better utilization and improving technology, leading to savings in operating costs, will also help shipping companies in dealing with the challenges faced. Operations research can assist shipping companies with all these aspects. As the shipping industry has a high degree of uncertainty related to the future development of demand, freight and fuel prizes and more, uncertainty should be handled. In operations research, two main approaches to handling uncertainty has developed; robust optimization and stochastic programming. A short introduction will be given in Chapter 3.

When looking at FSMP where there exists an initial fleet that is going to be replaced over a planning horizon spanning several years, this is referred to as a fleet renewal problem (FRP) (Pantuso et al., 2013). FRPs have received less attention than other categories of FSMPs. This report will consider the maritime fleet renewal problem (MFRP) for a case company which can be said to operate in a liner shipping mode. The case company transports goods in the roll-on/roll-off (ro-ro) category. This shipping category is to a very high degree affected by the changes in the world economy, as one of the major customer groups is car producers. Demand for shipping of cars is highly correlated with the world economy. Normally, freight agreements with car manufacturers are stated in share of production, and not absolute figures. This means that the amount of transported cars will vary with production, which is highly correlated with the world economy.

Current research for the shipping industry have focused more on the operational and tactical problems that the industry face, than the strategic problems. These have received less attention, perhaps due to a mismatch between the objectives that have been used in the models, and the objectives of the decision makers at the strategic level. Strategic problems cover a longer planning horizon, and may involve large capital investment decisions, such as in the MFRP. Because of the large investment decisions, it may be that traditional cost minimizing or profit maximizing model does not reflect the considerations done at the strategic decision level. To address this problem, a model will be developed for maximizing investment returns, trying to reflect the objectives of decision makers facing strategic problems in a better way than the traditional profit maximization and cost minimization models. It aims to improve the decision support from operations research for decisions maker at this level. There has been little or no work done earlier in operations research when it comes to measuring investment returns for MFRPs, and it may thus be considered of great interest to expand this field of research to assist better strategic decision support for the industry. To be able to compare the results form the new model, a profit maximizing model will also be developed. The model will use stochastic programming as an approach to handle the uncertainty of future development in the market.

This report is structured as follows: Chapter 2 will describe the MFRP in detail and discuss the use of investment returns in the objective function. In Chapter 3 an introduction to uncertainty and how this can be implemented in models will be given. Chapter 4 presents a short literature study for the problem. Chapter 5 will describe the two models developed for this report. Chapter 6 will present and discuss results of the computational study. Chapter 7 summarizes the key takeaways from the report.

Chapter 2

Problem Description

This chapter will start by describing the maritime fleet renewal problem (MFRP) in detail in Section 2.1, before discussing the use of different types of objective functions in Section 2.2.

2.1 The Maritime Fleet Renewal Problem

The maritime fleet renewal problem (MFRP) is considered a version of the maritime fleet size and mix problem (MFSMP). The MFSMP is the problem of optimizing the size and mix of a fleet of ships to meet demand requirements. MFSMP can range from tactical to strategic problems, depending on the length of the planning horizon. When taking into account multiple time periods and an initial fleet, the problem turns into the MFRP (Pantuso et al., 2013). The MFRP is the problem of deciding how many and which ship types to operate in each time period to efficiently meet demand. It includes decisions about how to acquire new ships and dispose of old ones.

To be able to solve the MFRP, one must also consider operational decisions to estimate how much capacity is needed to meet demand. The operating decisions can be solved as either a scheduling, routing or deployment problem. The highest level of detail is found in scheduling problems. The level of detail decreases with routing problems, and even more so in deployment problems, which is at higher level of abstraction. Without a good solution to the operating decisions, the solution to the MFRP will risk not being optimal because of false assumptions regarding the operating decisions. Problems at the strategic level often use high-level models, this may be because of several reasons: Tradeoffs between level of detail and computational efficiency, there might not be any more detailed data available, or the data available is so uncertain that it does not reason to use it for strategic planning.

The MFRP is solved each time the strategic planning is performed, so the emphasis of the problem is on the decisions to be taken here and now, i.e. which ships to buy and sell in the upcoming period. The decisions proposed for following periods are only meant as decision support, meaning that they explain how the solution outline the development of the fleet in the coming periods, but the final decisions for these periods are taken at a later time.

There are several ways of obtaining new ships, one of them building new ships (the building is done by a shipbuilder, but building will be used as term for new ships to differ from buying ships from the second hand market). New ships must be ordered from a shipbuilder, and have a lead time from order to delivery ranging from one year up to perhaps four at the most. In addition to building new ships, it might also be possible to buy ships in the second-hand market. When buying ships in the second-hand market, there is usually no lead time, given that the seller has received the ships in its fleet, and is ready to be delivered to the buyer as soon as the arrangements have been made. Besides buying a new or second hand ship, there is also a possibility of chartering in ships. Chartering in ships does not affect the composition of the fleet, but can provide flexibility in the operating decisions. Space, voyage, time and bareboat charter are the most common chartering types.

With space charter you pay for the space needed to transport a shipment, typically on a liner ship. Voyage charter means paying to have a ship perform one or more voyages between specified ports at an agreed rate. With time charter, you charter an entire boat for a specified period, paying a charter rate plus all sailing costs. Bareboat charter means hiring a ship, usually for a longer time period, paying an agreed rate and all fixed (capital costs, insurance, etc.) and sailing costs.

When it comes to disposing of ships, ships can be sold at the second-hand market, or ships can be scrapped. When a company decides to scrap a ship, it is usually paid a rate based on the current value of the steel in the ship. If a company does not want to scrap or sell a ship, but still does not plan to sail with it, the ship can be put in lay-up, saving the company for some of the costs due to less crew, and minimum engine activity, it may also be possible that they are able to reduce their insurance cost if a ship is in lay-up over a longer period. There is also the possibility of chartering out ships the same ways as one can charter them in, and selling them in the second hand market. The same way as with chartering in ships, this does not affect the composition of the fleet, but provides flexibility for the operational decisions.

The shipping company have a set of fixed costs, such as capital and operating costs. Capital costs are related to the amount paid when buying a ship, in addition to the way the ship is financed; through debt (loans) or equity (cash). Operating costs are connected to manning, insurance, maintenance and repair, and administrative costs. The possibility of putting ships in lay-up is a way of saving costs for the shipping company when it does not want to sell or scrap a ship. The variable costs incur when a ship is sailing, and consists of fuel costs, port and canal fees, and cargo handling costs at ports. The fuel costs will vary with the length of the route sailed and the operating speed, and typically make up most of the variable costs. In the case that the shipping company does not have enough capacity to transport a contracted demand, the company will need to use space charter to ensure transportation of the agreed amount.

In Ro-Ro shipping, one considers different types of cargoes, such as cars, high and heavy (HH) and break bulk (BB). The different ship types have different capacities for different types of cargoes. When taking different types of cargoes into consideration, the problem is further complicated.

2.2 Introducing a new objective function

Previous work on the MFRP have mostly used profit maximization or cost minimization as the objective function. These types of objective functions are very useful when the major decisions about what to do in the planning horizon are made. When considering the MFRP, the decisions made will possibly lead to large investments in terms of new ships. These are decisions that often are made by the top management or investors of a company.

When making strategic decisions, top management and investors will often be interested in the rate of return that investments will give, and the risk associated with the investments. Since the MFRP only consider investments for a shipping company, the risk rate can be considered equal for all proposed solutions, meaning that rate of return will be an important measure for decisions. To model this reality in a better way, the use of investment returns as objective function will give a new dimension to using operational research to solve the MFRP.

When measuring investment returns, a wide range of measures have been developed for this purpose. In this report, Return on capital employed (ROCE) will be used. ROCE considers the returns from operations compared to the necessary capital employed to perform the operations. In (Coles, 1997) ROCE is defined as

$$ROCE = \frac{\text{Operating profit}(P^{OP})}{\text{Capital employed } (C^{E})} \tag{2.1}$$

Capital Employed can have many definitions, one of which is the capital investment necessary for a business to function. When applying this to the MFRP it is reasonable to define Capital Employed as the capital investments in the shipping fleet. Other capital such as buildings will not be taken into consideration. When starting a planning horizon, the initial fleet will have an initial value for the capital employed, and as the planning horizon proceeds, this will be updated with the adjustments of the fleet.

The definition in Equation (2.1) considers ROCE for one time period. When considering ROCE for the MFRP, one should consider the average Capital Employed for the period. The definition of ROCE over the planning horizon used in this report will therefore be:

$$ROCE = \sum_{t \in T} \frac{P_t^{OP}}{C_t^{E}/(T+1)} \tag{2.2}$$

where P_t^{OP} is the operational profit in time t, C_t^{E} is the capital employed in period t, and T is the length of the planning horizon. This is more accurately named Return on Average Capital Employed (ROACE), but the term ROCE will be used for this report.

The formulation in Equation (2.2) gives a ROCE for the entire planning horizon. From this it is possible to estimate the compounded annual growth rate (CAGR) from the following formula (Luenberger, 2009):

$$
\begin{aligned}
(1+\text{CAGR})^T =& \text{ROCE}+1 \\
1+\text{CAGR} =& (\text{ROCE}+1)^{\frac{1}{T}} \\
\text{CAGR} =& (\text{ROCE}+1)^{\frac{1}{T}}-1
\end{aligned}
\tag{2.3}
$$

ROCE only measures the return as share of investment, so 1 is added to the right hand side of the equation. Omitting this 1 would lead to a ROCE of 1 giving a CAGR of 0%, which would not seem logical, considering that the returns have been positive. CAGR is used to define the return per year, if all returns are reinvested, for instance for money placed in a saving account. There is a difference between this type of investment and the investments considered in the MFRP; when investing by putting money in a bank account, the investment sum is guaranteed. When investing in a ship fleet, the investment is tied to the ship, and it cannot be expected to have the entire investment returned if one were to sell all the ships. Because of this, CAGR, as used in this report, can be considered the rate of capital employed returned per year. The ROCE measure can be said to measure how much of the capital employed that is returned over the planning horizon. It is important to realize that the CAGR presented in this report can not be compared directly to, for instance, the interest rate in a bank, or the CAGR of a stock investment.

Chapter 3

Modeling Uncertainty

As mentioned in Chapter 1, shipping companies solving the MFRP face a large degree of uncertainty over the planning horizon considered. In optimization, uncertainty is reflected in the parameters of the problem, which are influenced by random variables. Two examples of this are demand and cost of sailing. Demand will be influenced by the world economy, and the cost of sailing will be influenced by fuel prices, both the world economy and fuel prices being parameters which are uncertain. The set of possible outcomes of the random variables is referred to as a scenario tree. This chapter will give an introduction to how uncertainty can be modeled when solving the MFRP. Section 3.1 will give an introduction to stochastic programming which will be used for the models developed in Chapter 5. In Section 3.2, methods of building and evaluating scenario trees will be introduced.

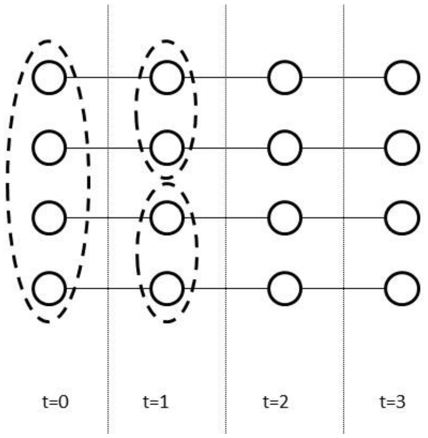
In operations research, there are different ways of approaching uncertainty. Two of the most commonly used approaches are stochastic programming and robust optimization. Stochastic programming, starting from a probability distribution of the random parameters, tries to describe the potential outcomes in the best possible way, using scenarios. Stochastic programming maximizes the expected objective value based on possible stochastic future outcomes of the random variable(s). Higle

(2005) gives a thorough introduction to stochastic programming. Robust optimization, on the other hand, can be said to consider uncertainty models that are not stochastic, but deterministic and set based. Bertsimas et al. (2011) give an introduction to robust optimization. Robust optimization ensures feasibility in all cases, which is specially important if there are not good recourse actions available. For this report, looking at the MFRP, stochastic programming is chosen because of the dynamics of the problem, and the possibility of recourse options, such as space charters.

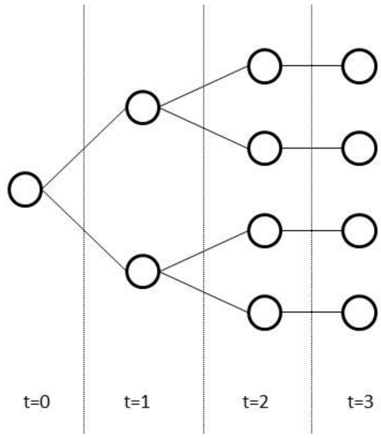
3.1 Stochastic programming

Solving problems without handling uncertainty means solving deterministic problems (DP). DP may be used because there are little, or no uncertainty involved in the problem faced. On the other hand, for problems such as the MFRP, which is considered to have a large degree of uncertainty, solving the problem as a DP usually means solving the expected value problem (EVP), which means that the expected value of the random variables are used. If the realization of the random variables is the same as the expected values, the EVP finds an optimal solution. The problem is that this solution has no guarantee as to how it will perform if the realization of the random variables differs from the expected value. Then the solution may behave very poorly, or even be infeasible.

Implementing the possible outcomes of the future is hard to do in a perfect way, as the realization often follows a continuous probability distribution. Think of future demand for transportation of cars for the MFRP that is studied in this paper. That demand is continuous over a possible very large interval. To implement the continuous probability distribution, will mean that solving an integral in the objective function will have to be done. By using a discretization of the contribution, this is avoided. The discretization is modeled by scenarios, and a higher number of scenar-



(a) Scenario representation



(b) Node representation

Figure 3.1: Representation of scenarios trees

ios will possibly give a more realistic approach to the problem, but will also increase the complexity of the problem.

The discretization of the uncertain variables is represented through a scenario tree, where the path of each leaf node correspond to a given realization of the random variables throughout the planning horizon. This scenario tree can be represented either with scenarios as in Figure 3.1a or with nodes as in Figure 3.1b. These pictures illustrate a scenario tree where one random variable can have a high or low development from the first time period to the second, and from the second time period to the third. This gives in total four scenarios. The scenario representation means that each possible leaf node has its own path through the planning horizon.

The examples shown in Figure 3.1 are called three-stage scenario trees. A stage is a time period where information is revealed or where a decision has to be made. The first stage here is in $t = 0$, where all scenarios have the same information about the random variables, which is the possible outcomes and their discrete distribution. The second stage is $t = 1$, where the realization of the random variables for $t = 1$ is known (high or low), the third stage is $t \geq 2$, where the realization of the random variables for $t = 2$ and onwards is revealed. The dotted circles in $t = 0$ and $t = 1$ in Figure

3.1a represents non-anticipativity constraints, where decisions made in scenarios in the same circle has to be equal at this time, since at this stage, all information about the future realization of the random variables is the same for these two scenarios. Both scenarios have experience a high development of the random variable from the first to second stage, but the future outcome is uncertain for both scenarios.

In stochastic programming, the term "recourse" is often used. Higle (2005) define recourse as the opportunity to adapt a solution to a specific outcome. In a two stage model, often some decisions has to be made about resources in the first stage, before demand is known in the second stage. This is the case for the maritime fleet renewal problem (MFRP), where investment and scrapping decisions has to be made in the first stage, before, for instance, demand and prices are known. Recourse actions in this case are, amongst other, the possibility to put ships on lay up in case of overcapacity in the fleet and use space charters or charter in ships in cases of undercapacity in the fleet.

3.2 Building and evaluating scenario trees

Kaut and Wallace (2003) present several methods of generating a scenario tree, dependent on the properties of the problem and available knowledge about the true distribution. These are conditional sampling, sampling from specified random variables and correlations, and moment matching, which will be briefly introduced here, as well as path-based methods and optimal discretization. Conditional sampling take samples in each node directly from the distribution, or by some explicit formula according to for instance the current stage in the scenario tree. Conditional sampling usually samples from each random variable, and combines them all-by-all, meaning that the scenario tree grows exponentially with each random variable. If s scenarios are being sampled for k random variables, the total number of scenarios ends up being s^k. Conditional sampling cannot handle correlations between the random variables. It is also possible to sample from specified random variables and correlations. In that case, the distribution of each random variable and the correlation between the random variables is known. If the distribution is not known, it is possible to sample using moments of the distribution for the random variables (mean, variance, skewness, kurtosis etc.) instead.

Once a method for building the scenario tree has been chosen and scenario trees are generated, it is possible to measure the quality of the scenario trees. Kaut and Wallace (2003) state that the quality of the scenario tree does not depend on the approximation of the distribution, but the quality of the decision it leads to. So instead of looking at the scenario tree, one considers the error of the approximation, which is defined as the difference between the value of the true objective function at the solutions of the true and approximated problem. From Pflug (2001) a definition of this error is given:

$$\begin{aligned} e_f(\tilde{\xi}_t, \breve{\xi}_t) &= F(\operatorname*{argmin}_x F(x; \breve{\xi}_t); \tilde{\xi}_t) - F(\operatorname*{argmin}_x F(x; \tilde{\xi}_t); \tilde{\xi}_t) \\ &= F(\operatorname*{argmin}_x F(x; \breve{\xi}_t); \tilde{\xi}_t) - \min_x F(x; \tilde{\xi}_t) \end{aligned} \tag{3.1}$$

Here, $\breve{\xi}_t$ is the "true" scenario tree, while $\tilde{\xi}_t$ is a generated scenario tree. The formulation in Equation (3.1) has one obvious problem; for most practical problems it is impossible to calculate. Kaut and Wallace (2003) decides to evaluate a scenario generation method, instead of trying to find the optimal scenario tree. They state that one would like to know that the scenario tree has stability, and no bias. The latter property is hard to test, because it involves solving the problem for the true (continuous) scenario tree, and if one could do that, there would be no need to generate other scenario trees. Stability is given both as in-sample stability and out-of-sample stability, which both desirable. In-sample stability is given by comparing the optimal solutions from using different scenario trees. With K scenario trees, it is defined as

$$F(x_k^*; \breve{\xi}_{tk}) \approx F(x_l^*; \breve{\xi}_{tl}) \qquad k, l \in 1...K \tag{3.2}$$

Out-of-sample stability is given by comparing the solutions from using different scenario trees. These solutions are then tested against the "true" scenario tree, referred to as $\tilde{\xi}$. With solutions from K scenario trees, it is defined as

$$F(x_k^*; \tilde{\xi}_t) \approx F(x_l^*; \tilde{\xi}_t) \qquad k, l \in 1...K \tag{3.3}$$

In-sample stability is far easier to test for than out-of-sample, since out-of-sample stability demands that one knows the true scenario tree, which may not be the case, and if it is known, may be practically impossible to test because of the complexity of the problem.

If the true distribution of the random variables is not known (which is often the case, since the future for most cases is impossible to predict), the quality of the scenario tree may be impossible to evaluate in a good way. Pantuso (2014) examines which properties of the uncertainty is important in stochastic programming. Pantuso (2014) finds that the correlations between the random variables and the shape of the distribution does not impact the solution to a large degree, so failing to describe these correctly does not necessarily have a large impact on the quality of the solutions from the scenario tree. The mean value of the variables on the other hands, will have larger impact on the solutions, if estimated incorrectly.

Chapter 4

Literature

This chapter provides an overview of literature relevant to the problem addressed in this report. This chapter will try to highlight how previous work done within the field of fleet size and mix problems (FSMP) relates to the work done in this report.

In Section 4.1, the methodology used for finding relevant literature will be presented. In Section 4.2, literature on FSMP will be presented. The maritime fleet renewal problem (MFRP) is a type of FSMP. To get a broader picture of research done within the field, a coverage of FSMP will be reviewed first, before Section 4.3 presents literature on the MFRP.

4.1 Methodology

Two survey papers have been used as sources for this review. Hoff et al. (2010) give a comprehensive picture of relevant literature for fleet composition and routing. They have a general focus on the problem, including road-based and maritime problems. The survey by Pantuso et al. (2013) focuses on papers about the maritime fleet size and mix problem (MFMSP), including the maritime fleet renewal problem (MFRP).

In addition to the survey papers by (Hoff et al., 2010) and (Pantuso et al., 2013), Scopus search has been used as an extra source to provide exhaustive coverage of all relevant literature. Scopus (Scopus) is considered one the largest and best bibliographic databases available.

For the fleet size and mix and fleet renewal problem, four Scopus searches were performed. The first search phrase required the words ”maritime”, ”fleet”, ”size” and ”mix” to be included in the title, abstract or keywords. This gave six results, whereof two were the survey papers by Pantuso et al. (2013) and Hoff et al. (2010), and two were other survey papers referenced in the former two. The final two papers were without relevance for the problem in this paper. Omitting the word ”maritime” gives 94 results, one of them being a literature survey on optimization in container liner shipping by Tran and Haasis (2013). This survey does conclude that there has been little interest in ship investments from the optimization field. The most recent result for this search is the survey paper by Pantuso et al. (2013). Substituting the word ”maritime” with ”ship” or ”shipping” gives five and seven results respectively.

For the fleet renewal problem, four Scopus searches were performed. The first search phrase required the words ”fleet” and ”renewal” to be included in the title, abstract or keywords. This gave 220 results. Including the word ”maritime” gives 10 results. Substituting the word ”maritime” with ”ship” or ”shipping” gives 48 and 17 results respectively.

The results from the Scopus searches were manually examined to check for relevance for this report. Most results could easily be omitted by reviewing the title and/or abstract, finding that they had no relevance for this study. The fact that Scopus search gave nearly no extra contribution compared to the literature presented in the surveys by Hoff et al. (2010) and Pantuso et al. (2013), show that there is not a large amount of literature regarding the MFSMP and MFRP available.

4.2 Fleet size and mix problems

Hoff et al. (2010) give a comprehensive review of relevant literature for fleet composition and routing. They have a general focus, including road-based and maritime problems. Hoff et al. (2010) find that most of the literature reviewed consider questions regarding the operational decisions about fleet composition and routing. They state that there is a general lack of literature regarding the tactical and strategic decisions. There might be several reasons as to why there is little work done on strategic and tactical problems regarding fleet composition and routing. One of them is the uncertainty involved when considering longer planning horizons, making a more high level approach to the routing problem suitable. Another is the fact that such problems may become far to complex to solve with the current solution methods. A third reason might be because of the misalignment between the models suggested and the objectives of the decision makers, as stated in Chapter 2.

The survey by Pantuso et al. (2013) focuses on papers about the maritime fleet size and mix problem (MFMSP). The survey finds that there is a general scarcity in the papers dealing with the MFSMP. In addition, very few papers consider an initial fleet to be renewed, and few papers explicitly treat uncertainty.

The papers reviewed by (Hoff et al., 2010) are categorized by method (exact or heuristic), problem type (fleet sizing, fleet composition, fleet composition and routing, or heterogenous fixed fleet routing) and modality (maritime, road-based or generic). For the purpose of this report, it is also interesting whether the papers consider the investment decisions or not.

The findings by Hoff et al. (2010) about most papers considering the operational decisions shows itself in the fact that most papers does not consider an initial fleet, or discuss the investments needed for the solutions found. Golden et al. (1984) is referred to as the first paper that reference the fleet size and mix vehicle routing problem. The paper consider the problem of finding the optimal fleet mix, and then solve the routing problem using this fleet as the available fleet. The paper does not consider an initial fleet, or expansion of the fleet, just finding the optimal fleet size and mix. The same findings are valid, for instance, for Sigurd et al. (2005). They solve a vehicle routing problem to estimate the best fleet for a ship scheduling problem. The model only considers the optimal fleet size and mix to minimize the cost of the routing problem, and does not explicitly consider the purchase price of a new ship.

Still, there are some papers that take a strategic perspective when solving the problem. Fagerholt et al. (2010) present a decision support methodology for strategic planning in maritime transportation. They use simulation to both generate scenarios and evaluate strategic decisions. The set of strategic decisions, such as which contracts to take and the fleet size and mix, are generated by experts before the model is run. The model maximizes profit, for each scenario and strategic decision, a short-term routing and scheduling problem is solved.

Another example of a paper that consider the strategic perspective is Pesenti (1995), which include a model for the organizational setup, in terms of a hierarchical model with cooperation between strategic, tactical and operational levels in the organization. The problem considers purchasing and use of container ships. The strategic level reports it decisions to the tactical level which then reports back to the strategic level with feedback on the decisions proposed from the strategic level. The model considers a fixed cost of buying and operating a ship. The strategic level objective function considers the cost and revenue from buying and selling ships, expected revenue from possible routes to service as well of cost of idle time and waiting time for customers (based on fleet utilization), maximizing the total profit.

4.3 Maritime fleet renewal problem

The survey paper by Pantuso et al. (2013) includes a section on the MFRP, where they find that very few of the papers reviewed consider the MFRP. All papers considering the MFRP, which explicitly handles the operating decisions, use some kind of higher level deployment model. The survey does not focus on how the objective function is constructed, most likely because all papers consider a cost minimizing or profit maximizing objective function. The papers that consider the MFRP have a strategic perspective by nature, and most of the papers that consider uncertainty explicitly looks at the MFRP. The reason for this is probably explained by the fact that uncertainty increases with the length of the planning horizon one considers. Papers considering strategic problems, such as the MFRP, will usually have a longer planning horizon than papers that considers operational or tactical problems.

Not all papers on the MFRP consider operational decisions at all. Jin and Kite-Powell (2000) present an analytical model for optimal fleet replacement and operations. The model does not take into consideration uncertainty or chartering, and does not consider the operational decisions. The objective function maximizes profit, and the model does not consider expansion of the fleet, only replacement.

Most papers have some way of modeling the deployment. Xinlian et al. (2000) present a dynamic model for fleet planning. In the model they first optimize deployment for different stages and fleet developments, before using these results to develop the optimal strategy for fleet development. They model the deployment between routes, much like the deployment model in this report, which will be presented in Chapter 5. The model minimizes cost of operational decisions and building up the fleet. Meng and Wang (2011) take a similar approach as Xinlian et al. (2000) as they introduce a scenario-based dynamic programming model. The model is intended for liner shipping, and solves the deployment problem as a MIP. The scenarios are generated by experts and describe different developments of the fleet and estimated demand. The operating decisions are taken on routes travelled, but where there is possibility of several pick up and delivery ports along a route. The model maximizes profit from operations.

Pantuso et al. (2014) present a model for the MFRP which handles uncertainty explicitly. The model has a cost minimizing objective function, with no optional demand or contracts. The operating decision is solved by deployment on trades, on which the model in this report to a large degree follows. The deployment formulation is similar to the one presented by Christiansen et al. (2013). Pantuso et al. (2014) finds that using stochastic programming to handle uncertainty will improve solutions when solving the MFRP.

Papers that handle uncertainty have different approaches as to how uncertainty is handled. Both Pantuso et al. (2014) and Meng and Wang (2011) use stochastic programming. Alvarez et al. (2011) on the other hand, present a robust optimization model which handle uncertainty explicitly. The model can deal with varying degrees of risk tolerance. The deployment is considered between markets for different commodities, much like the trades in this report. The model considers purchase price as a fixed up-front cost, and does not consider lead time for delivery of new ships. Alvarez et al. (2011) consider the investments needed in terms of a budget constraint which limit the total net amount that can be invested in the fleet during the planning horizon. However, they do not in any way discuss the solutions in terms of investment needed.

Chapter 5

Mathematical formulations

In this chapter four mathematical formulations of the MFRP will be presented. Modeling assumptions for the formulations will be discussed in Section 5.1. Then a model that maximizes profit will be presented with two formulations in section 5.2.1 and 5.2.2, one with a scenario representation and one with a node representation of the scenario tree, as discussed in Chapter 3. Then a model that maximizes ROCE will be presented with two formulations in section 5.3.1 and 5.3.3, one scenario and one node formulation.

For the ROCEMax formulations, transformations of the variables will be needed, as well as linearizations of the transformations for the integer and binary variables. The transformations and linearizations will not be shown in full for all variables. All formulations presented in this chapter can be found in full in Appendix A.

5.1 Modeling assumptions

This section will cover some of the modeling assumptions, as well as some case specific characteristics, before the formulations are presented in Sections 5.2 and 5.3.

It is assumed that new orders and scrappings of ships must be integer variables. It could be argued that it is possible to order parts of a ship by selling a part of the ship in the open market or entering an agreement with another company. This possibility will not be considered in this report. For ships put on lay-up, chartered in or out, these can be fractional. A fractional value of the variable will indicate that a ships was only put on lay-up or chartered in or out for parts of the period.

The model assumes that there will be demand for three types of cargo: Cars, HH and BB. Demand and capacity is given in the unit RT43, which is a standard measurement unit for cars cargo. One RT43 is equal to 9.1 cubic metres. HH is cargo that needs special placement because of height and/or weight, while BB is other cargo not compatible with standard cargo sizes. The ship has different capacities for each type of cargo, all given in RT43 units. The largest capacity of all cargo types is also considered the total capacity for the ship, normally cars. An assumption is made that the capacities fill up individually, up to the capacity for each type and the total capacity of a ship. This is not completely accurate, as there is usually is an extra restriction for the capacities of HHs and BBs. Implementing this would require a load variable for cargo. Since the demand for cars is by far the highest of the three cargo types, this restriction is not expected to have large impact on the solutions to the problem. Therefore, for sake of computational effectiveness, this restriction is omitted. Table 5.1 shows some examples of valid and invalid combinations of cargo load for an example ship type. Example 1 is valid, since the capacity for all cargo types is not exceeded, and the total cargo load is not above the total capacity for the ship. Example 2 is invalid, even though all capacity restriction for the different cargo types is not exceeded, the total load is above the total capacity of the ship. Example 3 is invalid, since the load for BB is above the BB capacity of the ship. The total capacity of the ship is respected, but that is not sufficient for the configuration to be valid.

This report will use a high level version to model the deployment decisions, very

	Cars	HH	BB	Valid/invalid
Capacities	2000	1000	500	
Example 1	1000	500	500	Valid
Example 2	1500	500	500	Invalid
Example 3	500	500	1000	Invalid

Table 5.1: Examples of valid and invalid configurations of cargo load

similar to the one presented in Pantuso et al. (2014). The deployment is considered by looking at transportation on trades. A trade consists of two geographical regions; origin and destination. Both origin and destination usually consist of several ports. The demand for cargo to be picked up at each origin port is aggregated into a total demand for the trade, and is delivered to the destination ports of the trade. An example of such a trade can be between Asia and North-America, as illustrated in Figure 5.1, where there are several origin ports in Asia and several destination ports in North-America. This is modeled as one trade with one demand.

