NTNU – Trondheim
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Risk modelling using Vine Copulas

Modelling an energy company portfolio

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Problem Description

The aim of this thesis is to investigate the predictive performance of a C - Vine Copula GARCH model on a portfolio of NordPool futures/forwards, benchmarked against DCC-GARCH, RiskMetrics and Historical Simulation.

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Abstract

In this paper, a method for calculating Value-at-Risk using GARCH and Vine Copula modelling with various marginals is implemented and tested on a set of eight electricity futures. The forecasts from this model are then compared to similar forecasts using a DCC-GARCH-model, RiskMetrics and historical simulation. These are all compared using the Kupiec and Christophersen tests. The comparison showed that at the 1%- and 99%-quantiles the Vine Copula method performs best, and the GARCH-based models generally outperformed the others. The Vine Copula performed worse than the benchmark models at the 5%- and 95%-quantiles. DCC-GARCH was able to predict all the quantiles fairly well in most of the portfolios.

Sammendrag

I oppgaven brukes det GARCH og Vine Copula modeller med ulike marginaler for å beregne Value-at-Risk på en portefølje bestående av åtte elektrisitets-futures. Resultatene fra denne modellen er sammenlignet med resultatene av lignende beregninger ved hjelp av DCC-GARCH, RiskMetrics og historisk simulering. Kupiec og Christophersen tester er brukt som sammenligningsgrunnlag. Vine Copula ga de beste resultatene på 1% - og 99% - kvantilen og modellene med GARCH-elementer presterte generelt bedre enn de andre modellene. For 5% - og 95% - kvantilene oppnådde Vine Copula dårligere resultater sammenlignet med de andre modellene. DCC-GARCH predikere alle kvantilene med akseptabel presisjon.

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I INTRODUCTION

Power companies trade electricity futures and forwards for both hedging and speculation purposes. In Europe, the increasingly interconnected electricity market has increased the liquidity of the contracts traded. This, combined with the political support for the introduction of more renewable electricity such as wind power has also given a larger (in volume) and more volatile market for electricity. With increased liquidity and more volatile markets there is a greater degree of risk. Risk models that can accurately predict the risk of such a portfolio are therefore of considerable value for power companies.

Value-at-Risk(VaR) is a common measure of risk. VaR allows for the quantification of probable losses over a period of time. The VaR forecasts, and quality thereof, are directly dependant on the model chosen to make the prediction. For banks, there has been a lot of pressure for testing and updating of VaR models, both regulatory (through the Basel Accord) and practical. Power companies, while they may have adopted many of the same risk systems as banks, have not faced similar pressure, and therefore may in many cases be using less accurate prediction models. This is part of the motivation for this paper: to investigate a potential improvement in the field of VaR forecasting for a power company portfolio.

A power company may hold a number of different future contracts so that their total risk profile is determined by the correlation between assets in their portfolio, a correlation that is also often time-varying. A good risk model therefore not only captures the behaviour of the individual assets in the portfolio, but also the time-varying dependencies between the assets. There are a number of possible ways to predict VaR, and in this paper we investigate a relatively new method of modelling: Vine Copulas. Copulas have become gradually more popular due to their ability to separate specification of marginals and dependency structure, and the Vine structure allows larger, more complex portfolios to be modelled without having to use simpler copula models due to practical constraints in calculation.

The aim of this thesis is therefore to investigate the accuracy of a C - Vine Copula GARCH model on a portfolio of NordPool futures/forwards contracts, benchmarked against a number of other models. Two of them have been chosen due to their widespread usage in the industry, Historical Simulation and RiskMetrics. To compare against a more complex model, DCC-GARCH was chosen as an example of a multivariate GARCH model. Multivariate GARCH - models have become quite popular as they can capture autoregressive and GARCH effects as well as enabling different marginal assumptions, however their choice of dependency structure is limited and not as flexible as that of a Vine Copula. The Vine Copula and DCC are tested with both a normal GARCH and a GJR-GARCH. We also test with different window sizes for the RiskMetrics, Vine Copula and DCC methods, as well as testing the effect of varying refit periods for the latter two models.

There has been some work done in testing Vine Copula-based models in other markets, as described in the next section, but there is a limited amount of data available for their usage for modelling electricity futures. Therefore, this thesis tests the applicability of this type of model to a portfolio modelled on that of a power company, specifically one trading within NordPool. NordPool is an energy market that has seen an increasing interconnection to other energy markets due to new connections having been built, and testing VaR models on historical data from this market will therefore give data on the accuracy of these models on a market experiencing change.

The dataset chosen for this paper was a set of 8 futures contracts, consisting of different positions within the NordPool market, chosen due to being highly liquid positions that may represent a large portion of an energy company portfolio. Newer products such as Contracts for Difference, CO_2-quotas and green certificates were not included, due to their lack of liquidity, leading to a lack of data.

The structure of the rest of this paper is as follows: Section 2 discusses existing literature, and Section 3 gives an overview of relevant theory. In Section 4, descriptive statistics for the dataset are given while in Section 5 methods for the different models are summarized. Section 6 shows the results of our backtesting while Section 7 evaluates the implication of these results, before suggesting future work in the area. In section 8 we give our recommendations and conclusions from our analysis.

II LITERATURE REVIEW

There has been done a lot of research on different types of Value-at-Risk models. This section covers articles relevant to our benchmarking models and our main Copula model.

The RiskMetrics methodology gained a lot of popularity after it was introduced. Evaluations of its accuracy were done in (Pafka and Kondor 2001) and (McMillan and Kambouroudis 2009), both papers considering financial markets. (Pafka and Kondor 2001) argue why RiskMetrics works, despite its somewhat inaccurate assumptions. They conclude that the method performs satisfactory well in one day ahead 5% VaR estimates, the reason being that the effect of fat tails is minor at this significance level. However, the method performs quite poorly at lower significance levels. (McMillan and Kambouroudis 2009) examines the forecasting performance of GARCH type VaR models against simpler models, including RiskMetrics. Their results support that of (Pafka and Kondor 2001): For the 5% VaR estimates the RiskMetrics forecasts are adequate, while it's outperformed by some of the GARCH models at lower quantiles.

Value-at-Risk estimates based on historical simulation are also very popular and widely used in the banking sector. (Hendricks 1996) looks at equally and exponentially weighted average (EWMA) models against historical simulation models with different in - sample sizes. He concludes that virtually all models produce accurate 5% forecasts, but have a hard time capturing the 1% quantile. Historical simulation estimates are somewhat higher at the 1% quantile than the other models, but the EWMA model is better at anticipating changes in risk over time. Later more popular historical simulation methods were introduced by (Boudoukh, Richardson, and Whitelaw 1998) and (Barone-Adesi, Bourgoin, and Giannopoulos 1998), as well as filtered historical simulation. (Pritsker 2006) examines the accuracy of these models and concludes that they respond poorly to changes in conditional volatility, but that the filtered historical simulation method looks promising.

Since they were introduced, GARCH models have gained a lot of momentum. Initially limited to the univariate case, multivariate models are becoming more and more popular. There are many multivariate GARCH models: VEC, BEKK, DCC, CCC, to name a few. Surveys of multivariate GARCH models can be found in (Bauwens, Laurent, and Rombouts 2006). (Bauwens, Laurent, and Rombouts 2006) argues that there's a dilemma between flexibility and parsimony of multivariate GARCH models. BEKK are very flexible models, but have too many parameters above 4 dimensions. Diagonal BEKK and VEC models are parsimonious, but restrictive in terms of their dependency structures. The DCC model allows for easy estimation on higher dimensional portfolios, but suffers from the restrictions since it imposes persistance in terms of covariance. (Malo and Kanto 2006) looks at a variety of multivariate GARCH models in the Nordic market in terms of their dynamic hedging performance. They conclude that all of their models are somewhat misspecified, either in terms of marginals or dependency structure, as the standard multivariate GARCH models compels one to use one distribution to model the whole covariance matrix, and argue that a combinatorial approach would be preferable.

Copulas have been widely used in risk management tools for several years. Their use has evolved from the bivariate case to multivariate copulas and finally, in recent years, Vine Copulas have been introduced. Vine Copulas allows for a more flexible dependency structure than previous multivariate copulas. The importance of this is obvious when concidering large portfolios where a single multivariate copula dependancy between all assets is an unlikely assumption. Vine Copulas are based on pair copula construction introduced by (Joe 1996), and further established in (Bedford and Cooke 2002), (Kurowicka and Cooke 2006) and (Aas, Czado, Frigessi, and Bakken 2009). As Vine Copulas are a relatively new concept most research papers are illustrative (see (Fischer, Köck, Schlüter, and Weigert 2009), (Berg and Aas 2009), (Min and Czado 2010) and (Czado, Schepsmeier, and Min 2012)) and papers in which the method is applied to portfolio theory are therefore somewhat limited. (de Melo Mendes, Semeraro, and Leal 2010) apply a D - Vine copula, with a set of four bivariate copulas, to a six - dimensional data set and discusses its use for portfolio management. (Brechmann and Czado 2012) considers different vine structures and a larger set of bivariate copulas on the Euro Stoxx 50. They conclude that Vine Copulas provide a good fit to the data and can accurately and efficiently forecast the Value-at-Risk of Euro Stoxx 50. (Emmanouil and Nikos) apply different C - Vine structures combined with EVT specified marginals to a large portfolio of Phelix futures, and conclude that all C - Vine models performs well statistically on both unconditional and conditional coverage tests.

III THEORY

Before introducing our method it may be necessary to define and discuss relevant theory. This part is divided into one section describing RiskMetrics, one discussing univariate and multivariate GARCH – models, another about Copulas and a final section describing Value-at-Risk.

A common assumption in risk models is that returns are independent and identically distributed (i.i.d). This assumption implies that the events that occurred yesterday don't affect today's events. Historical data of electricity futures have however shown signs of volatility clustering (Haugom, Westgaard, Solibakke, and Lien 2010). The observed volatility clustering depends on the frequency of the data, daily data often experiences clustering effects. Models mentioned below can take these effects into account.

3.1 RiskMetrics

The RiskMetrics model assumes that return series can be modeled by a stochastic process of the form

$$r_t = \sigma_t(\lambda) Z_t \tag{1}$$

where λ is a vector of smoothening constants, $\mu_t(\lambda) = E[r_t|I_{t-1}]$, $\sigma = E[\varepsilon_t|I_{t-1}]$ and Z_t is an i.i.d gaussian variable with zero mean and unit variance. I_t is the information set.

An exponentially weighted moving average (EWMA) method is used to forecast the average volatility over the next day:

$$\sigma_t^2(\lambda) = (1-\lambda) \sum_{j=1}^{t-1} \lambda^{j-1} r_{t-j}^2 \tag{2}$$

for a large t, it can be approximated by the following expression:

$$\sigma_t^2(\lambda) = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-j}^2 \tag{3}$$

Returns can then be forecasted assuming $r_t|I_{t-1} \sim N(0, \sigma_t^2)$.

3.2 GARCH

(Bollerslev 1986) and (Engle 1982) introduced the GARCH- and ARCH-models respectively. Below an AR-GARCH process is shown:

$$X_t = \phi_1 X_{n-1} + ... + \phi_1 X_{n-1} + \varepsilon_t \quad (4)$$

$$\varepsilon_t = \sigma_t v_t \quad (5)$$

where v_t is a strong white noise (iid(0,1)), $\phi_1 X_{n-1}$ is the autoregressive term and σ_t satisfies the recurrence equation

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2 \quad (6)$$

Thus Var($\varepsilon_t | \varepsilon_{t-1}$,$\varepsilon_{t-2}$)= σ_t^2, E(ε_t)=0, Cov(ε_t,ε_s)=0 (t$\neq$ s). The residuals ε_t satisfy (2), and (3) is a GARCH(p,q) process. If in (3) all $\beta_q = 0$, then ε_t is an ARCH(p) process.

The error term in the model is of specific interest for later analysis. When estimating model parameters it is possible to assume different distributions for the residuals. A specific version of the definition above is the AR(1)-GARCH(1,1) model :

$$X_t = \phi_1 X_{t-1} + \sigma_t v_t \quad (7)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (8)$$

In the model previous error terms and volatility are the main variables. The parameters α_1 and β, as seen in (4), determines the strength of respectively previous error terms and volatility. The first parameter, α_0, is the long-run average variance. The model parameters are determined by the maximum-likelihood method.

Electricity futures experience a mean-reverting process (Lucia and Schwartz 2002). Mean-reverting means that after a high or low price the price gradually reverts back to the mean. The parameters in the GARCH-model have some characteristics that capture mean reversion. An alpha above 0,1 implies that volatility is very sensitive to market events, while a beta value above 0,9 tells that volatility takes a long time to die out following a crisis in the market (Alexander 2008a).

A disadvantage of the standard GARCH-model is that it can't model the asymmetries of the volatility with respect to the sign of past shocks. As a consequence positive and negative returns with the same absolute value would give the same volatility. Extensions of the standard GARCH model that take this into account exist. The GJR - GARCH model is one of these, and captures asymmetric effects by including a new term into the volatility regression, $\gamma_1 \varepsilon_{t-1}^2 I_{t-1}$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \varepsilon_{t-1}^2 I_{t-1} ... + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2 \quad (9)$$

where I = 0 if $\varepsilon > 0$ and I = 1 if $\varepsilon < 0$.

3.3 Multivariate GARCH

A natural extension of the univariate GARCH - models is to develop a model for the multivariate domain. Consider the generalized stochastic vector process, x_t $\{t = 1,2...T\}$, of financial returns with dimension N x 1 and mean vector μ_t conditioned on the past information set

$$x_t|I_{t-1} = \mu_t + \varepsilon_t, \tag{10}$$

where the residuals are modelled as

$$\varepsilon_t = H_t^{1/2} z_t \tag{11}$$

H_t is the conditional covariance matrix of x_t^2 and z_t is an N x 1 i.i.d random vector. H_t may be classified into direct multivariate extensions, factor models and conditional correlation models. The trade-off between the models are model parametrization and dimensionality.

3.3.1 Dynamic Conditional Correlation Models (DCC)

DCC - models are estimated in a two-stage process where univariate and multivariate dynamics are separated. Standardized residuals from the estimated univariate GARCH models are used to compute the correlation matrix (Engle 2002). Decomposing the conditional covariance matrix into conditional standard deviations and correlations we have the following relationship

$$H_t = D_t R_t D_t \tag{12}$$

R_t (dynamic conditional correlation matrix) needs to be positive definite since it should be possible to invert the covariance matrix H_t. Positive definiteness is achieved by modeling a proxy process(Engle 2002)

$$\begin{aligned} Q_t &= \bar{Q} + a\,(z_{t-1}z'_{t-1} - \bar{Q}) + b\,(Q_{t-1} - \bar{Q}) \\ &= (1-a-b)\bar{Q} + a z_{t-1}z'_{t-1} + bQ_{t-1} \end{aligned} \tag{13}$$

Here a and b are non negative scalars and by introducing the constraint a + b < 1 we ensure stationarity and positive definiteness of Q_t. The a and b in DCC(a, b) are the news and decay term respectively, the latter of which quantifies the models sudden reaction to new information and the persistence after a market event. z_t is the standardized errors and $\bar{Q}$ is the unconditional matrix of the standardized errors. The proxy process is then used to obtain the correlation matrix:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \tag{14}$$

The advantage of using a two-stage process is the possibility of having varying specifications on the univarate GARCH process. The specifications are not limited to the standard GARCH model, but also asymmetric GARCH models are applicable. For further details on DCC-modelling see Appendix E.

3.4 Copulas

Copulas are multivariate distribution functions defined as $C(u_1, u_2 ... u_i)$ where $u_i \varepsilon [0, 1]$. Copulas were introduced by (Sklar 1959) and enable us to model the dependency structure and marginals separately. Sklar's theorem states that a given multivariate distribution can be expressed in terms of a copula:

$$F(x_1, x_2, ...x_i) = C(F_1^{-1}(x_1), F_2^{-1}(x_2), ...F_i^{-1}(x_i)) \tag{15}$$

or,

$$C(u_1, u_2) = F(F^{(-1)}(u_1), F_2^{(-1)}(u_2)) \tag{16}$$

Where $F_i^{-1}(u)$ is the inverse of the marginals' distribution functions.

