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Norwegian University of
Science and Technology

Oil Price Realized Volatility Forecasts: The Role of Implied Volatility, Past Returns, Bid-Ask Spread and Slope of the Futures Curve

Henrik Søyland Langeland

Industrial Economics and Technology Management
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Supervisor: Sjur Westgaard, IØT

Norwegian University of Science and Technology
Department of Industrial Economics and Technology Management

Preface

This thesis completes my master's degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The work has been carried out during the spring of 2013 under the supervision of Professor Sjur Westgaard.

I would like to thank Sjur for his guidance and his positive and encouraging nature. My gratitude also goes to Postdoc Peter Molnar whom has played an essential part in the work with this theses. His advice has never been more than seconds away independent of time of the day, and day of the week. Finally, Post-doctoral Fellow Erik Haugom has contributed substantially with his highly relevant expertise.

Henrik Langeland
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Abstract

Forecasting oil price volatility is necessary in order to perform tasks such as portfolio optimization, options and derivatives pricing, value-at-risk modeling and hedging. In this paper volatility forecasting in the WTI futures market is approached with a focus on identifying useful forecasting variables. A realized volatility (RV) time series based on high frequency data is constructed and used in a long-memory model. The predition model is expanded by including implied volatility (IV) and exogenous market variables (EX) including volume, open interest, daily returns, bid-ask spread and the slope of the futures curve. When testing different combinations of these variables in out-of-sample predictions we find that IV significantly improves daily and weekly volatility forecasts, and that the exogenous market variables significantly improves daily, weekly and monthly volatility forecasts. Of the exogenous market variables the daily returns contribute significant for all prediction horizons. The returns also show a V-shaped relation to volatility.

Sammendrag

Prediksjon av volatilitet er nødvendig for å utføre oppgaver som porteføljeoptimering, prising av opsjoner og derivater, value-at-risk-modellering og hedging. I denne artikkelen blir volatilitetspreksjon for terminkontraktsmarkedet for WTI råolje tilnærmet med fokus på hvilke variabler som inneholder nyttig informasjon. En realisert volatilitet (RV) tidsserie blir konstruert ved bruk av høyfrekvent markedsdata og brukt i en tilnærmet ”long-memory”-modell. Modellen utvides ved å indkludere implisert volatilitet (IV) og eksogene markedsvariabler inkluderer volum, open interest, daglig avkastning, bid-ask differansen og helningen på terminkontraktkurven. Når forskjellige kombinasjoner av disse variablene testes ”out-of-sample” finner vi at IV gir en signifikant forbedring av volatilitetsprediksjonene for daglige og ukentlige tidshorisonter. De eksogene variablene gir en signifikant forbedring av daglige, ukentlige og månedlige volatilitetsprediksjoner. Daglig avkastning forbedrer prediksjonene over alle de tre tidshorisontene og viser et V-formet forhold til volatiliteten.

Contents

Chapter 1

Introduction

Over the last century, energy consumption has followed the exponential growth of the world economy. Economic activity is now crucially dependent upon a large supply of crude oil, accounting for 33 %[1] of the world's demand for energy. This has positioned crude oil as the most important commodity in the world and its price has a significant impact on the overall macroeconomy (Guo and Kliesen, 2005).

Since so many subjects are exposed to this market, understanding its volatility must be given attention. Particularly important are *forecasts* of the price volatility. The accuracy of such forecasts will determine the precision of tasks such as portfolio optimization, options and derivatives pricing, value-at-risk modeling and hedging. Reliable forecasts of volatility will also be necessary for making decisions about investments in production capital, directly affecting the supply side of the market.

Forecasting volatility has traditionally been done using the generalized autoregressive conditional heteroscedasticity (GARCH) approach introduced by Bollerslev (1986) based on the work of Engle (1982). A vast number of volatility forecasting models based on this concept already exist for energy commodity markets (see e.g. Marzo and Zagaglia (2010) and Wei et al. (2010)). However, this approach does not measure volatility very precisely. A breakthrough in volatility measuring was provided when Andersen and Bollerslev (1998) introduced realized volatility as the sum of squared intra-daily returns. Since this made volatility essentially an observable variable, it can now be modeled more easily. However, this has not alleviated the search for increasingly complex volatility forecasting models in the literature.

[1] EIA numbers for 2008

It has long been recognized that there are other sources of information about future volatility than RV. A natural candidate is the market's expectation of future volatility, commonly referred to as implied volatility (IV). It has been found by some (e.g. Lamoureux and Lastrapes (1993); Jorion (1995); Agnolucci (2009)) to deliver both biased and inefficient forecasts of volatility. This is surprising because IV can incorporate known future events likely to affect volatility. Evidence that IV has a role in volatility forecasts has also been presented (e.g. Day and Lewis (1993); Szakmary et al. (2003); Doran and Ronn (2005); Agnolucci (2009)). According to Jorion (1995), a failure to unearth IV's predictive power can only be interpreted in two ways; inefficient information processing in options markets or misleading test procedures. In highly liquid and transparent markets such as the WTI futures market the former is unlikely. Left is the latter, and in particular the discussion about the bias of the Black-Scholes (BS) formula (see e.g. Doran and Ronn (2005)). A way to avoid this possible problem (and several others) is to use a volatility index which is based on the market price of variance. Such an index was introduced for the WTI futures market in 2008.

Volatility has also been linked to several other market variables. For instance, the relationship between volume and volatility is widely documented (e.g. Clark (1973) and Gallant et al. (1992)). In addition to possibly improving volatility forecasts, including additional variables in the analysis can generate an improved understanding of the market.

Relatively little work has been done investigating RV in the WTI futures market. This is especially true when considering the market's economic importance (Sadorsky, 2006). Wang et al. (2008) studied the realized correlation between oil and gas markets and found the use of RV in energy markets to be highly appropriate, especially in areas such as volatility forecasting. Martens and Zein (2004) compared RV with options-derived IV for the WTI futures market, finding that the RV makes IV redundant. Little work has been done regarding the WTI IV index, perhaps due to its recent inception. An exception is Padungsaksawasdi and Daigler (2013) who studied the return-IV relation, and concluded that IV increases with negative returns.

In this paper we deviate from the mainstream direction of volatility forecasting and its focus on comparisons of model specifications. Instead, we look at whether additional *variables* should be included when forecasting volatility for the WTI futures market. We chose a long-memory model based on the realized measure of volatility using high-frequency data. In addition, two fundamentally different types of variables are used in the model, the forward looking IV index and other exogenous market variables including volume, open interest, daily returns and the slope of the futures curve. The main findings can be summarized as follows. First, we find that including information from the OVX significantly improves the day-ahead and weekly volatility forecasts. Second, the exogenous market variables improve volatility forecasts for daily, weekly and monthly horizons. Of the exogenous market variables, especially the daily returns are found to improve volatility predictions.

The rest of the paper will unfold as follows. Chapter 2 presents the WTI crude oil market and its implied volatility index together with other market variables. Chapter 3 outlines the theoretical foundation for the RV and IV measures used in this paper. Chapter 4 describes the market data. Chapter 5 describes the model and evaluations of in- and out-of-sample predictions. Chapter 6 summarizes and concludes.

Chapter 2

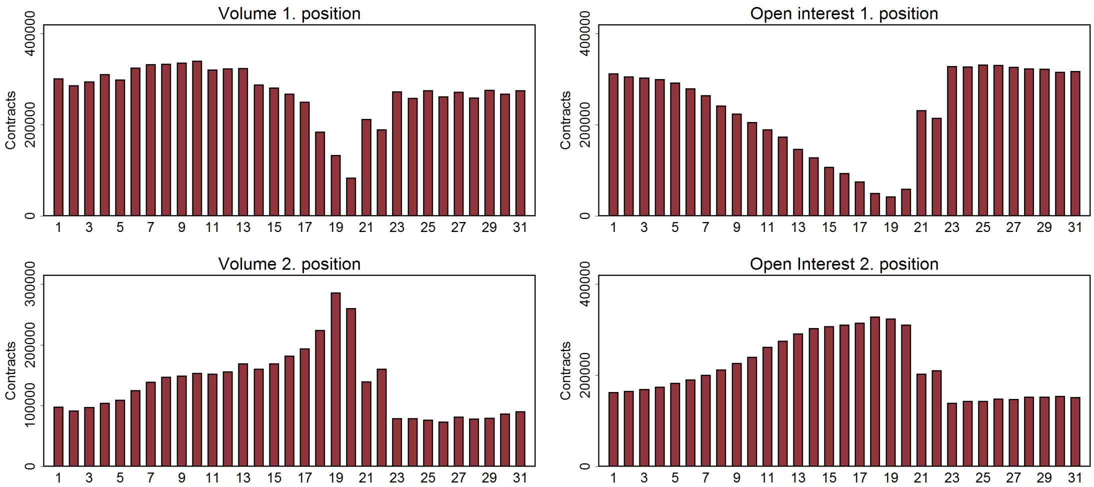
WTI crude oil futures market

Benchmark prices are crucial to a unified world oil market and makes formula pricing possible. The most liquid and widely used petroelum benchmark price is the West Texas Intermediate Light Sweet Crude Oil (WTI) futures contract (Downey, 2009). The main assumtion behind formula pricing is that changes in volatility for the petroleum market as a whole is bigger than specific products. The price volatility of this contract therefore reaches far beyond its specific grade of crude. The rest of this chapter gives an overview of WTI futures contract, its main derivatives and a description of important market variables linked to volatility.

2.1 WTI crude oil futures contracts

The WTI futures contracts are primarily traded through the CME Globex electronic trading platform, but also through the CME Clear Port and open outcry at the New York Mercantile Exchange. A single WTI futures contract represents 1000 bbl of oil for physical delivery in Cushing Oklahoma. Front month trading ends on the third business day prior to the 25th of the month prior to delivery. If the contract is held until expiry physical delivery must be given or taken during the following month according to the specifications in the contract.

The contracts' expiration gives rise to monthly seasonality. Figure 4.1.3 (a) shows how average volume for the first position changes throughout the month. The volume traded on the second position is a mirror image because of traders rolling their positions. The same mechanics seem to drive open interest as show in figure 4.1.3 (b). Yearly seasonality is usually not found in the crude oil prices even though several petroleum products show distinct seasonality. The reason is that the varying seasonality of all the petroleum products that exist, tend to even out



(a) Volume **(b)** Open interest

Figure 2.1.1: Daily average values during each day of the month for volume and open interest during the period 16/5/2007 to 15/5/2012

yearly seasonality in the crude oil markets (Downey, 2009).

Daily average total volume is 900,000 futures and options contracts and the largest open interest for all contracts has reached 7.5 million lots.[1] Trading starts 17:00 and ends 16:15 eastern time (ET) on weekdays, meaning that every day there is a 45 minute break. The trading week starts Sunday evening and ends Friday afternoon.

2.2 Options and the oil volatility index

CBOE began calculating the CBOE Crude Oil Volatility Index (OVX) in June 2008 (using data back to May 2007) according to the "VIX methodology" which will be discussed in detail in section 3.2. The index has become an important instrument for trading oil price volatility (Whaley, 2008).

The options underlying the OVX are options on the United States Oil fund (USO), an exchange traded fund (ETF) established to replicate the returns of the WTI benchmark price. This investment vehicle reduces transaction costs for investors

[1]Numbers for 2013. See www.cmegroup.com/trading/energy/

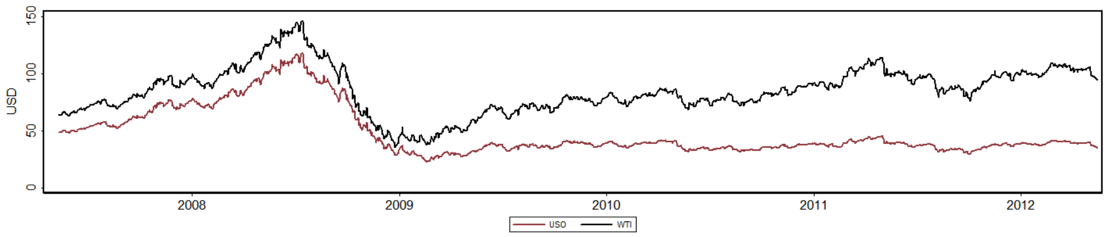


Figure 2.2.1: *The price of one share of the USO and the price of the WTI futures contracts rolling from 1. to 2. position the first business day prior to the 10. of each month for the period 16/5/2007 to 15/5/2012*

seeking exposure to the oil price. Since USO needs a management and it is exposed to hedging risks it is unable to completely replicate the price of oil. This is obvious by looking at figure 2.2.1 which shows that the price of one share has been unable to keep up with the price of the front month contracts. It is also seen that the short term variations are very similar. The daily returns of the USO have a 88 % correlation to the first futures position and a 94 % correlation to the second futures position. The difference is important since the OVX represents expectations of the volatility of the USO which in practice it will means the expectations of the volatility of the contracts held by the USO.