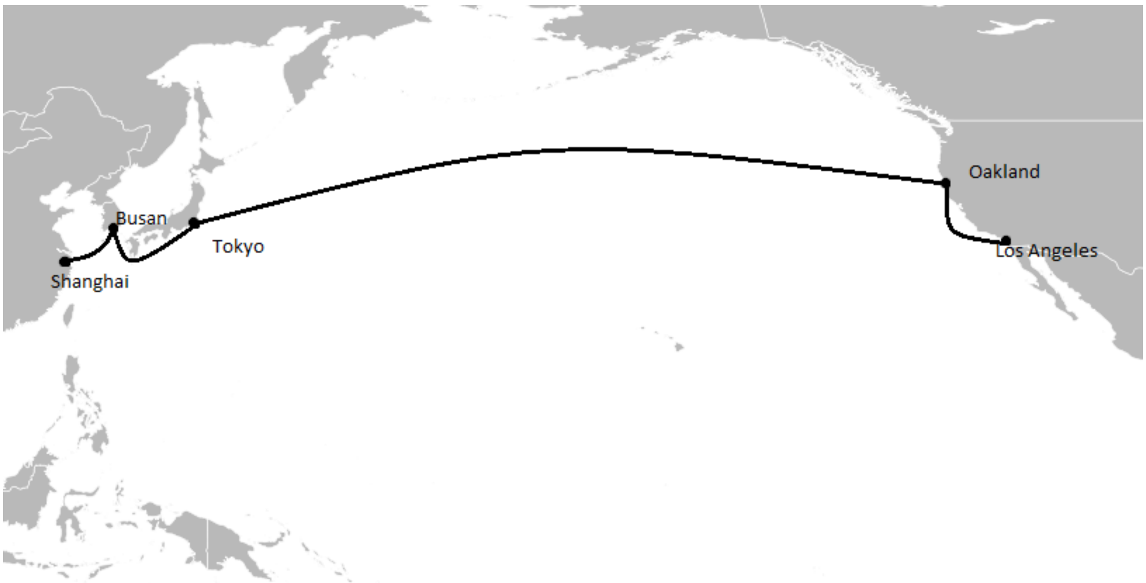


Figure 5.1: An example trade between Asia and North-America.

When modeling deployment on trades, each trade is considered a node. In a trade, all origin ports are aggregated into a single origin, and all destination ports are aggregated into a single destination, so that a visit to a trade node can be considered traveling from the origin to the destination carrying cargo, and sailing back to the destination in ballast. There might be some demand for cargo to be picked up in the destination ports to be transported on another trade. So instead of having to sail in ballast back to the origin and in ballast again to the destination, this time serving as the origin on the new trade, this is referred to as sailing on *loops*. A loop consists of serving one or several trades after another before sailing back in ballast to the first trade. Figure 5.2 shows two example loops. The number of loops performed during a given period can take fractional values. A fractional value of the number of loops will indicate that one or more loops may have started in the previous period, or will finish in the following period. When generating loops a maximum cardinality of two has been used, meaning that loops consisting of up to two trades will be generated. Pantuso et al. (2014) showed that when solving the MFRP using this deployment model, larger cardinalities does not improve the solution much, but give large increases to the solution time.

A shipping company usually have a set of contractual agreements for shipments on trades. The trades will be separated into two sets; one set consisting of the trades for which the shipping company have contractual agreements for demand for the entire planning horizon, referenced as contractual trades. The other sets will consist of trades where there are no contractual agreements at the start of the planning horizon, referenced as optional trades. The shipping company may choose to service these trades during the planning horizon. If the shipping company choose to service an optional trade in a period, it must continue to do so for the rest of the planning horizon.

When ships sail a loop, it services all trades on that loop, sailing in ballast between the trades on the loops. In the model, ballast sailing between loops is not included. This in an optimistic assumption about the total sailing. At the same time, ballast

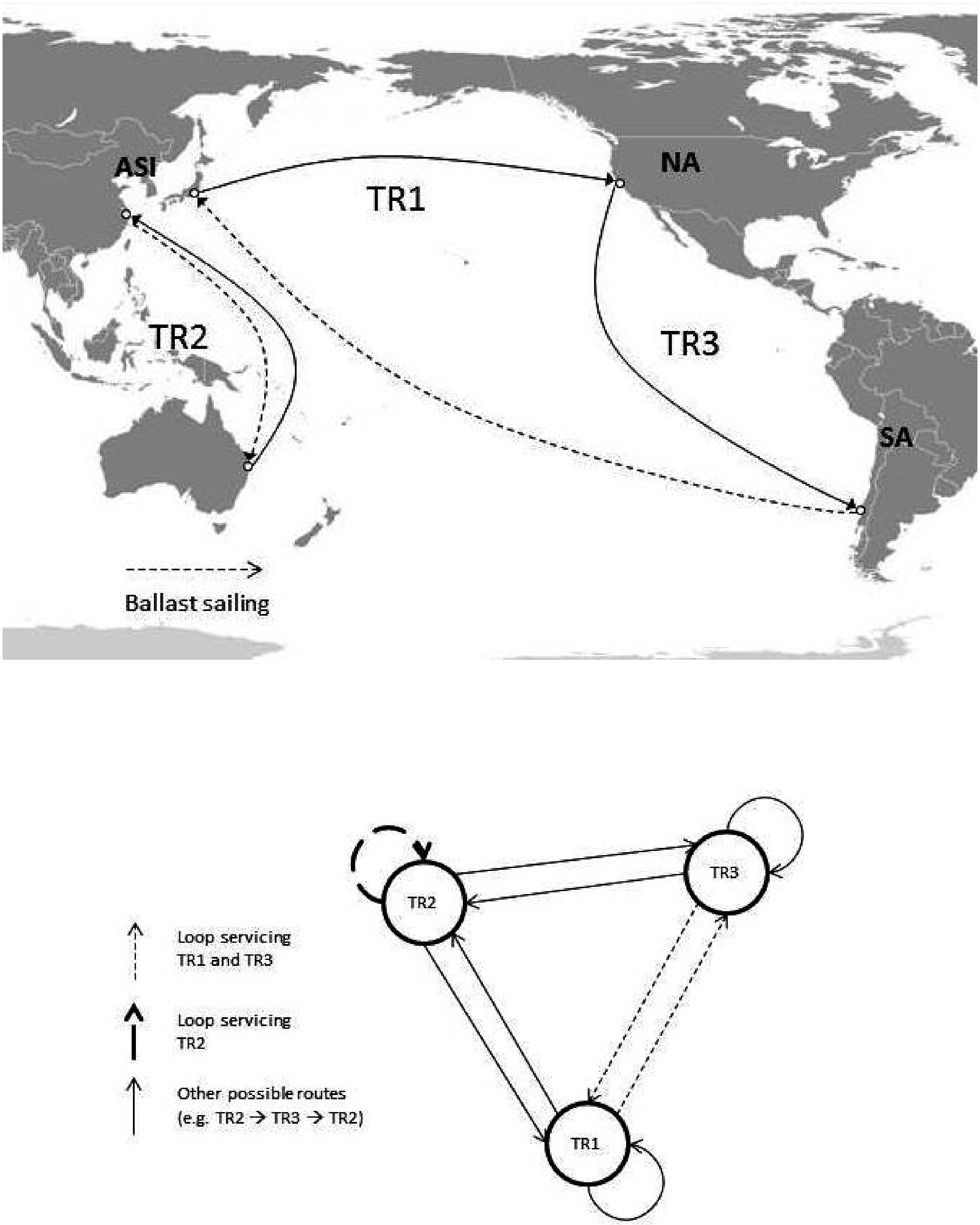


Figure 5.2: Two examples of loops, represented both on the map and as a graph.

sailing from the destination port of the last trade to the origin port of the first trade on a loop is included. If the next loop starts in a region other than the the origin of the first trade on the former loop, this sailing is not likely not be included in actual operating decisions. Thus a pessimistic assumption of the total sailing is made. The optimistic and pessimistic assumptions about total sailing are assumed to balance each other in a sufficient way for this model. A trade may include frequency requirements, meaning that the trade must be serviced at least a certain number of times during a time period.

The cost of buying a new ship is affected by the way the company chooses to finance the ship. The financing decision when buying a ship is not considered in this report, and all costs are considered in an up-front payment at the time the ship is ordered, whether this is the cash sum of the ship or the net present value of future repayments and interest.

The fleet consists of ships of different types, varying in capacities, speed and age. In each period, only two types of ships will be available to build.

It is assumed that the ships have a set lifetime, and that they will leave the fleet when the maximum age is reached. No considerations as to the possibility of extending the lifetime of a ship is taken.

5.2 ProfitMax formulations

5.2.1 ProfitMax scenario formulation

Let $T = \{0, ..., \bar{T}\}$ be the set of periods, indexed by t. $\bar{T}$ will then be the final period of the planning horizon. T^L is the lead time from order to delivery of a new ship. When ordering new ships, these can be delivered in period $t + T^L$ if ordered in period t. New ships are considered to enter the fleet at the start of a year, meaning that if a ships is ordered in time period $t = 0$, it is delivered at the start of period $t = T^L$. This means that new ships can only be ordered in periods $t \leq \bar{T} - T^L$. For period 0, deployment considerations are not taken into account.

S is the set of scenarios, indexed by s, each scenario has probability P_s of occurring, where $\sum_{s \in S} P_s = 1$. The set S^{NA}_{ts} consists of all scenarios that are connected to scenario s in time t, meaning that decisions made in scenario s in time t must be the same in all scenarios in S^{NA}_{ts}.

Let V_t be the set of ships available in the market in period t, indexed by v. The set of available ships that can be delivered in a period is denoted V^N_t. This means that ships that are part of V^N_t must be ordered in period $t - T^L$. In periods $t < T^L$ only ships that were ordered before the start of the planning horizon can be delivered. These are given by the parameter Y^{NB}_{vt}, which is the number of ships of type v ordered in the sunk period, delivered at the start of period t. A_{vt} is the age of a ship of type v in period t, and $\bar{A}$ the lifetime of ships. Ships are part of V_t as long as $0 \leq A_{vt} \leq \bar{A}$.

N_t is the set of all trades that the shipping company may operate in period t, indexed by i. The subset N^C_t of N_t is the set of contractual trades, which the shipping company have contractual obligations to service from the start of the planning horizon. N^O_t is then the set of optional trades. Let R_t be the set of all loops, indexed by r, with $R_{vt} \subset R_t$ being the loops on which a ship of type v can sail in period t. The subset R_{ivt} consists of all loops that can be sailed by ship v in period t that services trade i. G is the set of all cargo types, indexed by c.

Let R^D_{it} be the revenue made from transporting one unit of goods on trade i in period t, where D_{itcs} is the demand on trade i in period t for cargo c in scenario s. C^{SP}_{it} is the cost of transporting one unit of goods by space charter. C^{OP}_{vt} is the operating cost for a ship, the fixed costs for having a ship in the pool for a period. Let C^{CO}_{vts} be the cost of chartering in one ship of type v for period t in scenario s and R^{CO}_{vts} the revenue made from chartering out one ship of type v for period t in scenario s.

C_{rvt}^{TR} is the cost of sailing one loop of type r. R_{vt}^{LU} is the savings made from putting a ship on lay-up. The potential future profit of ships in the fleet at the end of the planning horizon is modeled by the sunset value of the ships. Let R_{vs}^{SV} be the sunset value of ships of type v in scenario s. This value is calculated by taking the price of a ship in the second hand market in the final period of the planning horizon, as this should incorporate the potential future profit from a ship. C_{vt}^{NB} is the cost of a new ship of type v ordered in period t and R_{vt}^{SC} is the revenue made when scrapping a ship. Let R_{vts}^{SE} be the revenue made from selling one ship of type v in the second hand market in time t and scenario s, while C_{vts}^{SH} is the cost of buying one ship of type v in the second hand market in time t and scenario s.

Q_{cv} is the capacity of cargo type c for a ship of type v. Let F_{it} be the frequency requirement on trade i in period t. Z_{rv} is the time it takes for a ship on type v to perform one loop r, and Z_v is the total available time for a ship of type v in a period. The limits on the maximum number of ships to sell or buy in the second hand market of a ship v in time t and scenario s is given by $\overline{SE}_{vts}$ and $\overline{SH}_{vts}$, while the limits on available ships of type v in time t and scenario s for chartering in and demand for chartering out is given by $\overline{CI}_{vts}$ and $\overline{CO}_{vts}$. $\overline{SE}_{ts}$, $\overline{SH}_{ts}$, $\overline{CI}_{ts}$ and $\overline{CO}_{ts}$ gives limit on the total number of ships to buy or sell in the second hand market and charter in or out, in each period t.

x_{vrt} is the variable stating how many loops of type r is performed by ships of type v in period t. y_{vt}^{P} is the pool variable stating how many ships are available in period t of type v. The pool variable is initially set in period 0 by the parameter Y_v^{IP} which gives the initial fleet. In later periods, adjustments to the fleet are made by the variables y_{vts}^{SC}, y_{vts}^{NB}, y_{vts}^{SE}, y_{vts}^{SH}, which are the scrappings, new ships built, and ships sold and bought in the second hand market, of type v in period t under scenario s. Ships can not be traded in the second hand market the same period as it can be delivered as a new build, but will be available from the period after. Ships that are scrapped in period t will not be available from the following period. The variables

l_{vt} says how many ships are put on lay-up. h^I_{vts} and h^O_{vts} are variables for the number of ships of type v to be chartered in and out respectively, in period t and scenario s. n_{its} is the variable for units of goods transported by space charter on trade i in time t and scenario s. Let δ_{its} be a binary variable set to 1 if the company services optional trade i in period t and scenario s.

Objective function

$$
\begin{aligned}
max z = & \sum_{s \in S} P_s \Bigg[\sum_{t \in T, t>0} \Big(\sum_{i \in N_t^O} \sum_{c \in G} R^D_{its} D_{itcs} \delta_{its} \\
& + \sum_{i \in N_t^C} \sum_{c \in G} (R^D_{its} D_{icts} - C^{SP}_{icts} n_{icts}) - \sum_{v \in V_t} (C^{OP}_{vts} y^P_{vts} \\
& + C^{CO}_{vts} h^I_{vts} - R^{CO}_{vts} h^O_{vt} \\
& + \sum_{r \in R_{vts}} C^{TR}_{vrts} x_{vrts} - R^{LU}_{vts} l_{vts}) \Big) \\
& - \sum_{t \in T, t \leq \bar{T} - T^L} \sum_{v \in V^N_{t+T^L}} C^{NB}_{vts} y^{NB}_{vts} \\
& + \sum_{t \in T} \sum_{v \in V_t} (R^{SC}_{vts} y^{SC}_{vts} \\
& + R^{SE}_{vts} y^{SE}_{vts} - C^{SH}_{vts} y^{SH}_{vts}) \\
& + \sum_{v \in V_{\bar{T}}} R^{SV}_{vs} y^P_{v\bar{T}s} \Bigg]
\end{aligned}
\tag{5.1}
$$

The objective function measure the operational profit, P^O, as described in Chapter 2, which is the numerator in the ROCE measure. The objective function first describe the revenue and cost from all deployment decisions from $t = 1$. The the cost and revenue made from decisions about obtaining or disposing of ships, before the sunset value is included last. The problem is subject to constraints (5.2)-(5.36):

Demand constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} x_{vrts} + n_{icts} \geq D_{itcs}, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, s \in S, \tag{5.2}$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} x_{vrts} \geq D_{itcs} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, s \in S, \tag{5.3}$$

Constraints (5.2) and (5.3) make sure that the demand on each trade for each cargo type is satisfied.

Capacity constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinG} Q_{cv} x_{vrts} + \sum_{c \in G} n_{cits} \geq \sum_{c \in G} D_{itcs}, \qquad t \in T \setminus \{0\}, i \in N_t^C, s \in S, \tag{5.4}$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinG} Q_{cv} x_{vrts} \geq \sum_{c \in G} D_{itcs} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S, \tag{5.5}$$

Constraints (5.4) and (5.5) make sure that the individual capacities for each ship, and the total capacity of the ships are not violated.

Frequency constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} x_{vrts} \geq F_{it}, \qquad t \in T \setminus \{0\}, i \in N_t^C, s \in S, \tag{5.6}$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} x_{vrts} \geq F_{it} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S, \tag{5.7}$$

Constraints (5.6) and (5.7) make sure that where a trade has frequency requirements, these are fulfilled.

Time constraints

$$\sum_{r\in R_v t} Z_{rv}x_{vrts} \leq Z_v(y^P_{vts} + h^I_{vt} - h^O_{vt} - l_{vts}), \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \qquad (5.8)$$

Constraints (5.8) make sure that the total sailing time of ships of type v does not exceed the available time in total for a ship of type v.

Optional trades constraints

$$\delta_{its} \leq \delta_{i,t+1,s}, \qquad t \in T \setminus \{0, \bar{T}\}, i \in N^O_t, s \in S, \qquad (5.9)$$

Constraints (5.9) ensures that when the company starts to service an optional trade, it must do so for the rest of the planning horizon.

Pool constraints

$$y^P_{vts} = y^P_{v,t-1,s} - y^{SC}_{v,t-1,s} + y^{SH}_{v,t-1,s} - y^{SE}_{v,t-1,s}, \qquad t \in T \setminus \{0\}, v \in V_t \setminus V^N_t, s \in S, \qquad (5.10)$$

$$y^P_{vts} = Y^{NB}_{vt}, \qquad t \in T : t < T^L, v \in V^N_t, s \in S, \qquad (5.11)$$

$$y^P_{vts} = y^{NB}_{v,t-T^L,s}, \qquad t \in T : t \geq T^L, v \in V^N_t, s \in S, \qquad (5.12)$$

$$y^P_{v0s} = Y^{IP}_v, \qquad v \in V_0, s \in S, \qquad (5.13)$$

$$y^P_{vts} = y^{SC}_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t \setminus V_{t+1}, s \in S, \qquad (5.14)$$

$$l_{vts} - h^I_{vts} + h^O_{vts} \leq y^P_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \qquad (5.15)$$

Constraints (5.10)-(5.14) take care of ships leaving and joining the pool. Constraints (5.10) represent the pool balance from year to year. Constraints (5.11) refer to ships delivered that was ordered in periods before the planning horizon. Constraints (5.12) refer to delivery of new orders of ships. Constraints (5.13) define the initial pool.

Constraints (5.14) make sure that ships that reach their lifetime are scrapped. Constraints (5.15) make sure that the number of ships on lay-up minus balance of ships the chartered in and out does not exceed the total number of available ships.

Charters and second hand market constraints

$$y^{SH}_{vts} \leq \overline{SH}_{vts}, \qquad t \in T, v \in V_t, s \in S, \tag{5.16}$$

$$y^{SE}_{vts} \leq \overline{SE}_{vts}, \qquad t \in T, v \in V_t, s \in S, \tag{5.17}$$

$$h^{I}_{vts} \leq \overline{CI}_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \tag{5.18}$$

$$h^{O}_{vts} \leq \overline{CO}_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \tag{5.19}$$

$$\sum_{v \in V_t \setminus V_t^N} y^{SH}_{vts} \leq \overline{SH}_{ts}, \qquad t \in T, s \in S, \tag{5.20}$$

$$\sum_{v \in V_t \setminus V_t^N} y^{SE}_{vts} \leq \overline{SE}_{ts}, \qquad t \in T, s \in S, \tag{5.21}$$

$$\sum_{v \in V_t \setminus V_t^N} h^{I}_{vts} \leq \overline{CI}_{ts}, \qquad t \in T \setminus \{0\}, s \in S, \tag{5.22}$$

$$\sum_{v \in V_t \setminus V_t^N} h^{O}_{vts} \leq \overline{CO}_{ts}, \qquad t \in T \setminus \{0\}, s \in S, \tag{5.23}$$

Constraints (5.16) and (5.18) make sure that ships chartered in and ships bought in the second hand market does node exceed the maximum number of available ships in the market of ship type v in time period t, and constraints (5.17) and(5.19) make sure that ships chartered out and ships sold in the second hand market does node exceed the maximum demand for ship type v in time period p in the market. Constraint (5.20)-(5.23) limits the total number of ships in each time period that can be bought or sold in the second hand market, or chartered in or out.

Non-anticipativity constraints

$$y^{SC}_{vts} = y^{SC}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, \tag{5.24}$$

$$y^{NB}_{vts} = y^{NB}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, \tag{5.25}$$

$$y^{SH}_{vts} = y^{SH}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, \tag{5.26}$$

$$y^{SE}_{vts} = y^{SE}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, \tag{5.27}$$

Constraints (5.24) and (5.27) ensure non-anticipativity.

Convexity and integer constraints

$$y^{NB}_{vts} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S, \tag{5.28}$$

$$y^{SC}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}, v \in V_t, s \in S, \tag{5.29}$$

$$y^{SH}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}, v \in V_t, s \in S, \tag{5.30}$$

$$y^{SE}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}, v \in V_t, s \in S, \tag{5.31}$$

$$y^{P}_{vts} \in \mathbb{R}^+, \qquad t \in T, v \in V_t, s \in S, \tag{5.32}$$

$$l_{vts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \tag{5.33}$$

$$h^{I}_{vts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \tag{5.34}$$

$$h^{O}_{vts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S, \tag{5.35}$$

$$x_{vrts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, s \in S, \tag{5.36}$$

$$\delta_{its} \in \{0, 1\}, \qquad t \in T \setminus \{0\}, i \in N^O_t, s \in S. \tag{5.37}$$

Constraints (5.28) and (5.29) impose non-negativity and integrality for new orders of ships and scrapping of ships. Constraints (5.32) - (5.37) restrict the related variables to real and non-negative values. Constraints (5.37) defines δits as a binary variable.

The ProfitMax Formulation can easily be transformed into a cost minimizing formulation by removing the possibility of the optional trades. The formulation will then solve the cost minimization problem for the setup of the fleet and trades to service at the start of the planning horizon.

5.2.2 ProfitMax node formulation

Instead of a set of scenarios, the node formulation for ProfitMax has a set N of nodes, indexed by n, where P_n is the probability of node n occurring. $n(t)$ gives all nodes n at time t in the scenario tree, and $a(n, \bar{t})$ is all ancestors of node n in the scenario tree in time $t - \bar{t}$, with $a(n, 1)$ written simply as $a(n)$. With these new sets and parameters, the ProfitMax formulation is exactly the same as the scenario formulation, except that the combination of t and s has been replaced with n, and there is no need for the non-anticipativity constraints (5.24) and (5.25). The full formulation will then be

Objective function

$$\begin{aligned} maxz = &\sum_{t \in T \setminus \{0\}} \sum_{n \in n(t)} P_n \Bigg[\sum_{i \in N_t^O} \sum_{c \in G} R_{in}^D D_{icn} \delta_{in} \\ &+ \sum_{i \in N_t^C} \sum_{c \in G} (R_{in}^D D_{icn} - C_{icn}^{SP} n_{icn}) - \sum_{v \in V_t} (C_{vn}^{OP} y_{vn}^P \\ &+ C_{vn}^{CO} h_{vn}^I - R_{vn}^{CO} h_{vn}^O \\ &+ \sum_{r \in R_{vn}} C_{vrn}^{TR} x_{vrn} - R_{vn}^{LU} l_{vn}) \Bigg] \\ &- \sum_{t \in T, t \leq \bar{T} - T^L} \sum_{v \in V_{t+T^L}^N} \sum_{n \in n(t)} P_n C_{vn}^{NB} y_{vn}^{NB} \\ &+ \sum_{t \in T} \sum_{v \in V_t} \sum_{n \in n(t)} P_n (R_{vn}^{SC} y_{vn}^{SC} \\ &+ R_{vn}^{SH} y_{vn}^{SE} - C_{vn}^{SH} y_{vn}^{SH}) \\ &+ \sum_{v \in V_{\bar{T}}} \sum_{n \in n(\bar{T})} P_n R_{vn}^{SV} y_{vn}^P \end{aligned} \tag{5.38}$$

The objective function (5.38) is the same as (5.1), with the changes in indexes as mentioned.

Demand constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} Q_{cv}x_{vrn} + n_{icn} \geq D_{icn}, \qquad t \in T\setminus\{0\}, i \in N_t^C, c \in G, n \in n(t), \quad (5.39)$$

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} Q_{cv}x_{vrn} \geq D_{icn}\delta_{in}, \qquad t \in T\setminus\{0\}, i \in N_t^O, c \in G, n \in n(t), \quad (5.40)$$

Capacity constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \max_{c\in G} Q_{cv}x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{itn}, \qquad t \in T\setminus\{0\}, i \in N_t^C, n \in n(t), \quad (5.41)$$

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \max_{c\in G} Q_{cv}x_{vrn} \geq \sum_{c\in G} D_{icn}\delta_{in}, \qquad t \in T\setminus\{0\}, i \in N_t^O, n \in n(t), \quad (5.42)$$

Constraints (5.39) - (5.42) ensures that demand is fulfilled, and that capacity restrictions of the fleet is not restricted, corresponding to constraints (5.2)-(5.5) in the scenario formulation.

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} x_{vrn} \geq F_{it}, \qquad t \in T\setminus\{0\}, i \in N_t^C, n \in n(t), \quad (5.43)$$

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} x_{vrn} \geq F_{it}\delta_{in}, \qquad t \in T\setminus\{0\}, i \in N_t^O, n \in n(t), \quad (5.44)$$

Time constraints

$$\sum_{r\in R_{vt}} Z_{rv}x_{vrn} \leq Z_v(y_{vn}^P + h_{vn}^I - h_{vn}^O - l_{vn}), \qquad t \in T\setminus\{0\}, v \in V_t, n \in n(t), \quad (5.45)$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{in}, \qquad t \in T \setminus \{0,1\}, i \in N_t^O, n \in n(t), \tag{5.46}$$

Constraints (5.43) - (5.46) take care of frequency requirements on the trades, the available sailing time for the fleet, and that when service on optional trades starts, it must continue to service this trade for the rest of the planning horizon, corresponding to constraints (5.6)-(5.9) in the scenario formulation.

Pool constraints

$$y_{vn}^P = y_{v,a(n)}^P - y_{v,a(n)}^{SC} + y_{v,a(n)}^{SH} - y_{v,a(n)}^{SE}, \qquad t \in T \setminus \{0\}, v \in V_t \setminus V_t^N, n \in n(t), \tag{5.47}$$

$$y_{vn}^P = Y_{vt}^{NB}, \qquad t \in T : t < \bar{T}^L, v \in V_t^N, n \in n(t), \tag{5.48}$$

$$y_{vn}^P = y_{v,a(n,T^L)}^{NB}, \qquad t \in T : t \geq \bar{T}^L, v \in V_t^N, n \in n(t), \tag{5.49}$$

$$y_{v0}^P = Y_v^{IP}, \qquad v \in V_0, \tag{5.50}$$

$$y_{vn}^P = y_{v,a(n)}^{SC}, \qquad t \in T \setminus 0, v \in V_t \setminus V_{t+1}, n \in n(t), \tag{5.51}$$

$$l_{vn} - h_{vn}^I + h_{vn}^O \leq y_{vn}^P, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.52}$$

Constraints (5.47)-(5.52) take care of the pool balances, corresponding to constraints (5.10)-(5.15) in the scenario formulation.

Charters and second hand market constraints

$$y_{vn}^{SH} \leq \overline{SH}_{vn}, \qquad t \in T, v \in V_t, \tag{5.53}$$

$$y_{vn}^{SE} \leq \overline{SE}_{vn}, \qquad t \in T, v \in V_t, n \in n(t), \tag{5.54}$$

$$h_{vn}^I \leq \overline{CI}_{vn}, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.55}$$

$$h_{vn}^O \leq \overline{CO}_{vn}, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.56}$$

$$\sum_{v \in V_t \setminus V_t^N} y_{vn}^{SH} \leq \overline{SH}_n, \qquad t \in T, n \in n(t), \tag{5.57}$$

$$\sum_{v \in V_t \setminus V_t^N} y_{vn}^{SE} \leq \overline{SE}_n, \qquad t \in T, n \in n(t), \tag{5.58}$$

$$\sum_{v \in V_t \setminus V_t^N} h_{vn}^{I} \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, n \in n(t), \tag{5.59}$$

$$\sum_{v \in V_t \setminus V_t^N} h_{vn}^{O} \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, n \in n(t), \tag{5.60}$$

Constraints (5.53)-(5.60) sets limits to the maximum number of ships that can be bought or sold in the second hand market, and chartered in and out in each period, corresponding to constraints (5.16)-(5.23) in the scenario formulation.

Convexity and integer constraints

$$y_{vn}^{NB} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V_{t+T^L}^N, n \in n(t), \tag{5.61}$$

$$y_{vn}^{SC} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.62}$$

$$y_{vn}^{SE} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.63}$$

$$y_{vn}^{SH} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.64}$$

$$y_{vn}^{P} \in \mathbb{R}^+, \qquad t \in T\, v \in V_t, n \in n(t), \tag{5.65}$$

$$l_{vn} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.66}$$

$$h_{vn}^{I} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.67}$$

$$h_{vn}^{O} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.68}$$

$$x_{vrn} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t), \tag{5.69}$$

$$\delta_{in} \in \{0, 1\}, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t). \tag{5.70}$$

Constraints (5.61)-(5.70) limits the variable to either integer, binary or positive real numbers, corresponding to constraints (5.28)-(5.37) in the scenario formulation. As mentioned, there in no need for the non anticipativity constraints (5.24)-(5.27) with the node formulation.