3.4.1 Bivariate Copulas

Copulas have become very popular in finance because of their ability to capture varying degrees of symmetric or asymmetric lower or upper tail dependencies. Lower and upper tail dependencies are defined (as shown in (Alexander 2008b)) as:

$$\lambda_{ij}^{l} = lim_{q->0} P(X_i < F_i^{-1}(q) | X_j < F_j^{-1}(q)) \tag{17}$$

$$\lambda_{ij}^{u} = lim_{q->1} P(X_i > F_i^{-1}(q) | X_j > F_j^{-1}(q)) \tag{18}$$

If $\lambda^l = \lambda^u$ the copula has symmetrical tail dependencies, if they are unequal it has asymmetric tail dependencies. There are a number of types of bivariate copulas, each with the capability to model different forms of asymmetric or symmetric tail dependencies. Bivariate copulas are often divided into two categories called elliptical and Archimedean copulas. Elliptical copulas are a family of copulas that have elliptic contoured distributions, such as the gaussian- and t-distributions, as a result these can only model symmetrical dependencies. Archimedean copulas, the most popular being the Clayton and Gumbel copulas which allow for asymmetrical tails. An overview of some bivariate copulas and their modelling capabilities is given in table 1.

	Gaussian	Student-t	Clayton	Gumbel
Positive dependence	x	x	x	x
Negative dependence	x	x		
Upper tail				x
Lower tail			x	

Table 1: *Commonly used copulas and their characteristics*

3.4.2 C - Vine

There are few multivariate copulas and they have limited applicability (Nikoloulopoulos, Joe, and Li 2012). The C - Vine methodology allows for modelling of multivariate distributions based on bivariate copulas. The models are built through pairwise copula construction, using the fact that copulas can model conditional joint density functions and that it is possible to decompose this into a cascade of pair copulas (Aas, Czado, Frigessi, and Bakken 2009). The problem is that there are several ways of decomposing the joint density functions. One way of dealing with this is by applying a Vine structure and its corresponding rules for decomposition.

3.4.3 C - Vine Inference/Calibration

To determine the structure it is necessary to identify the root node of each tree. Because of this there will be $\frac{d!}{2}$ possible C - vine structures, where d is the number of variables. One way of doing this was proposed by (Czado, Schepsmeier, and Min 2012) and it involves calculating Kendall's tau, τ, in absolute terms for all pairs and choosing the root with the largest sum of Kendall's taus. An example of a C-vine tree structure is shown in the figure below.

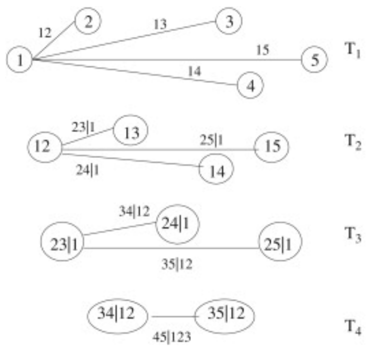


Figure 1: *Example C-Vine tree structure*

When the root notes have been established, the appropriate pair copulas are chosen in each tree sequentially. In the first tree its a simple matter of fitting bivariate copulas between the root node variable to all the other variables. In the next tree, one repeats the calculations involving the Kendall's tau matrix, but with the remaining variables conditioned on the root node in the previous tree, in order to choose the root node in the next tree. This is only possible if you can obtain the conditional distribution functions, but as it happens this can be established by using the pair copulas in the previous tree and applying the relationship:

$$h(x|v;\theta) = F(x|v) = \frac{\partial C_{x,v_j|v_{-j}}(F(x|v_{-j}),F(v_j|v_{-j}))}{\partial F(v_j|v_{-j})} \tag{19}$$

v is a vector containing the variables that have not been chosen as root nodes in the previous trees, whilst v_{-j} contains the root nodes, θ is the copula parameters.

The copulas in the second tree will be conditional copulas, given by $C_{x,v_j}(F(x|v_{-j}),F(v_j|v_{-j}))$. To determine the next trees you repeat the steps above until all unconditional and conditional pair copulas and their parameters have been established.

3.5 Value-at-Risk

Value-at-Risk is a central concept in modern financial risk management. With a specified portfolio, time horizon and probability, the Value-at-Risk is a threshold value so that the probability for that level loss on the portfolio is at the given probability. It is a given percentile of a predictive probability distribution of the size of a future financial loss on a specified time horizon and portfolio. Conditional Value-at-Risk is the expected loss at a given level. That is, if the loss exceeds the given loss threshold, the conditional Value-at-Risk is the probable amount that will actually be lost.

Value-at-Risk can be calculated using a number of methods, with the most popular being historical simulation, the variance-covariance method and Monte Carlo simulation. In a Monte Carlo simulation, a large number of possible portfolio values at the risk horizon are generated, and so also a large number of simulated returns. These are recalculated as present value, and then the $100(\alpha)$ VaR for the given horizon is found as minus the lower (α) quantile of the discounted portfolio return distribution (Alexander 2008c).

Two tests are commonly used when evaluating the accuracy of such a forecast. The unconditional coverage test, as proposed by (Kupiec 1995) is a likelihood ratio test based on the hypothesis that the model is accurate. The formula is listed below.

$$LR_{uc} = \frac{(1-\pi_{exp})^{n_0}\pi_{exp}^{n1}}{(1-\pi_{obs})^{n_0}\pi_{obs}^{n1}} \tag{20}$$

Heres is π_{exp} the expected proportion of returns that lie in the prescribed interval of the distribution, π_{obs} is the observed proportion of returns that lie in the prescribed interval, n_1 is the number of returns that lie inside the interval , and n_0 is the number of returns that lie outside the interval. This means that $n_1 + n_0$ is the total number of returns in the out-of-sample period. Also we have that

$$\pi_{obs} = \frac{n_1}{n} \tag{21}$$

The asymptotic distribution of -2 ln LR_{uc} is chi-squared with one degree of freedom. Kupiec's test says however nothing about whether several exceedances occur in rapid succession or whether they tend to be isolated.

Another test that takes this into account is the test introduced by (Christoffersen 1998).Christoffersen's test is also called the conditional coverage test since it measures whether a models forecasts produce clusters of exceedances. The test is based on the hypothesis that the forecast is accurate and that there is no clustering in exceedances. The formula is the following:

$$LR_{cc} = \frac{(1-\pi_{exp})^{n_0}\pi_{exp}^{n1}}{\pi_{01}^{n_{01}}(1-\pi_{01})^{n_{00}}\pi_{11}^{n_{11}}(1-\pi_{11})^{n_{10}}} \tag{22}$$

Here is n_{ij} the number of returns with indicator value i followed by indicator value j. For example is n_{01} defined as number of times a return outside the prescribed interval (denoted by 0) is followed by a return inside the prescribed interval (denoted by 1). In addition we have the following relationships:

$$\pi_{01} = \frac{n_{01}}{n_{00} + n_{01}} \quad and \quad \pi_{11} = \frac{n_{11}}{n_{10} + n_{11}} \tag{23}$$

The asymptotic distribution of $-2lnLR_{cc}$ is also chi-squared with one degree of freedom.

IV Descriptive Statistics

This paper considers four types of electricity futures/forwards: weekly, monthly, quarterly and yearly. Different positions are also considered ranging from 1. - 3. pos for weekly, 1.- 2. - pos for monthly and quarterly and 1. pos for yearly. Our data consists of future prices from 05.01.2007 - 16.01.2013. Due to low liquidity older data are not included. The analysis is based on the corresponding daily log – returns of the time series, and to avoid jumps in returns that are generated by the roll – over of the futures contracts, the returns series is cleaned by removing the returns at the roll - over dates. The data set is then split into an insample, 05.01.2007 - 18.01.2010, and an outsample, 19.01.2010 - 16.01.2013. This gives us an insample of 600 points, and an outsample of 550. The descriptive statistics for the insample is given below:

Table 2: *Descriptive Statistics for the various contracts*

	Week			**Month**		**Quarter**		**Year**
	Pos1	**Pos2**	**Pos3**	**Pos1**	**Pos2**	**Pos1**	**Pos2**	**Pos1**
Mean	-0,23 %	-0,05 %	0,11 %	0,05 %	0,06 %	0,06 %	0,05 %	0,08 %
Median	-0,29 %	-0,14 %	0,00 %	0,00 %	0,00 %	0,00 %	0,11 %	0,13 %
Min	-12,70 %	-10,01 %	-10,68 %	-9,53 %	-9,58 %	-8,41 %	-7,82 %	-8,99 %
Max	13,02 %	10,20 %	10,34 %	9,30 %	7,85 %	7,28 %	7,87 %	9,19 %
Kurt	5,21	4,01	3,96	3,64	3,58	3,60	4,51	6,44
Skew	0,14	0,26	0,24	0,23	0,12	0,00	-0,08	-0,17
AC(1)	0,18	0,14	0,12	0,13	0,07	0,01	0,01	-0,06
AC(2)	0,04	0,07	0,05	0,03	0,01	-0,01	-0,01	-0,01
AC(3)	0,05	0,08	0,07	0,05	0,04	0,03	0,02	0,02
AC(4)	0,00	-0,01	-0,02	-0,02	-0,03	-0,03	-0,06	0,00
AC(5)	0,08	0,03	-0,02	-0,04	-0,08	-0,08	-0,10	-0,08
Data points	600	600	600	600	600	600	600	600
Jarque Bera	124,20	32,35	28,77	15,80	9,82	9,14	57,56	298,59
ACF critical value	0,08	0,08	0,08	0,08	0,08	0,08	0,08	0,08
Q-statistic	25,39	19,00	13,26	13,17	8,73	4,81	8,99	6,16
Critical Q-value	11,07	11,07	11,07	11,07	11,07	11,07	11,07	11,07
Var 1%	-8,62 %	-7,70 %	-6,57 %	-6,31 %	-5,89 %	-6,35 %	-5,22 %	-5,04 %
VaR 5 %	-4,54 %	-4,72 %	-4,55 %	-4,35 %	-4,12 %	-4,07 %	-3,66 %	-3,01 %
VaR 95 %	5,10 %	5,63 %	5,13 %	5,26 %	4,90 %	4,30 %	3,31 %	2,94 %
VaR 99%	8,94 %	8,16 %	7,98 %	7,43 %	6,79 %	6,50 %	5,93 %	4,73 %

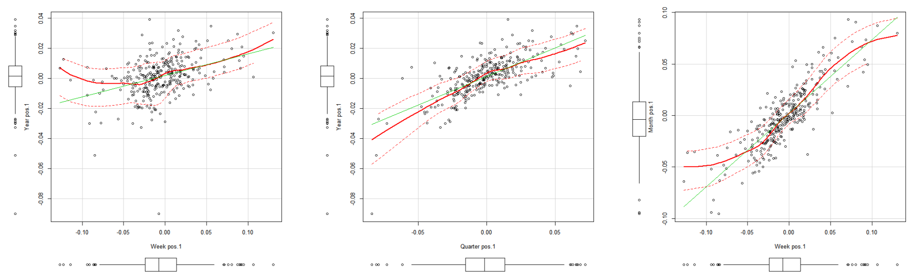
The weekly futures has the largest extreme returns, followed by monthly. Weekly pos 1 - pos 3, as well as monthly pos 1 show signs of autocorrelation. All our series exhibits excess kurtosis and a slight negative or positive skewness. As a consequence, normality is rejected by the Jarque – Bera test for all of our return series.

To test for heteroscedasticity an autoregressive model with one lag is fitted to the weekly and monthly futures return series and the corresponding residuals are squared and the Ljung - Box test is applied. Since the two other series didn't show any signs of significant autocorrelation, it's sufficient to test the squared returns. All of the series show signs of heteroscedastic behaviour, as can be seen in the table below:

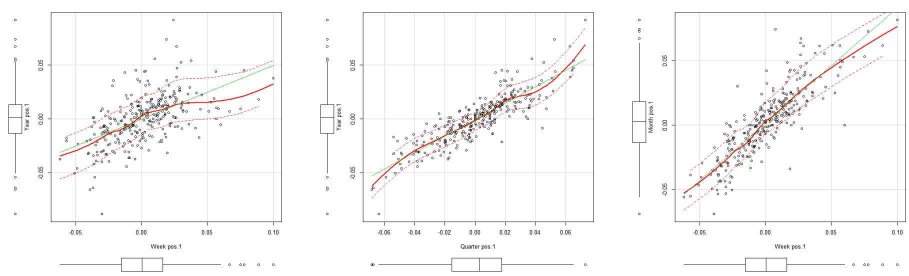
Table 3: *Ljung-Box test - squared returns*

	Ljung-Box							
Lag	W1	W2	W3	M1	M2	Q1	Q2	Y1
1	7,8E-05	1,14E-06	1,09E-05	0,003877	0,000927	0,000365	1,45E-06	0,006345
3	2E-07	5,06E-10	1,61E-10	1,36E-06	1,55E-06	5,26E-08	1,04E-09	1,08E-11
5	2,08E-13	6E-15	1,36E-12	2,16E-10	1,03E-08	2,04E-12	1,11E-16	0
10	0	0	0	1,73E-13	1,64E-11	4,22E-14	0	0
30	5,55E-16	0	0	4,13E-14	1,81E-14	1,11E-16	0	0

Scatterplots are a simple graphical way of depicting bivariate dependencies between variables. It's a good place to start if you're looking at the possibility of using a Copula to model the dependency structure. Below are three scatterplots from our portfolio. For more scatterplots see appendix B.



(a) Week pos. 1 and Year pos. 1 period 1 (b) Quarter pos. 1 and Year pos. 1 period 1 (c) Month pos. 1 and Week pos. 1 period 1



(d) Week pos. 1 and Year pos. 1 period 2 (e) Quarter pos. 1 and Year pos. 1 period 2 (f) Month pos. 1 and Week pos. 1 period 2

Figure 2: *Scatterplots in-sample*

In figure 2 data taken from first 300 data points in the in-sample are called period 1, while the last 300 days of the in-sample are called period 2. Tail non - linearity is most evident in the scatterplots with yearly forwards and quarterly forwards while the remaining of the futures/forwards have a fairly linear tail dependency. There is also some difference in trend between the two periods.

V Method

Our analysis is based around testing a variety of VaR forecasting methods so as to compare their efficiency. Our main model uses GARCH(1,1) filtering, fitting marginal distribution to the resulting residuals and inverting them over the distributions, modelling the results with a C-vine copula, then using the innovations made through simulations from the copula to calculate a VaR forecast. A number of Copula constructions are tested. Our method is inspired by the work of (Brechmann and Czado 2012). We compare this to three other models used as benchmarks, and use two different GARCH types for both of the GARCH based methods. For the remainder of this paper, we will refer to the standard GARCH model as sGARCH so as to more clearly differentiate it from general GARCH type models and gjrGARCH.

In the analysis we compare six different approaches to calculate Value-at-Risk.

1. Simulation using a copula with sGARCH: A C-Vine Copula is fitted to residuals filtered through sGARCH and AR-sGARCH for time series with autoregression, with t and skewed t marginals.
2. Simulation using a copula with gjrGARCH: A C-Vine Copula is fitted to residuals filtered through gjrGARCH and AR-gjrGARCH for time series with autoregression, with t and skewed t marginals.
3. DCC with sGARCH: Multivariate normal DCC(1,1) - model with gaussian filtered univariate sGARCH processes.
4. DCC with gjrGARCH: Multivariate normal DCC(1,1) - model with gaussian filtered univariate gjr-GARCH processes.
5. RiskMetrics
6. Historical Simulation with an expanding window.

5.1 Assumptions

In all our testing, we have assumed constant portfolio weights. As our forecasts are only for the next day Value-at-Risk, this assumption does not seem unreasonable. We have tested a set of five different portfolios, as seen in the table 4, with some variation between long, short and mixed portfolios. After calculating each data point of our outsample, we change our insample using a moving window, except for the historical simulation, which uses an expanding window.

Table 4: *Portfolio weights used for testing*

	Portfolios tested				
	Portfolio 1	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**
Week pos.1	0,2	0,05	-0,2	0,125	-0,125
Week pos.2	0,1	0,1	0,1	0,125	0,125
Week pos.3	0,1	-0,2	-0,1	0,125	-0,125
Month pos.1	0,2	-0,1	0,2	0,125	0,125
Month pos.2	0,05	0,2	0,05	0,125	0,125
Quarter pos.1	0,2	0,05	-0,2	0,125	-0,125
Quarter pos.1	0,05	-0,2	-0,05	0,125	-0,125
Year pos.1	0,1	0,1	0,1	0,125	0,125

5.2 Copula Simulation

5.2.1 GARCH filtering

To filter out autoregressive effects, as well as to model the heterodescasticity we can observe in our data, we run our data through an AR-GARCH model. This allows us to model the heterodescasticity observable in the in-sample series. To select a suitable GARCH model for use in filtering residuals to fit our copula to, we simulated using two different GARCH models, sGARCH and gjrGARCH, with three different window sizes (600, 400 and 200 data points), over our entire outsample. We refit the GARCH models for each data point. We test for autoregressive effects in our data using the Ljung–Box test before deciding whether or not to add a AR-element on each time series. (Ghalanos 2013b)

5.2.2 Marginal Distributions

We estimate suitable marginal distributions for the residuals using the maximum likelihood method, restricting ourselves to considering symmetric and skewed student-T distributions as well as the normal distribution, as those distributions capture a range of distributions.