The USO has no exposure to the spot price of oil and only holds futures contracts and other oil related derivatives. The main part of the fund's exposure to the oil market is contracts at the first and second position. The fund rolls these contracts during a 4–day window starting approximately 14 days before expiry of the first positions.[2] The rolling window for each month is publicly announced on the company website but further details about the rolling is not available to the market. It is therefore impossible to know the exact composition of the USO's contract holdings during this window other than it is likely to be shifting towards the second position. Over the course of one month the fund will therefore be mainly be holding second position contracts about half of the days and first position contracts the other half.

Because both the underlying and the options are traded side by side in a very liquid market with minimal transaction costs, prices are likely to be highly synchronized. Its therefore unlikely that the market will suffer from asymmetric information dissemination, which according to Jorion (1995) is a frequent source of measurement errors when investigating implied volatility.

[2]See www.unitedstatesoilfund.com/uso-rolldates.php

2.3 Market variables

Different market variables are found to have different links to volatility. Some of these links are found across markets and other are market specific. In this section, some very common market variables used in volatility analysis are introduced in the context of the WTI futures market.

2.3.1 Volume

The "Mixture of Distribution Model" originally proposed by Clark (1973) states that both traded volume and volatility are driven by the same underlying "news"-variable and will therefore be positively correlated. According to this theory, it is primarily the *number* of trades that will capture the content of this news variable, a relation that has been widely documented for stock markets for instance by Gallant et al. (1992) and Andersen (1996).

The contract specifications in the WTI futures market disturbs the information content of the volume variable because of the previously mentioned monthly cycle. Volume on first position will start to decline as the contract nears expiry and volume on second position contract will increase as it closes in on first position as shown in figure 2.1.1. Rolling the contracts further away from expiry such as USO does, does little in removing this cycle. Therefore the volume traded is driven mostly by the structure of the market.

An alternative description of the volume-volatility relation, based on the microstructure theory described by O'Hara (1995), is grounded in different types of traders with asymmetric information. The theory states that informed traders will prefer to trade larger quantities, therefore the trade size has information content for prices (Kim and Verrecchia, 1991). However, this information can potentially be "hidden" by devising tactics of stealth trading, by simply breaking a large trade into smaller parts which according to (Chakravarty, 2001) this is a widely used tactic.

Either way, it is possible that number of trades and the size of trades provide different information and are therefore linked differently to the volatility in the WTI futures market. We therefore decompose volume into these two components. Figure 2.3.1 plots number of trades and average size of trades calculated for each day of the month for a synthetic futures contract designed to replicate the positions held by USO. The figure shows that the cyclic variation in volume is mainly evident in the number of trades component. Secondly, the very low average size of trades

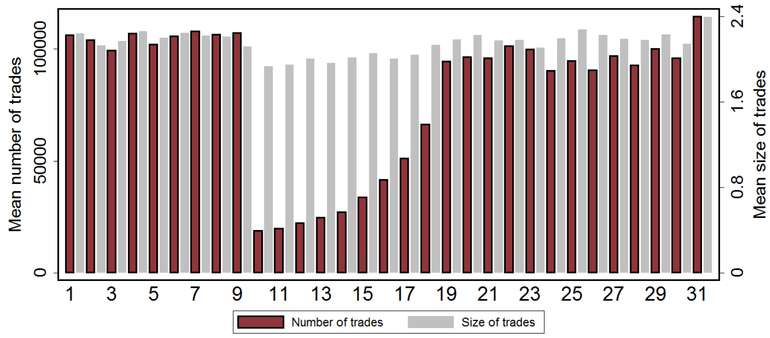


Figure 2.3.1: *The average size of trades and the average number of trades for each day of the month when rolling to second position the first business day prior to the 10th every month. Numbers are for the period 16/5/2007 to 15/5/2012*

(approximately 2 contracts) indicate that stealth trading is indeed widespread.

2.3.2 Open interest

Open interest is a variable which shows how many bets market participants already have taken, indicating depth and size of the market. High open interest means that a lot of subjects already have taken bets, and low open interest means that many subjects are not in the market yet. Therefore, this market characteristic might be relevant for volatility forecasting. Girma and Mougoue (2002) found that lagged values of open interest affect volatility and can be used for short term predictions of price movements in petroleum futures markets. Figlewski (1981) found evidence that open interest is positively correlated with volatility in futures markets using a monthly average of open interest.

Since taking physical delivery of the oil is certainly not the goal of many of the subjects trading oil futures, open interest is a highly cyclical variable depending on the days to maturity of the contract, as described in the previous section. This means that in order for the measure of open interest to provide information the cyclical component must be removed from this variable.

2.3.3 Daily returns

Doran and Ronn (2005) found a positive correlation between price returns and volatility in energy markets, contrary to the ”leverage effect” found in equity markets. Their explanation is it that higher commodity prices represents a threat to economic activity for energy importing countries. For oil markets this ”inverse leverage effect” has been partially confirmed by for instance Wei et al. (2010) and Cheong (2009) who found significant indications of its existence in the *ICE Brent* market. However, together with Wang et al. (2008), Agnolucci (2009) and Padungsaksawasdi and Daigler (2013) they failed to discover it in the WTI futures market. Cheong (2009) consequently suggested that the WTI futures market may have other forms of interrelationships to macroeconomic indicators and financial derivatives prices. Padungsaksawasdi and Daigler (2013) also conclude that although behavioral theories proved weak explanations for return-volatility relations in commodity markets they make more sense than the leverage hypothesis.

At a more fundamental level Kilian (2006) argues that changes in the oil price are primarily driven the demand side and makes the distinction between precautionary demand and aggregate demand, pointing out their different effects on the US economy. Precautionary demand with negative effects, and aggregate demand with positive effects. The author concludes that the price build up since late 1990s is mainly the result of aggregate demand while during instances of political instability prices are driven by precautionary demand.

We do not have strong expectations about the sign of the leverage effect. However, we expect that positive and negative returns might have different impact and we allow this in our estimation.

2.3.4 Bid-ask spread

The bid-ask spread (BAS) has been found to have a positive correlation to price volatility (e.g. Bollerslev and Melvin (1994); Roll (1984) and Wang and Yau (2000)). According to Amihud and Mendelson (1986) it is a measure of market illiquidity as the quoted ask offers a premium on immediate buying and the quoted bid offers a premium on immediate selling.

The BAS is commonly divided into three components which it must cover. The processing cost of orders, the cost for market makers of holding the futures (i.e. the cost of hedging) and the cost of adverse information. The cost of processing orders through the Globex electronic trading platform is minimal, and the instruments available to market makers make hedging their portfolio efficient. The variation in cost of hedging will therefore be driven by the cost of variance which is reflected in the OVX. One would therefore suspect some correlation between the BAS and the OVX.

Glosten and Milgrom (1985) identify adverse information as the main driver of

BAS. The cost incurred to market makers through adverse information is caused by trading with a superiorly informed customer. In order to protect them self to this loss they must widen the BAS. This component would therefore represent excess variation to the variation found in the OVX and could be interpreted as insecurity about the direction of the market.

Our data set does not contain bid and ask quotes, therefore we have to calculate the spread from actual trades. Schultz (2000) and Huang and Stoll (1996) showed that the Roll-estimator (Roll, 1984) is an appropriate measure of spreads when applied to intra-daily data for liquid markets. It is based on the recognition that if trades fluctuate between spreads, returns will be negatively autocorrelated. The bid-ask spread is defined as follows:

$$BAS = 2\sqrt{\frac{-\sum_{t=1}^{t} \Delta P_t \Delta P_{t-1}}{T-1}}, \tag{2.1}$$

where ΔP is the price difference between two consecutive trades and T is the number of trading pairs during the day.

This estimation is of the effective spread not the quoted spread. Meaning the spread investors actually face in the market. The assumptions underlying estimator are; that successive trades are independent, that the spread is constant during the day, that trade types do not contain information about future changes in value and that changes in true value do not contain information about futures trades (Roll, 1984).

2.3.5 Futures curve slope

According to Litzenberger and Rabinowitz (1995) the crude oil market is expected to exhibit backwardation. The cause is that owning an extractive resource is equal to owning a call option with a pay-off equal to the spot price and strike price equal to the extraction cost. The producer will therefore evaluate the price of this option against having oil out of the ground. Without backwardation, this option would not be exercised (hence, no production), just as an option on a stock without dividend would not be exercised before expiration. In times of high volatility this option becomes more valuable which in turn requires stronger backwardation for the option to be exercised. The price volatility should therefore positively correlated with the degree of backwardation (Litzenberger and Rabinowitz, 1995).

Kogan et al. (2009) expands on this theory based on a production economy framework. By observing that since capital investments for oil production are irreversible, the supply of oil would also be inelastic and futures prices volatile. Since spot prices are affected by the degree of optimality of the production capital stock, the absolute value of the slope will be larger when there is a large deviation from this optimality. This leads to a V-shaped relationship between volatility and the shape of the futures curve. In other words, an increasing degree of contango will also be positively correlated to volatility. The measure of the slope of the futures curve used by Kogan et al. (2009) is the following one:

$$SL_t = \ln\Big(\frac{P_{t,6}}{P_{t,3}}\Big), \tag{2.2}$$

where $P_{t,6}$ is the latest price tick observable at day t for the 6th position and $P_{t,3}$ for the 3rd position. In order to allow for the V-shaped relationship to be captured we will split the SL_t variable into SL_t^+ and SL_t^- after demeaning it:

$$SL_t^+ = max(SL_t, 0) \tag{2.3}$$

$$SL_t^- = min(SL_t, 0) \tag{2.4}$$

Chapter 3

Theory of volatility

This chapter will introduce the two most important theoretical concepts of volatility used in this paper. First is a description of the assumptions and calculation of the RV measure. Second is a description of how the implied volatility variable is calculated.

3.1 Realized volatility

First, let $p(t)$ denote the price of some asset, which is governed by the following process:

$$dp_t = \mu(t)\mathrm{d}t + \sigma(t)\,\mathrm{d}W(t) + \kappa(t)\mathrm{d}q(t), \tag{3.1}$$

where $\mu(t)$ is the drift, $\sigma(t)$ is the instantaneous volatility and $W(t)$ is a standard Brownian motion. $q(t)$ is a Poisson counting process with the corresponding time varying intensity function $\lambda(t)$, adding to the unobserved quadratic variation proportional to the number and sizes of the jumps. The theory of quadratic variation allows for the decomposition of the total variation into its continuous and jump parts which returns the following representation:

$$QVar_t = \int_0^t \sigma^2(s)\mathrm{d}s + \sum_{j=1}^{q(t)} \kappa^2(s_j) \tag{3.2}$$

In the absence of jumps the quadratic variation is equal to the integrated variance:

$$IVar_t = \int_0^t \sigma^2(s)\mathrm{d}s \tag{3.3}$$

Andersen and Bollerslev (1998) proposed to use the realized variation as a proxy for the integrated variance which can be written as:

$$\lim_{M\to\infty} RVar_t(M) = \int_0^t \sigma^2(s)\mathrm{d}s \tag{3.4}$$

where M is the frequency of intra-daily sampling, indicating that the accuracy of the measure will increase as the sampling frequency increases. With equally spaced intervals, intra-daily returns can be written as:

$$r_{t,j} = p_{t,j} - p_{t,j-1}, \quad j = 1, \cdots, M, \quad t = 1, \cdots, T \tag{3.5}$$

for T days. Assuming sufficiently high sampling frequency the drift in the Browninan motion, determining the price process, can be neglected. The result is that realized variance can be written as the sum of squared intra-daily returns:

$$RVar_t(M) = \sum_{j=1}^{M} r_{t,j}^2, \quad t = 1, \cdots, T. \tag{3.6}$$

Throughout this paper the measure realized volatility (RV) will be used, defined as:

$$RV_t(M) = \sqrt{RVar_t(M)} = \sqrt{\sum_{j=1}^{M} r_{t,j}^2}, \quad t = 1, \cdots, T. \tag{3.7}$$

3.2 Implied volatility and volatility indices

The need for a robust measure of expected volatility and a demand to hedge changes in volatility motivated the development of the Sigma Index (Brenner and Galai, 1989), fundamentally different from the BS-derived IV. The modified version of this original index, coined the ”VIX methodology”, has since its inception become the foundation for the most common volatility indices (Whaley, 2008). The core idea of the index is to calculate the square root of the *price of variance* by the construction of a portfolio of options which isolates the variance of the underlying.