5.3 ROCEMax formulations

5.3.1 ROCEMax scenario formulation

ROCE was defined as Operational Profit over Capital Employed in Chapter 2. The Profitmax formulation from the previous section measured Operational Profit. To be able to maximize ROCE, Capital Employed must be included in the objective function (5.1), by changing it to:

$$\begin{aligned} maxz = &\sum_{s\in S} P_s \Bigg[\Big(\sum_{t\in T, t>0} \Big(\sum_{i\in N_t^O} \sum_{c\in G} R_{its}^D D_{itcs} \delta_{its} \\ &+ \sum_{i\in N_t^C} \sum_{c\in G} (R_{its}^D D_{icts} - C_{its}^{SP} n_{icts}) - \sum_{v\in V_t} (C_{vts}^{OP} y_{vts}^P \\ &- C_{vts}^{CO} h_{vts}^I - R_{vts}^{CO} h_{vt}^O \\ &+ \sum_{r\in R_{vts}} C_{vrts}^{TR} x_{vrts} - R_{vts}^{LU} l_{vts}) \Big) \\ &- \sum_{t\in T, t\le \bar{T}-T^L} \sum_{v\in V_{t+T^L}^N} C_{vts}^{NB} y_{vts}^{NB} \\ &+ \sum_{t\in T} \sum_{v\in V_t} (R_{vts}^{SC} y_{vts}^{SC} \\ &+ R_{vts}^{SE} y_{vts}^{SE} - C_{vts}^{SH} y_{vts}^{SH}) \\ &+ \sum_{v\in V_{\bar{T}}} R_{vs}^{SV} y_{v\bar{T}s}^P \Big) / \Big(\sum_{t\in T} C_{ts}^E / (\bar{T}+1) \Big) \Bigg] \end{aligned} \tag{5.71}$$

Where C_{ts}^E is defined as

$$C_{ts}^E = \beta C_{t-1,s}^E + C_{vt}^{NB} y_{vts}^{NB} - R_{vts}^{SC} y_{vts}^{SC} + C_{vts}^{SH} y_{vts}^{SH} - R_{vts}^{SE} y_{vts}^{SE}, \qquad t \in T, \backslash\{0\}, s \in S, \tag{5.72}$$

$$C^{E}_{0s} = C^{EI} + C^{NB}_{v0s} y^{NB}_{v0s} - R^{SC}_{v0s} y^{SC}_{v0s} + C^{SH}_{v0s} y^{SH}_{v0s} - R^{SE}_{v0s} y^{SE}_{v0s}, \qquad s \in S. \tag{5.73}$$

C^{EI} is the value of the fleet of ships at the start of the planning horizon, and β is the yearly depreciation of the fleet.

The constraint set (5.2)-(5.37) does not change.

5.3.2 Linearization of the ROCEMax formulation

The problem is now a linear-fractional programming (LFP) problem, as shown by (Mørch, 2013). This means that it can not be solved directly as a mixed integer programming (MIP) problem. To be able to solve it using standard MIP solution methods, a transformation has to be done. The Charnes-Cooper transformation (Charnes and Cooper, 1962) will be applied.

A new variable, w is defined:

$$w = \frac{\bar{T} + 1}{\sum\limits_{s \in S} \sum\limits_{t \in T} P_s C^{E}_{ts}} \tag{5.74}$$

The variable w is used to transform the variables of the LFP problem, in the following way

$$\bar{x}_{vrts} = w x_{vrts}, \qquad t \in T \setminus \{0\}, v \in V_t, r \in R_{vt}, s \in S. \tag{5.75}$$

The transformation will be done in the same way for the rest of the decision variables.

The result of the transformation is inserted into the objective function and the constraints.

The transformations of δ_{its}, y^{NB}_{vts}, y^{SC}_{vts}, y^{SH}_{vts} and y^{SE}_{vts} are more complex than for the other variables, because of the binary restriction on δ_{its}, and integer restrictions on

y_{vts}^{NB}, y_{vts}^{SC}, y_{vts}^{SH} and y_{vts}^{SE}.

Linearization of the binary variables

For the transformation of δ_{it} it is possible to use the method described by Glover (1975). This involves expressing the relation in (5.75) through a set of linear constraints.

$\bar{\delta}_{its}^{O}$ should be 0 if $\delta_{its} = 0$, and w if $\delta_{its} = 1$. This is done by introducing the following set of constraints:

$$w - \bar{\delta}_{its}^{O} + \delta_{its} \leq 1, \qquad t \in T, i \in N_t, s \in S, \tag{5.76}$$

$$\bar{\delta}_{its}^{O} - w \leq 0, \qquad t \in T, i \in N_t, s \in S, \tag{5.77}$$

$$\bar{\delta}_{its}^{O} - \delta_{its} \leq 0 t \in T, i \in N_t, s \in S. \tag{5.78}$$

Constraints (5.76) ensure $\bar{\delta}_{its}^{O} \geq w$ if $\delta_{its} = 1$, while constraints (5.77) makes sure that $\bar{\delta}_{its}^{O} \leq w$. These two together means that $\bar{\delta}_{its}^{O} = w$ if $\delta_{its} = 1$. Constraints (5.78) sets $\bar{\delta}_{its}^{O} = 0$ if $\delta_{its} = 0$.

Linearization of the integer variables

To be able to keep the integer restrictions on variables y_{vts}^{NB}, y_{vts}^{SC}, y_{vts}^{SH} and y_{vts}^{SE}, it is not possible to use the same transformation as for δ_{its} directly. To be able to use the same transformation method, the variables must be rewritten so that binary restrictions can be used instead of integer restrictions. Assume that for each ship to be ordered, scrapped or bought or sold in the second hand market, a decision is made whether or not to buy that ship. modeled as a binary decision variable. With a fixed limit of how many decisions it is possible to make in each period to order, scrap, buy or sell ships, it is possible to rewrite the variables like this:

$$y^{NB}_{vts} = \sum_{j \in \bar{D}} y^{NB}_{vtsj}, \qquad t \in T, v \in V^N_{t+T^L}, s \in S. \qquad (5.79)$$

Only y^{NB}_{vts} is shown here, as it is done in the same way for y^{SC}_{vts}, y^{SH}_{vts} and y^{SE}_{vts}. Here $\bar{D}$ is the set of decisions that can be made for each ship type in each time period, with $|\bar{D}|$ the maximum number of ships to order, buy, scrap or sell in a period of type v, and y^{NB}_{vtj}, y^{SC}_{vtj}, y^{SH}_{vtj} and y^{SE}_{vtj} become the decisions of whether to make the jth decision to build, scrap, buy or sell a ship of type v in period t and scenario s, respectively. $|\bar{D}|$ is not meant to be a ship type dependent limit, but an upper bound to limit the decision space. If there are ship dependent restrictions on how many ships that can be ordered, scrapped, bought or sold, this has to be implemented through own restrictions.

By making this transformation, the integer restriction on the variables is replaced with binary restrictions, and it is possible to linearize these the same way as was done with δ_{its} in constraints (5.76)-(5.78)

This formulation will lead to symmetry in terms of possible solutions, because a decision of buying one ship made in any j will give the same solution (it does not matter if the decision is made in $j = 1$ or $j = 25$). This symmetry gives a large set of different solutions which will have the same objective value, and thus increasing the complexity of the formulation unnecessarily. To remove as much of this symmetry as possible, four remedies are presented here (shown only for y^{NB}_{vts}, since it can be applied for y^{SC}_{vt} y^{SH}_{vts} and y^{SE}_{vts} by replacing y^{NB}_{vt} with y^{SC}_{vt} y^{SH}_{vts} and y^{SE}_{vts}).

The first suggestion is to introduce a new set of constraints

$$y^{NB}_{vtsj} \leq y^{NB}_{v,t,s,j-1}, \qquad t \in T, v \in V^N_{t+T^L}, j \in \bar{D} \setminus \{1\}, s \in S \qquad (5.80)$$

Constraints (5.80) ensures that if j ships are to be built, only the j first y^{NB}_{vtsj}, for a given v, t and s, can be positive. This means that all other combinations of y^{NB}_{vtsj}

that will lead to the same result will not have to be evaluated, e.g. if a decision is made to order three new ships of type v in period t, then $y^{NB}_{vt1} = y^{NB}_{vts3} = y^{NB}_{vts2} = 1$, and $y^{NB}_{vts4} = ... = y^{NB}_{vt|\bar{D}|} = 0$. This formulation is referred to as the Anti-Symmetry Formulation.

The other suggestion is to use this rewriting of the variables

$$y^{NB}_{vts} = \sum_{j \in \bar{D}} j y^{NB}_{vtsj}, \qquad t \in T, v \in V^N_{t+T^L}, s \in S \qquad (5.81)$$

and make the set $\sum_{j \in \bar{D}} j y^{NB}_{vtsj}$ of type SOS1, which means that maximum one of the variables y^{NB}_{vtj} can take a positive value for all j, and the value of j will be the number of ships scrapped or sold. This formulation is referred to as the SOS1 Formulation.

The third formulation is a combination of the first two formulations. By making each j represent the option of buying or scrapping 2^j ships, there will be no need for anti-symmetry constraints or making the set $\sum_{j \in \bar{D}} j y^{NB}_{vtsj}$ of type SOS1. The formulation will then be

$$y^{NB}_{vts} = \sum_{j \in \bar{D}} 2^j y^{NB}_{vtsj}, \qquad t \in T, v \in V^N_{t+T^L}, s \in S \qquad (5.82)$$

This formulation is referred to as the Power Formulation.

The fourth formulation, denoted as the Pattern Formulation, takes into consideration the special characteristics of this problem, where only two types of ships are available to build each year. Patterns of different combinations of ships to buy, e.g. [0 0], [0 1], [1 0], [1 1], ..., [25 25], are introduced, where the first number gives the number of ships to build of the first ship type, and the second number gives the number of ships to build of the second ship type. All patterns are then in the set

P^P, indexed by p. The binary variable p^P_{tsp} defines which pattern to use in time t and scenario s. The output in terms of ships is given in the parameter P^O_{vp} which gives the number of ships of type v to buy with pattern p. This formulation is a version of a formulation presented in Rakke et al. (2014). In Rakke et al. (2014), there is no binary restriction on the patterns that are used, just integer restriction on the convex combination of patterns. This is not possible to implement for the ROCEMax formulation, so variables p^P_{tsp} will be binary. The variable y^{NB}_{vts} is then replaced by

$$y^{NB}_{vts} \quad = \quad \sum_{p \in P^P} P^O_{vp} p^P_{tsp}, \qquad t \in T, v \in V^N_{t+T^L}, s \in S. \tag{5.83}$$

A transformation will be made, and a set of constraints to keep the relation between w and p^p_{tsp} such as was done with δ_{its} in constraints (5.76)-(5.78)

The Power Formulation and Pattern Formulation for keeping integrality will be evaluated against the Anti-Symmetry Formulation proposed by (Mørch, 2013) in terms of computational efficiency in Chapter 6.

Linearization constraint

When all transformation of the variables have been made, it is needed to impose a constraint for the value of w, as defined in (5.74):

$$\sum_{t \in T} \sum_{s \in S} P_s C^E_{ts} w = \bar{T} + 1 \tag{5.84}$$

C^E_{ts} was defined as

$$C^E_t s = \beta C^E_{t-1,s} + C^{NB}_{vt} y^{NB}_{vts} - R^{SC}_{vts} y^{SC}_{vts} + C^{SH}_{vt} y^{SH}_{vts} - R^{SE}_{vts} y^{SE}_{vts} \tag{5.85}$$

where

$$\begin{aligned}C^E_{t-1,s} =& \beta C^E_{t-2,s} + C^{NB}_{v,t-1} y^{NB}_{v,t-1,s} - R^{SC}_{v,t-1,s} y^{SC}_{v,t-1,s} \\ &+ C^{SH}_{v,t-1} y^{SH}_{v,t-1,s} - R^{SE}_{v,t-1,s} y^{SE}_{v,t-1,s}\end{aligned} \tag{5.86}$$

This leads to

$$\begin{aligned}C^E_{ts} =& \beta(\beta C^E_{t-2,s} + C^{NB}_{v,t-1} y^{NB}_{v,t-1,s} - R^{SC}_{v,t-1,s} y^{SC}_{v,t-1,s} \\ &+ C^{SH}_{v,t-1} y^{SH}_{v,t-1,s} - R^{SE}_{v,t-1,s} y^{SE}_{v,t-1,s} \\ &+ C^{NB}_{vt} y^{NB}_{vts} - R^{SC}_{vts} y^{SC}_{vts} + C^{SH}_{vt} y^{SH}_{vts} - R^{SE}_{vts} y^{SE}_{vts}) \\ =& \beta^2 C^E_{t-2,s} + \beta(C^{NB}_{v,t-1} y^{NB}_{v,t-1,s} - R^{SC}_{v,t-1,s} y^{SC}_{v,t-1,s} \\ &+ C^{SH}_{v,t-1} y^{SH}_{v,t-1,s} - R^{SE}_{v,t-1,s} y^{SE}_{v,t-1,s} \\ &+ C^{NB}_{vt} y^{NB}_{vts} - R^{SC}_{vts} y^{SC}_{vts} + C^{SH}_{vt} y^{SH}_{vts} - R^{SE}_{vts} y^{SE}_{vts})\end{aligned} \tag{5.87}$$

From this it can be shown that C^E_{ts} can be written as

$$\begin{aligned}C^E_{ts} =& \beta^t C^{EI} + \beta^{t-1} * (C^{NB}_{v,1} y^{NB}_{v,1,s} - R^{SC}_{v,1,s} y^{SC}_{v,1,s} \\ &+ C^{SH}_{v,1} y^{SH}_{v,1,s} - R^{SE}_{v,1,s} y^{SE}_{v,1,s}) \\ &+ \beta^{t-2} * (C^{NB}_{v,2} y^{NB}_{v,2,s} - R^{SC}_{v,2,s} y^{SC}_{v,2,s} + C^{SH}_{v,2} y^{SH}_{v,2,s} - R^{SE}_{v,2,s} y^{SE}_{v,2,s}) \\ &+ ... + C^{NB}_{vt} y^{NB}_{vts} - R^{SC}_{vts} y^{SC}_{vts} + C^{SH}_{vt} y^{SH}_{vts} - R^{SE}_{vts} y^{SE}_{vts}\end{aligned} \tag{5.88}$$

From this it follows that $\sum_{t \in T} C^E_t s$ may be written as:

$$\sum_{t \in T} \Big(\beta^t C^{EI} + \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vt} y^{NB}_{vts} + R^{SC}_{vts} y^{SC}_{vts} + C^{SH}_{vt} y^{SH}_{vts} + R^{SE}_{vts} y^{SE}_{vts})\Big) \tag{5.89}$$

So constraint (5.84) can be written as

$$\sum_{t \in T} \big(\beta^t C^{EI} w + \sum_{s \in S} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} \\ P_s (C^{NB}_{vt} \bar{y}^{NB}_{vts} + R^{SC}_{vts} \bar{y}^{SC}_{vts} + C^{SH}_{vt} \bar{y}^{SH}_{vts} + R^{SE}_{vts} \bar{y}^{SE}_{vts})\big) = \bar{T} + 1 \tag{5.90}$$

The problem is now on linear form, and the solution is found by transforming the variables back to their original counterparts, such as in (5.75).

5.3.3 ROCEMax node formulation

The ROCEMax node formulation will have the same new sets from the scenario formulation as the ProfitMax formulation presented in section 5.2.2. The ROCEMax node formulation with the Anti-Symmetry Formulation is presented here. All formulations can be found in the Appendix A.

Objective function

$$\begin{aligned} maxz = & \sum_{t \in T \setminus \{0\}} \sum_{n \in n(t)} P_n \Big(\sum_{i \in N^O_t} \sum_{c \in G} R^D_{in} D_{icn} \bar{\delta}_{in} \\ & + \sum_{i \in N^C_t} \sum_{c \in G} (R^D_{in} D_{icn} w - C^{SP}_{icn} \bar{n}_{icn}) - \sum_{v \in V_t} (C^{OP}_{vn} \bar{y}^P_{vn} \\ & + C^{CO}_{vn} \bar{h}^I_{vn} - R^{CO}_{vt} \bar{h}^O_{vn} \\ & + \sum_{r \in R_{vn}} C^{TR}_{vrn} \bar{x}_{vrn} - R^{LU}_{vn} \bar{l}_{vn})\Big) \\ & + \sum_{n \in n(\bar{T})} \sum_{v \in V_{\bar{T}}} R^{SV}_{vn} \bar{y}^P_{vn} \\ & - \sum_{t \in T, t \leq \bar{T} - T^L} \sum_{n \in n(t)} \sum_{v \in V^N_{t+T^L}} P_n C^{NB}_{vn} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vnj} \\ & + \sum_{t \in T} \sum_{n \in n(t)} \sum_{v \in V_t} P_n (R^{SC}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vnj} \\ & + R^{SE}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vn} - C^{SH}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vn}) \end{aligned} \tag{5.91}$$

The objective function is as described in section 5.3.1, with the transformation of the variables and the new variable w. It can be noticed that the term $R^D_{in}D_{icn}$ has to be multiplied by w, since this term is only a scalar in the objective function. All other terms of the objective function includes a variable, which means that the transformation is included by replacing the variables with the transformed variable, e.g. n_{icn} is replaced by $\bar{n}_{icm}$

Demand constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv}\bar{x}_{vrn} + \bar{n}_{icn} \geq D_{icn}w, \qquad t \in T \setminus \{0\}, i \in N^C_t, c \in G, n \in n(t), \quad (5.92)$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv}\bar{x}_{vrn} \geq D_{in}\bar{\delta}_{icn}, \qquad t \in T \setminus \{0\}, i \in N^O_t, c \in G, n \in n(t), \quad (5.93)$$

The demand constraints which correspond to constraints (5.39) and (5.40).

Capacity constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinC} Q_{cv}x_{vrn} + \sum_{c \in G} n_{cin} \geq \sum_{c \in G} D_{icn}, \quad t \in T \setminus \{0\}, i \in N^C_t, n \in n(t), \quad (5.94)$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinC} Q_{cv}x_{vrn} \geq \sum_{c \in C} D_{icn}\bar{\delta}_{in}, \qquad t \in T \setminus \{0\}, i \in N^O_t, n \in n(t), \quad (5.95)$$

The capacity constraints which correspond to constraints (5.41) and (5.42).

Frequency constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t \in T \setminus \{0\}, i \in N^C_t, n \in n(t), \quad (5.96)$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{in}, \qquad t \in T \setminus \{0\}, i \in N^O_t, n \in n(t), \quad (5.97)$$

Frequency constraints, corresponding to (5.43)-(5.44).

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vn} + \bar{h}^I_{vn} - \bar{h}^O_{vn} - \bar{l}_{vn}), \qquad t \in T\setminus\{0\}, v \in V_t, n \in n(t), \quad (5.98)$$

Available time constraints, corresponding to (5.45).

Optional trades constraints

$$\bar{\delta}_{i,a(n)} \leq \bar{\delta}_{in}, \qquad t \in T\setminus\{0,1\}, i \in N^O_t, n \in n(t), \quad (5.99)$$

Constraint (5.99) correspond to (5.46).

Pool constraints

$$\bar{y}^P_{vn} = \bar{y}^P_{v,a(n)} - \sum_{j\in\bar{D}}(\bar{y}^{SC}_{v,a(n),j} - \bar{y}^{SH}_{v,a(n),j} + \bar{y}^{SE}_{v,a(n),j}), \qquad t \in T\setminus\{0\}\; v \in V_t\setminus V^N_t, n \in n(t), \quad (5.100)$$

$$\bar{y}^P_{vn} = Y^{NB}_{vt} w, \qquad t \in T : t < \bar{T}^L, v \in V^N_t, n \in n(t) \quad (5.101)$$

$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}} \bar{y}^{NB}_{v,(a(n,T^L),j}, \qquad t \in T : t \geq \bar{T}^L, v \in V^N_{t+T^L}, n \in n(t), \quad (5.102)$$

$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v \in V_0, n \in n(t), \quad (5.103)$$

$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}} \bar{y}^{SC}_{v,a(n),j}, \qquad t \in T\setminus 0, v \in V_t\setminus v \in V_{t+1}, n \in n(t), \quad (5.104)$$

$$\bar{l}_{vn} - \bar{h}^I_{vn} + \bar{h}^O_{vn} \leq \bar{y}^P_{vn}, \qquad t \in T\setminus\{0\}, v \in V_t, n \in n(t), \quad (5.105)$$

Constraints (5.100)-(5.105) correspond to constraints (5.47)-(5.52). It can be noticed that for the decision variables, the sum over $\bar{D}$ has to be included, since the decision variables now have binary restrictions, as compared to the integer restrictions for the corresponding variables in the ProfitMax formulation. The transformed variables in constraints (5.100)-(5.104) are forced to be w or 0 by constraints (5.114)-(5.128), which follow later.

Charters and second hand market constraints

$$\sum_{j\in\bar{D}} \bar{y}^{SH}_{vnj} \leq \overline{SH}_{vn} w, \qquad t \in T, v \in V_t, \tag{5.106}$$

$$\sum_{j\in\bar{D}} \bar{y}^{SE}_{vnj} \leq \overline{SE}_{vn} w, \qquad t \in T, v \in V_t, \tag{5.107}$$

$$\bar{h}^{I}_{vn} \leq \overline{CI}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t, \tag{5.108}$$

$$\bar{h}^{O}_{vn} \leq \overline{CO}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t, \tag{5.109}$$

$$\sum_{v\in V_t\setminus V_t^N} \sum_{j\in\bar{D}} y^{SH}_{vnj} \leq \overline{SH}_{n}, \qquad t \in T, n \in n(t), \tag{5.110}$$

$$\sum_{v\in V_t\setminus V_t^N} \sum_{j\in\bar{D}} y^{SE}_{vnj} \leq \overline{SE}_{n}, \qquad t \in T, n \in n(t), \tag{5.111}$$

$$\sum_{v\in V_t\setminus V_t^N} h^{I}_{vn} \leq \overline{CI}_{n}, \qquad t \in T \setminus \{0\}, n \in n(t), \tag{5.112}$$

$$\sum_{v\in V_t\setminus V_t^N} h^{O}_{vn} \leq \overline{CO}_{n}, \qquad t \in T \setminus \{0\}, n \in n(t), \tag{5.113}$$

Constraints (5.106)-(5.113) correspond to (5.53)-(5.60).

Transformation of binary variables constraints

$$w - \bar{\delta}^{O}_{in} + \delta^{O}_{in} \leq 1, \qquad t \in T \setminus \{0\}, i \in N_t, n \in n(t), \tag{5.114}$$

$$\bar{\delta}^{O}_{in} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t, n \in n(t), \tag{5.115}$$

$$\bar{\delta}^{O}_{in} - \delta^{O}_{in} \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t, n \in n(t), \tag{5.116}$$

$$w - \bar{y}^{NB}_{vnj} + y^{NB}_{vnj} \leq 1, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), j \in \overline{D}, \quad (5.117)$$

$$\bar{y}^{NB}_{vnj} - w \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), j \in \overline{D}, \quad (5.118)$$

$$\bar{y}^{NB}_{vnj} - y^{NB}_{vnj} \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), j \in \overline{D}, \quad (5.119)$$

$$w - \bar{y}^{SC}_{vnj} + y^{SC}_{vnj} \leq 1, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.120)$$

$$\bar{y}^{SC}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.121)$$

$$\bar{y}^{SC}_{vnj} - y^{SC}_{vnj} \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.122)$$

$$w - \bar{y}^{SH}_{vnj} + y^{SH}_{vnj} \leq 1, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.123)$$

$$\bar{y}^{SH}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.124)$$

$$\bar{y}^{SH}_{vnj} - y^{SH}_{vnj} \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.125)$$

$$w - \bar{y}^{SE}_{vnj} + y^{SE}_{vnj} \leq 1, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.126)$$

$$\bar{y}^{SE}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D}, \quad (5.127)$$

$$\bar{y}^{SE}_{vnj} - y^{SE}_{vnj} \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D} \quad (5.128)$$

Contraints (5.114)-(5.128) show the constraints setting the new binary decision variable to 0 or w, as described in constraints (5.76)-(5.78)

Linearization constraints

$$\sum_{t \in T} \sum_{n \in n(t)} P_n \big(\beta^t C^{EI} w + \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vn} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vnj} + R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vnj} + C^{SH}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vnj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vnj}) \big) = \bar{T} + 1 \quad (5.129)$$

Convexity, integer and binary constraints

$$y^{NB}_{vn} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), \quad (5.130)$$

$$y^{SC}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t), \quad (5.131)$$

$$y^{SH}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t), \quad (5.132)$$

$$y^{SE}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.133}$$

$$y^{NB}_{vnj} \in \{0,1\}, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), \tag{5.134}$$

$$y^{SC}_{vnj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.135}$$

$$y^{SH}_{vnj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.136}$$

$$y^{SE}_{vnj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.137}$$

$$\delta_{in} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, i \in N^O_t, n \in n(t), \tag{5.138}$$

$$\bar{\delta}_{in} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, i \in N^O_t, n \in n(t), \tag{5.139}$$

$$\bar{y}^{NB}_{vnj} \in \mathbb{R}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), \tag{5.140}$$

$$\bar{y}^{SC}_{vnj} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.141}$$

$$\bar{y}^{SH}_{vnj} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.142}$$

$$\bar{y}^{SE}_{vnj} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t), \tag{5.143}$$

$$y^{P}_{vn} \in \mathbb{R}^+, \qquad t \in T\, v \in V_t, n \in n(t), \tag{5.144}$$

$$l_{vn} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.145}$$

$$h^I_{vt} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.146}$$

$$h^O_{vt} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t), \tag{5.147}$$

$$x_{vrn} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t), \tag{5.148}$$

$$w \in \mathbb{R}^+. \tag{5.149}$$

Chapter 6

Computational Study

In this chapter the results from a computational study of the formulations presented in Chapter 5 will be presented. The results will be analyzed to discuss the performance of the formulations presented in Chapter 5, and to evaluate the value of maximizing investment returns in the objective function.

The computational study is based on implementation of the ProfitMax and ROCEMax formulations in commercial optimization software. Xpress Mosel has been the modeling language. The implementations has been solved using Xpress-IVE Version 1.24.00 64 bit with Xpress Optimizer Version 24.01.04. The code was run for different test instances, which are introduced in Section 6.1. All runs were performed on a computer running Windows 7 Enterprise 64-bit operating system, having an Intel® Core™ i7-3770 CPU @ 3.4 GHz and 16 GB RAM.

In Section 6.1 the test instances used for the computational study are presented. The performance of the formulations from Chapter 5 are evaluated in Section 6.2. The scenario generation method is presented together with an evaluation of the in-sample stability of different sizes of the scenario tree in Section 6.3. In Section 6.4 the results from ProfitMax and ROCEMax formulations are presented and discussed. The Value of Stochastic Solution (VSS) is presented in Section 6.5. Section 6.6 will test the impact on the formulations for changes in parameter values. Section 6.7 and 6.8 will test the formulations with inclusion of charters and second hand market, and using a three stage case as compared to a two stage case. Section 6.9 will discuss the solutions from the ROCEMax formulation if an adjustment is made as to how the ROCE is calculated.

6.1 Test instances

Different test instances were made for the computational study. The instances vary with the size of the fleet and the number of trades with contractual obligations at the start of the planning horizon. All data for ship types demand (i.e. new build cost, sailing cost) and trades(i.e. demand, length) are based on data from the case company. In total, three sets of test instances were made; small, medium and large. The sets are described in terms of fleet composition and trades in Tables 6.1 and 6.2. A set consists of input data with 10 different scenario trees.

For each set, the types of ships and available trades are the same. There are two ship types which will reach the maximum lifetime of a ship during the planning horizon. The capacity of these ships will have to be replaced in some manner during the planning horizon to be able to serve all trades with contractual obligations.

The three sets are meant to describe shipping companies of different sizes. The large set reflects the size of a large liner shipping company, as the case company.The large set has a fleet of 55 ships, serving 11 of 14 trades. The medium set describes a shipping company with a fleet of 35 ships, servicing 7 of 11 trades. The small set describes a shipping company with 27 ships, servicing 5 of 8 trades. The optional trades in each case is adjusted so that the different set sizes offer the same relative expansion opportunity in terms of optional trades.

Except for fleet configuration and trades, all data sets use the same input parameters.

		Capacity (Units RT43)			Service	Ships in initial fleet		
Ship type	Age	Car	BB	HH	Speed (knots)	Large	Medium	Small
PCC1	26	4975	300	2200	18.5	4	4	3
PCC2	12	6800	300	2500	18.5	6	3	2
PCTC1	9	6800	300	2500	19	8	4	3
LCTC1	4	6000	1500	2000	19	10	5	5
PCTC2	4	5450	900	2200	19	12	7	2
PCTC3	14	6150	200	1800	19.6	7	3	3
RORO1	28	4853	1500	3100	20.5	6	3	2
RORO2	-1	5660	2200	4000	20.8	2	0	1
LCTC2	0	6000	1500	2000	18.5	0	2	0
RORO3	-2	5660	2200	4000	20.8	0	0	0
LCTC3	-1	6000	1500	2000	18.5	0	0	0
RORO4	-3	5660	2200	4000	20.8	0	0	0
LCTC4	-2	6000	1500	2000	18.5	0	0	0
LCTC5	-3	6000	1500	2000	18.5	0	0	0

Table 6.1: Set of ship types. The columns contains ship type, age at the beginning of the planning horizon and capacity in cars, HHs and BBs, cruising speed, and number of ships in the fleet at the beginning of the planning horizon for each instance set. A negative age means that the ship can be delivered from year $t = -Age$.

Trade	Length	Demand (Units RT43)			Freq	Large	Medium	Small
		Car	HH	BB				
T1	13,500	435,213	77,047	5,762	48	C	C	C
T2	11,700	103,048	14,201	4,476	0	C	C	C
T3	7,800	41,036	24,420	3,404	0	C	C	C
T4	7,500	19,200	0	0	0	C	C	C
T5	13,000	89,406	21,427	7,425	24	C	C	C
T6	6,500	469,379	67,854	20,075	52	C	C	-
T7	14,500	35,331	66,607	36,845	0	C	C	-
T8	7,800	98,000	29,240	2,689	0	C	O	O
T9	4,900	119,397	44,121	2,167	0	C	O	-
T10	9,000	60,159	7,401	1939	48	O	O	O
T11	8,400	24,818	10,434	3,252	0	O	O	O
T12	6,500	14,0508	53,928	14,115	0	O	-	-
T13	15,021	266,855	55,474	16,776	0	C	-	-
T14	19,200	397,688	123,779	66,198	48	C	-	-

Table 6.2: Set of trades. The columns contains name of the trade, the length of each trade, demand for cars, HHs and BBs, frequency demand, and status at the beginning of the planning horizon for each instance set. A "C" means that the shipping company has contractual obligations, and must serve the trade, while an "O" means that the trade is optional to service.

The planning horizon is set to five years. Revenue made from shipping goods is set to 32 cents per nautical mile shipped, estimated by taking the trip cost per nautical of the least efficient ship type in the fleet, and adding a profit margin on top of this. Space charter cost is set to 2,000 USD per unit of goods transported by space charter. All ships costs, such as trip costs, operating costs, lay up revenue etc. is estimated using raw data from the case company. The second hand price of ships is given in the fleet data. Second hand prices are used to calculate sunset value and charter costs. All input values are properly discounted over the planning horizon, using a discount factor of 12%, as suggested in Stopford (2008).