5.2.3 Copula fitting

After having filtered our time series through AR-GARCH models, we must select suitable Copulas to model the relationship between the residuals. We test three different methods of choosing the tree structure, using the Kendall's tau maximization method (as discussed earlier in the theory section), as well as a tree structure based on the volatility of each contract (highest volatility first) and a tree structure based on the time horizon (with the closest and shortest term contract first, and the longest term contract last). Then, to choose a fitting Copula, we utilise the Akaike Information Criterion to select Copulas that explain the relationships between our residuals best. To do so, we construct C-Vine Copulas of our chosen structures, using every combination of each of our candidate Copula types, before then selecting the type of Copulas and construction thereof that gives us the best results under the AIC test. The Copulas we consider are: The Gaussian Copula. the Student t Copula, the Clayton Copula, the Gumbel Copula, the Frank Copula, the Joe Copula, the BB1 Copula, the BB6 Copula, the BB7 Copula and the BB8 Copula, as well as 90, 180 and 270 degree rotations of the above. Those Copulas capture a good range of possible dependencies. We test several different time intervals for how often the Copula is refit, intervals of 25, 50 and 100 time steps.

5.2.4 Simulation

Using a pseudorandom algorithm we generate variables between 0 and 1, inclusive. These are then used to generate values from the root node of the chosen C-Vine Copula. The values generated from the root node are then used to generate values from the next nodes in the tree, that are then used to generate values for the next, etc. In this way, we generate a value for each of the nodes, which are then inverted over the marginals. Thereafter, they are used as GARCH residuals, and reverse filtered through GARCH to give us possible next day returns.

5.2.5 Value-at-Risk

We calculate the Value-at-Risk through Monte Carlo simulation. By using the simulated values, acquired through the method described above, as possible returns from each part of our portfolio, we are able to calculate a high number of possible paths that our portfolio can take. With our simulations, we create 10000 different scenarios and then and calculate the relevant quantiles from our desired Value-at-Risk level thus find the value at risk from them. This is done for every point of our out-of-sample.

5.3 DCC

5.3.1 GARCH filtering

Similarly to the method outlined above, we choose suitable GARCH models to filter for heterodescasticity. (Venter and de Jongh 2002) found that changing the error distribution in the univariate GARCH models does not affect the results of the DCC-simulation. A quick test using our model confirmed this; changing between normal, skewed-t and t residuals did not affect the simulated quantiles from the DCC-model. As a consequence the errors are assumed to be normally distributed. We test our model using three different window sizes (600/400/200) and two different GARCH models, sGARCH and gjr-GARCH.

5.3.2 DCC-modelling

In (Engle and Sheppard 2001) varying news and decay term was tested. The DCC(1,1)-model was preferred compared to DCC-models with longer lags. We have therefore chosen a DCC(1,1)-model and the multivariate errors are assumed to be normally distributed. If the results from the

DCC(1,1)-model are poor, further lags will be tested. The DCC-models are tested using three different time intervals for refitting the DCC parameters: 25, 50 and 100 days.

5.3.3 Value-at-Risk

We multiply the portfolio weights into the covariance matrix before multiplying again with the transpose of the portfolio weights. This gives us the variances of the five portfolios. The expected means of the univariate time series are calculated as the mean of the window size used in the GARCH filtering. Here each time point is equally weighted. We then assume a normal distribution of returns, set the calculated variance as the variance of a distribution, and find the relevant quantiles which is then the Value-at-Risk for that portfolio at that point in time.

5.4 RiskMetrics

We use the RiskMetrics methodology to calculate Value-at-Risk. Using an exponentially weighted average model with different window sizes and a smoothening constant set to 0.94, the corresponding weighted covariance matrix is calculated. From this the portfolio variance,σ, is estimated and day of head returns are calculated by assuming r $\sim$ N(0, σ^2).

5.5 Historical Simulation

As a benchmark, we use a simple Historical Simulation to calculate a Value-at-Risk. Using bootstrapping, we select 10000 random selections from our in-sample and use them to calculate the relevant quantiles as a Value-at-Risk measure. This is done using an expanding window.

5.6 Testing

After having created forecasts for our entire outsample region using the methods explained above, we test the predictions using the Kuipec and Christophersen tests, focusing on the model giving the best results overall from each type of model. These results will then be analyzed further.

5.7 Implementation

The methods were implemented using the open-source software R connected to excel using the add-in RExcel (Baier and Neuwirth 2013). The packages rugarch (Ghalanos 2013b), rmgarch (Ghalanos 2013a) and CDVine (Brechmann and Schepsmeier 2013) were used for GARCH filtering, DCC modelling and Vine Copula modelling respectively.

VI RESULTS

We have tested a number of variants of each of the models in question, with a number of different parameters for such variables as GARCH window size, copula constructions and refit periods. We have included only the best results from each model category here.

For the Copula-GARCH - model a copula construction based on maximization of Kendall‘s tau, a GARCH window of 600 days and refitting the copula with an interval of 50 days gave the best results. To demonstrate the difference between the gjr - and s-GARCH model, we have chosen to show both models for the Copula-GARCH approach. DCC-GARCH had the best results with a gjr-GARCH model with a window size of 600 days and 100 days as refit window for the DCC-parameters. RiskMetrics proved to be insensitive to the choice of window size as long as the interval was longer than 100 days. To be consistent with the other methods, which had 200 days as the smallest GARCH window, we chose 200 days as the window size.

For the results of the other models see appendix D. This does not include the results of the 200-point window Copula-GARCH and 200-point DCC-GARCH results, as the GARCH models were unable to converge for a number of data points with that window size.

In this section, we have first presented the results found in setting up the models, compare the the forecasts each model gave directly for one portfolio, then compared the results found with the Kupiec and Christopher tests for each models forecasts. These results will then be analyzed in the next section.

6.1 Model Specifics

The results of the Ljung-Box test for the standardized residuals using AR(1)-GARCH(1,1) and GARCH(1,1) are in appendix D. The chosen structure, given the results, is an AR(1)-GARCH(1,1) model for the weekly and monthly contracts and a GARCH(1,1) model for the quarterly and the yearly contracts. The standardized residuals are t-distributed for all contracts except the yearly contract which is skewed t-distributed. See appendix C for quantile-quantile - plots.

6.2 Value-at-Risk Plots

We have here plotted the actual VaR forecasts against the actual returns for portfolio 3, as it is a good example of the general dynamics for each model. For similar plots for the other portfolios see appendix A. For each series of portfolio returns, we have plotted in the 1%, 5%, 95% and 99% VaR forecasts as well as the actual returns for each given day. The results for each model are shown separately. Any exceedances for the 1% quantile, as well as any non-exceedances for the 99% quantile, are highlighted with a circle.

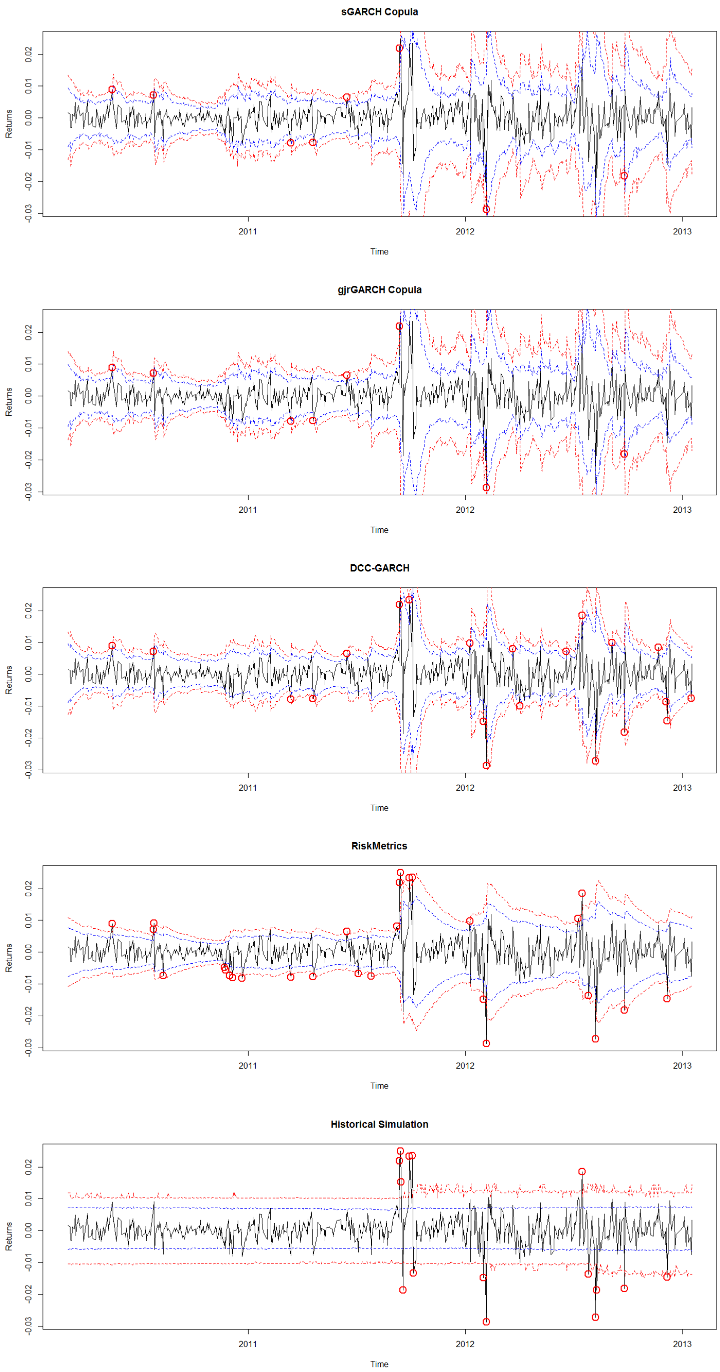


Figure 3: *Forecasts and real returns for portfolio 3*

As can be observed from figure 3, each model reacts differently. The GARCH-based models adjust faster after a spike than RiskMetrics and Historical Simulation, and the latter only reacts to larger sets of spikes.

6.3 Kupiec Test Results

We have here compared the Kupiec Test results for each quantile. For the precise exceedance numbers, see Appendix D.

Table 5: *Kupiec Test Results for each quantile*

1% quantile	sGARCH Copula	gjrGARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	0,499264923	0,241015225	0,313512104	0,002299	0,038000908
Portfolio 2	0,827727467	0,827727467	0,082718385	0,00078949	0,53738666
Portfolio 3	0,499264923	0,499264923	0,082718385	0,00025504	0,169526799
Portfolio 4	0,499264923	0,499264923	0,829353963	0,0160106	0,315720865
Portfolio 5	0,084505484	0,084505484	0,168072677	0,0160106	0,002298999
5% quantile					
Portfolio 1	0,499264923	0,027541598	0,914498951	0,39028265	0,484352084
Portfolio 2	0,123766721	0,078913068	0,490340486	0,21928166	0,158391254
Portfolio 3	0,185292203	0,123766721	0,774596102	0,76718643	0,111629117
Portfolio 4	0,003666054	0,014968597	0,626400369	0,50177897	0,484352084
Portfolio 5	0,007654348	0,007654348	0,626400369	0,50177897	0,013506685
95% quantile					
Portfolio 1	0,078913068	0,078913068	0,490340486	0,922294	0,26559127
Portfolio 2	0,921847957	0,921847957	0,188410442	0,12376672	0,922293996
Portfolio 3	0,185292203	0,078913068	0,626400369	0,92184796	0,390282652
Portfolio 4	0,078913068	0,26559127	0,626400369	0,92184796	0,078913068
Portfolio 5	0,078913068	0,078913068	0,015342786	0,61959749	0,390282652
99% quantile					
Portfolio 1	0,027541598	0,499264923	0,831017476	0,08357179	0,499264923
Portfolio 2	0,499264923	0,827727467	0,831017476	0,08357179	0,832749017
Portfolio 3	0,499264923	0,499264923	0,037549278	0,0160106	0,832749017
Portfolio 4	0,499264923	0,499264923	0,831017476	0,08357179	0,241015225
Portfolio 5	0,499264923	0,827727467	0,313512104	0,03800091	0,315720865

For the 1% quantile, the GARCH based models strongly outperform RiskMetrics and Historical Simulation on the Kupiec test. The Copula-based methods also have better results than the DCC-based method. This changes for the 5% quantile, where the Copula-based methods perform worse, performing worse than RiskMetrics on several portfolios, while DCC is the strongest model. For the 95% quantile the GARCH-based models perform worse than RiskMetrics and Historical Simulation, with the exception of Portfolio 2 (where the Copula models strongly outperform RiskMetrics and Historical Simulation and DCC outperforms Historical Simulation slightly) and portfolio 4 (where Historical Simulation performs weakly). For the 99% quantile, the gjrGARCH-Copula performs best, with both sGARCH copula and DCC models having the one weak portfolio. RiskMetrics performs poorly for this quantile, and Historical Simulation performs comparably to gjrGARCH Copula, but slightly weaker on several portfolios.

6.4 Christophersen Test Results

We have here compared the Christopher Test results for each quantile.

Table 6: *Christopher Test Results for each quantile*

1% quantile	sGARCH Copula	gjrGARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	0,772898645	0,494698578	0,534494499	0,00111716	0,006107452
Portfolio 2	0,932734086	0,932734086	0,085469162	0,00258182	0,161672415
Portfolio 3	0,772898645	0,772898645	0,184295201	0,00077108	0,335018896
Portfolio 4	0,772898645	0,772898645	0,914252346	0,00343997	0,537084755
Portfolio 5	0,224173352	0,224173352	0,33276775	0,0420442	0,006312606
5% quantile					
Portfolio 1	0,071193858	0,071193858	0,843975409	0,01800561	0,197893279
Portfolio 2	0,132651725	0,07978927	0,539938446	0,21322945	0,006852531
Portfolio 3	0,180364277	0,143585971	0,262748353	0,26265842	0,272923911
Portfolio 4	0,010451363	0,033041104	0,29578921	0,06473633	0,197893279
Portfolio 5	0,01872848	0,01872848	0,879530406	0,78287477	0,040518297
95% quantile					
Portfolio 1	0,196537717	0,07978927	0,539938446	0,16254343	0,01750817
Portfolio 2	0,409850274	0,409850274	0,209278491	0,0319754	0,469027657
Portfolio 3	0,055774095	0,017030592	0,071096198	6,4311E-05	6,49883E-08
Portfolio 4	0,017030592	0,09075607	0,660724042	0,40985027	0,017030592
Portfolio 5	0,017030592	0,017030592	0,001615166	0,0703579	8,83E-07
99% quantile					
Portfolio 1	0,772898645	0,772898645	0,933503783	0,08604977	0,772898645
Portfolio 2	0,772898645	0,932734086	0,933503783	0,00795491	0,915213788
Portfolio 3	0,772898645	0,772898645	0,091812818	0,0040873	0,003553404
Portfolio 4	0,772898645	0,772898645	0,933503783	0,08604977	0,494698578
Portfolio 5	0,772898645	0,094774629	0,153248574	0,00610745	0,1537026

For the 1% quantile, the GARCH based models greatly outperforms RiskMetrics and Historical Simulation on the Kupiec test. The Copula-based methods also have better results than the DCC-based method, with sGARCH having similar results as gjrGARCH, except for Portfolio 1. RiskMetrics performs worse than Historical Simulation, overall. This changes for the 5% quantile, where the Copula-based methods perform worse, performing comparably to Historical Simulation, with RiskMetrics somewhat better and DCC-GARCH the clearly strongest model. For the 95% quantile the GARCH-based models perform worse than RiskMetrics and Historical Simulation, with the exception of Portfolio 2 (where the GARCH models strongly outperform RiskMetrics, and the Copula models outperforms DCC). For the 99% quantile, the GARCH based models perform best, with sGARCH producing good results for all portfolios, DCC producing better results for three of the portfolios, but far worse on two, and gjrGARCH producing better results for one portfolio, but far worse for another. Historical Simulation has mixed results, but outperforms RiskMetrics.