The portfolio is centered around two strips of out-of the money calls and puts and its exposure to the risk of price variations is eliminated by delta hedging with a forward position in the underlying. A clean exposure to volatility risk, independent of the value of the underlying, is obtained by calibrating the options to yield a constant sensitivity to variance. If each option is weighted proportionally to the strikes on either side of the option's price and is inversely proportionally to the square of the option's strike price, the sensitivity of the portfolio to total variance is equal to one. Holding the portfolio to expiration therefore replicates the total variance (Demeterfi et al., 1999). The price of the variance σ^2 is defined as the forward price of a particular strip of options shown in the following equation (CBOE, 2003):

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{t}\left[\frac{F}{K_0} - 1\right]^2, \tag{3.8}$$

where T is the time to expiration, F is the forward price level of variance defined as the strike price at which the absolute difference between the call and put option is smallest, plus the discounted difference between the put and the call, K_0 is the first strike below F, ΔK_i is half the difference of the strikes on either side of K_i, R is the risk free interest rate to expiration and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i. i represents the selected out-of-the-money puts and calls on the underlying centered around K_0. Puts with $K_i < K_0$ and calls with $K_i > K_0$ and only options with a non-zero bid quote are used.

Two such portfolios are calculated and a third synthetic portfolio with a constant 30-days to expiry is constructed by linear interpolation of the near- and next-term positions shown the following formula:

$$IV = \sqrt{\left(T_1\sigma_1^2\left[\frac{T_2 - 30}{T_2 - T_1}\right] + T_2\sigma_2^2\left[\frac{30 - T_1}{T_2 - T_1}\right]\right)\frac{365}{30}}, \tag{3.9}$$

where T_1 and T_2 is the time to expiry of the near- and next term options, respectively. Maintaining a constant 30-days to expiry is preferred because the implied volatility changes with the options' time to expiry. When the near-term option has less than one week to expiration the index rolls over to the next positions.

The possibility of arbitrage implies that the forward price of variance must be equal to the forward price of the portfolio which replicates it. Observing that the forward positions of the underlying in the portfolio contributes nothing to its value, the forward price of variance reduces to the forward price of the strips of options. In this measure, the risk premium of volatility described by Chernov (2001) will be included. This is likely to cause implied volatility to be higher than realized volatility. Given a liquid options market the OVX is expected to be an accurate proxy of what investors' expectations of future volatility including the risk premium of volatility. The index is calculated on a minute-to-minute basis is published by the Chicago Board Options Exchange (CBOE).[1]

[1]When the index is published IV in equation 3.9is multiplied by 100.

Chapter 4

Data

The data used in this paper were purchased from the CME Group and contain all trades of WTI futures contracts made through the CME Globex electronic trading platform from 16/5/2007 to 15/5/2012, totaling 173.4 million ticks. From this data only information involving the contract position held by USO is kept.[1] Trades happening more than 24 hours before closing are also removed.[2] In addition, trading days with an early close (i.e before 16:15 ET) are removed. This leaves a total of 1246 trading days in the sample. All variables are generated from this data set, except the IV and open interest variables which were acquired through EcoWin Reuters database.

4.1 Realized measures

In order to generate a realized measure, the inhomogeneous time series was made homogeneous by extracting ticks closest to every minute-change. This leaves 1.51 million ticks over the 1246 trading days between 16/5/2007 and 15/5/2012. This period has a total of 1.74 million trading minutes (based on a 23.25 hour trading day) indicating that during the average trading hour in this sample, there are about 8 minutes without any contracts changing hands. The *previous tick method* described by Wasserfallen and Zimmermann (1985) is chosen as was suggested by Hansen and Lunde (2006), for time series where sampling may occur several times between observations. Another important appeal of this method, contrary to the linear interpolation method, is that it only uses information available at the time of calculation.

[1] Since USO rolling is impossible to observe, rolling is set to happen on the business day prior to the 9th every month.

[2] Extended trading days can happen under special circumstances, usually around holidays.

The choice of sampling frequency has a direct effect on the accuracy of the RV measure (which was defined in equation 3.7 as the square root of the realized variance). Equation 3.4 dictates that increasing the frequency is desirable and causes more information about the volatility of the underlying process to be captured. This can be seen in the last two columns of table 4.1; the realized measures based on higher frequencies display a higher autocorrelation.

There are, however, upper boundaries for sampling frequencies. Microstructure noise will increasingly disturb the calculated results as M increases. For instance, at a sufficiently high M realized volatility will increase due to the oscillations in prices caused by the bid-ask spread. When balancing the contradictory needs to capture information and avoid microstructure noise, Andersen et al. (2001) concluded that market liquidity is ultimately the deciding factor.

Andersen et al. (2001) argue that in a "liquid marked" such a point of balance is found at 5-minute sampling intervals, or 288 intra-daily observations. Admittedly, the WTI futures market is not as liquid as the FX market, which was the market studied by these authors, but that it is "highly liquid" was explained in section 2.1. A frequency of 5-minutes is therefore chosen in this paper. (As was done by for instance Bandi and Russell (2006), Andersen et al. (2007) and Patton (2011).)

Further justification of this choice is given on three accounts. First, microstructure noise can according to both Bandi and Russell (2008) and Bollerslev et al. (2008) be seen in a signature plot of average values for RV calculated at different frequencies. The average values displayed in table 4.1 show that microstructure effect starts to become significant at the 1-minute frequency, confirming that RV based on 5-minute intervals is not highly affected. A second justification is given by the standard deviations in table 4.1 showing minimum values for both 5 and 10 minute intervals. The standard deviation becomes higher for both higher and lower frequencies again probably because of microstructure noise. Third is that a very high kurtosis of intra-day returns could indicate problems. Figure 4.1.1 shows that the kurtosis of the returns increases with increasing frequencies and that it appears to level out at around 5 minute intervals.

Next we provide some characteristics of the RV time series. Figure 4.1.1 shows the slowly decaying autocorrelation of the 5-minute realized measure. This indicates that treating the time series as a long memory process can be justified. Futhermore, the RV time series over the period is stationary according to both the Dickey Fuller test and the Phillips-Perron test. When splitting the sample into five subsamples, however, the same tests can not reject the existence of a unit root for all parts of the time series.[3]. In other words, even though strictly speaking this time series does not have a unit root, it is quite close to being an I(1) process. Therefore, we later estimate a model for both levels of volatility and percentage changes in volatility.

Barndorff-Nielsen et al. (2008) found that for equity markets a decomposition of the RV measure based on positive and negative intra-daily returns contained different information. This was done in order to test if this was the case for the WTI futures market. Figure 4.1.1 reveals that this effect is not evident and that the regular RV measure will contain more information than any of these two measures.

Figure 4.1.3 (a) shows that the monthly cycles identified in section 2.1 are not seen in the RV measure. Further, the closing of the market during the weekend could cause additional volatility during for instance Mondays and Fridays, however according to figure 4.1.3 (b) there are no signs of this effect.

Although the WTI futures market is studied at a daily frequency in this paper, some additional information about the market can be given from the high frequency data. Figure 4.1.3 (b) shows that early morning and mid-day has higher activity and volatility than the hours from 17:00 to 01:00 ET. There is an increase in volume and volatility at around 02:00 ET, which could represent markets opening in Europe and as markets open in the US there is another, even more distinct, rise in volatility and market activity. The drop at around 11:00 ET matches well with both markets closing in Europe and lunchtime in the US. However, we do not investigate intra-daily variations further.

4.2 Other variables

Figure 4.2.1 displays the main variables[4] and the closing price of the front month contract from 16/5/2007 to 15/7/2012. It is apparent from this figure that this period was highly turbulent, as was the case in most markets during this period. In 2008 the price of oil reached record levels, only to be followed by a unprecedented decline and then a long recovery. These price fluctuations reveals an incoherent RV-price relation. At the beginning of the period they appear positively correlated but during the decline in price, volatility rises even more steeply. Then, RV seems to be negatively correlated to the price, but in a decreasing matter.

Comparing the graphs of the implied volatility measured by the OVX and the

[3]See appendix A

[4]For descriptive statistics see Appendix A

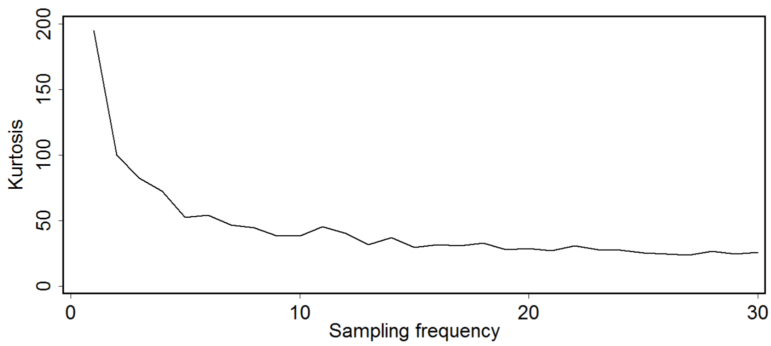


Figure 4.1.1: *Kurtosis of returns for sampling frequencies between 1 and 30 minutes.*

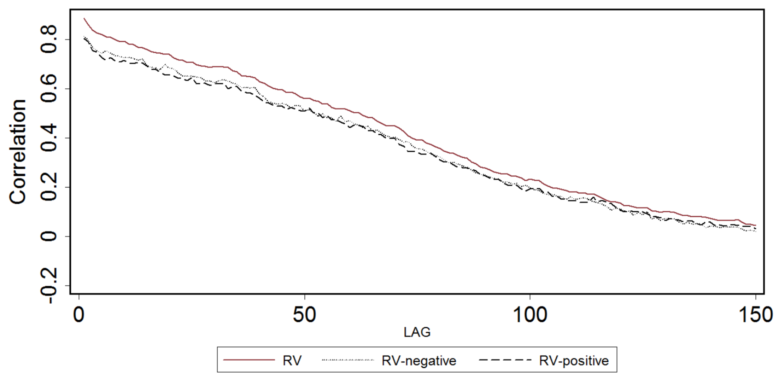
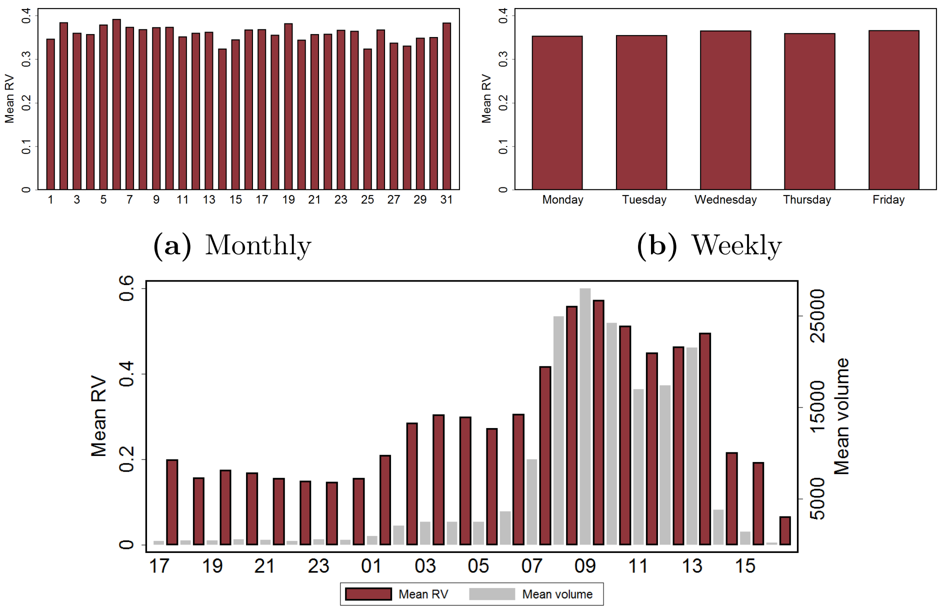