6.2 Choosing a formulation

In Chapter 5, a scenario formulation and a node formulation were presented for both the ProfitMax model and the ROCEMax model. In addition, four alternative formulations for how to preserve the integrality restriction on the decision variables for the ROCEMax model were proposed. The Anti-Symmetry Formulation and SOS1 Formulation were tested by Mørch (2013), who found that the Anti-Symmetry Formulation outperformed the SOS1 Formulation. Therefore, only three of them will be tested in this section, i.e. the Anti Symmetry Formulation, the Power Formulation and the Pattern Formulation.

To test the computational effectiveness of the ROCEmax formulations, the formulations were run on the full set using four scenarios. 10 scenario trees were generated, and the results shown are the averages of these solutions. The formulations were run with a time limit of 3,000 seconds. Four scenarios is chosen for this testing of computational effectiveness, since it is the smallest size of the scenario tree that is possible to generate using the method described in Section 6.3. It is assumed that the solution time of the formulations will scale with the complexity of the problem, such that the best formulation for a smaller instance will be as relatively good or

Formulation	Avg. gap	Avg solution time
Anti Symmetry + Scenarios	0.19%	2504 s
Anti Symmetry + Nodes	0.09%	1944 s
Power+ Scenarios	0.21%	2829 s
Power + Nodes	0.16%	1870 s
Pattern + Scenarios	39.07%	3024 s
Pattern + Nodes	6.01%	3006 s

Table 6.3: Comparison of performance for the ROCEMax formulation using the three formulations for keeping integrality discussed in Section 5.3.1 combined with a node and a scenario formulation, tested on the large set

better for larger instances.

Table 6.3 show the results from testing the Anti-Symmetry, Power and Pattern formulations combined with scenario and nodes formulations, in terms of performance. The tests were done with 10 scenario trees. Average gap is the average of the reported gap between the objective value and the best bound found. The average solution time shows the average time to find the optimal solution (with an upper bound of 3000 if the formulation does not find an optimal solution within the time limit).

As it can be seen from the results in Table 6.3, the Anti-Symmetry Formulation combined with a node formulation performs best, followed by the Power Formulation combined with a node formulation. When comparing the performance of a formulation, combined with a node formulation and a scenario formulation, the combination with a node formulation outperformed the combination with a scenario formulation in all cases.

The Pattern Formulation performs definitely worst of all formulations combined with both a scenario formulation and a node formulation. This is most likely because the ROCEMax formulation presented in this report does not allow the option of

	Avg. gap	Avg solution time
Large		
Anti symmetry + nodes	0.48%	381 s
Power + nodes	0.53%	1,278 s
Medium		
Anti symmetry + nodes	0.49%	767 s
Power + nodes	0.65%	2,034 s
Small		
Anti symmetry + nodes	0.47%	265 s
Power + nodes	0.51%	655 s

Table 6.4: Comparison of solution time to read 0.5% optimality for the Anti Symmetry Formulation and the Power Formulation

not having binary restrictions on the patterns used, and instead only using integer restrictions on the convex combination of patterns, as presented in Rakke et al. (2014).

The ProfitMax Formulation presented in Chapter 5 solves much quicker, solving to optimality within a couple of seconds for all sets with 6 scenarios.

The difference of performance between the Anti Symmetry Formulation and the Power Formulation, combined with a node formulation, is small, so these formulations have been further tested on data instances of the medium and small sets as well. These tests have run to 0.5% optimality gap, with a time limit of 3,000 seconds, since this will be used later on in the chapter. The results from these tests are shown in 6.4

It can be seen that the Anti Symmetry Formulations outperforms the Power Formulation in terms of reaching a 0.5% optimality gap in all cases. Therefore the

Anti Symmetry Formulation combined with a node formulation is used in what follows.

6.3 Scenario generation

To handle the uncertainty as discussed in Chapter 3, a two-stage stochastic programming model has been implemented. The discretization of the random variables has been done using a modified version of (Høyland et al., 2003), using distribution functions instead of moments to control the margins. This model generates the desired number of scenarios for the random variables, taking the distribution function of each random variable and the correlations as input. The scenarios then each consist of a vector with $\bar{u}$ elements with the different realization of the random variables in each scenario, $\bar{u}$ being the number of random variables.

The model consists of three random variables, the market status, steel price and fuel price. These are chosen as random variables since they affect the profit for shipping companies to a large degree, and the development can be assumed to be uncertain. For all $t \in T$, the correlations between the random variables are considered to be the same. The correlation matrix is shown in Table 6.5.

Market status is assumed to have large impact on freight revenue, demand, prices for chartering in and out ships, and second-hand prices, and some lesser impact on new build prices. Steel prices impact the scrapping revenue, whilst fuel prices impact the trip costs. It is assumed a very high correlation between market status and steel prices, and between steel prices and fuel prices. Market status and fuel prices are assumed to have a high correlation. The correlations are the same as used by Pantuso (2014)

The first stage decisions consists of the buying, selling and scrapping decisions made in the first time period. In this period, no tactical decisions (fleet deployment) are

	Market status	Steel price	Fuel price
Market status	1	0.8	0.75
Steel Price	0.8	1	0.8
Fuel Price	0.75	0.8	1

Table 6.5: Correlation matrix

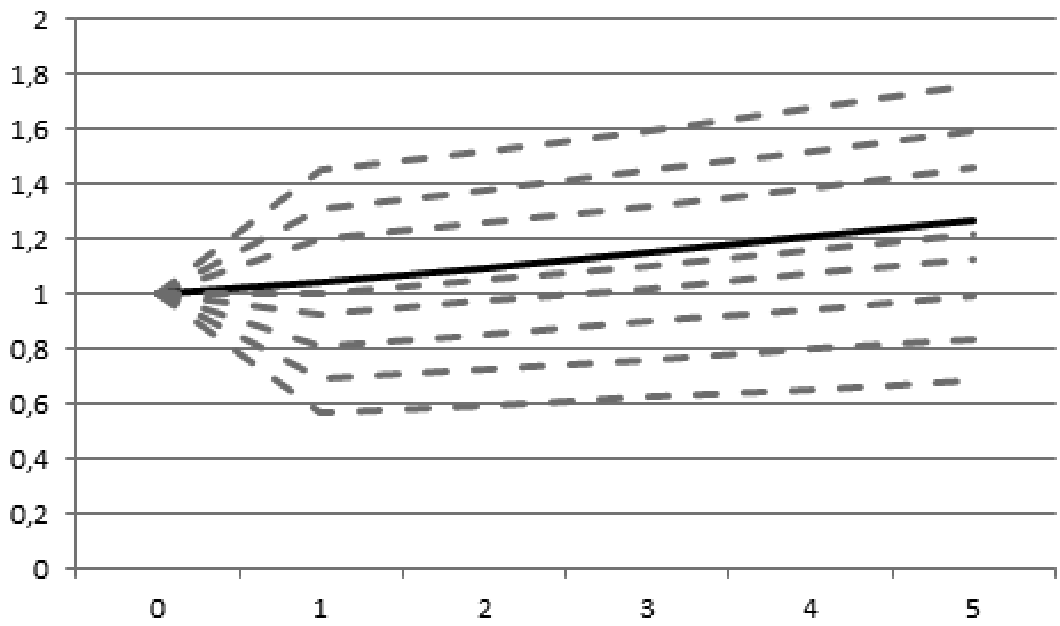


Figure 6.1: Example of two stage scenario tree, for one random variable

made. The second stage consists of all decisions made from $t = 1$ until the end of the planning horizon. At the beginning of the second stage, the actual development of the random variables is revealed. In Figure 6.1, an example of the development of a random variable in eight scenarios is shown. The bold line shows the expected value of the variable, while the dotted lines represent a scenario each. In the first stage ($t = 0$), all scenarios have the same information about current and future prices, while in the second stage ($t \geq 1$), the realization of the random variable is revealed.

The number of scenarios to use when running the formulations is of great impor-

Standard deviation

	4 scenarios	6 scenarios	8 scenarios	10 scenarios
ROCEMax	1.0%	0.6%	0.7%	0.3%
ProfitMax	0.7%	0.3%	0.4%	0.2%

Table 6.6: Standard deviations from the optimal solutions when run on 10 data instances

tance, since having too many scenarios might make the problem unnecessary hard to solve, while having to few scenarios might lead to unstable solutions, meaning that a given scenario tree might impact the solution so much that results are useless for comparison with other formulations. In-sample stability, as discussed in Chapter 3, is a measure to evaluate if the selected scenario generation method generates suitable scenarios trees. The ROCEMax and ProfitMax formulations have been run on 10 data instances of the large instance set, each for four, six, eight and 10 scenarios, to see if stability is achieved. In Table 6.6, the standard deviation in percent of the average optimal objective value is given.

From the results in Table 6.6 it is clear that increasing number of scenarios improves stability, in terms of standard deviation. The standard deviation decreases from four to six scenarios, increases when going from six to eight scenarios, before decreasing again when going to 10 scenarios. The increase from six to eight scenarios is most likely explained in the error from the scenario generation method. The standard deviation with both six and eight scenarios is within what can be accepted, as the difference in the Value of Stochastic Solution, presented in Section 6.5 is of a higher order of magnitude. The computational complexity increases with increasing number of scenarios, six scenarios is therefore used in what follows, to ensure both stability and that the complexity does not increase unnecessarily.

6.4 Comparison of ProfitMax and ROCEMax solutions

The results from testing the ROCEMAx and ProfitMax formulations are presented in Table 6.7. The tests are run using six scenarios, and the values shown are the average from 10 different instances of the scenario tree. A time limit of 10,000 seconds and an optimality gap of 0.5% is applied. The average gap from the results is lower than 0.6% for all sets.

Table 6.7 first shows the decisions made in the first period of the planning horizon ($t = 0$). The results show the total amount of both ship types bought. There is a clear trend in the results: The ROCEMax fomulation consistently buys much fewer ships in the first period than the ProfitMax fomulation.

The next row shows the expected total number of ships that are bought during the second stage ($t \geq 1$). The results are consistent with what could be expected, that solutions on the larger set buys more ships to renew and expand than the medium and small sets. The ROCEMax also plans to invest in fewer ships over the entire planning horizon compared to the ProfitMax Formulation. Compared to the number of ships leaving the fleet during the planning horizon, and the size of the fleet, the total investments scale reasonably between the set sizes. The results show that the ProfitMax fomulation tends to expand the fleet to a much larger degree than the ROCEMax fomulation, which leads to some interesting insight. The ROCEMax formulation will only choose to invest in a new ship if the investment will lead to an higher return than without the investment, which means that the extra profit gained over the capital needed for the investment must be higher the return without the investment. If it is only marginally lower, the investment will not be made. For the ProfitMax Formulation it is enough that the extra investment gives a net profit, as long as it is positive. So the ROCEMax Formulation can be said to have

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t = 0$	6.7	3.6	2.6	19.4	8.6	5.9
$t = 1, .., T$	12.1	6.4	4.4	11.9	8.5	5.6
Scrappings						
$t = 0$	-	-	-	-	-	-
$t = 1, .., T$	12.9	8.1	5.6	10.8	7.3	5.2
Optional trades						
$t = 1$	0.78	1.18	0.80	0.97	1.67	1.07
$t = 2$	0.78	1.18	0.80	1.65	2.50	1.57
$t = 3$	0.80	1.20	0.82	2.00	3.00	2.00
$t = 4$	0.80	1.20	0.82	2.00	3.00	2.00
$t = 5$	0.83	1.22	0.87	2.00	3.00	2.00
Profit (MUSD)	1,994	1,054	687	2,152	1,163	754
ROCE	194 %	191 %	185 %	169 %	168 %	163 %
CAGR	24.1 %	23.8 %	23.3 %	21.9 %	21.8 %	21.4 %
CAGR Extra	-	-	-	10.4 %	12.2 %	11.9 %
Extra Return	14.9 %	13.7 %	13.1 %	-	-	-
Extra Profit	-	-	-	7.9 %	10.3 %	9.8 %
VSS	11.9 %	15.9 %	21.7 %	12.6 %	3.3 %	11.7 %
Space charter cost						
SS	497	205	135	300	130	89
DS	920	470	354	628	169	184

Table 6.7: Comparison of the the results from the two formulations, ROCEMax and ProfitMax

a stricter restriction on the needed profit from an investment.

After the new build decisions, the scrapping decisions are shown. None of the formulations choose to scrap any ships in the first period for any of the set sizes, but there is a difference between the formulations in terms of the total number of ships to scrap during the planning horizon. The ProfitMax Formulation plan to scrap only the ships leaving the fleet, with one or a few extra ships of other types in some cases. The ROCEMax Formulation plan to scrap more of the other types ships in addition to the ones leaving the fleet during the planning horizon. This result builds to the structural difference found from the new build decisions, where the ProfitMax Formulation to a much larger degree choose to expand the fleet to meet increasing demand and take extra optional trades compared to the ROCEMax formulation. It can be seen that the ProfitMax formulation consistently choose to service all optional trades, except for one, for all set sizes. The trade that is never chosen is T10, which is explained by the relatively low demand and high frequency requirement.

Next, Table 6.7 shows the economic results from the two formulations. The results show that there is a consistent difference between the achieved ROCE and profit between the two formulations. The row "CAGR Extra" shows the return that the ProfitMax Formulation achieve for the extra capital employed needed to gain the extra profit compared to the ROCEMax Formulation. When looking at the CAGR that the ProfitMax Formulation achieve on the extra capital needed, this number is significantly lower than the ROCE achieved for both the ROCEMax Formulation and the ProfitMax Formulation. Next, the extra return from using ROCEMax compared to ProfitMax is given. This is calculated by taking the relative increase in ROCE from the solution of the ProfitMax Formulation to the solution of the ROCEMax Formulation.. An increase from 13.1% to 14.9% must be considered significant. It is also interesting to see that the relative gain in profit from ROCEMax to ProfitMax is smaller than the relative gain in ROCE the other way around. The Value of Stochastic Solution (VSS) will be discussed in the next section, 6.5.

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t = 0$	6.5	3.5	2.6	19.4	8.5	5.6
$t = 1, .., T$	11.9	6.2	4.2	9.3	5.7	4.1
Scrappings						
$t = 0$	-	-	-	-	-	-
$t = 1, .., T$	13.4	8.8	6.0	10.9	7.4	5.2
Profit (MUSD)	1,873	994	674	1,976	1,034	701
ROCE	194 %	188 %	183 %	168 %	167 %	163 %
CAGR	24.0 %	23.6 %	23.1 %	21.8 %	21.7 %	21.4 %
CAGR Extra	-	-	-	8.5 %	7.5 %	7.6 %
Extra Return	14.8 %	12.9 %	12.1 %	-	-	-
Extra Profit	-	-	-	5.5 %	4.0 %	4.0 %
VSS	9.7 %	16.1 %	22.0 %	19.7 %	25.9 %	42.7 %
Space charter cost						
SS	502	207	135	301	132	88
DS	889	474	354	861	450	384

Table 6.8: Results without optional trades

The last part of Table 6.7 show the space charter cost incurred for the formulations. This can be seen as an indicator of the willingness to use space charters instead of investing in new ships. It is obvious that the ROCEMax Formulation use this option far more often than the ProfitMax Formulation. No restriction has been set to the amount of units shipped by space charters, it can be discussed if it is realistic for the ROCEMax formulation to be able to use space charters to such a large degree.

Table 6.8 shows the results from running the formulations with no optional trades. For the ProfitMax formulation, this case can be considered a cost minimizing problem, as discussed in Chapter 5. These results show that the difference in results between the ROCEMax formulation and the ProfitMax formulations are structurally the same as in when including optional trades. This means that most of the difference in number of ships built, does not appear because of taking more optional trades, this difference is only a few ships. When looking at the space charter cost in Table 6.7, it can be seen that the ROCEMax formulation choose to use the space charter option more often than the ProfitMax Formulation. This means that the ProfitMax formulation sees that it is beneficial to buy a new ship rather than having to use space charter, because this is profitable. It is however not profitable enough for the ROCEMax formulation to make the same decision.

Even though the ROCEMax model performs significantly better than ProfitMax when considering ROCE (as one should expect), these results can not be taken as the optimal decision to the MFRP. The extra profit gained from the solutions from the ProfitMax model is of course of value. The solutions from the ROCEMax model can be considered to be better than the solutions from the ProfitMax Formulation if the decision makers have an alternative investment for the reduced capital demand. If there does not exist a viable alternative investment option, the extra profit gained from ProfitMax would be preferable over the extra returns from the ROCEMax model. Comparing these results give decision makers a better picture of the possible decisions and the payoff from those. The introduction of the ROCEMax model clearly adds an extra dimension to the MFRP that has not been available in earlier research.

The ProfitMax fomulation makes no consideration as to the investments needed when investing in ships, except for the decreasing net present value of the building cost. Therefore, it is more likely to replace ships leaving the fleet at an earlier time than the ROCEMax fomulation. The results from the tests show that the ProfitMax Formulation more often plan to scrap ships leaving the fleet before they reach their lifetime.

It is also important to evaluate the results in light of the assumptions made when modeling the problem. There has been made no considerations to the available amount of capital for the ship companies. The ProfitMax fomulation especially expands the fleet considerably during the planning horizon. It can be discussed if it is realistic for a company to make investments of this size, compared to the existing value of the fleet at the start of the planning horizon.

The results discussed in this section may also be sensitive to values of the input parameters. Even though the input parameters has been chosen to be as realistic as possible, it is interesting to see if changes in some of the input parameters, such as revenue or price of new ships will lead to large changes in the results. In Section 6.6 the results will be tested for sensitivity in terms of the input parameters.

6.5 Value of stochastic solution

In this section the Value of stochastic solution (VSS), as defined in Birge (1982), from using the stochastic implementations ROCEMax and ProfitMax will be presented. The VSS is calculated by comparing the results from the deterministic solution (DS) and stochastic solution (SS), when run on stochastic data instances. The DS is found taking the first stage decisions from solving the problem using expected values

	ROCEMax	ProfitMax
Large	2	8
Medium	1	7
Small	-	4

Table 6.9: First stage decisions for the DS

for random variables (expected value problem). These first stage decisions are then used as input when run on a stochastic instance with the scenario tree used for the SS. To compare the results from different sizes of the data instances, the results are presented as percentage of the DS. The VSS is presented in Table 6.7

The results in Table 6.7 show that the SS give a significant expected value compared to the DS in all cases, except for the ProfitMax fomulation run on the medium sized set. In the case of ROCEMax Formulation, the VSSs are remarkably higher, compared to the ProfitMax fomulation, for the medium and small sets. Table 6.7 also shows the space charter costs for the DS, the solution of the DS run on scenario trees, and the SS, for both the ProfitMax Formulation and the ROCE formulation.

The DSs will never wait and see, since all future development is given. The investment decisions made in $t = 0$ will therefore not be flexible with regards to different future development than the expected. The first stage solutions of the DS is shown in Table 6.9. The DS invests less in new builds, leading to lesser flexibility in case on increased demand. Since the DS only consider the expected value, no consideration as to the effects of under capacity in case of increased demand is taken. The SSs will consider the possibility of increased demand, since this is part of some of the scenarios. The result is that the SS choose to build more ships, and thus incurs less space charter costs because of too little capacity, as is shown in Table 6.7.

When considering the lower VSS for the ProfitMax formulation with the medium set instance space charter costs, an obvious reason is the relatively lower space charter cost in the DS as compared to the SS. For the medium set the increase in space charter cost is about 30% from SS to DS, while it is 105% and 96% for the large and small sets respectively. The space charter cost also explain why the VSS for the ROCEMax is higher than for the ProfitMax Formulation for the medium and small sets. The ROCEMax Formulation chooses to use space charter as an option more often than the ProfitMax Formulation, and for the medium and small sets, the increase in space charter costs is much higher than for the large set and all sets with the ProfitMax formulation. The smaller increase for the large set with the ROCEMax Formulation may be explained by the larger fleet given increased flexibility as compared to the smaller fleets in the medium and small sets.

When considering the case without optional trades, results shown in Table 6.8, the VSS increases for the ProfitMax Formulations, but stays the same for the ROCEMax Formulations, compared to the base case. The reason for the increase for the ProfitMax Formulation comes from the large increase in use of space charters. When optional trades are available, the ProfitMax Formulation choose to invest in more ships than without the optional trades, this is also valid for the DS. The investment is meant to give opportunity to service optional trades, but also gives increased flexibility for the DS when the demand turns out to be higher than expected in some scenarios. Some of the optional trades that were planned to be serviced can then be avoided, to have more capacity for the trades with contractual agreements, and avoid using space chartering on these. Without the optional trades, the DS does not invest as much, and space charters will have be to used in the scenarios where demand increases. The optional trades also give a possibility to use capacity on these trades in case of lower demand than expected. This option is not available when optional trades are not included.

6.6 Impact of varying input parameter values

As discussed in Section 6.4, the results from the fomulations may be sensitive to changes in some of the input parameters. For instance, the sunset value may not fully capture the future profit potential of ships in the fleet, a longer planning horizon might give more return on the investments made. The revenue from shipping goods will also be considered. This is uncertain in the model, but if the initial level is set too low, this might impact the results to a large degree.

In Section 6.4, the results showed that the ProfitMax Formulation chose to take all but one optional trade for all test instances, while the ROCEMax was much more conservative in choosing optional trades. It can be interesting to see what happens if there are more optional trades available for the medium and small instances. This might lead to larger differences between the solutions from the two formulations, or the increased optional trades might add flexibility that makes more optional trades profitable also for the ROCEMax Formulation.

Table 6.7 clearly shows that the VSSs were highly affected by the space charter costs in the DS. Setting this cost lower might give a lower VSS. It could also be seen that the ROCEMax Formulation chose to use the space charter option more than the ProfitMax Formulation. This might also change if the cost is set at a lower level.

In this section, the impact of changing the values of the freight revenue, space charter cost, the number of optional trades and the length of the planning horizon will be tested. The changes in parameters will be done for one parameter at a time. All tests have been done using a optimality condition of 1% and a maximum running time of 3,000 seconds. The results from Section 6.4 showed that the difference between the ROCEMax and ProfitMax formulations differed by more than 10%, when comparing the ROCE from the two formulations. Also, the computational tests showed that the largest decrease in gap when the gap was as low as 1% came from decreasing

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t = 0$	0.1	-	-	20.2	9.8	6.6
$t = 1, .., T$	15.8	8.7	5.9	13.3	9.9	7.1
Scrappings						
$t = 0$	-	-	-	-	-	-
$t = 1, .., T$	11.2	7.3	5.1	10.7	7.3	5.2
Optional trades						
$t = 1$	0.60	0.88	0.58	1.05	1.73	1.17
$t = 2$	0.60	0.90	0.58	1.67	2.58	1.72
$t = 3$	0.60	0.90	0.58	2.05	3.03	2.02
$t = 4$	0.60	0.95	0.58	2.13	3.07	2.03
$t = 5$	0.63	0.95	0.62	2.40	3.35	2.27
Profit (MUSD)	4,284	2,262	1,513	4,711	2,568	1,739
ROCE	467 %	454 %	440 %	373 %	354 %	348 %
CAGR	41.5 %	40.8 %	40.1 %	36.5 %	35.3 %	35.0 %
CAGR Extra	-	-	-	4.8 %	4.9 %	5.0 %
Extra Return	25.1 %	28.2 %	26.6 %	-	-	-
Extra Profit	-	-	-	10.0 %	13.5 %	14.9 %
VSS	3.7 %	5.0 %	5.4 %	3.2 %	0.0 %	0.7 %
Space charter cost						
SS (MUSD)	744	327	233	297	130	88
DS (MUSD)	939	537	366	576	149	110

Table 6.10: Results from running test with increased freight revenue

the upper bound, not by new solutions. Thus, using 1% optimality gap should still give good enough results to be able to compare solutions between the formulations.

6.6.1 Impact of increased freight revenue

The freight revenue parameter was based on data from the case company, calculated to reflect trip costs for the least efficient ship in the fleet, plus a posit margin. The level of freight revenue is important, because it will determine the possible profit, and thus what investments are profitable, both for the ROCEMax Formulation and the ProfitMax Formulation.

By increasing the freight revenue by 56%, from 32 cents to 50 cents per nautical mile a unit is shipped, it seems reasonable to believe that investments should be more profitable, and that the formulations will choose to buy more ships, and possibly service more optional trades for the ROCEMax formulation (remember that the ProfitMax formulation already service all optional trades, except one).

The results from running the same tests as in Section 6.4, using a higher freight revenue, are shown in Table 6.10

The results in Table 6.10 show that increasing the freight revenue increases the structural difference between the two formulations. The solutions from ProfitMax Formulation builds more ships, and choose to service more trades, since the profitability per cargo shipped has increased. The ROCEMax Formulation on the other hand choose to buy no ships in the first stage, and fewer ships over the entire planning horizon. The ROCEMax Formulation chooses to use the space charter option more than compared to the base case. This is the result of the space charter option being relatively cheaper when the freight revenue increases, and the space charter cost stays the same.

The ProfitMax Formulations also choose to service the last trade, which was not

	ROCEMax		ProfitMax	
	Med	Small	Med	Small
New builds ordered				
$t = 0$	3.8	3.1	13.6	12
$t = 1, .., T$	6.9	4.9	14.0	9.3
Scrappings				
$t = 0$	-	-	-	-
$t = 1, .., T$	7.1	5.1	7.2	5.2
Optional trades				
$t = 1$	1.20	0.67	1.85	1.42
$t = 2$	1.22	0.77	3.30	2.77
$t = 3$	1.22	0.78	5.00	4.00
$t = 4$	1.23	0.78	5.00	4.00
$t = 5$	1.25	0.80	5.00	4.00
Profit (MUSD)	1,072	733	1,369	983
ROCE	195 %	189 %	158 %	149 %
CAGR	24.1 %	23.6 %	20.8 %	20.1 %
CAGR Extra	-	-	16.0 %	17.0 %
Extra Return	23.5 %	26.2 %	-	-
Extra Profit	-	-	27.7 %	34.1 %
VSS	16.6 %	22.4 %	1.1 %	2.0 %
Space charter cost				
SS	202	127	131	89
DS	470	338	157	120

Table 6.11: Results from running tests with extra optional trades

serviced in any cases in the base case. The ROCEMax formulation, on the other, hand services fewer optional trades, but the trades chosen has a higher demand, so total units shipped increases.

6.6.2 Impact of increasing the number of optional trades

Optional trades are both a recourse action if the company ends up with overcapacity in the fleet, as well as an expansion option, if the company wants to expand the fleet by servicing more trades. In Section 6.4 it was shown that the ROCEMax formulation most likely used the optional trades as a recourse action if the market status turned out to be lower than expected. For the ProfitMax formulation, all optional trades, except one, were chosen for all tests. By increasing the number of optional trades available for the medium and small sets, it can be expected that the difference between the ROCEMax and ProfitMax formulation increases, especially in terms of number of ships built.

The new sets of optional trades for the medium set include T12 and T13, in addition to the ones listed in Table 6.2. The set of new optional trades for the small set include T9 and T13 in addition to the ones listed in Table 6.2. The result from running the tests with a new set of optional trades is shown in Table 6.11.

The results in Table 6.11 show that increasing the number of optional trades will increase the difference in solutions between the ROCEMax and ProfitMax formulations. The numbers of new build ships have increased slightly for the ROCEMax Formulation, while it has increased by approximately 10 ships for the entire planning horizon for both the medium and small sets for the ProfitMax Formulation. As for the results from the base case, the ProfitMax Formulation chooses to service all optional trades, except one, while the ROCEMax Formulation uses the optional trades mostly as a recourse action in cases of overcapacity in the fleet. There is actually a decrease in the number of optional trades for the small set compared to the base

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t = 0$	-	-	-	20.4	9.1	6.2
$t = 1, .., T$	15.9	8.8	6.5	10.9	9.9	7.8
Scrappings						
$t = 0$	-	-	-	-	-	-
$t = 1, .., T$	11.4	7.4	5.2	15.9	8.5	5.9
Optional trades						
$t = 1$	0.57	0.62	0.45	0.97	1.67	1.07
$t = 2$	0.57	0.62	0.47	1.63	2.35	1.53
$t = 3$	0.57	0.62	0.48	2.00	3.00	2.00
$t = 4$	0.58	0.63	0.55	2.00	3.00	2.00
$t = 5$	0.73	0.80	0.78	2.00	3.00	2.00
...						
$t = 10$	0.85	0.93	0.85	2.97	3.92	2.80
Profit (MUSD)	4 515	2 340	1 605	4 928	2 711	1 841
ROCE	288 %	276 %	268 %	243 %	232 %	219 %
CAGR	14.5 %	14.2 %	13.9 %	13.1 %	12.7 %	12.3 %
CAGR Extra	-	-	-	9.0 %	9.2 %	9.8 %
Extra Return	18.4 %	18.8 %	22.1 %	-	-	-
Extra Profit	-	-	-	9.2 %	15.9 %	14.7 %
VSS	3.7 %	5.0 %	5.5 %	4.6 %	1.6 %	4.4 %
Space charter cost						
SS	676	319	202	301	130	90
DS	931	523	355	704	190	206

Table 6.12: Results from running the test with a planning horizon of 10 years

case, but this is explained by the difference in demand for the optional trades, so units shipped has increased when increasing available optional trades also for the small set.

When looking at the VSS in Table 6.11, the results from the ROCEMax formulations is approximately the same, both the medium and large set both being slightly higher than for the base case. This results is as expected, since the other results did not differ much as well. For the ProfitMax Formulation, the VSS is significantly lower compared to the base case. This is best explained by considering the optional trades as recourse options. The DS will choose to invest planning to service extra optional trades. In cases where the demand increases, these investments give flexibility in the fleet, so that fewer optional trades can be serviced, to reduce the use of space charters on contractual trades. This is seen by the relative small increase in space charter cost from the SS to the DS.

6.6.3 Impact of increasing the planning horizon

The planning horizon was initially set to five years, where the potential future profit of the ships in the fleet at the end of the planning horizon was reflected in the sunset value. The sunset value may not necessarily be able to reflect this potential future profit in a good way. By increasing the planning horizon from five to 10 years, it is possible that this profit potential will be better reflected. By increasing the planning horizon from five to 10 years, nothing is changed about the problem. The uncertainty is still modeled the same way, and there are no ships leaving the fleet due to age from year six to 10. Therefore, increasing the planning horizon can be considered estimating the sunset value in a more detailed way.

The results from running the tests using a planning horizon of 10 years is shown in Table 6.12.

The ROCE listed in Table 6.12 is lower than the correct value. This is the result of the linearization constraint (5.90) being wrongly implemented in terms of estimating the average ROCE over the planning Horizon (the term $\bar{T}+1$ was not updated from a five year planning horizon). This error does however not influence the optimal solutions, as it is only a scaling issue, making the average Capital Employed too high. The error is only present in the results in this section.