VII Discussion

7.1 Accuracy

As we can see in the results section, there is a lot of variation in the accuracy of the various models. When looking at number of exceedances, the historical simulation was the least accurate model for the 1% and 99% quantiles, but performed adequately for the 5% and 95% quantiles. It did far worse on the Christophersen test. Similarly, RiskMetrics was outperformed by both the DCC-GARCH and Copula-GARCH models for the 1% quantile, but perfomed acceptably for the 5% quantile. However, once again, when taking into account the Christophersen test, it has more clustering of exceedances in general. Meanwhile, the Copula-GARCH models perform well for the 1% and 99% quantiles, but are less accurate in predicting the 5% and 95% quantiles. This is somewhat reversed for the DCC, though it manages to capture the 99% quantile quite well for several portfolios. In short, the more complex models are overall more precise. In general, one should be alert for overfitting in cases like this, but the large difference between the insample and the outsample is a contra-indication in this case.

The DCC-GARCH and Copula-GARCH models each has their bias, though. The DCC-GARCH tends to underestimate the risk, while the Copula-GARCH overestimates. For practical application, it may well be that looking at each of these models together may give you a more precise risk estimate than considering only one of them. The Copula was closer for the 1% quantile for most of our portfolios, but due to our small sample size, it is difficult to find any statistical validity. For the 5% quantile, however, the DCC outperformed the Copula, though this was the quantile most models had fairly good results for. Using a mixed set of these two models may give overall better predictions than relying on only one of them.

7.2 Dataset

We must take into consideration that this dataset is of eight different electricity futures, several of which are the same contract, but with different positions. Therefore, it should not be too surprising that the DCC modelling is able to simulate the correlations fairly well. It should be remarked that both the Copula and the DCC give good results, though. This means that using copulas, which can model complex dependencies, may be more than required for this task. However, they both outperform RiskMetrics and historical simulation in several areas and metrics.

It is quite interesting that our models were able to adapt to the changing circumstances quite well. Our initial in-sample and our out-sample had quite different characteristics, and several of the models were able to react quite effectively to this change. Due to our descriptive analysis considering only our in-sample, that means that we might have chosen marginal distributions that were less than optimal for the out-sample, but this does show that the tested models are at least somewhat rugged, and able to adapt to changing conditions, even when based on a set of initial conditions that does not accurately reflect the out-sample.

One challenge in this analysis is that our sample size is rather small. An out-sample of 550 datapoints makes it quite difficult to make any strong statements about accuracy on the 1% quantile. Analyzing the 5% quantile is more statistically valid, but as we are attempting to capture tail effects, it is difficult to do so without considering quantiles such as the 1% one.

7.3 Potential Future Work and Error Sources

There are a number of ways our model could be extended or otherwise improved which may correct some of the potential error sources, but which was outside the scope of this paper, and we shall attempt to indicate some of them here. These are largely divided into extensions to our Copula-GARCH model, improvements to our benchmarks, extensions and utilizations of our forecasts, and dataset considerations.

For the Copula-GARCH model, there are a number of ways it could be extended. There are a number of different GARCH models that could be used for filtering. Here we only tested two, but the model could potentially be improved by using a different form for GARCH. The lags and the autoregressive terms in the GARCH-model were chosen so that we had i.i.d standardized residuals. However, it could be interesting to see the relative performance of models with higher lags in both the autoregressive and GARCH parameters. We also held the types of marginal distributions for the standardized residuals fixed. However, there may be potential for model improvement if the marginal distributions were reconsidered regularly with a given interval, e.g every 50 days. Testing further types of marginal distributions may also lead to interesting results, as we restricted ourselves to only a few distributions in this paper. Furthermore, our window size was limited due to our sample size. Our tests with a window size of 600 data points were generally better than the models with 400 data points. It could therefore well be that even larger windows would further improve the model.

So far, only empirical selection procedures exist for a C-Vine decomposition. The choice of root node in the C-vine Copula fit is motivated by the fact that we want the highest dependencies in the bivariate conditional distributions estimated first. This means that later in the tree structure we have weaker dependencies. The entire C-Vine Copula structure may vary a lot based on the choice of root node. In addition to C-Vine there are other Vine Copula models such as D-vine and R-vine that may be employed. At the moment there is limited literature on the relative performance between different types of Vine Copulas, and in further analysis it could be interesting to compare results from the D-vine, C-vine and R-vine approaches, as it could be that a D- or R-Vine approach could outperform our current C-Vine Copula. Similarly, the choice of copula types was done based on estimations using the AIC-criteria. In further testing other selection criteria, such as BIC, could be employed. The BIC-criteria penalizes the effect of adding new parameters more than the AIC-criteria, which could potentially improve the result.

In improving the benchmarks, there are a number of ways this could be done. In the DCC-model we assumed a gaussian distribution for the multivariate dependency. It is possible that changing this to a t-distributed multivariate dependency may improve the forecasts from the

DCC-model. The DCC-model with normal dependency used as a benchmark had a tendency towards underestimating the risk at the 1%-quantile. However, in light of our results, we see that a t-distribution might be a better choice. In the DCC-model we used one lag for both the news term and the decay term. An alternative to this approach could be to test different lags and incorporate this into a procedure that refitted the DCC-model so that the lags of the news and the decay term could change. As pointed out in (Kring 2007) a drawback of the DCC model is that α and β in the model are scalars instead of matrices. This means that the elements of the conditional correlation matrix are influenced by the same coefficients. These conditions are necessary in order to maintain the positive definitiveness of the conditional correlation matrix, but the assumption may not be realistic. Other DCC-models, as proposed in (Billio, Caporin, and Gobbo 2003), relax this assumption so that the parameters could be matrices which is a more realistic approach. Further work could therefore consists of testing those DCC-models.

Also, there are a number of different multivariate GARCH-models. As a benchmark we chose to use a DCC-model, but further analysis could also test other diagonal VEC-models, BEKK-models, factor-models and generalized orthogonal models in comparison to a Copula approach. Testing how much of the improvement is due to the GARCH filtering could also be done by utilizing a GARCH filtered historical simulation.

Our current model assumes continual portfolio rebalancing, as it uses fixed weights of each of the different contracts. This may be a somewhat unrealistic assumption, but in the results section different portfolio weights were used and thereby we got an impression how changing weights impacted the results. Also, the time horizon of the Value-at-Risk is one day ahead, so that the bias is less compared to a Value-at-Risk calculation over longer horizons.

Our models may also be extended so that they may be used for portfolio optimization. This will allow it not only to be used in calculating the risk at a given set of weights, but also to optimize weights to minimize the Value-at-Risk (or conditional Value-at-Risk) at a chosen quantile. The model could also be extended to give Value-at-Risk estimates for a longer time horizon, or to calculate other risk measurements.

Finally, for this dataset, the DCC performed fairly well compared to the Copula model, possibly due to the high rate of intercorrelation between the elements of the portfolio. Conducting the same kind of analysis on a dataset with more complex relationships, e.g. equities, may provide interesting results.

VIII Conclusions

In this paper we have tested if a Vine Copula GARCH method was able to accurately model the Value-at-Risk for an energy company portfolio. Eight futures contracts on NordPool were investigated, consisting of weekly, monthly, quarterly and yearly futures. AR(1) - GARCH(1,1) models were used together with a C- Vine Copula structure to simulate day ahead returns for the portfolio. This was compared with a DCC-GARCH-based method, as well as RiskMetrics and Historical Simulation. The accuracy of the corresponding VaR was dependent on the choice of portfolio weights, and for the copula, the specific copula structures.

We found that though the Vine Copula was able to accurately model the 1% quantile, it performed worse than some simpler methods for the 5% quantile. This may be due to the choice of Copula structure, marginal distributions, or the method itself. However, the Vine Copula based methods still outperformed RiskMetrics and a pure Historical Simulation for some portfolios. The DCC-GARCH model was outperformed by the Copula-based model on the 1% quantile, but did fairly accurately on both the quantiles tested, unlike the other models.

Our analysis indicates that more advanced risk calculation methods are necessary for a suitable calculation of Value-at-Risk for the power industry, at least within NordPool, and for quantiles beyond 5%. Historical simulations and the RiskMetrics approach are not able to adequately predict the risk for this market. A GARCH approach greatly assists with changing market conditions, and a Copula structure is suitable for modelling the dependency structure, though a DCC-GARCH or similar model may be sufficient. Our DCC-GARCH model was in itself a significant improvement over the other benchmarks, and was only outperformed by the Copula on the 1% quantile, so further research may be required on the difference in performance of DCC- and Copula- based dependency structures in this market and for this type of modelling.

A limited number of copula structures and types of structures were tested, so further work may be done in testing a greater variety of such structures. It may be that better results may be achieved with D- or R- Vine structures, or with a different method for constructing a copula structure. However, a sufficient degree of accuracy in forecasting the Value-at-Risk for the 1% quantile was achieved that it seems that this may be a suitable method for calculating the VaR for outer quantiles.

In closing, the Copula-GARCH approach shows promise for modelling the outer Value-at-Risk quantiles of a NordPool electricity futures portfolio, but simpler models may still be able to model the 5% quantile better, and the DCC-GARCH model performed slightly worse for the 1% and 99% quantiles, but also performed acceptably on all quantiles tested.

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Appendix A: VaR forecast plots