Figure 4.1.2: *Autocorrelation of RV based on 5-minute returns. Two additional lines show the autocorrelation for RV time series constructed using only squared negative returns and RV time series constructed using only squared positive returns.*

Table 4.1: *RV calculated for 1, 3, 5, 10, 15, and 30 minute intervals. DF is the Dickey Fuller test and PP is the Phillips-Perron test for unit root. Values below -3.430 means rejection at the 1% significans level for both series.AC(1) and AC(10) are the autocorrelation for 1 and 10 lags respectively.*

Descriptive statistics of realized volatility measures

	Mean	Min	Max	Kurt	Skew	SD	DF	PP	AC_1	AC_{10}
RV_{1min}	0.373	0.129	1.326	7.121	2.015	0.184	-7.379	-5.245	0.916	0.820
RV_{3min}	0.363	0.110	1.313	7.233	1.994	0.180	-8.360	-5.470	0.894	0.791
RV_{5min}	0.359	0.107	1.131	6.603	1.906	0.176	-8.500	-5.201	0.891	0.791
RV_{10min}	0.354	0.111	1.158	6.532	1.881	0.175	-9.160	-6.148	0.874	0.782
RV_{15min}	0.353	0.105	1.311	7.251	1.986	0.179	-10.179	-7.859	0.847	0.762
RV_{30min}	0.346	0.104	1.313	7.510	1.999	0.181	-11.920	-7.806	0.796	0.725



(a) Monthly **(b)** Weekly

(c) Daily

Figure 4.1.3: Seasonality of RV.

RV based on 5 minute intervals, it is obvious that they are highly correlated. Its also apparent that the OVX measure is usually above the realized volatility. The OVX measured as the price of variance, described in section 3.2, shows an average implied volatility of 0.41 while the average RV is 0.36. The difference in values could represent what Chernov (2001) described as the risk premium on variance. The RV measure is also clearly noisier than the implied volatility measure. This should be expected considering equation 3.9 which defines the OVX measure as the 30-day expected volatility while the RV is the instantaneous daily volatility.

From the panel showing the slope of the futures curve it is apparent that over the period, the market was mainly in contango. Since the values have been demeaned, this graph has been shifted down and the dotted line shows the original zero-line. The variable appear to be well correlated to the RV and IV measures. It also indicates a positive correlation between the value of the slope and volatility, as was suggested by Kogan et al. (2009). During and after the large price decline in the second part of 2009 the steep slope of the futures curve would indicate that there was a large discrepancy between the actual capital stock and the optimal capital stock after the large decline in price.

The bid-ask spread appears to be correlated to the volatility variables from the beginning of the period until about the end of the first quarter of 2009 which can be explained by the high OVX levels, as an indication of market makers' cost of hedging. At that point the bid-ask spread settles at a low and narrow band. It is likely that this change is caused by a structural change in the market. For instance the entry of a more sophisticated market maker. This would also to some extent explain the simultaneous reduction in the size of the absolute returns as a better informed market maker would be able to reduce returns.

Another possible explanation is based on the observation that the reduction in bid-ask spreads happens at the beginning of the price recovery and when world markets displayed less turmoil. At this point it could be that market participants were less uncertain about the direction of the market and that the role of adverse information was reduced. At the end of the period is seems that there is some increase again in the BAS. At at that point in time prices had again reached historically high levels and the direction of the market might have appeared more certain.

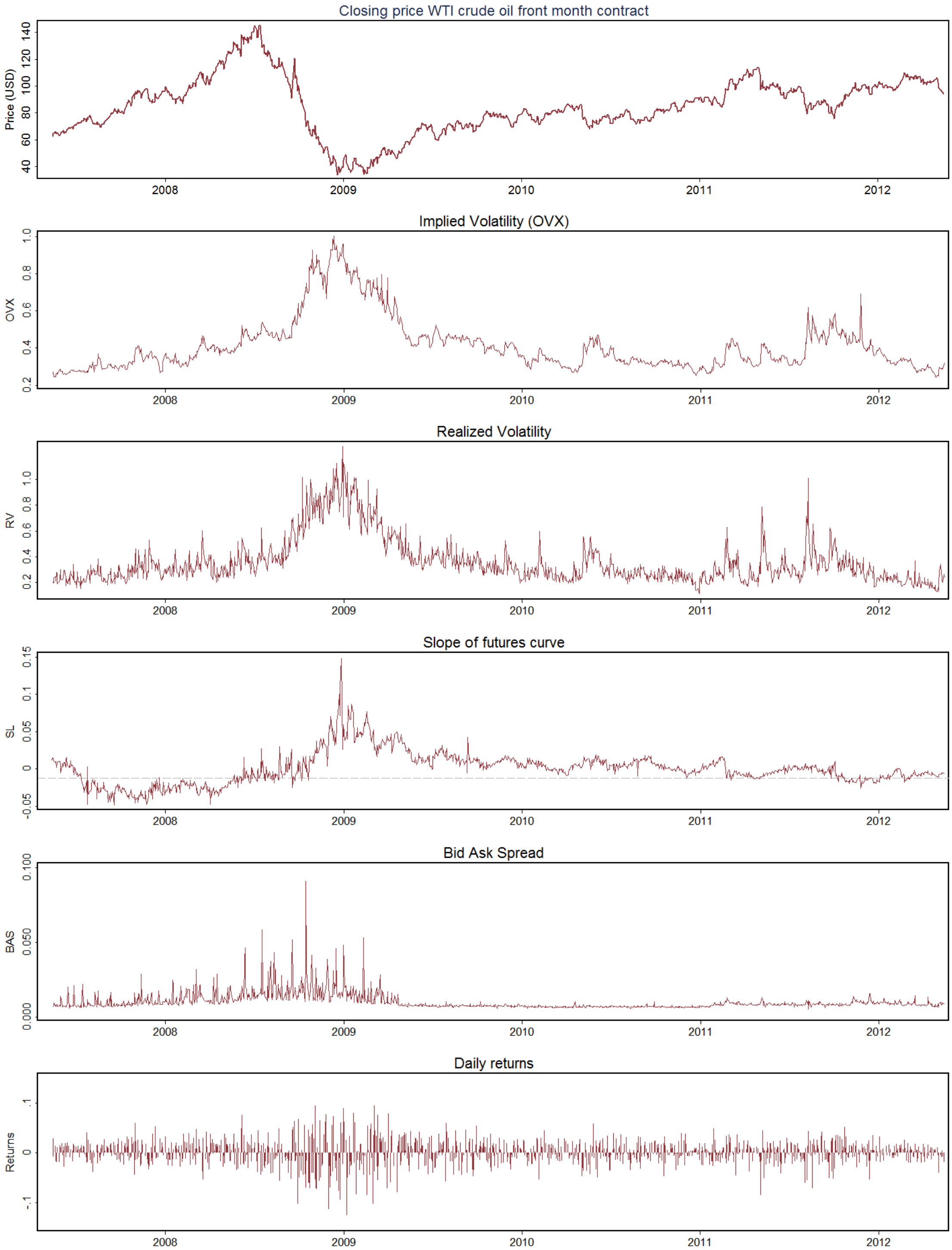


Figure 4.2.1: *Time series for the period 16/5/2007 to 15/5/2012*

Chapter 5

Results

This chapter first explains the HAR-RV model described by (Corsi, 2009). Then, the model is expanded by adding implied volatility and exogenous variables. The resulting models are fitted to the sample values of daily, weekly and monthly RV using OLS regression. Each regression is performed for both the levels of RV and the first difference of RV. The models are then evaluated for out-of-sample predictions, again for the three different time-horizons and for levels and first differences.

5.1 Model framework

The heterogeneous market hypothesis, postulated by Müller et al. (1997), claims that the asymmetric behavior of volatility is based on trader's different time-horizons. In brief, the short term trader will be influenced by both short term and long term volatility while a long term trader is not easily influenced by short term volatility. This gives rise to HAR-RV model, which is an approximate long-memory *cascading* model of realized volatility (Corsi, 2009). According to Andersen et al. (2007) the model has shown remarkably good forecasting performance comparable to the much more complicated long-memory ARFIMA model and steadily outperforms short-memory models.

Latent realized volatilities over different time-horizons are defined as a simple average of the daily quantities. Weekly realized volatility (with 5 trading days per week) is defined as:

$$RV_t^{(w)} = \frac{1}{5}\Big(RV_t^{(d)} + RV_{t-1d}^{(d)} + \cdots + RV_{t-4d}^{(d)}\Big), \tag{5.1}$$

where RV is the realized volatility measure defined by equation 3.4. Monthly volatility is defined analogously. The partial volatility process at each level of the cascade is assumed to be a function of past realized volatility at the same timescale and the expectation of the next period values of the longer term partial volatilities (except for the monthly timescale which only has the AR(1) structure). With latent *partial* volatility defined as $\tilde{\sigma}_t^{(\cdot)}$ the model is shown below:

$$\begin{aligned}
\tilde{\sigma}_{t+1m}^{(m)} =& c^{(m)} + \phi RV_t^{(m)} + \tilde{\omega}_{t+1m}^{(m)}, \\
\tilde{\sigma}_{t+1w}^{(w)} =& c^{(w)} + \phi RV_t^{(w)} + \gamma^{(w)} \mathbb{E}_t[\tilde{\sigma}_{t+1m}^{(m)}] + \tilde{\omega}_{t+1w}^{(w)}, \\
\tilde{\sigma}_{t+1d}^{(d)} =& c^{(d)} + \phi RV_t^{(d)} + \gamma^{(d)} \mathbb{E}_t[\tilde{\sigma}_{t+1w}^{(w)}] + \tilde{\omega}_{t+1d}^{(d)},
\end{aligned}$$

where $RV_t^{(d)}, RV_t^{(w)}$ and $RV_t^{(m)}$ are the daily, weekly and monthly volatilities, respectively, as defined in equation 5.1.

By recursive substitution of the partial volatilities and setting $\tilde{\sigma}_t^{(d)} = \sigma_t^{(d)}$, the model can be written as follows:

$$\sigma_{t+1d}^{(d)} = c + \beta^d RV_t^d + \beta^w RV_t^w + \beta_m RV_t^m + \tilde{\omega}_{t+1d}^{(d)} \tag{5.2}$$

From this process of latent volatility the time series model of realized volatility becomes:

$$RV_{t+1}^{(d)} = c + \beta^d RV_t^d + \beta^w RV_t^w + \beta_m RV_t^m + \omega_{t+1d}, \tag{5.3}$$

where $\omega_{t+1d} = \tilde{\omega}_{t+1d}^{(d)} - \omega_{t+1d}^{(d)}$. Equation 5.3 is a a three factor stochastic volatility model labeled HAR(3)-RV with a simple autoregressive structure enabling treatment of volatilities realized over different intervals. A benefit of having such a simple model is that it can easily be extended by adding additional regressors.