The results shown in Table 6.12 show that there is no major impact of increasing the length of the planning horizon. Since all values are discounted, the impact of profits in the later periods of the planning horizon has less impact on the final results in terms of profit and ROCE. On the other hand, these periods have a higher demand (since a slight increase in the market status is implemented in the mean value), and no investments are made, increasing the profit made in these periods.

The high profit show that the sunset value does not fully reflect the future profit potential of a ship. The ROCEMax formulation chooses to invest in fewer ships over the planning horizon, and none in the first stage. This result comes from the needed return on new investment is higher in this case, since the profitability of the ships in the fleet increase when the planning horizon increases.

The VSS is lower than for the base case. This is best explained by the longer planning horizon giving better possibilities of correcting bad decisions in the first stage, leading to the impact of these decisions being lower as the planning horizon is extended.

6.6.4 Impact of changing space charter cost

Initially, the space charter cost was set to 2,000 USD per unit of cargo shipped, as a high cost for a recourse option that is not to be used often. This approach is not necessarily always true. In this section, the space charter costs will be estimated by using an even higher cost, 5,000 USD per unit shipped, for HH and BB, because these may be difficult to ship in an space charter market, and this should not be a viable option unless it is an extreme case (to ensure feasibility). The space charter cost for cars is reduced, set at a slightly higher level then the revenue made from shipping goods, 40 cents as compared to 32 cents per nautical mile. The space charter cost for cars is also set to vary with the market status. The total effect of these changes in the space charter cost is most likely a reduction of the space charter cost for the shipping company, since demand for cars is the highest.

In Section 6.4 it was shown that the ROCEMax and ProfitMax formulation had different approaches to space charter costs vs new builds. The ROCEMax formulation was more likely to use space charter than the ProfitMax Formulation, instead of building more internal capacity. Also, Section 6.5 showed that the space charter cost had a high impact on the VSS.

The results shown in Table 6.13 show that the ROCEMax formulation chooses not to build any ships in the first stage, but waits until later in the planning horizon. The total number of ships bought is also reduced. This shows that when space charter cost is reduced, the ROCEMax formulation to an ever higher degree choose to use space chartering as an option instead of investing in new ships. The ProfitMax formulation also chooses to invest in fewer ships both in the first stage and the second stage, except for the small set, where the number of new build is higher for the second stage as compared to the base case. This may be because of the change in space charter cost, in the case of the small set, results in postponement of investment decisions being more profitable.

When looking at the VSS, this is significantly lower, as one would expect. The DS still needs to take a much higher space charter cost than the SS, but as this cost is reduced, the total impact of this increase is lower, and so the VSS ends up being much lower than for the base case.

The result of changing the structure of the space charter cost led to larger differences between the formulations, and lower VSS. The space charter cost was essentially reduced, since the cost was reduced for cars, which is the cargo type with the highest

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t = 0$	-	-	-	10.3	5.4	4
$t = 1, .., T$	1.3	0.9	-	13.1	10.2	7.7
Scrappings						
$t = 0$	3.7	0.8	-	-	-	-
$t = 1, .., T$	12.9	8.1	5.6	10.3	7.1	5.1
Optional trades						
$t = 1$	0.70	0.93	0.63	0.98	1.77	1.08
$t = 2$	0.72	0.93	0.63	1.23	2.03	1.33
$t = 3$	0.72	0.93	0.63	2.00	3.00	2.00
$t = 4$	0.72	0.93	0.63	2.00	3.00	2.00
$t = 5$	0.73	0.97	0.63	2.00	3.00	2.00
Profit (MUSD)	1,762	955	628	2,232	1,203	814
ROCE	259 %	250 %	250 %	194 %	190 %	178 %
CAGR	29.1 %	28.5 %	28.4 %	24.1 %	23.7 %	22.7 %
CAGR Extra	-	-	-	10.3 %	10.8 %	11.3 %
Extra Return	33.5 %	31.8 %	40.2 %	-	-	-
Extra Profit	-	-	-	26.7 %	26.0 %	29.6 %
VSS	1.5 %	1.7 %	2.2 %	0.3 %	0.7 %	0.2 %
Space charter cost						
SS	729	444	219	165	64	36
DS	880	508	241	257	69	71

Table 6.13: Results from runnings tests with updated space charter costs

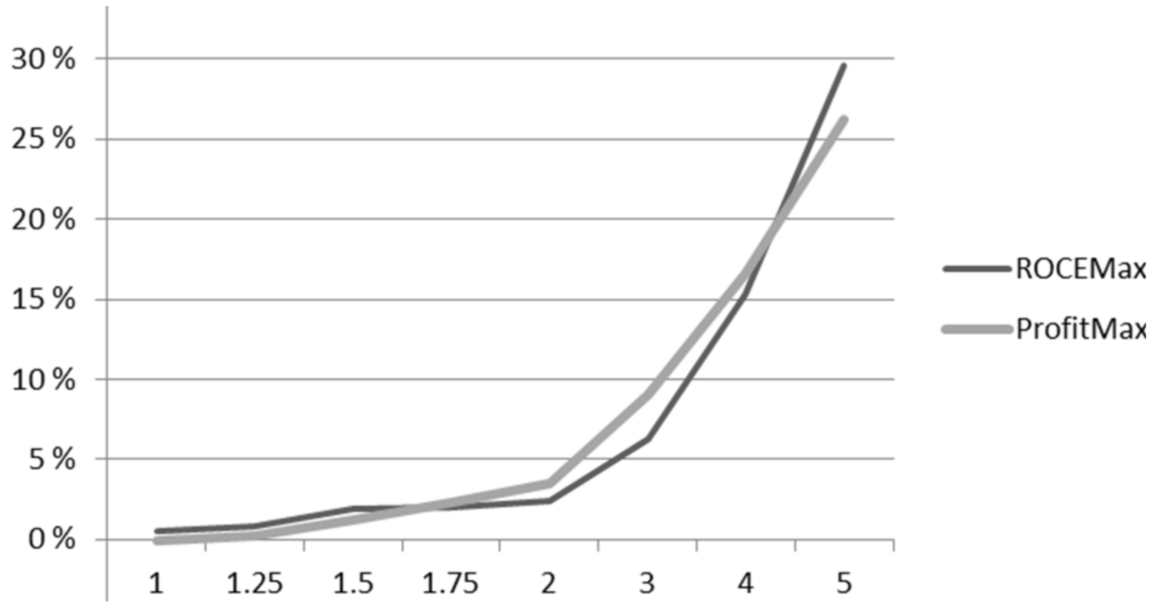


Figure 6.2: Development of VSS with increasing space charter cost

demand. It can be discussed whether the level of space chartering is realistic. No restrictions are put on the use of space chartering, other than the cost.

The result in Table 6.13 showed the effect of reducing the space charter cost. It can also be interesting to consider what happens if the space charter cost increases. A test was done, for one data instance of the large set, using 10 levels of space charter costs, to see how the results differ with the space charter cost. Only the space charter cost for cars was changed, for HHs and BBs, the cost of 10,000 USD used in this section is still applied. The space charter cost is set relative to the freight revenue, starting out at the same level, and gradually increasing in 10 steps up to 20 times the freight revenue, to test for an extreme case. The increase is small at first, and gradually increases. Figures 6.2 and 6.3 show the result from this test in terms of VSS and similarity between first stage decisions from the formulations. The x-axis shows the level of space charter cost for cars, compare to the freight revenue.

Figure 6.2 show that the VSS increase with increasing space charter cost. When the space charter cost passed went from five times to 10 times the freight revenue, the

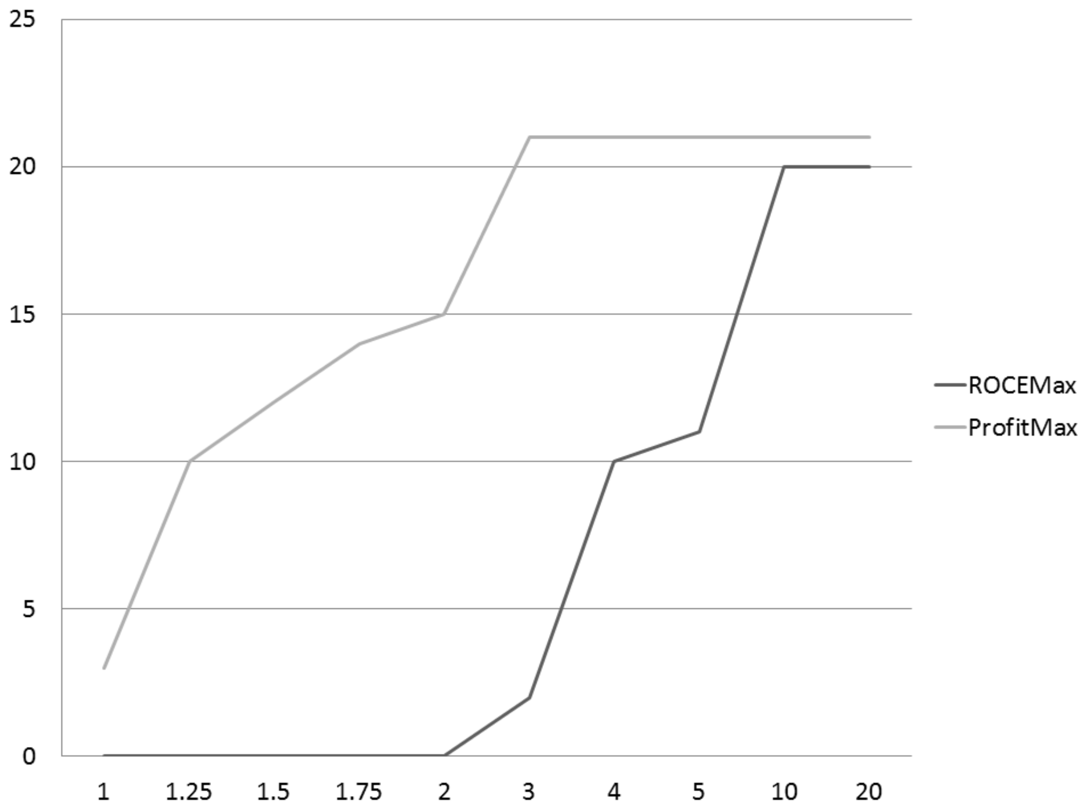


Figure 6.3: Comparison of new build decisions in the first stage between the ROCEMax and ProfitMax formulations with increasing space charter costs

VSS increased to 200% for the ROCEMax formulation, and 185% for the ProfitMax Formulation. At four times the level of the freight revenue, the VSS is approximately the same as for the base case.

Figure 6.3 compares the number of ships that the ROCEMax Formulation and ProfitMax Formulation build in the first stage. The results show that increasing space charter cost makes the solutions from the two formulations more similar to each other, in terms of the first stage solutions. The economic results also become more similar as the space charter cost increases, but it takes an extreme increase to make the solutions almost similar. The case of space charter cost at 20 times the freight revenue practically means that the space charter option is not to be used, except for ensuring feasibility in cases of extreme demand where there is no possibility of having enough capacity in the fleet. Giving a realistic value to the space charter cost is of great importance, especially when comparing the suggested solution from the two formulations.

6.6.5 Summary of impact of parameter values

The tests done in this section show that the impact of changing the values of parameters in the manner discussed in this section did not change the structural differences in the solutions proposed by the ROCEMax and ProfitMax Formulations. The changes in solutions from the base case only increased the difference in solutions between the two formulations, except for increasing the space charter cost. However, an extreme change in the space charter cost was necessary to end up with solutions which were not structurally different. This shows that the ROCEMax Formulation is far more conservative than the ProfitMax Formulation, and the structure of the solutions are not sensitive to changes in the parameter values.

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t=0$	3.4	0.6	-	16.6	5.8	3.8
$t=1,..,T$	12.3	6.0	3.7	14.7	12.1	9.8
Scrappings						
$t=0$	-	-	-	-	-	-
$t=1,..,T$	12.0	7.5	5.2	10.5	7.1	5.0
Optional trades						
$t=1$	0.78	0.87	0.63	1.17	2.08	1.42
$t=2$	0.78	0.87	0.63	1.63	2.32	1.58
$t=3$	0.78	0.90	0.65	2.00	3.00	2.00
$t=4$	0.78	0.90	0.65	2.00	3.00	2.00
$t=5$	0.78	0.90	0.67	2.00	3.00	2.00
Profit (MUSD)	1,991	1,103	769	2,174	1,226	857
ROCE	219 %	237 %	250 %	179 %	180 %	178 %
CAGR	26.1 %	27.5 %	28.5 %	22.8 %	22.9 %	22.7 %
CAGR Extra	-	-	-	14.2 %	13.2 %	12.7 %
Extra Return	22.3 %	31.6 %	40.0 %	-	-	-
Extra Profit				9.2 %	11.2 %	11.5 %
VSS	10.5 %	18.7 %	18.7 %	6.0 %	2.5 %	2.3 %
Space charter cost						
SS	289	77	49	96	13	9
DS	581	250	192	318	43	73

Table 6.14: Results from running tests with option of chartering in and out ships

6.7 Including charters and second hand market

The formulations presented in Chapter 5 included options for chartering in and out ships, and a second hand market for buying and selling ships. For the computational study in Chapter 6, these options were not included, by setting the limits to 0.

In this section, the option of chartering in and out ships, and buying and selling ships in a second hand market will be included. The limits for these each of options have been set to maximum one ship per type per period, and maximum a total of three ships per period.

Only PCTC and LCTC type of ships are available for chartering and in the second hand market. The exclusion of ships of type RORO comes from the fact that RORO ships have a very specific design, accustomed to ro-ro shipping. Because of this, it is not likely to find these type of ships available for charters or in the second hand market. A limit of one available ship of each type, and a total of three ships per time period to be chartered in or out is set.

When testing the formulations in Chapter 6, there existed only one recourse action in addition to investing and disposing of ships, namely the space charter option. By including charters of ships and a second hand market, these can be considered additional recourse actions. Pantuso et al. (2014) showed that increasing the number of ships available for charters decreased the value of the stochastic solution.

The inclusion of a second hand market means introducing new integer variables, which complicates the problem, and increase the solution time. For this reason, a space charter option is first included without a second hand market, before a second hand market is included as well. The results from running the ROCEMax and ProfitMax formulations with the possibility of chartering ships are shown in Table 6.14.

The results in Table 6.14 have the same structural differences as the base case and

	Charters in					Charters out				
$t =$	1	2	3	4	5	1	2	3	4	5
ROCE										
Large	1.8	2.1	1.6	2.1	2.5	1.9	1.8	1.4	1.0	0.6
Med	1.5	2.2	2.1	2.4	2.9	2.6	1.7	1.3	0.8	0.7
Small	1.5	2.2	2.1	2.3	2.8	2.2	1.2	0.9	0.6	0.5
Profitmax										
Large	1.9	1.1	0.5	0.4	0.1	1.9	2.7	3.4	2.8	2.0
Med	1.7	1.4	0.7	0.6	0.3	2.1	2.1	3.2	2.1	1.3
Small	1.7	1.2	0.4	0.2	0.1	1.8	2.3	3.2	2.0	1.0

Table 6.15: Charters in and out for the test with charter option

the tests done in the previous section. The ROCEMax Formulation also choose to use the charter options far more often and to a greater degree than the ProfitMax Formulation. The VSS decreases for the ProfitMax Formulation, which corresponds to the findings in Pantuso et al. (2014), but not for the ROCEMax Formulations. This is because the ROCEMax Formulation uses chartering of ships as an replacement for building new ships, while the ProfitMax Formulation use the chartering much less, and more as recourse option and while waiting for delivery of new builds, which can be seen from the results in Table 6.15. This means that the DS for the ROCEMax Formulation plan to charter ships, so these limits on charters are already reached, and chartering ships does not act as an recourse option for the DS.

When a second hand market is included as well, the formulations does not reach a 1% optimality within the time limit, the gap at this time varies from 1.5% for the small set to 5.5% for the large set. Because of this, the results from including a second hand market should not be considered accurate results. There is still reason to believe that the structure in the solutions proposed with this gap still reveal some traits about the problem when a second hand market is included. The results from

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t=0$	0.7	-	-	10.3	2.0	1.0
$t=1,..,T$	4.9	1.4	0.0	13.1	11.1	8.2
Scrappings						
$t=0$	-	-	-	-	0.1	0.4
$t=1,..,T$	10.3	7.0	5.0	10.2	7.0	3.3
Optional trades						
$t=1$	0.67	0.42	0.25	1.32	2.40	1.58
$t=2$	0.67	0.42	0.25	1.63	2.68	1.87
$t=3$	0.67	0.42	0.25	2	3	2
$t=4$	0.67	0.42	0.25	2	3	2
$t=5$	0.67	0.42	0.27	2	3	2
Profit (MUSD)	2,106	1,154	819	2,294	1,314	919
ROCE	250 %	289 %	319 %	201 %	213 %	208 %
CAGR	28.5 %	31.2 %	33.2 %	24.7 %	25.6 %	25.3 %
CAGR Extra	-	-	-	11.6 %	9.5 %	9.0 %
Extra Return	24.4 %	35.5 %	53.0 %	-	-	-
Extra Profit	-	-	-	8.9 %	13.8 %	12.1 %
VSS	15.0 %	12.9 %	20.0 %	7.4 %	3.1 %	1.4 %
Space charter cost						
SS	278	76	14	128	8	4
DS	824	287	220	390	56	24

Table 6.16: Results from running tests with charter options and a second hand market

	Charters in					Charters out				
$t =$	1	2	3	4	5	1	2	3	4	5
ROCE										
Large	1.6	1.9	1.9	2.1	2.8	2.1	1.3	0.8	0.5	0.1
Med	1.4	2.2	2.5	2.7	3.3	1.7	0.7	0.2	0.0	0.0
Small	1.3	2.0	2.4	2.4	2.8	2.4	0.7	0.1	0.0	0.0
Profitmax										
Large	1.8	1.2	0.6	0.4	0.1	2.0	2.4	2.6	2.0	1.0
Med	1.5	1.4	1.1	1.1	0.5	2.2	1.4	2.2	1.6	0.6
Small	1.1	1.1	0.7	0.5	0.1	2.7	2.9	3.1	2.2	0.5

Table 6.17: Charters in and out for the test with charter option and second hand market

	Second hand purchases					Second hand sales				
$t =$	0	1	2	3	4	0	1	2	3	4
ROCE										
Large	3.0	1.5	2.0	2.1	2.1	0.0	1.4	1.2	1.0	0.5
Med	2.0	1.4	1.9	2.4	2.6	1.7	2.0	1.6	1.0	0.4
Small	2.0	0.9	1.5	1.9	2.5	1.7	1.9	1.1	0.8	0.3
Profitmax										
Large	3.0	1.6	1.4	1.4	1.0	0.0	1.8	2.7	2.8	2.3
Med	3.0	2.3	1.8	1.9	1.3	0.0	1.7	2.6	2.4	1.8
Small	3.0	2.3	1.8	1.9	0.9	0.0	1.4	2.1	2.2	1.5

Table 6.18: Second hand purchases and sales

running the tests with a charter option and a second hand market is shown in Table 6.16.

The results in Table 6.16 show that the inclusion of a second hand market as well as a charter option does not change the difference between the solutions. Both formulations build fewer ships in the first stage and for the entire planning horizon, this is because more ships are bough in the second hand market, as well as the charter option reduces the need for investments in the fleet.

The same remarks about the VSS as for the case with only charter option can be made, except that the reduction is a little lower for the ProfitMax formulation, and the ROCEMax Formulation see an increase in VSS compared to the base case. This is most likely the result of the formulations using the second hand market both for investments and as a recourse option. The second hand market however, have a lead time of one period for deliveries of ships, as compared to the charters, which are available right away. Both Formulations buy and sell in the second hand market throughout the planning horizon. The ROCEMax Formulation buys more than it sells, meaning that it use the second hand market as recourse option when demand increases. The ProfitMax Formulation on the other hand sell more than in buys, meaning that it use the second hand market as a recourse option when demand is decreasing. These results can be seen in Tables 6.17 and 6.18

6.8 Going from a two-stage to a three-stage case

In Chapter 6, it was shown that by using a two-stage stochastic model, it was possible to achieve a VSS ranging from 11.9 to 21.7 %. In the two-stage stochastic model used in Chapter 6, the realization of all random variables for the rest of the planning horizon was known from time period $t = 1$. In this section, a three-stage model will be presented and tested. The three-stage model consists of an additional stage compare to the two-stage model used in Chapter 6. The three-stage model means

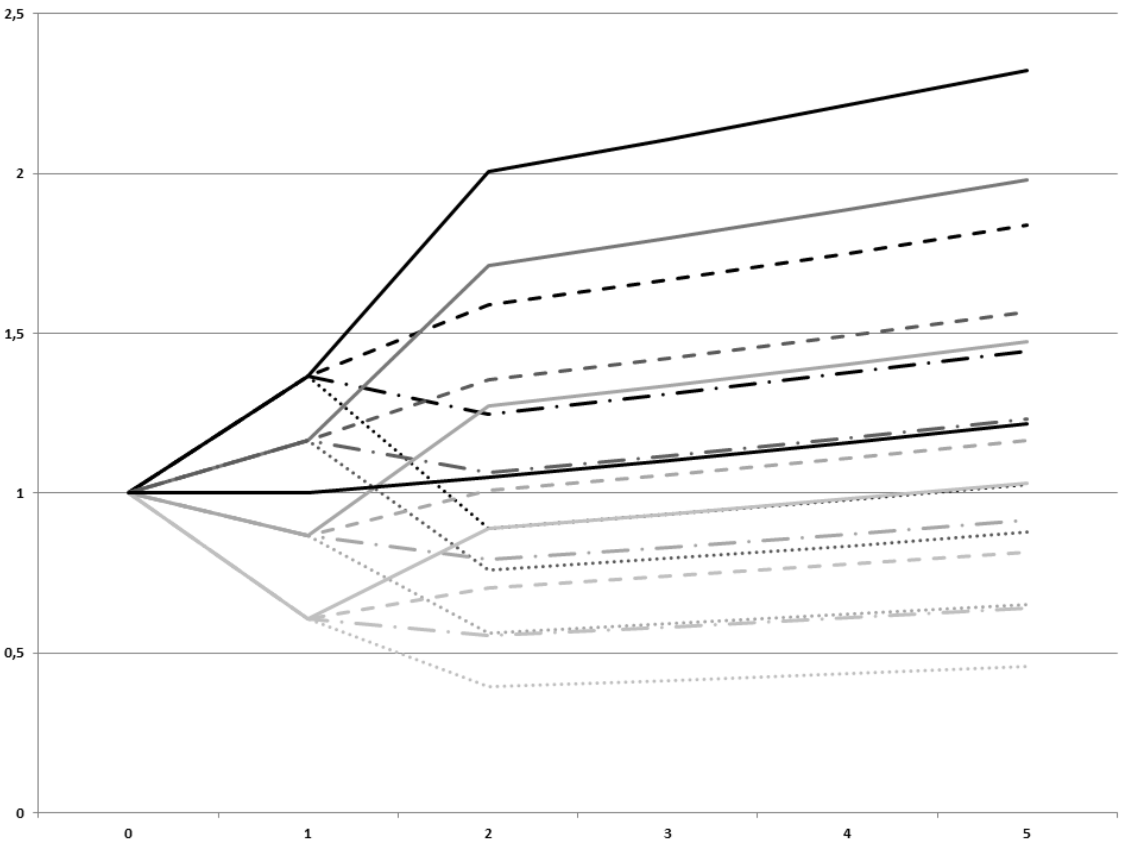


Figure 6.4: Example of three stage scenario tree, for one random variable. The expected value is the middle black line.

that in time $t = 1$, only a first realization will be discovered, the random variables further development is still uncertain for the rest of the planning horizon, this is discovered in time $t = 2$. Picture 6.4 shows an illustration of the scenario three in the three stage model.

The results from running the ROCEMax and Profit Max formulations on a three stage data instance is shown in Table 6.19

The results show that going from a two-stage model to a three stage model does not change the results significantly. The structural differences that were found in the results from the base case is still present. For multistage problems, the calculation of the VSS is more complex than for a two stage problem, as is presented in Escudero

	ROCEMax			ProfitMax		
	Large	Med	Small	Large	Med	Small
New builds ordered						
$t=0$	5.4	2.8	2.0	16.7	8.2	5.7
$t=1,..,T$	7.6	3.4	2.4	5.6	4.8	3.3
Scrappings						
$t=0$	-	-	-	-	-	-
$t=1,..,T$	9.2	5.1	3.6	8.1	5.2	3.5
Optional trades						
$t=1$	0.38	0.50	0.42	1.28	2.15	1.30
$t=2$	0.77	0.98	0.72	1.65	2.48	1.55
$t=3$	0.98	1.25	0.93	1.77	2.77	1.75
$t=4$	0.80	1.05	0.80	1.75	2.72	1.80
$t=5$	0.90	1.12	0.80	1.80	2.85	1.87
Profit (MUSD)	1,960	1,071	729	2,108	1,158	784
ROCE	205 %	205 %	201 %	185 %	183 %	178 %
CAGR	25.0 %	25.0 %	24.6 %	23.3 %	23.2 %	22.7 %
CAGR Extra	-	-	-	14.7 %	14.8 %	15.5 %
Extra Return	11.3 %	11.9 %	12.9 %	-	-	-
Extra Profit				7.5 %	8.1 %	7.6 %
VSS_1	7.3 %	9.0 %	11.8 %	6.4 %	1.9 %	6.2 %
Space charter cost						
SS	296	84	47	94	14	9
DS	576	251	192	318	42	74

Table 6.19: Results from running the tests using a three stage data instance

et al. (2007) . The calculation of the VSS as presented in Escudero et al. (2007) will not be done in here. The VSS presented in Table 6.19 is therefore indexed by 1, meaning that the decisions in $t = 0$ for the EVP have been locked when solving with the stochastic scenario tree. Escudero et al. (2007) show that VSS_1 is lower than VSS_2 for the problem (which is not calculated), so the reduced VSS_1 as compared to the VSS for the base case, seems reasonable.

6.9 Adjusting the calculation of ROCE

All tests so far have shown that the ROCEMax Formulation is far more conservative in building decisions as compared to the ProfitMax Formulation. When estimating the ROCE, it can be argued that the cost of building a ship acts as a double negative factor, since it both increases the capital employed and decreases the profit. The new build cost was included so that the ROCEMax formulation and ProfitMax Formulation could be compared easily (P^O as discussed in Chapter 2 is the profit from the ProfitMaxFormulation). In this section, tests where the new build cost has been omitted from the profit estimation in the ROCEMax Formulation will be done. Investing in new builds will then just effect the capital employed. The results from these tests are shown in Table 6.20.

The results in Table 6.20 show that omitting the cost of new building from the profit estimation in the ROCEMax Formulation does increase the number of ships built, as compared to the results from the base case, as presented in Section 6.4. The increase is however only slight, and the structural difference in decisions between the formulations is still clearly present. It should be noted that the reason this test is not run for the ProfitMax Formulation, is that omitting the new build cost from the ProfitMax Formulation would lead to an unbounded problem, which is not the case for the ROCEMax Formulation. The economic results in Table 6.20 cannot be compared directly to the results from the ProfitMax Formulation, since they have

	ROCEMax		
	Large	Med	Small
New builds ordered			
$t = 0$	10.1	4.3	3
$t = 1, .., T$	13.0	9.7	6.5
Optional trades			
$t = 1$	0.95	1.38	0.90
$t = 2$	1.03	1.40	0.93
$t = 3$	1.15	1.50	1.03
$t = 4$	1.25	1.68	1.167
$t = 5$	1.93	2.75	1.82
Profit (MUSD)	2,498	1,364	922
ROCE	239%	236%	213%
CAGR	27.7%	27.45%	27.1%
CAGR Extra	-	-	-
Extra Return	41.8%	42.9%	43.9%
Extra Profit	-	-	-
VSS	10.0%	12.2%	15.4%
Space charter cost			
SS	412	121	54
DS	812	392	228

Table 6.20: Results from omitting new build cost from profit calculation

different approaches to estimate profit and thus ROCE. The results suggest that investigating the calculation of investment returns further will be of value value in future research.

6.10 Summary of the computational study

The computational study has investigated the solutions from the two formulations developed in Chapter 5. The solutions were based on a base case, as well as testing for varying input parameters, including charters and a second hand market, expanding to a three-stage case, and adjusting the calculation of ROCE.

The main finding is that the ROCEMax Formulation is far more conservative compared to the ProfitMax Formulation for all cases. The solutions vary, but the structural differences between the formulations is present in all cases. The ROCEMax Formulation choose to use the space charter option to a lager degree than the ProfitMax Formulation, so modeling the use of this as realistic as possible will be important for the validity of the model. Adjusting the calculation of ROCE did also have a impact on the solutions suggested, so further studies into how this should be calculated should be done. Ideally, the calculation of ROCE, or another measure for investment returns, should be done in the same way as is done by investors.

Chapter 7

Concluding remarks

This thesis introduced a new model for solving the Maritime Fleet Renewal Problem (MFRP). This model maximizes investment returns in the objective function, in terms of ROCE, as compared to profit maximization or cost minimization which has been the only objective functions proposed by current literature about the MFRP.

The introduction of investment returns in the objective function gave a more complex formulation than the profit maximizing model that was also developed. The objective function ends up as a fraction, so a transformation must be made, and special solutions for handling the binary and integer variables were needed. Still, the new ROCEMax Formulation was possible to solve for the instances developed for this report, within reasonable time to be applicable for industry adaptation. More focus should still be given to improve the model in terms of computational effectiveness.

The results showed the the new ROCEMax Formulation gave structurally different solutions from the ProfitMax Formulation. The differences were not sensitive to adjustments of parameter values, so the new model can be considered giving a valuable contribution to the field of strategic maritime problems in operations research. The literature indicates that the use of investment returns has not been applied to these type of problems in operations research before. The dimension of considering the investments needed for the solutions suggested, is therefore a new addition to the field of strategic, maritime problems.

The ROCEMax Formulation should be developed to better reflect the return measures used by shipping companies. ROCE was chosen for measure for this thesis, but other measures may also be appropriate. The calculation of ROCE in the model should also be given attention in the further work with the model.