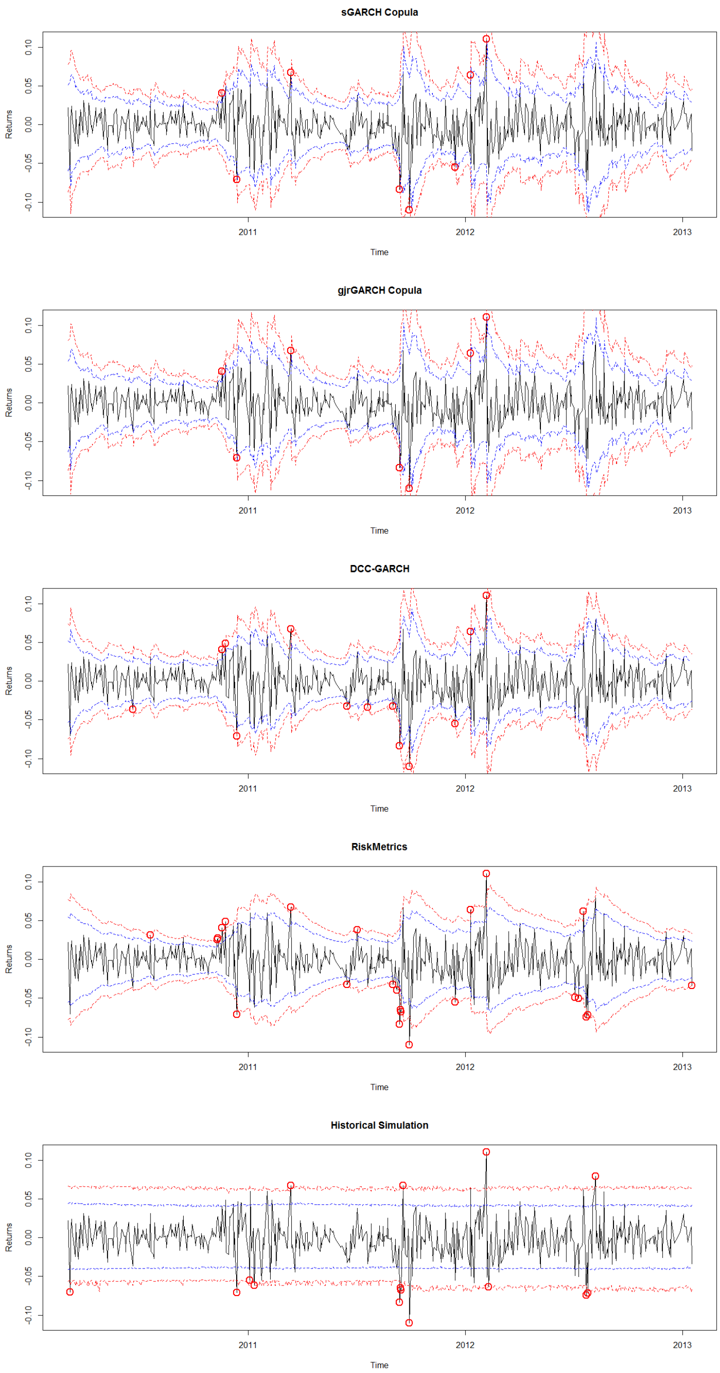


Figure 4: *Forecasts and real returns for portfolio 1*

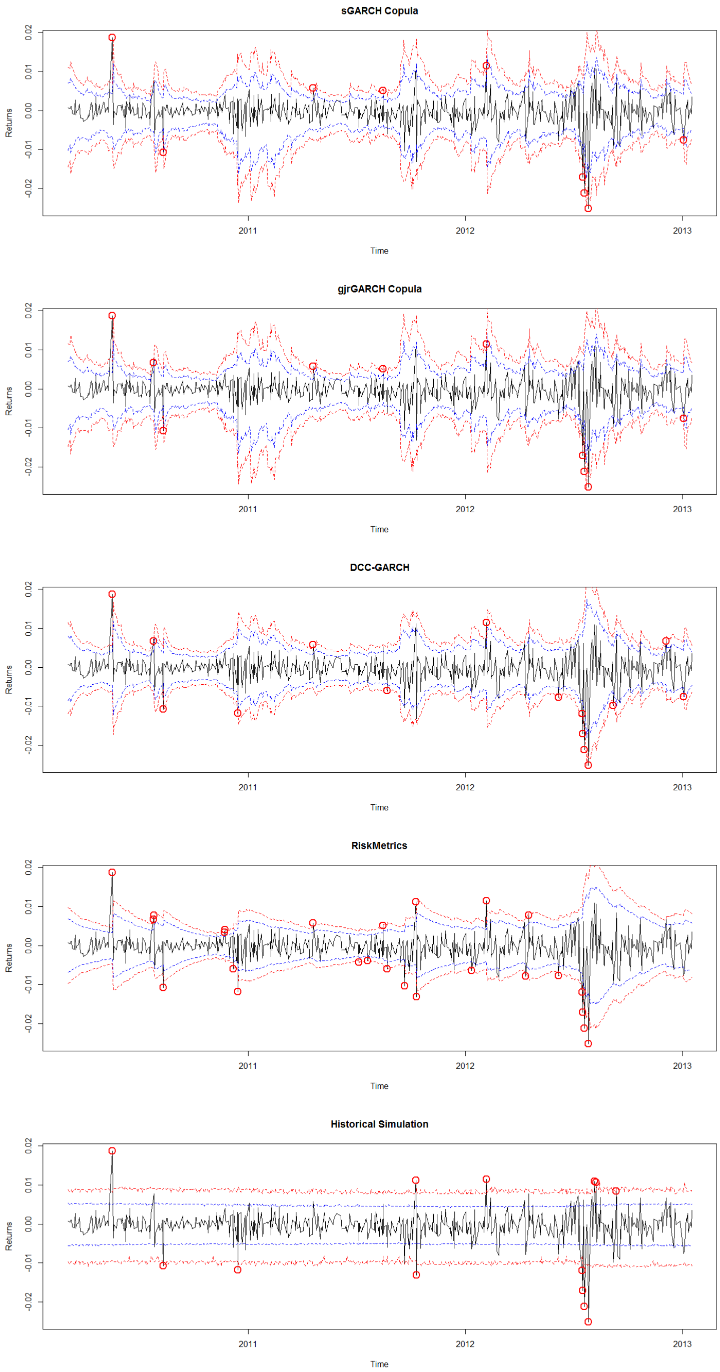


Figure 5: *Forecasts and real returns for portfolio 2*

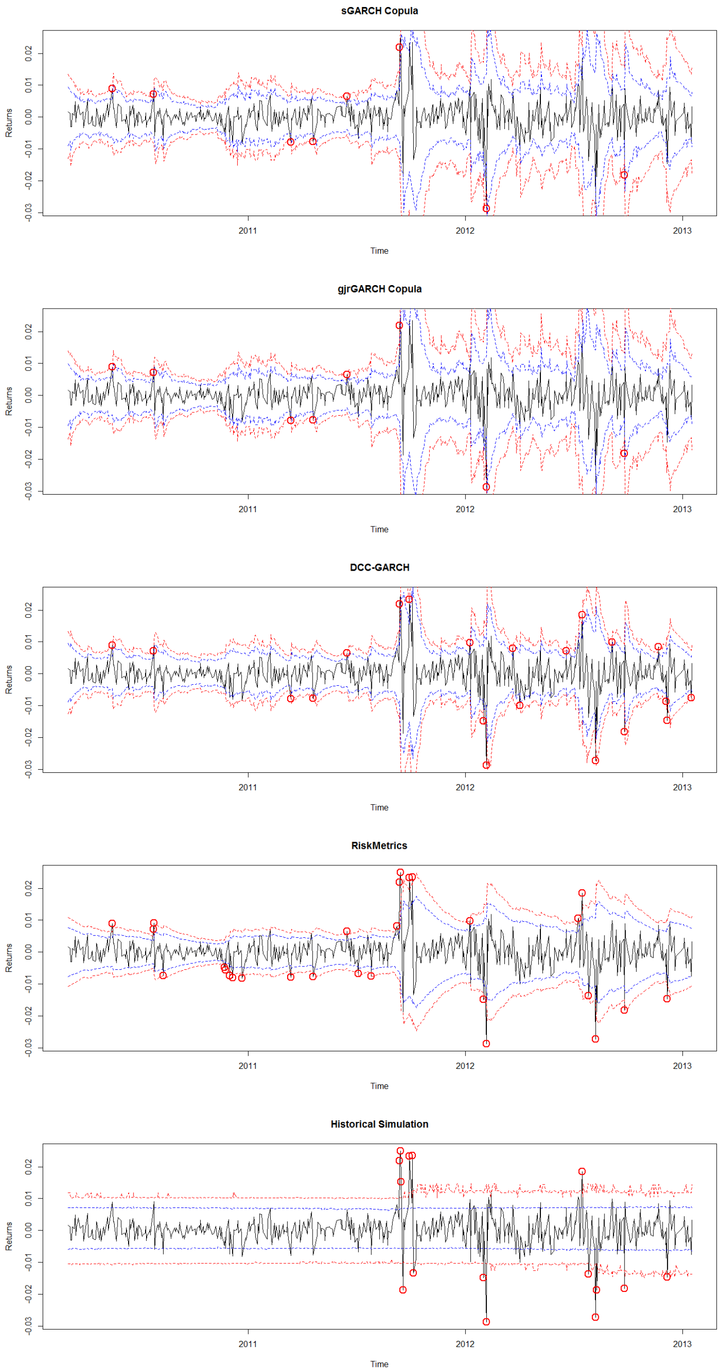


Figure 6: *Forecasts and real returns for portfolio 3*

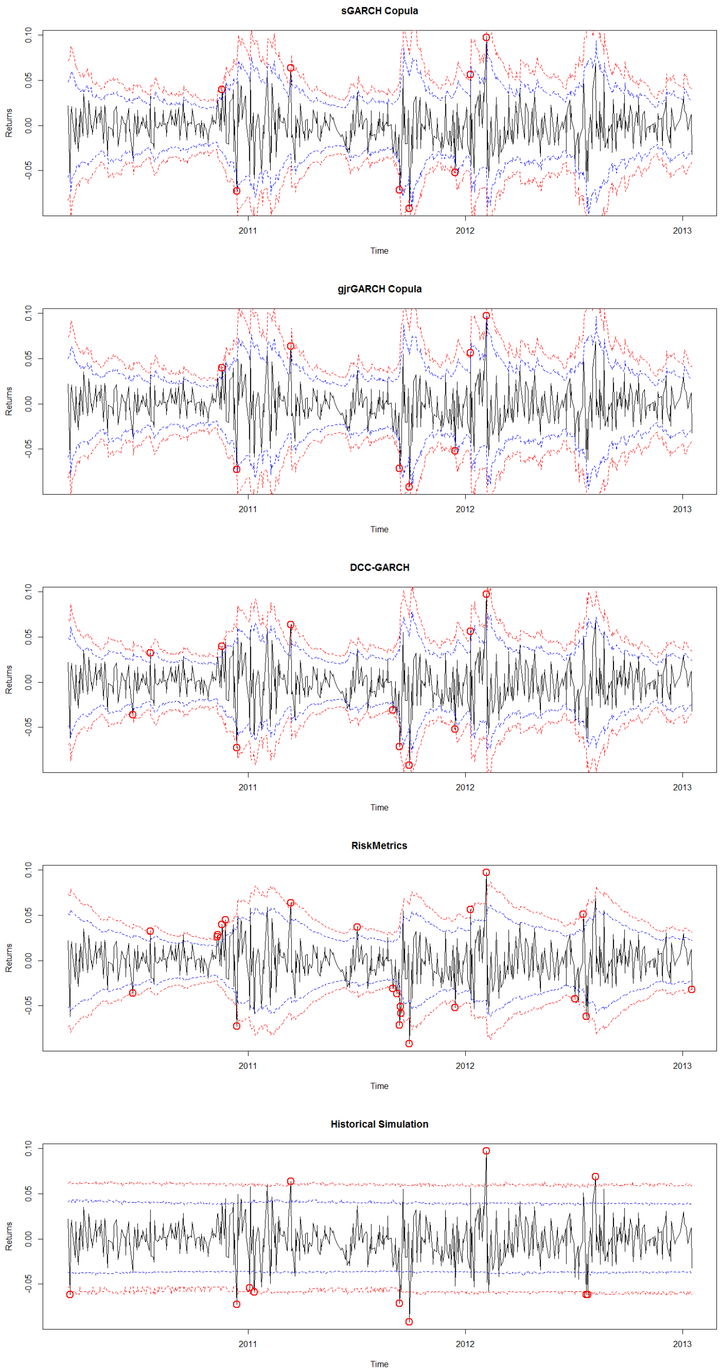


Figure 7: *Forecasts and real returns for portfolio 4*

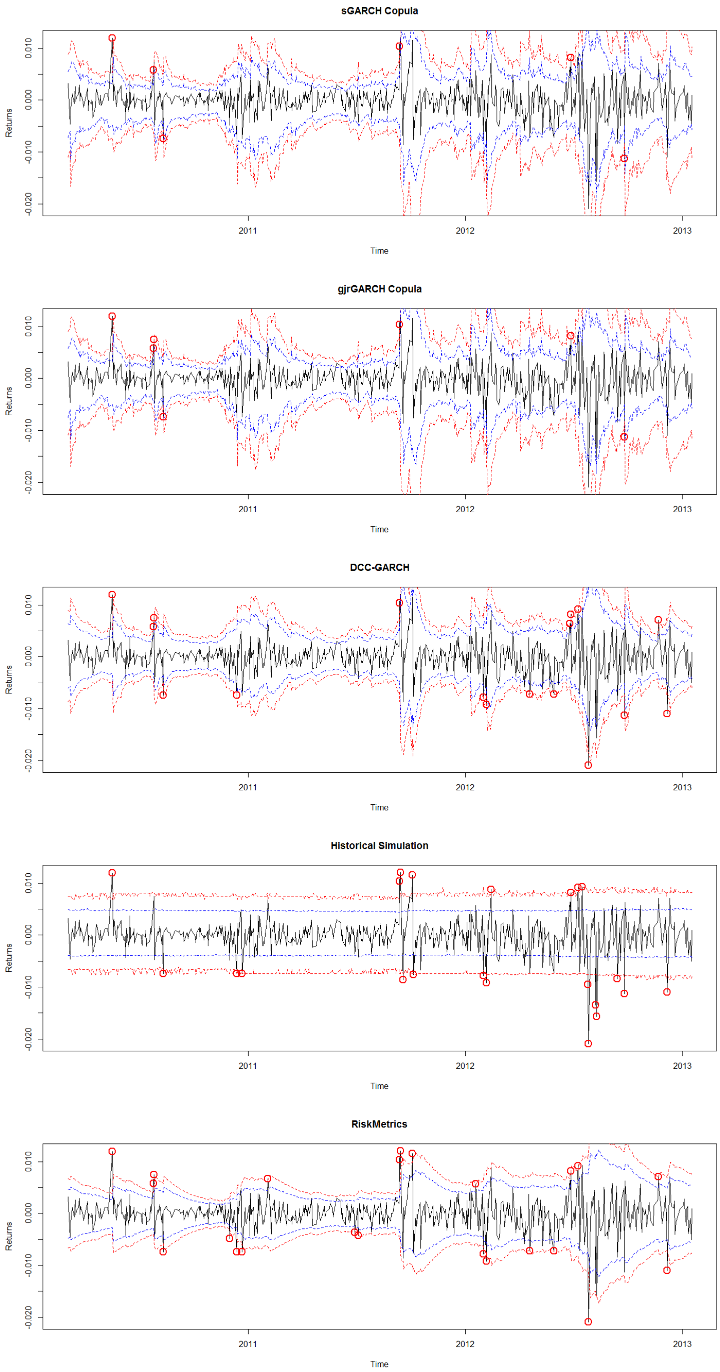


Figure 8: *Forecasts and real returns for portfolio 5*

APPENDIX B: SCATTERPLOTS

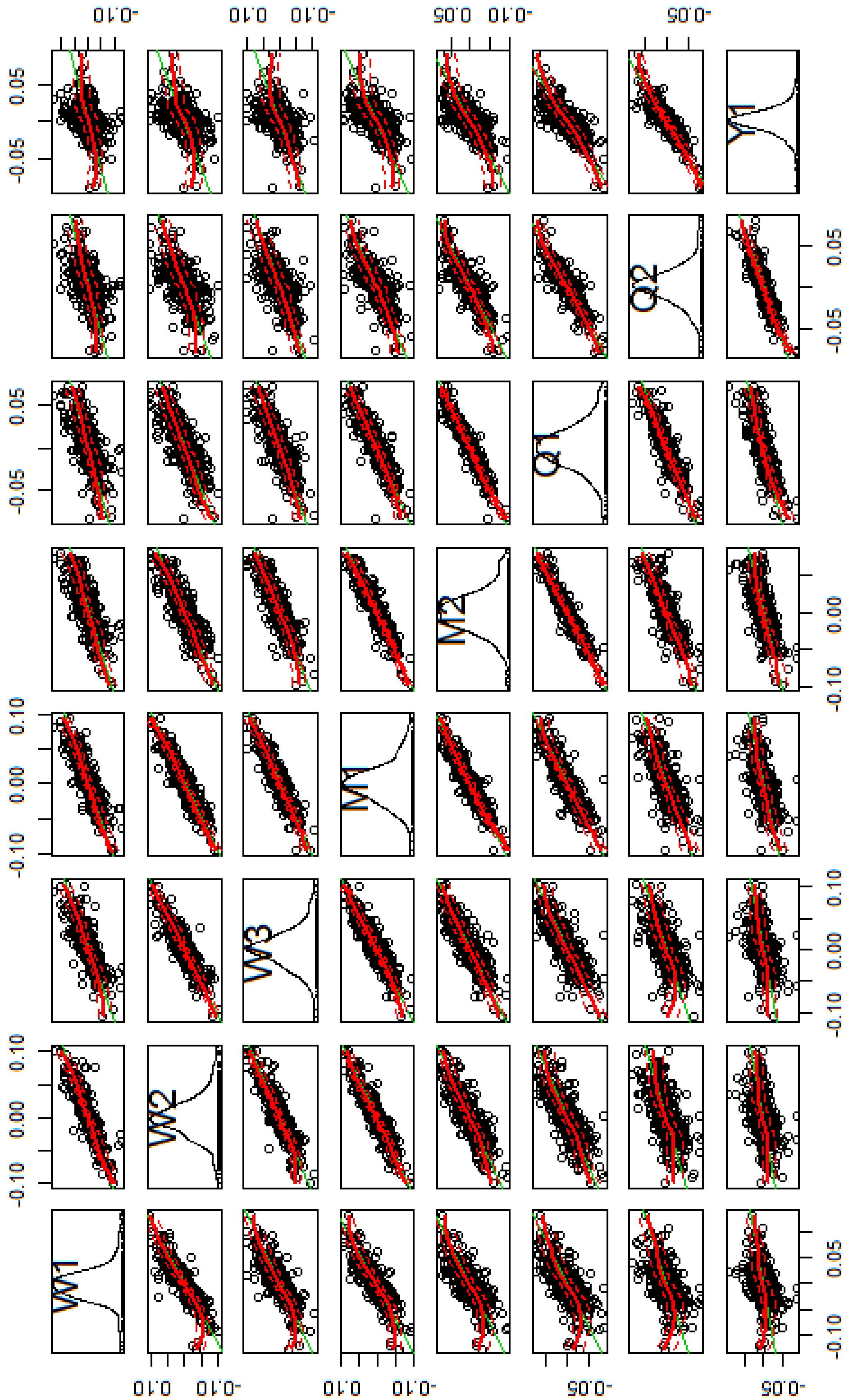
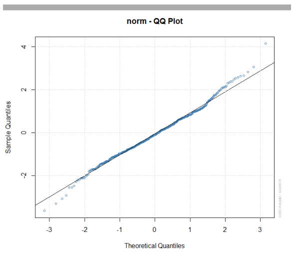
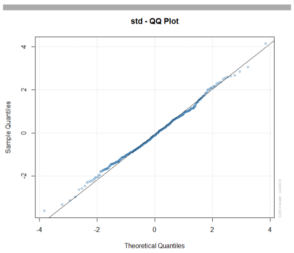


Figure 9: *Scatterplots in-sample*

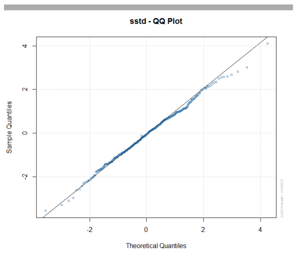
Appendix C: QQ - Plots



(a) Week 1 - normal

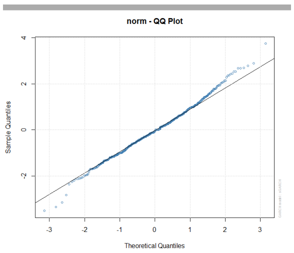


(b) Week 1 - t

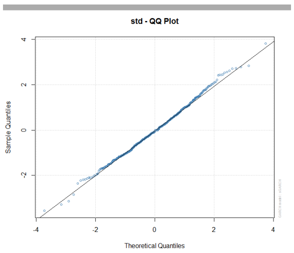


(c) Week 1 - skewed t

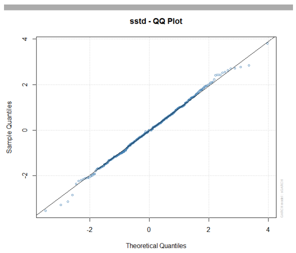
Figure 10: *Q-Q - plots standardized residuals - week position 1*



(a) Week 2 - normal

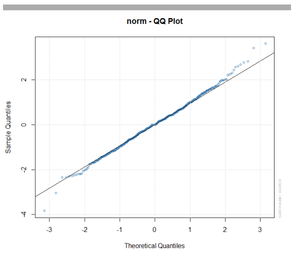


(b) Week 2 - t

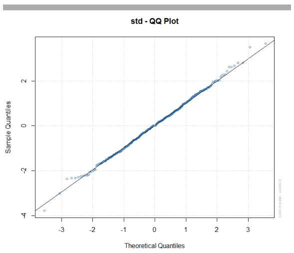


(c) Week 2- skewed t

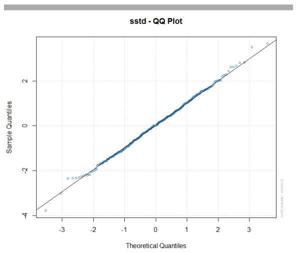
Figure 11: *Q-Q - plots standardized residuals - week position 2*



(a) Week 3 - normal

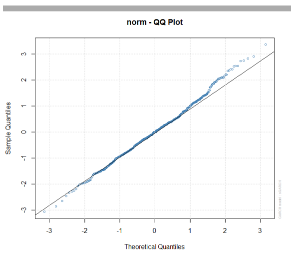


(b) Week 3 - t

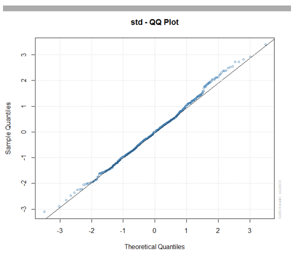


(c) Week 3 - skewed t

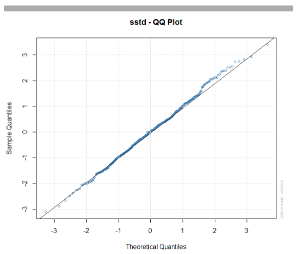
Figure 12: *Q-Q - plots standardized residuals - week position 3*



(a) Month 1 - normal

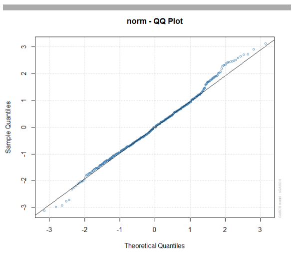


(b) Month 1 - t

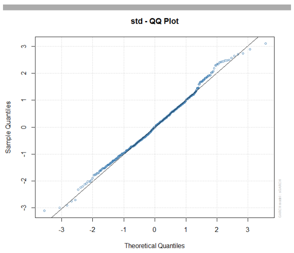


(c) Month 1 - skewed t

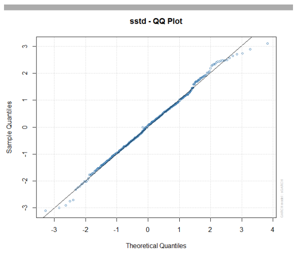
Figure 13: *Q-Q - plots standardized residuals - month position 1*