5.2 Implementation

The HAR-RV model is extended by adding additional regressors, similarly in principle to what was done by Haugom et al. (2011) for the electricity market. Two main extensions are made; one using the IV measure, and one using the exogenous variables discussed in section 2.3. The specifications are shown in the following equations:

HAR-RV-IV:

$$RV_{t+1} = \beta_0 + \beta_1 RV_t + \beta_2 RV_t^w + \beta_3 RV_t^m + \beta_4 IV_t + \epsilon_{t+1} \tag{5.4}$$

HAR-RV-EX:

$$\begin{aligned} RV_{t+1} = & \beta_0 + \beta_1 RV_t + \beta_2 RV_t^w + \beta_3 RV_t^m + \beta_4 SIZE_t + \beta_5 NTR_t + \\ & \beta_6 OI_t + \beta_7 RTN_t^+ + \beta_8 RTN_t^- + \beta_9 BAS_t + \\ & \beta_{10} SL_t^+ + \beta_{11} SL_t^- + \epsilon_{t+1} \end{aligned} \tag{5.5}$$

HAR-RV-IV-EX:

$$\begin{aligned} RV_{t+1} = & \beta_0 + \beta_1 RV_t + \beta_2 RV_t^w + \beta_3 RV_t^m + \beta_4 IV_t + \beta_5 SIZE_t + \beta_6 NTR_t + \\ & \beta_7 OI_t + \beta_8 RTN_t^+ + \beta_9 RTN_t^- + \beta_{10} BAS_t + \\ & \beta_{11} SL_t^+ + \beta_{12} SL_t^- + \epsilon_{t+1} \end{aligned} \tag{5.6}$$

RTN_t^+ defined as $max(RTN_t, 0)$, and RTN_t^- as $min(RTN_t, 0)$ with RTN being the percentage change in price from market close at $t-1$ to market close at t
In equation 5.6 and 5.4, IV_t is the implied volatility measured by the OVX. In equation 5.4 and 5.6, $SIZE_t$ is the daily average size of trades, NTR_t is the average number of contracts traded during day t adjusted for monthly cycle, OI_t is in number of open interests at day t adjusted for monthly cycle, RTN_t^+ defined as $max(RTN_t, 0)$, and RTN_t^- as $min(RTN_t, 0)$ with RTN being the percentage change in price from market close at $t-1$ to market close at t, BAS_t is the measure of average bid-ask spread during day t estimated using the Roll estimator, SL_t^+ and SL_t^- represents the slope of the futures curve as was specified in equations 2.3 and 2.4.

The left hand side of these equations are also changed from the next day values of RV displayed above. Additionally, the models are used to predict weekly and monthly RV. When doing so the same definition in equation 5.1 is used. In other words the models are used to predict next week's average volatility and next month's average volatility.

As was mentioned in section 4.1 the time series of realized volatility will partially appear like an integrated process. From a practical perspective, predicting the first difference of such time-series can be useful. All calculations done for the level of RV are therefore also done for the first difference of RV. What this means in practice for the daily horizon is simply the percentage change in RV from t to t+1. For the weekly horizon it means the percentage change from the last week's average volatility written $RV_t^{(w)}$ and the average volatility for the next week written $RV_{t+6}^{(w)}$. When having change over a monthly horizon on the left hand side of the equation it means the percentage change between last month's average RV, written $RV_t^{(m)}$, and next month's average RV, written $RV^{(m)}_{t+21}$.

5.3 In-sample modeling

The models described in the previous section, together with the original HAR-RV model from equation 5.3, were estimated for the levels of RV and the first differences of RV. This was done for three different horizons, next day, next week and next month. The results are shown in three tables; Table 5.1, Table 5.2 and Table 5.3.

Table 5.1 shows the results for fitting the models to next-day values. Firstly, it is clear that all three coefficients in the HAR-RV (1) model are highly significant. More weight is put on the variables with a shorter time horizon and it appears that one-day volatility has the strongest influence on the next day's volatility level in the WTI futures market. This would be expected according to the model's underlying assumption that short term traders are mainly concerned about short term volatility.

When adding the IV variable in model (2), it becomes the main explanatory variable both in terms of statistical significance and in coefficient size. A substantial reduction is observed in all three RV-coefficients, but by far the largest reduction is seen in the monthly measure. This variable goes from being highly statistically significant to losing all explanatory power. Since the OVX measures the market expectation of 30-day volatility the long horizon part of the RV measures should be embedded. In other words it is a representation of what the long term traders think about volatility.

Adding the exogenous variables in model (3) moderately decreases the estimated coefficient of the short term component. The information content of the EX variables is therefore mainly overlapping the information in the daily RV measure. One can also observe an increase in the R-squared values for both models (2) and (3) which indicates that adding the variables improves the original model.

Model (3) shows that there are variations in the contributions of the different EX-variables. The two return-variables show the highest statistical significance of the EX variables. The two variables show a substantial difference in the size of their coefficients with 1.8 times larger effect when the return is negative than positive. This means that large returns (both positive and negative) increase volatility, but the increase is larger for negative returns, indicating a leverage effect.

The BAS measure also shows statistical significance. However, as we can see from figure 4.2.1 there seems to be a structural change in the BAS at 1/5/2009. We therefore split the sample into two subsamples.[1] We find that the coefficient is significant in the first subsample, but not in the second. This should not be surprising since the behavior of the variable changed significantly at that time. This supports our assumption that the market structure changed at 1/5/2009. One possible explanation would be the entry of a new market maker.

When combining all the variables in model (4) the R-squared value indicates that *both* the IV *and* the EX measures contain additional information to that provided by the RV measures. By comparing the R-squared value for (2) and (3) it seems evident that the EX variables contain more information than the IV about next day volatility. Additionally, the increase in R-squared values induced by the IV variable is similar regardless of whether the model contains EX variables or not, implying that the IV variable contains separate information from the EX variables.

In general, the same effects from adding the IV and EX variables are seen in the RV coefficients when the models are fitted to the first difference of RV. Slightly different effects from adding the IV and EX variables can be inferred from the R-squared values. They suggest that the information content of the IV variable is larger when predicting change than levels. The same is seen for the EX variables when looking at the R-squared values in model (7).

That the slope of the futures curve is unable to explain changes in volatility in model (7) and (8), but is a highly significant explanatory variable for levels of volatility seen in models (3) and (4).

Fitting the same four models to average RV over the next five days is presented in table 5.2. It is clear from (1) that the weight of the coefficients and the statistical significance is shifted towards the weekly measure of past volatility compared to when the model was fitted to daily values. When adding the IV variable, seen in model (2) the reduction of the RV coefficients is largest for the monthly measure but also substantial for the weekly and daily measure. In contrary to the daily horizon the effect of the monthly RV variable remains highly significant. It should also be noted that the IV measure is again the most important explanatory variable with the highest coefficient and significance.

In the case of estimating the model for weekly averages of RV, adding the EX variables still reduces all the RV coefficient but to a lesser extent than the IV variable. From the R-squared values it is seen that the increase from (1) to either

[1] See appendix B

(2) or (3) is almost equal. This could indicate that the IV variable performs better relative to the EX variables when it comes to next weekly average levels of RV. The significance of the EX variables are largely similar to what was found for the daily horizon, with returns, BAS and slope of futures curve being statistically significant.

When fitting the models to the first difference of weekly RV, model (5) in table 5.2 shows that the weight has been shifted towards the weekly measure of RV. Adding the IV measure induces the same effect as seen when fitted to RV values and all thee RV-coefficients are reduced. Further, there is a slight improvement in the statistical significance of the effects of the EX variables, particularly in the OI variables.

The in-sample fittings of the models to monthly RV measures are presented in table 5.3. For the monthly time-horizon the effect of the long term component of the time- series is the one with the highest significance and highest coefficient. This is evident for both levels and the first difference of the monthly RV.

Adding the IV variables seen in model (2). This again reduces all coefficients and it becomes the main explanatory variable. The biggest reduction is now seen in the weekly RV measure but its statistical significance as well as the size of the coefficient is lower than when fitting the model to daily and weekly measures of RV.

The effect of adding the EX variables seen in model (3) of table 5.3. It reduces the RV coefficients to a less extent than for the daily and weekly horizon. The negative returns and the BAS variable are still highly significant. Additionally, the number of trades is becomes a significant variable. Of the return variables only the negative returns now have any explanatory power. In addition the number of trades is significant when the models are estimated to a monthly measure of volatility.

The model combining both IV and EX variables (4) displays a higher R-squared value than models (1), (2) and (3). But the increase from adding the IV variable, seen from (3) to (4), is much less than what was observed for the daily and weekly horizon. This could indicate that when it comes to a monthly horizon the IV variable has less additional information to that provided by the HAR-RV-EX model.

When it comes to the first difference of monthly volatility seen in models (5)–(8) in table 5.3, the IV variable is no longer as significant as it was for the shorter horizons. Additionally the EX variables are all, with the exception of open interest, more statistically significant. This is again an indication that the IV variable performs better for shorter horizons.

Table 5.1: Daily horizon: *In-sample fitting of models for next day levels of RV, and the change in average RV from t to t+1, during 16/5/2007 to 15/5/2012. Models (2)–(4) and (6)–(8) are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including;* $SIZE_t$ *as daily average size of trades,* NTR_t *as average number of contracts traded per trade,* OI_t *as open interests,* RTN_t^+ *as price return for* $max(RTN_t, 0)$*, and* RTN_t^- *for* $min(RTN_t, 0)$*,* BAS_t *as measure of average bid-ask spread,* SL_t^+ *as slope of the futures curve if positive and* SL_t^- *the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
$RV_t^{(d)}$	0.378***	0.311***	0.290***	0.223***	-1.374***	-1.518***	-1.494***	-1.646***
	(11.30)	(9.36)	(8.16)	(6.31)	(-16.13)	(-17.76)	(-16.02)	(-17.65)
$RV_t^{(w)}$	0.355***	0.227***	0.353***	0.229***	0.762***	0.485***	0.727***	0.446**
	(6.51)	(4.15)	(6.70)	(4.33)	(5.49)	(3.44)	(5.26)	(3.18)
$RV_t^{(m)}$	0.242***	0.037	0.208***	-0.007	0.551***	0.108	0.521***	0.033
	(5.54)	(0.78)	(4.39)	(-0.14)	(4.95)	(0.87)	(4.19)	(0.24)
OVX_t		0.464***		0.491***		1.007***		1.112***
		(8.98)		(8.92)		(7.57)		(7.64))
$SIZE_t$			0.013	0.027***			0.015	0.047**
			(1.93)	(4.06)			(0.83)	(2.64)
NTR_t			0.002	0.002*			0.000	0.001
			(1.87)	(2.16)			(0.17)	(0.37)
OI_t			-0.000	0.008			0.008	0.026
			(-0.01)	(1.44)			(0.51)	(1.75)
RTN_t^+			0.900***	0.765***			1.890***	1.583***
			(5.36)	(4.68)			(4.29)	(3.66)
RTN_t^-			-1.519***	-1.191***			-2.930***	-2.185***
			(-9.49)	(-7.47)			(-6.98)	(-5.18)
BAS_t			1.329**	1.209**			1.984	1.711
			(3.22)	(3.02)			(1.83)	(1.61)
SL_t^-			-0.007	0.115			0.277	0.553
			(-0.03)	(0.56)			(0.50)	(1.02)
SL_t^+			0.706**	0.854***			0.570	0.906
			(2.98)	(3.71)			(0.92)	(1.49)
β_0	0.009	-0.039***	-0.028	-0.137***	0.022	-0.083***	-0.035	-0.280***
	(1.84)	(-5.31)	(-1.27)	(-5.48)	(1.68)	(-4.40)	(-0.59)	(-4.25)
R^2	0.827	0.837	0.844	0.854	0.195	0.230	0.235	0.269

* p<0.05, ** p<0.01, *** p<0.001

Table 5.2: Weekly horizon: *In-sample fitting of models for next 5 day average levels of RV, and the percentage change in RV from recent 5 days to next 5 days, during 16/5/2007 to 15/5/2012. Models (2)–(4) and (6)–(8) are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including;* $SIZE_t$ *as daily average size of trades,* NTR_t *as average number of contracts traded per trade,* OI_t *as open interests,* RTN_t^+ *as price return for* $max(RTN_t, 0)$, *and* RTN_t^- *for* $min(RTN_t, 0)$, BAS_t *as measure of average bid-ask spread,* SL_t^+ *as slope of the futures curve if positive and* SL_t^- *the slope if negative.*