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Appendix A

Formulations

A.1 ProfitMax Formulations

A.1.1 Profit Max Scenario Formulation

Sets	
T	set of periods, indexed by t, final period $\bar{T}$
S	set of scenarios, indexed by s
V_{t}	$= \{0 \leq A_{vt} \leq \bar{A}\}$ set of ship types existing in the marked in period t, indexed by v
N_{t}^{C}	set of mandatory trades (where there are contractual obligations) operated in period t, indexed by i
N_{t}^{O}	set of optional trades (where there are no contractual obligations) in period t, indexed by i
N_{t}	set of all trades in period t, indexed by i
R_{t}	set of loops in period t, indexed by r
$R_{vt} \subseteq R_t$	set of loops which can be sailed by a ship of type v in period t, indexed by r

R_{ivt}	set of loops servicing trade i which can be sailed by a ship of type v in period t, indexed by r
G	set of cargo types, indexed by$c$

Parameters

$\bar{T}^{\mathrm{L}}$	lead time for new buildings
P_s	probability of scenario s occurring
$\bar{A}$	lifetime of a generic ship
A_{vt}	age of a ship of type v in period t
C^{NB}_{vts}	building cost of a ship of type v in period t under scenario s
C^{SH}_{vt}	cost of a ship of type v in period t in the second-hand market
C^{OP}_{vts}	fixed operating cost for ship of type v in period t under scenario s
C^{TR}_{vrts}	cost of performing a loop r for ship of type v in period t under scenario s
C^{SP}_{its}	cost of voyage charter on trade i in period t under scenario s
C^{CI}_{vts}	cost of chartering in a ship of type v in period t under scenario s
R^{LU}_{vts}	lay up savings for one period t for a ship of type v under scenario s
R^{CO}_{vts}	charter out revenue for one period t for a ship of type v under scenario s
R^{SC}_{vts}	scrapping revenue for a ship of type v in period t under scenario s
R^{SH}_{vts}	revenue from selling a ship of type v in period t under scenario s in the second-hand market
R^{SV}_{vs}	sunset value for a ship of type v under scenario s
F_{it}	frequency requirement for trade i in period t
Z_{rv}	total time it takes for a ship of type v to perform loop r
Z_v	total available time for a ship of type v in one period
Q_{cv}	total capacity for a ship of type v
$\overline{CI}_{vts}$	limit on number of available ships for chartering in of type v in period t under scenario s

$\overline{CO}_{vts}$	limit on number of possible ships for chartering out of type v in period t under scenario s
$\overline{SH}_{vts}$	limit on number of available ships in the second hand market of type v in period t under scenario s
$\overline{SE}_{vts}$	limit on number of possible ships that can be sold in the second hand market out of type v in period t under scenario s
$\overline{CI}_{ts}$	limit of total number of ships that can be chartered in in period t under scenario s
$\overline{CO}_{ts}$	limit of total number of ships that can be chartered out in period t under scenario s
$\overline{SH}_{ts}$	limit of total number of ships that can be bought in the second hand market in period t under scenario s
$\overline{SE}_{ts}$	limit of total number of ships that can be sold in the second hand market in period t under scenario s
D_{its}	agreed amount of cargo to be transported on trade i in period t under scenario s
Y_v^{IP}	initial pool of ship type v
Y_{vt}^{NB}	new ships of type v ordered in the sunk period, to be delivered in period t

Variables

y_{vts}^{NB}	number of new buildings ordered in period t under scenario s of ship type v
y_{vts}^{SC}	number of ships scrapped in period t under scenario s of type v
y_{vts}^{SH}	number of ships bought in the second-hand market in period t under scenario s of type v
y_{vts}^{SE}	number of ships sold in the second-hand market in period t under scenario s of type v
y_{vts}^{P}	number of ships in the pool at the end of period t under scenario s

x_{vrts}	number of loops r performed by ships of type v in period t under scenario s
l_{vts}	number of ships of type v on lay up for period t under scenario s
h^I_{vts}	number of ships of type v chartered in in period t under scenario s
h^O_{vts}	number of ships of type v chartered out in period t under scenario s
n_{its}	units of goods space chartered in on trade i in time t under scenario s
δ_{its}	binary variable, 1 if optional trade i is serviced in node n

Objective function

$$
\begin{aligned}
maxz = & \sum_{s \in S} P_s \Big(\sum_{t \in T, t>0} \big(\sum_{i \in N_t^O} \sum_{c \in G} R_{its}^D D_{itcs} \delta_{its} \\
& + \sum_{i \in N_t^C} \sum_{c \in G} (R_{its}^D Dicts - C_{icts}^{SP} n_{icts}) - \sum_{v \in V_t} (C_{vts}^{OP} y_{vts}^P \\
& + C_{vts}^{CO} h_{vts}^I - R_{vts}^{CO} h_{vt}^O \\
& + \sum_{r \in R_{vts}} C_{vrts}^{TR} x_{vrts} - R_{vts}^{LU} l_{vts}) \big) \\
& + \sum_{v \in V_{\bar{T}}} R_{vs}^{SV} y_{v\bar{T}s}^P \\
& - \sum_{t \in T, t \leq \bar{T} - T^L} \sum_{v \in V_{t+T^L}^N} C_{vts}^{NB} y_{vts}^{NB} \\
& + \sum_{t \in T} \sum_{v \in V_t} (R_{vts}^{SC} y_{vts}^{SC} \\
& + R_{vts}^{SE} y_{vts}^{SE} - C_{vts}^{SH} y_{vts}^{SH}) \Big)
\end{aligned}
$$

Demand constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} x_{vrts} + n_{icts} \geq D_{itcs}, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, s \in S,
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} x_{vrts} \geq D_{itcs} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, s \in S,
$$

Capacity constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinG} Q_{cv} x_{vrts} + \sum_{c \in G} n_{cits} \geq \sum_{c \in G} D_{itcs}, \qquad t \in T \setminus \{0\}, i \in N_t^C, s \in S,
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinG} Q_{cv} x_{vrts} \geq \sum_{c \in G} D_{itcs} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,
$$

Frequency constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} x_{vrts} \geq F_{it}, \qquad t \in T \setminus \{0\}, i \in N_t^C, s \in S,
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} x_{vrts} \geq F_{it} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,
$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv} x_{vrts} \leq Z_v(y^P_{vts} + h^I_{vt} - h^O_{vt} - l_{vts}), \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$

Optional trades constraints

$$\delta_{its} \leq \delta_{i,t+1,s} t \in T \setminus \{0, \bar{T}\}, i \in N^O_t, s \in S,$$

Pool constraints

$$y^P_{vts} = y^P_{v,t-1,s} - y^{SC}_{v,t-1,s} + y^{SH}_{v,t-1,s} - y^{SE}_{v,t-1,s}, \qquad t \in T \setminus \{0\}, v \in V_t \setminus V^N_t, s \in S,$$

$$y^P_{vts} = Y^{NB}_{vt}, \qquad t \in T : t < T^L, v \in V^N_t, s \in S,$$
$$y^P_{vts} = y^{NB}_{v,t-T^L,s}, \qquad t \in T : t \geq T^L, v \in V^N_t, s \in S,$$
$$y^P_{v0s} = Y^{IP}_v v \in V_0, s \in S,$$
$$y^P_{vts} = y^{SC}_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t \setminus v \in V_{t+1}, s \in S,$$
$$l_{vts} - h^I_{vts} + h^O_{vts} \leq y^P_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$

Charters and second hand market constraints

$$y^{SH}_{vts} \leq \overline{SH}_{vts}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SE}_{vts} \leq \overline{SE}_{vts}, \qquad t \in T, v \in V_t, s \in S,$$
$$h^I_{vts} \leq \overline{CI}_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$h^O_{vts} \leq \overline{CO}_{vts}, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$\sum_{v\in V_t \setminus V^N_t} y^{SH}_{vts} \leq \overline{SH}_{ts}, \qquad t \in T, s \in S,$$
$$\sum_{v\in V_t \setminus V^N_t} y^{SE}_{vts} \leq \overline{SE}_{ts}, \qquad t \in T, s \in S,$$
$$\sum_{v\in V_t \setminus V^N_t} h^I_{vts} \leq \overline{CI}_{ts}, \qquad t \in T \setminus \{0\}, s \in S,$$
$$\sum_{v\in V_t \setminus V^N_t} h^O_{vts} \leq \overline{CO}_{ts}, \qquad t \in T \setminus \{0\}, s \in S,$$

Non-anticipativity constraints

$$y^{SC}_{vts} = y^{SC}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts},$$
$$y^{NB}_{vts} = y^{NB}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts},$$
$$y^{SH}_{vts} = y^{SH}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts},$$
$$y^{SE}_{vts} = y^{SE}_{vt\bar{s}}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts},$$

Convexity and integer constraints

$$y^{NB}_{vts} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,$$
$$y^{SC}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\} \, v \in V_t, s \in S,$$
$$y^{SH}_{vt} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\} \, v \in V_t, s \in S,$$
$$y^{SE}_{vt} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\} \, v \in V_t, s \in S,$$
$$y^{P}_{vts} \in \mathbb{R}^+, \qquad t \in T \, v \in V_t, s \in S,$$
$$l_{vts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$h^{I}_{vt} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$h^{O}_{vt} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$x_{vrts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, s \in S.$$

A.1.2 ProfitMax node formulation

Set, parameters and variables

Sets

T	set of periods, indexed by t, final period $\bar{T}$
N	set of nodes in the scenario tree, indexed by n
V_{t}	$= \{0 \leq A_{vt} \leq \bar{A}\}$ set of ship types existing in the marked in period t, indexed by v
N_{t}^{C}	set of mandatory trades (where there are contractual obligations) operated in period t, indexed by i
N_{t}^{O}	set of optional trades (where there are no contractual obligations) in period t, indexed by i
N_{t}	set of all trades in period t, indexed by i
R_{t}	set of loops in period t, indexed by r
$R_{vt} \subseteq R_t$	set of loops which can be sailed by a ship of type v in period t, indexed by r
R_{ivt}	set of loops servicing trade i which can be sailed by a ship of type v in period t, indexed by r
G	set of cargo types, indexed by$c$

Parameters

$\bar{T}^{\mathrm{L}}$	lead time for new buildings
P_n	probability of being in node n
$n(t)$	all nodes n at time t in the scenario tree
$a(n,t)$	all ancestors of node n in the scenario tree in time t, $a(n,t)$ is written simply as $a(n)$
$\bar{A}$	lifetime of a generic ship
A_{vt}	age of a ship of type v in period t
C_{vn}^{NB}	building cost of a ship of type v in node n
C_{vn}^{SH}	cost of a ship of type v in node n in the second-hand market

C^{OP}_{vn}	fixed operating cost for ship of type v in node n
C^{TR}_{vrn}	cost of performing a loop r for ship of type v in node ns
C^{SP}_{in}	cost of voyage charter on trade i in node n
C^{CI}_{vn}	cost of chartering in a ship of type v in period n
R^{LU}_{vn}	lay up savings for one period t for a ship of type n
R^{CO}_{vn}	charter out revenue for one period t for a ship of type n
R^{SC}_{vn}	scrapping revenue for a ship of type v in period n
R^{SH}_{vn}	revenue from selling a ship of type v in period n in the second-hand market
R^{SV}_{vn}	sunset value for a ship of type v in node n
F_{it}	frequency requirement for trade i in period t
Z_{rv}	total time it takes for a ship of type v to perform loop r
Z_v	total available time for a ship of type v in one period
Q_{cv}	total capacity for a ship of type v
$\overline{CI}_{vn}$	limit on number of available ships for chartering in of type v in node n
$\overline{CO}_{vn}$	limit on number of possible ships for chartering out of type v in node n
$\overline{SH}_{vn}$	limit on number of available ships in the second hand market of type v in node n
$\overline{SE}_{vn}$	limit on number of possible ships that can be sold in the second hand market out of type v in node n
$\overline{CI}_{ts}$	limit of total number of ships that can be chartered in in node n
$\overline{CO}_{ts}$	limit of total number of ships that can be chartered out in node n
$\overline{SH}_{ts}$	limit of total number of ships that can be bought in the second hand market in node n
$\overline{SE}_{ts}$	limit of total number of ships that can be sold in the second hand market in node n
D_{in}	agreed amount of cargo to be transported on trade i in node n

Y_v^{IP}	initial pool of ship type v
Y_{vt}^{NB}	new ships of type v ordered in the sunk period, to be delivered in period t

Variables

y_{vn}^{NB}	number of new buildings ordered in node n of ship type v
y_{vn}^{SC}	number of ships scrapped in node n of type v
y_{vn}^{SH}	number of ships bought in the second-hand market in node n of type v
y_{vn}^{SE}	number of ships sold in the second-hand market in node n of type v
y_{vn}^{P}	number of ships in the pool at the end of the period in node n
x_{vn}	number of loops r performed by ships of type v in node n
l_{vn}	number of ships of type v on lay up for node n
h_{vn}^{I}	number of ships of type v chartered in in node n
h_{vn}^{O}	number of ships of type v chartered out in node n
n_{in}	units of goods space chartered in on trade i in node n
δ_{in}	binary variable, 1 if optional trade i is serviced in node n

Objective function

$$
\begin{aligned}
maxz = &\sum_{t\in T\setminus\{0\}}\sum_{n\in n(t)} P_n\Big(\sum_{i\in N_t^O}\sum_{c\in G} R_{in}^D D_{icn}\delta_{in}\\
&+\sum_{i\in N_t^C}\sum_{c\in G}(R_{in}^D D_{icn} - C_{icn}^{SP} n_{icn}) - \sum_{v\in V_t}(C_{vn}^{OP} y_{vn}^P\\
&+ C_{vn}^{CO} h_{vn}^I - R_{vn}^{CO} h_{vn}^O\\
&+ \sum_{r\in R_{vn}} C_{vrn}^{TR} x_{vrn} - R_{vn}^{LU} l_{vn})\Big)\\
&+ \sum_{v\in V_{\bar{T}}}\sum_{n\in n(\bar{T})} P_n R_{vn}^{SV} y_{vn}^P\\
&- \sum_{t\in T, t\leq \bar{T}-T^L}\sum_{v\in V_{t+T^L}^N}\sum_{n\in n(t)} P_n C_{vn}^{NB} y_{vn}^{NB}\\
&+ \sum_{t\in T}\sum_{v\in V_t}\sum_{n\in n(t)} P_n(R_{vn}^{SC} y_{vn}^{SC}\\
&+ R_{vn}^{SH} y_{vn}^{SE} - C_{vn}^{SH} y_{vn}^{SH})
\end{aligned}
$$

Demand constraints

$$
\sum_{v\in V_t}\sum_{r\in R_{ivt}} Q_{cv} x_{vrn} + n_{icn} \geq D_{icn}, \qquad t\in T\setminus\{0\}, i\in N_t^C, c\in G, n\in n(t),
$$

$$
\sum_{v\in V_t}\sum_{r\in R_{ivt}} Q_{cv} x_{vrn} \geq D_{icn}\delta_{in}, \qquad t\in T\setminus\{0\}, i\in N_t^O, c\in G, n\in n(t),
$$

Capacity constraints

$$
\sum_{v\in V_t}\sum_{r\in R_{ivt}} \max_{cinG} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{itcs}, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),
$$

$$
\sum_{v\in V_t}\sum_{r\in R_{ivt}} \max_{cinG} Q_{cv} x_{vrn} \geq \sum_{c\in G} D_{icn}\delta_{in}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),
$$

Frequency constraints

$$
\sum_{v\in V_t}\sum_{r\in R_{ivt}} x_{vrn} \geq F_{it}, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),
$$

$$
\sum_{v\in V_t}\sum_{r\in R_{ivt}} x_{vrn} \geq F_{it}\delta_{in}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),
$$

Time constraints

$$\sum_{r \in R_v t} Z_{rv} x_{vrn} \leq Z_v(y^P_{vn} + h^I_{vn} - h^O_{vn} - l_{vn}), \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{in}, \qquad t \in T \setminus \{0,1\}, i \in N^O_t, n \in n(t),$$

Pool constraints

$$y^P_{vn} = y^P_{v,a(n)} - y^{SC}_{v,a(n)} + y^{SH}_{v,a(n)} - y^{SE}_{v,a(n)}, \qquad t \in T \setminus \{0\}\; v \in V_t \setminus V^N_t, n \in n(t),$$

$$\begin{aligned}
y^P_{vn} &= Y^{NB}_{vt}, && t \in T : t < \bar{T}^L, v \in V^N_t, n \in n(t),\\
y^P_{vn} &= y^{NB}_{v,a(a(n))}, && t \in T : t \geq \bar{T}^L, v \in V^N_t, n \in n(t),\\
y^P_{v0} &= Y^{IP}_v, && v \in V_0,\\
y^P_{vn} &= y^{SC}_{v,a(n)}, && t \in T \setminus 0, v \in V_t \setminus v \in V_{t+1}, n \in n(t),\\
l_{vn} - h^I_{vn} + h^O_{vn} &\leq y^P_{vn}, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),
\end{aligned}$$

Charters and second hand market constraints

$$\begin{aligned}
y^{SH}_{vn} &\leq \overline{SH}_{vn}, && t \in T, v \in V_t,\\
y^{SE}_{vn} &\leq \overline{SE}_{vn}, && t \in T, v \in V_t, n \in n(t),\\
h^I_{vn} &\leq \overline{CI}_{vn}, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^O_{vn} &\leq \overline{CO}_{vn}, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
\sum_{v \in V_t \setminus V^N_t} y^{SH}_{vn} &\leq \overline{SH}_n, && t \in T, n \in n(t),\\
\sum_{v \in V_t \setminus V^N_t} y^{SE}_{vn} &\leq \overline{SE}_n, && t \in T, n \in n(t),
\end{aligned}$$

$$\sum_{v \in V_t \setminus V_t^N} h_{vn}^I \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

$$\sum_{v \in V_t \setminus V_t^N} h_{vn}^O \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

Convexity and integer constraints

$$y_{vn}^{NB} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V_{t+T^L}^N, n \in n(t),$$

$$y_{vn}^{SC} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$

$$y_{vn}^{SE} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$

$$y_{vn}^{SH} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$

$$y_{vn}^{P} \in \mathbb{R}^+, \qquad t \in T\, v \in V_t, n \in n(t),$$

$$l_{vn} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t),$$

$$h_{vn}^{I} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t),$$

$$h_{vn}^{O} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t),$$

$$x_{vrn} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t).$$

A.1.3 ROCEMax scenario formulations

Sets

T	set of periods, indexed by t, final period $\bar{T}$
S	set of scenarios, indexed by s
V_{t}	$= \{0 \leq A_{vt} \leq \bar{A}\}$ set of ship types existing in the marked in period t, indexed by v
N_{t}^{C}	set of mandatory trades (where there are contractual obligations) operated in period t, indexed by i
N_{t}^{O}	set of optional trades (where there are no contractual obligations) in period t, indexed by i
N_{t}	set of all trades in period t, indexed by i
R_{t}	set of loops in period t, indexed by r
$R_{vt} \subseteq R_t$	set of loops which can be sailed by a ship of type v in period t, indexed by r
R_{ivt}	set of loops servicing trade i which can be sailed by a ship of type v in period t, indexed by r
$\bar{D}$	set of decisions for building, scrapping, selling and buying ships, indexed by j
G	set of cargo types, indexed by$c$

Parameters

$\bar{T}^{\mathrm{L}}$	lead time for new buildings
P_s	probability of scenario s occurring
$\bar{A}$	lifetime of a generic ship
A_{vt}	age of a ship of type v in period t
C_{vts}^{NB}	building cost of a ship of type v in period t under scenario s
C_{vt}^{SH}	cost of a ship of type v in period t in the second-hand market
C_{vts}^{OP}	fixed operating cost for ship of type v in period t under scenario s

C^{TR}_{vrts}	cost of performing a loop r for ship of type v in period t under scenario s
C^{SP}_{its}	cost of voyage charter on trade i in period t under scenario s
C^{CI}_{vts}	cost of chartering in a ship of type v in period t under scenario s
R^{LU}_{vts}	lay up savings for one period t for a ship of type v under scenario s
R^{CO}_{vts}	charter out revenue for one period t for a ship of type v under scenario s
R^{SC}_{vts}	scrapping revenue for a ship of type v in period t under scenario s
R^{SH}_{vts}	revenue from selling a ship of type v in period t under scenario s in the second-hand market
R^{SV}_{vs}	sunset value for a ship of type v under scenario s
F_{it}	frequency requirement for trade i in period t
Z_{rv}	total time it takes for a ship of type v to perform loop r
Z_{v}	total available time for a ship of type v in one period
Q_{cv}	total capacity for a ship of type v
$\overline{CI}_{vts}$	limit on number of available ships for chartering in of type v in period t under scenario s
$\overline{CO}_{vts}$	limit on number of possible ships for chartering out of type v in period t under scenario s
$\overline{SH}_{vts}$	limit on number of available ships in the second hand market of type v in period t under scenario s
$\overline{SE}_{vts}$	limit on number of possible ships that can be sold in the second hand market out of type v in period t under scenario s
$\overline{CI}_{ts}$	limit of total number of ships that can be chartered in in period t under scenario s
$\overline{CO}_{ts}$	limit of total number of ships that can be chartered out in period t under scenario s
$\overline{SH}_{ts}$	limit of total number of ships that can be bought in the second hand market in period t under scenario s

$\overline{SE}_{ts}$	limit of total number of ships that can be sold in the second hand market in period t under scenario s
D_{its}	agreed amount of cargo to be transported on trade i in period t under scenario s
Y_v^{IP}	initial pool of ship type v
Y_{vt}^{NB}	new ships of type v ordered in the sunk period, to be delivered in period t

Variables

y_{vts}^{NB}	number of new buildings ordered in period t under scenario s of ship type v
y_{vts}^{SC}	number of ships scrapped in period t under scenario s of type v
y_{vts}^{SH}	number of ships bought in the second-hand market in period t under scenario s of type v
y_{vts}^{SE}	number of ships sold in the second-hand market in period t under scenario s of type v
y_{vts}^{P}	number of ships in the pool at the end of period t under scenario s
x_{vrts}	number of loops r performed by ships of type v in period t under scenario s
l_{vts}	number of ships of type v on lay up for period t under scenario s
h_{vts}^{I}	number of ships of type v chartered in in period t under scenario s
h_{vts}^{O}	number of ships of type v chartered out in period t under scenario s
n_{its}	units of goods space chartered in on trade i in time t under scenario s
δ_{its}	binary variable, 1 if optional trade i is serviced in node n

Anti Symmetry formulation

Objective function

$$
\begin{aligned}
max z = & \sum_{s \in S} P_s \Bigg(\sum_{t \in T \setminus \{0\}} \Big(\sum_{i \in N_t^O} \sum_{c \in G} R_{its}^D D_{icts} \bar{\delta}_{its} \\
& + \sum_{i \in N_t^C} \sum_{c \in G} (R_{its}^D D_{icts} w - C_{icts}^{SP} \bar{n}_{icts}) - \sum_{v \in V_t} (C_{vts}^{OP} \bar{y}_{vts}^P \\
& + C_{vts}^{CO} \bar{h}_{vts}^I - R_{vt}^{CO} \bar{h}_{vts}^O \\
& + \sum_{r \in R_{vts}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vts}^{LU} \bar{l}_{vts}) \Big) \\
& + \sum_{n \in n(\bar{T})} \sum_{v \in V_{\bar{T}}} R_{vts}^{SV} \bar{y}_{vts}^P \\
& - \sum_{t \in T, t \leq \bar{T} - T^L} \sum_{n \in n(t)} \sum_{v \in V_{t+T^L}^N} C_{vts}^{NB} \sum_{j \in \bar{D}} \bar{y}_{vtsj}^{NB} \\
& + \sum_{t \in T} \sum_{n \in n(t)} \sum_{v \in V_t} R_{vts}^{SC} \sum_{j \in \bar{D}} \bar{y}_{vtsj}^{SC} \\
& + R_{vts}^{SH} \sum_{j \in \bar{D}} \bar{y}_{vts}^{SE} - C_{vts}^{SH} \sum_{j \in \bar{D}} \bar{y}_{vts}^{SH} \Bigg)
\end{aligned}
$$

Demand constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} \bar{x}_{vrts} + \bar{n}_{icts} \geq D_{icts} w, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, s \in S,
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} \bar{x}_{vrts} \geq D_{its} \bar{\delta}_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, s \in S,
$$

Capacity constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinC} Q_{cv} x_{vrts} + \sum_{c \in G} n_{cin} \geq \sum_{c \in G} D_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^C, s \in S,
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinC} Q_{cv} x_{vrts} \geq \sum_{c \in C} D_{itcs} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,
$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrts} \geq F_{it}w, \qquad t \in T\setminus\{0\}, i \in N_t^C, s \in S,$$
$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrts} \geq F_{it}\bar{\delta}_{its}, \qquad t \in T\setminus\{0\}, i \in N_t^O, s \in S,$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vts} + \bar{h}^I_{vts} - \bar{h}^O_{vts} - \bar{l}_{vts}), \qquad t \in T\setminus\{0\}, v \in V_t, s \in S,$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{its}, \qquad t \in T\setminus\{0,1\}, i \in N_t^O, s \in S,$$

Pool constraints

$$\bar{y}^P_{vts} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(\bar{y}^{SC}_{v,t-1,j} - \bar{y}^{SH}_{v,t-1,j} + \bar{y}^{SE}_{v,t-1,j}), \qquad t \in T\setminus\{0\}\; v \in V_t \setminus V_t^N, s \in S,$$

$$\bar{y}^P_{vts} = Y^{NB}_{vt}, \qquad t \in T : t < \bar{T}^L, v \in V_t^N, s \in S$$

$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} \bar{y}^{NB}_{v,t-T^L,j}, \qquad t \in T : t \geq \bar{T}^L, v \in V^N_{t+T^L}, s \in S,$$
$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v \in V_0, s \in S,$$
$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} \bar{y}^{SC}_{v,t-1,j}, \qquad t \in T\setminus 0, v \in V_t \setminus v \in V_{t+1}, s \in S,$$
$$\bar{l}_{vts} - \bar{h}^I_{vts} + \bar{h}^O_{vts} \leq \bar{y}^P_{vts}, \qquad t \in T\setminus\{0\}, v \in V_t, s \in S,$$

Charters and second hand market constraints

$$\sum_{j\in\bar{D}} \bar{y}^{SH}_{vtsj} \leq \overline{SH}_{vts}w, \qquad t\in T, v\in V_t,$$

$$\sum_{j\in\bar{D}} \bar{y}^{SE}_{vtsj} \leq \overline{SE}_{vts}w, \qquad t\in T, v\in V_t,$$

$$\bar{h}^{I}_{vts} \leq \overline{CI}_{vts}w, \qquad t\in T\setminus\{0\}, v\in V_t,$$

$$\bar{h}^{O}_{vts} \leq \overline{CO}_{vts}w, \qquad t\in T\setminus\{0\}, v\in V_t,$$

$$\sum_{v\in V_t\setminus V^N_t}\sum_{j\in\bar{D}} y^{SH}_{vtsj} \leq \overline{SH}_n, \qquad t\in T, s\in S,$$

$$\sum_{v\in V_t\setminus V^N_t}\sum_{j\in\bar{D}} y^{SE}_{vtsj} \leq \overline{SE}_n, \qquad t\in T, s\in S,$$

$$\sum_{v\in V_t\setminus V^N_t} h^{I}_{vts} \leq \overline{CI}_n, \qquad t\in T\setminus\{0\}, s\in S,$$

$$\sum_{v\in V_t\setminus V^N_t} h^{O}_{vts} \leq \overline{CO}_n, \qquad t\in T\setminus\{0\}, s\in S,$$

Linearization of binary variables constraints

$$w-\bar{\delta}^{O}_{its}+\delta^{O}_{its} \leq 1, \qquad t\in T\setminus\{0\}, i\in N_t, s\in S,$$

$$\bar{\delta}^{O}_{its}-w \leq 0, \qquad t\in T\setminus\{0\}, i\in N_t, s\in S,$$

$$\bar{\delta}^{O}_{its}-\delta^{O}_{its} \leq 0, \qquad t\in T\setminus\{0\}, i\in N_t, s\in S,$$

$$w-\bar{y}^{NB}_{vtsj}+y^{NB}_{vtsj} \leq 1, \qquad t\in T: t\leq \bar{T}-T^L, v\in V^N_{t+T^L}, i\in N_t, s\in S, j\in \overline{D},$$

$$\bar{y}^{NB}_{vtsj}-w \leq 0, \qquad t\in T: t\leq \bar{T}-T^L, v\in V^N_{t+T^L}, i\in N_t, s\in S, j\in \overline{D},$$

$$\bar{y}^{NB}_{vtsj}-y^{NB}_{vtsj} \leq 0, \qquad t\in T: t\leq \bar{T}-T^L, v\in V^N_{t+T^L}, i\in N_t, s\in S, j\in \overline{D},$$

$$w-\bar{y}^{SC}_{vtsj}+y^{SC}_{vtsj} \leq 1, \qquad t\in T, v\in V_t, i\in N_t, s\in S, j\in \overline{D},$$

$$\bar{y}^{SC}_{vtsj}-w \leq 0, \qquad t\in T, v\in V_t, i\in N_t, s\in S, j\in \overline{D},$$

$$\bar{y}^{SC}_{vtsj}-y^{SC}_{vtsj} \leq 0, \qquad t\in T, v\in V_t, i\in N_t, s\in S, j\in \overline{D},$$

$$w-\bar{y}^{SH}_{vtsj}+y^{SH}_{vtsj} \leq 1, \qquad t\in T, v\in V_t, i\in N_t, s\in S, j\in \overline{D},$$

$$\bar{y}^{SH}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SH}_{vtsj} - y^{SH}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$w - \bar{y}^{SE}_{vtsj} + y^{SE}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SE}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SE}_{vtsj} - y^{SE}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D}$$

Anti symmetry constraints

$$\bar{y}^{NB}_{vtsj} \leq \bar{y}^{NB}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$

$$\bar{y}^{SC}_{vtsj} \leq \bar{y}^{SC}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$

$$\bar{y}^{SH}_{vtsj} \leq \bar{y}^{SH}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$

$$\bar{y}^{SE}_{vtsj} \leq \bar{y}^{SE}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$

Linearization of integer variables

$$y^{NB}_{vts} = \sum_{j \in \bar{D}} y^{NB}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SC}_{vts} = \sum_{j \in \bar{D}} y^{SC}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SH}_{vts} = \sum_{j \in \bar{D}} y^{SH}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SE}_{vts} = \sum_{j \in \bar{D}} y^{SE}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vts} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vtsj}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vtsj} + C^{SH}_{vts} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vtsj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vtsj}) = \bar{T} + 1$$

Non-anticipativity constraints

$$y^{SC}_{vtsj} = y^{SC}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{NB}_{vtsj} = y^{NB}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{SH}_{vtsj} = y^{SH}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{SE}_{vtsj} = y^{SE}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