(a) Month 2 - normal

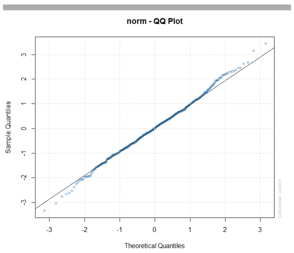


(b) Month 2 - t

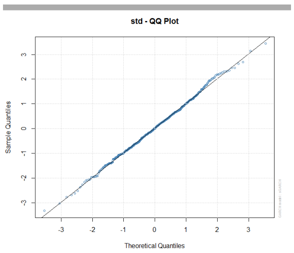


(c) Month 2 - skewed t

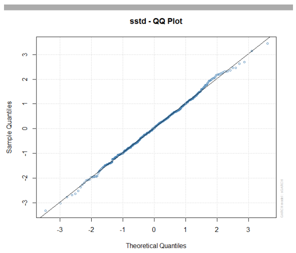
Figure 14: *Q-Q - plots standardized residuals - month position 2*



(a) Quarter 1 - normal

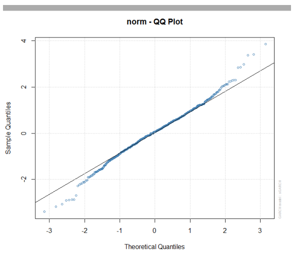


(b) Quarter 1 - t

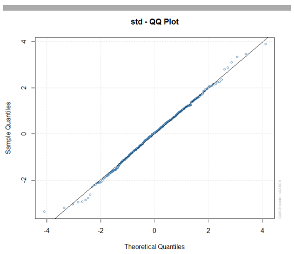


(c) Quarter 1 - skewed t

Figure 15: *Q-Q - plots standardized residuals - quarter position 1*



(a) Quarter 2 - normal



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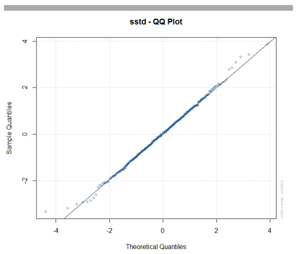
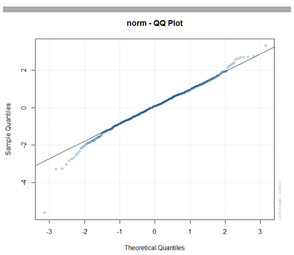
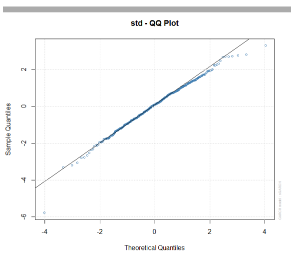


(c) Quarter 2 - skewed t

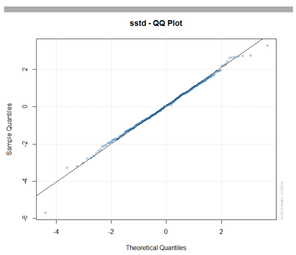
Figure 16: *Q-Q - plots standardized residuals - quarter position 2*



(a) Year 1 - normal



(b) Year 1 - t



(c) Year 1 - skewed t

Figure 17: *Q-Q - plots standardized residuals - year position 1*

APPENDIX D: FULL RESULTS

	Ljung - Box Test									
	Lag 1		Lag 3		Lag 5		Lag 10		Lag 30	
	Garch(1,1)	AR - Garch(1,1)	Garch(1,1)	AR - Garch(1,1)	Garch(1,1)	AR - Garch(1,1)	Garch(1,1)	AR - Garch(1,1)	Garch(1,1)	AR - Garch(1,1)
W1	0,000	0,697	0,000	0,650	0,000	0,505	0,000	0,039	0,000	0,016
W2	0,001	0,729	0,000	0,208	0,002	0,396	0,002	0,116	0,017	0,131
W3	0,004	0,977	0,006	0,355	0,024	0,573	0,029	0,236	0,095	0,219
M1	0,002	0,771	0,009	0,599	0,024	0,630	0,014	0,149	0,033	0,090
M2	0,068	0,562	0,218	0,661	0,130	0,329	0,275	0,410	0,491	0,470
Q1	0,824	0,360	0,879	0,668	0,451	0,347	0,623	0,513	0,763	0,652
Q2	0,769	0,368	0,943	0,749	0,117	0,100	0,237	0,205	0,700	0,668
Y1	0,126	0,436	0,438	0,804	0,296	0,491	0,723	0,867	0,109	0,134

Table 7: *Ljung-Box test - standardized residuals using either an AR(1)-GARCH(1,1) or a GARCH(1,1) - model*

Copula 1 - AR(1)*-sGARCH(1,1) - 1% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	4	0,4992	0,7728	7	0,5373	0,7553	3	0,2410	0,4946	3	0,2410	0,4946	1	0,0176	0,0599
600 & 25	3	0,2410	0,4946	7	0,5373	0,7553	4	0,4992	0,7728	3	0,2410	0,4946	3	0,2410	0,4946
400 & 50	3	0,2410	0,4946	5	0,8277	0,9327	4	0,4992	0,7728	3	0,2410	0,4946	1	0,0176	0,0599
600 & 50	4	0,4992	0,7728	5	0,8277	0,9327	4	0,4992	0,7728	4	0,4992	0,7728	2	0,0845	0,2242
400 & 100	3	0,2410	0,4946	6	0,8327	0,9152	4	0,4992	0,7728	3	0,2410	0,4946	1	0,0176	0,0599
600 & 100	3	0,2410	0,4946	6	0,8327	0,9152	4	0,4992	0,7728	3	0,2410	0,4946	2	0,0845	0,2242

Table 8: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*-sGARCH(1,1) - 5% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	17	0,0275	0,0529	22	0,2656	0,0908	18	0,0479	0,0767	15	0,0077	0,0193	10	0,000	0,0004
600 & 25	16	0,0150	0,0330	20	0,1237	0,1327	19	0,0789	0,1081	16	0,0150	0,0330	17	0,0275	0,0512
400 & 50	17	0,0275	0,0529	20	0,1237	0,1327	17	0,0275	0,0512	16	0,0150	0,0330	10	0,000	0,0004
600 & 50	17	0,4992	0,0712	20	0,1237	0,1327	21	0,1852	0,1804	14	0,0037	0,0105	15	0,0077	0,0187
400 & 100	21	0,1852	0,4013	22	0,2656	0,0908	19	0,0789	0,1081	17	0,0275	0,0529	9	0,000	0,0001
600 & 100	16	0,0150	0,0330	20	0,1237	0,1327	21	0,1852	0,1804	15	0,0077	0,0193	16	0,0150	0,0321

Table 9: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*-sGARCH(1,1) - 95% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	533	0,0275	0,0037	522	0,9223	0,4690	537	0,0016	0,0007	532	0,0479	0,0084	533	0,0275	0,0735
600 & 25	533	0,0275	0,0735	522	0,9223	0,4690	530	0,1237	0,0319	529	0,1852	0,0557	531	0,0789	0,0170
400 & 50	533	0,0275	0,0037	522	0,9223	0,4690	535	0,0077	0,0006	532	0,0479	0,0084	532	0,0479	0,0447
600 & 50	531	0,0789	0,1965	523	0,9218	0,4098	529	0,1852	0,0557	531	0,0789	0,0170	531	0,0789	0,0170
400 & 100	532	0,0479	0,0084	524	0,7671	0,9331	536	0,0037	0,0002	532	0,0479	0,0084	535	0,0077	0,0049
600 & 100	531	0,0789	0,1965	523	0,9218	0,4098	529	0,1852	0,0557	529	0,1852	0,0557	533	0,0275	0,0232

Table 10: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*-sGARCH(1,1) - 99% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	545	0,8277	0,9327	544	0,8327	0,9152	545	0,8277	0,0948	546	0,4992	0,7728	546	0,4992	0,0472
600 & 25	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	547	0,2410	0,4946	545	0,8277	0,0948
400 & 50	545	0,8277	0,9327	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,0472
600 & 50	546	0,0275	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728
400 & 100	545	0,8277	0,9327	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,0472
600 & 100	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	545	0,8277	0,0948

Table 11: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*-sGARCH(1,1) - 1% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	3	0,2410	0,4946	4	0,4992	0,7728	3	0,2410	0,4946	4	0,4992	0,7728	2	0,0845	0,2242
600 & 25	2	0,0845	0,2242	6	0,8327	0,9152	3	0,2410	0,4946	3	0,2410	0,4946	2	0,0845	0,2242
400 & 50	3	0,2410	0,4946	11	0,0380	0,0527	13	0,0063	0,0174	3	0,2410	0,4946	7	0,5374	0,7553
600 & 50	3	0,2410	0,4946	6	0,8327	0,9152	4	0,4992	0,7728	4	0,4992	0,7728	3	0,2410	0,4946
400 & 100	3	0,2410	0,4946	5	0,8277	0,9327	5	0,8277	0,9327	3	0,2410	0,4946	3	0,2410	0,4946
600 & 100	3	0,2410	0,4946	7	0,5374	0,7553	4	0,4992	0,7728	4	0,4992	0,7728	2	0,0845	0,2242

Table 12: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*-sGARCH(1,1) - 5% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	16	0,0150	0,0330	24	0,4844	0,535354	24	0,4844	0,26131	16	0,0150	0,0330	22	0,2656	0,2147
600 & 25	15	0,0077	0,01969	23	0,36555	0,663549	23	0,36555	0,2428	14	0,003666	0,010451	24	0,4844	0,26131
400 & 50	15	0,0077	0,0193	30	0,629556	0,216206	28	0,9223	0,220848	15	0,0077	0,0193	31	0,501779	0,646725
600 & 50	17	0,0275	0,0712	26	0,7672	0,3399	22	0,2656	0,2147	15	0,0077	0,0193	22	0,2656	0,5337
400 & 100	17	0,0275	0,0529	23	0,36555	0,41309	24	0,4844	0,26131	16	0,0150	0,0330	24	0,4844	0,26131
600 & 100	15	0,0077	0,0193	23	0,36555	0,41309	23	0,36555	0,2428	14	0,003666	0,010451	27	0,9218	0,246107

Table 13: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*-sGARCH(1,1) - 95% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	531	0,0789	0,0170	533	0,0275	0,073511	535	0,0077	0,000591	533	0,0275	0,0038	538	0,000677	0,001621
600 & 25	530	0,1238	0,13272	530	0,1238	0,13272	536	0,003666	0,009664	530	0,1238	0,0320	537	0,001636	0,004152
400 & 50	531	0,0789	0,0170	535	0,0077	0,020596	530	0,1238	0,0049	531	0,0789	0,0170	534	0,0150	0,011137
600 & 50	530	0,1238	0,13272	524	0,7672	0,3399	526	0,4844	0,0473	530	0,1238	0,0320	530	0,1238	0,0049
400 & 100	532	0,0480	0,0084	527	0,36555	0,41309	534	0,0150	0,011137	534	0,0150	0,00163	536	0,003666	0,009664
600 & 100	531	0,0789	0,0170	525	0,6196	0,2668	535	0,0077	0,004898	532	0,0480	0,0084	538	0,000677	0,001621

Table 14: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a Copula-sGARCH(1,1) -mode, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*-sGARCH(1,1) - 99% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	545	0,8277	0,9327	545	0,8277	0,9327	549	0,01768	0,059868	546	0,4992	0,7728	547	0,2410	0,4946
600 & 25	547	0,2410	0,4946	546	0,4992	0,7728	549	0,01768	0,059868	548	0,0845	0,2242	548	0,0845	0,2242
400 & 50	544	0,8327	0,9152	547	0,2410	0,4946	543	0,5374	0,7553	546	0,4992	0,7728	544	0,8327	0,9152
600 & 50	546	0,4992	0,7728	546	0,4992	0,7728	544	0,8327	0,9152	546	0,4992	0,7728	542	0,3157	0,1537
400 & 100	546	0,4992	0,7728	546	0,4992	0,7728	549	0,01768	0,059868	546	0,4992	0,7728	547	0,2410	0,4946
600 & 100	546	0,4992	0,7728	546	0,4992	0,7728	549	0,01768	0,059868	548	0,0845	0,2242	547	0,2410	0,4946

Table 15: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 - AR(1)*-sGARCH(1,1) - 1% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	4	0,4992	0,7728	3	0,2410	0,4946	4	0,4992	0,7728	4	0,4992	0,7728	3	0,2410	0,4946
600 & 25	3	0,2410	0,4946	6	0,8327	0,9152	5	0,8277	0,9327	3	0,2410	0,4946	3	0,2410	0,4946
400 & 50	3	0,2410	0,4946	3	0,2410	0,4946	4	0,4992	0,7728	3	0,2410	0,4946	3	0,2410	0,4946
600 & 50	4	0,4992	0,7728	6	0,8327	0,9152	5	0,8277	0,9327	4	0,4992	0,7728	3	0,2410	0,4946
400 & 100	4	0,4992	0,7728	6	0,8327	0,9152	6	0,8327	0,9152	4	0,4992	0,7728	3	0,2410	0,4946
600 & 100	4	0,4992	0,7728	6	0,8327	0,9152	7	0,5374	0,7553	4	0,4992	0,7728	3	0,2410	0,4946

Table 16: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 - AR(1)*-sGARCH(1,1) - 5% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	18	0,0480	0,0794	25	0,6196	0,2668	22	0,2656	0,2147	15	0,0077	0,0193	20	0,1238	0,1436
600 & 25	17	0,0275	0,0529	27	0,9218	0,4099	20	0,1238	0,1436	15	0,0077	0,0193	22	0,2656	0,5337
400 & 50	17	0,0275	0,0529	25	0,6196	0,2668	21	0,1853	0,1804	16	0,0150	0,0330	21	0,1853	0,1804
600 & 50	17	0,0275	0,0712	27	0,9218	0,4099	21	0,1853	0,1804	16	0,0150	0,0330	22	0,2656	0,5337
400 & 100	20	0,1238	0,2866	27	0,9218	0,4099	21	0,1853	0,1804	16	0,0150	0,0330	22	0,2656	0,2147
600 & 100	19	0,0789	0,1924	27	0,9218	0,4099	22	0,2656	0,2147	17	0,0275	0,0529	22	0,2656	0,2147

Table 17: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 - AR(1)*-sGARCH(1,1) - 95% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	532	0,0480	0,0084	529	0,1853	0,2062	531	0,0789	0,0170	533	0,0275	0,0038	531	0,0789	0,0170
600 & 25	532	0,0480	0,1244	524	0,7672	0,3399	528	0,2656	0,0175	531	0,0789	0,0170	530	0,1238	0,0049
400 & 50	532	0,0480	0,0084	528	0,2656	0,3008	531	0,0789	0,0170	533	0,0275	0,0038	530	0,1238	0,0049
600 & 50	532	0,0480	0,1244	524	0,7672	0,3399	526	0,4844	0,0473	531	0,0789	0,0798	529	0,1853	0,0096
400 & 100	530	0,1238	0,0320	522	0,9223	0,4690	526	0,4844	0,0473	530	0,1238	0,0320	529	0,1853	0,0096
600 & 100	530	0,1238	0,0320	522	0,9223	0,4690	526	0,4844	0,0473	530	0,1238	0,0320	529	0,1853	0,0096

Table 18: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 - AR(1)*-sGARCH(1,1) - 99% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	545	0,8277	0,9327	546	0,4992	0,7728	545	0,8277	0,9327	546	0,4992	0,7728	544	0,8327	0,1396
600 & 25	547	0,2410	0,4946	546	0,4992	0,7728	544	0,8327	0,9152	547	0,2410	0,4946	542	0,3157	0,1537
400 & 50	545	0,8277	0,9327	545	0,8277	0,9327	544	0,8327	0,1396	546	0,4992	0,7728	543	0,5374	0,1617
600 & 50	546	0,4992	0,7728	546	0,4992	0,7728	544	0,8327	0,9152	546	0,4992	0,7728	541	0,1695	0,1237
400 & 100	546	0,4992	0,7728	546	0,4992	0,7728	543	0,5374	0,7553	546	0,4992	0,7728	542	0,3157	0,1537
600 & 100	546	0,4992	0,7728	546	0,4992	0,7728	543	0,5374	0,7553	546	0,4992	0,7728	543	0,5374	0,7553