	$RV_{t+6}^{(w)}$				$\ln\left(\frac{RV_{t+6}^{(w)}}{RV_t^{(w)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
$RV_t^{(d)}$	0.242***	0.188***	0.208***	0.150***	0.489***	0.378***	0.442***	0.315***
	(9.04)	(7.07)	(7.23)	(5.26)	(6.29)	(4.81)	(5.20)	(3.68)
$RV_t^{(w)}$	0.387***	0.283***	0.371***	0.263***	-1.473***	-1.685***	-1.522***	-1.756***
	(8.86)	(6.46)	(8.68)	(6.15)	(-11.62)	(-13.03)	(-12.09)	(-13.71)
$RV_t^{(m)}$	0.325***	0.159***	0.274***	0.086*	0.855***	0.517***	0.718***	0.313*
	(9.30)	(4.14)	(7.11)	(2.05)	(8.42)	(4.56)	(6.32)	(2.49)
OVX_t		0.378***		0.428***		0.768***		0.926***
		(9.13)		(9.64)		(6.28)		(6.96)
$SIZE_t$			0.002	0.015**			0.001	0.028
			(0.44)	(2.73)			(0.08)	(1.73)
NTR_t			-0.001	-0.001			-0.004	-0.003
			(-1.01)	(-0.80)			(-1.63)	(-1.46)
OI_t			0.006	0.013**			0.027*	0.043**
			(1.22)	(2.82)			(1.98)	(3.12)
RTN_t^+			0.470***	0.352**			0.807*	0.551
			(3.45)	(2.66)			(2.01)	(1.39)
RTN_t^-			-1.033***	-0.746***			-2.183***	-1.562***
			(-7.95)	(-5.79)			(-5.70)	(-4.05)
BAS_t			1.300***	1.195***			3.212**	2.982**
			(3.88)	(3.69)			(3.25)	(3.07)
SL_t^-			0.135	0.241			0.249	0.473
			(0.79)	(1.46)			(0.49)	(0.95)
SL_t^+			0.604**	0.733***			1.514**	1.789**
			(3.14)	(3.95)			(2.67)	(3.21)
β_0	0.016***	-0.023***	0.006	-0.088***	0.047***	-0.034	0.005	-0.200***
	(3.97)	(-3.93)	(0.34)	(-4.39)	(3.89)	(-1.94)	(0.10)	(-3.30)
R^2	0.875	0.883	0.885	0.893	0.112	0.139	0.158	0.190

* p<0.05, ** p<0.01, *** p<0.001

Table 5.3: Monthly horizon: *In-sample fitting of models for next 20 day average levels of RV, and the change in RV from recent 20 days to next 20 days, during 16/5/2007 to 15/5/2012. Model (2)–(4) and (6)–(8) are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including;* $SIZE_t$ *as daily average size of trades,* NTR_t *as average number of contracts traded per trade,* OI_t *as open interests,* RTN_t^+ *as price return for* $max(RTN_t, 0)$*, and* RTN_t^- *for* $min(RTN_t, 0)$*,* BAS_t *as measure of average bid-ask spread,* SL_t^+ *as slope of the futures curve if positive and* SL_t^- *the slope if negative.*

	$RV_{t+21}^{(m)}$				$\ln\left(\frac{RV_{t+21}^{(m)}}{RV_t^{(m)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
$RV_t^{(d)}$	0.173***	0.131***	0.182***	0.136***	0.414***	0.366***	0.438***	0.357***
	(6.07)	(4.54)	(5.95)	(4.45)	(5.18)	(4.47)	(5.14)	(4.12)
$RV_t^{(w)}$	0.363***	0.281***	0.308***	0.224***	0.750***	0.657***	0.653***	0.503***
	(7.79)	(5.92)	(6.80)	(4.87)	(5.76)	(4.88)	(5.17)	(3.87)
$RV_t^{(m)}$	0.373***	0.243***	0.345***	0.201***	-1.397***	-1.543***	-1.627***	-1.883***
	(9.99)	(5.82)	(8.48)	(4.44)	(-13.38)	(-13.06)	(-14.25)	(-14.76)
OVX_t		0.296***		0.330***		0.334**		0.591***
		(6.60)		(6.90)		(2.61)		(4.37)
$SIZE_t$			-0.007	0.003			0.003	0.020
			(-1.17)	(0.49)			(0.21)	(1.20)
NTR_t			-0.004***	-0.004***			-0.010***	-0.009***
			(-5.30)	(-5.23)			(-4.43)	(-4.33)
OI_t			0.008	0.013**			0.062***	0.072***
			(1.54)	(2.67)			(4.44)	(5.14)
RTN_t^+			0.190	0.099			0.211	0.047
			(1.32)	(0.70)			(0.52)	(0.12)
RTN_t^-			-0.807***	-0.586***			-1.476***	-1.078**
			(-5.87)	(-4.23)			(-3.84)	(-2.75)
BAS_t			2.445***	2.364***			6.390***	6.237***
			(6.89)	(6.78)			(6.45)	(6.33)
SL_t^-			0.447*	0.529**			1.169*	1.303*
			(2.46)	(2.96)			(2.29)	(2.57)
SL_t^+			-0.035	0.065			2.119***	2.285***
			(-0.17)	(0.33)			(3.71)	(4.03)
β_0	0.033***	0.002	0.054**	-0.019	0.084***	0.048**	-0.008	-0.139*
	(7.47)	(0.30)	(2.80)	(-0.86)	(6.77)	(2.64)	(-0.14)	(-2.27)
R^2	0.846	0.852	0.861	0.866	0.158	0.162	0.242	0.254

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table 5.4: *Mean squared errors of predictions using rolling windows of different sizes. * indicates the best performing window.*

	RV_{t+1}		$\ln(RV_{t+1}/RV_t)$	
	HAR-RV	HAR-RV-IV-EX	HAR-RV	HAR-RV-IV-EX
w=125	0.00608	0.00638	0.0363	0.0408
w=250	0.00610	0.00556	0.0353	0.0333
w=500*	0.00430	0.00364	0.0362	0.0332
w=750	0.00494	0.00400	0.0382	0.0338

5.4 Out-of-sample forecasting

Since the goal of this paper is to improve volatility forecasting, the models must be tested for out-of-sample predictions. Out-of-sample predictions are used to make direct comparisons of the performance of the models relative to the actual values of the time series.

The choice of estimation-window for out-of-sample predictions will affect how the models are performing. On the one hand a larger window will make the model estimates more reliable as the underlying data sample increases and on the other hand a large window make the model less adaptable to changes in the market. Table 5.4 displays the mean squared error (MSE) for predictions made by two of the model specifications when using four different rolling windows. The numbers indicate that a two year window will produces the best results. Based on this crude comparison, a two year window rolling window is chosen for subsequent out-of-sample predictions.

In order to evaluate the relative performance of volatility models Mincer and Zarnowitz (1969) suggested running a regression as described by the following equation:

$$RV_{t+1} = \beta_0 + \beta_1 \hat{v}_{Model1,t} + \beta_2 \hat{v}_{Model2,t} + \epsilon_{t+1} \tag{5.7}$$

where RV_{t+1} is the observed realized volatility at $t+1$, $\hat{v}_{Model1,t}$ is the forecast obtained from one of the models and $\hat{v}_{Model2,t}$ is the forecast from a second model. The main benefit with this procedure is that it will directly give an indication of the differences between the models. The method is also applied to evaluations of weekly and monthly volatility predictions by exchanging RV_{t+1} with $RV^{(w)}_{t+6}$ and $RV^{(m)}_{t+21}$ respectively. The results from these comparisons are displayed in table 5.5.[2]

[2]Additional comparison statistics such as the mean square error (MSE) and the mean absolute error (MAE) for RV_{t+1} predictions can be found in Appendix C.

The regressions in table 5.5 show that when predicting RV one day ahead, adding both IV and EX variables significantly improves forecasting. When comparing the HAR-RV-IV model and the HAR-RV-EX the regression is inconclusive. This shows that the performance of the two models is comparable in making predictions. The coefficients indicate that the IV is slightly better at predicting the level of volatility, while the EX variables are slightly better at predicting the one-day change in volatility. When comparing the combined model to the other models the tests all show that the combined model outperforms the other models.

In the comparisons of models' predictions one week ahead, shown in panel B, a slightly different dynamic is seen. It does improves the model significantly when predicting the levels of RV, but when making predictions of the change in volatility the IV variable no longer significantly improves the original HAR-RV model. Adding the EX variables is clearly an improvement for predictions, and there is evidence that the HAR-RV-EX model outperforms the HAR-RV-IV model both in predictions of levels and in predictions of change. When comparing the combined model to the rest of the models, it still performs better than the simpler HAR-RV-EX model indicating that the IV variable does contain some information about the future weekly volatility levels. When it comes to predictions of change, adding the IV makes only a slight improvement significant at the 5 % level.

When comparing the monthly predictions seen in panel C of table 5.5 a similar pattern to the one in panel B is seen. The EX variables clearly contribute more to the precision of the predictions than the IV variables. When comparing the predictions of the HAR-RV-IV and the HAR-RV-EX it is clear that the latter is performing better for both levels and differences. Nevertheless, the IV variable is still an improvement to the HAR-RV-EX model when levels are predicted. When predicting changes the combined model does not perform any better than the simpler HAR-RV-EX model.

Table 5.5: Out-of-sample predictions: *Mincer and Zarnowitz (1969) regression comparison of predictions using a 500 day rolling window. The predictions in panel A are made one day ahead. In panel B predictions are made for next 5 day average levels of RV. In panel C predictions are made for the next 20-day average of RV. All predictions are done for both RV levels and first difference of RV levels. The HAR-RV model is the original model proposed by Corsi (2009) and the lines below show the model extensions described in section 5.2*

Panel A:

	(1)		(2)		(3)		(4)		(5)		(6)	
	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$
HAR-RV	0.245*	0.256	-0.065	-0.041			-0.066	0.131				
HAR-RV-IV	0.678***	0.656***			0.348***	0.656***			-0.137	0.030		
HAR-RV-EX			1.025***	0.872***	0.619***	0.470***					-0.050	0.033
HAR-RV-IV-EX							0.981***	0.851***	1.059***	0.859***	0.970***	0.859***
β_0	0.021*	-0.003	0.015	0.00	0.010	-0.008	0.028*	0.004	0.027**	0.005	0.026*	0.004

Panel B:

	(1)		(2)		(3)		(4)		(5)		(6)	
	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+6}^{(w)}$	$\ln\left(\frac{RV_{t+6}^{(w)}}{RV_t^{(w)}}\right)$	$RV_{t+6}^{(w)}$	$\ln\left(\frac{RV_{t+6}^{(w)}}{RV_t^{(w)}}\right)$	$RV_{t+6}^{(w)}$	$\ln\left(\frac{RV_{t+6}^{(w)}}{RV_t^{(w)}}\right)$	$RV_{t+6}^{(w)}$	$\ln\left(\frac{RV_{t+6}^{(w)}}{RV_t^{(w)}}\right)$	$RV_{t+6}^{(w)}$	$\ln\left(\frac{RV_{t+6}^{(w)}}{RV_t^{(w)}}\right)$
HAR-RV	0.185	0.548*	-0.233	0.030			-0.253*	0.171				
HAR-RV-IV	0.610***	0.397			0.171	0.149			-0.311*	0.016		
HAR-RV-EX			1.024***	0.993***	0.647***	0.894***					-0.194	0.418
HAR-RV-IV-EX							1.011***	0.885***	1.083***	1.009***	0.976***	0.622*
β_0	0.059***	-0.007	0.443	0.174	0.010	0.011	0.078***	0.010	0.074***	0.012	0.069***	0.013

Panel C:

	(1)		(2)		(3)		(4)		(5)		(6)	
	$RV_{t+1}^{(d)}$	$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$	$RV_{t+21}^{(m)}$	$\ln\left(\frac{RV_{t+21}^{(m)}}{RV_t^{(m)}}\right)$	$RV_{t+21}^{(m)}$	$\ln\left(\frac{RV_{t+21}^{(m)}}{RV_t^{(m)}}\right)$	$RV_{t+21}^{(m)}$	$\ln\left(\frac{RV_{t+21}^{(m)}}{RV_t^{(m)}}\right)$	$RV_{t+21}^{(m)}$	$\ln\left(\frac{RV_{t+21}^{(m)}}{RV_t^{(m)}}\right)$	$RV_{t+21}^{(m)}$	$\ln\left(\frac{RV_{t+21}^{(m)}}{RV_t^{(m)}}\right)$
HAR-RV	-0.046	0.500	-0.121	-0.164			-0.161	-0.113				
HAR-RV-IV	0.673***	0.146			-0.017	-0.177			-0.180*	-0.173		
HAR-RV-EX			0.779***	0.941***	0.690***	0.952***					-0.242	0.711
HAR-RV-IV-EX							0.807***	0.899***	0.834***	0.948***	0.906***	0.105
β_0	0.109***	-0.015*	0.109***	0.020**	0.103***	0.020**	0.113***	0.018*	0.111***	0.020**	0.105***	0.015*

Chapter 6

Conclusion

The aim of this paper has been to improve time series forecasting of realized volatility (RV) in the WTI futures market by including additional explanatory variables. The approach chosen was to combine realized volatility with implied volatility (IV) and other exogenous market variables (EX) in a forecasting model based on the HAR-RV model described by Corsi (2009). The realized measure of volatility was calculated using high frequency data. As a proxy for IV the oil volatility index published by the CBOE was used. Market variables added to the model are volume, open interest, daily returns, the bid-ask spread and the slope of the futures curve.