Convexity and integer constraints

$$y^{NB}_{vts} \in \mathbb{Z}^+, \qquad t \in T : t \le \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,$$
$$y^{SC}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{SH}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{SE}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{NB}_{vtsj} \in \{0,1\}, \qquad t \in T : t \le \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,$$
$$y^{SC}_{vtsj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{SH}_{vtsj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{SE}_{vtsj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$\delta^{O}_{its} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,$$
$$\bar{\delta}^{O}_{its} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,$$
$$\bar{y}^{NB}_{vtsj} \in \mathbb{R}^+, \qquad t \in T : t \le \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,$$
$$\bar{y}^{SC}_{vtsj} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$\bar{y}^{SH}_{vtsj} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$\bar{y}^{SE}_{vtsj} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{P}_{vts} \in \mathbb{R}^+, \qquad t \in T\, v \in V_t, s \in S,$$
$$l_{vts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$h^{I}_{vt} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$h^{O}_{vt} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, v \in V_t, s \in S,$$
$$x_{vrts} \in \mathbb{R}^+, \qquad t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, s \in S,$$

$w \in \mathbb{R}^+.$

SOS1 formulation

Objective function

$$
\begin{aligned}
maxz = &\sum_{s\in S} P_s \Bigg(\sum_{t\in T\setminus\{0\}} \Big(\sum_{i\in N_t^O} \sum_{c\in G} R_{its}^D D_{icts} \bar{\delta_{its}} \\
&+ \sum_{i\in N_t^C} \sum_{c\in G} (R_{its}^D D_{icts} w - C_{icts}^{SP} \bar{n}_{icts}) - \sum_{v\in V_t} (C_{vts}^{OP} \bar{y}_{vts}^P \\
&+ C_{vts}^{CO} \bar{h}_{vts}^I - R_{vt}^{CO} \bar{h}_{vts}^O \\
&+ \sum_{r\in R_{vts}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vts}^{LU} \bar{l}_{vts}) \Big) \\
&+ \sum_{n\in n(\bar{T})} \sum_{v\in V_{\bar{T}}} R_{vts}^{SV} \bar{y}_{vts}^P \\
&- \sum_{t\in T, t\leq \bar{T}-T^L} \sum_{n\in n(t)} \sum_{v\in V_{t+T^L}^N} C_{vts}^{NB} \sum_{j\in\bar{D}} j \bar{y}_{vtsj}^{NB} \\
&+ \sum_{t\in T} \sum_{n\in n(t)} \sum_{v\in V_t} R_{vts}^{SC} \sum_{j\in\bar{D}} j \bar{y}_{vtsj}^{SC} \\
&+ R_{vts}^{SH} \sum_{j\in\bar{D}} j \bar{y}_{vts}^{SE} - C_{vts}^{SH} \sum_{j\in\bar{D}} j \bar{y}_{vts}^{SH} \Bigg)
\end{aligned}
$$

Demand constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t \in T\setminus\{0\}, i\in N_t^C, c\in G, n\in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{its} \bar{\delta}_{icts}, \qquad t \in T\setminus\{0\}, i\in N_t^O, c\in G, n\in n(t),
$$

Capacity constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{icts}, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c\in C} D_{itcs} \delta_{its}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),
$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),$$
$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{its}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vts} + \bar{h}^I_{vts} - \bar{h}^O_{vts} - \bar{l}_{vts}), \qquad t\in T\setminus\{0\}, v\in V_t, n\in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{its}, \qquad t\in T\setminus\{0,1\}, i\in N_t^O, s\in S,$$

Pool constraints

$$\bar{y}^P_{vts} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(j\bar{y}^{SC}_{v,t-1,j} - j\bar{y}^{SH}_{v,t-1,j} + j\bar{y}^{SE}_{v,t-1,j}), \qquad t\in T\setminus\{0\}\; v\in V_t\setminus V_t^N, s\in S,$$

$$\bar{y}^P_{vts} = Y^{NB}_{vt}, \qquad t\in T : t<\bar{T}^L, v\in V_t^N, s\in S$$

$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} j\bar{y}^{NB}_{v,t-T^L,j}, \qquad t\in T : t\geq\bar{T}^L, v\in V^N_{t+T^L}, s\in S,$$
$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v\in V_0, s\in S,$$
$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} j\bar{y}^{SC}_{vtj}, \qquad t\in T\setminus 0, v\in V_t\setminus v\in V_{t+1}, s\in S,$$
$$\bar{l}_{vts} - \bar{h}^I_{vts} + \bar{h}^O_{vts} \leq \bar{y}^P_{vts}, \qquad t\in T\setminus\{0\}, v\in V_t, s\in S,$$

Charters and second hand market constraints

$$\sum_{j \in \bar{D}} j \bar{y}^{SH}_{vtsj} \leq \overline{SH}_{vts} w, \qquad t \in T, v \in V_t,$$

$$\sum_{j \in \bar{D}} j \bar{y}^{SE}_{vtsj} \leq \overline{SE}_{vts} w, \qquad t \in T, v \in V_t,$$

$$\sum_{j \in \bar{D}} j \bar{h}^{I}_{vtsj} \leq \overline{CI}_{vts} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$

$$\bar{h}^{O}_{vtsj} \leq \overline{CO}_{vts} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$

$$\sum_{v \in V_t \setminus V_t^N} \sum_{j \in \bar{D}} j y^{SH}_{vtsj} \leq \overline{SH}_n, \qquad t \in T, s \in S,$$

$$\sum_{v \in V_t \setminus V_t^N} \sum_{j \in \bar{D}} j y^{SE}_{vtsj} \leq \overline{SE}_n, \qquad t \in T, s \in S,$$

$$\sum_{v \in V_t \setminus V_t^N} h^{I}_{vts} \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, s \in S,$$

$$\sum_{v \in V_t \setminus V_t^N} h^{O}_{vts} \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, s \in S,$$

Linearization of binary variables constraints

$$w - \bar{\delta}^{O}_{its} + \delta^{O}_{its} \leq 1, \qquad t \in T \setminus \{0\}, ii \in N_t^O, s \in S,$$

$$\bar{\delta}^{O}_{its} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,$$

$$\bar{\delta}^{O}_{its} - \delta^{O}_{its} \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,$$

$$w - \bar{y}^{NB}_{vtsj} + y^{NB}_{vtsj} \leq 1, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{NB}_{vtsj} - w \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{NB}_{vtsj} - y^{NB}_{vtsj} \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, s \in S, j \in \overline{D},$$

$$w - \bar{y}^{SC}_{vtsj} + y^{SC}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SC}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SC}_{vtsj} - y^{SC}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$w - \bar{y}^{SH}_{vtsj} + y^{SH}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SH}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SH}_{vtsj} - y^{SH}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$w - \bar{y}^{SE}_{vtsj} + y^{SE}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SE}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SE}_{vtsj} - y^{SE}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D}$$

Linearization of integer variables

$$y^{NB}_{vts} = \sum_{j \in \bar{D}} j y^{NB}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SC}_{vts} = \sum_{j \in \bar{D}} j y^{SC}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SH}_{vts} = \sum_{j \in \bar{D}} j y^{SH}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SE}_{vts} = \sum_{j \in \bar{D}} j y^{SE}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

Non-anticipativity constraints

$$y^{SC}_{vtsj} = y^{SC}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{NB}_{vtsj} = y^{NB}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{SH}_{vtsj} = y^{SH}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{SE}_{vtsj} = y^{SE}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vts} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vtsj}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vtsj} + C^{SH}_{vts} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vtsj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vtsj}) = \bar{T} + 1$$

Convexity and integer constraints

$$
\begin{aligned}
y^{NB}_{vts} &\in \mathbb{Z}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
y^{NB}_{vts} &\in \mathbb{Z}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
y^{SC}_{vts} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SH}_{vts} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SE}_{vts} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{NB}_{vtsj} &\in \{0,1\}, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
y^{SC}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SH}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SE}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
j y^{NB}_{vtsj} &\text{ is SOS1 for } j, && , t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
j y^{SC}_{vtsj} &\text{ is SOS1 for } j, && , t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
j y^{SH}_{vtsj} &\text{ is SOS1 for } j, && , t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
j y^{SE}_{vtsj} &\text{ is SOS1 for } j, && , t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\delta^{O}_{its} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,\\
\bar{\delta}^{O}_{its} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,\\
\bar{y}^{NB}_{vtsj} &\in \mathbb{R}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
\bar{y}^{SC}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\bar{y}^{SH}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\bar{y}^{SE}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{P}_{vts} &\in \mathbb{R}^+, && t \in T\, v \in V_t, s \in S,\\
l_{vts} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
h^{I}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
h^{O}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
x_{vrts} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, s \in S,
\end{aligned}
$$

$$w \in \mathbb{R}^+.$$

Power Formulation

Objective function

$$
\begin{aligned}
max z = &\sum_{s\in S} P_s \Bigg(\sum_{t\in T\setminus\{0\}} \Big(\sum_{i\in N_t^O} \sum_{c\in G} R_{its}^D D_{icts} \bar{\delta}_{its} \\
&+ \sum_{i\in N_t^C} \sum_{c\in G} (R_{its}^D D_{icts} w - C_{icts}^{SP} \bar{n}_{icts}) - \sum_{v\in V_t} (C_{vts}^{OP} \bar{y}_{vts}^P \\
&+ C_{vts}^{CO} \bar{h}_{vts}^I - R_{vt}^{CO} \bar{h}_{vts}^O \\
&+ \sum_{r\in R_{vts}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vts}^{LU} \bar{l}_{vts}) \Big) \\
&+ \sum_{n\in n(\bar{T})} \sum_{v\in V_{\bar{T}}} R_{vts}^{SV} \bar{y}_{vts}^P \\
&- \sum_{t\in T, t\leq \bar{T}-T^L} \sum_{n\in n(t)} \sum_{v\in V_{t+T^L}^N} C_{vts}^{NB} \sum_{j\in \bar{D}} 2^j \bar{y}_{vtsj}^{NB} \\
&+ \sum_{t\in T} \sum_{n\in n(t)} \sum_{v\in V_t} R_{vts}^{SC} \sum_{j\in \bar{D}} 2^j \bar{y}_{vtsj}^{SC} \\
&+ R_{vts}^{SH} \sum_{j\in \bar{D}} 2^j \bar{y}_{vts}^{SE} - C_{vts}^{SH} \sum_{j\in \bar{D}} 2^j \bar{y}_{vts}^{SH} \Bigg)
\end{aligned}
$$

Demand constraints

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t\in T\setminus\{0\}, i\in N_t^C, c\in G, n\in n(t),$$

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{its} \bar{\delta}_{icts}, \qquad t\in T\setminus\{0\}, i\in N_t^O, c\in G, n\in n(t),$$

Capacity constraints

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{icts}, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),$$

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c\in C} D_{itcs} \delta_{its}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),$$

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{its}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vts} + \bar{h}^I_{vts} - \bar{h}^O_{vts} - \bar{l}_{vts}), \qquad t\in T\setminus\{0\}, v\in V_t, n\in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{its}, \qquad t\in T\setminus\{0,1\}, i\in N_t^O, s\in S,$$

Pool constraints

$$\bar{y}^P_{vts} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(2^j\bar{y}^{SC}_{v,t-1,j} - 2^j\bar{y}^{SH}_{v,t-1,j} + 2^j\bar{y}^{SE}_{v,t-1,j}), \qquad t\in T\setminus\{0\}\, v\in V_t\setminus V_t^N, s\in S,$$

$$\bar{y}^P_{vts} = Y^{NB}_{vt}w, \qquad t\in T: t<\bar{T}^L, v\in V_t^N, s\in S$$

$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} 2^j\bar{y}^{NB}_{v,t-T^L,j}, \qquad t\in T: t\geq\bar{T}^L, v\in V^N_{t+T^L}, s\in S,$$

$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v\in V_0, s\in S,$$

$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} 2^j\bar{y}^{SC}_{v,t-1,j}, \qquad t\in T\setminus 0, v\in V_t\setminus v\in V_{t+1}, s\in S,$$

$$\bar{l}_{vts} - \bar{h}^I_{vts} + \bar{h}^O_{vts} \leq \bar{y}^P_{vts}, \qquad t\in T\setminus\{0\}, v\in V_t, s\in S,$$

Charters and second hand market constraints

$$\bar{y}^{SH}_{vts} \leq \overline{SH}_{vts} w, \qquad t \in T, v \in V_t,$$
$$\bar{y}^{SE}_{vts} \leq \overline{SE}_{vts} w, \qquad t \in T, v \in V_t,$$
$$\bar{h}^{I}_{vts} \leq \overline{CI}_{vts} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\bar{h}^{O}_{vts} \leq \overline{CO}_{vts} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\sum_{v \in V_t \setminus V_t^N} y^{SH}_{vts} \leq \overline{SH}_n, \qquad t \in T, s \in S,$$
$$\sum_{v \in V_t \setminus V_t^N} y^{SE}_{vts} \leq \overline{SE}_n, \qquad t \in T, s \in S,$$
$$\sum_{v \in V_t \setminus V_t^N} h^{I}_{vts} \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, s \in S,$$
$$\sum_{v \in V_t \setminus V_t^N} h^{O}_{vts} \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, s \in S,$$

Linearization of binary variables constraints

$$w - \bar{\delta}^{O}_{its} + \delta^{O}_{its} \leq 1, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,$$
$$\bar{\delta}^{O}_{its} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,$$
$$\bar{\delta}^{O}_{its} - \delta^{O}_{its} \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, s \in S,$$

$$w - \bar{y}^{NB}_{vtsj} + y^{NB}_{vtsj} \leq 1, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{NB}_{vtsj} - w \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{NB}_{vtsj} - y^{NB}_{vtsj} \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, s \in S, j \in \overline{D},$$

$$w - \bar{y}^{SC}_{vtsj} + y^{SC}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SC}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SC}_{vtsj} - y^{SC}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$w - \bar{y}^{SH}_{vtsj} + y^{SH}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SH}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SH}_{vtsj} - y^{SH}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$w - \bar{y}^{SE}_{vtsj} + y^{SE}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SE}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SE}_{vtsj} - y^{SE}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D}$$

Linearization of integer variables

$$y^{NB}_{vts} = \sum_{j \in \bar{D}} 2^j y^{NB}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SC}_{vts} = \sum_{j \in \bar{D}} 2^j y^{SC}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SH}_{vts} = \sum_{j \in \bar{D}} 2^j y^{SH}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SE}_{vts} = \sum_{j \in \bar{D}} 2^j y^{SE}_{vtsj}, \qquad t \in T, v \in V_t, s \in S,$$

Non-anticipativity constraints

$$y^{SC}_{vtsj} = y^{SC}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

$$y^{NB}_{vtsj} = y^{NB}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

$$y^{SH}_{vtsj} = y^{SH}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

$$y^{SE}_{vtsj} = y^{SE}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vts} \sum_{j \in \bar{D}} 2^j \bar{y}^{NB}_{vtsj}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} 2^j \bar{y}^{SC}_{vtsj} + C^{SH}_{vts} \sum_{j \in \bar{D}} 2^j \bar{y}^{SH}_{vtsj} + R^{SE}_{vt} \sum_{j \in \bar{D}} 2^j \bar{y}^{SE}_{vtsj}) = \bar{T} + 1$$

Convexity and integer constraints

$$
\begin{aligned}
y^{NB}_{vts} &\in \mathbb{Z}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
y^{SC}_{vts} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SH}_{vts} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SE}_{vts} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{NB}_{vtsj} &\in \{0,1\}, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
y^{SC}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SH}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SE}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\delta^{O}_{its} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,\\
\bar{\delta}^{O}_{its} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,\\
\bar{y}^{NB}_{vtsj} &\in \mathbb{R}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
\bar{y}^{SC}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\bar{y}^{SH}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\bar{y}^{SE}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{P}_{vts} &\in \mathbb{R}^+, && t \in T\, v \in V_t, s \in S,\\
l_{vts} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
h^{I}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
h^{O}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
x_{vrts} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, s \in S,\\
w &\in \mathbb{R}^+.
\end{aligned}
$$

Pattern Formulation

Objective function

$$
\begin{aligned}
maxz = & \sum_{s \in S} P_s \Bigg(\sum_{t \in T \setminus \{0\}} \Big(\sum_{i \in N_t^O} \sum_{c \in G} R_{its}^D D_{icts} \bar{\delta}_{its} \\
& + \sum_{i \in N_t^C} \sum_{c \in G} (R_{its}^D D_{icts} w - C_{icts}^{SP} \bar{n}_{icts}) - \sum_{v \in V_t} (C_{vts}^{OP} \bar{y}_{vts}^P \\
& + C_{vts}^{CO} \bar{h}_{vts}^I - R_{vt}^{CO} \bar{h}_{vts}^O \\
& + \sum_{r \in R_{vts}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vts}^{LU} \bar{l}_{vts}) \Big) \\
& + \sum_{n \in n(\bar{T})} \sum_{v \in V_{\bar{T}}} R_{vts}^{SV} \bar{y}_{vts}^P \\
& - \sum_{t \in T, t \leq \bar{T} - T^L} \sum_{n \in n(t)} \sum_{v \in V_{t+T^L}^N} C_{vts}^{NB} \sum_{p \in P^P} P_{vp}^O \bar{p}_{tsp}^P \\
& + \sum_{t \in T} \sum_{n \in n(t)} \sum_{v \in V_t} R_{vts}^{SC} \sum_{j \in \bar{D}} \bar{y}_{vtsj}^{SC} \\
& + R_{vts}^{SH} \sum_{j \in \bar{D}} \bar{y}_{vts}^{SE} - C_{vts}^{SH} \sum_{j \in \bar{D}} \bar{y}_{vts}^{SH} \Bigg)
\end{aligned}
$$

Demand constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, n \in n(t),
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{its} \bar{\delta}_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, n \in n(t),
$$

Capacity constraints

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c \in G} n_{cin} \geq \sum_{c \in G} D_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^C, n \in n(t),
$$

$$
\sum_{v \in V_t} \sum_{r \in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c \in C} D_{itcs} \delta_{its}, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),
$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t \in T\setminus\{0\}, i \in N_t^C, n \in n(t),$$
$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{its}, \qquad t \in T\setminus\{0\}, i \in N_t^O, n \in n(t),$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vts} + \bar{h}^I_{vts} - \bar{h}^O_{vts} - \bar{l}_{vts}), \qquad t \in T\setminus\{0\}, v \in V_t, n \in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{its}, \qquad t \in T\setminus\{0,1\}, i \in N_t^O, s \in S,$$

Pool constraints

$$\bar{y}^P_{vts} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(\bar{y}^{SC}_{v,t-1,j} - \bar{y}^{SH}_{v,t-1,j} + \bar{y}^{SE}_{v,t-1,j}), \qquad t \in T\setminus\{0\}\; v \in V_t\setminus V_t^N, s \in S,$$

$$\bar{y}^P_{vts} = Y^{NB}_{vt}w, \qquad t \in T : t < \bar{T}^L, v \in V_t^N, s \in S$$

$$\bar{y}^P_{vts} = \sum_{p\in P^P} P^O_{vp}\bar{p}^P_{t-1,s,p}, \qquad t \in T : t \geq \bar{T}^L, v \in V^N_{t+T^L}, s \in S,$$
$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v \in V_0, s \in S,$$
$$\bar{y}^P_{vts} = \sum_{j\in\bar{D}} \bar{y}^{SC}_{v,t-1,j}, \qquad t \in T\setminus 0, v \in V_t\setminus v \in V_{t+1}, s \in S,$$
$$\bar{l}_{vts} - \bar{h}^I_{vts} + \bar{h}^O_{vts} \leq \bar{y}^P_{vts}, \qquad t \in T\setminus\{0\}, v \in V_t, s \in S,$$

Charters and second hand market constraints

$$\bar{y}^{SH}_{vts} \leq \overline{SH}_{vts} w, \qquad t \in T, v \in V_t,$$
$$\bar{y}^{SE}_{vts} \leq \overline{SE}_{vts} w, \qquad t \in T, v \in V_t,$$
$$\bar{h}^{I}_{vts} \leq \overline{CI}_{vts} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\bar{h}^{O}_{vts} \leq \overline{CO}_{vts} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\sum_{v \in V_t \setminus V^N_t} y^{SH}_{vts} \leq \overline{SH}_n, \qquad t \in T, s \in S,$$
$$\sum_{v \in V_t \setminus V^N_t} y^{SE}_{vts} \leq \overline{SE}_n, \qquad t \in T, s \in S,$$
$$\sum_{v \in V_t \setminus V^N_t} h^{I}_{vts} \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, s \in S,$$
$$\sum_{v \in V_t \setminus V^N_t} h^{O}_{vts} \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, s \in S,$$

Linearization of binary variables constraints

$$w - \bar{\delta}^{O}_{its} + \delta^{O}_{its} \leq 1, \qquad t \in T \setminus \{0\}, i \in N^O_t, s \in S,$$
$$\bar{\delta}^{O}_{its} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N^O_t, s \in S,$$
$$\bar{\delta}^{O}_{its} - \delta^{O}_{its} \leq 0, \qquad t \in T \setminus \{0\}, i \in N^O_t, s \in S,$$

$$w - \bar{p}^{P}_{tsp} + p^{P}_{tsp} \leq 1, \qquad t \in T, s \in S, p \in P^O,$$
$$\bar{p}^{P}_{tsp} - w \leq 0, \qquad t \in T, s \in S, p \in P^O,$$
$$\bar{p}^{P}_{tsp} - p^{P}_{tsp} \leq 0, \qquad tt \in T, s \in S, p \in P^O,$$

$$w - \bar{y}^{SC}_{vtsj} + y^{SC}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SC}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SC}_{vtsj} - y^{SC}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$w - \bar{y}^{SH}_{vtsj} + y^{SH}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SH}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$

$$\bar{y}^{SH}_{vtsj} - y^{SH}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$w - \bar{y}^{SE}_{vtsj} + y^{SE}_{vtsj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SE}_{vtsj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D},$$
$$\bar{y}^{SE}_{vtsj} - y^{SE}_{vtsj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, s \in S, j \in \overline{D}$$

Transformation of patterns to new buildings constraints

$$y^{NB}_{vts} = \sum_{p \in P^P} P^O_{vp} p^P_{tsp}, \qquad t \in T, v \in V^N_t, s \in S, \tag{A.1}$$
$$\sum_{p \in P^P} p^P_{tsp} = 1, \qquad t \in T, s \in S. \tag{A.2}$$

Non-anticipativity constraints

$$y^{SC}_{vtsj} = y^{SC}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{NB}_{vtsj} = y^{NB}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{SH}_{vtsj} = y^{SH}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$
$$y^{SE}_{vtsj} = y^{SE}_{vt\bar{s}j}, \qquad t \in T, v \in V_t, s \in S, \bar{s} \in S^{NA}_{ts}, j \in \bar{D}$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vts} \sum_{p \in P^P} P^O_{vp} \bar{p}^P_{tsp}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vtsj} + C^{SH}_{vts} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vtsj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vtsj}) = \bar{T} + 1$$

Convexity and integer constraints

$$y^{NB}_{vts} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,$$
$$y^{SC}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{SH}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$
$$y^{SE}_{vts} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,$$

$$
\begin{aligned}
y^{NB}_{vtsj} &\in \{0,1\}, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
y^{SC}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SH}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{SE}_{vtsj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\delta^{O}_{its} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,\\
\bar{\delta}^{O}_{its} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, i \in N^O_t, s \in S,\\
\bar{y}^{NB}_{vtsj} &\in \mathbb{R}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, s \in S,\\
\bar{y}^{SC}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\bar{y}^{SH}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
\bar{y}^{SE}_{vtsj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, s \in S,\\
y^{P}_{vts} &\in \mathbb{R}^+, && t \in T\, v \in V_t, s \in S,\\
l_{vts} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
h^{I}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
h^{O}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, s \in S,\\
x_{vrts} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, s \in S,\\
\bar{p}^{P}_{tsp} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, p \in P^O, s \in S,\\
p^{P}_{tsp} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, p \in P^O s \in S,\\
w &\in \mathbb{R}^+.
\end{aligned}
$$

A.1.4 ROCEMax node formulation

Sets

T	set of periods, indexed by t, final period $\bar{T}$
N	set of nodes in the scenario tree, indexed by n
V_{t}	$= \{0 \leq A_{vt} \leq \bar{A}\}$ set of ship types existing in the marked in period t, indexed by v
N_{t}^{C}	set of mandatory trades (where there are contractual obligations) operated in period t, indexed by i
N_{t}^{O}	set of optional trades (where there are no contractual obligations) in period t, indexed by i
N_{t}	set of all trades in period t, indexed by i
R_{t}	set of loops in period t, indexed by r
$R_{vt} \subseteq R_t$	set of loops which can be sailed by a ship of type v in period t, indexed by r
R_{ivt}	set of loops servicing trade i which can be sailed by a ship of type v in period t, indexed by r
$\bar{D}$	set of decisions for building, scrapping, selling and buying ships, indexed by j
G	set of cargo types, indexed by$c$

Parameters

$\bar{T}^{\mathrm{L}}$	lead time for new buildings
P_n	probability of being in node n
$n(t)$	all nodes n at time t in the scenario tree
$a(n,t)$	all ancestors of node n in the scenario tree in time t, $a(n,t)$ is written simply as $a(n)$
$\bar{A}$	lifetime of a generic ship
A_{vt}	age of a ship of type v in period t

C^{NB}_{vn}	building cost of a ship of type v in node n
C^{SH}_{vn}	cost of a ship of type v in node n in the second-hand market
C^{OP}_{vn}	fixed operating cost for ship of type v in node n
C^{TR}_{vrn}	cost of performing a loop r for ship of type v in node ns
C^{SP}_{in}	cost of voyage charter on trade i in node n
C^{CI}_{vn}	cost of chartering in a ship of type v in period n
R^{LU}_{vn}	lay up savings for one period t for a ship of type n
R^{CO}_{vn}	charter out revenue for one period t for a ship of type n
R^{SC}_{vn}	scrapping revenue for a ship of type v in period n
R^{SH}_{vn}	revenue from selling a ship of type v in period n in the second-hand market
R^{SV}_{vn}	sunset value for a ship of type v in node n
F_{it}	frequency requirement for trade i in period t
Z_{rv}	total time it takes for a ship of type v to perform loop r
Z_v	total available time for a ship of type v in one period
Q_{cv}	total capacity for a ship of type v
$\overline{CI}_{vn}$	limit on number of available ships for chartering in of type v in node n
$\overline{CO}_{vn}$	limit on number of possible ships for chartering out of type v in node n
$\overline{SH}_{vn}$	limit on number of available ships in the second hand market of type v in node n
$\overline{SE}_{vn}$	limit on number of possible ships that can be sold in the second hand market out of type v in node n
$\overline{CI}_n$	limit of total number of ships that can be chartered in in node n
$\overline{CO}_n$	limit of total number of ships that can be chartered out in node n
$\overline{SH}_n$	limit of total number of ships that can be bought in the second hand market in node n

$\overline{SE}_n$	limit of total number of ships that can be sold in the second hand market in node n
D_{in}	agreed amount of cargo to be transported on trade i in node n
Y_v^{IP}	initial pool of ship type v
Y_{vt}^{NB}	new ships of type v ordered in the sunk period, to be delivered in period t

Variables

y_{vn}^{NB}	number of new buildings ordered in node n of ship type v
y_{vn}^{SC}	number of ships scrapped in node n of type v
y_{vn}^{SH}	number of ships bought in the second-hand market in node n of type v
y_{vn}^{SE}	number of ships sold in the second-hand market in node n of type v
y_{vn}^{P}	number of ships in the pool at the end of the period in node n
x_{vn}	number of loops r performed by ships of type v in node n
l_{vn}	number of ships of type v on lay up for node n
h_{vn}^{I}	number of ships of type v chartered in in node n
h_{vn}^{O}	number of ships of type v chartered out in node n
n_{in}	units of goods space chartered in on trade i in node n
δ_{in}	binary variable, 1 if optional trade i is serviced in node n

Anti Symmetry formulation

Objective function

$$
\begin{aligned}
maxz = \sum_{t\in T\setminus\{0\}} \sum_{n\in n(t)} P_n \Big(& \sum_{i\in N_t^O} \sum_{c\in G} R_{in}^D D_{icn} \bar{\delta_{in}} \\
& + \sum_{i\in N_t^C} \sum_{c\in G} (R_{in}^D D_{icn} w - C_{icn}^{SP} \bar{n}_{icn}) - \sum_{v\in V_t} (C_{vn}^{OP} \bar{y}_{vn}^P \\
& + C_{vn}^{CO} \bar{h}_{vn}^I - R_{vt}^{CO} \bar{h}_{vn}^O \\
& + \sum_{r\in R_{vn}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vn}^{LU} \bar{l}_{vn}) \Big) \\
& + \sum_{n\in n(\bar{T})} \sum_{v\in V_{\bar{T}}} R_{vn}^{SV} \bar{y}_{vn}^P \\
& - \sum_{t\in T, t\leq \bar{T}-T^L} \sum_{n\in n(t)} \sum_{v\in V_{t+T^L}^N} P_n C_{vn}^{NB} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{NB} \\
& + \sum_{t\in T} \sum_{n\in n(t)} \sum_{v\in V_t} P_n (R_{vn}^{SC} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SC} \\
& + R_{vn}^{SH} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SE} - C_{vn} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SH})
\end{aligned}
$$

Demand constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, n \in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{in} \bar{\delta}_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, n \in n(t),
$$

Capacity constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^C, n \in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c\in C} D_{itcs} \delta_{in}, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),
$$

Frequency constraints

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \bar{x}_{vrn} \geq F_{it} w, \qquad t \in T \setminus \{0\}, i \in N_t^C, n \in n(t),$$
$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} \bar{x}_{vrn} \geq F_{it} \bar{\delta}_{in}, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

Time constraints

$$\sum_{r \in R_v t} Z_{rv} \bar{x}_{vrn} \leq Z_v (\bar{y}_{vn}^P + \bar{h}_{vn}^I - \bar{h}_{vn}^O - \bar{l}_{vn}), \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{in}, \qquad t \in T \setminus \{0, 1\}, i \in N_t^O, n \in n(t),$$