Table 19: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a Copula-sGARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*- GJR-GARCH(1,1) - 1% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	4	0,4992	0,7728	5	0,8277	0,9327	4	0,4992	0,7728	3	0,2410	0,4946	2	0,0845	0,2242
600 & 25	3	0,2410	0,4946	7	0,5373	0,7553	4	0,4992	0,7728	3	0,2410	0,4946	2	0,0845	0,2242
400 & 50	5	0,8277	0,9327	4	0,4992	0,7728	4	0,4992	0,7728	4	0,4992	0,7728	1	0,0177	0,0599
600 & 50	3	0,2410	0,4946	5	0,8277	0,9327	4	0,4992	0,7728	4	0,4992	0,7728	2	0,0845	0,2242
400 & 100	4	0,4992	0,7728	5	0,8277	0,9327	4	0,4992	0,7728	3	0,2410	0,4946	1	0,0177	0,0599
600 & 100	3	0,2410	0,4946	5	0,8277	0,9327	3	0,2410	0,4946	3	0,2410	0,4946	2	0,0845	0,2242

Table 20: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*- GJR-GARCH(1,1) - 5% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	19	0,0789	0,1924	21	0,1852	0,0557	19	0,0789	0,1081	17	0,0275	0,0529	10	0,000	0,0004
600 & 25	18	0,0478	0,1211	20	0,1237	0,1327	20	0,1237	0,1436	15	0,0077	0,0193	17	0,0275	0,0512
400 & 50	19	0,0789	0,1924	20	0,1237	0,1327	20	0,1237	0,1436	17	0,0275	0,0529	10	0,000	0,0004
600 & 50	17	0,0275	0,0712	19	0,0789	0,0797	20	0,1237	0,1436	16	0,0149	0,0330	15	0,0077	0,0187
400 & 100	19	0,0789	0,1924	19	0,0789	0,1965	20	0,1237	0,1436	17	0,0275	0,0529	10	0,000	0,0004
600 & 100	19	0,0789	0,1924	20	0,1237	0,1327	20	0,1237	0,1436	16	0,0149	0,0330	18	0,0478	0,0767

Table 21: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*- GJR-GARCH(1,1) - 95% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	534	0,0149	0,0015	523	0,9218	0,4098	537	0,0016	0,0041	531	0,0789	0,0170	533	0,0275	0,0735
600 & 25	532	0,0478	0,0447	524	0,7671	0,3398	530	0,1237	0,0319	530	0,1237	0,0319	531	0,0789	0,0797
400 & 50	534	0,0149	0,0015	524	0,7671	0,7634	536	0,0037	0,0002	533	0,0275	0,0037	533	0,0275	0,0735
600 & 50	531	0,0789	0,0797	523	0,9218	0,4098	531	0,0789	0,0170	528	0,2656	0,0908	531	0,0789	0,0170
400 & 100	532	0,0478	0,0084	523	0,9218	0,9480	535	0,0077	0,0049	532	0,0478	0,0084	533	0,0275	0,0232
600 & 100	530	0,1237	0,0319	522	0,9223	0,4690	529	0,1852	0,0557	530	0,1237	0,0319	531	0,0789	0,0170

Table 22: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 1 - AR(1)*- GJR-GARCH(1,1) - 99% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	545	0,8277	0,9327	545	0,8277	0,9327	545	0,8277	0,0948	546	0,4992	0,7728	546	0,4992	0,0472
600 & 25	546	0,4992	0,7728	545	0,8277	0,9327	546	0,4992	0,7728	546	0,4992	0,7728	545	0,8277	0,0948
400 & 50	544	0,8327	0,9152	545	0,8277	0,9327	545	0,8277	0,0948	545	0,8277	0,9327	546	0,4992	0,0472
600 & 50	546	0,4992	0,7728	545	0,8277	0,9327	546	0,4992	0,7728	546	0,4992	0,7728	545	0,8277	0,0948
400 & 100	545	0,8277	0,9327	545	0,8277	0,9327	546	0,4992	0,7728	545	0,8277	0,9327	546	0,4992	0,0472
600 & 100	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	546	0,4992	0,7728	545	0,8277	0,0948

Table 23: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on maximum Kendalls Tau. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*- GJR-GARCH(1,1) - 1% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	4	0,4993	0,7729	3	0,2410	0,4947	5	0,8277	0,9327	3	0,2410	0,4947	2	0,0845	0,22417
600 & 25	2	0,0845	0,22417	5	0,8277	0,9327	5	0,8277	0,9327	2	0,0845	0,22417	3	0,2410	0,4947
400 & 50	3	0,2410	0,4947	5	0,8277	0,9327	6	0,8328	0,9152	4	0,4993	0,7729	1	0,01768	0,05987
600 & 50	2	0,0845	0,22417	7	0,5374	0,7553	5	0,8277	0,9327	3	0,2410	0,4947	2	0,0845	0,22417
400 & 100	3	0,2410	0,4947	6	0,8328	0,9152	8	0,3157	0,5371	3	0,2410	0,4947	3	0,2410	0,4947
600 & 100	3	0,2410	0,4947	9	0,1695	0,33502	6	0,8328	0,9152	3	0,2410	0,4947	3	0,2410	0,4947

Table 24: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*- GJR-GARCH(1,1) - 5% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	18	0,0479	0,1211	22	0,2656	0,5337	24	0,4844	0,26131	17	0,0275	0,0529	24	0,4844	0,26131
600 & 25	16	0,0150	0,0330	22	0,2656	0,5337	22	0,2656	0,2147	16	0,0150	0,0330	28	0,9223	0,91985
400 & 50	18	0,0479	0,1211	23	0,3655	0,41309	25	0,6196	0,26809	17	0,0275	0,0529	24	0,4844	0,26131
600 & 50	17	0,0275	0,07119	22	0,2656	0,30085	22	0,2656	0,2147	16	0,0150	0,0330	30	0,62956	0,85375
400 & 100	19	0,0789	0,1924	25	0,6196	0,65648	23	0,3655	0,2428	17	0,0275	0,0529	28	0,9223	0,22085
600 & 100	17	0,0275	0,07119	23	0,3655	0,41309	22	0,2656	0,2147	15	0,00765	0,01927	30	0,62956	0,85375

Table 25: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*- GJR-GARCH(1,1) - 95% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	530	0,1238	0,0320	532	0,0479	0,0447	534	0,0150	0,01114	531	0,0789	0,0170	539	0,00026	0,0005
600 & 25	529	0,1853	0,0558	530	0,1238	0,1327	536	0,00367	0,00966	530	0,1238	0,0320	537	0,00164	0,00415
400 & 50	530	0,1238	0,0320	531	0,0789	0,0798	534	0,0150	0,01114	533	0,0275	0,0038	538	0,00068	0,00162
600 & 50	529	0,1853	0,2062	528	0,2656	0,30085	534	0,0150	0,01114	531	0,0789	0,0170	538	0,00068	0,00162
400 & 100	530	0,1238	0,0320	527	0,3655	0,41309	534	0,0150	0,01114	533	0,0275	0,0038	538	0,00068	0,00162
600 & 100	530	0,1238	0,0320	526	0,4844	0,53535	534	0,0150	0,01114	531	0,0789	0,0170	538	0,00068	0,00162

Table 26: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 2 - AR(1)*- GJR-GARCH(1,1) - 99% quantile															
	Portfolio 1			Portfolio 2			Portfolio 3			Portfolio 4			Portfolio 5		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	545	0,8277	0,9327	545	0,8277	0,9327	549	0,01768	0,05987	546	0,4993	0,7729	548	0,0845	0,22417
600 & 25	546	0,4993	0,7729	546	0,4993	0,7729	549	0,01768	0,05987	546	0,4993	0,7729	548	0,0845	0,22417
400 & 50	544	0,8328	0,9152	545	0,8277	0,9327	549	0,01768	0,05987	546	0,4993	0,7729	547	0,2410	0,4947
600 & 50	546	0,4993	0,7729	546	0,4993	0,7729	549	0,01768	0,05987	548	0,0845	0,22417	547	0,2410	0,4947
400 & 100	545	0,8277	0,9327	545	0,8277	0,9327	549	0,01768	0,05987	545	0,8277	0,9327	547	0,2410	0,4947
600 & 100	546	0,4993	0,7729	546	0,4993	0,7729	549	0,01768	0,05987	548	0,0845	0,22417	547	0,2410	0,4947

Table 27: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on volatility, with the highest volatility contract first. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 -AR(1)*- GJR-GARCH(1,1) - 1% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	5	0,8277	0,9327	3	0,2410	0,4947	5	0,8277	0,9327	4	0,4993	0,7729	3	0,2410	0,4947
600 & 25	3	0,2410	0,4947	6	0,8328	0,9152	6	0,8328	0,9152	4	0,4993	0,7729	3	0,2410	0,4947
400 & 50	4	0,4993	0,7729	4	0,4993	0,7729	4	0,4993	0,7729	3	0,2410	0,4947	3	0,2410	0,4947
600 & 50	4	0,4993	0,7729	6	0,8328	0,9152	5	0,8277	0,9327	4	0,4993	0,7729	3	0,2410	0,4947
400 & 100	4	0,4993	0,7729	4	0,4993	0,7729	6	0,8328	0,9152	4	0,4993	0,7729	3	0,2410	0,4947
600 & 100	3	0,2410	0,4947	6	0,8328	0,9152	8	0,3157	0,5371	4	0,4993	0,7729	5	0,8277	0,9327

Table 28: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 - AR(1)*- GJR-GARCH(1,1) - 5% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	19	0,0789	0,1924	25	0,6196	0,0704	23	0,3655	0,2428	16	0,0150	0,0330	20	0,1238	0,1436
600 & 25	20	0,1238	0,2866	28	0,9223	0,4690	23	0,3655	0,2428	16	0,0150	0,0330	21	0,1853	0,4059
400 & 50	19	0,0789	0,1924	25	0,6196	0,2668	22	0,2656	0,2147	17	0,0275	0,0529	21	0,1853	0,18036
600 & 50	19	0,0789	0,1924	27	0,9219	0,4099	21	0,1853	0,18036	17	0,0275	0,0529	21	0,1853	0,4059
400 & 100	20	0,1238	0,2866	25	0,6196	0,0704	22	0,2656	0,2147	18	0,0479	0,0794	20	0,1238	0,1436
600 & 100	18	0,0479	0,1211	27	0,9219	0,4099	22	0,2656	0,2147	16	0,0150	0,0330	22	0,2656	0,5337

Table 29: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 -AR(1)*- GJR-GARCH(1,1) - 95% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	532	0,0479	0,0084	530	0,1238	0,1327	530	0,1238	0,0320	531	0,0789	0,0170	529	0,1853	0,0558
600 & 25	530	0,1238	0,0320	524	0,7672	0,3399	528	0,2656	0,0908	531	0,0789	0,0170	529	0,1853	0,0558
400 & 50	534	0,0150	0,0016	529	0,1853	0,2062	532	0,0479	0,0447	533	0,0275	0,0038	529	0,1853	0,0558
600 & 50	531	0,0789	0,0798	524	0,7672	0,3399	527	0,3655	0,1383	531	0,0789	0,0170	529	0,1853	0,0558
400 & 100	533	0,0275	0,0038	529	0,1853	0,4059	531	0,0789	0,0170	533	0,0275	0,0038	529	0,1853	0,0558
600 & 100	531	0,0789	0,0170	523	0,9219	0,4099	526	0,4844	0,1979	530	0,1238	0,0320	528	0,2656	0,0175

Table 30: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Copula 3 - AR(1)*- GJR-GARCH(1,1) - 99% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	544	0,8328	0,9152	546	0,4993	0,7729	544	0,8328	0,1396	545	0,8277	0,9327	543	0,5374	0,1617
600 & 25	546	0,4993	0,7729	546	0,4993	0,7729	546	0,4993	0,7729	546	0,4993	0,7729	542	0,3157	0,1537
400 & 50	544	0,8328	0,9152	545	0,8277	0,9327	543	0,5374	0,1617	545	0,8277	0,9327	543	0,5374	0,1617
600 & 50	546	0,4993	0,7729	546	0,4993	0,7729	545	0,8277	0,9327	546	0,4993	0,7729	541	0,1695	0,1237
400 & 100	544	0,8328	0,9152	546	0,4993	0,7729	544	0,8328	0,9152	543	0,5374	0,7553	544	0,8328	0,1396
600 & 100	546	0,4993	0,7729	546	0,4993	0,7729	544	0,8328	0,9152	546	0,4993	0,7729	540	0,0836	0,08605

Table 31: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a Copula-GJR-GARCH(1,1) -model, with a copula structure based on contract horizon and position. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-GJR-GARCH(1,1) - 1% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	8	0,3157	0,5371	10	0,0836	0,0860	14	0,0023	0,0066	8	0,3157	0,5371	9	0,1695	0,3350
600 & 25	8	0,3157	0,5371	10	0,0836	0,0860	11	0,0380	0,0928	7	0,5374	0,7553	9	0,1695	0,3350
400 & 50	8	0,3157	0,5371	10	0,0836	0,0860	13	0,0063	0,0174	8	0,3157	0,5371	9	0,1695	0,3350
600 & 50	8	0,3157	0,5371	10	0,0836	0,0860	11	0,0380	0,0928	7	0,5374	0,7553	9	0,1695	0,3350
400 & 100	8	0,3157	0,5371	10	0,0836	0,0860	12	0,0160	0,0420	7	0,5374	0,7553	9	0,1695	0,3350
600 & 100	8	0,3157	0,5371	10	0,0836	0,0860	10	0,0836	0,1859	6	0,8327	0,9152	9	0,1695	0,3350

Table 32: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a DCC(1,1) - AR(1)*-GJR-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-GJR-GARCH(1,1) - 5% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	30	0,6296	0,0553	24	0,4844	0,5354	26	0,7672	0,2627	28	0,9223	0,0344	25	0,6196	0,8756
600 & 25	30	0,6296	0,8412	24	0,4844	0,5354	26	0,7672	0,2627	25	0,6196	0,2815	24	0,4844	0,2613
400 & 50	29	0,7711	0,0449	24	0,4844	0,5354	27	0,9218	0,2461	27	0,9218	0,3809	25	0,6196	0,8756
600 & 50	28	0,9223	0,8662	24	0,4844	0,5354	26	0,7672	0,2627	25	0,6196	0,2815	25	0,6196	0,8756
400 & 100	30	0,6296	0,0553	24	0,4844	0,5354	26	0,7672	0,2627	28	0,9223	0,0344	25	0,6196	0,8756
600 & 100	28	0,9223	0,8662	24	0,4844	0,5354	26	0,7672	0,2627	25	0,6196	0,2815	25	0,6196	0,8756

Table 33: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a DCC(1,1) - AR(1)*-GJR-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-GJR-GARCH(1,1) - 95% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	524	0,7672	0,3398	527	0,3655	0,1383	524	0,7672	0,0202	524	0,7672	0,3398	532	0,0479	0,0084
600 & 25	524	0,7672	0,3398	529	0,1853	0,2062	525	0,6196	0,0704	524	0,7672	0,3398	534	0,0150	0,0016
400 & 50	526	0,4844	0,5354	527	0,3655	0,1383	525	0,6196	0,0131	526	0,4844	0,5354	532	0,0479	0,0084
600 & 50	525	0,6196	0,6565	529	0,1853	0,2062	525	0,6196	0,0704	525	0,6196	0,6565	534	0,0150	0,0016
400 & 100	525	0,6196	0,6565	528	0,2656	0,0908	525	0,6196	0,0131	526	0,4844	0,5354	533	0,0275	0,0038
600 & 100	526	0,4844	0,5354	529	0,1853	0,2062	525	0,6196	0,0704	525	0,6196	0,6565	534	0,0150	0,0016

Table 34: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a DCC(1,1) - AR(1)*-GJR-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-GJR-GARCH(1,1) - 99% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	545	0,8277	0,9327	543	0,5374	0,1617	538	0,0160	0,0041	544	0,8327	0,9152	542	0,3157	0,1537
600 & 25	545	0,8277	0,9327	545	0,8277	0,9327	539	0,0380	0,0928	545	0,8277	0,9327	542	0,3157	0,1537
400 & 50	545	0,8277	0,9327	544	0,8327	0,9152	538	0,0160	0,0041	545	0,8277	0,9327	541	0,1695	0,0089
600 & 50	545	0,8277	0,9327	545	0,8277	0,9327	539	0,0380	0,0928	545	0,8277	0,9327	542	0,3157	0,1537
400 & 100	545	0,8277	0,9327	543	0,5374	0,1617	538	0,0160	0,0041	545	0,8277	0,9327	542	0,3157	0,1537
600 & 100	545	0,8277	0,9327	545	0,8277	0,9327	539	0,0380	0,0928	545	0,8277	0,9327	542	0,3157	0,1537