Our results show that the HAR-RV model fits the RV time series significantly better when both the IV and EX variables are added to the model. The effect of adding the IV variable was strongest when the model was fitted to next-day levels and weakest when the model was fitted to next-month changes in RV levels. The effect of adding the EX variables was more statistically significant for the longer horizons than the short. Of the exogenous variables the daily return variables were the only variables to be highly statistically significant effects for both RV levels and differences for all time-horizons. The bid-ask spread and the slope of the futures curve were also found to have significant effects.

Out-of-sample testing shows that time series predictions of volatility are improved the most by adding both the IV and EX variables. Implied volatility improves predictions most significantly for short-term predictions, whereas other market variables (and particularly the bid-ask spread) had a more significant effect than implied volatility for long-term forecasts.

This work shows that including implied volatility and other market variables improves volatility forecasts for the WTI futures market. Additional finding in this paper is that leverage effects (the relationship between past returns and volatility) is very different from the one found in equity markets. It has a V-shape, meaning that large returns (both positive and negative) increase volatility in this market. Therefore, we suggest the leverage effect for oil, as well as other commodities, to be investigated further.

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Appendices

Appendix A

Descriptives

Table A.1: *Phillips-Perron test and autocorrelation for subsamples of RV based on 5-minute intervals. Values below -3.430 means rejection of unit root at the 1% significans level. AC(1) and AC(10) are the autocorrelation for 1 and 10 lags respectively.*

	PP	AC_1	AC_{10}
RV_{1-200}	-9.044	0.426	0.286
$RV_{200-400}$	-2.480	0.873	0.723
$RV_{401-600}$	-3.209	0.843	0.657
$RV_{601-800}$	-6.183	0.656	0.333
$RV_{801-1000}$	-6.299	0.662	0.087
$RV_{1000-1200}$	-4.994	0.759	0.319

Table A.2: *Descriptive statistics for regression variables. NTR and OI are adjusted for monthly cycle and are divided by 10,000 and 100,000 respectively.*

Variable	Obs	Mean	Std. Dev.	Min	Max
$\ln(\frac{RV_{t+1}}{RV_t})$	1246	0.000	0.208	-0.712	0.757
RV_t	1246	0.360	0.176	0.115	1.256
RV_{t5}	1246	0.360	0.166	0.138	1.018
RV_{t20}	1246	0.360	0.160	0.173	0.960
IV_{OVX}	1246	0.413	0.144	0.243	1.004
NTR	1246	11.098	0.689	9.063	13.39
$SIZE$	1246	2.142	0.441	1.202	3.537
OI	1246	3	0.405	0.479	4.143
RTN^+	1246	0.0090	0.0141	0.0000	0.0947
RTN^-	1246	-0.0090	0.0156	-0.1267	0.0000
RG^*	1246	0.021	0.013	0.005	0.126
BAS	1246	0.0141	0.008	0.007	0.129
SL^-	1246	-0.0076	0.0109	-0.0488	0.0000
SL^+	1246	0.0077	0.0147	0.0000	0.1478

Appendix B

In-sample modeling

Table B.1: Daily horizon subsample for observations 1-489: *In-sample fitting of models for next day levels of RV, and the change in average in RV from t to t+1, during 15/5/2009 to 1/5/2009. Model 2-4 are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including;* $SIZE_t$ *as daily average size of trades,* NTR_t *as average number of contracts traded per trade,* OI_t *as number of open interests,* RTN_t^+ *as price return if positive and* RTN_t^- *the return if negative,* BAS_t *as measure of average bid-ask spread,* SL_t^+ *as slope of the futures curve if positive and* SL_t^- *the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
RVt	0.207***	0.156**	0.179**	0.130*	-1.316***	-1.552***	-1.642***	-1.992***
	(3.80)	(2.93)	(3.14)	(2.31)	(-10.87)	(-12.52)	(-10.98)	(-13.18)
RVt5	0.502***	0.293**	0.433***	0.281**	0.630***	0.403*	0.801***	0.433*
	(5.13)	(2.92)	(4.34)	(2.78)	(3.44)	(2.21)	(4.30)	(2.33)
RVt20	0.269***	0.096	0.245*	0.097	0.337*	-0.290	0.306	-0.533**
	(3.37)	(1.18)	(2.48)	(0.97)	(2.09)	(-1.55)	(1.84)	(-2.74)
OVX		0.494***		0.464***		1.258***		1.754***
		(6.12)		(5.24)		(6.25)		(7.65)
SIZE			-0.003	0.023			0.037	0.063*
			(-0.19)	(1.40)			(1.42)	(2.49)
NTRADES			0.001	0.001			0.002	0.004
			(0.28)	(0.54)			(0.61)	(1.49)
OI			-0.010	-0.003			0.004	0.033
			(-0.76)	(-0.23)			(0.21)	(1.76)
RTNup			0.997***	0.829***			1.542*	1.235
			(3.95)	(3.34)			(2.17)	(1.80)
RTNdown			-1.142***	-0.839***			-4.482***	-3.375***
			(-4.69)	(-3.44)			(-6.62)	(-5.05)
BAS			1.203*	1.043*			-4.981	-5.279
			(2.23)	(1.99)			(-0.75)	(-0.83)
SLminus			-0.085	-0.046			-1.012	3.382
			(-0.21)	(-0.12)			(-0.55)	(1.80)
SLplus			0.549	0.533			1.624	0.909
			(1.52)	(1.51)			(1.20)	(0.70)
constant	0.012	-0.033**	0.050	-0.092	0.104***	-0.029	0.039	-0.263**
	(1.41)	(-2.95)	(0.86)	(-1.48)	(3.34)	(-0.78)	(0.44)	(-2.76)
R-sqr	0.864	0.873	0.874	0.880	0.165	0.206	0.230	0.286

* p<0.05, ** p<0.01, *** p<0.001

Table B.2: Daily horizon subsample for observations 489-1246: *In-sample fitting of models for next day levels of RV, and the change in average in RV from t to t+1, during 1/5/2009 to 15/5/2012. Model 2-4 are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including; $SIZE_t$ as daily average size of trades, NTR_t as average number of contracts traded per trade, OI_t as number of open interests, RTN_t^+ as price return if positive and RTN_t^- the return if negative, BAS_t as measure of average bid-ask spread, SL_t^+ as slope of the futures curve if positive and SL_t^- the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
RVt	0.539***	0.456***	0.382***	0.262***	-1.316***	-1.552***	-1.642***	-1.992***
	(13.18)	(10.91)	(7.78)	(5.30)	(-10.87)	(-12.52)	(-10.98)	(-13.18)
RVt5	0.193**	0.113	0.272***	0.146*	0.630***	0.403*	0.801***	0.433*
	(3.11)	(1.83)	(4.45)	(2.40)	(3.44)	(2.21)	(4.30)	(2.33)
RVt20	0.142**	-0.079	0.134*	-0.153*	0.337*	-0.290	0.306	-0.533**
	(2.60)	(-1.26)	(2.45)	(-2.41)	(2.09)	(-1.55)	(1.84)	(-2.74)
OVX		0.444***		0.600***		1.258***		1.754***
		(6.53)		(8.00)		(6.25)		(7.65)
SIZE			0.016	0.025**			0.037	0.063*
			(1.85)	(2.99)			(1.42)	(2.49)
NTRADES			0.001	0.002*			0.002	0.004
			(1.16)	(2.10)			(0.61)	(1.49)
OI			0.002	0.012			0.004	0.033
			(0.33)	(1.96)			(0.21)	(1.76)
RTNup			0.822***	0.717**			1.542*	1.235
			(3.52)	(3.19)			(2.17)	(1.80)
RTNdown			-1.992***	-1.614***			-4.482***	-3.375***
			(-8.97)	(-7.38)			(-6.62)	(-5.05)
BAS			-0.698	-0.799			-4.981	-5.279
			(-0.32)	(-0.38)			(-0.75)	(-0.83)
SLminus			-0.271	1.231*			-1.012	3.382
			(-0.45)	(2.00)			(-0.55)	(1.80)
SLplus			0.650	0.406			1.624	0.909
			(1.47)	(0.95)			(1.20)	(0.70)
constant	0.038***	-0.009	-0.004	-0.107***	0.104***	-0.029	0.039	-0.263**
	(3.57)	(-0.74)	(-0.14)	(-3.45)	(3.34)	(-0.78)	(0.44)	(-2.76)
R-sqr	0.568	0.591	0.624	0.654	0.165	0.206	0.230	0.286

* p<0.05, ** p<0.01, *** p<0.001

Table B.3: Weekly horizon subsample for observations 1-489: *In-sample fitting of models for next day levels of RV, and the change in average in RV from t to t+1, during 1/5/2009 to 15/5/2012. Model 2-4 are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including;* $SIZE_t$ *as daily average size of trades,* NTR_t *as average number of contracts traded per trade,* OI_t *as number of open interests,* RTN_t^+ *as price return if positive and* RTN_t^- *the return if negative,* BAS_t *as measure of average bid-ask spread,* SL_t^+ *as slope of the futures curve if positive and* SL_t^- *the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
RVt	0.129***	0.081*	0.150***	0.103**	0.218*	0.131	0.270**	0.180*
	(3.40)	(2.26)	(3.76)	(2.69)	(2.53)	(1.57)	(2.96)	(2.02)
RVt5	0.591***	0.393***	0.537***	0.392***	-0.782***	-1.135***	-0.863***	-1.139***
	(8.64)	(5.80)	(7.70)	(5.68)	(-5.05)	(-7.18)	(-5.41)	(-7.08)
RVt20	0.245***	0.081	0.164*	0.024	0.479***	0.187	0.352*	0.089
	(4.40)	(1.47)	(2.38)	(0.35)	(3.79)	(1.45)	(2.22)	(0.56)
OVX		0.469***		0.443***		0.838***		0.841***
		(8.61)		(7.33)		(6.58)		(5.98)
SIZE			-0.037***	-0.013			-0.078**	-0.032
			(-3.37)	(-1.16)			(-3.09)	(-1.24)
NTRADES			-0.004*	-0.003			-0.008*	-0.006
			(-2.12)	(-1.88)			(-1.98)	(-1.74)
OI			0.009	0.015			0.068**	0.080***
			(0.97)	(1.79)			(3.28)	(3.98)
RTNup			0.548**	0.387*			0.799*	0.493
			(3.10)	(2.29)			(1.97)	(1.25)
RTNdown			-0.711***	-0.422*			-1.197**	-0.650
			(-4.17)	(-2.53)			(-3.06)	(-1.68)
BAS			0.474	0.322			0.564	0.273
			(1.26)	(0.90)			(0.65)	(0.33)
SLminus			0.353	0.390			1.210	1.248
			(1.26)	(1.47)			(1.84)	(1.96)
SLplus			0.304	0.288			0.031	-0.008
			(1.20)	(1.20)			(0.05)	(-0.01)
constant	0.019**	-0.024**	0.130**	-0.006	0.047***	-0.031	0.121	-0.137
	(3.08)	(-3.20)	(3.22)	(-0.14)	(3.37)	(-1.71)	(1.31)	(-1.38)
R-sqr	0.927	0.936	0.932	0.939	0.066	0.143	0.115	0.177