Pool constraints

$$\bar{y}_{vn}^P = \bar{y}_{v,t-1}^P - \sum_{j \in \bar{D}} (\bar{y}_{v,t-1,j}^{SC} - \bar{y}_{v,t-1,j}^{SH} + \bar{y}_{v,t-1,j}^{SE}), \qquad t \in T \setminus \{0\}\ v \in V_t \setminus V_t^N, n \in n(t),$$

$$\bar{y}_{vn}^P = Y_{vt}^{NB}, \qquad t \in T : t < \bar{T}^L, v \in V_t^N, n \in n(t)$$

$$\bar{y}_{vn}^P = \sum_{j \in \bar{D}} \bar{y}_{v,t-T^L,j}^{NB}, \qquad t \in T : t \geq \bar{T}^L, v \in V_{t+T^L}^N, n \in n(t),$$
$$\bar{y}_{v0s}^P = Y_v^{IP} w, \qquad v \in V_0, n \in n(t),$$
$$\bar{y}_{vn}^P = \sum_{j \in \bar{D}} \bar{y}_{v,t-1,j}^{SC}, \qquad t \in T \setminus 0, v \in V_t \setminus v \in V_{t+1}, n \in n(t),$$
$$\bar{l}_{vn} - \bar{h}_{vn}^I + \bar{h}_{vn}^O \leq \bar{y}_{vn}^P, \qquad t \in T \setminus \{0\}, v \in V_t, n \in n(t),$$

Charters and second hand market constraints

$$\bar{y}^{SH}_{vn} \leq \overline{SH}_{vn} w, \qquad t \in T, v \in V_t,$$

$$\bar{y}^{SE}_{vn} \leq \overline{SE}_{vn} w, \qquad t \in T, v \in V_t,$$

$$\bar{h}^{I}_{vn} \leq \overline{CI}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$

$$\bar{h}^{O}_{vn} \leq \overline{CO}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$

$$\sum_{v \in V_t \setminus V^N_t} y^{SH}_{vn} \leq \overline{SH}_n, \qquad t \in T, n \in n(t),$$

$$\sum_{v \in V_t \setminus V^N_t} y^{SE}_{vn} \leq \overline{SE}_n, \qquad t \in T, n \in n(t),$$

$$\sum_{v \in V_t \setminus V^N_t} h^{I}_{vn} \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

$$\sum_{v \in V_t \setminus V^N_t} h^{O}_{vn} \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

Linearization of binary variables constraints

$$w - \bar{\delta}_{in} + \delta_{in} \leq 1, \qquad t \in T \setminus \{0\}, i \in N^O_t, n \in n(t),$$

$$\bar{\delta}_{in} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N^O_t, n \in n(t),$$

$$\bar{\delta}_{in} - \delta_{in} \leq 0, \qquad t \in T \setminus \{0\}, i \in N^O_t, n \in n(t),$$

$$w - \bar{y}^{NB}_{vnj} + y^{NB}_{vnj} \leq 1, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L} n \in n(t), j \in \overline{D},$$

$$\bar{y}^{NB}_{vnj} - w \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{NB}_{vnj} - y^{NB}_{vnj} \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t), j \in \overline{D},$$

$$w - \bar{y}^{SC}_{vnj} + y^{SC}_{vnj} \leq 1, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SC}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SC}_{vnj} - y^{SC}_{vnj} \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$

$$w - \bar{y}^{SH}_{vnj} + y^{SH}_{vnj} \leq 1, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SH}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SH}_{vnj} - y^{SH}_{vnj} \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$
$$w - \bar{y}^{SE}_{vnj} + y^{SE}_{vnj} \leq 1, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SE}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SE}_{vnj} - y^{SE}_{vnj} \leq 0, \qquad t \in T, v \in V_t, n \in n(t), j \in \overline{D},$$

Anti symmetry constraints

$$\bar{y}^{NB}_{vnj} \leq \bar{y}^{NB}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$
$$\bar{y}^{SC}_{vnj} \leq \bar{y}^{SC}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$
$$\bar{y}^{SH}_{vnj} \leq \bar{y}^{SH}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$
$$\bar{y}^{SE}_{vnj} \leq \bar{y}^{SE}_{vts,j-1}, \qquad t \in T, v \in V_t, s \in S, , jin\bar{D} \setminus \{1\},$$

Linearization of integer variables

$$y^{NB}_{vn} = \sum_{j \in \bar{D}} y^{NB}_{vnj}, \qquad t \in T, v \in V_t, n \in n(t),$$
$$y^{SC}_{vn} = \sum_{j \in \bar{D}} y^{SC}_{vnj}, \qquad t \in T, v \in V_t, n \in n(t),$$
$$y^{SH}_{vn} = \sum_{j \in \bar{D}} y^{SH}_{vnj}, \qquad t \in T, v \in V_t, n \in n(t),$$
$$y^{SE}_{vn} = \sum_{j \in \bar{D}} y^{SE}_{vnj}, \qquad t \in T, v \in V_t, n \in n(t),$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vn} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vnj}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vnj} + C^{SH}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vnj} + R^{SE}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vnj}) = \bar{T} + 1$$

Convexity, integer and binary constraints

$$
\begin{aligned}
y^{NB}_{vn} &\in \mathbb{Z}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),\\
y^{SC}_{vn} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{SH}_{vn} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{SE}_{vn} &\in \mathbb{Z}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{NB}_{vnj} &\in \{0,1\}, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),\\
y^{SC}_{vnj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{SH}_{vnj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{SE}_{vnj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
\delta_{in} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; i \in N^O_t, n \in n(t),\\
\bar{\delta}_{in} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; i \in N^O_t, n \in n(t),\\
\bar{y}^{NB}_{vnj} &\in \mathbb{R}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),\\
\bar{y}^{SC}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
\bar{y}^{SH}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
\bar{y}^{SE}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{P}_{vn} &\in \mathbb{R}^+, && t \in T\; v \in V_t, n \in n(t),\\
l_{vn} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{I}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{O}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
x_{vrn} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t),\\
w &\in \mathbb{R}^+.
\end{aligned}
$$

SOS1 formulation

Objective function

$$\begin{aligned} maxz = \sum_{t\in T\setminus\{0\}} \sum_{n\in n(t)} P_n \Big(& \sum_{i\in N_t^O} \sum_{c\in G} R_{in}^D D_{icn} \bar{\delta_{in}} \\ & + \sum_{i\in N_t^C} \sum_{c\in G} (R_{in}^D D_{icn} w - C_{icn}^{SP} \bar{n}_{icn}) - \sum_{v\in V_t} (C_{vn}^{OP} \bar{y}_{vn}^P \\ & + C_{vn}^{CO} \bar{h}_{vn}^I - R_{vt}^{CO} \bar{h}_{vn}^O \\ & + \sum_{r\in R_{vn}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vn}^{LU} \bar{l}_{vn}) \Big) \\ & + \sum_{n\in n(\bar{T})} \sum_{v\in V_{\bar{T}}} R_{vn}^{SV} \bar{y}_{vn}^P \\ & - \sum_{t\in T, t\leq \bar{T}-T^L} \sum_{n\in n(t)} \sum_{v\in V_{t+T^L}^N} P_n C_{vn}^{NB} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{NB} \\ & + \sum_{t\in T} \sum_{n\in n(t)} \sum_{v\in V_t} P_n (R_{vn}^{SC} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SC} \\ & + R_{vn}^{SH} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SE} - C_{vn} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SH}) \end{aligned}$$

Demand constraints

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, n \in n(t),$$

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{in} \bar{\delta}_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, n \in n(t),$$

Capacity constraints

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^C, n \in n(t),$$

$$\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c\in C} D_{itcs} \delta_{in}, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),$$
$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{in}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vn} + \bar{h}^I_{vn} - \bar{h}^O_{vn} - \bar{l}_{vn}), \qquad t\in T\setminus\{0\}, v\in V_t, n\in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{in}, \qquad t\in T\setminus\{0,1\}, i\in N_t^O, n\in n(t),$$

Pool constraints

$$\bar{y}^P_{vn} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(j\bar{y}^{SC}_{v,t-1,j} - j\bar{y}^{SH}_{v,t-1,j} + j\bar{y}^{SE}_{v,t-1,j}), \qquad t\in T\setminus\{0\}\; v\in V_t\setminus V_t^N, n\in n(t),$$

$$\bar{y}^P_{vn} = Y^{NB}_{vt}, \qquad t\in T: t<\bar{T}^L, v\in V_t^N, n\in n(t)$$

$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}} j\bar{y}^{NB}_{v,t-T^L,j}, \qquad t\in T: t\geq\bar{T}^L, v\in V^N_{t+T^L}, n\in n(t),$$
$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v\in V_0, n\in n(t),$$
$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}} j\bar{y}^{SC}_{vtj}, \qquad t\in T\setminus 0, v\in V_t\setminus v\in V_{t+1}, n\in n(t),$$
$$\bar{l}_{vn} - \bar{h}^I_{vn} + \bar{h}^O_{vn} \leq \bar{y}^P_{vn}, \qquad t\in T\setminus\{0\}, v\in V_t, n\in n(t),$$

Charters and second hand market constraints

$$\bar{y}^{SH}_{vnj} \le \overline{SH}_{vn} w, \qquad t \in T, v \in V_t,$$

$$\bar{y}^{SE}_{vnj} \le \overline{SE}_{vn} w, \qquad t \in T, v \in V_t,$$

$$\bar{h}^{I}_{vnj} \le \overline{CI}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$

$$\bar{h}^{O}_{vnj} \le \overline{CO}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$

$$\sum_{v \in V_t \setminus V_t^N} y^{SH}_{vn} \le \overline{SH}_n, \qquad t \in T, n \in n(t),$$

$$\sum_{v \in V_t \setminus V_t^N} y^{SE}_{vn} \le \overline{SE}_n, \qquad t \in T, n \in n(t),$$

$$\sum_{v \in V_t \setminus V_t^N} h^{I}_{vn} \le \overline{CI}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

$$\sum_{v \in V_t \setminus V_t^N} h^{O}_{vn} \le \overline{CO}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

Linearization of binary variables constraints

$$w - \bar{\delta}_{in} + \delta_{in} \le 1, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

$$\bar{\delta}_{in} - w \le 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

$$\bar{\delta}_{in} - \delta_{in} \le 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

$$w - \bar{y}^{NB}_{vnj} + y^{NB}_{vnj} \le 1, \qquad t \in T : t \le \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{NB}_{vnj} - w \le 0, \qquad t \in T : t \le \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{NB}_{vnj} - y^{NB}_{vnj} \le 0, \qquad t \in T : t \le \bar{T} - T^L, v \in V^N_{t+T^L}, i \in N_t, n \in n(t), j \in \overline{D},$$

$$w - \bar{y}^{SC}_{vnj} + y^{SC}_{vnj} \le 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SC}_{vnj} - w \le 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SC}_{vnj} - y^{SC}_{vnj} \le 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$w - \bar{y}^{SH}_{vnj} + y^{SH}_{vnj} \le 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SH}_{vnj} - w \le 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SH}_{vnj} - y^{SH}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$w - \bar{y}^{SE}_{vnj} + y^{SE}_{vnj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SE}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SE}_{vnj} - y^{SE}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D}$$

Linearization of integer variables

$$y^{NB}_{vn} = \sum_{j \in \bar{D}} j y^{NB}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SC}_{vn} = \sum_{j \in \bar{D}} j y^{SC}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SH}_{vn} = \sum_{j \in \bar{D}} j y^{SH}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$

$$y^{SE}_{vn} = \sum_{j \in \bar{D}} j y^{SE}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vn} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vnj}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vnj} + C^{SH}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vnj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vnj}) = \bar{T} + 1$$

Convexity and integer constraints

$$y^{NB}_{vn} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$

$$y^{NB}_{vn} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$

$$y^{SC}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\} \, v \in V_t, n \in n(t),$$

$$y^{SH}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\} \, v \in V_t, n \in n(t),$$

$$y^{SE}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\} \, v \in V_t, n \in n(t),$$

$$y^{NB}_{vnj} \in \{0, 1\}, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$

$$
\begin{aligned}
y^{SC}_{vnj} &\in \{0,1\}, & & t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
y^{SH}_{vnj} &\in \{0,1\}, & & t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
y^{SE}_{vnj} &\in \{0,1\}, & & t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
j y^{NB}_{vnj} &\text{ is SOS1 for } j, & & , t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
j y^{SC}_{vnj} &\text{ is SOS1 for } j, & & , t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
j y^{SH}_{vnj} &\text{ is SOS1 for } j, & & , t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
j y^{SE}_{vnj} &\text{ is SOS1 for } j, & & , t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
\delta_{in} &\in \{0,1\}, & & t \in T \setminus \{\bar{T}\}\, i \in N^O_t, n \in n(t),\\
\bar{\delta}_{in} &\in \mathbb{R}^+, & & t \in T \setminus \{\bar{T}\}\, i \in N^O_t, n \in n(t),\\
\bar{y}^{NB}_{vnj} &\in \mathbb{R}^+, & & t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),\\
\bar{y}^{SC}_{vnj} &\in \mathbb{R}^+, & & t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
\bar{y}^{SH}_{vnj} &\in \mathbb{R}^+, & & t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
\bar{y}^{SE}_{vnj} &\in \mathbb{R}^+, & & t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
y^{P}_{vn} &\in \mathbb{R}^+, & & t \in T\, v \in V_t, n \in n(t),\\
l_{vn} &\in \mathbb{R}^+, & & t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{I}_{vt} &\in \mathbb{R}^+, & & t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{O}_{vt} &\in \mathbb{R}^+, & & t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
x_{vrn} &\in \mathbb{R}^+, & & t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t),\\
w &\in \mathbb{R}^+.
\end{aligned}
$$

Power Formulation

Objective function

$$
\begin{aligned}
maxz = & \sum_{t\in T\setminus\{0\}} \sum_{n\in n(t)} P_n \Big(\sum_{i\in N_t^O} \sum_{c\in G} R_{in}^D D_{icn} \bar{\delta_{in}} \\
& + \sum_{i\in N_t^C} \sum_{c\in G} (R_{in}^D D_{icn} w - C_{icn}^{SP} \bar{n}_{icn}) - \sum_{v\in V_t} (C_{vn}^{OP} \bar{y}_{vn}^P \\
& + C_{vn}^{CO} \bar{h}_{vn}^I - R_{vt}^{CO} \bar{h}_{vn}^O \\
& + \sum_{r\in R_{vn}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vn}^{LU} \bar{l}_{vn}) \Big) \\
& + \sum_{n\in n(\bar{T})} \sum_{v\in V_{\bar{T}}} R_{vn}^{SV} \bar{y}_{vn}^P \\
& - \sum_{t\in T, t\leq \bar{T}-T^L} \sum_{n\in n(t)} \sum_{v\in V_{t+T^L}^N} P_n C_{vn}^{NB} \sum_{j\in\bar{D}} \bar{y}_{vnj}^{NB} \\
& + \sum_{t\in T} \sum_{n\in n(t)} \sum_{v\in V_t} P_n (R_{vn}^{SC} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SC} \\
& + R_{vn}^{SH} \sum_{j\in\bar{D}} \bar{y}_{vnj}^{SE} - C_{vn} \sum_{j\in\bar{D}} \bar{y}_{vnj}^{SH})
\end{aligned}
$$

Demand constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t\in T\setminus\{0\}, i\in N_t^C, c\in G, n\in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{in} \bar{\delta}_{icts}, \qquad t\in T\setminus\{0\}, i\in N_t^O, c\in G, n\in n(t),
$$

Capacity constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{icts}, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c\in C} D_{itcs} \delta_{in}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),
$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t \in T\setminus\{0\}, i \in N_t^C, n \in n(t),$$
$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{in}, \qquad t \in T\setminus\{0\}, i \in N_t^O, n \in n(t),$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vn} + \bar{h}^I_{vn} - \bar{h}^O_{vn} - \bar{l}_{vn}), \qquad t \in T\setminus\{0\}, v \in V_t, n \in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{in}, \qquad t \in T\setminus\{0,1\}, i \in N_t^O, n \in n(t),$$

Pool constraints

$$\bar{y}^P_{vn} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(\bar{y}^{SC}_{v,t-1,j} - \bar{y}^{SH}_{v,t-1,j} + \bar{y}^{SE}_{v,t-1,j}), \qquad t \in T\setminus\{0\}\; v \in V_t\setminus V_t^N, n \in n(t),$$

$$\bar{y}^P_{vn} = Y^{NB}_{vt}, \qquad t \in T : t < \bar{T}^L, v \in V_t^N, n \in n(t)$$

$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}} \bar{y}^{NB}_{v,t-T^L,j}, \qquad t \in T : t \geq \bar{T}^L, v \in V^N_{t+T^L}, n \in n(t),$$
$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v \in V_0, n \in n(t),$$
$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}} \bar{y}^{SC}_{v,t-1,j}, \qquad t \in T\setminus 0, v \in V_t \setminus v \in V_{t+1}, n \in n(t),$$
$$\bar{l}_{vn} - \bar{h}^I_{vn} + \bar{h}^O_{vn} \leq \bar{y}^P_{vn}, \qquad t \in T\setminus\{0\}, v \in V_t, n \in n(t),$$

Charters and second hand market constraints

$$\bar{y}_{vn}^{SH} \leq \overline{SH}_{vn} w, \qquad t \in T, v \in V_t,$$
$$\bar{y}_{vn}^{SE} \leq \overline{SE}_{vn} w, \qquad t \in T, v \in V_t,$$
$$\bar{h}_{vn}^{I} \leq \overline{CI}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\bar{h}_{vn}^{O} \leq \overline{CO}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\sum_{v \in V_t \setminus V_t^N} y_{vn}^{SH} \leq \overline{SH}_{n}, \qquad t \in T, n \in n(t),$$
$$\sum_{v \in V_t \setminus V_t^N} y_{vn}^{SE} \leq \overline{SE}_{n}, \qquad t \in T, n \in n(t),$$
$$\sum_{v \in V_t \setminus V_t^N} h_{vn}^{I} \leq \overline{CI}_{n}, \qquad t \in T \setminus \{0\}, n \in n(t),$$
$$\sum_{v \in V_t \setminus V_t^N} h_{vn}^{O} \leq \overline{CO}_{n}, \qquad t \in T \setminus \{0\}, n \in n(t),$$

Linearization of binary variables constraints

$$w - \bar{\delta}_{in} + \delta_{in} \leq 1, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$
$$\bar{\delta}_{in} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$
$$\bar{\delta}_{in} - \delta_{in} \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

$$w - \bar{y}_{vnj}^{NB} + y_{vnj}^{NB} \leq 1, \qquad t \in T : t \leq \bar{T} - T^L, v \in V_{t+T^L}^N, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}_{vnj}^{NB} - w \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V_{t+T^L}^N, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}_{vnj}^{NB} - y_{vnj}^{NB} \leq 0, \qquad t \in T : t \leq \bar{T} - T^L, v \in V_{t+T^L}^N, i \in N_t, n \in n(t), j \in \overline{D},$$

$$w - \bar{y}_{vnj}^{SC} + y_{vnj}^{SC} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}_{vnj}^{SC} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}_{vnj}^{SC} - y_{vnj}^{SC} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$w - \bar{y}_{vnj}^{SH} + y_{vnj}^{SH} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}_{vnj}^{SH} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SH}_{vnj} - y^{SH}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$w - \bar{y}^{SE}_{vnj} + y^{SE}_{vnj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SE}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SE}_{vnj} - y^{SE}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D}$$

Linearization of integer variables

$$y^{NB}_{vn} = \sum_{j \in \bar{D}} 2^j y^{NB}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SC}_{vn} = \sum_{j \in \bar{D}} 2^j y^{SC}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SH}_{vn} = \sum_{j \in \bar{D}} 2^j y^{SH}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$
$$y^{SE}_{vn} = \sum_{j \in \bar{D}} 2^j y^{SE}_{vnj}, \qquad t \in T, v \in V_t, s \in S,$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vn} \sum_{j \in \bar{D}} \bar{y}^{NB}_{vnj}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vnj} + C^{SH}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vnj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vnj}) = \bar{T} + 1$$

Convexity and integer constraints

$$y^{NB}_{vn} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$
$$y^{SC}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$
$$y^{SH}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$
$$y^{SE}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$
$$y^{NB}_{vnj} \in \{0,1\}, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$
$$y^{SC}_{vnj} \in \{0,1\}, \qquad t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),$$

$$
\begin{aligned}
y^{SH}_{vnj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{SE}_{vnj} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
\delta_{in} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\; i \in N^O_t, n \in n(t),\\
\bar{\delta}_{in} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; i \in N^O_t, n \in n(t),\\
\bar{y}^{NB}_{vnj} &\in \mathbb{R}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),\\
\bar{y}^{SC}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
\bar{y}^{SH}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
\bar{y}^{SE}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\; v \in V_t, n \in n(t),\\
y^{P}_{vn} &\in \mathbb{R}^+, && t \in T\; v \in V_t, n \in n(t),\\
l_{vn} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{I}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{O}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
x_{vrn} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t),\\
w &\in \mathbb{R}^+.
\end{aligned}
$$

Pattern Formulation

Objective function

$$
\begin{aligned}
maxz = & \sum_{t\in T\setminus\{0\}} \sum_{n\in n(t)} P_n \Big(\sum_{i\in N_t^O} \sum_{c\in G} R_{in}^D D_{icn} \bar{\delta_{in}} \\
& + \sum_{i\in N_t^C} \sum_{c\in G} (R_{in}^D D_{icn} w - C_{icn}^{SP} \bar{n}_{icn}) - \sum_{v\in V_t} (C_{vn}^{OP} \bar{y}_{vn}^P \\
& + C_{vn}^{CO} \bar{h}_{vn}^I - R_{vt}^{CO} \bar{h}_{vn}^O \\
& + \sum_{r\in R_{vn}} C_{vrn}^{TR} \bar{x}_{vrn} - R_{vn}^{LU} \bar{l}_{vn}) \Big) \\
& + \sum_{n\in n(\bar{T})} \sum_{v\in V_{\bar{T}}} R_{vn}^{SV} \bar{y}_{vn}^P \\
& - \sum_{t\in T, t\leq \bar{T}-T^L} \sum_{n\in n(t)} \sum_{v\in V_{t+T^L}^N} P_n C_{vn}^{NB} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{NB} \\
& + \sum_{t\in T} \sum_{n\in n(t)} \sum_{v\in V_t} P_n (R_{vn}^{SC} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SC} \\
& + R_{vn}^{SH} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SE} - C_{vn} \sum_{j\in \bar{D}} \bar{y}_{vnj}^{SH})
\end{aligned}
$$

Demand constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} + \bar{n}_{icts} \geq D_{icts} w, \qquad t \in T \setminus \{0\}, i \in N_t^C, c \in G, n \in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} Q_{cv} \bar{x}_{vrn} \geq D_{in} \bar{\delta}_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^O, c \in G, n \in n(t),
$$

Capacity constraints

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} + \sum_{c\in G} n_{cin} \geq \sum_{c\in G} D_{icts}, \qquad t \in T \setminus \{0\}, i \in N_t^C, n \in n(t),
$$

$$
\sum_{v\in V_t} \sum_{r\in R_{ivt}} \max_{cinC} Q_{cv} x_{vrn} \geq \sum_{c\in C} D_{itcs} \delta_{in}, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),
$$

Frequency constraints

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}w, \qquad t\in T\setminus\{0\}, i\in N_t^C, n\in n(t),$$
$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{x}_{vrn} \geq F_{it}\bar{\delta}_{in}, \qquad t\in T\setminus\{0\}, i\in N_t^O, n\in n(t),$$

Time constraints

$$\sum_{r\in R_v t} Z_{rv}\bar{x}_{vrn} \leq Z_v(\bar{y}^P_{vn} + \bar{h}^I_{vn} - \bar{h}^O_{vn} - \bar{l}_{vn}), \qquad t\in T\setminus\{0\}, v\in V_t, n\in n(t),$$

Optional trades constraints

$$\delta_{i,a(n)} \leq \delta_{in}, \qquad t\in T\setminus\{0,1\}, i\in N_t^O, n\in n(t),$$

Pool constraints

$$\bar{y}^P_{vn} = \bar{y}^P_{v,t-1} - \sum_{j\in\bar{D}}(\bar{y}^{SC}_{v,t-1,j} - \bar{y}^{SH}_{v,t-1,j} + \bar{y}^{SE}_{v,t-1,j}), \qquad t\in T\setminus\{0\}\; v\in V_t\setminus V_t^N, n\in n(t),$$

$$\bar{y}^P_{vn} = Y^{NB}_{vt}, \qquad t\in T: t<\bar{T}^L, v\in V_t^N, n\in n(t)$$

$$\bar{y}^P_{vn} = \sum_{p\in P^P} P^O_{vp}\bar{p}^P_{t-1,s,p}, \qquad t\in T: t\geq\bar{T}^L, v\in V^N_{t+T^L}, n\in n(t),$$
$$\bar{y}^P_{v0s} = Y^{IP}_v w, \qquad v\in V_0, n\in n(t),$$
$$\bar{y}^P_{vn} = \sum_{j\in\bar{D}}\bar{y}^{SC}_{v,t-1,j}, \qquad t\in T\setminus 0, v\in V_t\setminus v\in V_{t+1}, n\in n(t),$$
$$\bar{l}_{vn} - \bar{h}^I_{vn} + \bar{h}^O_{vn} \leq \bar{y}^P_{vn}, \qquad t\in T\setminus\{0\}, v\in V_t, n\in n(t),$$

Charters and second hand market constraints

$$\bar{y}^{SH}_{vn} \leq \overline{SH}_{vn} w, \qquad t \in T, v \in V_t,$$
$$\bar{y}^{SE}_{vn} \leq \overline{SE}_{vn} w, \qquad t \in T, v \in V_t,$$
$$\bar{h}^{I}_{vn} \leq \overline{CI}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\bar{h}^{O}_{vn} \leq \overline{CO}_{vn} w, \qquad t \in T \setminus \{0\}, v \in V_t,$$
$$\sum_{v \in V_t \setminus V_t^N} y^{SH}_{vn} \leq \overline{SH}_n, \qquad t \in T, n \in n(t),$$
$$\sum_{v \in V_t \setminus V_t^N} y^{SE}_{vn} \leq \overline{SE}_n, \qquad t \in T, n \in n(t),$$
$$\sum_{v \in V_t \setminus V_t^N} h^{I}_{vn} \leq \overline{CI}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$
$$\sum_{v \in V_t \setminus V_t^N} h^{O}_{vn} \leq \overline{CO}_n, \qquad t \in T \setminus \{0\}, n \in n(t),$$

Linearization of binary variables constraints

$$w - \bar{\delta}_{in} + \delta_{in} \leq 1, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$
$$\bar{\delta}_{in} - w \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$
$$\bar{\delta}_{in} - \delta_{in} \leq 0, \qquad t \in T \setminus \{0\}, i \in N_t^O, n \in n(t),$$

$$w - \bar{p}^{P}_{np} + p^{P}_{np} \leq 1, \qquad t \in T, n \in n(t), p \in P^O,$$
$$\bar{p}^{P}_{np} - w \leq 0, \qquad t \in T, n \in n(t), p \in P^O,$$
$$\bar{p}^{P}_{np} - p^{P}_{np} \leq 0, \qquad tt \in T, n \in n(t), p \in P^O,$$

$$w - \bar{y}^{SC}_{vnj} + y^{SC}_{vnj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SC}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SC}_{vnj} - y^{SC}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$w - \bar{y}^{SH}_{vnj} + y^{SH}_{vnj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SH}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$

$$\bar{y}^{SH}_{vnj} - y^{SH}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$w - \bar{y}^{SE}_{vnj} + y^{SE}_{vnj} \leq 1, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SE}_{vnj} - w \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D},$$
$$\bar{y}^{SE}_{vnj} - y^{SE}_{vnj} \leq 0, \qquad t \in T, v \in V_t, i \in N_t, n \in n(t), j \in \overline{D}$$

Transformation of patterns to new buildings constraints

$$y^{NB}_{vn} = \sum_{p \in P^P} P^O_{vp} p^P_{np}, \qquad t \in T, v \in V^N_t, n \in n(t), \tag{A.3}$$
$$\sum_{p \in P^P} p^P_{np} = 1, \qquad t \in T, n \in n(t). \tag{A.4}$$

Linearization constraints

$$\sum_{t \in T} (\beta^t C^{EI}) w + \sum_{t \in T} \sum_{v \in V_t} \sum_{\bar{t} \in T, \bar{t} < (T-t)} \beta^{\bar{t}} (C^{NB}_{vn} \sum_{p \in P^P} P^O_{vp} \bar{p}^P_{np}$$
$$+ R^{SC}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SC}_{vnj} + C^{SH}_{vn} \sum_{j \in \bar{D}} \bar{y}^{SH}_{vnj} + R^{SE}_{vt} \sum_{j \in \bar{D}} \bar{y}^{SE}_{vnj}) = \bar{T} + 1$$

Convexity and integer constraints

$$y^{NB}_{vn} \in \mathbb{Z}^+, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$
$$y^{SC}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t),$$
$$y^{SH}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t),$$
$$y^{SE}_{vn} \in \mathbb{Z}^+, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t),$$
$$y^{NB}_{vnj} \in \{0, 1\}, \qquad t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),$$
$$y^{SC}_{vnj} \in \{0, 1\}, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t),$$
$$y^{SH}_{vnj} \in \{0, 1\}, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t),$$
$$y^{SE}_{vnj} \in \{0, 1\}, \qquad t \in T \setminus \{\bar{T}\}\ v \in V_t, n \in n(t),$$
$$\delta_{in} \in \{0, 1\}, \qquad t \in T \setminus \{\bar{T}\}\ i \in N^O_t, n \in n(t),$$
$$\bar{\delta}_{in} \in \mathbb{R}^+, \qquad t \in T \setminus \{\bar{T}\}\ i \in N^O_t, n \in n(t),$$

$$
\begin{aligned}
\bar{y}^{NB}_{vnj} &\in \mathbb{R}^+, && t \in T : t \leq \bar{T} - T^L, v \in V^N_{t+T^L}, n \in n(t),\\
\bar{y}^{SC}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
\bar{y}^{SH}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
\bar{y}^{SE}_{vnj} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, v \in V_t, n \in n(t),\\
y^{P}_{vn} &\in \mathbb{R}^+, && t \in T\, v \in V_t, n \in n(t),\\
l_{vn} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{I}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
h^{O}_{vt} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, v \in V_t, n \in n(t),\\
x_{vrn} &\in \mathbb{R}^+, && t \in T \setminus \{0\}, r \in R_{vt}, v \in V_t, n \in n(t),\\
\bar{p}^{P}_{np} &\in \mathbb{R}^+, && t \in T \setminus \{\bar{T}\}\, p \in P^O, n \in n(t),\\
p^{P}_{np} &\in \{0,1\}, && t \in T \setminus \{\bar{T}\}\, p \in P^O n \in n(t),\\
w &\in \mathbb{R}^+.
\end{aligned}
$$

Appendix B

Attachments

Included as a ZIP file can be found:

- Mosel code for the Anti Symmetry, Power and Pattern Formulations, with node and scenario formulation
- Three input files, one for each set size; Large, Medium and Small