Table 35: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a DCC(1,1) - AR(1)*-GJR-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-s-GARCH(1,1) - 1% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	8	0,3157	0,5371	11	0,0380	0,0527	12	0,0160	0,0420	8	0,3157	0,5371	8	0,3157	0,5371
600 & 25	8	0,3157	0,5371	10	0,0836	0,0860	11	0,0380	0,0928	7	0,5374	0,7553	8	0,3157	0,5371
400 & 50	8	0,3157	0,5371	10	0,0836	0,0860	12	0,0160	0,0420	8	0,3157	0,5371	8	0,3157	0,5371
600 & 50	8	0,3157	0,5371	10	0,0836	0,0860	11	0,0380	0,0928	7	0,5374	0,7553	8	0,3157	0,5371
400 & 100	8	0,3157	0,5371	10	0,0836	0,0860	10	0,0836	0,1859	7	0,5374	0,7553	8	0,3157	0,5371
600 & 100	8	0,3157	0,5371	10	0,0836	0,0860	10	0,0836	0,1859	6	0,8327	0,9152	8	0,3157	0,5371

Table 36: *Christophersen p.value, Kupiec p.value and actual exceedances for the 1% quantile using a DCC(1,1) - AR(1)*-s-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-s-GARCH(1,1) - 5% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	30	0,6296	0,5012	24	0,4844	0,5354	26	0,7672	0,2627	29	0,7711	0,0449	26	0,7672	0,2627
600 & 25	30	0,6296	0,8412	24	0,4844	0,5354	24	0,4844	0,2613	27	0,9218	0,9600	25	0,6196	0,2681
400 & 50	31	0,5018	0,0647	24	0,4844	0,5354	26	0,7672	0,2627	29	0,7711	0,0449	28	0,9223	0,9199
600 & 50	30	0,6296	0,8412	24	0,4844	0,5354	23	0,3655	0,2428	25	0,6196	0,2815	25	0,6196	0,2681
400 & 100	31	0,5018	0,0647	24	0,4844	0,5354	25	0,6196	0,2681	29	0,7711	0,0449	27	0,9218	0,9480
600 & 100	31	0,5018	0,7749	24	0,4844	0,5354	23	0,3655	0,2428	25	0,6196	0,2815	25	0,6196	0,2681

Table 37: *Christophersen p.value, Kupiec p.value and actual exceedances for the 5% quantile using a DCC(1,1) - AR(1)*-s-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-s-GARCH(1,1) - 95% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	523	0,9218	0,4099	527	0,3655	0,1383	524	0,7672	0,0202	524	0,7672	0,3398	530	0,1238	0,0320
600 & 25	524	0,7672	0,3398	529	0,1853	0,2062	524	0,7672	0,0202	524	0,7672	0,3398	533	0,0275	0,0038
400 & 50	524	0,7672	0,3398	527	0,3655	0,1383	525	0,6196	0,0131	526	0,4844	0,5354	530	0,1238	0,0320
600 & 50	525	0,6196	0,6565	530	0,1238	0,1327	524	0,7672	0,0202	525	0,6196	0,6565	533	0,0275	0,0038
400 & 100	524	0,7672	0,3398	527	0,3655	0,1383	525	0,6196	0,0131	526	0,4844	0,5354	531	0,0789	0,0170
600 & 100	526	0,4844	0,5354	530	0,1238	0,1327	525	0,6196	0,0131	525	0,6196	0,6565	533	0,0275	0,0038

Table 38: *Christophersen p.value, Kupiec p.value and actual exceedances for the 95% quantile using a DCC(1,1) - AR(1)*-s-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

DCC(1,1) - AR(1)*-s-GARCH(1,1) - 99% quantile															
	Portfolio 1			**Portfolio 2**			**Portfolio 3**			**Portfolio 4**			**Portfolio 5**		
(Window & Refit)	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p	Exc.	Kup.p	Chr.p
400 & 25	544	0,8327	0,9152	542	0,3157	0,0082	538	0,0160	0,0041	545	0,8277	0,9327	541	0,1695	0,0089
600 & 25	545	0,8277	0,9327	545	0,8277	0,9327	537	0,0063	0,0141	545	0,8277	0,9327	542	0,3157	0,1537
400 & 50	544	0,8327	0,9152	543	0,5374	0,0062	538	0,0160	0,0041	545	0,8277	0,9327	541	0,1695	0,0089
600 & 50	545	0,8277	0,9327	545	0,8277	0,9327	537	0,0063	0,0141	545	0,8277	0,9327	542	0,3157	0,1537
400 & 100	544	0,8327	0,9152	543	0,5374	0,0062	538	0,0160	0,0041	545	0,8277	0,9327	541	0,1695	0,0089
600 & 100	545	0,8277	0,9327	544	0,8327	0,1396	537	0,0063	0,0141	545	0,8277	0,9327	542	0,3157	0,1537

Table 39: *Christophersen p.value, Kupiec p.value and actual exceedances for the 99% quantile using a DCC(1,1) - AR(1)*-s-GARCH(1,1) -model. *AR(1)-terms is used only for the weekly and monthly future contracts.*

Specific number of exceedences for the models presented in section 6:

Table 40: *Specific number of exceedences for the 1% quantile*

1%	sGARCH Copula	GJR-GARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	4	3	8	14	11
Portfolio 2	5	5	10	15	7
Portfolio 3	4	4	10	16	9
Portfolio 4	4	4	6	12	8
Portfolio 5	2	2	9	12	14

Table 41: *Specific number of exceedences for the 5% quantile*

5%	sGARCH Copula	GJR-GARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	17	17	28	32	24
Portfolio 2	20	19	24	34	35
Portfolio 3	21	20	26	26	36
Portfolio 4	14	16	25	31	24
Portfolio 5	15	15	25	31	41

Table 42: *Specific number of exceedences for the 95% quantile*

95%	sGARCH Copula	GJR-GARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	531	531	525	522	528
Portfolio 2	523	523	528	530	522
Portfolio 3	529	531	524	523	518
Portfolio 4	531	528	524	523	531
Portfolio 5	531	531	533	525	518

Table 43: *Specific number of exceedences for the 99% quantile*

99%	sGARCH Copula	GJR-GARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	546	546	544	540	546
Portfolio 2	546	545	544	540	544
Portfolio 3	546	546	538	538	544
Portfolio 4	546	546	544	540	547
Portfolio 5	546	545	541	539	542

We here present the average size of an exceedence for each quantile, model and portfolio presented in section 6. We calculate this by finding the difference between the return and the forecast for each exceedence, then calculating the average of these.

Table 44: *Average exceedence levels for each quantile*

1% quantile	sGARCH Copula	GJR-GARCH Copula	DCC GARCH	RiskMetrics	Historical Simulation
Portfolio 1	-0,014217	-0,0174779	-0,01023	-0,01077	-0,01377
Portfolio 2	-0,003427	-0,0035783	-0,00219	-0,00301	-0,00614
Portfolio 3	-0,002901	-0,0037629	-0,00478	-0,0031	-0,00782
Portfolio 4	-0,010215	-0,0099134	-0,01138	-0,00966	-0,01101
Portfolio 5	-0,002282	-0,0025869	-0,00179	-0,00216	-0,00294
5% quantile					
Portfolio 1	-0,011565	-0,0114648	-0,01109	-0,01416	-0,02123
Portfolio 2	-0,002971	-0,0031328	-0,00277	-0,00283	-0,0034
Portfolio 3	-0,002969	-0,0032885	-0,0037	-0,00409	-0,00457
Portfolio 4	-0,011466	-0,0097993	-0,01044	-0,01198	-0,01758
Portfolio 5	-0,001967	-0,0020498	-0,00215	-0,00216	-0,00293
95% quantile					
Portfolio 1	-0,044729	-0,0452426	-0,04151	-0,04109	-0,0456
Portfolio 2	-0,005916	-0,0059234	-0,00605	-0,00603	-0,00555
Portfolio 3	-0,008961	-0,0088815	-0,0079	-0,00749	-0,00745
Portfolio 4	-0,040976	-0,0416963	-0,03888	-0,03791	-0,04266
Portfolio 5	-0,005396	-0,005407	-0,0055	-0,00514	-0,00516
99% quantile					
Portfolio 1	-0,063653	-0,0647064	-0,05558	-0,05501	-0,06568
Portfolio 2	-0,008806	-0,0089047	-0,0082	-0,00815	-0,009
Portfolio 3	-0,01441	-0,0144415	-0,01075	-0,01024	-0,01125
Portfolio 4	-0,057809	-0,0588078	-0,05201	-0,05098	-0,06151
Portfolio 5	-0,008784	-0,0087187	-0,00753	-0,00699	-0,00796

APPENDIX E: THEORY

DCC

The log-likelihood function is expressed as

$$\begin{aligned} LL &= \frac{1}{2}\sum_{i=1}^{T}\left(N\log(2\pi)+2\log|D_t|+\log|R_t|+z_t'R_t^{-1}z_t'\right) \\ &= \frac{1}{2}\sum_{i=1}^{T}\left(N\log(2\pi)+2\log|D_t|+\varepsilon'_tD_t^{-1}D_t^{-1}\varepsilon_t\right)-\frac{1}{2}\sum_{i=1}^{T}\left(z'_tz_t+\log|R_t|+z'_tR_t^{-1}z'_t\right) \\ &= LL_V(\theta_1)+LL_R(\theta_1,\theta_2) \end{aligned} \tag{24}$$

$LL_V(\theta_1)$ is the volatility component and $LL_R(\theta_1,\theta_2)$ is the correlation component.

(Engle and Sheppard 2001) introduced a way of forecasting the correlation matrix. The multi-step ahead evolution of the proxy process

$$Q_{t+n} = (1-\alpha-\beta)\bar{Q}+\alpha E_t\left[z_{t+n-1}z'_{t+n-1}\right]+\beta Q_{t+n-1} \tag{25}$$

is used in the forecasting of the correlation matrix:

$$E_t\left[R_{t+n}\right] = \sum_{i=0}^{n-2}(1-\alpha-\beta)\bar{R}(\alpha+\beta)^i+(\alpha+\beta)^{n-1}R_{n+1} \tag{26}$$

Here is $E_t\left[z_{t+n-1}z'_{t+n-1}\right]=R_{t+n-1}$, $R_{t+n}=diag(Q_{t+n})^{-1/2}Q_{t+n}diag(Q_{t+n})^{-1/2}$, $\bar{Q}\approx R$ and $E_t\left[Q_{t+1}\right]=E_t\left[R_{t+1}\right]$.

COPULA

The Gaussian copula with dependence parameter matrix Σ is given by

$$C_\Sigma(\mathbf{u}) = \Phi_\Sigma\left(\Phi^{-1}(u_1),\ \ldots\ \Phi^{-1}(u_D)\right), \tag{27}$$

where $\Phi^{-1}(\cdot)$ is the inverse of the univariate standard normal distribution function and $\Phi_\Sigma(\cdot)$ is the joint cumulative distribution function of a multivariate normal distribution with zero means and covariance matrix Σ(correlation parameter ρ. Its density is given by

$$c_\Sigma(\mathbf{u}) = |\Sigma|^{-\frac{1}{2}}\ \exp\left(-\frac{1}{2}\begin{pmatrix}\Phi^{-1}(u_1)\\ \vdots \\ \Phi^{-1}(u_D)\end{pmatrix}^T(\Sigma^{-1}-\mathbf{I})\begin{pmatrix}\Phi^{-1}(u_1)\\ \vdots \\ \Phi^{-1}(u_D)\end{pmatrix}\right).$$

The D-dimensional Student copula distribution as given by (Demarta and McNeil 2005):

$$C_{\Sigma,\nu}(\mathbf{u}) = \int_{-\infty}^{t_\nu^{-1}(u_1)}\cdots\int_{-\infty}^{t_\nu^{-1}(u_D)}\frac{\Gamma\left(\frac{\nu+D}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi\nu)^D|\Sigma|}}\left(1+\frac{\mathbf{x}'\Sigma^{-1}\mathbf{x}}{\nu}\right)^{-\frac{\nu+D}{2}}d\mathbf{x}, \tag{28}$$

where $t_\nu^{-1}()$ is the inverse univariate Student-t distribution with ν degrees of freedom. Its density can be derived to be

$$c_{\Sigma,\nu}(\mathbf{u}) = \frac{f_{\Sigma,\nu}(t_\nu^{-1}(u_1),\ldots,t_\nu^{-1}(u_D))}{\prod_{i=1}^{D} f_\nu(t_\nu^{-1}(u_i))}, \tag{29}$$

where $f_{\Sigma,\nu}$ is the joint density of a D-dimensional random vector from a multivariate Student distribution with ν degrees of freedom and covariance matrix Σ and f_ν is the density of a univariate Student-t distribution with ν degrees of freedom. Both the gaussian and the Student-t copulas are symmetric and has therefore the same upper and lower tail dependencies. In the table below is properties for those two copulas given.

Family	**Parameter range**	**Kendall's τ**	**Tail dependence**
Gaussian	$\rho \in (-1,1)$	$\frac{2}{\pi}arcsin(\rho)$	0
Student-t	$\rho \in (-1,1), \nu > 2$	$\frac{2}{\pi}arcsin(\rho)$	$2t_{\nu+1}(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}})$

Table 45: *Properties of bivariate elliptical copulas*

The independence copula is defined by:

$$C(u_1, u_2, \cdots, u_{n_U}) = \prod_{i=1}^{n_U} u_i$$

Archimedean copulas are characterized by a single dependence parameter and the following representation:

$$C(u_1,\ u_2,\ \ldots\ , u_D) = \psi(\psi^{-1}(u_1) + \ldots + \psi^{-1}(u_D)),$$

where $\psi(\cdot)$ is the *generator* (Nelsen 1999) of the copula. The generators for the the Archimedean copulas used in this paper are displayed in the table at the next page.

The parameters of the marginal processes and dependence copula can be estimated with the maximum likelihood estimation (MLE) method.

Family	Generator function	Parameter range	Kendall‘s τ	Tail dependence(lower, upper)
Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta > 0$	$\frac{\theta}{\theta+2}$	$(2^{-\frac{1}{\theta}},0)$
Joe	$-log[1-(1-t)^{\theta}]$	$\theta > 1$	$1+\frac{4}{\theta^2}\int_0^1 t\,log(t)(1-t)^{\frac{2(1-\theta)}{\theta}}dt$	$(0,2-2^{\frac{1}{\theta}})$
Frank	$-log[\frac{e^{-\theta t}-1}{e^{-\theta}-1}]$	$\theta > 0$	$1-\frac{4}{\theta}+4\frac{\int_0^{\theta}\frac{\frac{c}{\theta}}{exp(x)-1}dx}{\theta}$	(0,0)
Gumbel	$(-logt)^{\theta}$	$\theta \geq 1$	$1-\frac{1}{\theta}$	$(0,2-2^{\frac{1}{\theta}})$
Clayton-Gumbel	$(t^{-\theta}-1)^{\delta}$	$\theta \geq 0, \delta \geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$(2^{\frac{-1}{\theta\delta}},2-2^{\frac{1}{\delta}})$
Joe-Gumbel	$(-log[1-(1-t)^{\theta}])^{\delta}$	$\theta \geq 1, \delta \geq 1$	$1+4\int_0^1\left(-log(-(1-t)^{\theta}+1))\times\frac{1-t-(1-t)^{-\theta}+t(1-t)^{-\theta})}{\theta\delta}\right)dt$	$(0,2-2^{\frac{1}{(\theta\delta)}})$
Joe-Clayton	$[1-(1-t)^{\theta})]^{-\delta}-1$	$\theta \geq 1, \delta > 0$	$1-\frac{2}{\delta(2-\theta)}+\frac{4}{\theta^2\delta}\int_0^1 t^{\frac{2-\theta}{\theta}+1}(t-1)^{(\delta+1)}$	$(2^{\frac{-1}{\delta}},2-2^{\frac{1}{\theta}})$
Joe-Frank	$-log\left[\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}\right]$	$\theta \geq 1, 0 < \delta \leq 1$	$1+4\int_0^1\left(-log\left(\frac{(1-t\delta)^{\theta}-1)}{(1-\delta)^{\theta}-1}\right)\times\frac{1-t\delta-(1-t\delta)^{-\theta}+t\delta(1-t\delta)^{-\theta}}{\theta\delta}\right)dt$	$(0,0^{a})$

Table 46: *Properties of archimedean copula generators.[a] When δ = 1, the upper tail dependence coeffcient is* $2-2^{\frac{1}{\theta}}$.