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table B.4: Weekly horizon subsample for observations 489-1246: *In-sample fitting of models for next day levels of RV, and the change in average in RV from t to t+1, during 1/5/2009 to 15/5/2012. Model 2-4 are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including; $SIZE_t$ as daily average size of trades, NTR_t as average number of contracts traded per trade, OI_t as number of open interests, RTN_t^+ as price return if positive and RTN_t^- the return if negative, BAS_t as measure of average bid-ask spread, SL_t^+ as slope of the futures curve if positive and SL_t^- the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
RVt	0.332***	0.279***	0.187***	0.093*	0.673***	0.543***	0.372*	0.125
	(9.17)	(7.43)	(4.28)	(2.10)	(5.74)	(4.44)	(2.57)	(0.84)
RVt5	0.210***	0.159**	0.290***	0.191***	-2.025***	-2.150***	-1.843***	-2.101***
	(3.83)	(2.88)	(5.32)	(3.50)	(-11.39)	(-11.94)	(-10.25)	(-11.50)
RVt20	0.220***	0.079	0.172***	-0.052	0.549***	0.203	0.339*	-0.249
	(4.56)	(1.40)	(3.54)	(-0.92)	(3.51)	(1.10)	(2.11)	(-1.30)
OVX		0.282***		0.469***		0.693***		1.230***
		(4.64)		(6.96)		(3.49)		(5.45)
SIZE			0.034***	0.040***			0.107***	0.125***
			(4.43)	(5.47)			(4.27)	(5.05)
NTRADES			0.001	0.002			0.001	0.002
			(0.99)	(1.79)			(0.19)	(0.82)
OI			-0.001	0.007			-0.008	0.012
			(-0.15)	(1.26)			(-0.46)	(0.65)
RTNup			0.279	0.197			0.435	0.218
			(1.34)	(0.98)			(0.63)	(0.32)
RTNdown			-1.210***	-0.914***			-2.608***	-1.828**
			(-6.11)	(-4.65)			(-3.98)	(-2.78)
BAS			2.423	2.344			2.404	2.199
			(1.25)	(1.25)			(0.38)	(0.35)
SLminus			-0.389	0.786			-2.934	0.144
			(-0.72)	(1.42)			(-1.64)	(0.08)
SLplus			1.355***	1.164**			5.032***	4.535***
			(3.43)	(3.03)			(3.86)	(3.54)
constant	0.071***	0.041***	-0.007	-0.088**	0.239***	0.166***	0.063	-0.150
	(7.64)	(3.68)	(-0.26)	(-3.13)	(7.91)	(4.53)	(0.72)	(-1.59)
R-sqr	0.525	0.538	0.581	0.607	0.211	0.224	0.279	0.306

* p<0.05, ** p<0.01, *** p<0.001

Table B.5: Monthly horizon subsample for observations 1-489: *In-sample fitting of models for next day levels of RV, and the change in average in RV from t to t+1, during 1/5/2009 to 15/5/2012. Model 2-4 are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including;* $SIZE_t$ *as daily average size of trades,* NTR_t *as average number of contracts traded per trade,* OI_t *as number of open interests,* RTN_t^+ *as price return if positive and* RTN_t^- *the return if negative,* BAS_t *as measure of average bid-ask spread,* SL_t^+ *as slope of the futures curve if positive and* SL_t^- *the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
RVt	0.127**	0.072	0.137**	0.089*	0.227*	0.139	0.234*	0.147
	(2.91)	(1.74)	(3.05)	(2.04)	(2.39)	(1.50)	(2.32)	(1.48)
RVt5	0.702***	0.474***	0.547***	0.398***	1.247***	0.884***	1.098***	0.820***
	(8.93)	(6.09)	(6.95)	(5.06)	(7.31)	(5.05)	(6.24)	(4.58)
RVt20	0.091	-0.098	0.195*	0.050	-1.677***	-1.971***	-1.631***	-1.883***
	(1.42)	(-1.53)	(2.50)	(0.65)	(-12.07)	(-13.83)	(-9.24)	(-10.59)
OVX		0.539***		0.454***		0.858***		0.835***
		(8.60)		(6.60)		(6.04)		(5.33)
SIZE			-0.040**	-0.015			-0.049	-0.003
			(-3.18)	(-1.17)			(-1.71)	(-0.11)
NTRADES			-0.004*	-0.004*			-0.005	-0.004
			(-2.38)	(-2.17)			(-1.23)	(-1.02)
OI			-0.014	-0.007			0.055*	0.067**
			(-1.39)	(-0.75)			(2.36)	(2.97)
RTNup			0.238	0.074			0.328	0.021
			(1.19)	(0.38)			(0.73)	(0.05)
RTNdown			-0.767***	-0.470*			-1.071*	-0.530
			(-3.98)	(-2.48)			(-2.48)	(-1.23)
BAS			0.791	0.634			1.709	1.444
			(1.86)	(1.55)			(1.80)	(1.56)
SLminus			0.267	0.305			2.028*	2.008**
			(0.85)	(1.01)			(2.56)	(2.61)
SLplus			-0.782**	-0.798**			-0.844	-0.914
			(-2.74)	(-2.92)			(-1.31)	(-1.46)
constant	0.041***	-0.009	0.212***	0.073	0.113***	0.030	0.134	-0.127
	(5.77)	(-1.03)	(4.64)	(1.50)	(7.14)	(1.49)	(1.30)	(-1.14)
R-sqr	0.898	0.911	0.908	0.916	0.273	0.326	0.310	0.351

* p<0.05, ** p<0.01, *** p<0.001

Table B.6: Monthly horizon subsample for observations 489-1246: *In-sample fitting of models for next day levels of RV, and the change in average in RV from t to t+1, during 1/5/2009 to 15/5/2012. Model 2-4 are extensions of the model proposed by Corsi (2009). IV represents the implied volatility measure. EX represents exogenous variables including; $SIZE_t$ as daily average size of trades, NTR_t as average number of contracts traded per trade, OI_t as number of open interests, RTN_t^+ as price return if positive and RTN_t^- the return if negative, BAS_t as measure of average bid-ask spread, SL_t^+ as slope of the futures curve if positive and SL_t^- the slope if negative.*

	$RV_{t+1}^{(d)}$				$\ln\left(\frac{RV_{t+1}^{(d)}}{RV_t^{(d)}}\right)$			
	HAR-RV (1)	HAR-RV-IV (2)	HAR-RV-EX (3)	HAR-RV-IV-EX (4)	HAR-RV (5)	HAR-RV-IV (6)	HAR-RV-EX (7)	HAR-RV-IV-EX (8)
RVt	0.332***	0.279***	0.187***	0.093*	0.673***	0.543***	0.372*	0.125
	(9.17)	(7.43)	(4.28)	(2.10)	(5.74)	(4.44)	(2.57)	(0.84)
RVt5	0.210***	0.159**	0.290***	0.191***	-2.025***	-2.150***	-1.843***	-2.101***
	(3.83)	(2.88)	(5.32)	(3.50)	(-11.39)	(-11.94)	(-10.25)	(-11.50)
RVt20	0.220***	0.079	0.172***	-0.052	0.549***	0.203	0.339*	-0.249
	(4.56)	(1.40)	(3.54)	(-0.92)	(3.51)	(1.10)	(2.11)	(-1.30)
OVX		0.282***		0.469***		0.693***		1.230***
		(4.64)		(6.96)		(3.49)		(5.45)
SIZE			0.034***	0.040***			0.107***	0.125***
			(4.43)	(5.47)			(4.27)	(5.05)
NTRADES			0.001	0.002			0.001	0.002
			(0.99)	(1.79)			(0.19)	(0.82)
OI			-0.001	0.007			-0.008	0.012
			(-0.15)	(1.26)			(-0.46)	(0.65)
RTNup			0.279	0.197			0.435	0.218
			(1.34)	(0.98)			(0.63)	(0.32)
RTNdown			-1.210***	-0.914***			-2.608***	-1.828**
			(-6.11)	(-4.65)			(-3.98)	(-2.78)
BAS			2.423	2.344			2.404	2.199
			(1.25)	(1.25)			(0.38)	(0.35)
SLminus			-0.389	0.786			-2.934	0.144
			(-0.72)	(1.42)			(-1.64)	(0.08)
SLplus			1.355***	1.164**			5.032***	4.535***
			(3.43)	(3.03)			(3.86)	(3.54)
constant	0.071***	0.041***	-0.007	-0.088**	0.239***	0.166***	0.063	-0.150
	(7.64)	(3.68)	(-0.26)	(-3.13)	(7.91)	(4.53)	(0.72)	(-1.59)
R-sqr	0.525	0.538	0.581	0.607	0.211	0.224	0.279	0.306

* p<0.05, ** p<0.01, *** p<0.001

Appendix C

Out-of-sample predictions

Table C.1: Test statistics for out-of sample predictions of next day levels and differences. Mean squared errors (MAE) and mean absolute errors (MAE) compared to acutal values are shown for the four different model specifications. * indicates statistically significant difference in the MSE values of the marked model compared to the MSE values for the basic HAR-RV model according to the Diebold and Mariano (2002) test.

	HAR-RV	HAR-RV-IV	HAR-RV-EX	HAR-RV-IV-EX
RV_{t+1}:				
MSE	0.004113	0.00416	0.003914*	0.003461*
MAE	0.0450	0.0451	0.04296	0.044192
$\ln(\frac{RV_{t+1}}{RV_t})$:				
MSE	0.03617	0.03511	0.03512	0.0332*
MAE	0.1460	0.145	0.1435	0.1387

* p<0.05, ** p<0.01, *** p<0.001

Table C.2: *Weekly horizon*

Table C.3: Test statistics for out-of sample predictions of next week levels and differences. Mean squared errors (MAE) and mean absolute errors (MAE) compared to acutal values are shown for the four different model specifications. * indicates statistically significant difference in the MSE values of the marked model compared to the MSE values for the basic HAR-RV model according to the Diebold and Mariano (2002) test.

	HAR-RV	HAR-RV-IV	HAR-RV-EX	HAR-RV-IV-EX
RV_{t+1}:				
MSE	0.00406	0.004064	0.0038177*	0.003707*
MAE	0.04268	0.04269	0.03987	0.03908
$\ln(\frac{RV_{t+1}}{RV_t})$:				
MSE	0.03622	0.0364	0.0348	0.03468
MAE	0.1389	0.1389	0.1337	0.1328

* p<0.05, ** p<0.01, *** p<0.001

Table C.4: Test statistics for out-of sample predictions of next month levels and differences. Mean squared errors (MAE) and mean absolute errors (MAE) compared to acutal values are shown for the four different model specifications. * indicates statistically significant difference in the MSE values of the marked model compared to the MSE values for the basic HAR-RV model according to the Diebold and Mariano (2002) test.

	HAR-RV	HAR-RV-IV	HAR-RV-EX	HAR-RV-IV-EX
RV_{t+1}:				
MSE	0.003521	0.003539	0.003189	0.003163
MAE	0.04227	0.04539	0.04040	0.04020
$\ln(\frac{RV_{t+1}}{RV_t})$:				
MSE	0.03667	0.03679	0.03189	0.0320
MAE	0.1542	0.1541	0.1352	0.1352

* p<0.05, ** p<0.01, *** p<0.001